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Author
Butter, Daniel

Publication Date
2009-08-18

UCB-PTH-09/19
June 2009

ANOMALY STRUCTURE OF SUPERGRAVITY AND ANOMALY CANCELLATION*

Daniel Butter *and* Mary K. Gaillard

Department of Physics
and
Theoretical Physics Group, Lawrence Berkeley National Laboratory,
University of California, Berkeley, California 94720

Abstract

We display the full anomaly structure of supergravity, including new D-term contributions to the conformal anomaly. This expression has the super-Weyl and chiral $U(1)_K$ transformation properties that are required for implementation of the Green-Schwarz mechanism for anomaly cancellation. We outline the procedure for full anomaly cancellation. Our results have implications for effective supergravity theories from the weakly coupled heterotic string theory.

*This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract DE-AC02-05CH11231, in part by the National Science Foundation under grants PHY-0457315 and PHY05-51164.

Disclaimer

When compactified from ten to four space-time dimensions, the weakly coupled heterotic string theory [1] has an invariance under a discrete group of transformations known as "T-duality" or "target space modular invariance" [2]. The effective four dimensional (4d) theory includes several important "moduli" chiral supermultiplets: the dilaton supermultiplet S, whose vacuum value determines the gauge coupling constant and the θ-parameter of the 4d gauge theory, and "Kähler moduli" T^i whose vacuum values determine the radii of compactification. The T-duality invariance of the effective 4d supergravity theory results in several desirable features [3]: 1) it assures that the Kähler moduli, or "T-moduli" are generically stabilized at self-dual points, with vanishing vacuum values for their auxiliary fields, so that supersymmetry breaking is dilaton dominated and no large flavor mixing is induced; 2) it protects a symmetry known as "R-symmetry" that assures that the mass of the axion (pseudoscalar) component of the dilaton supermultiplet remains sufficiently small to offer a solution to the strong CP problem; and 3) it may provide a residual discrete symmetry at low energy that plays the role of R-parity, needed to preserve lepton and baryon number conservation and the stability of the lightest supersymmetric partner, which makes the latter an attractive candidate for dark matter. This symmetry can be stronger than R-parity and thus forbid higher dimension operators that could otherwise generate too large an amplitude for proton decay.

At the quantum level of the effective theory, T-duality is broken by quantum anomalies, as is, generically, an Abelian $U(1)_X$ gauge symmetry, both of which are exact symmetries of string perturbation theory. It was realized some time ago that these symmetries could be restored by a combination of 4-d counterparts [4] of the Green-Schwarz (GS) mechanism in 10 dimensions [5] and string threshold corrections [6]. However anomaly cancellation has been demonstrated explicitly only for the coefficient of the Yang-Mills superfield strength bilinear. The entire supergravity chiral anomaly has in fact been determined [7], but the complete superfield form of the anomaly is required to fully implement anomaly cancellation. The anomaly arises from linear and logarithmic divergences in the effective supergravity theory, and is ill-defined in an unregulated theory. We use Pauli Villars (PV) regulation, which has been shown [8] to require only massive chiral multiplets and Abelian gauge multiplets as PV regulator fields, thereby preserving, for example, BRST invariance.

T-duality acts as follows on chiral (antichiral) superfields $Z^p = T^i, \Phi^a$ ($\bar{Z}^{\bar{p}} = \bar{T}^{\bar{\imath}}, \bar{\Phi}^{\bar{a}}$):

$$T^i \to h(T^j), \qquad \Phi^a \to f(q_i^a, T^j)\Phi^a, \qquad \bar{T}^{\bar{\imath}} \to h^*(\bar{T}^{\bar{\jmath}}), \qquad \bar{\Phi}^{\bar{a}} \to f^*(q_i^a, \bar{T}^{\bar{\jmath}})\bar{Z}^{\bar{a}}, \tag{1}$$

where q_i^a are the modular weights of Φ^a, and, under $U(1)_X$ transformations,

$$V_X \to V_X + \Lambda_X + \bar{\Lambda}_X, \qquad \Phi^a \to e^{-q_X^a \Lambda_X}\Phi^a, \qquad \bar{\Phi}^a \to e^{-q_X^a \bar{\Lambda}_X}\bar{\Phi}^a, \tag{2}$$

where V_X is the $U(1)_X$ vector superfield, with Λ_X ($\bar{\Lambda}_X$) chiral (antichiral). In the regulated theory the anomalous part of the Lagrangian takes the form [9]

$$\mathcal{L}_{\rm anom} = \frac{1}{64\pi^2}\int d^4\theta \mathrm{Tr}\left(\Omega_m \ln \mathcal{M}^2\right), \tag{3}$$

where $\mathcal{M}^2$ is a real superfield whose lowest component is the PV squared mass matrix:

$$\mathcal{M}^2\Big| = |m(z, \bar{z}, V_X|)|^2, \tag{4}$$

with $z, \bar{z}, V_X|$ the lowest components, respectively, of $Z, \bar{Z}, V_X$. Under a general anomalous transformation the logarithm in (3) shifts by an amount

$$\Delta \ln \mathcal{M}^2 = H_m(T^i, \Lambda_X) + \bar{H}_m(\bar{T}^{\bar{\imath}}, \bar{\Lambda}_X), \tag{5}$$

with H_m a (matrix-valued) chiral superfield. The resulting anomaly is given by [9, 10]

$$\Delta\mathcal{L}_{\rm anom} = \frac{1}{64\pi^2}\int d^4\theta \mathrm{Tr}\left[\Omega_m H_m(T, \Lambda_X)\right] + \text{h.c.}, \tag{6}$$

$$\Omega_m = \frac{1}{6}\left[\mathcal{M}^2(\mathcal{D}^2 - 8\bar{R})\mathcal{M}^{-2}R^m + \text{h.c.}\right] + \frac{1}{3}G_m^{\alpha\dot{\beta}}G^m_{\alpha\dot{\beta}} + \frac{4}{3}R^m\bar{R}^m + \frac{8}{3}\tilde{\Omega}_W - 4\Omega_J, \tag{7}$$

where the operators in (7) are defined by

$$R^m = -\frac{1}{8}\mathcal{M}^{-2}(\bar{\mathcal{D}}^2 - 8R)\mathcal{M}^2, \qquad G^m_{\alpha\dot{\beta}} = \frac{1}{2}\mathcal{M}[\mathcal{D}_\alpha, \mathcal{D}_{\dot{\beta}}]\mathcal{M}^{-1} + G_{\alpha\dot{\beta}}, \tag{8}$$

$$(\bar{\mathcal{D}}^2 - 8R)\tilde{\Omega}_W = W^{\alpha\beta\gamma}W_{\alpha\beta\gamma} - \frac{1}{12}X^\alpha X_\alpha, \qquad X_\alpha = -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha K, \tag{9}$$

$$(\bar{\mathcal{D}}^2 - 8R)\Omega_J = J^\alpha J_\alpha, \qquad J_\alpha = T_a W_\alpha^a - \frac{1}{16}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\alpha \ln \mathcal{M}^2. \tag{10}$$

The superfields R and $G_{\alpha\dot{\beta}}$ are related to elements of the super-Riemann tensor; their lowest components are auxiliary fields of the supergravity supermultiplet. K is the Kähler potential, and $W_{\alpha\beta\gamma}$ and W^a_α are the superfield strengths for, respectively, spacetime curvature and the Yang-Mills gauge group with generators T_a. We are working in Kähler $U(1)_K$ superspace [11], where the superdeterminant of the supervielbien E is related to the superdeterminant E_0 of conventional superspace by a superWeyl transformation: $E = E_0\, e^{-\frac{1}{3}K(Z,\bar{Z})}$, so that the Lagrangian for the supergravity and chiral supermultiplet kinetic energy is

$$\mathcal{L}_{\rm kin} = -3\int E_0\, e^{-\frac{1}{3}K(Z,\bar{Z})} = -3\int E. \tag{11}$$

In the $U(1)_K$ superspace formulation, one obtains a canonical Einstein term with no need for further Weyl transformations on the component fields. The structure group of Kähler U(1) geometry contains the Lorentz, $U(1)_K$, Yang-Mills and chiral multiplet reparameterization groups. Chiral multiplets Z^i are *covariantly* chiral: $\mathcal{D}_{\dot\alpha}Z^i = \mathcal{D}_\alpha \bar{Z}^{\bar\imath} = 0$, where the covariant spinorial derivatives $\mathcal{D}_\alpha, \mathcal{D}_{\dot\alpha}$ contain the $U(1)_K$, Yang-Mills, spin and reparameterization connections. The superfield Ω_J in (10) can be explicitly constructed [9] following the procedure used to construct [12] the Yang-Mills Chern-Simons superfield $\Omega_{\rm YM}:\ \ (\bar{\mathcal{D}}^2 - 8R)\Omega_{\rm YM} = W^\alpha_a W^a_\alpha$.

The result (6), (7) has been obtained using both component field [9] and superfield [10] calculations. It can be shown [9] that PV regulation can be done in such a way that a) gauge and superpotential couplings that contribute to the renormalization of the Kähler potential $K(Z,\bar{Z})$, as well as all dilaton couplings, can be regulated in a T-duality and $U(1)_X$ invariant manner, and b) the remaining anomaly can be absorbed into the masses of chiral PV superfields with a very simple, T-duality and $U(1)_X$ invariant, Kähler metric. Given these results, it suffices to calculate the contribution from the latter set of PV fields to obtain the anomaly. The new "D-terms", that is, the first three terms in (7), can be obtained most easily in superspace, by first working in superconformal supergravity, and then fixing the gauge to $U(1)_K$ superspace [10].

Anomaly cancellation is most readily implemented using the linear multiplet formulation for the dilaton [13]. A linear supermultiplet is a real supermultiplet that satisfies

$$(\mathcal{D}^2 - 8\bar{R})L = (\bar{\mathcal{D}}^2 - 8R)L = 0. \tag{12}$$

It has three components: a scalar, the dilaton $\ell = L|$, a spin-$\frac{1}{2}$ fermion, the dilatino χ, and a two-form $b_{\mu\nu}$ that is dual to the axion Ims, and no auxiliary field. For the purpose of anomaly cancellation we want instead to use a real superfield that satisfies the *modified* linearity condition:

$$(\bar{\mathcal{D}}^2 - 8R)L = -\Phi, \qquad (\mathcal{D}^2 - 8\bar{R})L = -\bar{\Phi}, \tag{13}$$

where Φ is a chiral multiplet with $U(1)_K$ and Weyl weights [11] $w_K(\Phi) = 2$, $w_W(\Phi) = 1$. Consider a theory defined by the Kähler potential K and the kinetic Lagrangian $\mathcal{L}$:

$$K = k(L) + K(Z, \bar{Z}), \qquad \mathcal{L} = -3\int d^4\theta\, E\, F(Z, \bar{Z}, V_X, L). \tag{14}$$

When a (modified) linear superfield L is included, the condition (11) for a canonical Einstein term in $U(1)_K$ superspace is replaced by

$$F - L\frac{\partial F}{\partial L} = -L^2\frac{\partial}{\partial L}\left(\frac{1}{L}F\right) = 1 - \frac{1}{3}L\frac{\partial k}{\partial L}, \tag{15}$$

with the solution:

$$F(Z, \bar{Z}, V_X, L) = 1 + \frac{1}{3}LV(Z, \bar{Z}, V_X) + \frac{1}{3}L\int\frac{\partial L}{L}\frac{\partial k(L)}{\partial L}, \tag{16}$$

where V is a constant of integration, and therefore independent of L. If we take

$$V = -bV(Z, \bar{Z}) + \delta_X V_X, \tag{17}$$

such that under an anomalous transformation $\Delta V = H(T, \Lambda_X) + \bar{H}(\bar{T}, \bar{\Lambda}_X)$, then

$$\Delta\mathcal{L} = \frac{1}{8}\int d^4\theta\frac{E}{R}(\bar{\mathcal{D}}^2 - 8R)LH + \text{h.c.} = -\frac{1}{8}\int d^4\theta\frac{E}{R}\Phi H + \text{h.c.}, \tag{18}$$

since the term involving $\bar{\mathcal{D}}^2$ vanishes identically [11]. The anomaly (6) will be canceled: $\Delta\mathcal{L} = -\Delta\mathcal{L}_{\text{anom}}$, provided (6) reduces to the form

$$\Delta\mathcal{L}_{\text{anom}} = \frac{1}{8}\int d^4\theta\Omega H(T, \Lambda_X) + \text{h.c.}, \tag{19}$$

$$\Omega = \text{Tr}\left[\frac{c_d}{6}\left\{\mathcal{M}^2(\mathcal{D}^2 - 8\bar{R})\mathcal{M}^{-2}R^m + \text{h.c.}\right\} + \frac{c_g}{3}G_m^{\alpha\dot\beta}G^m_{\alpha\dot\beta} + \frac{4c_r}{3}R^m\bar{R}^m\right] + \frac{8c_w}{3}\tilde{\Omega}_W - 4\text{Tr}\left(c_j\Omega_J\right), \tag{20}$$

where the (matrix valued) constants $c_n = c_n(q_i, q_X)$ depend on the modular weights q_i and the $U(1)_X$ charges q_X of the PV fields. They are determined by the requirement that linear and logarithmic divergences cancel, and will be given explicitly in [9]. The resulting component expression includes the standard results for the coefficients of the operators

$$r^{\mu\nu\rho\sigma}\left(r_{\mu\nu\rho\sigma} - i\tilde{r}_{\mu\nu\rho\sigma}\right), \qquad F^a_{\mu\nu}\left(F^{\mu\nu}_a - i\tilde{F}^{\mu\nu}_a\right), \;\; a \neq X, \qquad F^X_{\mu\nu}\left(F^{\mu\nu}_X - i\tilde{F}^{\mu\nu}_X\right). \tag{21}$$

In the present approach, the factor 1/3 in the coefficient of the last operator in (21), relative to that of the second one, comes from a combination of both operators appearing in J_α in (10) and the operator $G^m_{\alpha\dot{\beta}}$ in (8).
Now consider the following Lagrangian

$$\mathcal{L}_{\rm lin} = -3\int d^4\theta\, E\left[F(Z,\bar{Z},V_X,L) + \frac{1}{3}(L+\Omega)(S+\bar{S})\right], \tag{22}$$

where S ($\bar{S}$) is chiral (antichiral):

$$S = (\bar{\mathcal{D}}^2 - 8R)\Sigma, \qquad \bar{S} = (\mathcal{D}^2 - 8\bar{R})\Sigma^\dagger, \qquad \Sigma \neq \Sigma^\dagger, \tag{23}$$

with Σ unconstrained; $L = L^\dagger$ is real but otherwise unconstrained, and Ω is the anomaly coefficient (20):

$$(\bar{\mathcal{D}}^2 - 8R)\Omega = \Phi, \qquad (\mathcal{D}^2 - 8\bar{R})\Omega = \bar{\Phi}. \tag{24}$$

If we vary the Lagrangian (22) with respect to the unconstrained superfields $\Sigma, \Sigma^\dagger$, we recover the modified linearity condition (13). This results in the term proportional to $S+\bar{S}$ dropping out from (22), which reduces to (14), with

$$F(Z,\bar{Z},V_X,L) = 1 - \frac{1}{3}\left[2Ls(L) - V(Z,\bar{Z},V_X)\right], \qquad s(L) = -\frac{1}{2}\int\frac{dL}{L}\frac{\partial k(L)}{\partial L}, \tag{25}$$

where the vacuum value $\langle\, s(L)|\rangle = \langle s(\ell)\rangle = g_s^{-2}$ is the gauge coupling constant at the string scale.
Alternatively, we can vary the Lagrangian (22) with respect to L, which determines L as a function of $\left(S+\bar{S}+V\right)$, subject to the condition

$$F + \frac{1}{3}L(S+\bar{S}) = 1, \tag{26}$$

which assures that once the (modified) linear multiplet is eliminated, the form (11), with a canonically normalized Einstein term, is recovered. Together with the equation of motion for L, the condition (26) is equivalent to the condition (15), and the Lagrangian (22) becomes

$$\mathcal{L}_{\rm lin} = -3\int d^4\theta\, E - \int d^4\theta\, E(S+\bar{S})\Omega = -3\int d^4\theta\, E + \frac{1}{8}\left(\int d^4\theta \frac{E}{R}S\Phi + {\rm h.c.}\right). \tag{27}$$

Since $L = L(S + \bar{S} + V)$ is invariant under T-duality and $U(1)_X$, we require $\Delta S = -H$, so the variation of (27) is again given by (18). The above duality transformation can be performed only if the real superfield Ω, with Kähler weight $w_K(\Omega) = 0$, has Weyl weight $w_W(\Omega) = -w_W(E) = 2$, so that $E\Omega = E_0\Omega_0$ is independent of K and therefore Weyl invariant and independent of $k(L)$. The operator (20) indeed satisfies this requirement, as has been verified [10] by identifying the Weyl invariant operators in conformal superspace, and then gauge-fixing to $U(1)_K$ superspace.

The Lagrangian (27) includes new tree level couplings that generate new ultraviolet divergences. We expect that these can be regulated by PV fields with modular and $U(1)_X$ invariant masses, as was shown [9] to be the case for the dilaton coupling to Φ_{YM}, so they will not contribute to the anomaly. These new terms are in fact expected from superstring-derived supergravity. The Lagrangian depends on the 2-form $b_{\mu\nu}$ only through the 3-form $h_{\mu\nu\rho}$. For a linear multiplet, the 3-form is just the curl of the 2-form: $h_{\mu\nu\rho} = \partial_{[\mu}b_{\nu\rho]}$. This is modified by (13). In 10d supergravity we have

$$H_{LMN} = \partial_{[L}B_{MN]} + \omega^{\rm YM}_{MNL} + \omega^{\rm Lor}_{MNL}, \qquad M, N, \ldots = 0, \ldots 9, \tag{28}$$

where $\omega^{\rm YM}$ and $\omega^{\rm Lor}$ are, respectively, the 10d Yang-Mills and Lorentz Chern-Simons forms. When this theory is compactified to 4d supergravity, we obtain the 4d counterparts of the Yang-Mills and Lorentz Chern-Simons forms, as well as additional terms that arise from indices $m, n, \ldots = 4, \ldots, 9$, in the compact 6d space:

$$h_{\mu\nu\rho} = \partial_{[\mu}b_{\nu\rho]} + \omega^{\rm YM}_{\mu\nu\rho} + \omega^{\rm Lor}_{\mu\nu\rho} + \text{scalar derivatives} + \ldots, \qquad \mu, \nu, \ldots = 0, \ldots, 3. \tag{29}$$

To conclude, we have determined the general form of the supergravity anomaly, and described how it may be canceled by a generalized Green-Schwarz mechanism. In many compactifications the anomaly is not completely canceled by the GS mechanism and string loop threshold corrections play a role; these can easily be incorporated into the present formalism by introducing [9] a dependence on the T-moduli in the superpotential for the massive PV fields: $W_{PV} = \mu(T^i)Z_{PV}Z'_{PV}$. Phenomenological applications of our results as well as a more precise connection to the underlying string theory will be explored elsewhere.

Acknowledgments. One of us (MKG) acknowledges the hospitality of the Kavli Institute for Theoretical Physics, where part of this work was performed. This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract DE-AC02-05CH11231, in part by the National Science Foundation under grants PHY-0457315 and PHY05-51164.

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