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Improving Supplier Compliance Through Joint and Shared Audits

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Abstract

When suppliers (i.e., contract manufacturers) fail to comply with environmental or safety regulations, several non-governmental agencies and consumer activists put pressure on the buyers (customers) to take necessary actions to improve supplier compliance. Due to concerns over negative image and public boycotts, many buyers are conducting costly audits to improve supplier compliance. By considering a common practice that calls for *independent* audits (i.e., each buyer performs its own audit) as a benchmark, we examine the implications of two new audit mechanisms in this paper. The first mechanism is called the *joint* audit mechanism under which buyers conduct joint audits by sharing the joint audit cost and impose a collective penalty if the supplier fails their joint audit. The second mechanism is referred to as the *shared* audit mechanism under which each buyer conducts its own audit independently but shares its audit report with the other buyer. By sharing the audit reports, the buyers impose a collective penalty on the supplier if the supplier fails any one of the audits. Using a game-theoretic model with 2 buyers and 1 supplier, our analysis reveals that the joint audit mechanism is beneficial in two important ways. It can make the supplier increase its compliance level in equilibrium. Also, when the audit cost is below a certain threshold, the joint audit mechanism can increase the supply chain profit so that it is Pareto-improving. Moreover, we find that the shared audit mechanism is beneficial in a similar manner when the audit cost lies within a certain range. Ultimately, our analysis reveals that joint audits can be Pareto-improving when the buyer's audit cost is low, shared audits can be Pareto-improving when the buyer's cost is medium, and independent audits turn out to a practical mechanism when the buyer's audit cost is high.

Keywords: Supply Chain Risk, Supplier Compliance, Audits, Collective Penalty

1. Introduction

Rising labor costs in the West have encouraged more firms to gradually source their products from other low-cost countries in the East. As more contract manufacturers compete for orders at lower price in these countries many factory owners cut corners to reduce their upfront investments and operating costs. In some cases, factory owners may even sacrifice product or process safety by not complying with product regulations or environmental and work safety codes. In terms of product safety violations, Tang and Babich (2014) report that some Chinese manufacturers

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committed product adulteration by using unsafe product materials. Examples include the use of melamine in milk and pet food products, ethanol in alcohol, lead tainted paints in toys, etc. In terms of environmental and work safety violations in China, the reader is referred to various reports developed by the Institute of Public and Environmental Affairs (IPE) for details (<http://www.ipe.org.cn/en/about/report.aspx>).

Bangladesh is an attractive country for western companies (e.g., Walmart, H&M, Mango, and Addidas) to source apparel products due to its low labor cost (US\$ 2 per day). However, without strong enforcement from the Bangladesh government and without strong commitment from buyers, many factory owners simply ignore health and safety issues at their factories. For example, due to the negligence of the factory owner, the collapse of Rana Plaza in Bangladesh killed over 1,000 apparel factory workers in 2013. While many international brands (Tommy Hilfiger, Gap, and several others) have contributed towards a fund for victims' relatives, the negative publicity caused major concerns for these companies that source from Bangladesh. Donaldson (2014) commented that there is a perception that 20% of the factories in Bangladesh are unsafe in terms of building structure safety, fire safety, electrical safety, etc. Besides Bangladesh, many developing countries such as China, Cambodia, and Vietnam are facing similar challenges from non-compliant suppliers with unsafe factories. In 2013, a shoe factory collapsed killing 3 workers in Cambodia (Fuller and Bradsher 2013). Later in 2014, a car parts factory explosion near Shanghai killed 68 workers (Demick 2013). In August 2015, a Tianjin warehouse overloaded with toxic chemicals such as sodium cyanide exploded killing over 114 people and injuring over 700 people (Wong and Fung 2015).

While these international brands are not directly and legally responsible for their suppliers' workers' safety, there is a perceived collateral damage to their image. As articulated in Tang (2013), these brands face a dilemma. If they stop sourcing from Bangladesh, millions of poor Bangladeshi workers will be out of work especially because the garment industry accounts for 80% of the country's exports. If they continue to source from Bangladesh, there is a moral obligation to improve work safety at various factories. However, ensuring compliance is challenging as there are thousands of factories that are involved in different supply chain operations ranging from weaving, dyeing, cutting, to sewing. Recently, to address these challenges, many companies are forming specific units to ensure workplace safety at their suppliers' factories by conducting *independent* audits. For example, PVH Corp., the parent company of Calvin Klein, Tommy Hilfiger, etc., increased their efforts in auditing supplier factories. Since 2012, PVH audits 84% of its tier-1 suppliers at least once per year and reports the non-compliant health and safety issues on its website (www.pvhcsr.com). While it is common for firms (or buyers) to conduct independent audits and penalize those non-compliant factories, this mechanism has two drawbacks: (a) the audit process

can be costly and time consuming; and (b) the penalty imposed by an individual buyer may not be severe enough to entice suppliers to increase compliance especially when the supplier has many customers (i.e., buyers).

To overcome these two drawbacks and to show commitment for improving supplier compliance, firms are now considering forming consortiums and conducting *joint* audits so that they can share the audit cost and they can impose a collective penalty to those non-compliant suppliers. One such example is the Accord on Fire and Building Safety in Bangladesh (bangladeshaccord.org). The Accord is a legally binding agreement signed in May 2013 by 166 apparel corporations from 20 countries in Europe, North America, Asia and Australia, along with numerous Bangladeshi unions and NGOs (e.g., Workers Rights Consortium, International Labor Organization). The goal of the Accord is to improve workplace safety of over 2 million workers at 1,800 factories (Kapner and Banjo 2013).¹ Specifically, the Accord represents a consortium of companies and on their behalf selects impartial inspectors (with fire and building safety expertise), conducts thorough safety inspections of supplier factories, releases inspection reports to the public, imposes corrective actions to those non-compliant factories, and “jointly terminates” the business relationship when a non-compliant supplier is found committing serious safety violations, or when a non-compliant supplier fails to participate fully in the inspection and remediation (Caro and Tang 2014).

There are trade-offs between independent audits and joint audits. Independent audits enable each firm to fully control its own audit effort, but an individual firm can only impose a limited individual penalty on the non-compliant supplier, especially when the supplier conducts business with multiple firms. On the other hand, joint audits can enable a group of firms to impose a severe collective penalty that can put a non-compliant supplier out of business. However, sharing the audit cost by the buyers may create an extra incentive for them to shirk the joint audit effort (due to free-riding). These trade-offs motivated us to examine the following questions when each buyer (firm) is concerned about its brand or collateral damage due to supplier’s non-compliance:

1. Under independent audits, what audit level should each buyer exert and what compliance level should a supplier adopt?
2. Under joint audits, what joint audit level should the consortium exert and compliance level should a supplier adopt?
3. Relative to individual audits, will joint audits result in higher supplier compliance level? Will they result in lower buyer audit effort?

¹To reduce the exposure to broad legal liability, Walmart, Target, and other U.S. retailers are developing a different accord for improving factory safety.

4. Which audit mechanism will generate higher payoffs for the firms, the supplier, and for the entire supply chain?

While joint audits enable buyers to share the audit cost, the joint audit mechanism can involve cumbersome and potentially lengthy negotiations among buyers. To avoid the associated transaction costs, we consider a practical alternative that is referred to as *shared* audits. Under shared audits, each buyer conducts its own audit independently but shares the audit report with other buyers. Akin to the joint audit mechanism, a non-compliant supplier will be penalized collectively by *all* buyers if the supplier fails any one of the audits. Hence, from the buyer's perspective, the shared mechanism can be appealing because it preserves the independence (as if the buyers adopted independent audits) without the need to go through the negotiation process. We then examine a basic question: how would the shared audit mechanism perform relative to the independent and joint audits?

To study these questions we develop a stylized model that involves 3 players (2 buyers and 1 supplier) and captures the essence of both independent and joint audits. For each audit mechanism, we consider a sequential move game in which the 2 buyers will first select their audit levels simultaneously under the independent audits (and jointly under the joint audits). Upon observing (or anticipating) the buyer's audit level, the supplier selects its compliance level. Our analysis of the equilibrium outcomes reveals that relative to the independent mechanism, the joint audit can make the supplier increase its compliance level. Also, when the buyer's audit cost is below a certain threshold, we find that the joint audit can increase the supply chain profit (i.e., the total profit of the buyers and supplier) so that one can devise a transfer payment system to ensure that joint audits is Pareto-improving (i.e., all parties are better off under the joint audit).

When we extend our analysis to examine the shared audits, we find that this mechanism possesses similar properties as the joint-audit mechanism. Also, when the buyer's audit cost lies within a certain range, we find that the shared audits can make the supplier increase its compliance level and it can increase the supply chain profit so that it is also Pareto-improving. Therefore, from the buyer's perspective, the shared mechanism can be a practical alternative to the joint audit mechanism because it enables each buyer to conduct its own audit independently without undertaking negotiations with the other buyers.

Ultimately, our analysis reveals that joint audits can be Pareto-improving when the buyer's audit cost is low, shared audits can be Pareto-improving when the buyer's cost is medium, and independent audits are practical to adopt when the buyer's audit cost is high. This result appears to be counterintuitive because one would have guessed that by splitting the joint audit cost or by sharing audit reports, the joint or the shared mechanism would dominate the independent

mechanism. However, due to the strategic interaction among the buyers and the supplier, we find that this intuition turns out to be incorrect. These findings give guidance and highlight the supply chain role played by industry consortiums that can either do joint audits – as in the Bangladesh Accord – or share audits among members as in the Electronic Industry Citizenship Coalition (EICC).²

Our paper falls within supply chain risk management – a new research stream that has drawn significant interests among practitioners and researchers in recent years (Sodhi et al. 2012). The rising interest in supply chain risk management is triggered by three types of supply chain disruptions. The first type is due to disruptions caused by natural disasters (Japan’s Tōhoku earthquake and tsunami, Thailand’s major flood, etc.) and man-made disasters such as the September 11 attacks. Chopra and Sodhi (2004) examine different mechanisms to mitigate various types of supply chain disruptions, Tomlin (2006) examines the implications of dual sourcing when one of the suppliers is unreliable, and Tang (2006) provides different strategies for mitigating supply chain risks. Hendricks and Singhal (2005) examine the impact of supply chain disruptions on the firm’s stock returns. The reader is referred to a recent book by Sodhi and Tang (2012) for a comprehensive discussion on this kind of supply chain disruptions. The second type of disruptions is due to major financial crisis (e.g., Asian currency devaluations in 1997, the sub-prime financial crisis in 2008) that can disrupt supplier’s operations. In the Operations Management (OM) literature, Babich et al. (2007) is one of the first to examine the issue of managing a portfolio of suppliers who face default risks.

Our paper considers the third type of supply chain disruptions that are caused by a deliberate act committed by the supplier. Some recent research examines the issue of product adulteration that occurs when suppliers use unsafe materials to produce certain products that can cause physical harm to the consumers. Well-publicized examples include Mattel’s lead tainted toys in 2007, melamine tainted milk in 2008, and Baxter’s adulterated Heperin in 2008. In the OM literature, Babich and Tang (2012) present a model to show that a firm can deter suppliers from committing product adulteration by deferring some of its payments so that the supplier can claim these payments only when no adulteration is found within a certain period of time. Rui and Lai (2015) find that the deferred payment strategy continues to be effective under more general conditions. More recently, after the IPE in Beijing exposed various factories dumping toxic waste in the water system in China and after the collapse of the Rana Plaza in 2013, the public expressed serious concern about suppliers’ compliance of environmental and work safety regulations. This has put pressure on many Western firms to take action to improve supplier compliance. In this setting, Plambeck and Taylor

²The EICC website states that “A key initial impetus for the founding of the EICC, and one of its continued benefits to its members today, is the practice of sharing audits.” See <http://www.eiccoalition.org/standards/validated-audit-process/>, accessed Oct 22, 2015.

(2015) use a game-theoretic model to explore the interactions between one buyer’s audit level and one supplier’s compliance and deception levels. By examining the equilibrium outcomes (supplier’s compliance level, supplier’s deception level, and buyer’s audit level) they show that, when a supplier deceives the auditors by hiding certain critical information, the buyer’s actions (increasing audit level, paying a higher price, etc.) could motivate the supplier to cause more harm. In the context of environmental violations, Kim (2015) examines the interactions between the regulator’s inspection policy and the firm’s non-compliance disclosure timing decisions. By considering the case when environmental violations are stochastic, he shows that there are conditions under which periodic inspection can be more effective than random inspection. Orsdemir et al. (2015) investigate how vertical integration can be used as a strategy to ensure compliance. They examine the scenario of two supply chains, one of which is vertically integrated, and highlight that the presence of a supply chain partnership plays a key role in determining supplier compliance. They argue that, in the absence of a partnership, overly tight violation scrutiny can backfire and degrade compliance when negative reporting externalities are high. In the presence of a supply chain partnership, the vertically-integrated supply chain will cease to share responsibly sourced components with the non-integrated supply chain, despite that the former benefits substantially from the exposure of the violations of the latter.

While our paper also deals with the issue of supplier compliance, it is fundamentally different from those in the extant literature on supply chain risk management in two ways. First, the above listed papers primarily focus on the strategic interaction between one buyer and one supplier. Instead, we examine and compare three different audit mechanisms (independent, joint, and shared) by capturing the strategic interactions among two buyers and one potentially non-compliant supplier. Second, we recognize the issue of a non-compliant supplier and employ the notion of “collective penalty” imposed by both buyers when the non-compliant supplier fails (i) the joint audit under the joint mechanism, or (ii) one of the audits under the shared mechanism. Our contribution is to examine the implications of a collective penalty facilitated by the joint and shared mechanisms.

This paper is organized as follows. In Section 2 we present our modeling framework and the resulting equilibrium outcomes associated with the independent and the joint audit mechanisms. In Section 3 we compare the equilibrium outcomes associated with the independent and the joint audit mechanisms. In Section 4 we extend our analysis to the case of the shared mechanism along with a comparison across all three mechanisms: independent, joint, and shared. We present our conclusions in Section 5. All proofs are provided in the Appendix.

2. The Model

Consider a supply chain comprising of two buyers ($i = 1, 2$) and one supplier s . For ease of exposition, we consider the case when buyer i sells one unit of its product at price p_i and pays the supplier a wholesale price w_i . We denote the supplier's unit cost by c_i . Since our focus is on the audit mechanism, we consider p_i , w_i and c_i to be exogenous so that the value of these parameters do not depend on the audit mechanism adopted by the buyers. In other words, the strategic intent of different audit mechanisms is to encourage the supplier to improve its compliance level, but not to increase selling prices or reduce wholesale prices (e.g., Van Mieghem 1999) or both. This seems reasonable in the context of outsourcing agreements between Western firms and suppliers located in developing countries because reducing the wholesale price would create public concerns about the firm's moral and ethical standards. Also, for the reason of tractability and given that the focus of our paper is to examine and compare three different audit mechanisms, we shall defer the case of endogenous wholesale price w_i to future research.

We use a sequential move game to model the dynamics between the buyers and the supplier: the buyers (e.g., international brands) are the leaders and the supplier is the follower that decides whether to comply or not. This sequence of events is representative of many global supply chains in which the buyers have a stronger position to set the sourcing terms. First, under independent audits, each buyer i selects its audit level z_i , $i = 1, 2$ simultaneously and incurs an audit cost of αz_i^2 . Here, z_i represents the probability that buyer i will conduct the audit. This notion of audit probability is commonly used in the literature (e.g., Babich and Tang 2012 and Plambeck and Taylor 2015). Under the joint audits, both buyers exert the joint audit level z and split the audit cost αz^2 . Upon observing (or anticipating) the buyer's audit level, the supplier selects its compliance level x and incurs a compliance cost γx^2 , where $\gamma > 0$ and $x \in [0, 1]$. Here, x represents the probability that the supplier complies with the (environmental, workplace and product safety) regulations. To facilitate the comparison of the supplier's compliance level and the supply chain profit across three different audit mechanisms, we shall assume that the audit cost α remains the same across all three mechanisms (even though the same approach can be applied to examine the case when the audit cost depends on the underlying audit mechanism). Also, for tractability, we do not consider the issue of deception, i.e., the supplier choosing the effort level to deceive the buyer, that was introduced by Plambeck and Taylor (2015).

Regardless of the audit mechanism adopted by the buyers, all parties face the following risks (Figure 1). First, if a non-compliant supplier is identified by buyer i , buyer i will reject the unit without paying, and the supplier will incur a goodwill cost g_i associated with contract termination imposed by buyer i . Second, if a non-compliant supplier is not identified by buyer i , buyer i will

accept the unit and pays the supplier w_i . However, there is a chance that this non-compliance will be exposed to the public. In that case, buyer i will incur an expected “collateral damage” d_i due to the spillover effect of the non-compliant supplier. Throughout this paper, we shall assume that the collateral damage d_i is severe enough so that there is incentive for the buyer to audit its supplier. For this reason, we make two assumptions that provide motivation for the supplier to care about compliance and for the buyer to care about auditing:

Assumption 1. *The supplier’s goodwill cost g_i associated with contract termination imposed by buyer i is higher than the supplier’s profit margin so that $g_i > (w_i - c_i)$ for $i = 1, 2$.*

Assumption 2. *The buyer i ’s damage cost d_i associated with a non-compliant supplier is higher than the buyer’s profit margin so that $d_i > (p_i - w_i) \equiv m_i$ for $i = 1, 2$.*

2.1 Independent Audits (I)

We now analyze the sequential move game under the independent audits by using backward induction. Specifically, we first analyze the supplier’s best response compliance level $x^*(z_1, z_2)$ for any given audit levels (z_1, z_2) selected by the buyers. Anticipating the supplier’s best response, we analyze a non-cooperative game in which both buyers select their own audit levels (z_1, z_2) simultaneously. We can then obtain the equilibrium outcomes via substitutions.

From Figure 1 we observe that the supplier will fail buyer i ’s audit with probability $z_i(1 - x)$ under independent audits. By considering the wholesale price w_i , the goodwill cost g_i , and the compliance cost γx^2 , the supplier’s problem for any given audit level (z_1, z_2) is:

$$\begin{aligned}\pi_s(z_1, z_2) &= \max_{x \in [0,1]} \sum_{i=1}^2 [w_i(1 - z_i(1 - x)) - g_i z_i(1 - x) - c_i] - \gamma x^2 \\ &= \max_{x \in [0,1]} \sum_{i=1}^2 (w_i - c_i) - \sum_{i=1}^2 (w_i + g_i) z_i(1 - x) - \gamma x^2.\end{aligned}\tag{1}$$

Figure 1: The independent audit: buyer i ’s audit level z_i and supplier’s compliance level x .

To ensure that the supplier has incentive to fully comply and that the compliance level captures the entire range from 0 to 1, we assume that the supplier’s profit margin is high enough so that the

supplier's expected profit is non-negative under full compliance (i.e., when $x = 1$). By considering the objective function given in (1), this assumption can be stated as:

Assumption 3. *The supplier's total profit margin is higher than his full compliance cost so that $\sum_{i=1}^2 (w_i - c_i) \geq \gamma$.*

Before determining the supplier's best response, we first consider the case when the buyers conduct full audit so that $(z_1, z_2) = (1, 1)$. In this case, the derivative of the supplier's profit given in (1) with respect to its compliance level x is equal to $\sum_{i=1}^2 (w_i + g_i) - 2\gamma x$. Hence, if we interpret $(w_i + g_i)$ as the supplier's gain for increasing its compliance level by investing 2γ per unit of compliance, then we can interpret the term $r_i \equiv \frac{w_i + g_i}{2\gamma}$ as the supplier's "rate of return on compliance from buyer i ." Also, by applying Assumptions 1 and 3, it is easy to check that $\sum_{i=1}^2 g_i \geq \sum_{i=1}^2 (w_i - c_i) \geq \gamma$ and that $\sum_{i=1}^2 w_i \geq \gamma$. Hence, we can conclude that: $\sum_{i=1}^2 \frac{w_i + g_i}{2\gamma} = r_1 + r_2 \geq 1$. As we shall see, r_i will be useful in interpreting our results later and the condition $r_1 + r_2 \geq 1$ will be employed in some of the proofs.

By considering the first-order condition associated with (1), the supplier's best response compliance level $x^*(z_1, z_2)$ and the corresponding payoff $\pi_s(z_1, z_2)$ are given by:

$$x^*(z_1, z_2) = \min \left\{ \frac{\sum_{i=1}^2 (w_i + g_i) z_i}{2\gamma}, 1 \right\} = \min \left\{ \sum_{i=1}^2 r_i z_i, 1 \right\}. \quad (2)$$

$$\pi_s(z_1, z_2) = \sum_{i=1}^2 (w_i - c_i) - \gamma + \gamma(1 - x^*)^2 \geq 0, \quad (3)$$

where the last inequality is due to Assumption 3. Observe from (2) that, for any given audit levels (z_1, z_2) , the supplier's compliance level $x^*(z_1, z_2)$ is based only on the rate of returns on compliance r_i and the buyer's audit level z_i . Hence, the higher the audit level z_i each buyer is willing to employ, the higher is the supplier's compliance level. Thus, it follows from Assumption 3 and (3) that the supplier's participation constraint $\pi_s(z_1, z_2) \geq 0$ is always satisfied. This result is due to the fact that the supplier can always select its compliance level x to ensure that its profit is non-negative.

Given the supplier's best response, we now analyze the buyer's problem in which both buyers select their audit levels (z_1, z_2) simultaneously in a non-cooperative manner. Buyer i maximizes its expected profit and selects its audit level z_i by anticipating the audit level z_j of buyer j . Upon investing αz_i^2 , buyer i earns m_i (i.e., the profit margin $(p_i - w_i)$) if the supplier passes the audit with probability $(1 - z_i(1 - x^*))$. At the same time, buyer i is exposed to the collateral damage d_i if the non-compliant supplier passes the audit with probability $(1 - z_i)(1 - x^*)$. For any given audit level z_j , buyer i 's problem can be formulated as follows

$$\begin{aligned} \Pi_i(z_j) &= \max_{z_i \in [0,1]} \{m_i(1 - z_i(1 - x^*)) - \alpha z_i^2 - d_i(1 - z_i)(1 - x^*)\} \\ &\text{s.t. (2).} \end{aligned} \quad (4)$$

Since $\alpha > 0$, the objective function is concave in z_i . By examining the first order condition, Equation (2), and the upper bound on z_i , the buyer i 's best response audit level $z_i^*(z_j)$ is given by:

$$z_i^*(z_j) = \min \left\{ \frac{d_i r_i + (d_i - m_i)(1 - r_j z_j)}{2(\alpha + (d_i - m_i)r_i)}, \frac{1 - r_j z_j}{r_i}, 1 \right\}.$$

Observe that the audit efforts are substitutes: buyer i 's audit level $z_i^*(z_j)$ decreases when buyer j 's audit level z_j increases. In addition, it is easy to check that, when α is high enough, buyer i 's audit level is an interior solution so that:

$$z_i^* = \frac{2(d_i r_i + (d_i - m_i)(\alpha + (d_j - m_j)r_j) - (d_i - m_i)r_j \cdot (d_j r_j + (d_j - m_j)))}{4(\alpha + (d_i - m_i)r_i)(\alpha + (d_j - m_j)r_j) - (d_i - m_i)r_i \cdot (d_j - m_j)r_j}, \text{ for } i = 1, 2. \quad (5)$$

The complex expression (5) is not amenable to close form analysis. For this reason, we consider the identical buyer case so that $p_i = p_j = p, d_i = d_j = d, w_i = w_j = w, c_i = c_j = c, m_i = m_j = m$, and $r_i = r_j = r$. Thus, $r_1 + r_2 \geq 1$ is now simplified to $2r \geq 1$. In this case, Equations (5) and (2) imply that, in equilibrium, the buyer's audit level z^I and the supplier's compliance level x^I under the independent audit mechanism can be expressed as:³

$$z^I = \begin{cases} \frac{dr + (d-m)}{2\alpha + 3r(d-m)} & \text{if } \alpha \geq \beta \\ \frac{1}{2r} & \text{if } \alpha < \beta \end{cases} \quad \text{and} \quad x^I = \begin{cases} 2r \frac{dr + (d-m)}{2\alpha + 3r(d-m)} & \text{if } \alpha \geq \beta \\ 1 & \text{if } \alpha < \beta, \end{cases} \quad (6)$$

where

$$\beta \equiv \frac{2dr^2 - r(d-m)}{2}. \quad (7)$$

Using Assumptions 1-3 to examine the buyer's equilibrium audit level z^I and the supplier's equilibrium compliance level x^I given in (6), and using the fact that $2r \geq 1$ when both buyers are identical, we obtain the following results:

Lemma 1. *Under the independent audit mechanism I, the buyer's audit level z^I and the supplier's compliance level x^I given in (6) possess the following properties:*

1. *The supplier's compliance level is higher than the buyer's audit level: $x^I = 2rz^I \geq z^I$.*
2. *Both z^I and x^I are increasing in the buyer's damage cost d .*
3. *Both z^I and x^I are decreasing in the buyer's audit cost α .*
4. *The supplier's compliance level x^I is decreasing in the supplier's compliance cost γ .*
5. *When the buyer's audit cost α is low (high), the buyer's audit level z^I increases (decreases) as the supplier's compliance cost γ increases.*

³Throughout this paper, we shall use the superscripts I, J, S to denote the equilibrium outcomes under independent, joint, and shared audits, respectively.

Lemma 1 has the following implications. Because the supplier's compliance level $x^I = 2r \cdot z^I$, it suffices to focus on the buyer's audit level z^I given in (6). The first statement reveals that, under the independent audit mechanism, the buyer's audit has an amplifying effect: it can trigger the supplier to increase its compliance level according to the factor of $2r(\geq 1)$ (i.e., twice the rate of return on compliance). Consequently, the first statement implies that the buyer can encourage the supplier to comply fully without conducting full audits (i.e., $z_i < 1$). The second statement is intuitive. Due to concerns over the damage cost d , the buyers will increase their audit levels as d increases, which will in turn force the supplier to increase its compliance level. In the same vein, the audit cost has a dampening effect: higher audit cost will force the buyers to reduce their audit levels, which leads to the supplier reducing its compliance level. The fourth statement can be interpreted in the same way.

The fifth statement requires some discussion. First, when the audit cost is low, the buyer can afford to increase its audit level to ensure that the supplier will sustain its (full) compliance level as γ increases. However, when α is high, the buyer is concerned about the audit cost. Under independent audits, each buyer has incentive to “free ride” on the other buyer's audit level due to the underlying substitution effect as observed in the best response function $z_i^*(z_j)$. This effect leads each buyer to shirk and reduce its audit level as γ increases to compensate for the higher expected collateral damage due to the supplier's lower compliance level.

By substituting z^I given in (6) into (4) and (3), we obtain the buyer's profit $\Pi^I(z^I)$ and the supplier's profit $\pi_s^I(z^I)$ under the independent audit mechanism, as follows:

$$\Pi^I(z^I) = m(1 - z^I(1 - 2r \cdot z^I)) - \alpha(z^I)^2 - d(1 - z^I)(1 - 2r \cdot z^I). \quad (8)$$

$$\pi_s^I(z^I) = 2(w - c) - \gamma + \gamma(1 - 2r \cdot z^I)^2. \quad (9)$$

We use $x^I, z^I, \Pi^I(z^I)$, and $\pi_s^I(z^I)$ as the benchmarks associated with the independent mechanism and next examine the joint audit mechanism.

2.2 Joint Audits (J)

We now analyze the sequential-move game associated with joint audits using backward induction. Consider the supplier's problem for any given joint audit level z that is simultaneously selected by both buyers. In this case, the supplier will fail the joint audit with probability $z(1 - x)$. Upon failing the joint audit, the supplier receives no payment and will be subject to the collective penalty $(g_1 + g_2)$ imposed by both buyers. Hence, the supplier solves:

$$\pi_s(z) = \max_{x \in [0,1]} \{[(w_1 + w_2)(1 - z(1 - x)) - (g_1 + g_2)z(1 - x) - (c_1 + c_2)] - \gamma x^2\}.$$

From the first-order condition, the supplier's best response $x(z)$ and its corresponding payoff $\pi_s(z)$ can be expressed as:

$$x^*(z) = \min\{(r_1 + r_2)z, 1\} \quad (10)$$

$$\pi_s(z) = \sum_{i=1}^2 (w_i - c_i) - \gamma + \gamma(1 - x^*)^2 \geq 0, \quad (11)$$

where $r_i \equiv \frac{(w_i + g_i)}{2\gamma}$ for $i = 1, 2$.

Analogous to the independent audit case, (10) reveals that the supplier's best response $x^*(z)$ is equal to the rate of return on compliance times the joint audit level. Also in line with the independent case, Equation (11) shows that the supplier's participation constraint $\pi_s(z) \geq 0$ is always satisfied due to Assumption 3. Comparing Equations (2) and (10) we notice that the supplier's compliance level will be the same under both independent and joint audit mechanisms if $z = z_1 = z_2$.

Now, we examine the buyers' problem under joint audits. Due to the complexity of the buyers' problem, we shall focus on the case of identical buyers. In this case, it is easy to check from (10) that $x^* = 2rz$ because there is no incentive for the buyer to set the audit level $z > \frac{1}{2r}$.

Akin to the independent audit mechanism, the buyers join a consortium to maximize their individual payoffs and independently decide on the joint audit level z and the apportionment of the audit cost αz^2 between themselves. Following Harsanyi (1982b), we shall model the audit level decision and the audit cost allocation as a simultaneous-move non-cooperative unanimity game in which each buyer i proposes an audit level and a split of the audit cost. If the buyers' proposed audit levels are identical, and the sum of the buyers' shares of the audit cost equals 1, the joint audit mechanism will be implemented; otherwise, the negotiations break down and buyers will resort to independent audits and earn Π^I given in (8). More formally, let z_i and θ_i represent the audit level and the share of audit cost proposed by buyer i . By considering each buyer's payoff using (4) and the fact that $x^* = 2rz$, we can check that the equilibrium $(\tilde{z}_i, \tilde{\theta}_i)$ in the simultaneous move non-cooperative unanimity game is given by:

$$(\tilde{z}_i, \tilde{\theta}_i) = \arg \max \{U(z_i, \theta_i; z_j, \theta_j) : z_i, \theta_i \in [0, 1]\}, \text{ where}$$

$$U(z_i, \theta_i; z_j, \theta_j) = \begin{cases} m(1 - z_i(1 - 2rz_i) - d(1 - z_i)(1 - 2rz_i)) - \theta_i \alpha z_i^2 & \text{if } z_i = z_j, \theta_i + \theta_j = 1; \\ \Pi^I & \text{otherwise,} \end{cases}$$

for $i = 1, 2$ and $i \neq j$. Due to the condition $z_1 = z_2$ and $\theta_1 + \theta_2 = 1$, we know that there exists an infinite number of equilibrium points to the above unanimity game. To select choose one equilibrium point, we adopt the payoff-dominance selection rule proposed by Harsanyi and Selten

(Harsanyi 1982a).⁴ Specifically, Harsanyi (1982b) shows that the payoff-dominant solution to this non-cooperative game solves the following optimization problem:

$$\max_{z, \theta_1 \in [0,1]} [m(1 - z(1 - 2rz)) - d(1 - z)(1 - 2rz) - \theta_1 \alpha z^2 - \Pi^I] \cdot [m(1 - z(1 - 2rz)) - d(1 - z)(1 - 2rz) - (1 - \theta_1) \alpha z^2 - \Pi^I], \quad (12)$$

where θ_1 is the share of the joint audit cost to be borne by buyer 1.⁵ In this case, it is easy to check that the optimal share is $\theta_1^* = 0.5$ and the equilibrium audit level z^J and the corresponding compliance level x^J satisfy:

$$z^J = \begin{cases} \frac{2dr+(d-m)}{\alpha+4r(d-m)} & \text{if } \alpha \geq 4\beta \\ \frac{1}{2r} & \text{if } \alpha < 4\beta \end{cases} \quad \text{and} \quad x^J = \begin{cases} 2r \frac{2dr+(d-m)}{\alpha+4r(d-m)} & \text{if } \alpha \geq 4\beta \\ 1 & \text{if } \alpha < 4\beta, \end{cases} \quad (13)$$

where β is given in (7).

Using Assumptions 1-3 to examine the buyer's joint audit level z^J and the supplier's compliance level x^J given in (13), we obtain the following results:

Lemma 2. *Under the joint audit mechanism J, the buyer's joint audit level z^J and the supplier's compliance level x^J given in (13) possess the following properties:*

1. *The supplier's compliance level is higher than the buyer's audit level: $x^J = 2rz^J \geq z^J$.*
2. *Both z^J and x^J are increasing in the buyer's damage cost d .*
3. *Both z^J and x^J are decreasing in the buyer's audit cost α .*
4. *The supplier's compliance level x^J is decreasing in the supplier's compliance cost γ .*
5. *When the buyer's audit cost α is low (high), the buyer's audit level increases (decreases) as the supplier's compliance cost γ increases.*

Because Lemma 2 is analogous to Lemma 1, we can interpret the results the same way as before. Also, because the supplier's compliance level $x^J = 2r \cdot z^J$, it suffices to focus on buyer's joint audit

⁴As defined by Harsanyi (1982b), a "payoff-dominant" equilibrium is Pareto superior to all other equilibria. Therefore, when faced with a choice among equilibria, the payoff-dominance selection rule assumes that all players would agree on the payoff dominant equilibrium since it offers to each player at least as much payoff as the other equilibria. The rule is also shown to be risk dominant.

⁵To maintain the consistency of each buyer's self-interest, our non-cooperative unanimity game enables us to preserve the non-cooperative framework throughout this paper. If we were to adopt the Nash Bargaining (NB) solution concept in a cooperative framework, then it is easy to observe that this optimization problem will yield the same Nash Bargaining solution, see Harsanyi (1982b).

level z^J given in (13). Using Equations (13) and (11), we obtain each buyer's profit $\Pi^J(z^J)$ and the supplier's profit $\pi_s^J(z^J)$ under the joint audit mechanism as:

$$\Pi^J(z^J) = m(1 - z^J(1 - 2rz^J)) - d(1 - z^J)(1 - 2rz^J) - \frac{\alpha}{2}(z^J)^2. \quad (14)$$

$$\pi_s^J(z^J) = 2(w - c) - \gamma + \gamma(1 - 2r \cdot z^J)^2. \quad (15)$$

After establishing the expressions for $x^J, z^J, \Pi^J(z^J)$, and $\pi_s^J(z^J)$, we can now compare the equilibrium outcomes between the joint and the independent audit mechanisms.

3. Independent vs. Joint Audits Equilibrium Comparison

First, we use Assumptions 1-3 and the outcomes presented in (6) and (13) to compare the buyer's audit level and the supplier's compliance level under both mechanisms. We obtain the following result:

Proposition 3. *Relative to the independent audit mechanism, the buyer's audit level and the supplier's compliance level are higher under the joint audit mechanism: $z^J \geq z^I$ and $x^J \geq x^I$.*

Proposition 3 is intuitive. Under the joint mechanism, the buyers can afford to exert a higher joint audit level because the audit cost is shared. On the other hand, the supplier must commit to a higher compliance level under joint audits in response to an increased audit level and the higher (collective) penalty of non-compliance.

Second, we compare the supplier's profits as given in (9) and (15) to obtain the following result:

Proposition 4. *Relative to the independent audit mechanism, the supplier obtains a lower profit under the joint audit mechanism: $\pi_s^J(z^J) \leq \pi_s^I(z^I)$.*

Proposition 4 has the following implications. The supplier is worse off when the buyers exert a higher audit level and impose the collective penalty under the joint audit mechanism. We next examine the buyer's profit under joint audits. By direct comparison of each buyer's profit Π^J given in (14) and Π^I given in (8), we get:

Proposition 5. *Relative to the independent audit mechanism, the buyer obtains a higher profit under the joint mechanism: $\Pi^J(z^J) \geq \Pi^I(z^I)$.*

Proposition 5 shows that each buyer obtains a higher profit under the joint audit mechanism by sharing the audit cost and by imposing the collective penalty on the non-compliant supplier.

The contrasting results as stated in Propositions 4 and 5 create a challenge for the buyers to adopt joint audits. Even if the supplier is forced to participate in the joint audit mechanism, one

would question the buyer's moral standard and the public may pressurize the buyers to treat the supplier fairly to ensure that the supplier is not worse off. Hence, the joint audit mechanism is viable only when it can be Pareto-improving so that the buyers and the supplier will not be worse off under joint audits.

The joint audit mechanism is Pareto-improving when the the supply chain profit (i.e., the total profit of both buyers and the supplier) is higher than under independent audits so that there exists a payment transfer scheme from the buyers to the supplier to ensure that the supplier will not be worse off. Thus, we need to examine whether the supply chain profit will be higher under the joint audit mechanism. When $\alpha \leq \beta$, we know from Equations (6) and (13) that $z^I = z^J = \frac{1}{2r}$ and it can be shown that there is a gain of $\frac{\alpha}{4r^2}$ in the supply chain profit under the joint audit mechanism compared to the profit with independent audits. Hence, intuitively when the auditing cost α is sufficiently low, the supply chain profit will be higher in the joint audit regime. The immediate question is what happens when α is high? The following proposition provides an answer. In preparation, notice that the supply chain profit under the joint mechanism is equal to $2 \cdot \Pi^J(z^J) + \pi_s^J(z^J)$, where $\Pi^J(z^J)$ and $\pi_s^J(z^J)$ are given in (14) and (15), and the supply chain profit under the independent audit mechanism is equal to $2 \cdot \Pi^I(z^I) + \pi_s^I(z^I)$, where $\Pi^I(z^I)$ and $\pi_s^I(z^I)$ are given in (8) and (9). Through direct comparison, we obtain the following result:

Proposition 6. *When the buyer's audit cost α is sufficiently high and the damage cost d and margin m are sufficiently small, the supply chain profit under the joint audit mechanism is lower than under independent audits, i.e., $2 \cdot \Pi^J(z^J) + \pi_s^J(z^J) < 2 \cdot \Pi^I(z^I) + \pi_s^I(z^I)$.*

We already argued that when the buyer's audit cost α is sufficiently low, the joint audit mechanism will increase the supply chain profit. With a higher supply chain profit, it is always possible for the buyers to work with the supplier to come up with a transfer payment scheme to ensure everyone is better off. Combining this observation along with Proposition 5 we can conclude that the joint audit mechanism can entice the supplier to increase its compliance level and it can be Pareto-improving as long as the buyer's audit cost α is below a certain threshold. When the buyer's audit cost α is high, Proposition 6 reveals that the joint audits may yield a lower supply chain profit. With a lower supply chain profit, the joint audit mechanism cannot be Pareto-improving and its implementation can be problematic. Therefore, despite the fact that the buyers split the audit cost, the joint audit mechanism is not always beneficial for the entire supply chain. Fortunately, for a wide range of parameter values the joint mechanism does improve the supply chain profit, as we show next.

We now illustrate our result numerically. Throughout the paper we set $w = 1100, g = 1110, c = 0, d = 1000, m = 800$, and we only vary the auditing cost α and the cost of compliance γ , which are

the key parameters in the model. For $\gamma = 1700$, Figures 2, 3 and 4 illustrate the results stated in Propositions 4, 5 and 6; respectively. To interpret Figures 2, 3 and 4, first consider the case when the buyer's audit cost α is low. Specifically, when $\alpha < \beta$, (6) and (13) reveal that the supplier will fully comply so that $x^I = x^J = 1$ and each buyer will use the same audit level $z^I = z^J = \frac{1}{2r}$ under both audit mechanisms. With the same audit level under both audit mechanisms, Figure 2 confirms that the supplier's profits are the same under both mechanisms. However, Figure 3 shows that each buyer's profit is higher under the joint audit mechanism. This is because both buyers split the joint audit cost under joint audits instead of each buyer paying for its own audit cost under the independent audits.

Next, we consider the case when the buyer's audit cost α is high; for instance, when $\alpha > 4\beta$. In this case, Lemmas 1 and 2 imply that, when the buyer's audit cost α is high, the buyer will audit less and so the supplier will comply less. In particular, (6) and (13) reveal that the supplier will reduce its compliance level so that $x^I < x^J < 1$. At the same time, the buyer will reduce its audit level so that $0 < z^I < z^J < \frac{1}{2r}$. From the supplier's perspective, the supplier will earn a much higher profit under the independent audit because the probability that the supplier will fail the audit under the independent audit is lower than that of under the joint audit when the buyer's audit cost becomes higher. Specifically, when $0 < z^I < z^J < \frac{1}{2r}$, the probability that the supplier will fail the audit is lower under the independent audit because $z^I(1 - x^I) = z^I(1 - 2rz^I) < z^J(1 - 2rz^J) = z^J(1 - x^J)$. This observation is reflected in Figure 2, which illustrates that the supplier will earn a higher profit under the independent audit when the buyer's audit cost α is high.

From the buyer's perspective, the benefit of the joint audit mechanism over the independent audit becomes less significant at high values of α for two reasons: (a) the benefit of sharing the audit cost under the joint audit mechanism becomes less significant when the buyer audits less (because α is high); and (b) the benefit of the collective penalty under the joint audit mechanism becomes less significant when the buyer audits less (because α is high). Figure 3 shows that the buyer will experience a profit gain under the joint audit, but this profit gain becomes smaller when α is high.

When α is high, the loss in the supplier's profit (due to the joint audits) appears to dominate the gain in the buyer's profit (due to the joint audits). Figure 4 confirms the statement of Proposition 6. The supply chain profit is lower under the joint audit mechanism when the buyer's audit cost α is sufficiently high. Hence, the joint audit mechanism cannot be Pareto-improving when auditing is too costly. However, when α is below a certain threshold, Figure 4 shows that the joint audit mechanism can be Pareto-improving.

Figure 2: Supplier's profits under I and J ($\gamma = 1700$).

Figure 3: Buyers' profits under I and J ($\gamma = 1700$).

Figure 4: Supply chain profits under I and J ($\gamma = 1700$).

4. Independent Audits With Shared Information (S)

As discussed in the last section, the joint audit mechanism can make the supplier increase its compliance level and it can be Pareto-improving when the buyer's audit cost α is low. While the joint audit mechanism has its appeal, some buyers may be reluctant to adopt the joint audit mechanism because: (a) buyers have less freedom in selecting auditors and the audit level; (b) buyers typically mistrust each other and do not want to reveal firm specific information; and (c) buyers may find it challenging to reach unanimous agreements on ways to share the audit cost. In view of these non-quantifiable factors, we consider a practical alternative shared audit mechanism that provides independence and yet it captures the spirit underlying the joint audit.

Under the shared mechanism, each buyer conducts its own audit, but they share their audit reports with the other buyer. Because audit reports are shared, a non-compliant supplier will be exposed to both buyers if it fails any one of the buyer's audits. Therefore, for any audit levels (z_1, z_2) , it is easy to check from Figure 5 that the supplier with compliance level x will fail buyer i 's audit with probability $(z_i + (1 - z_i)z_j)(1 - x)$ for $i = 1, 2$. By noting that the supplier will fail buyer i 's audit with probability $z_i(1 - x)$ under the independent audits (Figure 1), we can conclude that, through sharing audit reports, the shared mechanism enables buyer i to identify a non-compliant supplier with an "additional probability" of $z_j(1 - z_i)(1 - x)$. This additional probability plays an important role in the subsequent analysis.

Figure 5: The shared audit: buyer i 's audit level z_i ($i = 1, 2$) and supplier's compliance level x .

As before, we analyze the sequential game associated with the shared mechanism by using

backward induction. First, we consider the supplier's problem. By considering the case of identical buyers and by noting that the probability that the supplier will fail buyer i 's audit is equal to $(z_i + z_j - z_i z_j)(1 - x)$, the supplier's problem can be formulated as:

$$\begin{aligned}
\pi_s(z_1, z_2) &= \max_{x \in [0,1]} \pi_s(z_1, z_2, x) \\
&= \max_{x \in [0,1]} \sum_{i=1}^2 [w(1 - (z_i + z_j - z_i z_j)(1 - x)) - g(z_i + z_j - z_i z_j)(1 - x) - c] - \gamma x^2 \\
&= \max_{x \in [0,1]} [2(w - c) - \sum_{i=1}^2 (g + w)(z_i + z_j(1 - z_i)(1 - x))] - \gamma x^2.
\end{aligned} \tag{16}$$

Next, we compare the supplier's profit function $\pi_s(z_1, z_2, x)$ under the shared mechanism as given above to the supplier's profit function under the independent audit as given in (1) for the case when the buyers are identical. It is easy to verify that for any given audit levels (z_1, z_2) , the supplier's profit is always lower under the shared mechanism.

By considering the first-order condition and by recalling that $r = \frac{g+w}{2\gamma}$, the supplier's best response (in terms of its compliance level) $x^*(z_1, z_2)$ and the corresponding payoff $\pi_s(z_1, z_2)$ under the shared mechanism are given as:

$$x^*(z_1, z_2) = \min\{2r(z_1 + z_2 - z_1 z_2), 1\}. \tag{17}$$

$$\pi_s(z_1, z_2) = 2(w - c) - \gamma + \gamma(1 - x^*)^2 \geq 0, \tag{18}$$

where the last inequality is due to Assumption 3. Observe that the supplier's best response $x^*(z_1, z_2)$ and the corresponding payoff $\pi_s(z_1, z_2) \geq 0$ under the shared mechanism are similar in structure to those under the independent audits, as stated in Equations (2) and (3). We now compare the supplier's best response across all three mechanisms (independent, joint, and shared). It follows immediately from (2), (10) (17) that:

Lemma 7. *For any given buyer's audit level (z_1, z_2) under independent (or shared) and z under joint audits, the supplier's best response compliance level under shared audits is higher than that of under independent audits. Also, the supplier's best response compliance level under shared audits is lower than that of under joint audits if and only if $z > z_1 + z_2 - z_1 z_2$.*

The first statement of Lemma 7 suggests that, by sharing audit reports, each buyer can leverage the "additional probability" to exert additional pressure on the supplier to increase its compliance level. However, relative to joint audits, the relative impact of the shared and the joint mechanisms would depend on the corresponding buyer's audit levels. We shall examine this issue further after we determine the buyers' audit levels in equilibrium.

Anticipating the supplier's best response given in (17), we now analyze the non-cooperative game engaged by the buyers. Recall that the supplier with compliance level x will fail buyer i 's audit with probability $(z_i + z_j - z_i z_j)(1 - x)$ and that the buyer will incur the damage d for failing to identify a non-compliant supplier with probability $(1 - z_i)(1 - z_j)(1 - x)$. Therefore, for any given buyer j 's audit level z_j , we can use the supplier's best response x^* given in (17) to formulate the buyer i 's problem as:

$$\begin{aligned} \Pi_i(z_j) = & \max_{z_i \in [0,1]} \{m(1 - (z_i + z_j - z_i z_j)(1 - x^*)) - \alpha z_i^2 - d(1 - z_i)(1 - z_j)(1 - x^*)\} \\ & \text{s.t. (17).} \end{aligned} \quad (19)$$

By examining the first order condition of (17) and the upper bound on z_i , the buyer i 's best response audit level $z_i^*(z_j)$ satisfies:

$$z_i^*(z_j) = \min \left\{ (1 - z_j) \cdot \frac{dr + (d - m)(0.5 - 2rz_j)}{\alpha + 2(d - m)r(1 - z_j)^2}, \frac{1 - 2rz_j}{2r(1 - z_j)}, 1 \right\}, \quad (20)$$

Observe that the buyer i 's best response audit level $z_i^*(z_j)$ under the shared mechanism resembles the buyer's best response under independent audits. However, there is a fundamental difference. Unlike in independent audits where the buyers' best responses are substitutes in the sense that $z_i^*(z_j)$ is decreasing in z_j , the buyers' best responses under shared audits can be "complements" or "substitutes", depending on the underlying parameter values. Also, notice that the more complex expression for the buyer's best response given in (20) is due to the fact that the supplier's response under the shared mechanism given in (17) involves an additional multiplicative interaction term $z_i z_j$. Nevertheless, we can determine the buyer's equilibrium audit level as follows:

Lemma 8. *Under the shared mechanism, the buyer's equilibrium audit level z^S and the supplier's equilibrium compliance level x^S can be expressed as follows:*

$$z^S = \begin{cases} \hat{z} & \text{if } \alpha \geq \beta^S \\ 1 - \sqrt{\frac{2r-1}{2r}} & \text{if } \alpha < \beta^S. \end{cases} \quad \text{and} \quad x^S = \begin{cases} 2r\hat{z}(2 - \hat{z}) & \text{if } \alpha \geq \beta^S \\ 1 & \text{if } \alpha < \beta^S \end{cases} \quad (21)$$

where $\hat{z}$ is the unique root of the following cubic equation:

$$U(z) \equiv \alpha z + 2(d - m)rz(1 - z)^2 - (1 - z)dr - (d - m)(1 - z)(0.5 - 2rz) = 0, \quad (22)$$

and $\beta^S = \beta \cdot \left\{ \sqrt{\frac{2(2r-1)}{r}} + \frac{2r-1}{r} \right\}$ with β as given in (7).

Although the buyer's audit level x^S and the supplier's compliance level z^S given in (21) are more complex under the shared mechanism, the underlying structure resembles the result as in the case of independent audits (and joint audits) as stated in Lemma 1 (and Lemma 2).

4.1 Comparing the shared mechanism against independent and joint audits

Recall from Lemma 7 that, for any given audit levels, the supplier's best response compliance level is higher under shared audits than under independent audits. However, this can be higher or lower than under joint audits. We now examine this property when each buyer exerts the equilibrium audit level z^S given in (21). We compare z^S , z^I and z^J given in (21), (6), and (13), and obtain the following result:

Proposition 9. *Under the shared mechanism, the buyer's audit level z^S and the supplier's compliance level x^S possess the following properties:*

1. *When the buyer's audit cost α is sufficiently low (i.e., $\alpha < \min\{\beta, \beta^S\}$) the buyer's audit level z^S satisfies: $z^S = 1 - \sqrt{\frac{2r-1}{2r}} < \frac{1}{2r} = z^I = z^J$. Also, the supplier's compliance level x^S satisfies: $x^S = 1 = x^I = x^J$.*
2. *When the buyer's audit cost α is sufficiently high, the buyer's audit level z^S satisfies: $z^I < z^S < z^J$. Also, the supplier's compliance level x^S satisfies: $x^I < x^S$.*

Proposition 9 can be explained as follows. First, consider the case when the buyer's audit cost α is low. When α is sufficiently low, each buyer can afford to choose an audit level that is high enough to ensure the supplier will comply fully (i.e., $x^I = x^J = x^S = 1$). Under independent and joint audits, the buyer sets audit level to $z^I = z^J = \frac{1}{2r} \leq 1$. However, because each buyer can leverage the extra report provide by the other buyer to exert more pressure on the supplier to comply fully so that $x^S = 1$, each buyer can lower the audit level under the shared mechanism.

Second, when the buyer's audit cost α is high, each buyer has to lower its audit level under all mechanisms. However, relative to independent audits, each buyer can afford to leverage the "savings" generated from sharing audit reports and choose a slightly higher audit level under the shared mechanism than under the independent mechanism so that $z^S > z^I$ when α is high. When this happens, the supplier has to respond by complying more under the shared mechanism than under the independent mechanism, so that $x^S > x^I$. However, the audit level under the shared mechanism is still lower than that under joint audits. This is because there are greater savings and consequently profits accrued due to joint audits and these savings in turn allow higher audit levels. Figures 6 and 7 illustrate the results as stated in Proposition 9. In particular, Figure 7 shows that, relative to independent audits, the shared mechanism can make the supplier increase its compliance level (i.e., $x^S > x^I$) when α exceeds a certain threshold. This threshold is non-trivial (i.e., positive) when the compliance cost is higher ($\gamma = 2200$). As expected, the joint audit mechanism dominates the other two in terms of compliance. In other words, if compliance is the main goal, then the supply chain should adopt the joint audit mechanism.

Figure 6: Audit levels under I, S and J, with $\gamma = 1700$ (left) and $\gamma = 2200$ (right).

Figure 7: Compliance levels under I, S and J, with $\gamma = 1700$ (left) and $\gamma = 2200$ (right).

Next, we compare the supplier's profit under the shared and the independent mechanisms. In preparation, we first determine the supplier's profit under the shared mechanism. By substituting $x^S = 2rz^S(2 - z^S)$ into (16), it is easy to check that the supplier's profit under the shared mechanism can be simplified as:

$$\pi_s^S = 2(w - c) - \frac{g + w}{2r} \cdot x^S(2 - x^S). \quad (23)$$

Also, for ease of comparison, we use (6) in (3) to rewrite the supplier's profit under the independent mechanism as:

$$\pi_s^I = 2(w - c) - \frac{g + w}{2r} \cdot x^I(2 - x^I). \quad (24)$$

In this case, we obtain the following result:

Proposition 10. *Under the shared mechanism, the supplier's profit π_s^S given in (23) satisfies the following properties:*

1. When the buyer's audit cost α is sufficiently low, (i.e., $\alpha < \min\{\beta, \beta^S\}$) $\pi_s^S = \pi_s^I = \pi_s^J = 2(w - c) - \frac{g+w}{2r}$.
2. When the buyer's audit level α is sufficiently high, $\pi_s^S < \pi_s^I$.

When the buyer's audit cost α is sufficiently high, Proposition 9 reveals that the buyer's audit level is higher under the shared mechanism than under independent audits (i.e., $z^S > z^I$). Consequently, the supplier has to increase its compliance level so that $x^S > x^I$, which will cause its profit to decrease. This explains the second statement of Proposition 10. Figure 8 illustrates the result stated in Proposition 10. One interesting remark in Figure 8 is that when the compliance cost is high ($\gamma = 2200$) there is a small range of α in which the supplier's profit under shared audits is higher than under independent audits ($\pi_s^S > \pi_s^I$). This peculiarity is again due to the shirking of auditing by the buyers.

Figure 8: Supplier profits under I, S and J, with $\gamma = 1700$ (left) and $\gamma = 2200$ (right).

The next proposition examines the buyer's profit under the shared mechanism:

Proposition 11. *When the buyer's audit cost α is sufficiently low, the buyer's profit under the shared information mechanism has the following properties:*

1. Relative to the independent audits, each buyer obtains a higher profit under the shared mechanism so that $\Pi^S > \Pi^I$.
2. Relative to joint audits, each buyer obtains a higher profit under the shared mechanism so that $\Pi^S > \Pi^J$ if and only if $r > \frac{1}{4(\sqrt{2}-1)}$.

Since the buyers share their audit reports and impose collective penalty under shared audits, the buyers can enforce full supplier compliance $x^S = x^I = 1$ by exerting a lower audit level so that $z^S = 1 - \sqrt{\frac{2r-1}{2r}} < \frac{1}{2r} = z^I$. Consequently, it is intuitive that the buyer's profit is higher under the

shared mechanism than under independent audits. This explains the first statement of Proposition 11. The second statement identifies a simple condition under which the total profit of both buyers under the shared mechanism is higher than under the joint audits. This condition $r > \frac{1}{4(\sqrt{2}-1)}$ holds when the supplier's compliance cost γ is low or when the goodwill cost g is high. Essentially, under this condition, the savings associated with the lower audit level under the shared mechanism can dominate the effect of splitting the audit costs under joint audits. We next construct a numerical example (Figure 9) to illustrate Proposition 11.

Figure 9: Buyer profits under I, S and J, with $\gamma = 1700$ (left) and $\gamma = 2200$ (right). In the left graph $r > \frac{1}{4(\sqrt{2}-1)}$ whereas in the right graph $r < \frac{1}{4(\sqrt{2}-1)}$.

As shown in Figure 9, we find that the shared mechanism enables the buyers to obtain a higher profit than independent audits (and joint audits) when the buyer's audit cost is low and when $r > \frac{1}{4(\sqrt{2}-1)}$. This verifies Proposition 11. Likewise, when the buyer's audit cost α is sufficiently high, Figure 9 reveals that the buyer's profit is higher under the shared audits than that under independent audits. However, it is interesting to note that, regardless of r , when α is sufficiently high the buyer's profit under shared audits is similar to – if not better than – the profit under joint audits. This can be explained as follows. When α is high, the buyer's audit level is low under both joint and shared audits. Due to the low audit level, the savings generated from splitting the joint audit cost becomes less significant under the joint audits. Consequently, the buyers would obtain similar profits under both the joint and the shared audits. Also recall that, like in the joint audit, a collective penalty is imposed in the shared audit mechanism when the supplier fails at least one of the two audits. Therefore, the buyers can adopt the shared mechanism so that they can maintain individual control without dealing with a potentially complicated negotiation process that could include the selection of joint auditors, determining the joint audit level, and determining the cost sharing agreements.

We also compare numerically the supply chain profit across all three mechanisms. As shown in Figure 10, we find that the supply chain profit under the shared mechanism is higher than that under the independent mechanism when the buyer's audit cost α is below a certain threshold. Hence, by combining this observation with the observation from Figure 7 that the shared mechanism can make the supplier increase its compliance level (i.e., $x^S > x^I$) when α exceeds a certain threshold, we can conclude that, relative to independent audits, the shared mechanism can make the supplier increase its compliance level and it can be Pareto-improving when the buyer's audit cost α lies within a certain range. When the cost of compliance is higher ($\gamma = 2200$), the range in which the shared and joint mechanisms are Pareto-improving diminishes. This observation is intuitive since the shared and joint mechanisms increase the compliance level, which is costlier to the supplier when γ is higher.

Figure 10: Supply chain profits under I, S and J, with $\gamma = 1700$ (left) and $\gamma = 2200$ (right).

5. Conclusions

In this paper, we investigate the impact of three different audit mechanisms (independent, joint, and shared) on the supplier's compliance level by considering a stylized model that involves 2 buyers and 1 supplier. Based on our examination of the equilibrium outcomes, we obtain the following results:

- Relative to the independent audit mechanism, the supplier will increase its compliance level under the joint mechanism (and under the shared audit mechanism when the buyer audit is sufficiently high), and the supplier will obtain a lower profit under the joint and shared mechanisms.
- Relative to the independent audit mechanism, the audit level is higher under the joint mech-

anism (and under the shared audit mechanism when the buyers' audit costs are sufficiently high), and the buyer will obtain a higher profit under both the joint and shared mechanisms.

- While the joint audit appears to be appealing, we find that the joint audit can cause harm to the supply chain payoff (i.e., channel profit) especially when the buyer's audit cost is high.
- Relative to the joint mechanism, the buyer can earn a higher profit under the shared mechanism when the buyer's audit cost is sufficiently low.
- Relative to the joint mechanism, the profit gap between the joint and the shared mechanism reduces to zero when the buyer's audit cost is sufficiently high.

In addition to gaining a better understanding about the impact of different audit mechanisms on the buyer's audit level and the supplier's compliance level, the above results have three practical implications:

1. When the audit cost is low, the joint mechanism can entice the supplier to increase its compliance level and this can be Pareto-improving (i.e., there exists a transfer payment scheme so that all parties will not be worse off.)
2. When the audit cost is medium, the shared mechanism can make the supplier increase its compliance level and this can also be Pareto-improving. The shared mechanism can be a practical alternative to joint audits because it can be easily implemented by simply modifying the (commonly observed) independent mechanism by adding a clause that requires all buyers to share their audit reports. This way, the buyers can derive the benefits of sharing information without the need to engage other buyers into potentially complicated or lengthy negotiations that are required for implementing joint audits.
3. When the audit cost is high, the independent mechanism is the most practical option because neither the joint or the shared mechanism can be Pareto-improving.

Future research could consider alternate audit mechanisms and settings where our modeling assumptions do not apply. These include settings where the buyers are non-identical (different price/cost structure, different bargaining power, etc.), or settings where information about price and cost structure is not perfectly known to all parties. Given the current concerns over supplier compliance, addressing those questions could be worthwhile avenues for future research.

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6. Appendix – Proofs

Proof of Lemma 1. The first statement follows from the fact that $2r \geq 1$ and (6). To prove the second statement, first observe that $z^I = \frac{x^I}{2r}$. Hence, it suffices to show the result for z^I and

when $\alpha \geq \beta$ (otherwise, z^I is constant). In preparation, we claim that $\beta \geq \frac{3mr^2}{2(1+r)}$. To prove this claim, we apply (7) to show this equality holds if and only if $d(2r^2 + r - 1) + m(1 - 2r) \geq 0$. By using the fact that $2r \geq 1$ and by applying Assumption 2, we prove the claim by showing that $d(2r^2 + r - 1) + m(1 - 2r) \geq (d - m)(2r - 1) \geq 0$. Now we prove z^I is increasing in d for any $\alpha \geq \beta \geq \frac{3mr^2}{2(1+r)}$. By differentiating z^I with respect to d , z^I is indeed increasing in d because $\alpha \geq \frac{3mr^2}{2(1+r)}$. This proves the second statement. The third, fourth and the fifth statements can be proven by direct differentiation with respect to α and γ respectively. We omit the details. ■

Proof of Lemma 2. The proof follows the same approach as the proof for Lemma 1. We omit the details. ■

Proof of Proposition 3. Because $x^I = 2rz^I$ and $x^J = 2rz^J$, it suffices to show that $z^J \geq z^I$. From (6) and (13), $z^J \geq z^I$ if and only if $\frac{2dr+(d-m)}{\alpha+4r(d-m)} \geq \frac{dr+(d-m)}{2\alpha+3r(d-m)}$. This inequality holds when $3dr\alpha + \alpha(d-m) + r(d-m)(m+d(2r-1)) \geq 0$. This last inequality holds because $r \geq 1/2$ due to Assumptions 1-3. ■

Proof of Proposition 4. Observe from (15) and (9) that, after some algebra,

$$\begin{aligned}\pi_s^J(z^J) - \pi_s^I(z^I) &= 2(g+w)z^I - 4r(g+w)z^{I^2} - 2(g+w)z^J + 4r(g+w)z^{J^2} + 4r^2(z^{I^2} - z^{J^2})\gamma, \\ &= 2(g+w)[z^I(1-rz^I) - z^J(1-rz^J)] \leq 0.\end{aligned}\tag{25}$$

The last inequality follows immediately by using three facts: (a) the parabola $y(1-ry)$ attains its maximum when $y = \frac{1}{2r}$; (b) $z^I \leq z^J$ (Proposition 3); and (c) both z^I and z^J are less than $\frac{1}{2r}$ (c.f. Equations (13) and (6)). ■

Proof of Proposition 5. By the assumption of individual rationality, the buyers operate under the joint audit mechanism only if $\Pi^J(z^J) \geq 0$.

Now, suppose that $0 \leq \Pi^J(z^J) < \Pi^J(z^I)$. Then $(\Pi^J(z^J) - \Pi^I)^2 < (\Pi^J(z^I) - \Pi^I)^2$, where Π^I are the profits if negotiations fail. But this would be a contradiction because z^J is the optimal solution to (12). Hence, $\Pi^J(z^J) \geq \Pi^J(z^I) = \Pi^I(z^I) + \frac{\alpha}{2}(z^I)^2 \geq \Pi^I(z^I)$ and the proof is complete. ■

Proof of Proposition 6. To compare the supply chain profit under both mechanisms, it suffices to examine the supply chain profit gap Δ_{SC} , where $\Delta_{SC} \equiv [2\Pi^J(z^J) + \pi_s^J(z^J)] - [2\Pi^I(z^I) + \pi_s^I(z^I)]$.

After some algebra, we have that:

$$\begin{aligned}
\Delta_{SC} &= \alpha(z^I)^2 + (z^J - z^I)\{2(d - m + 2dr - 2r\gamma) - (z^J + z^I)(\alpha + 4r(d - m - r\gamma))\} \\
&= \alpha[\sqrt{2} \cdot z^I - z^J][\sqrt{2} \cdot z^I + z^J] + (z^J - z^I)\{2(d - m + 2dr - 2r\gamma) - (z^J + z^I)(4r(d - m - r\gamma))\} \\
&= (z^J - z^I) \left[\alpha[\sqrt{2} \cdot z^I + z^J] \left(\frac{\sqrt{2} \cdot z^I - z^J}{z^J - z^I} \right) + 2(d - m + 2dr - 2r\gamma) - (z^J + z^I)(4r(d - m - r\gamma)) \right]
\end{aligned}$$

Hence, the sign of Δ_{SC} depends on the term in squared brackets since from Proposition 3 we know that $z^J \geq z^I$. It can be shown that

$$\begin{aligned}
&\lim_{\alpha \rightarrow \infty} \left[\alpha[\sqrt{2} \cdot z^I + z^J] \left(\frac{\sqrt{2} \cdot z^I - z^J}{z^J - z^I} \right) + 2(d - m + 2dr - 2r\gamma) - (z^J + z^I)(4r(d - m - r\gamma)) \right] \\
&= \frac{d^2(1 + 4r + 5r^2) - 2d(m + 2mr + 2r(1 + 3r)\gamma) + m(m + 4r\gamma)}{d - m + 3dr} = \frac{f(d)}{d - m + 3dr}
\end{aligned}$$

where $f(d) = d^2(1 + 4r + 5r^2) - 2d(m + 2mr + 2r(1 + 3r)\gamma) + m(m + 4r\gamma)$, a quadratic in d with roots $\frac{2r(\gamma + m) + m + 6\gamma r^2 \pm r\sqrt{-m^2 + 4\gamma m(r + 1) + 4(\gamma + 3\gamma r)^2}}{1 + r(5r + 4)}$. The roots are real if and only if, $g(m) = -m^2 + 4m(1 + r)\gamma + 4(\gamma + 3r\gamma)^2 > 0$. Note that $g(0) > 0$, so f has real roots for m sufficiently small. Note also that $f(m) = mr^2(5m - 12\gamma)$ and $f'(m) = 2r(2m - 2\gamma + 5mr - 6r\gamma)$, so we have that

$$\begin{aligned}
f(m) > 0 &\Rightarrow 5m - 12\gamma > 0 \Rightarrow 2m - 2\gamma + 5mr - 6r\gamma > 2m - 2\gamma + 12r\gamma - 6r\gamma = 2m - 2\gamma + 6r\gamma \\
&= 2m + 2\gamma(3r - 1) > 0 \Rightarrow f'(m) > 0,
\end{aligned}$$

where we have used the fact that $2r > 1$. This implies that $f(d)$ has a single root d^* in the region $d > m$ because $f(m) > 0$ and $f'(m) < 0$ cannot hold simultaneously as shown above. Finally, if $d < d^*$, then $f(d) < 0$. This proves the result. ■

Proof of Lemma 7. The first statement follows immediately from (2) and (17) because

$$\begin{aligned}
x^S - x^I &= r(z_1 + z_2) - 2r(z_1 + z_2 - z_1 z_2) \\
&= r(z_1 + z_2 - 2z_1 z_2) = r[z_1 + z_2 - 2(\sqrt{z_1 z_2})^2] \\
&> r[z_1 + z_2 - 2\sqrt{z_1 z_2}] > 0.
\end{aligned}$$

The second statement follows immediately from (10) and (17). ■

Proof of Lemma 8. First, consider the interior solution. The symmetric interior equilibrium resulting from (20) will satisfy:

$$z = (1 - z) \cdot \frac{dr + (d - m)(0.5 - 2rz)}{\alpha + 2(d - m)r(1 - z)^2}.$$

Upon rearranging the term, the interior buyer's audit will satisfy (22). To show that $U(z)$ has a unique root over $(0, 1)$, observe that $U(0) < 0$, $U(1) > 0$, and $U(z)$ is concave over $(0, 1)$. Hence, we can conclude that $U(z)$ has a unique solution $\hat{z} \in (0, 1)$.

Second, consider the boundary solution of the buyer's audit in equilibrium. First, we determine the boundary audit level z that will entice fully supplier compliance so that $x^*(z, z) = 1$. It follows from (17) that the supplier's compliance level $x^S = 1$ when $2r(2z - z^2) = 1$, which occurs when the buyer's boundary audit level is equal to $z = 1 - \sqrt{\frac{2r-1}{2r}}$. In order for the audit level $z = 1 - \sqrt{\frac{2r-1}{2r}}$ qualify as the boundary solution, $z \geq \hat{z}$ so that $U(1 - \sqrt{\frac{2r-1}{2r}}) > U(\hat{z}) = 0$. After some algebra, one can show that the condition for $U(1 - \sqrt{\frac{2r-1}{2r}}) > 0$ can be simplified as $\alpha < \beta^S$. This completes the proof. ■

Proof of Proposition 9. The first statement follows immediately from (21), (6), and (13) when α is small along with the fact that $1 - \sqrt{\frac{2r-1}{2r}} < \frac{1}{2r}$. Second, we note that when α is sufficiently high, the buyer will opt for the interior audit solution under all three mechanism. Therefore, the buyer's audit level z^S under the shared mechanism satisfies $U(z^S) = 0$, where $U(z)$ is given in (22). Next, let us substitute z^I and z^J from (6) and (13) into $U(z)$ given in (22), it can be shown after some algebra that $U(z^I) < 0$ and $U(z^J) > 0$. By using the argument that the function $U(z)$ is increasing and concave in z over $[0, 1]$, we can conclude that $z^I < z^S < z^J$. Finally, observe from (6) and (21) along with the fact that $z^I < z^S$ and that $y < y(2 - y)$ for any $y \in (0, 1)$, it is easy to check that $x^I = 2rz^I < 2rz^S < 2rz^S(2 - z^S) = x^S$. This completes the proof. ■

Proof of Proposition 10. When α is low, we have that $x^S = x^I = x^J = 1$. Hence, we can show that $\pi_s^S = \pi_s^I$ by substituting $x^S = x^I = 1$ into (23) and (24). We can use the same approach to show that $\pi_s^S = \pi_s^J$. We omit the details. Next, observe that the function $2(w - c) - \frac{g+w}{2r} \cdot y(2 - y)$ is a decreasing function of y over the range $y \in [0, 1]$. Combine this observation along with Proposition 9 so that $x^S > x^I$ when α is high, then we obtain the desired result through direct comparison of the supplier's profit functions as given in (23) and (24). ■

Proof of Proposition 11. When α is low, Equation (6) and Proposition 9 state that $x^S = x^I = x^J = 1$ and $z^S = 1 - \sqrt{\frac{2r-1}{2r}} < \frac{1}{2r} = z^I = z^J$. By substituting these values into (17) and (8), we can apply the fact that $1 - \sqrt{\frac{2r-1}{2r}} < \frac{1}{2r}$ to prove the first statement because:

$$\Pi^S = m - \alpha \left(1 - \sqrt{\frac{2r-1}{2r}} \right)^2 > m - \alpha \left(\frac{1}{2r} \right)^2 = \Pi^I.$$

To prove the second statement, substitute $x^S = x^I = x^J = 1$ and $z^S = 1 - \sqrt{\frac{2r-1}{2r}} < \frac{1}{2r} = z^I = z^J$

into (17) and (14). It can be shown that

$$\Pi^S - \Pi^J = \left(m - \alpha \left(1 - \sqrt{\frac{2r-1}{2r}} \right)^2 \right) - \left(m - \frac{\alpha}{2} \left(\frac{1}{2r} \right)^2 \right) = \frac{\alpha}{2} \left(\left(\frac{1}{2r} \right)^2 - 2 \left(1 - \sqrt{\frac{2r-1}{2r}} \right)^2 \right).$$

Hence, we can conclude that $\Pi^S - \Pi^J > 0$ if and only if $\frac{1}{2r} > \sqrt{2} \cdot \left(1 - \sqrt{\frac{2r-1}{2r}} \right)$. We obtain the desired by noting that the latter is equivalent to $r > \frac{1}{4(\sqrt{2}-1)}$. This completes the proof. ■