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DETERMINATION OF THE FORM FACTORS IN $K_{\mu 3}$ DECAYS

Berkeley, California

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UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

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ERRATA

To: All recipients of UCRL-11327

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The definition of $b_2(\xi)$ in Eq. 7 should read:

$$b_2(\xi) = - [2(p_\nu p_K) + (\text{Re } b(q^2))(q^2 - m_\mu{}^2)] .$$

A term has been omitted in Eq. 8, which should read:

$$\vec{d} = E_K(\vec{p}_\mu \times \vec{p}_\pi) + E_\mu(\vec{p}_\pi \times \vec{p}_K) + E_\pi(\vec{p}_K \times \vec{p}_\mu) + \left[\frac{\vec{p}_\mu \cdot (\vec{p}_K \times \vec{p}_\pi)}{E_\mu + m_\mu}\right] \vec{p}_\mu \quad .$$

We wish to thank Dr. D. Bartlett for calling our attention to these errors. Some additional corrections may be found in Physics Letters 11, 360 (1964).

Phys. Letters

UCRL-11327 +
errata

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Nicola Cabibbo and Alexander Maksymowicz

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We discuss briefly the relevance of a measurement of the muon polarization in the case of completely specified kinematics for the determination of the form factors in $K_{\mu 3}$ decays.

It can be shown that once we have specified the kinematics, the muon is completely polarized along <u>some</u> direction.[1] This follows from the zero spins of the pion and K meson and from the definite helicity of the neutrino. The direction of the polarization vector depends on the assumed form of the decay amplitude, and its measurement will give information about the amplitude.

To be definite, let us consider the matrix element for $K^+ \to \mu^+ \nu \pi^0$ within the frame of the V-A theory of weak interactions;[2] it can be written as[3]

$$\left[\bar{\mu}\, \gamma_\lambda (1 + \gamma_5)\nu\right] f_1(q^2) \left[(p_K + p_\pi)_\lambda + \xi(q^2)(p_K - p_\pi)_\lambda\right] .$$

The form factors $f_1(q^2)$ and $\xi(q^2)$ are functions of the invariant mass of the μ-ν system. From this expression we can evaluate the polarization vector,[4] $\mathbf{P}$, in the rest frame of the K meson:

$$\mathbf{P} = \mathbf{A}/|\mathbf{A}| , \qquad (1)$$

where

$$\underset{\sim}{A} = a_1(\xi)\underset{\sim}{p}_\mu - a_2(\xi)\left\{\underset{\sim}{p}_\mu\left[(M_K - E_\pi)/m_\mu + (\underset{\sim}{p}_\pi \cdot \underset{\sim}{p}_\mu)(E_\mu - m_\mu)/|\underset{\sim}{p}_\mu|^2\right] + \underset{\sim}{p}_\pi\right\}$$

$$+ M_K \ \mathrm{Im}\ \xi(q^2)(\underset{\sim}{p}_\pi \times \underset{\sim}{p}_\mu) \ . \qquad (2)$$

In the above equation we have set

$$a_1(\xi) = 2M_K^2\left[E_\nu + \mathrm{Re}\ b(q^2)(E_\pi^* - E_\pi)\right]/m_\mu \ ,$$

$$a_2(\xi) = M_K^2 + 2\ \mathrm{Re}\ b(q^2)M_K E_\mu + |b(q^2)|^2 m_\mu^2 \ , \qquad (3)$$

$$b(q^2) = \left[\xi(q^2) - 1\right]/2 \ ,$$

and

$$E_\pi^* = (M_K^2 + m_\pi^2 - m_\mu^2)/2M_K \ . \qquad (4)$$

If time-reversal invariance holds, ξ is real, and the polarization vector lies in the decay plane and is completely described by the angle Θ that it makes with the muon momentum vector (see Fig. 1). We define Θ to be positive if $\underset{\sim}{P}$ and the pion momentum lie in the same half-plane with respect to the muon momentum (the situation shown in Fig. 1), but negative otherwise. In all kinematic configurations $\underset{\sim}{P}$ rotates through 2π radians as ξ is varied from $-\infty$ to $+\infty$,[5] giving a one-to-one correspondence between Θ and ξ.

Figure 2 shows the dependence of Θ on the pion energy, E_π, for 200-MeV muons and various values of ξ, from which the dependence of Θ on ξ for fixed E_π can easily be obtained.

We deem it worthwhile to draw attention to the measurement of muon polarization for fully specified kinematics (in spite of the fact that some calculations have already appeared in the literature[1,4]), because we think it can lead to a better determination of ξ and its dependence on q^2 than would be possible with other methods. The method proposed here requires collection of a sample of well-reconstruced events in which the muon is stopped in a nondepolarizing material and the direction of the decay electron observed. From these events one can then determine $\xi(q^2)$ by a statistical analysis.[6] One advantage of this method is that it measures ξ directly, without requiring any knowledge of the q^2 dependence of $f_1(q^2)$ (or the assumption that this dependence is negligible), as is the case with other methods. A further advantage is that the measurement is not spoiled by any experimental biases, except for those which may be connected with the detection of the electron from the muon decay. An example of such "allowed" biases is the loss of events in certain kinematical regions due to nondetection or impossibility of unambiguous reconstruction.

For $K^o \to \mu^+ \pi^- \nu$ and $\overline{K}^o \to \mu^+ \pi^- \nu$ decays, the same formulas are valid in the K-meson rest frame, if we take $f_1(q^2)$ and $\xi(q^2)$ as the appropriate form factors for these decays[7] (for the analogous decays with the emission of negative muons the polarization changes sign except for the component perpendicular to the decay plane). For these decays it is also useful to have the corresponding formulas for the polarization of a muon emitted by a K meson with momentum $\underset{\sim}{p}_K$ in the laboratory system. These are

$$\underset{\sim}{P}_{lab} = \underset{\sim}{B}/|\underset{\sim}{B}|$$

and (5)

$$\underset{\sim}{B} = b_1(\xi)\left[\frac{\underset{\sim}{p}_\mu}{m_\mu}\left(\frac{\underset{\sim}{p}_\nu \cdot \underset{\sim}{p}_\mu}{E_\mu + m_\mu} - E_\nu\right) + \underset{\sim}{p}_\nu\right]$$

$$+ b_2(\xi)\left[\frac{\underset{\sim}{p}_\mu}{m_\mu}\left(\frac{\underset{\sim}{p}_K \cdot \underset{\sim}{p}_\mu}{E_\mu + m_\mu} - E_K\right) + \underset{\sim}{p}_K\right] - (\mathrm{Im}\ \xi)\underset{\sim}{d}\ , \quad (6)$$

where

$$b_1(\xi) = M^2 + m_\mu{}^2|b(q^2)|^2 + 2(\mathrm{Re}\ b(q^2))(p_\mu\ p_K)\ ,$$

(7)

$$b_2(\xi) = -\left[2(p_\nu\ p_K) + 2(\mathrm{Re}\ b(q^2))(q^2 - m_\mu{}^2)\right]\ ,$$

$$\underset{\sim}{d} = E_K(\underset{\sim}{p}_\mu \times \underset{\sim}{p}_\pi) + E_\mu(\underset{\sim}{p}_\pi \times \underset{\sim}{p}_K) + E_\pi(\underset{\sim}{p}_K \times \underset{\sim}{p}_\mu)\ , \quad (8)$$

and $b(q^2)$ has been defined in Eq. (4). The metric is $(AB) = A^oB^o - \underset{\sim}{A}\cdot\underset{\sim}{B}$.

We would like to thank Dr. Rae Stiening for his interest in this work and for many useful discussions.

FOOTNOTES AND REFERENCES

* This work was performed under the auspices of the U. S. Atomic Energy Commission.

1. This fact was noted by J. Werle, Nucl. Phys. 6, 1 (1958) and S. W. McDowell, Nuovo Cimento 9, 258 (1958).

2. R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958) and E. C. G. Sudarshan and R. E. Marshak, Proceedings of Padua-Venice Conference on Mesons and Newly Discovered Particles, September, 1957 (Societa Italiana di Fisica, Padua-Venice, 1958).

3. Our notation for the form factors is the usual one; sometimes one uses the two quantities f_1 and $f_2 = f_1 \xi$.

4. This formula has also been obtained by R. Gatto, Phys. Rev. 111, 1426 (1958) and J. Nilsson, Nucl. Phys. 14, 639 (1960).

5. The only exception occurs when the three decay particles are exactly collinear.

6. A two-parameter fit to $\xi(q^2)$ should be sufficient. The extension of the Goldberger-Treiman relations to $K_{\mu 3}$ decays suggests a parametrization of the form $\xi(q^2) = \alpha/(q^2 + \beta)$ and in fact predicts $\alpha = M_K^2 - m_\pi^2$ [J. Bernstein and S. Weinberg, Phys. Rev. Letters 5, 481 (1960)].

7. The $\Delta I = \frac{1}{2}$ rule for leptonic decays requires $\xi(q^2)$ to be the same for $K^+ \to \pi^0 \mu^+ \nu$ and $K^0 \to \pi^- \mu^+ \nu$.

FIGURE LEGENDS

Fig. 1. Specification of kinematics for $K^+ \to \pi^0 \mu^+ \nu$ decay.

Fig. 2. Muon polarization as a function of pion energy for various values of ξ at a muon energy of 200 MeV.

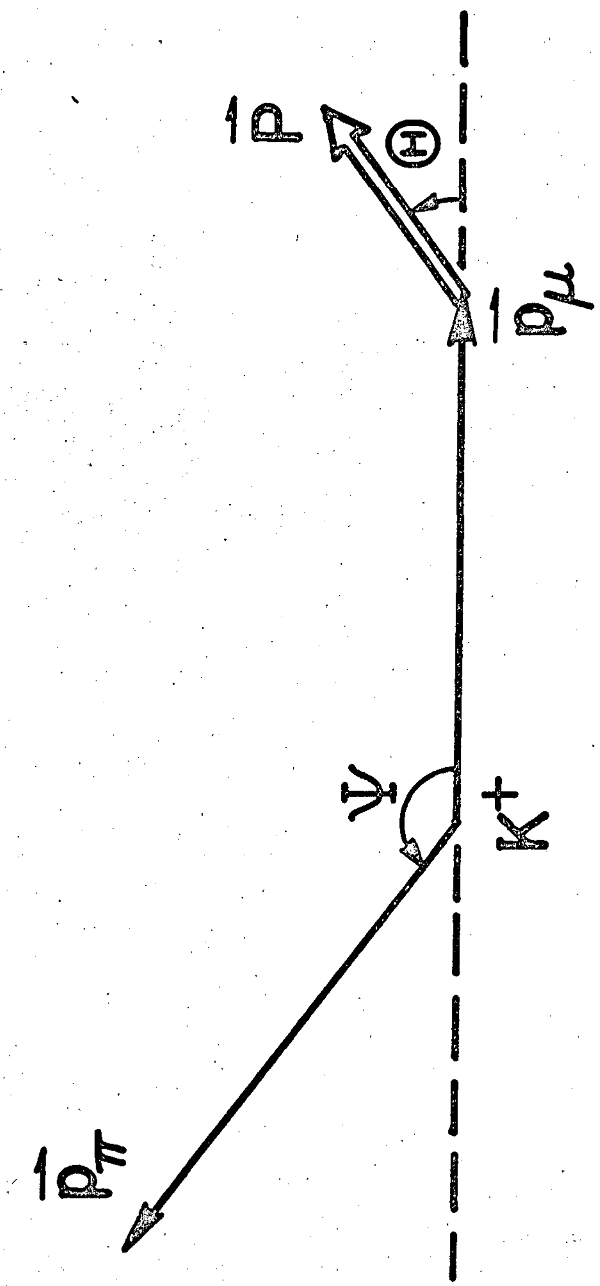

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Fig. 1

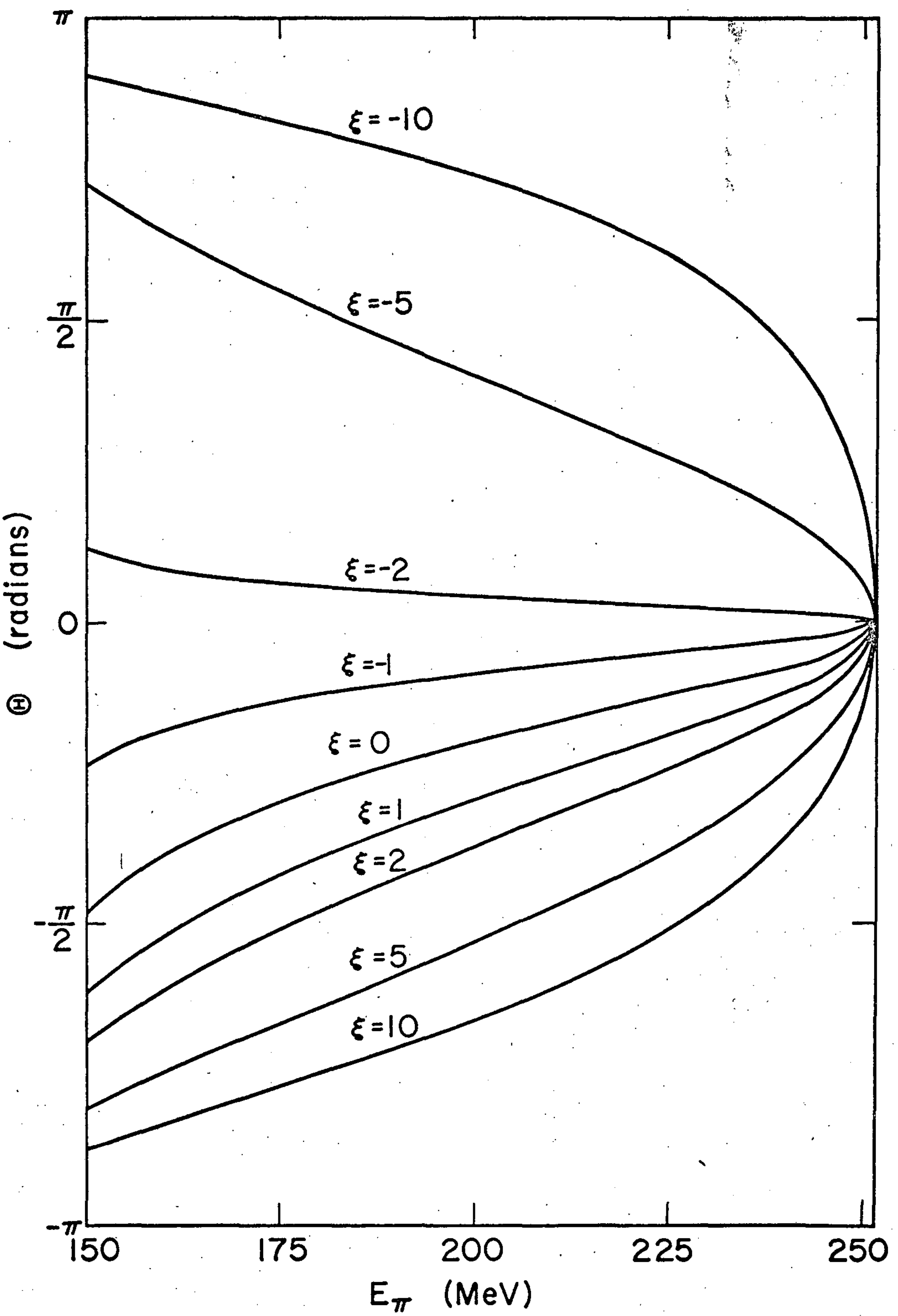

Fig. 2

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