SESM 66-22a

STRUCTURES AND MATERIALS RESEARCH
DEPARTMENT OF CIVIL ENGINEERING

REFINED FINITE ELEMENT ANALYSIS OF LINEAR AND NONLINEAR TWO-DIMENSIONAL STRUCTURES

-APPENDICES-

by
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Report to
National Science Foundation
NSF Grant GK-75

OCTOBER, 1966

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APPENDIX I

CONSTITUTIVE EQUATIONS FOR LINEAR ELASTIC MATERIALS

1. THREE-DIMENSIONAL EQUATIONS

The material is assumed to be compressible and linearly hyper-elastic. With respect to a rectangular cartesian system (x_1, x_2, x_3) the thermoelastic constitutive law for infinitesimal deformations may be written

$$\epsilon_{ij} = S_{ijkl}\,\tau_{kl} + \alpha_{ij}\,\theta \tag{A1-1a}$$

$$\tau_{ij} = C_{ijkl}\,(\epsilon_{kl} - \alpha_{kl}\,\theta) \tag{A1-1b}$$

where S_{ijkl} : elastic compliances

C_{ijkl} : elastic moduli

α_{kl} : dilatation coefficients

θ : temperature variation over a reference level.

To express Equations (A1-1a) and (A1-1b) in matrix form we arrange the components of the stress, strain and dilatation tensors as (6x1) vectors, and the fourth-order tensors of material constants as (6x6) matrices:

$$\begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ 2\epsilon_{12} \\ 2\epsilon_{23} \\ 2\epsilon_{31} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ & & S_{33} & S_{34} & S_{35} & S_{36} \\ & & & S_{44} & S_{45} & S_{46} \\ & \text{symm.} & & & S_{55} & S_{56} \\ & & & & & S_{66} \end{bmatrix} \begin{Bmatrix} \tau_{11} \\ \tau_{22} \\ \tau_{33} \\ \tau_{12} \\ \tau_{23} \\ \tau_{31} \end{Bmatrix} + \theta \begin{Bmatrix} \alpha_{11} \\ \alpha_{22} \\ \alpha_{33} \\ 2\alpha_{12} \\ 2\alpha_{23} \\ 2\alpha_{31} \end{Bmatrix}$$

$$\begin{Bmatrix} \tau_{11} \\ \tau_{22} \\ \tau_{33} \\ \tau_{12} \\ \tau_{23} \\ \tau_{31} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ & & C_{33} & C_{34} & C_{35} & C_{36} \\ & & & C_{44} & C_{45} & C_{46} \\ & \text{symm.} & & & C_{55} & C_{56} \\ & & & & & C_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{11} - \alpha_{11}\theta \\ \epsilon_{22} - \alpha_{22}\theta \\ \epsilon_{33} - \alpha_{33}\theta \\ 2\epsilon_{12} - 2\alpha_{12}\theta \\ 2\epsilon_{23} - 2\alpha_{23}\theta \\ 2\epsilon_{31} - 2\alpha_{31}\theta \end{Bmatrix}$$

or

$$\epsilon = S\,\tau + \alpha\,\theta \tag{A1-2a}$$

$$\tau = C(\epsilon - \alpha\theta) = C\epsilon - t\,\theta \tag{A1-2b}$$

with $$C S = I \qquad t = C\alpha$$

The coefficients affecting shear strains allow us to express the internal work W_i per unit of volume in the following forms

$$W_i = \tau^T \epsilon = \epsilon^T \tau = \epsilon^T C \epsilon - \theta t^T \epsilon = \tau^T S \tau + \theta \alpha^T \tau \tag{A1-3}$$

Consider now a new cartesian frame $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$. If the transformation laws for stresses and strains are

$$\bar{\epsilon} = T_\epsilon\,\epsilon \qquad \bar{\tau} = T_\tau\,\tau \tag{A1-4}$$

then from the invariance of the internal work as given by the last two forms in Equation (A1-3), we get the following transformation laws for the material constants

$$\begin{aligned} \bar{C} &= T_\epsilon^T C\, T_\epsilon \\ \bar{S} &= T_\tau^T S\, T_\tau \\ \bar{\alpha} &= T_\tau^T \alpha \\ \bar{t} &= T_\epsilon^T t \end{aligned} \tag{A1-5}$$

It should be noted that T_τ and T_ϵ are similar but not equal because of the factors affecting shear strains.

2. TWO DIMENSIONAL EQUATIONS

For two-dimensional problems we assume that (x_1, x_2) is a plane of elastic symmetry. Therefore $c_{ik} = s_{ij} = c_{ki} = s_{ki} = 0$ for $i = 1,2,3,4$ and $k = 4,5,6$; shear strains and shear stresses in the x_3 direction decouple. We consider only four components

$$\begin{Bmatrix} \tau_{11} \\ \tau_{22} \\ \tau_{33} \\ \tau_{12} \end{Bmatrix} \quad \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ & c_{22} & c_{23} & c_{24} \\ & & c_{33} & c_{34} \\ \text{symm.} & & & c_{44} \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ 2\epsilon_{12} \end{Bmatrix} - \theta \begin{Bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{Bmatrix} \qquad \text{(A1-6a)}$$

The inverse law may be written in terms of the so-called technical constants as

$$\begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ 2\epsilon_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & \frac{\nu_{12}}{E_1} & \frac{\nu_{13}}{E_1} & \frac{\mu_{14}}{E_1} \\ \frac{\nu_{21}}{E_2} & \frac{1}{E_2} & \frac{\nu_{23}}{E_2} & \frac{\mu_{24}}{E_2} \\ \frac{\nu_{31}}{E_3} & \frac{\nu_{32}}{E_3} & \frac{1}{E_3} & \frac{\mu_{34}}{E_3} \\ \frac{\mu_{41}}{G_{12}} & \frac{\mu_{42}}{G_{12}} & \frac{\mu_{43}}{G_{12}} & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \tau_{11} \\ \tau_{22} \\ \tau_{33} \\ \tau_{12} \end{Bmatrix} + \theta \begin{Bmatrix} \alpha_{11} \\ \alpha_{22} \\ \alpha_{33} \\ 2\alpha_{12} \end{Bmatrix} \qquad \text{(A1-6b)}$$

where

- E_K : Young's moduli in the x_K direction;
- G_{12} : shear modulus associated with the (x_1, x_2) axes;
- ν_{Kl} : Poisson's ratios associated with (x_K, x_l);
- μ_{Kl} : Coefficients of influence between extensional and shear strains and stresses.

Since the matrix $\mathbf{S}$ is symmetric, there are 6 relations between the 16 technical constants ;

$$E_j \nu_{ji} = E_i \nu_{ij} \qquad E_j \mu_{ji} = G_{12} \mu_{ij} \qquad (\text{no sum})$$

3. TWO-DIMENSIONAL TRANSFORMATION MATRICES

With respect to the system $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$ obtained by a rotation φ of x_1 and x_2 about x_3, we have the following transformation matrices for strains and stresses

$$\mathbf{T}_\epsilon = \begin{bmatrix} \alpha^2 & \alpha\beta & \cdot & -\alpha\beta \\ \alpha\beta & \beta^2 & \cdot & \alpha\beta \\ \cdot & \cdot & 1 & \cdot \\ \alpha\beta & -\alpha\beta & \cdot & \alpha^2-\beta^2 \end{bmatrix} \qquad \mathbf{T}_\tau = \begin{bmatrix} \alpha^2 & \alpha\beta & \cdot & 2\alpha\beta \\ \alpha\beta & \beta^2 & \cdot & -2\alpha\beta \\ \cdot & \cdot & 1 & \cdot \\ \alpha\beta & -\alpha\beta & \cdot & \alpha^2-\beta^2 \end{bmatrix} \qquad (\text{A1-7})$$

where $\alpha = \cos\varphi$, $\beta = \sin\varphi$ and φ is positive in the counter-clockwise sense. Transformed material constants follow from (A1-5).

4. ORTHOTROPIC MATERIAL

If we take (x_1, x_2) along the principal elastic directions the stress-strain matrix has the form

$$\mathbf{C}_p = \begin{bmatrix} c_{11}^p & c_{12}^p & c_{13}^p & \cdot \\ & c_{22}^p & c_{23}^p & \cdot \\ & & c_{33}^p & \cdot \\ \text{symm.} & & & c_{44}^p \end{bmatrix} \qquad (\text{A1-8})$$

In terms of technical constants let

$$\chi = 1 + \nu_{21}\nu_{32}\nu_{13} + \nu_{12}\nu_{23}\nu_{31} - \nu_{23}\nu_{32} - \nu_{12}\nu_{21} - \nu_{31}\nu_{13}$$

then

$$
\begin{aligned}
c_{11}^{p} &= E_1\,(1-\nu_{23}\nu_{32})\,/\,\chi \\
c_{22}^{p} &= E_2\,(1-\nu_{13}\nu_{31})\,/\,\chi \\
c_{33}^{p} &= E_3\,(1-\nu_{12}\nu_{21})\,/\,\chi \\
c_{44}^{p} &= G_{12} \\
c_{12}^{p} &= E_1(\nu_{12}-\nu_{32}\nu_{13})/\chi = E_2(\nu_{21}-\nu_{32}\nu_{13})/\chi \\
c_{23}^{p} &= E_2(\nu_{23}-\nu_{21}\nu_{13})/\chi = E_3(\nu_{32}-\nu_{12}\nu_{31})/\chi \\
c_{31}^{p} &= E_3(\nu_{31}-\nu_{21}\nu_{32})/\chi = E_1(\nu_{13}-\nu_{12}\nu_{23})/\chi
\end{aligned}
\tag{A1-9}
$$

For axes (x_1, x_2) not oriented in the elastic directions, the matrix $\mathbf{C}$ is full like in Equation (A1-6). Their elements may be evaluated in terms of the elements of $\mathbf{C}_p$ by using the Equations (A1-5) and (A1-7).

Let φ = angle (x_{3p}, x_1) measured from x_1;

$\alpha = \cos\varphi \quad \beta = \sin\varphi$;

then

$$
\begin{aligned}
c_{11} &= c_{11}^{p}\alpha^4 + (2c_{12}^{p}+c_{44}^{p})\,\alpha^2\beta^2 + c_{22}^{p}\,\beta^4 \\
c_{22} &= c_{11}^{p}\beta^4 + (2c_{12}^{p}+c_{44}^{p})\,\alpha^2\beta^2 + c_{22}^{p}\,\alpha^4 \\
c_{44} &= c_{44}^{p} + (c_{11}^{p}+c_{22}^{p}-2c_{12}^{p}-4c_{44}^{p})\,\alpha^2\beta^2 \\
c_{12} &= c_{12}^{p} + (c_{11}^{p}+c_{22}^{p}-2c_{12}^{p}-4c_{44}^{p})\,\alpha^2\beta^2 \\
c_{24} &= \left[\, c_{22}^{p}\beta^2 - c_{11}^{p}\alpha^2 + (c_{12}^{p}+2c_{44}^{p})(\alpha^2-\beta^2) \,\right]\alpha\beta \\
c_{34} &= \left[\, c_{22}^{p}\alpha^2 - c_{11}^{p}\beta^2 + (c_{12}^{p}+2c_{44}^{p})(\alpha^2-\beta^2) \,\right]\alpha\beta \\
c_{i3} &= c_{i3}^{p} \qquad (i = 1,2,3)
\end{aligned}
\tag{A1-10}
$$

The following combinations are invariant

$$c_{11} + c_{22} + 2c_{12} = c_{11}^P + c_{22}^P + 2c_{12}^P$$
$$c_{44} - c_{12} = c_{44}^P - c_{12}^P$$

5. ISOTROPIC MATERIAL

Here $E_i = E$, $\nu_{ik} = \nu$, $\chi = (1-\nu)^2(1-2\nu)$ and

$$\mathbf{C} = \frac{E}{(1-2\nu)(1+\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & \cdot \\ \nu & 1-\nu & \nu & \cdot \\ \nu & \nu & \nu & \cdot \\ \cdot & \cdot & \cdot & \frac{1}{2}-\nu \end{bmatrix} \tag{A1-11}$$

If the material is also thermally isotropic $\alpha_{ij} = \delta_{ij}\alpha$ and

$$\mathbf{t} = \frac{E\alpha}{1-2\nu} \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{Bmatrix} \tag{A1-12}$$

6. PLANE STRESS

We set $\tau_{33} = 0$ and condense the stress-strain matrix to

$$\begin{Bmatrix} \tau_{11} \\ \tau_{22} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} \bar{c}_{11} & \bar{c}_{12} & \bar{c}_{14} \\ & \bar{c}_{22} & \bar{c}_{24} \\ \text{symm.} & & \bar{c}_{44} \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ 2\epsilon_{12} \end{Bmatrix} - \theta \begin{Bmatrix} \bar{t}_1 \\ \bar{t}_2 \\ \bar{t}_4 \end{Bmatrix} \tag{A1-13}$$

plus

$$\epsilon_{33} = \langle \gamma_{31} \quad \gamma_{32} \quad \gamma_{34} \rangle \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ 2\epsilon_{12} \end{Bmatrix} + \bar{\alpha}_3 \theta$$

where

$$\gamma_{3i} = -c_{3i}/c_{33} \qquad \bar{\alpha}_3 = t_3/c_{33} \qquad \bar{c}_{ij} = c_{ij} - c_{i3}\gamma_{3j}$$

$$\bar{t}_i = t_i - \alpha_3 t_3 \qquad (i = 1,2,4)$$

For orthotropic material referred to the elastic axes

$$\bar{c}_{14} = \bar{c}_{24} = 0 \quad . \quad \chi = 1-\nu_{21}\nu_{12} \ , \ \bar{c}_{11} = E_1/\chi \ , \ \bar{c}_{22} = E_2/\chi$$
$$\bar{c}_{44} = G_{12} \qquad \bar{c}_{12} = E_1\nu_{12}/\chi = E_2\,\nu_{21}/\chi \tag{A1-14}$$

and we see that these coefficients do not depend on the properties in the x_3 direction.

For elastically and thermally isotropic solid

$$\begin{Bmatrix} \tau_{11} \\ \tau_{22} \\ \tau_{12} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & \cdot \\ \nu & 1 & \cdot \\ \cdot & \cdot & \frac{1-\nu}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ 2\epsilon_{12} \end{Bmatrix} - \frac{E\alpha}{1-\nu} \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} \tag{A1-15}$$

7. PLANE STRAIN

Since $\epsilon_{33} = 0$ we may delete directly the 3rd column in Equation (A1-6a); for orthotropic material we may use Equations (A1-8), (A1-9) and (A1-10); the coefficients depend on the material properties in the x_3 direction. Finally for isotropic material we may delete the 3rd row of $\mathbf{C}$ in Equation (A1-11).

APPENDIX II

COMPARISON BETWEEN DIFFERENT TYPES OF QUADRILATERALS ASSEMBLED WITH TRIANGULAR ELEMENTS

1. MOTIVATION

Quadrilaterals formed by combinations of several triangles are often used as basic blocks for the finite element discretization of two-dimensional structures. Their main advantages for a production program are:

(a) simplification of mesh description;

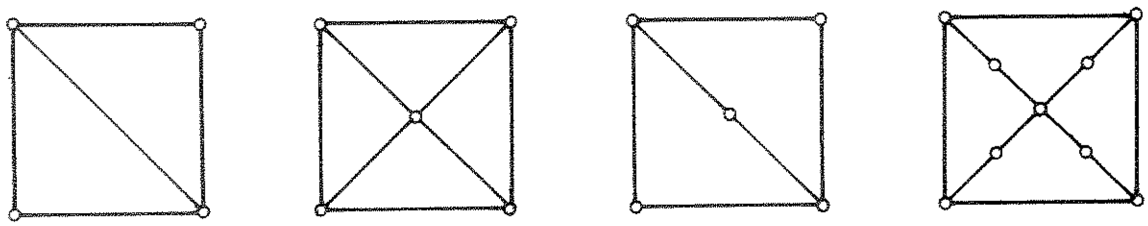
(b) reduction of degrees of freedom and connectivity by the previous condensation of internal nodes;

(c) improved internal stress values, when obtained by averaging over subtriangles.

In order to establish a direct comparison between quadrilaterals formed with different combinations of linear and constant strain triangles, eight types were selected to assemble a square of side unity in plane stress (Fig. A2.1).

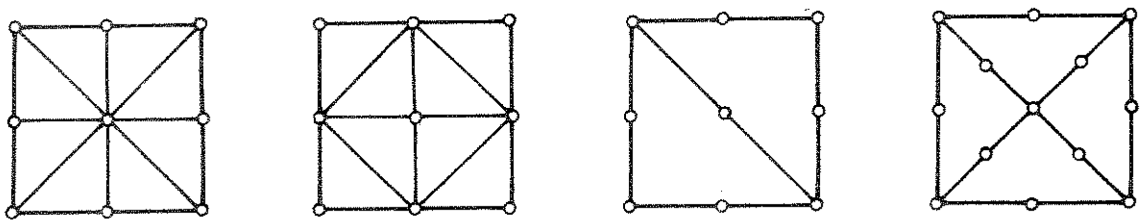
Stiffness matrices for 4 and 5 nodal point triangles, needed for Types No. 3 and 4, were computed from the expression (III-28) valid for the 6 nodal point element, but where the corner strain-nodal displacement submatrices $\mathbf{U}$ and $\mathbf{V}$ must be condensed to (3x4) and (3x5), respectively. This is accomplished by combining appropriated columns when a midpoint is eliminated. For instance, if we impose a linear displacement constraint on side 3 to form a 5 nodal point triangle, $u_4 = (u_1+u_2)/2$ and similarly for v_4; therefore, we must add one-half of column 5 to columns 1 and 2 in the expressions (III-15).

All internal degrees of freedom were eliminated by condensation. Invariant properties of the external stiffness matrices are presented in Table 6.



Type 1 Type 2 Type 3 Type 4

Quadrilaterals with 4 External Nodal Points



Type 5 Type 6 Type 7 Type 8

Quadrilaterals with 8 External Nodal Points.

Fig. A2.1 - Comparison of **Several Quadrilateral Types.**

Lower bounds for the stiffness coefficients for 4 nodal point rectangles were obtained by Pian [38] by assumed stress distribution, i.e., constructing equilibrium models. Their properties are also reproduced in Table 6.

2. CONCLUSIONS

(a) The little practical value of the proposed "improvement" of stiffness matrices by introduction of a large number of additional modes (discussed in II.1.3) is illustrated here by the properties of the 4 nodal point quadrilaterals. The inclusion of a single quadratic deformation mode for type No. 3 produces already a stiffness matrix not very far from the exact solution as demonstrated by the lower bounds. The constraint imposed by the linear restraint on the external sides is seen to dominate immediately. On the other hand, for the 8 nodal point quadrilaterals, the improvement obtained by selecting a more refined fundamental mode pattern is such that no convergence of the stiffness invariants can be observed between types 7 and 8.

(b) Of all 4 nodal point quadrilaterals, type No. 3, assembled with two 4 nodal point triangles, seems to provide the best balance between accuracy and assembly time; an interesting feature is that its 8 diagonal elements are equal (for a square). Of all 8 nodal point quadrilaterals, type No. 8 is by far the best one.

(c) Quadrilaterals constructed with CST's offer no advantages concerning either stiffness properties or formation time; their stress pattern is even worse and especially detrimental for problems involving plastic or incompressible distortions. Therefore its use is not recommended.

Table 6. Properties of Stiffness Matrices of a Square of Side = 1., E = 1., ν = 1/3, h = 1., formed by Different Combinations of Triangular Elements.

8 Fundamental Degrees of Freedom (Linear Edge Displacements)							
Type No.	No. of Trian.	Nodes per Trian.	Total No. of Nodal Pnts.	Addit. Nodal Pnts.	Smallest Nonzero Eigenvalue	Trace	Time to form, sec. IBM 7094
1	2	3	4	-	0.7500	6.0000	0.033
2	4	3	5	1	0.5625	4.1250	0.066
3	2	4	5	1	0.4167	3.8667	0.045
4	4	5	9	5	0.4018	3.8036	0.155
Lower Bounds (Equilibrium Models):							
Linear stress expansion					0.3333	3.6667	
Quadratic stress expansion					0.3750	3.7500	
Cubic stress expansion					0.3892	3.7783	
16 Fundamental Degrees of Freedom							
5	8	3	9	1	0.4755	15.7500	0.120
6	8	3	9	1	0.3750	16.1250	0.120
7	2	6	9	1	0.2980	19.0666	0.099
8	4	6	13	5	0.1929	12.7321	0.282

APPENDIX III

COMPUTER PROGRAM FOR ELASTIC PLANE STRESS AND PLANE STRAIN ANALYSIS

1. IDENTIFICATION

PSE-LST - Plane Stress Elastic Analysis Using Linear Strain Triangles.

Programmed: Carlos A. Felippa, March 1965 (this version June 1966).

2. PURPOSE

The purpose of this program is the solution of general plane stress or plane strain static, linear elastic problems using linear strain triangles combined to form an efficient mesh-generating unit. Surface loads, body forces and thermal effects may be considered.

3. PROGRAMMING INFORMATION

The program is written in FORTRAN IV (version 13) for the IBM 7094 computer. It is subdivided into 6 links and makes use of the Overlay feature of the IBSYS Loader.

4. TAPE AND DISK USAGE

FORTRAN logical units 1,2,8 and 9 and the scratch area of a 1301 disk are used for temporary storage. SHARE routine I9 BC DISK is used for direct random access to the disk. Logical unit 3 is Overlay Link residence.

5. BASIC MESH UNITS

The basic mesh element is a quadrilateral composed of four 6 nodal point linear strain triangles (Fig. A3.1), the center point being the centroid. Internal points 9 to 13 are eliminated by condensation hereby reducing the number of degrees of freedom from 26 to 16.

Single 6 nodal point triangles may also be specified to facilitate fitting of certain shapes.

6. CAPACITY

The mesh input is subjected to the following limitations for a computer with 32 K core storage:

Max. number of elements*	350
Max. number of external** nodal points	1050
Max. number of restrained components***	250
Max. difference of nodal point numbers for the same element	79

* The work "elements" refers to basic mesh units;

** Internal quadrilateral points excluded;

*** One for roller, two for fixed point.

These limits are dictated by the stress computation and not by the equation solver. The maximum number of degrees of freedom is then 2100+350x10 =5600.

7. PROGRAM STRUCTURE

The link structure is shown in Fig. A3.2, where each rectangle represents a subroutine. Their functions are:

MAIN remains in core during execution and controls calling sequence;

SETUP inputs, prints and checks mesh data and evaluates element stiffnesses;

STQUAD assembles and condenses quadrilateral stiffness;

STLST6 computes stiffness of a six nodal point triangle;

FORMK assembles the complete stiffness matrix;

LDINPT inputs load case and reduces surface, body and thermal loads to equivalent forces on external nodal points;

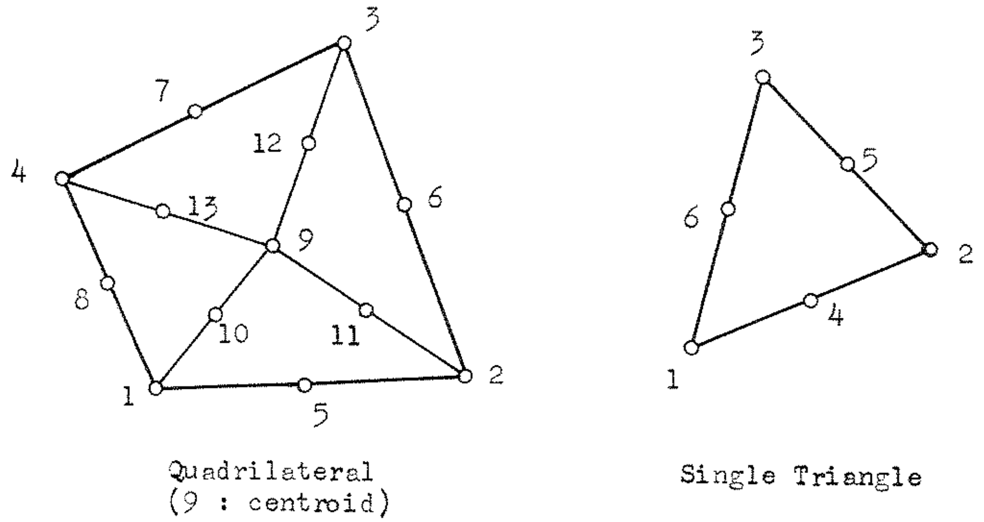


Fig. A3.1 - Basic Mesh Elements.

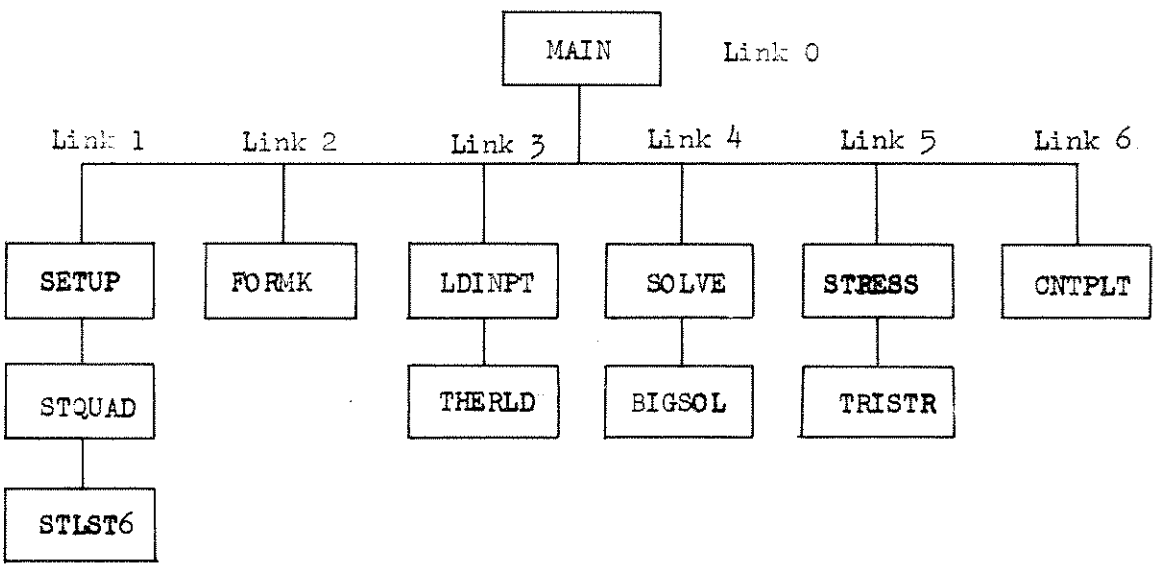


Fig. A3.2 - Link Structure.

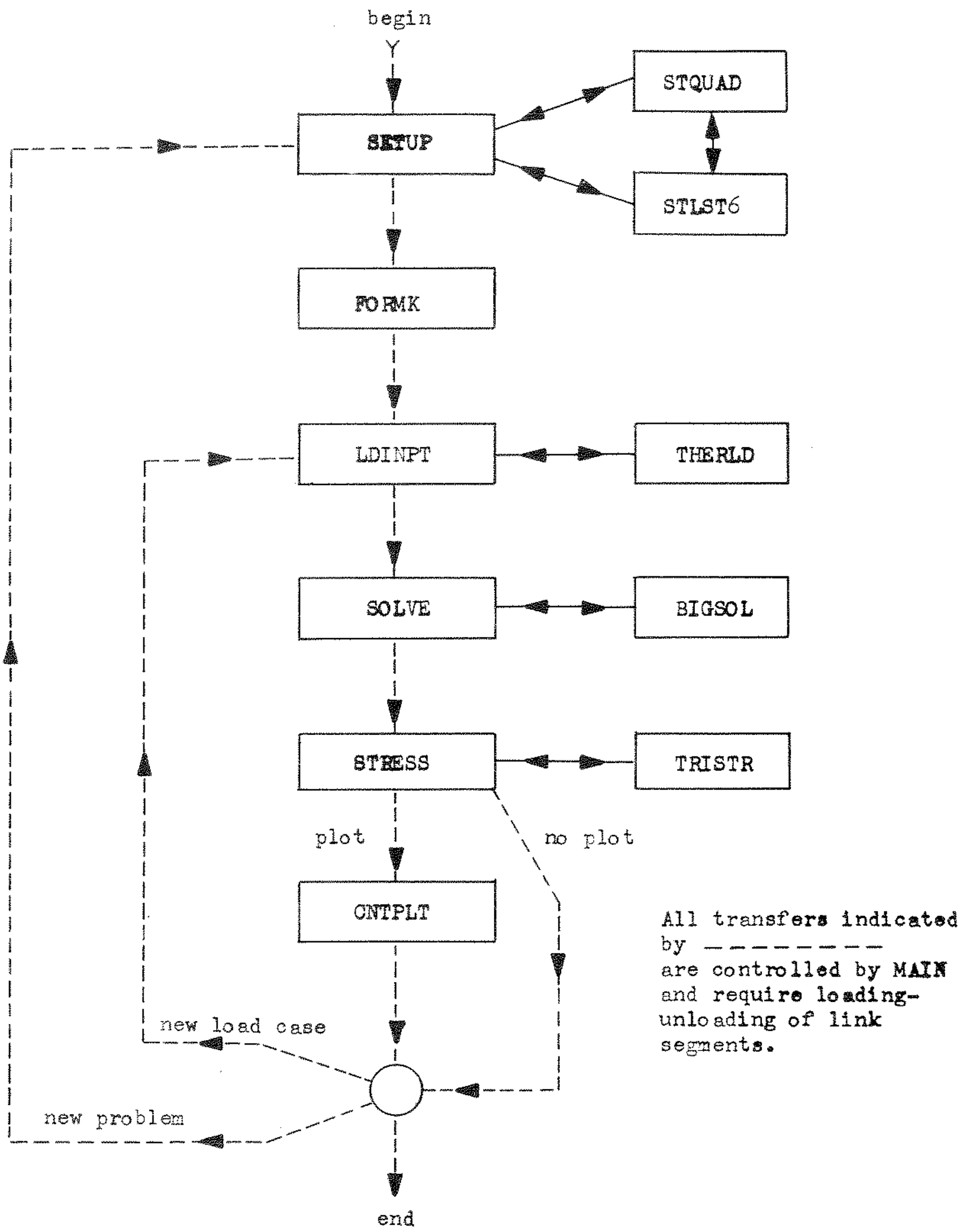


Fig. A3.3 - Subroutine Flow Chart.

THERLD computes initial thermal forces for a single triangle;

SOLVE obtains nodal point displacements from BIGSOL;

BIGSOL large capacity band solver subroutine;

STRESS evaluates and prints element and nodal stresses;

TRISTR computes stresses for a single triangle;

CNTPLT produces printer plots of stress contour lines;

The subroutine flow chart is presented in Fig. A3.3.

8. SEQUENCE OF OPERATIONS

In the ensuing description, only operations related with the generation and assembly of individual stiffness matrices are described in some detail.

Notation:

n = number of equations = 2 x (No. of external nodes);

m = half band width, including diagonal = 2 x (maximum element nodal difference) + 2.

(a) Description of Structure: Numerical data defining geometric and physical characteristics of the structure are read in, printed and organized by SETUP. Possible mesh errors are checked and the band width computed.

(b) Computation of Element Stiffnesses.

(I) Quadrilateral: the position of the centroid is evaluated by the program and the complete stiffness K_Q(26x26) assembled after 4 calls to STLST6. K_Q may be imagined to be partitioned as follows:

$$K_Q = \left[\begin{array}{c|c} K_{11} & K_{12} \\ \hline K_{21} & K_{22} \end{array}\right] \begin{array}{l} \text{16 rows} \\ \text{10 rows} \end{array} \qquad \text{(A3-1)}$$

This matrix is reduced by symmetric backward Gauss elimination:

$$k_{ij} = k_{ij} - \frac{k_{iq}\, k_{qj}}{k_{qq}} = k_{ij} - k_{iq}\, c_{qj} \quad \text{(no sum on } q\text{)} \tag{A3-2}$$

for q = 26, 25,....17; i,j = 1,2,... (q-1).

The multiplier c_{iq} is overwritten on K_{iq}. At the end of this process we have

$$\left[\begin{array}{c|c} K & K_{12} \\ \hline X_{21} & L_{22} D_2 \end{array}\right] \tag{A3-3}$$

where

$$K_{22} = L_{22}\, D_2\, L_{22}^T \tag{A3-4}$$

represents the backward Gauss decomposition of K_{22} (L_{22} = unit lower triangle, D_2 = diagonal),

$$X_{21} = D_2^{-1}\, L_{22}^{T-1}\, K_{21} \tag{A3-5}$$

and

$$K = K_{11} - K_{12}\, L_{22}^{-1}\, X_{21} \tag{A3-6}$$

is the condensed (16x16) stiffness matrix which is written on a disk track. X_{21} and the decomposed K_{22} are stored on the next track.

(II) Single triangle: its (12x12) stiffness matrix is obtained by a single call to STLST6.

(c) <u>Assembly of the Complete Stiffness Matrix</u>: This operation is carried out in FORMK. The complete stiffness is subdivided into "k" blocks of "r" equations (i.e., r/2 nodal points) each, where

$$(k-1)\, r < n \leq k\, r \quad \text{and} \quad r = 17600/m$$

The last block contains only n- (k-1)r equations. Previously, "k" arrays specifying the numbers of the elements which contribute to each block are computed and stored. The formation of the first block involves the following steps:

(I) Stiffness matrices of contributing elements are read from disk tracks and added by the direct stiffness procedure into a one-dimensional array S(17600), where the first "r" columns of the upper band stiffness are stored in compact form, i.e.,

$$\text{1st block} = \langle \mathbf{c}_1^T \;\; \mathbf{c}_2^T \; \ldots\ldots \; \mathbf{c}_r^T \rangle$$

where

$$\mathbf{c}_1^T = \langle k_{11} \quad 0 \quad 0 \quad \cdots\cdots\cdots \; 0 \rangle$$

$$\mathbf{c}_2^T = \langle k_{22} \quad k_{12} \quad 0 \quad \cdots\cdots\cdots \; 0 \rangle$$

$$\vdots$$

$$\mathbf{c}_r^T = \langle k_{rr} \quad k_{r-1,r} \quad \cdots\cdots\cdots \quad k_{r-m+1,r} \rangle$$

(II) Displacement boundary conditions constraining $r_i = 0$ are imposed by setting the off-diagonal entries of the i-th row and column to zero and the diagonal element to 1. If a point is constrained to move along a line x' not parallel to any of the global axes (X,Y), a tensor transformation is previously applied to select x' as local X-axis.

(III) The entire block is written on tape, each column c_i constituting a single physical record of length "m".

The second block comprises columns c_{r+1} to c_{2r}, etc; this process continues until all columns have been assembled.

(d) Load Input: Any number of load cases may be processed sequentially. Subroutine LDINPT accepts the following loading conditions:

1) concentrated nodal forces;

2) distributed forces (parabolic variation over a side);

3) gravity loading;

4) thermal increments (linear variation over subtriangles).

In cases (3) and (4), equivalent nodal forces arise at internal quadrilateral points, i.e., $\mathbf{R}_2 \neq \mathbf{0}$ in:

$$\begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{bmatrix} \begin{Bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \end{Bmatrix} = \begin{Bmatrix} \mathbf{R}_1 \\ \mathbf{R}_2 \end{Bmatrix} \tag{A3-7}$$

In this case, $\mathbf{X}_{21}$ and the decomposed $\mathbf{K}_{22}$ are read from disk and the lower portion of (A3-3) assembled. A backward reduction from rows 26 to 17 is performed on the load vector, after which we have

$$\text{on } \mathbf{R}_1 : \quad \mathbf{R} = \mathbf{R}_1 - \mathbf{L}_{22}^{-1} \mathbf{X}_{21} \mathbf{R}_2 \tag{A3-8}$$

$$\text{on } \mathbf{R}_2 : \quad \mathbf{X}_2 = \mathbf{D}_2^{-1} \mathbf{L}_{22}^{T-1} \mathbf{R}_2 \tag{A3-9}$$

The condensed load vector $\mathbf{R}$ is added to the external force vector and $\mathbf{X}_2$ is stored on the same track of $\mathbf{X}_{21}$ and $\mathbf{L}_{22}$. After the complete external force vector is formed, B.C.'s are imposed.

(e) Displacement Solution: nodal point displacements are obtained by BIGSOL, a large capacity linear equation solver for symmetric band matrices, with optional double-precision residual improvement.

(f) Stress Computation: to recover displacements of internal quadrilateral points, we set up again

$$\left[\begin{array}{c|c} & \\ \hline X_{21} & L_{22} \end{array}\right] \left\{\begin{array}{c} r_1 \\ \hline r_2 \end{array}\right\} \quad \left\{\begin{array}{c} \\ \hline X_2 \end{array}\right\} \text{ 10 rows} \tag{A3-10}$$

and perform forward-substitution on the displacement vector from rows 17 to 26 to obtain

$$r_2 = L_{22}^{-1}(X_2 - X_{21} r_1) \tag{A3-11}$$

The coordinate stress components σ_x, σ_y and τ_{xy} are computed by utilizing Equations (III-14) and (III-15) at the four subtriangles, and averaged at centroid and corners. These values are considered "element stresses" and may be printed if desired.

"Nodal point stresses" at external nodes are obtained as arithmetic means over contributing elements (these values have significance only if their material properties are the same.) Principal stresses and directions are also evaluated and printed.

(g) Stress Plots: if stress graphs are specified, convenient spacings are computed by STRESS; then CNTPLT proceeds to generate and print stress contour lines. For this purpose, quadrilaterals are again subdivided into 4 subtriangles and the stress values at their vertices (quadrilateral centroid and corners) linearly interpolated.

9. TIMING

Some representative execution times (IBM 7094) for nxn quadrilateral meshes (no residual iteration, 6 stress graphs), are given for illustration purposes:

Mesh	4x4	8x8	16x16
External degress of freedom (No. of equations to solve)	130	450	1666
Half band width	34	58	106
Total No. of degrees of freedom	290	1090	4226
Time for one load case (complete)	0.6 min.	2.4 min.	14 min.
Each extra load case	0.2 min.	0.8 min.	2 min.

Notes:

(1) The overhead execution time (load-unload 6 links) is approximately 0.25 min;

(2) Each extra load case requires only the substitution of the load vector, since the complete stiffness was decomposed in the first pass.

10. AUXILIARY PROGRAMS

(a) Mesh generator: generates and punches element and nodal arrays for four-sided shapes;

(b) Mesh plot routine: produces a CALCOMP mesh plot to check input;

(c) Stress plot routine: produces graphs of stress contours and principal directions for the X-Y CALCOMP Plotter using a punched stress deck.

11. INPUT DATA INSTRUCTIONS

The following sequence of cards define numerically the structure:

A - STRUCTURE DATA

(a) Start Card (A6): with the word START punched in cols. 1-5. This card must precede the input data deck of any problem.

(b) Title Card (13A6): Alphameric information in cols. 1-78 to identify output.

(c) Control Card (8I4,5L2)

Columns	Variable Name	Meaning
1- 4	NUMEL	Number of elements (≤ 350);
5- 8	NUMCP	Number of corner points;
9-12	NUMNP	Number of external nodal points (≤ 1050);
13-16	NUMBC	Number of restrained points;
17-20	NUMPB	Number of defining boundary points, see (e);
21-24	NLOAD	Number of load cases, set = 1 if left blank;
25-28	NMAT	Number of different materials (≤ 6), set to 1 if left blank;
29-32	MAXIT	Maximum number of residual iterations in the displacement solution; punch a 1 or 2 for large, ill-conditioned problems (see note 1).

The next five fields are for logical flags; if a T is punched, the indicated option takes place:

33-34	T1	All quadrilaterals have the same stiffness matrix (see note 2);
35-36	T2	Punching of external nodal point coordinates and displacements (I4,2F8.3,2E14.5);

37-38	T3	Punching of averaged σ_x, σ_y and τ_{xy} at external nodes and quadrilateral centroids (I4,3E18.6);
39-40	T4	Print of element stresses (see note 3);
41-42	T5	Another problem follows.

Notes:

1) An ill-conditioned problem is one for which the complete stiffness matrix (with B.C.'s) is nearly singular, i.e., straining displacement mode amplitudes are small in comparison with the total displacements. Examples: (i) slender structures; (ii) two or more materials with very different elastic moduli.

2) All quadrilaterals have the same stiffness if they can be superimposed by a translation; nodal point numbering must correspond.

3) Element stresses should be printed in problems involving several material types, since average stresses and their plots do not display actual interface discontinuities.

(d) Material Property Table (I4,4F10.3). One card per material type (total NMAT cards):

Cols.	1- 4	Material type number;
	5-14	Elastic modulus;
	15-24	Poisson's ratio;
	25-34	Specific weight;
	35-44	Coefficient of thermal expansion.

For plane strain, reduced values must be used:

$$E' = E/(1-\nu^2) \quad , \quad \nu' = \nu/(1-\nu) \quad , \quad \alpha' = \alpha\,(1+\nu) .$$

(e) Defining Boundary Array (20I4). For stress graph plotting, NUMPB corner points which define the boundary of the region to be plotted as a series of straight lines must be punched in cyclic order, 20 per card. The initial point and the sense may be arbitrary. Holes in multiple connected bodies cannot be plotted separately.

(f) Element Array (10I4,F10.3). One card per element.

Cols. 1 - 4 Element number;

5 -36 Nodal point numbers:

(I) for quadrilaterals: external nodal points in counter-clockwise order I-J-K-L-M-N-O-P (Fig. A3.4). The starting corner is arbitrary, except when equal stiffnesses are implied (T1 = .TRUE. in control card);

(II) for single triangles: punch nodes I-J-K-L-M-N (Fig. A3.4), leave cols. 29-36 blank.

37-40 Element material type, set to 1 if left blank.

41-50 Element thickness, set to 1.00 if left blank.

Note: if a quadrilateral is not convex (not recommended), the entrant corner must be either J or K.

(g) Coordinate Array (I4,2F8.3). One card per corner point (total NUMCP cards).

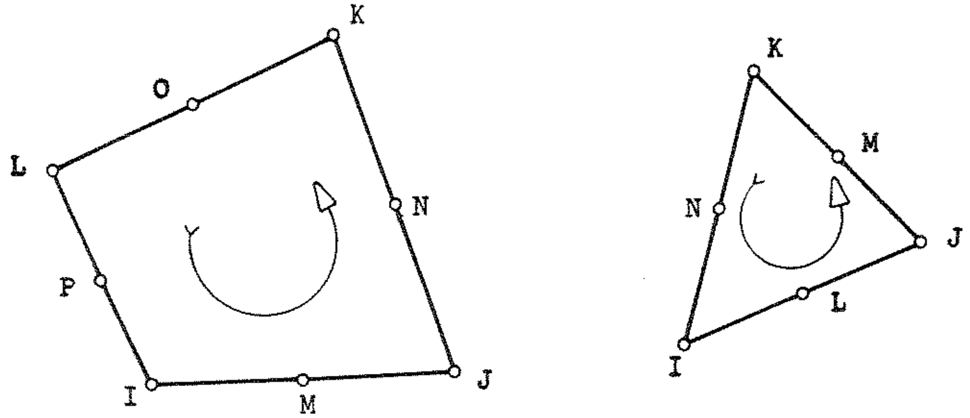
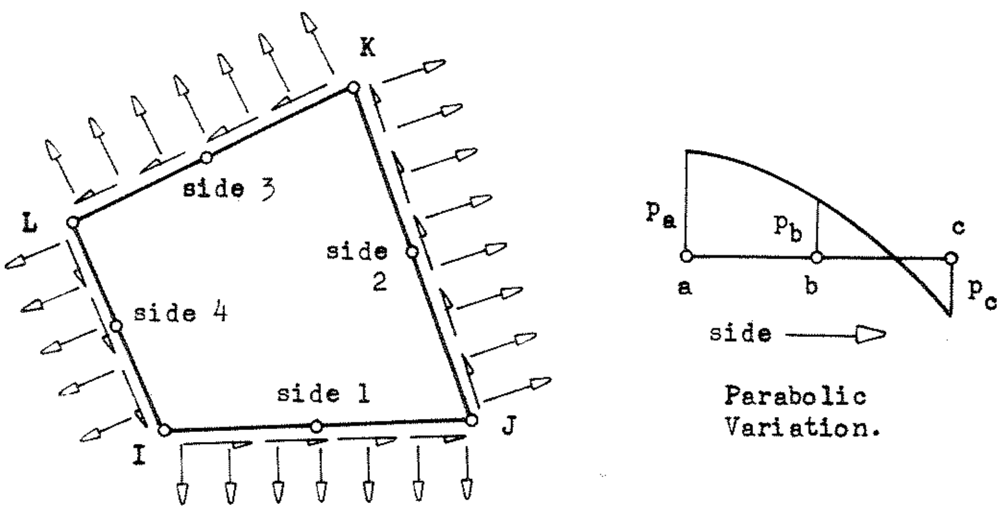


Fig. A3.4 - Nodal Point Numbering.



Positive Sense Indicated.
(For single triangles, J-K is
side no. 3).

Fig. A3.5 - Conventions for Element Side Loading.

Cols. 1 - 4 Corner point number;

5 -14 X-coordinate;

15-24 Y-coordinate.

(h) Boundary Condition Array (2I4,F10.3). One card per restrained point.

Cols. 1 - 4 Nodal point number;

5 - 8 Tag = 0 if point is fixed in both directions;

= 1 if point is fixed in the X-direction;

= 2 if point is free to move along a line forming angle φ with the X axis;

9 -18 Angle in degrees, positive counterclockwise (for type 2 of boundary condition only)

Note: tag 2 with $\varphi = 90^\circ$ and tag 1 are equivalent, but it is recommended to use the second one to avoid an extra transformation.

B - LOADING DATA

Each load case must be specified by a data deck initiated by a LOADNG card; this package follows the structure data deck. A load deck consists of the following cards;

(i) Loading Card (A6): with the word LOADNG punched in cols. 1-6.

(j) Title Card (13A6): alphameric information in cols. 1-78 to identify load case.

(k) Control Card (3I4,L2)

Columns	Variable Name	Meaning
1- 4	NPLD	Number of nodal points loaded with concentrated forces;
5- 8	NELD	Number of element sides loaded with distributed forces;
9-12	NTLD	Number of elements undergoing thermal increments;
13-14	DENS	Logical flag for gravity loading: if a T is punched, gravity forces acting along the (-Y) direction are considered.

(1) Graph Indicator Card (7I4)

The first six fields specify, by a positive integer punch, that the corresponding stress graph will be printed:

Columns	Graph
1- 4	Sigma xx
5- 8	Sigma yy
9-12	Tau xy
13-16	Sigma max
17-20	Sigma min
21-24	Max shear

The last field (cols. 24-28) indicates the number NSK $\leqslant$ 50 of elements to be skipped from the plots. If NSK $>$ 0, additional cards must follow, specifying skipped element numbers (20I4). Element skipping may be used for two different purposes:

1) to eliminate small regions of high stress gradients which cannot be accurately described by a printer plot;

2) to plot a portion of the structure, which is then amplified; in this case the array of boundary points (e) must specify the boundary of the subregion.

(m) <u>Nodal Point Forces</u> (I4,2F8.3). One card per node loaded with a concentrated force (no cards if NPLD = 0):

Cols.	1- 4	Nodal point number;
	5-12	X-load;
	13-20	Y-load.

(n) <u>Element Side Loads</u> (I4,6F8.3). One card per element side under surface tractions (no cards if NELD = 0). The convention for positive pressure and shear is indicated in Fig. A3.5. The side variation is assumed to be parabolic and specified by the values at points a, b and c (in counterclockwise sense). For instance, for side 2 of a quadrilateral: a = corner J , b = midpoint N, c = corner K.

Cols.	1- 4	Element number
	5- 8	Side number (see Fig. A3.5);
	9-16	Normal pressure at a;
	17-24	Normal pressure at b;
	25-32	Normal pressure at c;
	33-40	Surface shear at a;
	41-48	Surface shear at b;
	49-56	Surface shear at c.

These values must be specified per unit of length of the boundary and per unit of thickness.

(n) Thermal Increments (I4,4F10.3). One card per element undergoing temperature changes (no cards if NTLD = 0):

Cols.	1- 4	Element number;
	5-14	Temperature variation at corner i;
	15-24	Temperature variation at corner j;
	25-34	Temperature variation at corner k;
	35-44	Temperature variation at corner l (leave blank for single triangle).

Note: the thermal increment at the centroid is assumed to be the mean of the corner values, and a linear variation assumed over each subtriangle.

12. NEW PROBLEM

The input of a new problem must follow the last load deck for the previous one. For safety, any number of blank cards may be inserted before the START card.

13. OUTPUT

The printed output consists of

Echo check of structure data;

Echo check of load input and final load vector;

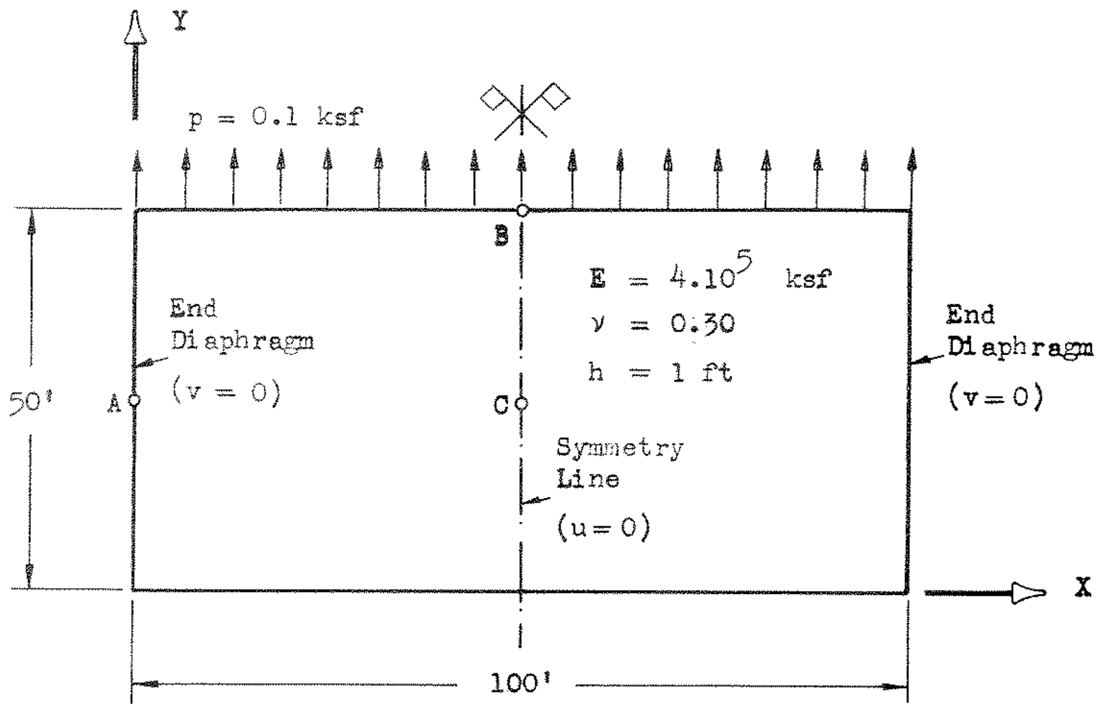
Nodal point displacements;

Element stresses (optional);

Averaged nodal point stresses (plus maxima over contributing elements for σ_x, σ_y and τ_{xy});

Stress graphs (optional).

Other values of interest might be output by the user by inserting appropriate PRINT statements.



Mesh of square elements for one-half of plate	2 x 2	4 x 4	8 x 8
Degrees of Freedom:			
unconstrained	42	290	1090
after B.C.s	32	272	1056
Deflection $v_C \times 10^5$	4.8297	4.8487	4.8515
Normal Stress σ_{xB}	0.3190	0.3207	0.3214
Shear Stress τ_{xyA}	0.1550	0.1380	0.1333
Run time, min.	0.3	0.5	2.1

Fig. A3.6 - Example: Folded Plate Member.

14. ERROR EXITS

Several error conditions caused by either wrong input data or exceeded array capacity are checked by SETUP; error messages are self-explanatory and may be complemented by examination of the input data printout. The program does not stop until all error conditions have been tested. If another problem follows and an error detected in SETUP, the program searches for the next START card at which point execution is continued.

15. EXAMPLE

To illustrate the speed and accuracy of this version, a 2:1 folded plate member under uniform in-plane load on top (Fig. A3.6) was analyzed by subdividing one-half into meshes of nxn quadrilaterals (squares in this case). This example was treated in [54] by using several types of rectangular elements with 3 degrees of freedom at each corner (u, v and ω); very fine meshes were needed in order to achieve 2-3 significant places for the displacements; moreover, the element selected as giving best results was not completely compatible.

A comparison of the typical values reproduced in Fig. A3.6 with their Aitken's extrapolations ($v_C = 4.8516\ 10^{-5}$, $\sigma_{xB} = 0.3215$, $\tau_{xyA} = 0.132$) shows that the 8x8 mesh provided 5 decimal places for the displacements, 4 for σ_x and 3 for τ_{xy}. Actually, the program has capacity for an 18x18 mesh (5500 degrees of freedom) if necessary. The consistency of the stress values is reflected by the fact that the maximum discrepancy over contributing elements did not exceed 0.004 for σ_x and σ_y, and 0.006 for τ_{xy} in the case of the 8x8 mesh.

```
$IBJOB PSELST  MAP,LOGIC
$IBFTC MAIN    DECK
C
C     ******************************************************************
C     PSE-LST -  LINEAR ELASTIC ANALYSIS OF PLANE STRESS OR PLANE
C     STRAIN PROBLEMS USING LINEAR STRAIN TRIANGLES (LST)
C     THE BASIC MESH INPUT UNIT IS A QUADRILATERAL FORMED WITH
C     FOUR LST, HAVING 16 EXTERNAL (FUNDAMENTAL) DEGREES OF FREEDOM
C     AND 10 INTERNAL (ADDITIONAL) DEGREES OF FREEDOM
C     ******************************************************************
C     CARLOS A. FELIPPA, JUNE 1966
C
      COMMON
     1 NUMEL, NUMCP, NUMNP, NUMBC, NLOAD, MAXIT, NEQ,
     2 IBANDW, NEQBC, NRUN, NIRA, NSKEWB, LNQ(6,4),
     3 T1, T2, T3, T4, T5, THERL
      LOGICAL PGRAPH, T1, T2, T3, T4, T5
      COMMON /CMATPR/  YM(6), PR(6), RHO(6), ALFA(6)
      COMMON /CELMAR/  NP(350,8), NEBC(250), BANGLE(250)
      COMMON /CNTARG/  NUMPB, NSK, PGRAPH, IGRTAG(6), SPACNG(6),
     2 GRHEAD(2,6), NPB(50), NELSKP(50)
  100 CALL SETUP
      CALL FORMK
  200 CALL LDINPT
      CALL SOLVE
      CALL STRESS
      IF (PGRAPH)  CALL CATPLT
      NRUN = NRUN + 1
      IF (NRUN.LE.NLOAD)  GO TO 200
      IF (T5)  GO TO 100
      STOP
      END
```

```
$ORIGIN        ALPHA,SYSUT3
$IBFTC SETP    DECK,LIST
      SUBROUTINE SETUP
C
C     ******************************************************************
C     SETUP INPUTS DATA AND EVALUATES ELEMENT STIFFNESSES
C     ******************************************************************
C
      COMMON
     1 NUMEL, NUMCP, NUMNP, NUMBC, NLOAD, MAXIT, NEQ,
     2 IBANDW, NEQBC, NRUN, NITRA, NSKEWB, LNQ(6,4),
     3 T1, T2, T3, T4, T5, THERL
      COMMON /CMATPR/  YM(6), PR(6), RHO(6), ALFA(6)
      COMMON /CELMAR/  NP(350,8), NEBC(250), BANGLE(250)
      COMMON /CSETUP/
     1 XORD(1050), YORD(1050), XCENT(350), YCENT(350),
     2 MAT(350), TH(350)
      COMMON /QUADRG/ S11(16,16), S21(10,20), X(5), Y(5), E, XU, THICK
      COMMON /STFARG/ ST(16,16), B(3), A(3), AREA, ET, NU, THIK
      COMMON /CNTARG/  NUMPB, NSK, PGRAPH, IGRTAG(6), SPACNC(6),
     2 GRHEAD(2,6), NPB(50), NELSKP(50)
      DIMENSION  HEAD(13), GRTITL(2,6), IPERM(3), IPERM4(4),
     1 QUDR(2), TRNG(2), DUM(1)
      EQUIVALENCE (DUM,ST)
      REAL NU
      LOGICAL T1, T2, T3, T4, T5, OD
      DATA IPERM /2,3,1/, IPERM4/2,3,4,1/
      DATA FLAG /6HSTART /, TEST /O777777770001/
      DATA QUDR(1) /12H QUADRLTERAL/, TRNG(1) /12H  TRIANGLE  /
      DATA GRTITL(1,1) /12H SIGMA XX    /,
     1      GRTITL(1,2) /12H SIGMA YY    /,
     2      GRTITL(1,3) /12H TAU XY      /,
     3      GRTITL(1,4) /12H SIGMA MAX   /,
     4      GRTITL(1,5) /12H SIGMA MIN   /,
     5      GRTITL(1,6) /12H MAX SHEAR   /
C
C     ******************************************************************
C     INITIALIZATION
C     ******************************************************************
C
  100 NRUN = 1
      IFLAG = 0
      MAXEL = 350
      MAXNP = 1050
      MAXPD = 80
      MAXBC = 250
      DO 110  I = 1,4
      J = IPERM4(I)
      LNQ(1,I) = I
      LNQ(2,I) = J
      LNQ(3,I) = 9
      LNQ(4,I) = I + 4
      LNQ(5,I) = J + 9
  110 LNQ(6,I) = I + 9
      DO 120  I = 1,2
      DO 120  J = 1,6
```

```
  120 GRHEAD(I,J) = GRTITL(I,J)
C
C     ******************************************************************
C     READ AND PRINT OF INPUT DATA
C     ******************************************************************
C
  140 READ 10, CHECK
      IF (CHECK.NE.FLAG)  GO TO 140
      READ 10,  HEAD
      PRINT 11, HEAD
   10 FORMAT (13A6)
   11 FORMAT (1H1,13A6)
C
C     CONTROL PARAMETERS
C
      READ 15,  NUMEL, NUMCP, NUMNP, NUMBC, NUMPB, NLOAD, NMAT,
     1 MAXIT, T1, T2, T3, T4, T5
      PRINT 16, NUMEL, NUMCP, NUMNP, NUMBC, NUMPB, NLOAD, NMAT,
     1 MAXIT, T1, T2, T3, T4, T5
   15 FORMAT (8I4, 5L2)
   16 FORMAT (//
     1 35H NUMBER OF ELEMENTS  . . . . . . . .      I8 /
     2 35H NUMBER OF CORNER POINTS . . . . . .      I8 /
     3 35H NUMBER OF NODAL POINTS  . . . . . .      I8 /
     4 35H NUMBER OF BOUNDARY CONDITIONS . . .      I8 /
     5 35H NUMBER OF DEF. BOUNDARY POINTS   .      I8 /
     6 35H NUMBER OF OF LOAD CASES . . . . . .      I8 /
     7 35H NUMBER OF DIFFERENT MATERIALS . . .      I8 /
     8 35H MAX NO OF RESIDUAL ITERATIONS . . .      I8 //
     9 35H FLAG EQUAL TYPE QUADRILATERALS. .       L8 /
     1 35H FLAG PUNCH DISPLACEMENTS. . . . .       L8 /
     2 35H FLAG PUNCH STRESSES . . . . . . . .      L8 /
     3 35H FLAG PRINT ELEMENT STRESSES . . .       L8 /
     4 35H FLAG NEW JOB FOLLOWS  . . . . . . .      L8 )
C
C     MATERIAL PROPERTIES
C
      READ 20,  (I, YM(I), PR(I), RHO(I), ALFA(I), L=1,NMAT)
      PRINT 22, (I, YM(I), PR(I), RHO(I), ALFA(I), I=1,NMAT)
   22 FORMAT (20H-MATERIAL PROPERTIES // 9H MAT. NO., 5X,
     1 15HELASTIC MODULUS, 5X, 15HPOISSON'S RATIO, 11X, 7HDENSITY,
     2 6X, 17HDILATATION COEFF.  //(I9,E20.5,2F19.5,E22.5))
   20 FORMAT (I4,4F10.3)
      READ 24,  (NPB(I), I=1,NUMPB)
      PRINT 26, (NPB(I), I=1,NUMPB)
   24 FORMAT (20I4)
   26 FORMAT (25H-DEFINING BOUNDARY POINTS  //(20I5))
C
C     ELEMENT ARRAY
C
      READ 30,  (N, (NP(N,I),I=1,8), MAT(N), TH(N), L=1,NUMEL)
      PRINT 31
      DO 150  N = 1,NUMEL
      IF (MAT(N).LE.0)  MAT(N) = 1
      IF (TH(N).LE.0.)  TH(N) = 1.
      QD = NP(N,7).GT.0
```

```
      IF (QD)        PRINT 32, QUDR,N,(NP(N,I),I=1,8),MAT(N),TH(N)
  150 IF (.NOT.QD) PRINT 33, TRNG,N,(NP(N,I),I=1,6),MAT(N),TH(N)
   30 FORMAT (10I4,F10.3)
   31 FORMAT (14H1ELEMENT ARRAY //4X, 7HELEMENT, 10X, 1HI, 5X, 1HJ,
     1 5X, 1HK, 5X, 1HL, 5X, 1HM, 5X, 1HN, 5X, 1HO, 5X, 1HP,
     2 4X, 8HMAT.TYPE, 5X,9HTHICKNESS /1X)
   32 FORMAT (2A6,I4,8I6,I11,F14.4)
   33 FORMAT (A6,A5,I4,1X,6I6,12X,I11,F14.4)
C
C     NODAL POINT COORDINATES
C
      DO 200  N = 1,NUMNP
      XORD(N) = TEST
  200 YORD(N) = TEST
      PRINT 35
   35 FORMAT (25H1CORNER POINT COORDINATES // 6H POINT, 7X, 5HX-CRD,
     1 7X, 5HY-CRD /1X)
      DO 220  M = 1,NUMCP
      READ 36,  N, XORD(N), YORD(N)
  220 PRINT 38, N, XORD(N), YORD(N)
   36 FORMAT (I4, 2F8.4)
   38 FORMAT (I6, 2F12.4)
C
C     BOUNDARY CONDITIONS
C
      DO 240  N = 1,MAXBC
  240 BANGLE(N) = 0.
      J = 0
      NSKEWB = 0
      PRINT 45
   45 FORMAT (20H-BOUNDARY CONDITIONS  //
     1 6H POINT, 3X, 3HTAG, 6X, 6HBANGLE /1X)
      DO 250  N = 1,NUMBC
      READ 50,  M, L, ANGLE
      PRINT 52, M, L, ANGLE
      K2 = 2*M
      K1 = K2 - 1
      IF (L-1)  242,244,246
  242 J = J + 2
      NEBC(J) = K2
      NEBC(J-1) = K1
      GO TO 250
  244 J = J + 1
      NEBC(J) = K1
      GO TO 250
  246 J = J + 1
      NEBC(J) = K2
      IF (ANGLE.NE.0.)  NSKEWB = 1
      BANGLE(J) = ANGLE/57.29578
  250 CONTINUE
   50 FORMAT (2I4,F10.3)
   52 FORMAT (2I6,F12.5)
      NEQBC = J
      NEQ = 2*NUMNP
      IF (NUMEL.GT.MAXEL)  GO TO 1000
  260 IF (NUMNP.GT.MAXNP)  GO TO 1010
```

```
  270 IF (NEQBC.GT.MAXBC)  GO TO 1020
C
C     ******************************************************************
C     DETERMINATION OF BAND WIDTH
C     ******************************************************************
C
  280 K = 0
      DO 320  N = 1,NUMEL
      DO 320  I = 1,7
      K1 = NP(N,I)
      IF (K1.LE.0)  GO TO 320
      II = I + 1
      DO 300  J = II,8
      K2 = NP(N,J)
      IF (K2.LE.0)  GO TO 300
      M = IABS(K2-K1)
      IF (M.GT.K)  K = M
      IF (M.LE.MAXPD)    GO TO 300
      PRINT 60, MAXPD, N
  300 CONTINUE
  320 CONTINUE
      IBANDW = 2*K + 2
      PRINT 62, IBANDW
   60 FORMAT (31HOMAX. NODAL POINT DIFFERENCE OF I5,
     1 21H  EXCEEDED, ELEMENT = I5)
   62 FORMAT (13HOBAND WIDTH = I5)
C
C     ******************************************************************
C     COMPUTATION OF CENTROID COORDINATES FOR QUADRILATERALS
C     AND CHECK FOR INPUT MESH ERRORS
C     ******************************************************************
C
      DO 400  N = 1,NUMEL
      IF (NP(N,7).LE.0)  GO TO 360
      DO 340  I = 1,4
      J = IPERM4(I)
      K1 = NP(N,I)
      K2 = NP(N,J)
      X(I) = XORD(K1)
      Y(I) = YORD(K1)
      IF (X(I).EQ.TEST.OR.Y(I).EQ.TEST)  GO TO 1040
      M = NP(N,I+4)
      XORD(M) = 0.5*(XORD(K1)+XORD(K2))
  340 YORD(M) = 0.5*(YORD(K1)+YORD(K2))
      X1 = X(1)
      Y1 = Y(1)
      X3 = X(3)
      Y3 = Y(3)
      X24 = X(2) + X(4)
      Y24 = Y(2) + Y(4)
      X124 = (X1+X24)/3.
      X234 = (X3+X24)/3.
      Y124 = (Y1+Y24)/3.
      Y234 = (Y3+Y24)/3.
      A124 = (X(2)-X1)*(Y(4)-Y1) - (X(4)-X1)*(Y(2)-Y1)
      A234 = (X(4)-X3)*(Y(2)-Y3) - (X(2)-X3)*(Y(4)-Y3)
```

```
      AREA = A124 + A234
      IF (A124.LT.0..OR.A234.LT.0.)  GO TO 1050
      R1 = A124/AREA
      R2 = A234/AREA
      XCENT(N) = X124*R1 + X234*R2
      YCENT(N) = Y124*R1 + Y234*R2
      GO TO 400
  360 DO 380  I = 1,3
      J = IPERM(I)
      K1 = NP(N,I)
      K2 = NP(N,J)
      X(I) = XORD(K1)
      Y(I) = YORD(K1)
      IF (X(I).EQ.TEST.OR.Y(I).EQ.TEST)  GO TO 1040
      M = NP(N,I+3)
      XORD(M) = 0.5*(XORD(K1)+XORD(K2))
  380 YORD(M) = 0.5*(YORD(K1)+YORD(K2))
      A2 = X(3)-X(1)
      A3 = X(2)-X(1)
      B2 = Y(3)-Y(1)
      B3 = Y(2)-Y(1)
      AREA = A3*B2-B3*A2
      IF (AREA.LT.0.)  GO TO 1050
  400 CONTINUE
      IF (IFLAG.NE.0)  STOP
      IF (IFLAG.EQ.0)  GO TO 420
      IF (.NOT.T5)  STOP
      GO TO 100
  420 REWIND 8
      WRITE (8)  (XORD(N),N=1,NUMNP),(XCENT(N),N=1,NUMEL),
     1           (YORD(N),N=1,NUMNP),(YCENT(N),N=1,NUMEL)
      WRITE (8) (MAT(N),N=1,NUMEL)
      WRITE (8) (TH(N), N=1,NUMEL)
C
C     ******************************************************************
C     COMPUTATION OF ELEMENT STIFFNESSES
C     ******************************************************************
C
      L = 0
      NTRACK = 0
      DO 450  I = 1,256
  450 DUM(I) = 0.
      DO 700  N = 1,NUMEL
      M = MAT(N)
      IF (NP(N,7).LE.0)  GO TO 600
C
C     QUADRILATERAL
C
      IF (T1.AND.L.GT.0)  GO TO 550
      L = L + 1
      DO 520  I = 1,4
      K = NP(N,I)
      X(I) = XORD(K)
  520 Y(I) = YORD(K)
      X(5) = XCENT(N)
      Y(5) = YCENT(N)
```

```
      THICK = TH(N)
      XU = PR(M)
      E =  YM(M)
      CALL STQLAD
  550 CALL WRDISK (NTRACK,S11,256)
      NTRACK = NTRACK + 1
      CALL WRDISK (NTRACK,S21,260)
      NTRACK = NTRACK + 1
      GO TO 700
C
C     TRIANGLE
C
  600 DO 620  I = 1,3
      J = IPERM(I)
      L = IPERM(J)
      K1 = NP(N,I)
      K2 = NP(N,J)
      A(L) = XORD(K2)-XORD(K1)
  620 B(L) = YORD(K1)-YORD(K2)
      AREA = A(3)*B(2)-A(2)*B(3)
      ET = YM(M)
      NU = PR(M)
      THIK = TH(N)
      CALL STLST6
      CALL WRDISK (NTRACK,ST,256)
      NTRACK = NTRACK + 2
  700 CONTINUE
      J = (NTRACK+38)/40
      NTRA = 40*J
  800 RETURN
C
C     ******************************************************************
C     ERROR EXITS
C     ******************************************************************
C
 1000 PRINT 1001
 1001 FORMAT (30HOMAX. NO. OF ELEMENTS EXCEEDED)
      IFLAG = 1
      GO TO 260
 1010 PRINT 1011
 1011 FORMAT (34HOMAX. NO. OF NODAL POINTS EXCEEDED)
      IFLAG = 1
      GO TO 270
 1020 PRINT 1021
 1021 FORMAT (33HOMAX. NO. OF CONSTRAINTS EXCEEDED)
      IFLAG = 1
      GO TO 280
 1040 PRINT 1041, K1
 1041 FORMAT (28HOMISSING COORDINATE, POINT =  I5)
      IFLAG = 1
      GO TO 400
 1050 PRINT 1051, N
 1051 FORMAT (34HONEGATIVE TRIANGLE AREA, ELEMENT = I5)
      IFLAG = 1
      GO TO 400
      END
```

```
$IBFTC QUAD    DECK
      SUBROUTINE STQUAD
C
C     ******************************************************************
C     THIS SUBROUTINE ASSEMBLES AND CONDENSES THE STIFFNESS MATRIX OF
C     A QUADRILATERAL FORMED BY FOUR LST ELEMENTS
C     ******************************************************************
C
      COMMON /QUADRG/ S11(16,16), S21(10,26), X(5), Y(5), E, XU, THICK
      COMMON /STFARG/ ST(16,16), B(3), A(3), AREA, ET, NU, THIK
      REAL NU
      DIMENSION S(26,26), LOC(12,4), IPERM4(4), DUM(1)
      EQUIVALENCE (DUM,S)
      DATA IPERM4/2,3,4,1/
      DATA LOC / 1, 2, 3, 4,17,18,  9,10,21,22,19,20,
     1           3, 4, 5, 6,17,18, 11,12,23,24,21,22,
     2           5, 6, 7, 8,17,18, 13,14,25,26,23,24,
     3           7, 8, 1, 2,17,18, 15,16,19,20,25,26 /
C
      ET = E
      THIK = THICK
      NU = XU
      DO 100  I = 1,676
  100 DUM(I) = 0.
C
C     ASSEMBLY OF FOUR SUBTRIANGLE STIFFNESSES
C
      DO 160  N = 1,4
      M = IPERM4(N)
      A(1) = X(5)-X(M)
      A(2) = X(N)-X(5)
      A(3) = X(M)-X(N)
      B(1) = Y(M)-Y(5)
      B(2) = Y(5)-Y(N)
      B(3) = Y(N)-Y(M)
      AREA = A(3)*B(2)-A(2)*B(3)
      CALL STLST6
      DO 160  I = 1,12
      K = LOC(I,N)
      DO 160  J = 1,12
      L = LOC(J,N)
  160 S(K,L) = S(K,L) + ST(I,J)
C
C     CONDENSATION OF INTERNAL NODAL POINTS
C
      DO 200  N = 1,10
      K = 26 - N
      L = K + 1
      PIVOT = S(L,L)
      DO 200  J = 1,K
      C = S(L,J)/PIVOT
      S(L,J) = C
      DO 200  I = J,K
      S(I,J) = S(I,J) - C*S(I,L)
  200 S(J,I) = S(I,J)
      DO 220  I = 1,16
```

```
      DO 220  J = 1,16
  220 S11(I,J) = S(I,J)
      DO 250  I = 1,10
      DO 250  J = 1,26
  250 S21(I,J) = S(I+16,J)
      RETURN
      END
```

```
$IBFTC STL6    DECK
      SUBROUTINE STLST6
C
C     *****************************************************************
C     ELEMENT STIFFNESS SUBROUTINE
C     LINEARLY VARYING STRAIN TRIANGLE WITH SIX NODAL POINTS
C     LINEAR ELASTIC ISOTROPIC MATERIAL
C     *****************************************************************
C
      COMMON /STFARG/ ST(16,16), B(3), A(3), AREA, ET, NU, THIK
      REAL NU, NUH
      DIMENSION CX1(3), CX2(3), CX3(3), CY1(3), CY2(3), CY3(3),
     1 U(3,6), V(3,6), UV(3,6,2), BA(3,2), IPERM(3)
      EQUIVALENCE (BA,B),  (UV,U), (UV(19),V)
      DATA IPERM /2,3,1/
      NUH = 0.5*(1.-NU)
      ER = ET/(1.-NU*NU)
      COMM = ER*THIK/(24.*AREA)
      DO 150  L = 1,3
      L1 = IPERM(L)
      L2 = IPERM(L1)
      L3 = L + 3
      DO 150  N = 1,2
      D0 = BA(L,N)
      D1 = BA(L1,N)
      UV(L ,L,N) = 3.*D0
      UV(L1,L,N) =   -D0
      UV(L2,L,N) =   -D0
      UV(L ,L3,N) = 4.*D1
      UV(L1,L3,N) = 4.*D0
  150 UV(L2,L3,N) = 0.
      DO 300  I = 1,6
      DO 200  L = 1,3
      CX1(L) = (U(1,I)+U(2,I)+U(3,I)+U(L,I))*COMM
      CY2(L) = (V(1,I)+V(2,I)+V(3,I)+V(L,I))*COMM
      CX2(L) = CX1(L)*NU
      CY1(L) = CY2(L)*NU
      CX3(L) = CY2(L)*NUH
  200 CY3(L) = CX1(L)*NUH
      K2 = 2*I
      K1 = K2 - 1
      DO 300  J = 1,6
      L2 = 2*J
      L1 = L2 - 1
      X1 = 0.
      X2 = 0.
      X3 = 0.
      X4 = 0.
      DO 280  K = 1,3
      X = U(K,J)
      Y = V(K,J)
      X1 = X1 + CX1(K)*X + CX3(K)*Y
      X2 = X2 + CX2(K)*Y + CX3(K)*X
      X3 = X3 + CY1(K)*X + CY3(K)*Y
  280 X4 = X4 + CY2(K)*Y + CY3(K)*X
      ST(K1,L1) = X1
```

```
      ST(L1,K1) = X1
      ST(K1,L2) = X2
      ST(L2,K1) = X2
      ST(K2,L1) = X3
      ST(L1,K2) = X3
      ST(K2,L2) = X4
  300 ST(L2,K2) = X4
      RETURN
      END
```

```
$ORIGIN        ALPHA,SYSUT3
$IBFTC FRMK    DECK,LIST
      SUBROUTINE FORMK
C
C     ******************************************************************
C     FORMK ASSEMBLES THE COMPLETE STIFFNESS MATRIX
C     ******************************************************************
C
      COMMON
     1 NUMEL, NUMCP, NUMNP, NUMBC, NLOAD, MAXIT, NN, MM, NEQBC
      COMMON /CELMAR/  NP(350,8), NEBC(250), BANGLE(250)
      COMMON /CFORMK/  ST(16,16), NEL(150), S(17600)
      DIMENSION NE(150,20), NEB(20), IND(20)
      EQUIVALENCE (NE,NEL)
C
C
C     ******************************************************************
C     FIND ALL ELEMENTS CONTRIBUTING TO EACH BLOCK OF 'NPBL'
C     NODAL POINTS
C     ******************************************************************
C
      NDIMS = 17600
      NEQBL = NDIMS/MM
      NPBL = NEQBL/2
      MM1 = MM + 1
      MM2 = MM + MM
      NB = 1 + (NUMNP-1)/NPBL
      DO 120  I = 1,NB
      NEB(I) = 0
      DO 120  N = 1,150
  120 NE(N,I) = 0
      DO 140  N = 1,NUMEL
      DO 130  I = 1,NB
  130 IND(I) = 0
      DO 140  I = 1,8
      K = NP(N,I)
      IF (K.LE.0)  GO TO 140
      M = 1 + (K-1)/NPBL
      IF (IND(M).NE.0)  GO TO 140
      NEB(M) = NEB(M) + 1
      L = NEB(M)
      NE(L,M) = N
      IND(M) = 1
  140 CONTINUE
      IF (NB.LE.1)  GO TO 160
      REWIND 9
      DO 150  M = 2,NB
      L = NEB(M)
  150 WRITE (9)  L, (NE(I,M), I=1,L)
      REWIND 9
C
C     ******************************************************************
C     GENERATION OF COMPLETE STIFFNESS MATRIX
C     ******************************************************************
C
  160 LL = NEB(1)
```

```
      REWIND 1
      DO 400  M = 1,NB
C
C     ASSEMBLE 'M-TH' BLOCK
C
      IF (M.GT.1)  READ (9)  LL, (NEL(I),I=1,LL)
      N1 = 1 + (M-1)*NPBL
      N2 = MIN0 (N1+NPBL-1,NUMNP)
      NPTB = N2 - N1 + 1
      NEQB = 2*NPTB
      NE1 = 2*N1 - 1
      NE2 = 2*N2
      N = MM*NEQB
      DO 180  I = 1,N
  180 S(I) = 0.
      DO 220  NV = 1,LL
      N = NEL(NV)
      NTRACK = 2*(N-1)
      CALL RCDISK (NTRACK,ST,256)
      DO 220  J = 1,8
      L = NP(N,J)
      IF (L.LT.N1.OR.L.GT.N2)  GO TO 220
      NC = MM2*(L-N1) + 1
      JJ = J + J - 1
      DO 200  I = 1,8
      K = NP(N,I)
      IF (K.LE.0.OR.K.GT.L)  GO TO 200
      L1 = NC + 2*(L-K)
      L2 = L1 + MM
      II = I + I - 1
      S(L1)   = S(L1)   + ST(II  ,JJ  )
      S(L2)   = S(L2)   + ST(II+1,JJ+1)
      S(L2+1) = S(L2+1) + ST(II  ,JJ+1)
      IF (K.EQ.L)  GO TO 200
      S(L1-1) = S(L1-1) + ST(II+1,JJ  )
  200 CONTINUE
  220 CONTINUE
C
C     IMPOSE BOUNDARY CONDITICNS
C
      DO 360  I = 1,NEQBC
      N = NEBC(I)
      NC = N - NE1 + 1
      IF (NC.LE.0.OR.NC.GT.NEQB)  GO TO 300
      L = MM*(NC-1) + 1
      PHI = BANGLE(I)
      IF (PHI.EQ.0.)  GO TO 270
      CN = COS(PHI)
      SN = SIN(PHI)
      L1 = L - MM
      S(L1) = S(L1)*CN*CN + 2.*S(L+1 )*SN*CN + S(L)*SN*SN
      L2 = L + 1
      DO 260  J = 3,MM
      L1 = L1 + 1
      L2 = L2 + 1
  260 S(L1) = S(L1)*CN + S(L2)*SN
```

```
  270 S(L) = 1.
      DO 280  J = 2,MM
      L = L + 1
  280 S(L) = 0.
  300 NCMIN = MAX0 (NC+1,1)
      NCMAX = MIN0 (NC+MM-1,NEQB)
      IF (NCMAX.LT.NCMIN)  GO TO 360
      LL = MM1*NCMIN - MM2 - NC
      IF (PHI.EQ.0.)  GO TO 340
      L = LL + 1
      K = NCMAX - 1
      IF (NCMAX.EQ.NEQB)  K = NCMAX
      DO 330  J = NCMIN,K
      L = L + MM1
  330 S(L) = S(L)*CN + S(L-1)*SN
  340 L = LL
      DO 350  J = NCMIN,NCMAX
      L = L + MM1
  350 S(L) = 0.
  360 CONTINUE
C
C     WRITE BLOCK OF EQUATIONS ONTO TAPE
C
      L1 = 1
      L2 = MM
      DO 380  N = 1,NEQB
      WRITE (1)  (S(I), I=L1,L2)
      L1 = L1 + MM
  380 L2 = L2 + MM
  400 CONTINUE
      RETURN
      END
```

```
$ORIGIN        ALPHA,SYSUT3
$IBFTC LCAD    DECK,LIST
      SUBROUTINE LDINPT
C
C     ******************************************************************
C     LDINPT INPUTS LOAD CASE AND REDUCES THERMAL, GRAVITY AND
C     IN-PLANE DISTRIBUTED LOADS TO KINEMATICALLY EQUIVALENT
C     NODAL POINT FORCES
C     ******************************************************************
C
      COMMON
     1 NUMEL, NUMCP, NUMNP, NUMBC, NLOAD, MAXIT, NEQ,
     2 IBANDW, NEQBC, NRUN, NTRA, NSKEWB, LNQ(6,4),
     3 T1, T2, T3, T4, T5, THERL
      COMMON /CMATPR/  YM(6), PR(6), RHO(6), ALFA(6)
      COMMON /CELMAR/  NP(350,8), NEBC(250), BANGLE(250)
      COMMON /CNTARG/  NUMPB, NSK, PGRAPH, IGRTAG(6), SPACNG(6),
     1 GRHEAD(2,6), NPB(50), NELSKP(50)
      COMMON /CLOAD /  XLOAD(1050), YLOAD(1050), ELOAD(350,26),
     1 XORD(1050), YORD(1050), XCENT(350), YCENT(350), NEI(350),
     2 MAT(350), TH(350), DT(350,5), S21(10,16), S22(10,10), R(26)
      COMMON /CTHERM/ B(3), A(3), DLT(3), COMM, FX(6), FY(6)
      DIMENSION  HEAD(13), CF(3,3), P(3,2), PC(3,2), PN(3), PT(3),
     1 NOD(3), DELT(4), IPERM4(4), IPERM(3)
      EQUIVALENCE (PN,PC), (PT,PC(4))
      LOGICAL THERL, DENS
      DATA EFLAG /6HSTART /, FLAG /6HLOADNG/
      DATA IPERM /2,3,1/, IPERM4 /2,3,4,1/
      DATA CF /4.,2.,-1., 2.,16.,2., -1.,2.,4./
C
  100 READ 10, CHECK
      IF (CHECK.EQ.EFLAG)  GO TO 1000
      IF (CHECK.NE.FLAG)  GO TO 100
   10 FORMAT (13A6)
C
C     INITIALIZE
C
      REWIND 8
      REWIND 9
      READ (8)   (XORD(N),N=1,NUMNP),(XCENT(N),N=1,NUMEL),
     1           (YORD(N),N=1,NUMNP),(YCENT(N),N=1,NUMEL)
      READ (8)   (MAT(N), N=1,NUMEL)
      READ (8)   (TH(N),  N=1,NUMEL)
      THERL = .FALSE.
      DO 110  N = 1,50
  110 NELSKP(N) = 0
      DO 120  N = 1,NUMNP
      XLOAD(N) = 0.
  120 YLOAD(N) = 0.
      DO 140  N = 1,NUMEL
      NEI(N) = 0
      DO 130  I = 1,5
  130 DT(N,I) = 0.
      DO 140  I = 1,26
  140 ELOAD(N,I) = 0.
C
```

```
      NOD(3) = K2
      PRINT 50, N, I, (NOD(J),P(J,1),P(J,2),J=1,3)
   50 FORMAT (1H0,2I7,I8,2F14.4/(15X,I8,2F14.4))
      X = XORD(K1)-XORD(K2)
      Y = YORD(K2)-YORD(K1)
      DO 250  K = 1,2
      DO 250  I = 1,3
      PC(I,K) = C.
      DO 240  J = 1,3
  240 PC(I,K) = PC(I,K) + CF(I,J)*P(J,K)
  250 PC(I,K) = PC(I,K)*TH(N)/30.
      DO 270  I = 1,3
      K = NOD(I)
      XLOAD(K) = XLOAD(K) + PN(I)*Y - PT(I)*X
  270 YLOAD(K) = YLOAD(K) + PN(I)*X + PT(I)*Y
  280 CONTINUE
C
C     ********************************************************************
C     GRAVITY LOADS
C     ********************************************************************
C
  300 IF (.NOT.DENS)  GO TO 400
      DO 380  N = 1,NUMEL
      M = MAT(N)
      IF (NP(N,7).LE.0)  GO TO 360
      NEI(N) = 1
      DO 350  I = 1,4
      J = IPERM4(I)
      K1 = NP(N,I)
      K2 = NP(N,J)
      A3 = XORD(K2)-XORD(K1)
      B3 = YORD(K1)-YORD(K2)
      A2 = XORD(K1)-XCENT(N)
      B2 = YCENT(N)-YORD(K1)
      AREA = A3*B2-A2*B3
      COMM = RHO(M)*AREA/6.
      DO 350  L = 4,6
      K = 2*LNC(L,I)
  350 ELOAD(N,K) = ELOAD(N,K) - COMM
      GO TO 380
  360 K1 = NP(N,1)
      K2 = NP(N,2)
      K3 = NP(N,3)
      A3 = XORD(K2)-XORD(K1)
      B3 = YORD(K1)-YORD(K2)
      A2 = XORD(K1)-XORD(K3)
      B2 = YORD(K3)-YORD(K1)
      AREA = A3*B2 - A2*B3
      COMM = RHO(M)*AREA/6.
      DO 370  L = 4,6
      K = NP(N,L)
  370 YLOAD(K) = YLOAD(K) - COMM
  380 CONTINUE
C
C     ********************************************************************
C     THERMAL LOADS
```

```
C      CONTROL CARD AND GRAPH SPECIFICATIONS
C
       READ 10,  HEAD
       READ 20, NPLD, NELD, NTLD, DENS
   20 FORMAT (3I4,L4)
       PRINT 25, NRUN, HEAD, NPLD, NELD, NTLD, DENS
   25 FORMAT (14H1LOAD CASE NO.  I5 // 1X, 13A6 //
     1 35H NO. OF NODAL POINT LOAD CARDS . .    I8 /
     2 35H NO. OF ELEMENT LOAD CARDS . . . .    I8 /
     3 35H NO. OF THERMALLY LOADED ELEMENTS     I8 /
     4 35H FLAG FOR GRAVITY LOADING  . . . .    L8)
       READ 28, IGRTAG, NSK
   28 FORMAT (20I4)
       PRINT 30, (GRHEAD(1,N),GRHEAD(2,N), IGRTAG(N), N=1,6)
   30 FORMAT (7H- GRAPH, 13X, 3HTAG/ 13X, 18H(1=PLOT,0=NO PLOT) /
     1 (1X,2A6,I9))
       IF (NSK.LE.0)  GO TO 150
       READ 28,  (NELSKP(I),I=1,NSK)
       PRINT 31, (NELSKP(I),I=1,NSK)
   31 FORMAT (20H0SKIPPED ELEMENTS = 25I4 /(20X,25I4))
C
C      ****************************************************************
C      CONCENTRATED NODAL POINT FORCES
C      ****************************************************************
C
  150 IF (NPLD.LE.0)  GO TO 200
       PRINT 33
   33 FORMAT (26H-CONCENTRATED NODAL FORCES //
     1 6H POINT, 8X, 6HX-LOAD, 8X, 6HY-LOAD /1X)
       DO 180  L = 1,NPLD
       READ 35,  N, XLOAD(N), YLOAD(N)
  180 PRINT 38, N, XLOAD(N), YLOAD(N)
   35 FORMAT (I4, 2F8.3)
   38 FORMAT (I6, 2F14.5)
C
C      ****************************************************************
C      ELEMENT SIDE LOADING (NORMAL PRESSURE AND SURFACE SHEAR)
C      ****************************************************************
C
  200 IF (NELD.LE.0)  GO TO 300
       PRINT 40
   40 FORMAT (20H-ELEMENT SIDE FORCES  // 8H ELEMENT, 3X, 4HSIDE,
     1 4X, 4HNODE, 3X, 11HN. PRESSURE, 3X, 11HSURF. SHEAR)
       DO 280  L = 1,NELD
       READ 45, N, I, P
   45 FORMAT (2I4, 6F8.3)
       K1 = NP(N,I)
       IF (NP(N,7).LE.0)  GO TO 220
       J = IPERM4(I)
       M = NP(N,I+4)
       GO TO 230
  220 J = IPERM(I)
       M = NP(N,I+3)
  230 K2 = NP(N,J)
       NOD(1) = K1
       NOD(2) = M
```

```
C     ******************************************************************
C
  400 IF (NTLD.LE.0)  GO TO 500
      PRINT 60
   60 FORMAT (19H-THERMAL INCREMENTS // 8H ELEMENT, 6X, 8HCORNER I,
     1 6X, 8HCORNER J, 6X, 8HCORNER K, 6X, 8HCORNER L /1X)
      DO 480  L = 1,NTLD
      THERL = .TRUE.
      READ 65,  N, DELT
      PRINT 68, N, DELT
   65 FORMAT (I4,4F10.3)
   68 FORMAT (I8,4F14.4)
      M = MAT(N)
      COMM = ALFA(M)*YM(M)*TH(N)/(6.*(1.-PR(M)))
      DO 420  I = 1,4
  420 DT(N,I) = DELT(I)
      IF (NP(N,7).LE.0)  GO TO 450
      NEI(N) = 1
      AVDT = (DELT(1)+DELT(2)+DELT(3)+DELT(4))/4.
      DT(N,5) = AVDT
      DLT(3) = AVDT
      DO 440  I = 1,4
      J = IPERM4(I)
      K1 = NP(N,I)
      K2 = NP(N,J)
      A(1) = XCENT(N)-XORD(K2)
      A(2) = XORD(K1)-XCENT(N)
      A(3) = XORD(K2)-XORD(K1)
      B(1) = YORD(K2)-YCENT(N)
      B(2) = YCENT(N)-YORD(K1)
      B(3) = YORD(K1)-YORD(K2)
      DLT(1) = DELT(I)
      DLT(2) = DELT(J)
      CALL THERLD
      DO 440  J = 1,6
      K = 2*LNC(J,I)
      ELOAD(N,K-1) = ELOAD(N,K-1) + FX(J)
  440 ELOAD(N,K)   = ELOAD(N,K)   + FY(J)
      GO TO 480
  450 DO 460  I = 1,3
      J = IPERM(I)
      K1 = NP(N,I)
      K2 = NP(N,J)
      M = IPERM(J)
      A(M) = XORD(K2)-XORD(K1)
      B(M) = YORD(K1)-YORD(K2)
  460 DLT(I) = DELT(I)
      CALL THERLD
      DO 470  J = 1,6
      K = NP(N,J)
      XLOAD(K) = XLOAD(K) + FX(J)
  470 YLOAD(K) = YLOAD(K) + FY(J)
  480 CONTINUE
      WRITE (9)  ((DT(N,I),I=1,5),N=1,NUMEL)
C
C     ******************************************************************
```

```
C     CONDENSATION OF INTERNAL QUADRILATERAL LOADS
C     ******************************************************************
C
  500 DO 600  N = 1,NUMEL
      IF (NEI(N).LE.0)  GO TO 600
      NTRACK = 2*N - 1
      CALL RDDISK (NTRACK,S21,260)
      DO 510  I = 1,26
  510 R(I) = ELOAD(N,I)
      DO 530  II = 1,10
      K = 26 - II
      L = K + 1
      M = L - 16
      DO 520  I = 1,K
  520 R(I) = R(I) - S21(M,I)*R(L)
  530 R(L) = R(L)/S22(M,M)
      CALL WRDISK (NTRACK,S21,286)
C
C     TRANSFER TO EXTERNAL NODE FORCE VECTOR
C
      DO 540  I = 1,8
      J = 2*I
      K = NP(N,I)
      XLOAD(K) = XLOAD(K) + R(J-1)
  540 YLOAD(K) = YLOAD(K) + R(J)
  600 CONTINUE
      WRITE (9) (NEI(N),N=1,NUMEL)
C
C     ******************************************************************
C     PRINT COMPLETE FORCE VECTOR, IMPOSE B.C. AND STORE ON TAPE
C     ******************************************************************
C
      REWIND 2
      PRINT 80, (N,XLOAD(N),YLOAD(N), N=1,NUMNP)
   80 FORMAT (19H1NODAL FORCE VECTOR  // 2(6H POINT, 8X,
     1 6HX-LOAD, 8X, 6HY-LOAD, 12X) // (I6,2F14.5,I18,2F14.5))
      DO 700  M = 1,NEBC
      N = NEBC(M)
      L = (N+1)/2
      PHI = BANGLE(M)
      IF (PHI.NE.0.)  XLOAD(L) = XLOAD(L)*COS(PHI) + YLOAD(L)*SIN(PHI)
      K = 2*L - N
      IF (K.EQ.1)  XLOAD(L) = 0.
  700 IF (K.EQ.0)  YLOAD(L) = 0.
      WRITE (2)  (XLOAD(N),YLOAD(N), N=1,NUMNP)
      RETURN
C
C     ERROR EXIT
C
 1000 PRINT 1001
 1001 FORMAT (46H0START CARD FOUND WHEN LOOKING FOR LOADNG CARD)
      STOP
      END
```

```
$IBFTC THLD    DECK,LIST
      SUBROUTINE THERLD
C
C     ******************************************************************
C     THIS SUBROUTINE COMPUTES RESTRAINT THERMAL FORCES FOR
C     A SIX NODAL POINT TRIANGLE
C     ******************************************************************
C
      COMMON /CTHERM/ B(3), A(3), DLT(3), COMM, FX(6), FY(6)
      DIMENSION IPERM(3)
      DATA IPERM /2,3,1/
      DO 120  I = 1,12
  120 FX(I) = 0.
      DO 150  I = 1,3
      J = IPERM(I)
      K = IPERM(J)
      DT3 = DLT(K)
      DT5 = DLT(I) + 2.*DLT(J) + DT3
      DT6 = DLT(J) + 2.*DLT(I) + DT3
      X = B(K)*COMM
      Y = A(K)*COMM
      FX(K) = X*DT3
      FY(K) = Y*DT3
      FX(J+3) = FX(J+3) + X*DT5
      FY(J+3) = FY(J+3) + Y*DT5
      FX(K+3) = FX(K+3) + X*DT6
  150 FY(K+3) = FY(K+3) + Y*DT6
      RETURN
      END
```

```
$ORIGIN        ALPHA,SYSUT3
$IBFTC SOLV    LIST,DECK
      SUBROUTINE SOLVE
C
C     *****************************************************************
C     SOLVE OBTAINS NODAL POINT DISPLACEMENTS FROM THE LARGE
C     CAPACITY BAND SOLVER BIGSOL
C     *****************************************************************
C
      COMMON
     1 NUMEL, NUMCP, NUMNP, NUMBC, NLOAD, MAXIT, NEQ,
     2 IBANDW, NEQBC, NRUN, NTRA, NSKEWB
      COMMON /CELMAR/  NP(350,8), NEBC(250), BANGLE(250)
      COMMON /BANARG/ NN, MM, MAXIR, TOLER, NTR, NITER, WS(15500)
      DIMENSION R(2100)
      EQUIVALENCE (WS,R)
      MM = IBANDW
      NN = NEQ
      MAXIR = MAXIT
      TOLER = 0.001
      NTR = NTRA
      KKK = 0
      IF (NRUN.GT.1)  KKK = 2
      CALL BIGSOL (KKK)
C
C     TRANSFORM SKEW DISPLACEMENTS, IF ANY, TO THE X Y GLOBAL SYSTEM
C
      IF (NSKEWB.LE.0)  GO TO 200
      DO 150  M = 1,NEQBC
      N = NEBC(M)
      PHI = BANGLE(M)
      IF (PHI.EQ.0.)  GO TO 150
      L = N - 1
      R(N) = R(L)*SIN(PHI)
      R(L) = R(L)*COS(PHI)
  150 CONTINUE
C
C     PUT DISPLACEMENTS ON TAPE
C
  200 REWIND 2
      WRITE (2)  (R(I),I=1,NN)
      RETURN
      END
```

```
$IBFTC BGSL     DECK,LIST
      SUBROUTINE BIGSOL(KKK)
C
C     ******************************************************************
C     LINEAR EQUATION SOLVER FOR LARGE SYMMETRIC BAND MATRICES
C     ******************************************************************
C     CARLOS A. FELIPPA, JULY 1966.
C
C     INPUT
C
C     NN     = NUMBER OF EQUATIONS.
C     MM     = HALF BAND WIDTH.
C     MAXIT  = MAX. NO. OF ITERATIONS ON RESIDUALS.
C     TOLER  = ACCURACY TEST (VALID ONLY IF MAXIT GT 0).
C     NTR    = NUMBER OF DISK TRACK STARTING AT WHICH THE REDUCED MATRIX
C              IS STORED - TRACKS 'NTR' TO 'NTR+NREC', WHERE NREC =
C              (NN+NC-1)*NBUFF/NC, NC = 460*NBUFF/MM AND NBUFF = 5 IN
C              THIS VERSION, ARE USED FOR SUCH PURPOSE - WHEN RESIDUAL
C              ITERATIONS ARE PERFORMED, THE NEXT CYLINDER IS USED
C              TO STORE SUCCESIVE ITERATES OF THE SOLUTION VECTOR.
C     UPPER HALF BAND OF INPUT MATRIX IS READ COLUMN-WISE FROM
C     LOGICAL UNIT 1, ONE COLUMN PER RECORD.
C     INPUT VECTOR IS READ FROM LOGICAL UNIT 2.
C
C     STORAGE
C
C     WS     = WORKING SPACE OF LENGTH 'NWS' (SEE WRITE-UP) - WS CONTAINS
C       A   =  STORAGE OF AN UPPER TRIANGLE OF BAND DURING REDUCTION.
C       D   =  DOUBLE PRECISION VECTOR FOR ITERATION ON RESIDUALS.
C       R1  =  SINGLE PRECISION VECTOR, EQUIVALENT TO D.
C       F   =  STORAGE OF FIRST ROW DURING REDUCTION.
C       ND  =  INDEXING ARRAY FOR THE UPPER TRIANGLE IN A.
C       X   =  BUFFER STORAGE FOR DISK I/O.
C
C     OUTPUT
C
C     R      = SOLUTION VECTOR, STORED IN THE FIRST NN LOCATIONS OF WS.
C     NITER  = RETURNS NUMBER OF RESIDUAL ITERATIONS PERFORMED.
C
C     ARGUMENT KKK  SPECIFIES THE FOLLOWING OPTIONS
C        KKK = 0   MATRIX REDUCTION AND SUBSTITUTION OF INPUT VECTOR.
C        KKK = 1   MATRIX REDUCTION ONLY.
C        KKK = 2   SUBSTITUTION OF INPUT VECTOR ONLY.
C
C     THE LENGTH OF WS HERE CORRESPONDS TO A MAX. BAND WIDTH MM = 160
C
      COMMON /BANARG/ NN, MM, MAXIT, TOLER, NTR, NITER, WS(15500)
      DIMENSION A(1), F(1), ND(1), R(1), R1(1), X(2300)
      DOUBLE PRECISION  D(1), D1, D2, D3
      EQUIVALENCE (WS,A,R), (WS(4401),D,R1), (WS(12881),F),
     1 (WS(13041),ND), (WS(13201),X)
      LOGICAL T1
      NR = NN - 1
      NM = NN - MM
      NN1 = NN + 1
      MM1 = MM + 1
```

```
      NITER = 0
      T1 = .FALSE.
      NBUFF = 5
      NC = (460*NBUFF)/MM
      NW = NC*MM
      NREC1 = (NN+NC-2)/NC -1
      IF (KKK.GE.2)  GO TO 210
C
C     ******************************************************************
C     DECOMPOSITION OF BAND MATRIX
C     ******************************************************************
C
      DO 110  J = 1,MM
  110 ND(J) = (J*(J+1))/2
C
C     SET UP FIRST TRIANGULAR BLOCK IN A
C
      REWIND 1
      DO 130  N = 1,MM
      LC1 = ND(N) - N + 1
      LC2 = LC1 + MM - 1
  130 READ (1) (A(I),I=LC1,LC2)
      NX = 0
      NTRACK = NTR
      DO 200  N = 1,NR
C
C     TRIANGLE IS SIMULTANEOUSLY REDUCED AND SHIFTED
C     PIVOTS AND MULTIPLIERS ARE TRANSFERRED TO X
C
      MR = MIN0 (MM,NN1-N)
      JJ = NX*MM + 1
      NX = NX + 1
      PIVOT = A(1)
      X(JJ) = PIVOT
      DO 150  J = 2,MR
      L = ND(J)
  150 F(J) = A(L)
      DO 160  J = 2,MR
      C = F(J)/PIVOT
      JJ = JJ + 1
      X(JJ) = C
      L  = ND(J)
      L1 = ND(J-1) + 1
      DO 160  I = 2,J
      L = L - 1
      L1 = L1 - 1
  160 A(L1) = A(L) - C*F(I)
      IF (N.GT.NM)  GO TO 190
C
C     STORE NEXT COLUMN
C
      READ (1) (A(I),I=LC1,LC2)
  190 IF (NX.LT.NC)  GO TO 200
C
C     'NC' REDUCED ROWS ARE WRITTEN ON 'NBUFF' DISK TRACKS
C
```

```
      CALL WRDISK (NTRACK,X,NW)
      NTRACK = NTRACK + NBUFF
      NX = 0
  200 CONTINUE
      JJ = NX*MM + 1
      X(JJ) = A(1)
      CALL WRDISK (NTRACK,X,JJ)
      IF (KKK.EQ.1)  RETURN
C
C     ******************************************************************
C     SUBSTITUTION OF INPUT VECTOR
C     ******************************************************************
C
  210 REWIND 2
      READ (2)  (R(I), I=1,NN)
C
C     FORWARD REDUCTION
C
  220 NTRACK = NTR
      NX = NC
      DO 240  N = 1,NR
      MR = MINO (MM,NN1-N)
      IF (NX.LT.NC)  GO TO 230
      CALL RDDISK (NTRACK,X,NW)
      NTRACK = NTRACK + NBUFF
      NX = 0
  230 JJ = NX*MM + 1
      NX = NX + 1
      C = R(N)
      R(N) = C/X(JJ)
      I1 = N + 1
      I2 = I1 + MR - 2
      DO 240  I = I1,I2
      JJ = JJ + 1
  240 R(I) = R(I) - C*X(JJ)
      JJ = NX*MM + 1
      ALAST = X(JJ)
      IF (NX.GE.NC)  CALL RDDISK (NTRACK,ALAST,1)
      R(NN) = R(NN)/ALAST
      NTRES = ((NTRACK+39)/40)*40
C
C     BACK SUBSTITUTION
C
      NTRACK = NTRACK - NBUFF
      CALL RDDISK (NTRACK,X,NW)
      NX = NN - NREC1*NC - 1
      DO 260  L = 2,NN
      N = NN1 - L
      MR = MINO (MM,L)
      NX = NX - 1
      IF (NX.GE.0)  GO TO 250
      NTRACK = NTRACK - NBUFF
      CALL RDDISK (NTRACK,X,NW)
      NX = NC - 1
  250 JJ = NX*MM + 1
      I1 = N + 1
```

```
      I2 = I1 + MR - 2
      DO 260  I = I1,I2
      JJ = JJ + 1
  260 R(N) = R(N) - X(JJ)*R(I)
      IF (T1)  GO TO 400
      IF (MAXIT.LE.0)  RETURN
C
C     ******************************************************************
C     ITERATION ON RESIDUALS
C     ******************************************************************
C
  280 NITER = NITER + 1
      CALL WRDISK (NTRES,R,NN)
      REWIND 1
      READ (1)  (X(I),I=1,MM)
      D1 = X(1)
      D3 = R(1)
      D(1) = D1*D3
      DO 350  N = 2,NN
      MR = MINO(N,MM)
      READ (1)  (X(I),I=1,MM)
      D1 = X(1)
      D3 = R(N)
      D(N) = D1*D3
      K = N
      DO 350  J = 2,MR
      K = K - 1
      D1 = X(J)
      D2 = R(K)
      D(K) = D(K) + D1*D3
  350 D(N) = D(N) + D1*D2
      REWIND 2
      READ (2)  (R(I), I=1,NN)
      DO 360  N = 1,NN
      D1 = R(N)
  360 R(N) = D1 - D(N)
      T1 = .TRUE.
      GO TO 220
  400 CALL RDDISK (NTRES,R1,NN)
C
C     CHECK ACCURACY OF SOLUTION
C
      RNORM = 0.
      DELR = 0.
      DO 450  N = 1,NN
      RNORM = RNORM + R1(N)**2
      DELR  = DELR  + R (N)**2
  450 R(N) = R1(N) + R(N)
      EPS = SQRT (DELR/RNORM)
      IF (EPS.LE.TOLER) RETURN
      IF (NITER.LT.MAXIT) GO TO 280
      PRINT 99, NITER, EPS
   99 FORMAT (35H0SPECIFIED ACCURACY NOT ATTAINED IN I5, 24H ITERATIONS,
     1 LAST EPS = E14.4)
      RETURN
      END
```

```
$ORIGIN         ALPHA,SYSUT3
$IBFTC STRS     DECK,LIST
      SUBROUTINE STRESS
C
C     ******************************************************************
C     STRESS COMPUTES AND PRINTS ELEMENT AND NODAL POINT STRESSES
C     ******************************************************************
C
      COMMON
     1 NUMEL, NUMCP, NUMNP, NUMBC, NLOAD, MAXIT, NEQ,
     2 IBANDW, NEQBC, NRUN, NTRA, NSKEWB, LNQ(6,4),
     3 T1, T2, T3, T4, T5, THERL
      COMMON /CELMAR/  NP(350,8), NEBC(250), BANGLE(250)
      COMMON /CMATPR/  YM(6), PR(6), RHO(6), ALFA(6)
      COMMON /CSTRES/
     1 SIG(1050,7), SIGC(350,7), SIGM(1050,3), COUNT(1050),
     2 XORD(1050), YORD(1050), XCENT(350), YCENT(350),
     3 S21(10,26), R(26)
      COMMON /CTRIST/ ER, G, NU, CODIL, DELT(3), B(3), A(3),
     1 RX(6), RY(6), ESIG(6,3)
      COMMON /CNTARG/  NUMPB, NSK, PGRAPH, IGRTAG(6), SPACNG(6),
     2 GRHEAD(2,6), NPH(50), NELSKP(50)
      DIMENSION  DSX(1050), DSY(1050), ANGLE(1050), ANGLEC(350),
     1 MAT(350), NEI(350), DT(350,3), D(26), SIGQ(9,3), COEF(4),
     2 FMAX(10), IPERM(3), IPERM4(4)
      EQUIVALENCE  (DSX,SIG(3151)), (DSY,DSX(1051)), (DT,DSY(1051)),
     1 (MAT,SIGC(1051)), (NEI,MAT(351)), (ANGLE,SIG(6301)),
     2 (ANGLEC,SIGC(2101))
      REAL NU
      LOGICAL T1, T2, T3, T4, T5, THERL, PGRAPH
      DATA IPERM /2,3,1/, IPERM4 /2,3,4,1/
      DATA COEF /0.50,0.50,0.25,1.00/
      DATA FMAX /10.,15.,20.,25.,30.,40.,50.,60.,80.,100./
C
C     ******************************************************************
C     PRINTOUT OF DISPLACEMENTS
C     ******************************************************************
C
      REWIND 2
      READ (2)  (DSX(I),DSY(I),I=1,NUMNP)
      PRINT 15
   15 FORMAT (26H1NODAL POINT DISPLACEMENTS  // 2(6H POINT, 7X,
     1 5HX-DIS, 9X, 5HY-DIS, 14X) /1X)
      PRINT 16, (N, DSX(N), DSY(N), N=1,NUMNP)
   16 FORMAT (I6,2F14.8,I18,2F14.8)
C
C     PUNCH OF DISPLACEMENTS
C
      IF (T2)  PUNCH 3, (N,XORD(N),YORD(N),DSX(N),DSY(N), N=1,NUMNP)
    3 FORMAT (I4,2F8.4,2E14.5)
C
C     ******************************************************************
C     INITIALIZE FOR STRESS COMPUTATION
C     ******************************************************************
C
      REWIND 8
```

```
      REWIND 9
      DO 120  N = 1,NUMNP
      COUNT(N) = 0.
      DO 120  I = 1,3
      SIG (N,I) = 0.
  120 SIGM(N,I) = 0.
      DO 130  N = 1,NUMEL
      DO 125  I = 1,5
  125 DT(N,I) = 0.
      DO 130  I = 1,3
  130 SIGC (N,I) = 0.
      READ (8)  (XORD(N),N=1,NUMNP),(XCENT(N),N=1,NUMEL),
     1          (YORD(N),N=1,NUMNP),(YCENT(N),N=1,NUMEL)
      READ (8)  (MAT(I),I=1,NUMEL)
      IF (THERL)  READ (9)  ((DT(N,I),I=1,5),N=1,NUMEL)
      READ (9)  (NEI(N),N=1,NUMEL)
C
C     ****************************************************************
C     COMPUTATION OF ELEMENT STRESSES
C     ****************************************************************
C
      IF (T4)  PRINT 20
   20 FORMAT (17H1ELEMENT STRESSES // 10H   ELEMENT,  13X,
     1 8HN' POINT, 9X, 6HSIG-XX, 8X, 6HSIG-YY, 8X, 6HTAU-XY)
      DO 300  N = 1,NUMEL
      M = MAT(N)
      NU = PR(M)
      ER = YM(M)/(1.-NU**2)
      G = 0.5*ER*(1.-NU)
      COOIL = ALFA(M)
      IF (NP(N,7).LE.0)  GO TO 250
C
C     QUADRILATERAL ELEMENT
C
C     RECOVER DISPLACEMENTS OF INTERNAL POINTS
C
      DO 150  I = 17,26
  150 R(I) = 0.
      DO 160  I = 1,8
      K = NP(N,I)
      L = 2*I
      D(L-1) = DSX(K)
  160 D(L)  =  DSY(K)
      NTRACK = 2*N - 1
      NW = 260
      IF (NEI(N).GT.0)  NW = 286
      CALL RDDISK (NTRACK,S21,NW)
      DO 180  I = 1,10
      L = I + 16
      K = L - 1
      D(L) = R(L)
      DO 180  J = 1,K
  180 D(L) = D(L) - S21(I,J)*D(J)
C
C     STRESSES ARE NOW EVALUATED AT EACH SUBTRIANGLE
C     AND AVERAGED FOR THE QUADRILATERAL
```

```
C
      DO 200  I = 1,27
  200 SIGQ(I,1) = 0.
      DO 220  I = 1,4
      J = IPERM4(I)
      K1 = NP(N,I)
      K2 = NP(N,J)
      A(1) = XCENT(N)-XORD(K2)
      A(2) = XORD(K1)-XCENT(N)
      A(3) = XORD(K2)-XORD(K1)
      B(1) = YORD(K2)-YCENT(N)
      B(2) = YCENT(N)-YORD(K1)
      B(3) = YORD(K1)-YORD(K2)
      DO 210  L = 1,6
      K = 2*LNQ(L,I)
      RX(L) = D(K-1)
  210 RY(L) = D(K)
      DELT(1) = DT(N,I)
      DELT(2) = DT(N,J)
      DELT(3) = DT(N,5)
      CALL TRISTR
      DO 220  K = 1,4
      M = LNQ(K,I)
      DO 220  J = 1,3
  220 SIGQ(M,J) = SIGQ(M,J) + COEF(K)*ESIG(K,J)
      IF (T4)  PRINT 25, N, (NP(N,I),(SIGQ(I,J),J=1,3),I=1,4),
     1 (SIGQ(9,J),J=1,3)
   25 FORMAT (14HOQUADRILATERAL I4, 4X, 6HCORNER I4, 3F14.4 /
     3 3(22X, 6HCORNER I4, 3F14.4/), 22X, 10HCENTROID   ,3F14.4)
      DO 230  I = 1,8
      K = NP(N,I)
      COUNT(K) = COUNT(K) + 1.
      DO 230  J = 1,3
      X = SIGQ(I,J)
      IF (ABS(X).GT.ABS(SIGM(K,J)))  SIGM(K,J) = X
  230 SIG(K,J) = SIG(K,J) + X
      DO 240  J = 1,3
  240 SIGC(N,J) = SIGQ(9,J)
      GO TO 300
C
C     SINGLE TRIANGLE
C
  250 DO 260  I = 1,3
      J = IPERM(I)
      M = IPERM(J)
      DELT(I) = DT(N,I)
      K1 = NP(N,I)
      K2 = NP(N,J)
      A(M) = XORD(K2)-XORD(K1)
  260 B(M) = YORD(K1)-YORD(K2)
      DO 270  I = 1,6
      K = NP(N,I)
      RX(I) = DSX(K)
  270 RY(I) = DSY(K)
      CALL TRISTR
      DO 280  I = 1,6
```

```
      K = NP(N,I)
      COUNT(K) = COUNT(K) + 1.
      DO 280  J = 1,3
      X = ESIG(I,J)
      IF (ABS(X).GT.ABS(SIGM(K,J)))  SIGM(K,J) = X
  280 SIG(K,J) = SIG(K,J) + X
      IF (T4)  PRINT 26, N, (NP(N,I),(ESIG(I,J),J=1,3),I=1,3)
   26 FORMAT (9HOTRIANGLE I4, 9X, 6HCORNER I4, 3F14.5 /
     1 (22X, 6HCORNER I4, 3F14.5))
  300 CONTINUE
C
C     ******************************************************************
C     NODAL POINT AVERAGE STRESSES
C     ******************************************************************
C
      DO 400  N = 1,NUMNP
      DO 320  I = 1,3
  320 SIG(N,I) = SIG(N,I)/COUNT(N)
      X = SIG(N,1)
      Y = SIG(N,2)
      XY= SIG(N,3)
      C = 0.5*(X+Y)
      DIF = 0.5*(X-Y)
      R = SQRT(DIF**2+XY**2)
      SIG(N,4) = C + R
      SIG(N,5) = C - R
      SIG(N,6) = R
      ANGLE (N) = 45.
      IF (DIF.NE.0.)  ANGLE(N) = 28.647890*ATAN(XY/DIF)
  400 CONTINUE
      DO 420  N = 1,NUMEL
      IF (NP(N,7).LE.0)  GO TO 420
      X =  SIGC(N,1)
      Y =  SIGC(N,2)
      XY = SIGC(N,3)
      C = 0.5*(X+Y)
      DIF = 0.5*(X-Y)
      R = SQRT(DIF**2+XY**2)
      SIGC(N,4) = C + R
      SIGC(N,5) = C - R
      SIGC(N,6) = R
      ANGLEC(N) = 45.
      IF (DIF.NE.0.)  ANGLEC(N) = 28.644890*ATAN(XY/DIF)
  420 CONTINUE
C
C     ******************************************************************
C     PRINT OF AVERAGED NODAL POINT STRESSES
C     ******************************************************************
C
      PRINT 30
      DO 440  N = 1,NUMNP
  440 PRINT 32,  N, XORD(N),YORD(N),SIG(N,1),SIGM(N,1),SIG(N,2),
     1 SIGM(N,2),SIG(N,3),SIGM(N,3),(SIG(N,I),I=4,7)
      PRINT 36, NUMNP
      DO 450  N = 1,NUMEL
      IF (NP(N,7).LE.0)  GO TO 450
```

```
      L = N + NUMNP
      PRINT 38, L, XCENT(N),YCENT(N),(SIGC(N,I),I=1,7)
  450 CONTINUE
   30 FORMAT (30H1AVERAGED NODAL POINT STRESSES // 6H POINT,3X,
     1 11HCOORDINATES, 9X, 8HSIGMA-XX, 13X, 8HSIGMA-YY, 13X, 8H TAU-XY ,
     2 10X, 7HSIG-MAX, 4X, 7HSIG-MIN, 2X, 9HMAX.SHEAR, 5X, 5HANGLE /
     3 10X, 1HX, 7X, 1HY, 2X, 3(4X,7HAVERAGE,3X,7HMAXIMUM), 36X,
     4 11H(SIG-MAX,X) /1X)
   32 FORMAT (I5, 2F8.3, 3(F11.4,F10.4), 1X, 4F11.4)
   36 FORMAT (49H-STRESSES AT QUADRILATERAL CENTROIDS (POINT NO. = I5,
     1 15H + ELEMENT NO.)/1X)
   38 FORMAT (I5, 2F8.3, 3(F16.4, 5X ), 1X,  4F11.4)
C
C     PUNCH OF STRESSES
C
      IF (.NOT.T3)  GO TO 500
      PUNCH 5, (N,(SIG(N,I),I=1,3), N=1,NUMNP)
      DO 480  N = 1,NUMEL
      L = NUMNP + N
  480 PUNCH 5, L,(SIGC(N,I),I=1,3)
    5 FORMAT (I4,3E18.6)
C
C     ******************************************************************
C     COMPUTATION OF GRAPH SPACINGS
C     ******************************************************************
C
  500 REWIND 9
      PGRAPH = .FALSE.
      DO 520  I = 1,6
      SPACNG(I) = 0.
      IF (IGRTAG(I).LE.0)  GO TO 520
      WRITE (9)  (SIG(N,I),N=1,NUMNP), (SIGC(N,I),N=1,NUMEL)
      PGRAPH = .TRUE.
  520 CONTINUE
      IF (.NOT.PGRAPH)  GO TO 800
      IF (NSK.LE.0)  GO TO 600
      DO 550  I = 1,NSK
      N = NELSKP(I)
      NEP = 8
      IF (NP(N,7).LE.0)  NEP = 6
      DO 550 J = 1,NEP
      K = NP(N,J)
      DO 550  L = 1,6
  550 SIG(K,L) = 0.
  600 DO 700  I = 1,6
      IF (IGRTAG(I).LE.0)  GO TO 700
      SGMAX = 0.
      DO 620  N = 1,NUMNP
      C = ABS(SIG(N,I))
  620 IF (C.GT.SGMAX)  SGMAX = C
      NF = ALOG10(SGMAX)
      N = 2
      IF (SGMAX.GE.1.)  N = 1
      F = 10.**(N-NF)
      C = F*SGMAX
      DO 630  L = 1,10
```

```
      IF (FMAX(L).GT.C)  GO TO 650
  630 CONTINUE
  650 SPACNG(I) =  0.1*FMAX(L)/F
  700 CONTINUE
C
  800 RETURN
      END
```

```
$IBFTC TRST     DECK,LIST
      SUBROUTINE TRISTR
C
C     ******************************************************************
C     TRISTR COMPUTES STRESSES FOR A 6 NODAL POINT TRIANGLE
C     ******************************************************************
C
      COMMON /CTRIST/ ER, G, NU, CODIL, DELT(3), B(3), A(3),
     1 RX(6), RY(6), ESIG(6,3)
      DIMENSION BA(3,2), UV(3,6,2), U(3,6), V(3,6), EPSX(3), EPSY(3),
     1 GMXY(3), IPERM(3)
      EQUIVALENCE (BA,B), (UV,U), (UV(19),V)
      REAL NU
      DATA IPERM /2,3,1/
      AREA = A(3)*B(2)-A(2)*B(3)
      DO 120  L = 1,3
      L1 = IPERM(L)
      L2 = IPERM(L1)
      L3 = L + 3
      DO 120  N = 1,2
      D0 = BA(L,N) /AREA
      D1 = BA(L1,N)/AREA
      UV(L ,L ,N) = 3.*D0
      UV(L1,L ,N) =   -D0
      UV(L2,L ,N) =   -D0
      UV(L ,L3,N) = 4.*D1
      UV(L1,L3,N) = 4.*D0
  120 UV(L2,L3,N) =    0.
      DO 150  I = 1,3
      THERM = CODIL*DELT(I)
      EPSX(I) = 0.
      EPSY(I) = 0.
      GMXY(I) = 0.
      DO 140  J = 1,6
      X = RX(J)
      Y = RY(J)
      C = U(I,J)
      D = V(I,J)
      EPSX(I) = EPSX(I) + C*X
      EPSY(I) = EPSY(I) + D*Y
  140 GMXY(I) = GMXY(I) + C*Y + D*X
      ESIG(I,1) = ER*(EPSX(I)+NU*EPSY(I)) - THERM
      ESIG(I,2) = ER*(EPSY(I)+NU*EPSX(I)) - THERM
  150 ESIG(I,3) =  G* GMXY(I)
      DO 180  I = 1,3
      J = IPERM(I)
      DO 180  K = 1,3
  180 ESIG(I+3,K) = 0.5*(ESIG(I,K)+ESIG(J,K))
      RETURN
      END
```

```
$ORIGIN         ALPHA,SYSUT3
$IBFTC CNPL     LIST,DECK
      SUBROUTINE CNTPLT
C
C     ****************************************************************
C     CNTPLT PRINTS STRESS CONTOUR GRAPHS
C     ****************************************************************
C
      COMMON NUMEL, NUMCP, NUMNP
      COMMON /CELMAR/  NP(350,8)
      COMMON /CNTARG/  NUMPB, NSK, PGRAPH, IGRTAG(6), SPACNG(6),
     1 GRHEAD(2,6), NPB(50), NELSKP(50)
      COMMON /CGRAPH/ P(101,101), XORD(1400), YORD(1400), F(1400)
      DIMENSION XLAB(11), S(3), NR(2,3), NPT(3), IPERM4(4)
      LOGICAL  T0, T1, T2, T3
      DATA ASTRK /6H*     /, BLANK /6H      /,
     1 XLAB(1) /66H0      1       2       3       4       5       6       7       8
     3 9       D        /
      DATA IPERM4 /2,3,4,1/
      DATA NR /2,3, 1,3, 1,2/
C
      REWIND 8
      REWIND 9
      NTOTP = NUMNP + NUMEL
      READ (8)  (XORD(N),N=1,NTOTP), (YORD(N),N=1,NTOTP)
C
C     PREPARE GRAPH PARAMETERS
C
      I = NPB(1)
      XMIN = XORD(1)
      YMIN = YORD(1)
      YMAX = YMIN
      XMAX = XMIN
      DO 100  N = 2,NUMPB
      I = NPB(N)
      Y = YORD(I)
      X = XORD(I)
      IF (XMIN.GT.X)  XMIN = X
      IF (YMIN.GT.Y)  YMIN = Y
      IF (XMAX.LT.X)  XMAX = X
  100 IF (YMAX.LT.Y)  YMAX = Y
      XD = XMAX - XMIN
      YD = (YMAX-YMIN)*.6
      XM = XD
      IF (YD.GT.XD)  XM = YD
      DX = XM/100.
      XR = XMIN - DX
      DY = DX/.6
      YR = YMIN - DY
      XS = XR - DX/2.
      YS = YR - DY/2.
      NCOL1 = (YMAX - YS)/DY + 1.
      NRP = 101
      NCP = NCOL1-1
      NG = 0
C
```

```
C     CHECK IF GRAPH IS TO BE PLOTTED
C
  105 NG = NG + 1
      IF (NG.GT.6)  GO TO 600
      IF (IGRTAG(NG).LE.0)  GO TO 105
      READ (9)  (F(N),N=1,NTOTP)
      SPACE = SPACNG(NG)
      FMAX = 10.*SPACE
      DO 110  I = 1,NRP
      DO 110  J = 1,NCP
  110 P(I,J) = BLANK
C
C     PLOT BOUNDARY
C
      DO 160  N = 1,NUMPB
      K = NPB(N)
      L = NPB(N+1)
      IF (N.EQ.NUMPB)  L = NPB(1)
      X1 = XORD(K)
      X2 = XORD(L)
      Y1 = YORD(K)
      Y2 = YORD(L)
      XD = X2 - X1
      YD = Y2 - Y1
      T0 = ABS(XD).GE.ABS(YD)
      T1 = .NOT.T0
      T2 = X1.LT.X2
      T3 = Y1.LT.Y2
      IF (T0.AND.T2.OR.T1.AND.T3)  GO TO 120
      TEMP = X1
      X1 = X2
      X2 = TEMP
      TEMP = Y1
      Y1 = Y2
      Y2 = TEMP
  120 IF (T1)  GO TO 140
      N1 = (X1-XS)/DX
      N2 = (X2-XS)/DX
      DO 130  NX= N1,N2
      X = FLOAT(NX)*DX + XR
      Y = Y1 + YD*(X-X1)/XD
      NY = (Y-YS)/DY
      NY = NCOL1 - NY
  130 P(NX,NY) = ASTRK
      GO TO 160
  140 N1 = (Y1-YS)/DY
      N2 = (Y2-YS)/DY
      DO 150  NY = N1,N2
      Y = FLOAT(NY)*DY + YR
      X = X1 + XD*(Y-Y1)/YD
      NX = (X-XS)/DX
      NYY = NCOL1 - NY
  150 P(NX,NYY) = ASTRK
  160 CONTINUE
C
C     INTERNAL CONTOUR LINES
```

```
C
      NSC = 1
      NES = NELSKP(1)
      DO 420  N = 1,NUMEL
      IF (N.NE.NES)  GO TO 170
      NSC = NSC + 1
      NES = NELSKP(NSC)
      GO TO 420
  170 IF (NP(N,7).LE.0)  GO TO 180
      NUMT = 4
      NPT(3) = NUMNP + N
      GO TO 190
  180 NUMT = 1
      NPT(3) = NP(N,3)
  190 DO 400  II = 1,NUMT
      JJ = IPERM4(II)
      NPT(1) = NP(N,II)
      NPT(2) = NP(N,JJ)
C
C     SORT FUNCTION VALUES
C
      DO 200  I = 1,3
      J = NPT(I)
  200 S(I) = F(J)
      NP1 = NPT(1)
      L = 1
      S1 = S(1)
      DO 220  I = 2,3
      IF(S(I).GE.S1)  GO TO 220
      NP1 = NPT(I)
      S1 = S(I)
      L = I
  220 CONTINUE
      L1 = NR(1,L)
      L2 = NR(2,L)
      IF (S(L1).GT.S(L2))  GO TO 240
      NP2 = NPT(L1)
      NP3 = NPT(L2)
      GO TO 250
  240 NP2 = NPT(L2)
      NP3 = NPT(L1)
  250 S2 = F(NP2)
      S3 = F(NP3)
      IF (S1.GT.FMAX.OR.S3.LT.-FMAX)  GO TO 400
      I = (S1+FMAX)/SPACE
      M = I - 9
      VALUE = FLOAT(M)*SPACE
      DIF13 = S3 - S1
      IF (DIF13.EQ.0.)  GO TO 400
  260 IF (VALUE.GT.S3)  GO TO 400
      NF = IABS(M) + 1
      IF (NF.GT.11)  GO TO 400
C
C     FIND END COORDINATES OF CONTOUR LINE SEGMENT
C
      XF = (VALUE-S1)/DIF13
```

```
      X = XORD(NP1)
      Y = YORD(NP1)
      X1 = X + XF*(XORD(NP3)-X)
      Y1 = Y + XF*(YORD(NP3)-Y)
      NTT = NP1
      IF (VALUE.GT.S2)  NTT = NP3
      ST = F(NTT)
      DIFT2 = ST - S2
      IF (ABS(DIFT2).LT.1.E-8)  GO TO 350
      XF = (VALUE-S2)/DIFT2
      X = XORD(NP2)
      Y = YORD(NP2)
      X2 = X + XF*(XORD(NTT)-X)
      Y2 = Y + XF*(YORD(NTT)-Y)
      XD = X2 - X1
      YD = Y2 - Y1
      TO = ABS(XD).GE.ABS(YD)
      T1 = .NOT.TO
      T2 = X1.LT.X2
      T3 = Y1.LT.Y2
      IF (TO.AND.T2.OR.T1.AND.T3)   GO TO 270
      TEMP = X1
      X1 = X2
      X2 = TEMP
      TEMP = Y1
      Y1 = Y2
      Y2 = TEMP
C
C     STORE SIGNAL INTO P ARRAY
C
  270 IF (T1)   GO TO 300
      IF (XD.EQ.0.)  GO TO 350
      N1 = (X1-XS)/DX
      N2 = (X2-XS)/DX
      DO 280  NX = N1,N2
      X = FLOAT(NX)*DX + XR
      Y = Y1 + YD*(X-X1)/XD
      NY = (Y-YS)/DY
      NY = NCOL1 - NY
  280 P(NX,NY) = XLAB(NF)
      GO TO 350
  300 N1 = (Y1-YS)/DY
      N2 = (Y2-YS)/DY
      DO 320  NY = N1,N2
      Y = FLOAT(NY)*DY + YR
      X = X1 + XD*(Y-Y1)/YD
      NX = (X-XS)/DX
      NYY = NCOL1 - NY
  320 P(NX,NYY) = XLAB(NF)
  350 M = M + 1
      VALUE = VALUE + SPACE
      GO TO 260
  400 CONTINUE
  420 CONTINUE
C
C     PRINT GRAPH
```

```
C
      PRINT 10, GRHEAD(1,NG),GRHEAD(2,NG)
   10 FORMAT (1H1,50X,2A6)
      NULL = 0
      PRINT 12, NULL, (I, I=10,100,10)
   12 FORMAT (1H0,I7,10I10 /1X)
      DO 450  J = 1,NCP
      L = J - 1
  450 PRINT 14, L, (P(I,J), I=1,NRP), L
   14 FORMAT (1HZ,I4,2X,101A1,I4)
      PRINT 12, NULL, (I, I=10,100,10)
      PRINT 16
   16 FORMAT (11H-REFERENCES // 11H CONT. LINE, 14X, 6HVALUES/1X)
      DO 500  I = 1,11
      W = FLOAT(I-1)*SPACE
      WN = - W
  500 PRINT 18, XLAB(I), W, WN
   18 FORMAT (9X, A1, 2F14.5)
      PRINT 20, ASTRK
   20 FORMAT (1H0,8X,A1, 8X,15HBOUNDARY POINTS)
      GO TO 105
  600 RETURN
      END
```

APPENDIX IV

COMPUTER PROGRAM FOR ELASTOPLASTIC PLANE STRESS ANALYSIS

1. IDENTIFICATION

PSP-LST: Plane Stress Plastic Analysis using Linear Strain Triangles

Programmed: Carlos A. Felippa, Dec. 1965.

2. PURPOSE

Step-by-step displacement analysis of elastoplastic plane stress problems using linear strain triangles as finite elements. Infinitesimal or finite displacements may be considered.

3. USAGE

The program was written in FORTRAN IV (version 13) for the IBM 7094 computer; it is subdivided into 3 links and must run under the supervision of the IBSYS Overlay Loader.

4. CAPACITY

The mesh input is subjected to the following limitations for a computer with 32 K storage:

Max. number of elements	80
Max. number of nodal points	160
Max. difference of nodal point numbers for the same element	21

These limits could be easily increased for a production program by a more extensive use of tape or disk storage and a larger capacity equation solver.

5. TAPE UNITS

Logical units 1 and 2 are used for temporary storage. Logical unit 3 is Overlay Link residence.

6. PROGRAM STRUCTURE

The link structure is shown in Fig. A4.1, where each subroutine deck is represented by a rectangle. Their functions are:

MAIN controls the calling sequence;

PRINSL outputs displacements, forces, strains and stresses;

CNTPLT produces contour line printer plots;

The three previous decks remain in core at all times.

SETUP inputs and organizes input data describing the problem and prepares for elastic solution;

SETIC evaluates strains and stresses from elastic displacements, scales solution to the first yield and prepares first nonlinear step;

NLSTEP evaluates elastic and plastic strains, stresses, plastic work, etc., from the last incremental step and arranges the next one;

STEPSL assembles complete instantaneous stiffness and solves for incremental displacements;

STFNS computes element stiffness matrix.

The flow chart of the execution is presented in Fig. A4.2.

7. MATERIAL ASSUMPTIONS

In order to simplify coding and input, the following specific assumptions were made:

(a) The material is homogeneous, isotropic, temperature and strain-rate independent for both elastic and plastic stages;

(b) Von Mises yield criterion

$$\bar{\sigma}^2 = 3 J_2 = 3 K^2 \qquad \text{(A4-1)}$$

(in terms of actual stresses in case of finite displacements).

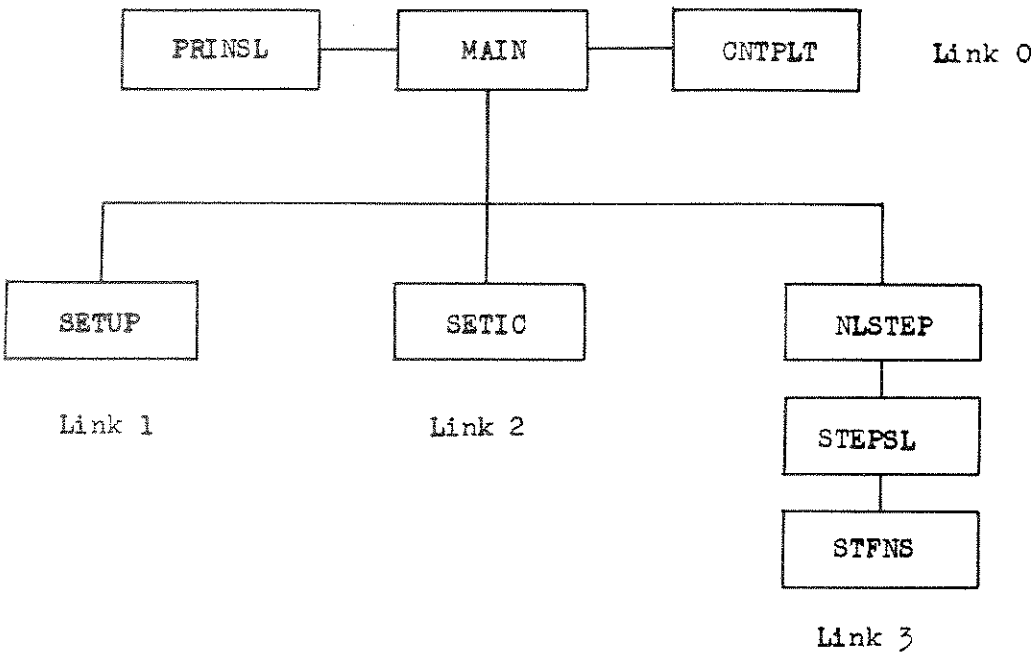


Fig. A4.1 - Link Structure.

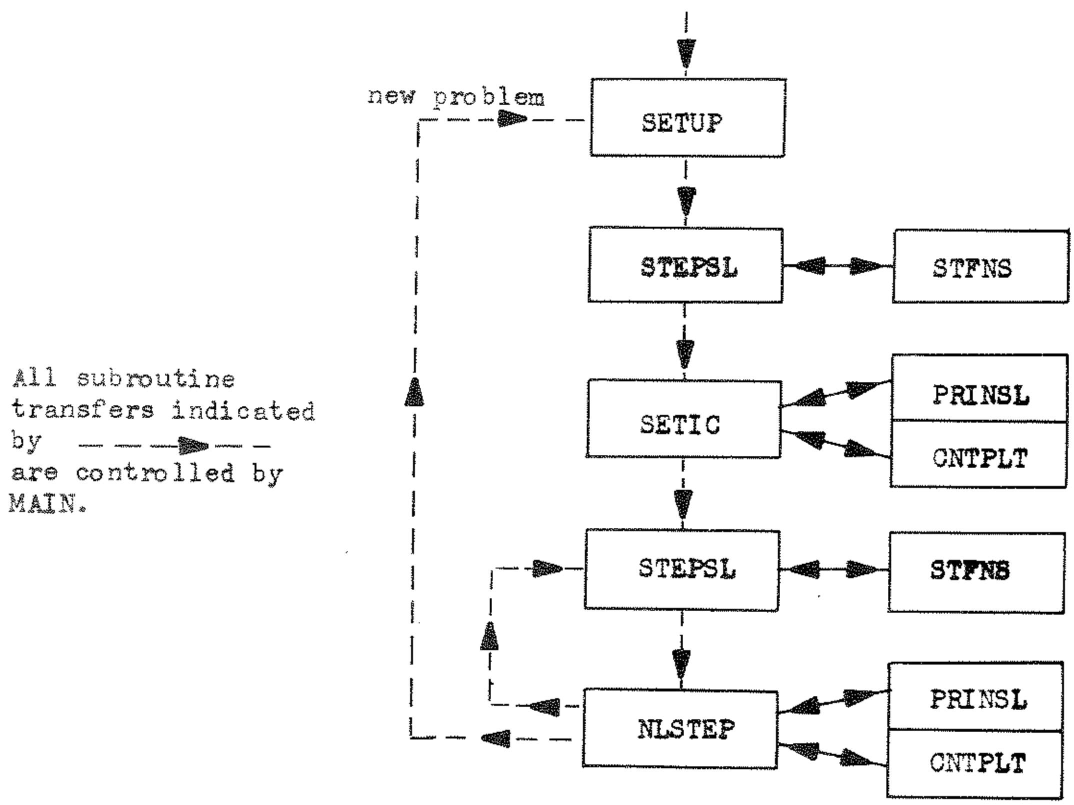


Fig. A4.2 - Subroutine Flow Chart.

(c) Isotropic Linear hardening, function of the total plastic work from the annealed state. The simple tension test (Fig. A4.4) gives

$$\epsilon^P = \frac{1-\chi}{E_p}(\sigma - Y) \quad \text{where} \quad \chi = E_p/E \tag{A4-2}$$

Since the total plastic work from yield is

$$W_p = \int_Y^{\sigma} \sigma \, d\epsilon^P = \frac{1-\chi}{2E_p}(\sigma^2 - Y^2) \tag{A4-3}$$

and $\bar{\sigma} = \sigma$, the invariant work-hardening law, valid for two or three dimensional problems, is

$$\bar{\sigma}^2 = 3\kappa^2 = Y^2 + \frac{2E_p}{1-\chi} W_p = 3F^2(W_p) = 3H(W_p) \tag{A4-4a}$$

therefore

$$H' = \frac{dH}{dW_p} = 2FF' = \frac{2E_p}{3(1-\chi)} = \frac{2\chi E}{3(1-\chi)} \tag{A4-4b}$$

These assumptions permit a very simple characterization of the material: in addition to the elastic constants E and ν , only Y and χ must be supplied.

Arbitrary yield and hardening criteria, anisotropic plasticity, dependence of elastic and plastic parameters on temperature, strain-rate (viscoplasticity) or stress rates could be incorporated without basic difficulties. Dynamic (inertial) effects might also be naturally included in the incremental procedure.

8. INTEGRATION PROCEDURE

The midpoint rule mentioned in IV.1.5 is used for the step-by-step integration. The sequence of operations for one step is detailed in Fig. A4.5. Two solutions are required per step:

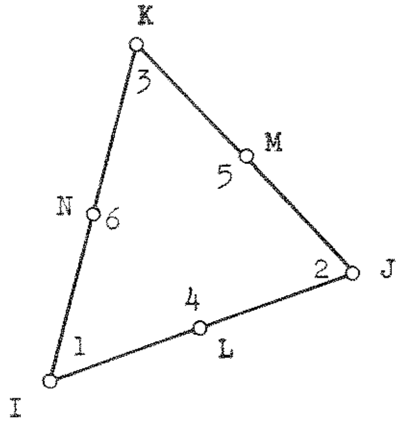
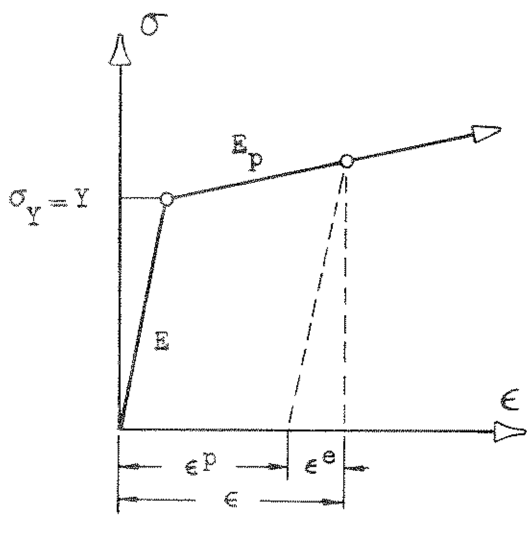
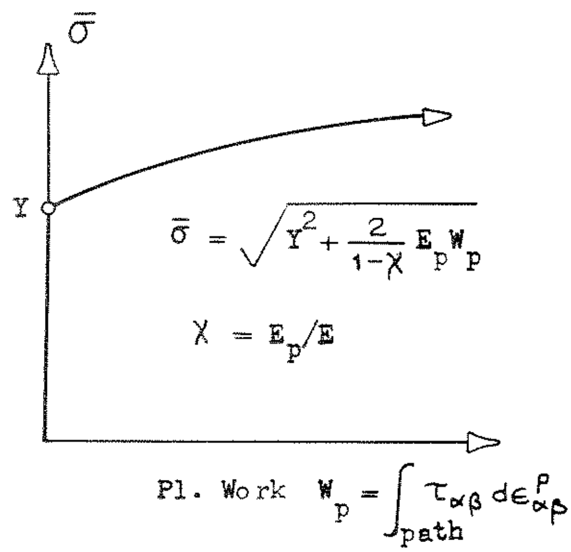


Fig. A4.3 - Basic Input Element.



Tension Test



Invariant Hardening Law

Fig. A4.4 - Assumed Work-Hardening Law.

(1) from point A of previous step to B using stiffness K_A at A;

(2) from point A to C using the "averaged" stiffness at the midpoint $M = (A+B)/2$.

It may be noticed that applied displacement increments (whenever possible) should be preferred. For elastic loading or unloading, only one step is needed. The sequence (1)-(2) is governed by the logical variable IFLAG:

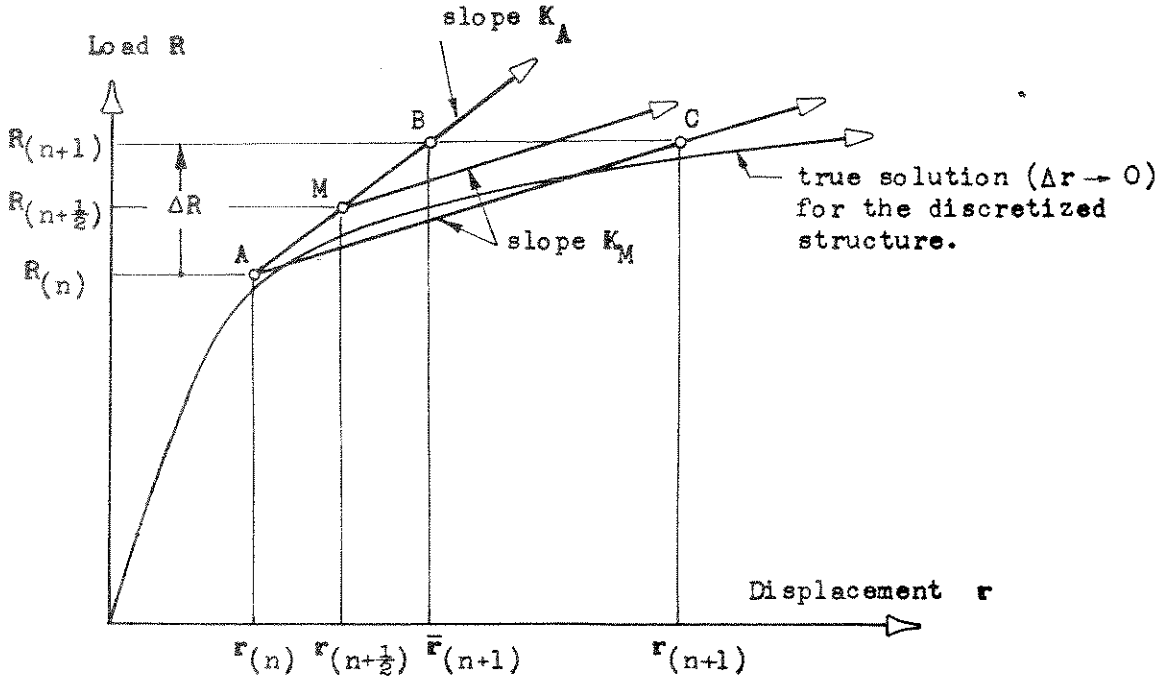
IFLAG = .TRUE. for the first solution (A to B);

IFLAG = .FALSE. for the second solution (A to C).

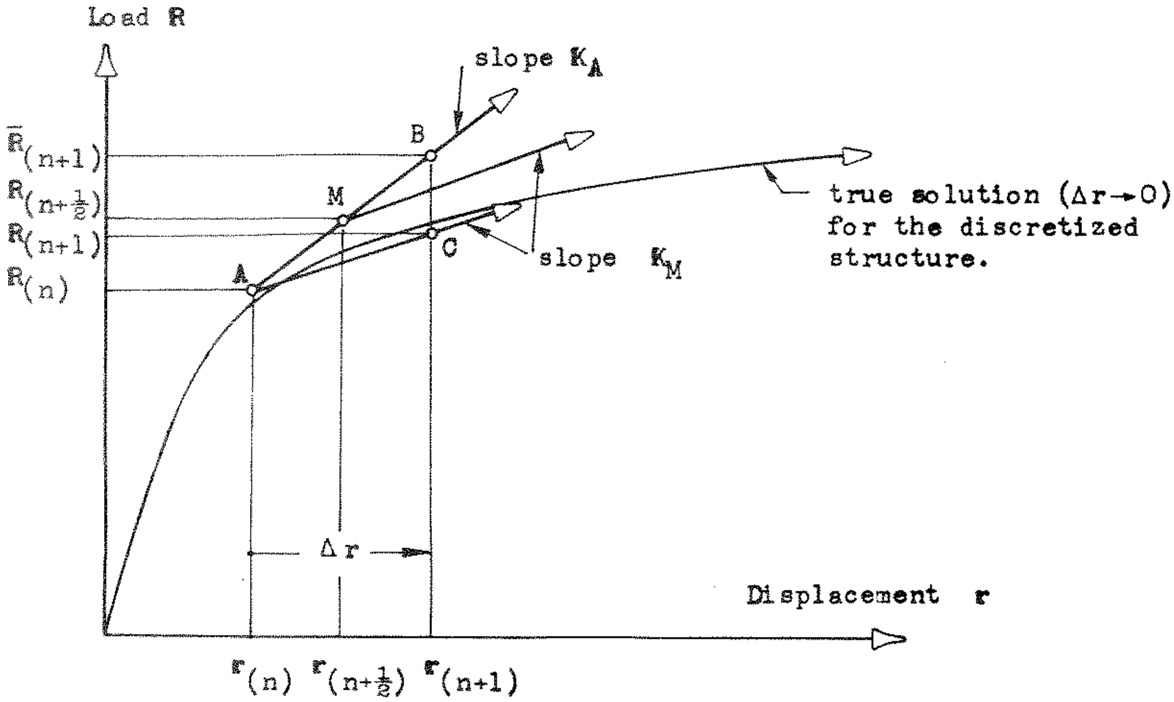
By elimination of the cards setting IFLAG = .TRUE. in the subroutines SETIC and NLSTEP and of the early RETURN in NLSTEP, the program will perform a direct incremental solution (Euler's method), i.e., point B is taken as the incremented solution. However, this procedure is not recommended since it generally gives a load-displacement curve well above the actual one unless very small increments are used. It has been found that the larger volume of operations per step is more than compensated by the possibility of using large intervals without appreciable effect on the solution.

9. LOADING PROGRAM

Initially specified loads or displacements are of arbitrary magnitude. The elastic solution is automatically scaled to first yield; then the specified parameters are incremented proportionally in the plastic range by a dimensionless factor ξ . Load increments may be halved or displacement increments doubled at specified step numbers. When a reference displacement component exceeds a certain ratio with respect to the yield value, or a maximum number of steps is exceeded, the loading process is interrupted and a very small negative increment is applied, thus reverting the entire structure to the elastic condition.



(a) For Load Increment.



(b) For Displacement Increment.

Fig. A4.5 - Midpoint Rule of Integration.

The next and final step is a complete unloading.

Non-proportional post-yield loading is also available as an option.

Any other arbitrary load program could be easily built-in or coded so as to be specified from input data. It must be noted, however, that the assumption of isotropic hardening is not realistic if load reversals are considered.

10. FINITE DISPLACEMENT ANALYSIS

If finite displacement analysis specified, the following extra operations are carried out:

(a) Actualization of coordinates and thicknesses after each step. The element LST-P3, described in IV.2.4.4, which accounts for a parabolic thickness variation, is used for the conventional stiffness matrix.

(b) Geometric stiffness is added to the conventional incremental stiffness. The existing stress variation inside the element is assumed to be parabolic and determined by the averaged nodal point values at corners and midpoints (see IV.4.3); average thickness is used.

(c) Incremented or Kirchoff stresses are transformed to actual stresses after each step. This is done element by element using the linearized transformation equation (IV-11) before averaging for nodal point stresses. Transformation on the nodal loads is not performed, i.e., they are referred to initial area and rotated axes.

According to the governing variational principle, incremented strains and stresses are referred to and must be evaluated in the geometry of the configuration at which the instantaneous stiffness was computed; i.e., geometry of A for strains at B and M, and geometry of M for strains at C (Fig. A4.5).

11. SEQUENCE OF OPERATIONS

(a) Input data describing the problem is read, printed and organized by SETUP, which prepares for the elastic solution.

(b) The complete elastic stiffness $\mathbf{K}$ is assembled in STEPSL by the direct stiffness procedure, using the element stiffnesses produced by STFNS. The upper portion of the band is stored as a rectangular array row-wise, i.e., element k_{ij} goes to A(I,J-I+1). The complete unaltered $\mathbf{K}$ is stored on logical unit 2. Boundary conditions are imposed by setting to zero the nodal force and all off-diagonal elements of the corresponding row and column; the diagonal element is set to 1. A skew roller is handled by rotating the non-constrained row and column and projecting the nodal force components. If nonzero displacements $\mathbf{r}_2$ are prescribed, the load vector must be modified as follows:

$$\begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{bmatrix} \begin{Bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \end{Bmatrix} = \begin{Bmatrix} \mathbf{R}_1 \\ \mathbf{R}_2 \end{Bmatrix} \longrightarrow \begin{bmatrix} \mathbf{K}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \end{Bmatrix} = \begin{Bmatrix} \mathbf{R}_1 - \mathbf{K}_{12}\mathbf{r}_2 \\ \mathbf{r}_2 \end{Bmatrix} \quad \text{(A4-5)}$$

The band structure is then preserved. The resulting set of equations is solved by symmetric Gauss elimination. Finally the unaltered matrix is read back from unit 2 and multiplied by the displacements to yield applied loads plus reactions.

(c) SETIC evaluates element strains and stresses and the nodal point averages. The maximum equivalent stress $\bar{\sigma}_{max}$ is found and the solution scaled so that $\bar{\sigma}_{max} = Y$. All nodal points for which $\bar{\sigma} > 0.999\, Y$ are considered to be in the plastic region.

If load increments are specified, the first incremental load vector is $\Delta \mathbf{R} = \xi \mathbf{R}_Y$. If displacements are prescribed, the incremental prescribed displacement in Equation (A4-5) is $\Delta \mathbf{r}_2 = \xi \mathbf{r}_{2Y}$ and $\Delta \mathbf{R}_1 = 0$. Element strains and stresses, loads and displacements are stored on logical unit 1.

(d) Control is transferred again to STEPSL, which computes and assembles the instantaneous stiffness and solves for incremental displacements as explained in (b). After the first solution of the midpoint rule, incremental displacements are divided by 2.

(e) NLSTEP computes elastic, plastic and total incremental strains and stresses; they are added to the previous values read from logical unit 1. The incremental plastic work per unit of volume is evaluated from

$$\Delta W_p \doteq \left(\tau_{\alpha\beta} + \frac{1}{2} \Delta \tau_{\alpha\beta} \right) \Delta \epsilon_{\alpha\beta} \tag{A4-6}$$

Nodal point strains and stresses are obtained by averaging over the contributing elements. The following plasticity test is applied at each nodal point:

if $\Delta W_p > 0$; plastic

if $\Delta W_p = 0 \begin{cases} \frac{1}{\sqrt{3}} \overline{\sigma} \geq 0.999\, F(W_p) & : \text{plastic} \\ \frac{1}{\sqrt{3}} \overline{\sigma} < 0.999\, F(W_p) & : \text{elastic} \end{cases}$

if $\Delta W_p < 0$: elastic

where $F(W_p)$ is computed from Equation (A4-4a). If the point is in the plastic region, all stress components are divided by a coefficient $\mho$ so that $\frac{\overline{\sigma}}{\mho} = K\sqrt{3}$, i.e., the point lies exactly on the yield surface. These scaling coefficients are printed and their departure from 1 serves as a measure of the accuracy of the integration procedure.

To perform the second solution of the midpoint rule, the same increment is used. For a new step, incremental loads or displacements are selected as described in (c); element strains and stresses, load and displacements and total plastic work are again stored on tape 1. The program now returns to STEPSL and proceeds as described in (d).

(f) Final unloading is performed as indicated in LOADING PROGRAM.

(g) Total loads and displacements are printed after each step; the nodal forces have been obtained by accumulation of the products $\mathbf{K}\,\Delta\mathbf{r}$ and include reactions. The satisfaction of equilibrium gives an idea of the accuracy of the direct solution. Strains and stresses, and contour graphs thereof, are printed at specified intervals.

12. NUMERICAL LIMITATIONS

Since elastic strains are obtained as difference of total and plastic strains, numerical instability may occur for very large plastic distorsions because of cancellation error. It is not recommended to go beyond 50-60 times the elastic deformations when using an eight-digit machine like the IBM 7094 for a finite displacement analysis (for small displacements, such problem has not been observed). To treat processes like metal forming, such limit might be raised by:

(a) Using a computer which carries more significant figures;

(b) Improving the accuracy of the displacement solution by residual correction and performing the computation of total and plastic strains and their difference in double precision.

13. TIMING

For the coarse idealizations that may be treated with this program, the displacement solution time is a small proportion of the total step time; computation of strains and stresses, printouts, plots, I/O, etc., is dominant. Therefore the step time is roughly proportional to the number of nodal points. For the maximum-sized system (160 nodal points) and small displacements, the total step time (2 solutions) is approximately 1 minute in the IBM 7094. Finite displacement analysis is about 15% slower.

14. INPUT DATA INSTRUCTIONS

The following sequence of cards describes the problem:

(a) Start Card (A6): with the word START punched on columns 1-5. This card must precede the data deck of any problem.

(b) Title Card (13A6): alphameric information in columns 1-78 to identify output.

(c) Control Card (11I4,2F8.3,3I4,3L2);

Columns	Variable Name	Meaning
1- 4	NUMEL	Number of elements ($\leq$ 80);
5- 8	NUMCP	Number of corner points;
9-12	NUMNP	Number of nodal points ($\leq$ 160);
13-16	NUMBC	Number of restrained points;
17-20	NUMBP	Number of defining boundary points (see (e));
21-24	NLOAD	Number of loaded points (must be 0 if NID > 0);
25-28	NID	Number of specified displacement components (must be 0 if NLOAD > 0);
29-32	NSPIN	Strain-stress print interval;
33-36	NGPIN	Graph print interval;
37-40	NEQRED	Number of equation for reference displacement;
41-44	NEQREF	Number of equation for reference force;
45-52	ALFA	Bound for reference displacement (with respect to first yield value);
53-60	XI	Initial parameter increment;
61-64	MAXST	Maximum number of steps (set to 20 if blank);
65-68	NDUPI	Step number for first interval change: XI is halved if load are imposed, doubled if displacements are imposed. Not performed if left blank;
69-72	NDUP2	Step number for second interval change; not performed if left blank.

The following three fields are for logical flags: if a T is punched, the indicated options will be carried out:

73-74	IPIN2	Print of element strains and stresses;
75-76	SMALD	Finite displacement analysis is performed;
77-78	INPL	Non proportional post-yield loading (see (1)).

Note: the equation number for the X-component of nodal point "n" is $n_x = 2n-1$; for the Y-component, $n_y = 2n$. The reference force is used only to compute and print an indicative ratio with respect to its first yield value.

(d) Material and Geometric Properties (5F10.4). These values hold for all elements:

Columns	Variable Name	Meaning
1-10	EM	Elastic modulus;
11-20	XU	Poisson's ratio;
21-30	TH	Initial thickness;
31-40	XI	Plastic/elastic modulus ratio;
41-50	YP	Initial yield point.

(e) Defining Boundary Points (20I4). For contour line graphs, NUMBP nodal points which define the boundary of the region to be plotted as a series of straight lines must be punched in cyclic order, 20 per card. The initial point and the sense is arbitrary. If no graphs will be plotted, a blank card may be inserted. For finite displacement analysis, all boundary points should be included since the actual deformed shape is plotted.

(f) Graph Spacings and Skipped Elements (7F5.1,15I3). The first seven fields (7F5.1) specify the spacing between contour lines for the following graphs:

Columns	Graph
1- 5	Sigma xx
5-10	Sigma yy
11-15	Tau xy
16-20	Sigma max.
21-25	Sigma min.
26-30	Equivalent stress.
31-35	Plastic work

If a spacing field is left blank, the corresponding graph is skipped. If the nonzero spacing is "s", the graph range is $\pm$ 10s, therefore 10s should exceed the maximum value expected. The last two graphs ($\bar{\sigma}$ and W_p) are useful to establish the extension of the plastic region.

The following fields (15I3) may be used to specify number of elements to be skipped from the plots (must be numbered in increasing order).

Note: the plots are generated by dividing each element into four subtriangles by joining the midpoints; the values at their vertices are interpolated linearly over them.

(g) Element Array (7I4). One card per element.

Cols.	1- 4	Element number;
	5-28	Nodal point numbers in the counterclockwise cyclic order I-J-K-L-M-N (Fig. A4.3).

(h) Coordinate Array (I4,2F8.3). One card per corner point.

Cols. 1- 4 Corner point number;

5-12 X-coordinate;

13-20 Y-coordinate.

(i) Boundary Condition Array (2I4,F10.3). One card per restrained point.

Cols. 1- 4 Nodal point number;

5- 8 Tag = 0 if point is fixed in both directions;

1 if point is fixed in X-direction;

2 if point is free to move along a line forming angle φ with the X-axis.

9-18 Angle φ in degrees, positive counterclockwise (for type 2 of boundary condition only).

(j) Nodal Point Loads (I4,2F8.3). One card per loaded point (no cards if NLOAD = 0).

Cols. 1- 4 Nodal point number;

5-12 X-load;

13-20 Y-load.

Note: the magnitude of the loads may be arbitrary, since they are scaled to first yield.

(k) Specified Displacements (2I4,F10.3). One card per specified displacement component (no cards if NID = 0).

Cols. 1- 4 Nodal point number;

5- 8 Component number = 1 for X-component;

= 2 for Y-component;

9-18 Displacement value.

The magnitude of the displacements is arbitrary.

(l) Non proportional Post-yield Loading: optional, only valid if flag INPL of control card was set .TRUE.:

Control Card: (2I4): specifies NLOAD, NID; these values are read after the elastic solution is completed and supersede the previous ones. Again only one of them may be > 0.

If NLOAD > 0, NLOAD load cards must follow with the format specified in (j);

If NID > 0, NID specified displacement cards must follow conforming to (k).

The increments in the plastic range are now based on these loads or displacements as yield values. Actual values must be specified, since they are not scaled.

15. RUN OF SEVERAL PROBLEMS

Any number of jobs may be run consecutively; each problem deck must be preceded by a START card. The program stops when an $EOF card is read.

```
$JOB    4696                          FELIPPA
$IBJOB PSPLST
$IBFTC MAIN     LIST,DECK
C
C     *****************************************************************
C     ELASTOPLASTIC ANALYSIS OF PLANE STRESS PROBLEMS
C     USING LST-P3 ELEMENT (SIX NODAL POINT TRIANGLE)
C     SMALL OR FINITE DISPLACEMENT ANALYSIS
C     *****************************************************************
C     C A FELIPPA, NOVEMBER 1965
C
      COMMON
     1 NUMEL, NUMCP, NUMNP, NUMBC, NEQ, NEQBC, NLOAD, NID, NEQRED,
     2 NEQREF, MAXST, NDUP1, NDUP2, NSTEP, NSPIN, NPRINT, NGPIN,
     3 NGRAPH, IBANDW, MAXBW, NSKEWD, MFLAG, IPIN1, IPIN2, INPL,
     4 IFLAG, SMALD, ALFA, ERDIS, ERFOR, COMM, COMF, CHI, PSI,
     5 EM, XU, TH, YP, YP2, XI, ER, G, FAC, A1, A2, A3, A4,
     6 EXTRAS(5), NP(80,6), XCRD(160), YORD(160), RT(160)
      COMMON /SOLARG/
     1 BETA(160), SIGP(160,3), DR(320), DF(320), NEBC(50),
     2 BANGLE(50), ND(10), DSD(10), SDYP(10)
      COMMON /NLARG / CLENGT(13440)
C
      COMMON /CNPARG/ NUMBP, XR, YR, XS, YS, NCOL1, DX, DY,
     1 SPACNG(7), GRHEAD(2,7), NPB(15), NELSKP(15)
C
  100 MAXBW = 42
      CALL SETUP
      CALL STEPSL
      CALL SETIC
  120 CALL STEPSL
      CALL NLSTEP
      IF (MFLAG.LT.3)  GO TO 120
      GO TO 100
      END
```

```
$IBFTC PRIN    LIST,DECK
      SUBROUTINE PRINSL
C
C     ******************************************************************
C     THIS SUBROUTINE OUTPUTS DISPLACEMENTS, STRAINS AND STRESSES
C     ******************************************************************
C
      COMMON
     1 NUMEL, NUMCP, NUMNP, NUMBC, NEQ, NEQBC, NLOAD, NID, NEQRED,
     2 NEQREF, MAXST, NDUP1, NDUP2, NSTEP, NSPIN, NPRINT, NGPIN,
     3 NGRAPH, IBANDW, MAXBW, NSKEWD, MFLAG, IPIN1, IPIN2, INPL,
     4 IFLAG, SMALD, ALFA, ERDIS, ERFOR, COMM, COMF, CHI, PSI,
     5 EM, XU, TH, YP, YP2, XI, ER, G, FAC, A1, A2, A3, A4,
     6 EXTRAS(5), NP(80,6), XCRD(160), YORD(160), RT(160)
      COMMON /NLARG /
     1 SIG(160,6), WP(160), YEQSG(160), SCAL(160), DWP(160),
     2 TST(160,4), PST(160,4), EST(160,4), COUNT(160),
     3 ETST(80,6,4), EPST(80,6,4), EEST(80,6,4), ESIG(80,6,3),
     4 EWP(80,6), FOR(320), FYP(320), DIS(320), DYP(320),
     5 BA(3,2), UV(6,3,2), DUMMY(658)
      DIMENSION F(320), R(320)
      EQUIVALENCE (F,FOR), (R,DIS)
      LOGICAL IPIN1, IPIN2
C
      PRINT 10, NSTEP
   10 FORMAT (49H1DISPLACEMENT, STRAIN AND STRESS PRINTOUT, STEP = I5)
      IF (NLOAD.GT.0)    PRINT 12, PSI
      IF (NID   .GT.0)    PRINT 13, PSI
   12 FORMAT (17H0LOAD PARAMETER = F10.4)
   13 FORMAT (25H0DISPLACEMENT PARAMETER = F11.5)
      PRINT 14, COMM, COMF
   14 FORMAT (25H0REF. DISPLAC. INCREASE = F11.5 /
     1          22H REF. FORCE INCREASE =     F14.5 )
      PRINT 15
   15 FORMAT (37H0NODAL POINT FORCES AND DISPLACEMENTS // 1X, 2(5HPOINT,
     1 8X, 5HX-DIS, 8X, 5HY-DIS, 7X, 6HX-LOAD, 7X, 6HY-LOAD, 8X) /1X)
      PRINT 16, (N,R(2*N-1),R(2*N),F(2*N-1),F(2*N), N=1,NUMNP)
   16 FORMAT (I6, 2F13.6, 2F13.4,  7X, I6, 2F13.6, 2F13.4)
      IF (IPIN1)  RETURN
      IF (IPIN2)  GO TO 200
      PRINT 20
   20 FORMAT (28H1ELEMENT STRAIN AND STRESSES// 35X,15HELASTIC STRAINS,
     1 42X, 15HPLASTIC STRAINS //8H ELEMENT, 2X, 5HPOINT, 2(8X, 5HEPS-X,
     2 8X, 5HEPS-Y, 7X, 6HEPS-XY, 8X, 5HEPS-Z, 5X))
      DO 140  N = 1,NUMEL
  140 PRINT 22, N, (NP(N,I), (EEST(N,I,J),J=1,4), (EPST(N,I,J),J=1,4),
     1 I=1,6)
   22 FORMAT (1H0, 2I7,2(4F13.6,5X) / (8X,I7,2(4F13.6,5X)))
      PRINT 25
   25 FORMAT (1H-,34X, 15H TOTAL STRAINS , 56X,8HSTRESSES //
     1 8H ELEMENT,2X, 5HPOINT, 8X, 5HEPS-X, 8X, 5HEPS-Y, 7X, 6HEPS-XY,
     2 8X, 5HEPS-Z, 6X, 8HPL. WORK, 11X,5HSIG-X,9X,5HSIG-Y,8X,6HTAU-XY)
      DO 150  N = 1,NUMEL
  150 PRINT 27, N, (NP(N,I), (ETST(N,I,J),J=1,4), EWP(N,I),
     1 (ESIG(N,I,J),J=1,3), I=1,6)
   27 FORMAT (1H0,2I7,4F13.6,F14.5,2X,3F14.4 / (8X,I7,4F13.6,F14.5,
```

```
      1 2X,3F14.4))
  200 PRINT 30
   30 FORMAT (14H-POINT STRAINS // 20X, 15HELASTIC STRAINS, 27X,
     1 15HPLASTIC STRAINS, 27X, 15H TOTAL STRAINS //6H POINT,
     2 3(5X, 5HEPS-X, 5X, 5HEPS-Y, 4X, 6HEPS-XY, 5X, 5HEPS-Z,2X)/1X)
      PRINT 32, (N, (EST(N,I),I=1,4), (PST(N,I),I=1,4), (TST(N,I),
     1 I=1,4), N=1,NUMNP)
   32 FORMAT (I6,4F10.6,2X,4F10.6,2X,4F10.6)
      PRINT 35
   35 FORMAT (32H-POINT STRESSES AND PLASTIC WORK // 6H POINT, 5X,
     1 5HX-ORD, 5X, 5HY-ORD, 3X, 7HR.THICK, 5X, 5HSIG-X, 5X, 5HSIG-Y,
     2 4X, 6HTAU-XY, 3X, 7HSIG-MAX, 3X, 7HSIG-MIN, 2X, 9HEQ.STRESS,
     3 3X, 7HSCALING, 3X, 7HPL.WORK, 7X, 3HDWP /1X)
      PRINT 38, (N, XORD(N), YORD(N), RT(N), (SIG(N,I),I=1,6), SCAL(N),
     1 WP(N), DWP(N), N=1,NUMNP)
   38 FORMAT (I6,8F10.4,1X,4F10.4)
      RETURN
      END
```

```
$IBFTC CNPL     LIST,DECK
      SUBROUTINE CNTPLT
C
C     ******************************************************************
C     THIS SUBROUTINE PRINTS STRESS CONTOURS FOR A 6 NP TRIANGULAR MESH
C     ******************************************************************
C
      COMMON
     1 NUMEL, NUMCP, NUMNP, NUMBC, NEQ, NEQBC, NLOAD, NID, NEQRED,
     2 NEQREF, MAXST, NDUP1, NDUP2, NSTEP, NSPIN, NPRINT, NGPIN,
     3 NGRAPH, IBANDW, MAXBW, NSKEWD, MFLAG, IPIN1, IPIN2, INPL,
     4 IFLAG, SMALD, ALFA, ERDIS, ERFOR, COMM, COMF, CHI, PSI,
     5 EM, XU, TH, YP, YP2, XI, ER, G, FAC, A1, A2, A3, A4,
     6 EXTRAS(5), NP(80,6), XCRD(160), YORD(160), RT(160)
      COMMON /NLARG/  SIG(160,7), P(101,101), F(160), DUMMY(1959)
      COMMON /CNPARG/ NUMBP, XR, YR, XS, YS, NCOL1, DX, DY,
     1 SPACNG(7), GRHEAD(2,7), NPB(15), NELSKP(15)
C
      LOGICAL  T0, T1, T2, T3, SMALD
      DIMENSION  XLAB(11), S(3), NR(2,3), NSUB(3,4), NPT(3)
      DATA ASTRK /6H*     /, BLANK /6H      /,
     1 XLAB(1) /66H0      1      2      3      4      5      6      7      8
     3 9      0      /
      DATA NR /2,3, 1,3, 1,2/
      DATA NSUB /1,4,6, 2,5,4, 3,6,5, 4,5,6/
C
      IF (SMALD)  GO TO 110
      XMIN = C.
      XMAX = 0.
      YMIN = 0.
      YMAX = 0.
      DO 105  N = 1,NUMBP
      I = NPB(N)
      Y = YORC(I)
      X = XORD(I)
      IF (XMIN.GT.X)  XMIN = X
      IF (YMIN.GT.Y)  YMIN = Y
      IF (XMAX.LT.X)  XMAX = X
  105 IF (YMAX.LT.Y)  YMAX = Y
      XD = XMAX - XMIN
      YD = 0.6*(YMAX-YMIN)
      XM = XD
      IF (YD.GT.XD)  XM = YD
      DX = XM/100.
      DY = DX/0.6
      XR = XMIN - DX
      YR = YMIN - DY
      XS = XR - DX/2.
      YS = YR - DY/2.
      NCOL1 = (YMAX - YS)/DY + 1.
  110 NRP = 101
      NCP = NCOL1 - 1
      NG = 1
  112 IF (SPACNG(NG).LE.0.)  GO TO 600
      SPACE = SPACNG(NG)
      DO 115  I = 1,NRP
```

```
      DO 115  J = 1,NCP
  115 P(I,J) = BLANK
      FMAX = 10.*SPACE
C
C     BOUNDARY
C
      DO 160  N = 1,NUMBP
      K = NPB(N)
      L = NPB(N+1)
      IF (N.EQ.NUMBP)  L = NPB(1)
      X1 = XORD(K)
      X2 = XORD(L)
      Y1 = YORD(K)
      Y2 = YORD(L)
      XD = X2 - X1
      YD = Y2 - Y1
      T0 = ABS(XD).GE.ABS(YD)
      T1 = .NOT.T0
      T2 = X1.LT.X2
      T3 = Y1.LT.Y2
      IF (T0.AND.T2.OR.T1.AND.T3)  GO TO 120
      TEMP = X1
      X1 = X2
      X2 = TEMP
      TEMP = Y1
      Y1 = Y2
      Y2 = TEMP
  120 IF (T1)  GO TO 140
      N1 = (X1-XS)/DX
      N2 = (X2-XS)/DX
      DO 130  NX= N1,N2
      X = FLOAT(NX)*DX + XR
      Y = Y1 + YD*(X-X1)/XD
      NY = (Y-YS)/DY
      NY = NCOL1 - NY
  130 P(NX,NY) = ASTRK
      GO TO 160
  140 N1 = (Y1-YS)/DY
      N2 = (Y2-YS)/DY
      DO 150  NY = N1,N2
      Y = FLOAT(NY)*DY + YR
      X = X1 + XD*(Y-Y1)/YD
      NX = (X-XS)/DX
      NYY = NCOL1 - NY
  150 P(NX,NYY) = ASTRK
  160 CONTINUE
C
C     INTERNAL CONTOUR LINES
C
      DO 170  N = 1,NUMNP
  170 F(N) = SIG(N,NG)
      NSK = 1
      NELS = NELSKP(NSK)
      DO 420  N = 1,NUMEL
      IF (N.NE.NELS)  GO TO 180
      NSK = NSK + 1
```

```
      NELS = NELSKP(NSK)
      GO TO 420
  180 DO 400  NS = 1,4
C
C     SORT FUNCTION VALUES
C
      DO 200  I = 1,3
      L = NSUB(I,NS)
      J = NP(N,L)
      NPT(I) = J
  200 S(I) = F(J)
      NP1 = NPT(1)
      L = 1
      S1 = S(1)
      DO 220  I = 2,3
      IF(S(I).GE.S1)  GO TO 220
      NP1 = NPT(I)
      S1 = S(I)
      L = I
  220 CONTINUE
      L1 = NR(1,L)
      L2 = NR(2,L)
      IF (S(L1).GT.S(L2))  GO TO 240
      NP2 = NPT(L1)
      NP3 = NPT(L2)
      GO TO 250
  240 NP2 = NPT(L2)
      NP3 = NPT(L1)
  250 S2 = F(NP2)
      S3 = F(NP3)
      IF (S1.GT.FMAX.OR.S3.LT.-FMAX)  GO TO 400
      I = (S1+FMAX)/SPACE
      M = I - 9
      VALUE = FLOAT(M)*SPACE
      DIF13 = S3 - S1
      IF (DIF13.EQ.0.)  GO TO 400
  260 IF (VALUE.GT.S3)  GO TO 400
      NF = IABS(M) + 1
      IF (NF.GT.11)  GO TO 400
C
C     FIND END COORDINATES OF CONTOUR LINE SEGMENT
C
      XF = (VALUE-S1)/DIF13
      X = XORD(NP1)
      Y = YORD(NP1)
      X1 = X + XF*(XORD(NP3)-X)
      Y1 = Y + XF*(YORD(NP3)-Y)
      NPT = NP1
      IF (VALUE.GT.S2)  NPT = NP3
      ST = F(NPT)
      DIFT2 = ST - S2
      IF (ABS(DIFT2).LT.1.E-8)  GO TO 350
      XF = (VALUE-S2)/DIFT2
      X = XORD(NP2)
      Y = YORD(NP2)
      X2 = X + XF*(XORD(NPT)-X)
```

```
      Y2 = Y + XF*(YORD(NPT)-Y)
      XD = X2 - X1
      YD = Y2 - Y1
      T0 = ABS(XD).GE.ABS(YD)
      T1 = .NOT.T0
      T2 = X1.LT.X2
      T3 = Y1.LT.Y2
      IF (T0.AND.T2.OR.T1.AND.T3)   GO TO 270
      TEMP = X1
      X1 = X2
      X2 = TEMP
      TEMP = Y1
      Y1 = Y2
      Y2 = TEMP
C
C     STORE SIGNAL INTO P ARRAY
C
  270 IF (T1)   GO TO 300
      IF (XD.EQ.0.)  GO TO 350
      N1 = (X1-XS)/DX
      N2 = (X2-XS)/DX
      DO 280  NX = N1,N2
      X = FLOAT(NX)*DX + XR
      Y = Y1 + YD*(X-X1)/XD
      NY = (Y-YS)/DY
      NY = NCOL1 - NY
  280 P(NX,NY) = XLAB(NF)
      GO TO 350
  300 N1 = (Y1-YS)/DY
      N2 = (Y2-YS)/DY
      DO 320  NY = N1,N2
      Y = FLOAT(NY)*DY + YR
      X = X1 + XD*(Y-Y1)/YD
      NX = (X-XS)/DX
      NYY = NCOL1 - NY
  320 P(NX,NYY) = XLAB(NF)
  350 M = M + 1
      VALUE = VALUE + SPACE
      GO TO 260
  400 CONTINUE
  420 CONTINUE
C
C     PRINT GRAPH
C
      PRINT 10,GRHEAD(1,NG), GRHEAD(2,NG)
   10 FORMAT (1H1,45X,2A6)
      NULL = 0
      PRINT 12, NULL, (I, I=10,100,10)
   12 FORMAT (1H-,I7,10I10 /1X)
      DO 450  J = 1,NCP
      L = J - 1
  450 PRINT 14, L, (P(I,J), I=1,NRP), L
   14 FORMAT (1X,I4,2X,101A1,I4)
      PRINT 12, NULL, (I, I=10,100,10)
      PRINT 16
   16 FORMAT (11H-REFERENCES // 11H CONT. LINE, 14X, 6HVALUES/1X)
```

```
      DO 500  I = 1,11
      W = FLOAT(I-1)*SPACE
      WN = - W
500    PRINT 18, XLAB(I), W, WN
 18   FORMAT (9X, A1, 2F14.3)
      PRINT 20, ASTRK
 20   FORMAT (1H0,8X,A1, 8X,15HBOUNDARY POINTS)
600   NG = NG + 1
      IF (NG.GT.7)  RETURN
      GO TO 112
      END
```

```
$ORIGIN        LOC1,SYSUT3
$IBFTC SETP    DECK,LIST
      SUBROUTINE SETUP
C
C     ******************************************************************
C     SETUP READS, PRINTS AND ORGANIZES INPUT DATA
C     ******************************************************************
C
      COMMON
     1 NUMEL, NUMCP, NUMNP, NUMBC, NEQ, NEQBC, NLOAD, NID, NEQRED,
     2 NEQREF, MAXST, NDUP1, NDUP2, NSTEP, NSPIN, NPRINT, NGPIN,
     3 NGRAPH, IBANDW, MAXBW, NSKEWD, MFLAG, IPIN1, IPIN2, INPL,
     4 IFLAG, SMALD, ALFA, ERCIS, ERFOR, COMM, COMF, CHI, PSI,
     5 EM, XU, TH, YP, YP2, XI, ER, G, FAC, A1, A2, A3, A4,
     6 EXTRAS(5), NP(80,6), XORD(160), YORD(160), RT(160)
      COMMON /SOLARG/
     1 BETA(160), SIGP(160,3), DR(320), DF(320), NEBC(50),
     2 BANGLE(50), ND(10), DSD(10), SDYP(10)
C
      COMMON /CNPARG/ NUMBP, XR, YR, XS, YS, NCOL1, DX, DY,
     1 SPACNG(7), GRHEAD(2,7), NPB(15), NELSKP(15)
C
      DIMENSION R(320), F(320), TITLE(13), IPERM(3), GRTITL(2,7)
      EQUIVALENCE (R,DR), (F,DF)
      LOGICAL IPIN1,IPIN2, INPL, SMALD, IFLAG
      DATA FLAG /6HSTART /
      DATA IPERM /2,3,1/
      DATA GRTITL(1,1) /12H SIGMA XX    /,
     1     GRTITL(1,2) /12H SIGMA YY    /,
     3     GRTITL(1,3) /12H TAU XY      /,
     4     GRTITL(1,4) /12H SIGMA MAX   /,
     5     GRTITL(1,5) /12H SIGMA MIN   /,
     6     GRTITL(1,6) /12H EQ. STRESS  /,
     7     GRTITL(1,7) /12HPLASTIC WORK/
C
      NSTEP = 1
      NSKEWD = 0
      DO 110  I = 1,2
      DO 110  J = 1,7
  110 GRHEAD(I,J) = GRTITL(I,J)
C
C     READ AND PRINT OF INPUT DATA
C
  120 READ 10, CHECK
      IF (CHECK.NE.FLAG)  GO TO 120
      READ 10, TITLE
   10 FORMAT (13A6)
      PRINT 11, TITLE
   11 FORMAT (1H1,13A6)
      READ 15,  NUMEL, NUMCP, NUMNP, NUMBC, NUMBP, NLOAD, NID,
     1 NSPIN, NGPIN, NEQRED, NEQREF, ALFA, CHI, MAXST, NDUP1,
     2 NDUP2, IPIN2, SMALD, INPL
      IF (MAXST.LE.0)   MAXST = 20
      IF(NDUP1.LE.0)   NDUP1 = MAXST + 5
      IF(NDUP2.LE.0)   NDUP2 = NDUP1
      PRINT 16, NUMEL, NUMCP, NUMNP, NUMBC, NUMBP, NLOAD, NID,
```

```
  1 NSPIN, NGPIN, NEQRED, NEQREF, ALFA, CHI, MAXST, NDUP1,
  2 NDUP2, IPIN2, SMALD, INPL
15 FORMAT (11I4, 2F8.3, 3I4,3L2)
16 FORMAT (//
  1 35H NO. OF ELEMENTS . . . . . . . . . .   I5 /
  2 35H NO. OF CORNER POINTS  . . . . . . .   I5 /
  3 35H NO. OF NODAL POINTS . . . . . . . .   I5 /
  4 35H NO. OF BOUND. CONDITIONS. . . . .   I5 /
  5 35H NO. OF DEFINING BOUND. POINTS . .   I5 /
  6 35H NO. OF POINTS LOADED  . . . . . .   I5 /
  7 35H NO. OF IMPOSED DISPLACEMENTS. . .   I5 //
  8 35H STRESS PRINT INTERVAL . . . . . .   I5 /
  9 35H GRAPH  PRINT INTERVAL . . . . . .   I5 //
  1 35H REFERENCE DISPLACEMENT EQUATION .   I5 /
  2 35H REFERENCE FORCE EQUATION. . . . .   I5 /
  3 35H REF. DISPLACEMENT BOUND . . . . .   F8.2/
  4 35H LOAD OR DIS. PARAM. INCREMENT . .   F8.4//
  5 35H MAX. NO. OF STEPS . . . . . . . .   I5 /
  6 35H STEP FOR 1ST INTERVAL CHANGE  . .   I5 /
  7 35H STEP FOR 2ST INTERVAL CHANGE  . .   I5 //
  8 35H FLAG FOR ELEMENT PRINTING . . . .   L5 /
  9 35H FLAG FOR FINITE DISPL. ANALYSIS .   L5 /
  1 35H FLAG FOR 2 PARAMETER LOADING  . .   L5 )
   IPIN2 = .NOT.IPIN2
   SMALD = .NOT.SMALD
   READ 18,  EM, XU, TH, XI, YP
   PRINT 19, EM, XU, TH, XI, YP
18 FORMAT (5F10.4)
19 FORMAT (30H-MATERIAL AND GEOM. PROPERTIES //
  1 34H ELASTIC MODULUS . . . . . . . . . .   F9.2 /
  2 35H POISSON RATIO . . . . . . . . . . .   F8.3 /
  3 35H INITIAL THICKNESS . . . . . . . . .   F8.3 /
  4 34H PLASTIC/ELASTIC MODULUS RATIO . . 1PE9.2/
  5 35H INITIAL YIELD POINT . . . . . . . . 0PF8.2)
   NEQ = 2*NUMNP
   COMM = 2.*XI/(3.*(1.-XI))
   A1 = 2.*(1.-XU**2)*COMM/3.
   A2 = (5.-4.*XU)/9.
   A3 = 2.*(5.*XU-4.)/9.
   A4 = 2.*(1.-XU)
   FAC = 3.*EM*COMM
   YP2 = YP**2
   READ 20,  (NPB(I), I=1,NUMBP)
   PRINT 21, (NPB(I), I=1,NUMBP)
20 FORMAT (20I4)
21 FORMAT (25H-DEFINING BOUNDARY POINTS // (1X,20I5))
   READ 22, SPACNG, NELSKP
22 FORMAT (7F5.1, 15I3)
   PRINT 23, ((GRTITL(I,N), I=1,2), SPACNG(N), N=1,7)
23 FORMAT (7H- GRAPH, 11X, 7HSPACING // (1X,2A6, F12.3))
   IF (NELSKP(1).GT.0)  PRINT 24, NELSKP
24 FORMAT (29H0SKIPPED ELEMENTS IN GRAPHS = 15I4)
   READ 25,  (N, (NP(N,I),I=1,6), L=1,NUMEL)
   PRINT 26, (N, (NP(N,I),I=1,6), N=1,NUMEL)
25 FORMAT (7I4)
26 FORMAT (15H1ELEMENT ARRAY  // 8H ELEMENT, 5X, 1HI, 5X, 1HJ,
```

```
   1 5X, 1HK, 5X, 1HL, 5X, 1HM, 5X, 1HN // (2X, 7I6))
      DO 125  I = 1,50
  125 BANGLE(I) = 0.
      DO 130  I = 1,NEQ
  130 F(I) = 0.
      PRINT 29
   29 FORMAT (19H-CORNER POINT ARRAY // 6H POINT, 10X, 5HX-ORD,
   1 10X, 5HY-ORD /1X)
      DO 140  I = 1,NUMCP
      READ 30, N, XORD(N), YORD(N)
  140 PRINT 31,N, XORD(N), YORD(N)
   30 FORMAT (I4, 2F8.3)
   31 FORMAT (I6, 2F15.5)
      DO 145  N = 1,NUMEL
      DO 145  I = 1,3
      J = IPERM(I)
      K = NP(N,I)
      L = NP(N,J)
      M = NP(N,I+3)
      XORD(M) = 0.5*(XORD(K)+XORD(L))
  145 YORD(M) = 0.5*(YORD(K)+YORD(L))
      IF (NLOAD.LE.0)   GO TO 160
      PRINT 35
   35 FORMAT (18H-NODAL POINT LOADS // 6H POINT,  9X, 6HX-LOAD,
   1 9X, 6HY-LOAD /1X)
      DO 150  I = 1,NLOAD
      READ 30,  N, X, Y
      PRINT 31, N, X, Y
      J = 2*N
      F(J-1) = X
  150 F(J)    = Y
  160 J = 0
      PRINT 40
   40 FORMAT (20H-BOUNDARY CONDITIONS // 6H POINT, 2X, 4HNFIX, 6X,
   1 6HBANGLE /1X)
      DO 200  I = 1,NUMBC
      READ 42,  N, NFIX, ANGLE
      PRINT 44, N, NFIX, ANGLE
   42 FORMAT (2I4,F10.4)
   44 FORMAT (2I6,F12.5)
      JY = 2*N
      JX = JY - 1
      IF (NFIX-1)  170,175,180
  170 J = J + 2
      NEBC(J-1) = JX
      NEBC(J) = JY
      GO TO 200
  175 J = J + 1
      NEBC(J) = JX
      GO TO 200
  180 J = J + 1
      NEBC(J) = JY
      IF (ANGLE.NE.0.)  NSKEWD = NSKEWD + 1
      BANGLE(J) = ANGLE/57.29578
  200 CONTINUE
      NEQBC = J
```

```
      IF (NID.LE.0)   GO TO 220
      PRINT 60
   60 FORMAT (22H-IMPOSED DISPLACEMENTS //  6H POINT, 2X, 4HCOMP,
     1 7X, 5HVALUE /1X)
      DO 210  I = 1,NID
      READ 42,  N, J, DIS
      PRINT 44, N, J, DIS
      L = 2*(N-1) + J
      ND(I) = L
  210 DSD(I) = DIS
C
C     CHECK FOR ZERO OR NEGATIVE ELEMENT AREAS
C     AND COMPUTE BAND WIDTH
C
  220 NPD = 0
      IERROR = 0
      DO 250  N = 1,NUMEL
      I = NP(N,1)
      J = NP(N,2)
      K = NP(N,3)
      X = XORD(I)
      Y = YORD(I)
      AREA = (XORD(J)-X)*(YORD(K)-Y) - (XORD(K)-X)*(YORD(J)-Y)
      IF (AREA.GT.0.)  GO TO 240
      IERROR = 1
      PRINT 62, N
   62 FORMAT (33H0NEGATIVE OR ZERO AREA, ELEMENT = I5)
  240 DO 250  I = 1,5
      K = I + 1
      DO 250  J = K,6
      M = NP(N,I) - NP(N,J)
      L = IABS(M)
      IF (L.GT.NPD)  NPD = L
  250 CONTINUE
      IBANDW = 2*NPD + 2
      PRINT 65, IBANDW
   65 FORMAT (13H0BAND WIDTH = I5)
      IF (IBANDW.LE.MAXBW)  GO TO 280
      IERROR = 1
      PRINT 70, MAXBW
   70 FORMAT (18H0MAX BAND WIDTH OF I3, 9H EXCEEDED)
  280 IF (IERROR.NE.0)  STOP
C
C
C     PREPARE FOR ELASTIC SOLUTION
C
      IFLAG = .FALSE.
      MFLAG = 1
      DO 300  I = 1,NUMNP
      RT(I) = 1.
      BETA(I) = 0.
      DO 300  J = 1,3
  300 SIGP(I,J) = 0.
C
C     SET UP GRAPH PARAMETERS
C
```

```
      XMIN = 0.
      XMAX = 0.
      YMIN = 0.
      YMAX = 0.
      DO 350  N = 1,NUMBP
      I = NPB(N)
      Y = YORD(I)
      X = XORD(I)
      IF (XMIN.GT.X)  XMIN = X
      IF (YMIN.GT.Y)  YMIN = Y
      IF (XMAX.LT.X)  XMAX = X
  350 IF (YMAX.LT.Y)  YMAX = Y
      XD = XMAX - XMIN
      YD = .6*(YMAX-YMIN)
      XM = XD
      IF (YD.GT.XD)  XM = YD
      DX = XM/100.
      DY = DX/.6
      XR = XMIN - DX
      YR = YMIN - DY
      XS = XR - DX/2.
      YS = YR - DY/2.
      NCOL1 = (YMAX - YS)/DY + 1.
      RETURN
      END
```

```
$ORIGIN         LCC1,SYSUT3
$IBFTC STIC     DECK,LIST
      SUBROUTINE SETIC
C
C     ****************************************************************
C     SETIC EVALUATES STRAINS AND STRESSES FROM ELASTIC SOLUTION
C     PREPARES FIRST NON LINEAR STEP
C     ****************************************************************
C
      COMMON
     1 NUMEL, NUMCP, NUMNP, NUMBC, NEQ, NEQBC, NLOAD, NID, NEQRED,
     2 NEQREF, MAXST, NDUP1, NDUP2, NSTEP, NSPIN, NPRINT, NGPIN,
     3 NGRAPH, IBANDW, MAXBW, NSKEWD, MFLAG, IPIN1, IPIN2, INPL,
     4 IFLAG, SMALD, ALFA, ERCIS, ERFOR, COMM, COMF, CHI, PSI,
     5 EM, XU, TH, YP, YP2, XI, ER, G, FAC, A1, A2, A3, A4,
     6 EXTRAS(5), NP(80,6), XCRD(160), YORD(160), RT(160)
      COMMON /SOLARG/
     1 BETA(160), SIGP(160,3), DR(320), DF(320), NEBC(50),
     2 BANGLE(50), ND(10), DSD(10), SDYP(10)
      COMMON /NLARG /
     1 SIG(160,6), WP(160), YEQSG(160), SCAL(160), DWP(160),
     2 TST(160,4), PST(160,4), EST(160,4), COUNT(160),
     3 ETST(80,6,4), EPST(80,6,4), EEST(80,6,4), ESIG(80,6,3),
     4 EWP(80,6), FOR(320), FYP(320), DIS(320), DYP(320),
     5 BA(3,2), UV(3,6,2), DUMMY(758)
      DIMENSION B(3), A(3), U(3,6), V(3,6), IPERM(3), ROT(6),
     1 EQSG(160), R(320), F(320)
      EQUIVALENCE (BA,B), (BA(4),A), (UV,U), (UV(19),V),
     1 (EQSG,SIG( 801)), (R,DIS), (F,FOR)
      LOGICAL IPIN1, IPIN2, INPL, SMALD, IFLAG
      DATA IPERM /2,3,1/
C
C     INITIALIZATION
C
      PSI = 1.00
      MFLAG = 0
      NPRINT = NSPIN
      NGRAPH = NGPIN
  120 DO 130  N = 1,NUMNP
  125 COUNT(N) = 0.
      DO 130  J = 1,4
      TST(N,J) = 0.
  130 SIG(N,J) = 0.
C
C     ELEMENT ELASTIC STRAINS AND STRESSES
C
      ER = EM/(1.-XU**2)
      G = 0.5*EM/(1.+XU)
      DO 250  NV = 1,NUMEL
      DO 140  I = 1,3
      J = IPERM(I)
      M = IPERM(J)
      K1 = NP(NV,I)
      K2 = NP(NV,J)
      A(M) = XCRD(K2) - XCRD(K1)
  140 B(M) = YORD(K1) - YORD(K2)
```

```
      AREA = A(3)*B(2) - A(2)*B(3)
      DO 150  L = 1,3
      L1 = IPERM(L)
      L2 = IPERM(L1)
      L3 = L + 3
      DO 150  N = 1,2
      D0 = BA(L, N)/AREA
      D1 = BA(L1,N)/AREA
      UV(L, L, N) = 3.*D0
      UV(L1,L, N) =    -D0
      UV(L2,L, N) =    -D0
      UV(L ,L3,N) = 4.*D1
      UV(L1,L3,N) = 4.*D0
  150 UV(L2,L3,N) =  0.
      DO 180  I = 1,3
      ETST(NV,I,1) = 0.
      ETST(NV,I,2) = 0.
      ETST(NV,I,3) = 0.
      ROT(I) = 0.
      DO 170  J = 1,6
      K = 2*NP(NV,J)
      X = DR(K-1)
      Y = DR(K)
      ETST(NV,I,1) = ETST(NV,I,1) + U(I,J)*X
      ETST(NV,I,2) = ETST(NV,I,2) + V(I,J)*Y
      ETST(NV,I,3) = ETST(NV,I,3) + V(I,J)*X + U(I,J)*Y
  170 ROT(I) = ROT(I) + V(I,J)*X - U(I,J)*Y
      ETST(NV,I,4) = XU*(ETST(NV,I,1) + ETST(NV,I,2))/(XU-1.)
      ESIG(NV,I,1) = ER*(ETST(NV,I,1) + XU*ETST(NV,I,2))
      ESIG(NV,I,2) = ER*(ETST(NV,I,2) + XU*ETST(NV,I,1))
  180 ESIG(NV,I,3) =  G*ETST(NV,I,3)
C
      DO 195  I = 1,3
      J = IPERM(I)
      L = I + 3
      ROT(L) = (ROT(I)+ROT(I1))/2.
      DO 190  K = 1,3
      ETST(NV,L,K) = (ETST(NV,I,K) + ETST(NV,J,K))/2.
  190 ESIG(NV,L,K) = (ESIG(NV,I,K) + ESIG(NV,J,K))/2.
  195 ETST(NV,L,4) = (ETST(NV,I,4) + ETST(NV,J,4))/2.
      DO 250  I = 1,6
C
C      ACTUAL STRESSES FOR FINITE DISPLACEMENT ANALYSIS
C
      IF (SMALD)  GO TO 210
      E11 = ETST(NV,I,1)
      E22 = ETST(NV,I,2)
      E12 = 0.5*ETST(NV,I,3)
      E33 = ETST(NV,I,4)
      OMEGA = ROT(I)
      C1 = E12 + OMEGA
      C2 = E12 - OMEGA
      S11 = ESIG(NV,I,1)
      S22 = ESIG(NV,I,2)
      S12 = ESIG(NV,I,3)
      ESIG(NV,I,1) = (1.-E22-E33)*S11 + C1*S12
```

```
      ESIG(NV,I,2) = (1.-E11-E33)*S22 + C2*S12
      ESIG(NV,I,3) = 0.5*(C2*S11+C1*S22) + (1.-0.5*(E11+E22)-E33)*S12
  210 K = NP(NV,I)
      COUNT(K) = COUNT(K) + 1.
      DO 220  J = 1,4
  220 TST(K,J) = TST(K,J) + ETST(NV,I,J)
      DO 250  J = 1,3
  250 SIG(K,J) = SIG(K,J) + ESIG(NV,I,J)
C
C     POINT STRESSES
C
      EQSMAX = 0.
      DO 310  N = 1,NUMNP
      CNT = COUNT(N)
      DO 280  J = 1,4
  280 TST(N,J) = TST(N,J)/CNT
      DO 300  J = 1,3
  300 SIG(N,J) = SIG(N,J)/CNT
      X = SIG(N,1)
      Y = SIG(N,2)
      XY = SIG(N,3)
      EQSG(N) = (X-Y)**2 + X*Y + 3.*XY**2
      IF (EQSG(N).GT.EQSMAX)   EQSMAX = EQSG(N)
  310 CONTINUE
      EQSMAX = SQRT(EQSMAX)
      COMM = YP/EQSMAX
C
C     SCALE SOLUTION TO INITIAL YIELD POINT
C
      DO 320 I = 1,NEQ
      FOR(I) = DF(I)*COMM
      FYP(I) = FOR(I)
      DF(I) = 0.
      DIS(I) = DR(I)*COMM
  320 DYP(I) = DIS(I)
      ERDIS = DIS(NEQRED)
      IF (ABS(ERDIS).LT.1.E-15)  ERDIS = 1.
      ERFOR = FOR(NEQREF)
      IF (ABS(ERFOR).LT.1.E-5)  ERFOR = 1.
  325 IF (NID.LE.0)  GO TO 340
      DO 330  I = 1,NID
  330 SDYP(I) = DSD(I)*COMM
  340 IF (.NOT.INPL)  GO TO 390
C
C     TWO PARAMETER LOADING (NON PROPORTIONAL POST YIELD LOADING)
C
      READ 10, NLOAD, NID
   10 FORMAT (2I4)
      PRINT 15, NLOAD, NID
   15 FORMAT (36H1NON PROPORTIONAL POST YIELD LOADING //
     1 35H NO. OF POINTS LOADED  . . . . . .  I5 /
     2 35H NO. OF IMPOSED DISPLACEMENTS. . .  I5 )
      IF (NLOAD.LE.0)  GO TO 360
      DO 345  I = 1,NEQ
  345 FYP(I) = 0.
      PRINT 25
```

```
   25 FORMAT (18H-NODAL POINT LOADS // 6H POINT,  9X, 6HX-LOAD,
     1 9X, 6HY-LOAD /1X)
      DO 350  I = 1,NLOAD
      READ 30,  N, X, Y
      PRINT 31, N, X, Y
      J = 2*N
      FYP(J-1) = X
  350 FYP(J)   = Y
   30 FORMAT (I4, 2F8.3)
   31 FORMAT (I6, 2F15.5)
  360 IF (NID.LE.0)  GO TO 390
      PRINT 40
   40 FORMAT (22H-IMPOSED DISPLACEMENTS //  6H POINT, 2X, 4HCOMP,
     1 7X, 5HVALUE /1X)
      DO 370  I = 1,NID
      READ 42,  N, J, X
      PRINT 44, N, J, X
      L = 2*(N-1) + J
      ND(I) = L
  370 SDYP(I) = X
   42 FORMAT (2I4,F10.4)
   44 FORMAT (2I6,F12.5)
C
C     SCALING OF STRAINS AND STRESSES
C
  390 DO 400  NV = 1,NUMEL
      DO 400  I = 1,6
      EWP(NV,I) = -0.000001
      DO 395  J = 1,4
      ETST(NV,I,J) = ETST(NV,I,J)*COMM
      EEST(NV,I,J) = ETST(NV,I,J)
  395 EPST(NV,I,J) = 0.
      DO 400  J = 1,3
  400 ESIG(NV,I,J) = ESIG(NV,I,J)*COMM
      YPR = 0.999*YP
      DO 500  N = 1,NUMNP
      DWP(N) = 0.
      WP(N) =  0.
      SCAL(N) = 0.
      EQSG(N) = SQRT(EQSG(N))*COMM
      DO 420  I = 1,4
      TST(N,I) = TST(N,I)*COMM
      EST(N,I) = TST(N,I)
  420 PST(N,I) = 0.
      DO 430  I = 1,3
      SIG(N,I) = SIG(N,I)*COMM
  430 SIGP(N,I) = SIG(N,I)
      RT(N) = 1. + TST(N,4)
      X =  SIG(N,1)
      Y =  SIG(N,2)
      XY = SIG(N,3)
      C = 0.5*(X+Y)
      DIF = 0.5*(X-Y)
      RAD = SQRT(DIF**2+XY**2)
      SIG(N,4) = C + RAD
      SIG(N,5) = C - RAD
```

```
C
C     PREPARE FOR FIRST NON LINEAR STEP
C
      IF (EQSG(N).LT.YPR)  GO TO 500
      DENM = A1*EQSG(N)**2 + A2*(X**2+Y**2) + A3*X*Y + A4*XY**2
      BETA(N) = 1./DENM
  500 CONTINUE
      XCHI = CHI
      IF (NLOAD.LE.0)  GO TO 530
      DO 520  I = 1,NEQ
  520 DF(I) = XCHI*FYP(I)
  530 IF (NID.LE.0)  GO TO 560
      DO 540  I = 1,NID
  540 DSD(I) = XCHI*SDYP(I)
  560 CONTINUE
      IF (SMALD)  GO TO 700
C
C     MODIFICATION OF COORDINATES FOR FINITE DISPLACEMENT ANALYSIS
C
      DO 600  N = 1,NUMNP
      L2 = 2*N
      XORD(N) = XORD(N) + COMM*DR(L2-1)
  600 YORD(N) = YORD(N) + COMM*DR(L2)
C
C     STORE INFORMATION ON TAPE
C
  700 REWIND 1
      WRITE (1) ((EWP(N,I),(ESIG(N,I,J),J=1,3),I=1,6),N=1,NUMEL),
     1 ( COUNT(N),N=1,NUMNP)
      WRITE (1) (((ETST(N,I,J),EPST(N,I,J),EEST(N,I,J),J=1,4),I=1,6),
     1 N=1,NUMEL),(F(I),R(I),FYP(I),I=1,NEQ)
      IF (.NOT.SMALD) WRITE (1)  XORD,YORD,RT
  799 IFLAG = .TRUE.
C
C     PRINT ELASTIC SOLUTION
C
      COMM = 1.
      COMF = 1.
      IPIN1 = .FALSE.
      CALL PRINSL
C
C     PLOT ELASTIC SOLUTION
C
      CALL CNTPLT
C
      PSI = PSI + XCHI
      RETURN
      END
```

```
$ORIGIN        LOC1,SYSUT3
$IBFTC NLST    DECK,LIST
      SUBROUTINE NLSTEP
C
C     ****************************************************************
C     NLSTEP COMPUTES STRAINS AND STRESSES FROM LAST NON LINEAR
C     STEP AND PREPARES INPUT FOR THE NEXT ONE
C     ****************************************************************
C
      COMMON
     1 NUMEL, NUMCP, NUMNP, NUMBC, NEQ, NEQBC, NLOAD, NID, NEQRED,
     2 NEQREF, MAXST, NDUP1, NDUP2, NSTEP, NSPIN, NPRINT, NGPIN,
     3 NGRAPH, IBANDW, MAXBW, NSKEWD, MFLAG, IPIN1, IPIN2, INPL,
     4 IFLAG, SMALD, ALFA, ERDIS, ERFOR, COMM, COMF, CHI, PSI,
     5 EM, XU, TH, YP, YP2, XI, ER, G, FAC, A1, A2, A3, A4,
     6 EXTRAS(5), NP(80,6), XORD(160), YORD(160), RT(160)
      COMMON /SOLARG/
     1 BETA(160), SIGP(160,3), DR(320), DF(320), NEBC(50),
     2 BANGLE(50), ND(10), DSD(10), SDYP(10)
      COMMON /NLARG /
     1 SIG(160,6), WP(160), YEQSG(160), SCAL(160), DWP(160),
     2 TST(160,4), PST(160,4), EST(160,4), COUNT(160),
     3 ETST(80,6,4), EPST(80,6,4), EEST(80,6,4), ESIG(80,6,3),
     4 EWP(80,6), FOR(320), FYP(320), DIS(320), DYP(320),
     5 BA(3,2), UV(3,6,2), DETST(6,4), DRT(160), DUMMY(574)
      DIMENSION B(3), A(3), U(3,6), V(3,6), IPERM(3), S(3), ROT(6),
     1 DEPST(4), DEEST(4), DESIG(3), EQSG(160), R(320), F(320)
      EQUIVALENCE  (B,BA), (A,BA(4)), (U,UV), (V,UV(19)),
     1 (EQSG,SIG( 801)), (R,DIS), (F,FOR)
      REAL NUT, NUH
      LOGICAL T1, T2, IPIN1, IPIN2, SMALD, LARGD, IFLAG
      DATA IPERM /2,3,1/
C
C     INITIALIZATION
C
      NUT = 1. - XU
      NUH = NUT/2.
      LARGD = .NOT.SMALD
      T1 = LARGD.AND..NOT.IFLAG
      T2 = SMALD.AND.IFLAG
      REWIND 1
      READ (1)  ((EWP(N,I),(ESIG(N,I,J),J=1,3),I=1,6),N=1,NUMEL),
     1  (COUNT(N),N=1,NUMNP)
      DO 120  N = 1,NUMNP
      WP(N) = 0.
      DWP(N) = 0.
      DRT(N) = 0.
      X = SIGP(N,1)
      Y = SIGP(N,2)
      COMM = (X+Y)/3.
      SIGP(N,1) = X - COMM
      SIGP(N,2) = Y - COMM
      SIGP(N,3) = 2.*SIGP(N,3)
      DO 120  J = 1,4
      SIG(N,J) = 0.
      TST(N,J) = 0.
```

```
      PST(N,J) = 0.
  120 EST(N,J) = 0.
      IF (IFLAG)  GO TO 135
C
C     TOTAL FORCES AND DISPLACEMENTS
C
      READ (1)  (((ETST(N,I,J),EPST(N,I,J),EEST(N,I,J),J=1,4),I=1,6),
     1 N=1,NUMEL),(F(I),R(I),FYP(I),I=1,NEQ)
      DO 130  N = 1,NEQ
      DIS(N) = DIS(N) + DR(N)
      F(N) = F(N) + DF(N)
  130 DF(N) = 0.
C
C     DO 300 . .  IS THE LOOP WHICH PROCESSES ELEMENT BY ELEMENT
C
  135 DO 300  NV = 1,NUMEL
C
C     INCREMENTAL ELEMENT STRAINS
C
      DO 150  I = 1,3
      DETST(I,1) = 0.
      DETST(I,2) = 0.
      DETST(I,3) = 0.
      ROT(I) = 0.
      K1 = NP(NV,I)
      J = IPERM(I)
      K2 = NP(NV,J)
      M = IPERM(J)
      A(M) = XORD(K2) - XORD(K1)
  150 B(M) = YORD(K1) - YORD(K2)
      AREA = A(3)*B(2) - A(2)*B(3)
      DO 160  L = 1,3
      L1 = IPERM(L)
      L2 = IPERM(L1)
      L3 = L + 3
      DO 160  N = 1,2
      D0 = BA(L, N)/AREA
      D1 = BA(L1,N)/AREA
      UV(L, L, N) = 3.*D0
      UV(L1,L, N) =   -D0
      UV(L2,L, N) =   -D0
      UV(L ,L3,N) = 4.*D1
      UV(L1,L3,N) = 4.*D0
  160 UV(L2,L3,N) =  0.
      DO 180  J = 1,6
      K = 2*NP(NV,J)
      X = DR(K-1)
      Y = DR(K)
      DO 180  I = 1,3
      C = U(I,J)
      D = V(I,J)
      DETST(I,1) = DETST(I,1) + C*X
      DETST(I,2) = DETST(I,2) + D*Y
      DETST(I,3) = DETST(I,3) + D*X + C*Y
  180 ROT(I) = ROT(I) + D*X - C*Y
      DO 190  I = 1,3
```

```
      I1 = IPERM(I)
      L = I + 3
      ROT(L) = (ROT(I)+ROT(I1))/2.
      DO 190  J = 1,3
  190 DETST(L,J) = (DETST(I,J)+DETST(I1,J))/2.
C
C     PLASTIC STRAIN INCREMENT
C
      DO 300  I = 1,6
      N = NP(NV,I)
      X = BETA(N)
      DO 220  J = 1,3
      DEPST(J) = 0.
  220 DEEST(J) = DETST(I,J)
      IF (X.EQ.0.)  GO TO 250
      S11 = SIGP(N,1)
      S22 = SIGP(N,2)
      S(1) = S11 + XU*S22
      S(2) = S22 + XU*S11
      S(3) = NUH*SIGP(N,3)
      DO 240  J = 1,3
      Y = SIGP(N,J)
      DO 230  K = 1,3
  230 DEPST(J) = DEPST(J) + DETST(I,K)*S(K)*X*Y
C
C     ELASTIC STRAIN AND STRESS INCREMENTS
C
  240 DEEST(J) = DEEST(J) - DEPST(J)
  250 DEEST(4) = XU*(DEEST(1)+DEEST(2))/(XU-1.)
      DEPST(4) = -DEPST(1)-DEPST(2)
      DETST(I,4) = DEEST(4)+DEPST(4)
      DESIG(1) = ER*(DEEST(1)+XU*DEEST(2))
      DESIG(2) = ER*(DEEST(2)+XU*DEEST(1))
      DESIG(3) =  G*DEEST(3)
C
C     INCREMENTAL PLASTIC WORK AND ELEMENT STRESSES
C
      DEWP = 0.
      DO 270  J = 1,3
      X = ESIG(NV,I,J)
      DX = DESIG(J)
      DEWP = DEWP + (X+.5*DX)*DEPST(J)
  270 ESIG(NV,I,J) = X + DX
      IF (DEWP.GT.0.)  EWP(NV,I) = EWP(NV,I) + DEWP
      WP(N) = WP(N) + EWP(NV,I)
      DWP(N) = DWP(N) + DEWP
      IF (T2)  GO TO 275
      DRT(N) = DRT(N) + DETST(I,4)
C
C     TOTAL ELEMENT STRAINS
C
  275 IF (IFLAG)  GO TO 290
      DO 280  J = 1,4
      ETST(NV,I,J) = ETST(NV,I,J) + DETST(I,J)
      EPST(NV,I,J) = EPST(NV,I,J) + DEPST(J)
      EEST(NV,I,J) = EEST(NV,I,J) + DEEST(J)
```

```
      TST(N,J) = TST(N,J) + ETST(NV,I,J)
      PST(N,J) = PST(N,J) + EPST(NV,I,J)
  280 EST(N,J) = EST(N,J) + EEST(NV,I,J)
C
C     COMPUTE ACTUAL STRESSES FOR FINITE DISPLACEMENT ANALYSIS
C
  290 IF (SMALD)  GO TO 295
      E11 = DETST(I,1)
      E22 = DETST(I,2)
      E12 = 0.5*DETST(I,3)
      E33 = DETST(I,4)
      OMEGA = ROT(I)
      C1 = E12 + OMEGA
      C2 = E12 - OMEGA
      S11 = ESIG(NV,I,1)
      S22 = ESIG(NV,I,2)
      S12 = ESIG(NV,I,3)
      ESIG(NV,I,1) = (1.-E22-E33)*S11 + C1*S12
      ESIG(NV,I,2) = (1.-E11-E33)*S22 + C2*S12
      ESIG(NV,I,3) = 0.5*(C2*S11+C1*S22) + (1.-0.5*(E11+E22)-E33)*S12
  295 DO 300  J = 1,3
      SIG(N,J) = SIG(N,J) + ESIG(NV,I,J)
  300 CONTINUE
      IF (T1)  READ (1)  XORD,YORD,RT
C
C     DO 400 . . . IS THE LOOP PROCESSING NODAL POINTS
C
C     AVERAGE NODE VALUES
C
      DO 400  N = 1,NUMNP
      CNT = COUNT(N)
      WP(N) = WP(N)/CNT
      DWP(N) = DWP(N)/CNT
      RT(N) = RT(N) + DRT(N)/CNT
      IF (IFLAG)  GO TO 315
      DO 310  J = 1,4
      TST(N,J) = TST(N,J)/CNT
      PST(N,J) = PST(N,J)/CNT
  310 EST(N,J) = EST(N,J)/CNT
  315 DO 320  J = 1,3
  320 SIG(N,J) = SIG(N,J)/CNT
      X =  SIG(N,1)
      Y =  SIG(N,2)
      XY2 = SIG(N,3)**2
      XY = X*Y
      DIF2 = (X-Y)**2
      C = 0.5*(X+Y)
      RAD = SQRT(0.25*DIF2+XY2)
      SIG(N,4) = C + RAD
      SIG(N,5) = C - RAD
      EQSIG = SQRT(DIF2+XY+3.*XY2)
      EQSG(N) = EQSIG
C
C     CHECK AND ENFORCE POINT YIELD CONDITION
C
      YEQSG(N) = SQRT(YP2+FAC*WP(N))
```

```
      SCAL(N) = 0.
      COMM = EQSIG/YEQSG(N)
      BETA(N) = 0.
      IF (DWP(N))  380,340,350
  340 IF (COMM.LE.0.999)   GO TO 380
  350 SCAL(N) = COMM
      DO 360  I = 1,6
  360 SIG(N,I) = SIG(N,I)/COMM
      DENM = A1*EQSIG**2 + A2*(X**2+Y**2) + A3*XY + A4*XY2
      BETA(N) = COMM**2/DENM
  380 DO 390  I = 1,3
  390 SIGP(N,I) = SIG(N,I)
  400 CONTINUE
      IF (SMALD)  GO TO 420
C
C     MODIFICATION OF COORDINATES FOR LARGE DISPLACEMENT ANALYSIS
C
      DO 410  N = 1,NUMNP
      K2 = 2*N
      XORD(N) = XORD(N) + DR(K2-1)
  410 YORD(N) = YORD(N) + DR(K2)
C
C     PREPARE NEXT STEP
C
  420 IF (.NOT.IFLAG)  GO TO 430
      IFLAG = .FALSE.
      RETURN
  430 NSTEP = NSTEP + 1
      IFLAG = .TRUE.
      IF (NSTEP.NE.NDUP2.AND.NSTEP.NE.NDUP1)   GO TO 440
      IF (NLOAD.GT.0)  CHI = CHI/2.
      IF (NID  .GT.0)  CHI = 2.*CHI
  440 RDIS = DIS(NEQRED)
      COMM = ABS(RDIS/ERDIS)
      IF (MFLAG.GT.0)  GO TO 450
      IF (COMM.GE.ALFA.OR.NSTEP.GE.MAXST)  MFLAG = 1
      GO TO 460
  450 MFLAG = MFLAG + 1
  460 L = MFLAG + 1
      GO TO (500,510,600,700), L
  500 XCHI = CHI
      GO TO 520
  510 XCHI = -CHI/100.
  520 IF (NLOAD.LE.0)  GO TO 550
      DO 540  N = 1,NEQ
  540 DF(N) = XCHI*FYP(N)
  550 IF (NID.LE.0)   GO TO 580
      DO 560  N = 1,NID
  560 DSD(N) = XCHI*SDYP(N)
  580 GO TO 680
  600 XCHI = - PSI
      DO 640  N = 1,NEQ
  640 DF(N) = -F(N)
  650 IF (NID.LE.0)  GO TO 680
      NLOAD = NID
      NID = 0
```

```
  680 REWIND 1
      WRITE (1) ((EWP(N,I),(ESIG(N,I,J),J=1,3),I=1,6),N=1,NUMEL),
     1 ( CCUNT(N),N=1,NUMNP)
      WRITE (1) (((ETST(N,I,J),EPST(N,I,J),EEST(N,I,J),J=1,4),I=1,6),
     1 N=1,NUMEL),(F(I),R(I),FYP(I),I=1,NEQ)
      IF (LARGD)  WRITE (1)  XORD, YORD, RT
      RFOR = FOR(NEQREF)
      COMF = RFOR/ERFOR
C
C     PRINT OF STRAINS AND STRESSES
C
      IPIN1 = .TRUE.
  700 IF(NSTEP.LT.NPRINT.AND.MFLAG.NE.1.AND.MFLAG.NE.3)  GO TO 750
      IPIN1 = .FALSE.
      NPRINT = NPRINT + NSPIN
  750 CALL PRINSL
C
C     STRESS CCNTOUR PLOT
C
  800 IF(NSTEP.LT.NGRAPH.AND.MFLAG.NE.1.AND.MFLAG.NE.3)  GO TO 900
      NGRAPH = NGRAPH + NGPIN
      CALL CNTPLT
C
  900 PSI = PSI + XCHI
      IF (MFLAG.GE.1)  IFLAG = .FALSE.
      RETURN
      END
```

```
$IBFTC STSL     LIST,DECK
      SUBROUTINE STEPSL
C
C     ****************************************************************
C     THIS SUBROUTINE ASSEMBLES THE COMPLETE INSTANTANEOUS STIFFNESS
C     MATRIX AND SOLVES FOR INCREMENTAL DISPLACEMENTS
C     ****************************************************************
C
      COMMON
     1 NUMEL, NUMCP, NUMNP, NUMBC, NEQ, NEQBC, NLOAD, NID, NEQRED,
     2 NEQREF, MAXST, NDUP1, NDUP2, NSTEP, NSPIN, NPRINT, NGPIN,
     3 NGRAPH, IBANDW, MAXBW, NSKEWD, MFLAG, IPIN1, IPIN2, INPL,
     4 IFLAG, SMALD, ALFA, ERDIS, ERFOR, COMM, COMF, CHI, PSI,
     5 EM, XU, TH, YP, YP2, XI, ER, G, FAC, A1, A2, A3, A4,
     6 EXTRAS(5), NP(80,6), XCRD(160), YORD(160), RT(160)
      COMMON /SOLARG/
     1 BETA(160), SIGP(160,3), DR(320), DF(320), NEBC(50),
     2 BANGLE(50), ND(10), DSC(10), SDYP(10)
      COMMON /STFARG /  ST(12,12), Y(3), X(3), ET, NU, THICK,
     1 BITA(6), SGP(6,3), SIGMA(6,3), RTH(6), EFLAG, SMLDIS
      COMMON /NLARG/ A(320,42)
      DIMENSION R(320), F(320), IPERM(3)
      EQUIVALENCE (F,DF), (R,DR)
      DATA IPERM /2,3,1/
      LOGICAL IFLAG, EFLAG, SMALD, SMLDIS
      REAL NU
C
C     INITIALIZE FOR ELEMENT STIFFNESS
C
      NN = NEQ
      MM = IBANDW
      ET = EM
      NU = XU
      SMLDIS = SMALD
      DO 110  I = 1,NN
      R(I) = F(I)
      DO 110  J = 1,MM
  110 A(I,J) = 0.
      DO 155  NV = 1,NUMEL
      DO 120  I = 1,3
      J = IPERM(I)
      K =  NP(NV,I)
      K1 = NP(NV,J)
      M = IPERM(J)
      X(M) = XCRD(K1) - XORD(K)
  120 Y(M) = YCRD(K) - YORD(K1)
      THICK = TH
      EFLAG = .TRUE.
      DO 130  I = 1,6
      K = NP(NV,I)
      RTH(I) = RT(K)
      BITA(I) = BETA(K)
      IF (BITA(I).GT.0.)  EFLAG = .FALSE.
      S11 = SIGP(K,1)
      S22 = SIGP(K,2)
      HYDR = (S11+S22)/3.
```

```
      SGP(I,1) = S11 - HYDR
      SGP(I,2) = S22 - HYDR
      SGP(I,3) =  2.*SIGP(K,3)
      DO 130  J = 1,3
  130 SIGMA(I,J) = SIGP(K,J)
C
C     COMPUTE ELEMENT STIFFNESS
C
      CALL STFNS
C
C     ADD TO TOTAL STIFFNESS MATRIX
C
      DO 150  I = 1,6
      IK = 2*I - 1
      L = NP(NV,I)
      NR = 2*L - 1
      DO 150  J = 1,6
      JK = 2*J - 1
      M = NP(NV,J)
      IF (L.GT.M)  GO TO 150
      NC = 2*(M-L) + 1
      A(NR,NC)   = A(NR,NC)    + ST(IK,JK)
      A(NR,NC+1) = A(NR,NC+1)  + ST(IK,JK+1)
      A(NR+1,NC) = A(NR+1,NC)  + ST(IK+1,JK+1)
      IF (NC.EQ.1)  GO TO 150
      A(NR+1,NC-1) = A(NR+1,NC-1) + ST(IK+1,JK)
  150 CONTINUE
  155 CONTINUE
      IF (IFLAG)  GO TO 165
      REWIND 2
      WRITE (2)  ((A(I,J), J=1,MM), I=1,NN)
C
C     MODIFICATION FOR IMPOSED DISPLACEMENTS
C
  165 IF (NID.LE.0)   GO TO 200
      DO 190  I=1,NID
      NR = ND(I)
      A(NR,1) = 1.
      DO 180  J=2,MM
      L = NR + J - 1
      IF (NN.LT.L)  GO TO 170
      R(L) = R(L) - A(NR,J)*DSD(I)
  170 A(NR,J) = 0.
      K = NR - J + 1
      IF (K.LE.0)  GO TO 180
      R(K) = R(K) - A(K,J)*DSD(I)
      A(K,J) = 0.
  180 CONTINUE
  190 R(NR) = DSD(I)
C
C     BOUNDARY CONDITIONS
C
  200 DO 240  M = 1,NEQBC
      NR = NEBC(M)
      PHI = BANGLE(M)
      IF (PHI.EQ.0.)  GO TO 220
```

```
      C = COS(PHI)
      S = SIN(PHI)
      NR1 = NR - 1
      A(NR1,1) = A(NR1,1)*C*C + 2.*A(NR1,2)*S*C + A(NR,1)*S*S
      R(NR1) = R(NR1)*C + R(NR)*S
      L = NR1
      DO 210  J = 3,MM
      A(NR1,J) = A(NR1,J)*C + A(NR,J-1)*S
      L = L - 1
      IF (L.LE.0)  GO TO 210
      A(L,J-1) = A(L,J-1)*C + A(L,J)*S
  210 CONTINUE
  220 A(NR,1) = 1.
      R(NR) = 0.
      DO 230  J = 2,MM
      A(NR,J) = 0.
      L = NR - J + 1
      IF (L.LE.0)  GO TO 230
      A(L,J) = 0.
  230 CONTINUE
  240 CONTINUE
C
C     REDUCTION OF MATRIX A AND VECTOR R
C
      NR = NN - 1
      DO 350  N = 1,NR
      M = N - 1
      PIVOT = A(N,1)
      MR = MIN0 (MM,NN-M)
      DO 320  L = 2,MR
      C = A(N,L)/PIVOT
      I = M + L
      J = 0
      DO 300  K = L,MR
      J = J + 1
  300 A(I,J) = A(I,J) - C*A(N,K)
      R(I) = R(I) - C*R(N)
  320 A(N,L) = C
  350 R(N) = R(N)/PIVOT
      R(NN) = R(NN)/A(NN,1)
C
C     BACK SUBSTITUTION
C
      DO 400  I = 2,NN
      M = NN - I
      N = M + 1
      MR = MIN0 (MM,I)
      DO 400  K = 2,MR
      L = M + K
  400 R(N) = R(N) - A(N,K)*R(L)
      IF (NSKEWD.LE.0)  GO TO 580
C
C     TRANSFORM SKEW DISPLACEMENTS TO X Y GLOBAL SYSTEM
C
      DO 550  M = 1,NEQBC
      NR = NEBC(M)
```

```
      PHI = BANGLE(M)
      IF (PHI.EQ.0.)  GO TO 550
      NR1 = NR - 1
      R(NR)  = R(NR1)*SIN(PHI)
      R(NR1) = R(NR1)*COS(PHI)
  550 CONTINUE
  580 IF (MFLAG.GE.1)  IFLAG = .FALSE.
      IF (IFLAG)  GO TO 800
C
C     RECOVER FORCE VECTOR
C
      REWIND 2
      READ  (2)  ((A(I,J), J=1,MM), I=1,NN)
      DO 750  N = 1,NN
      F(N) = A(N,1)*R(N)
      DO 750  J = 2,MM
      L = N + J - 1
      IF (L.GT.NN)  GO TO 730
      F(N) = F(N) + A(N,J)*R(L)
  730 K = N - J + 1
      IF (K.LE.0) GO TO 750
      F(N) = F(N) + A(K,J)*R(K)
  750 CONTINUE
      RETURN
  800 DO 850   N = 1,NN
  850 R(N) = 0.5*R(N)
      RETURN
      END
```

```
$IBFTC STFN    LIST,DECK
      SUBROUTINE STFNS
C
C     ******************************************************************
C     ELEMENT STIFFNESS SUBROUTINE FOR LST-P3
C     SMALL OR LARGE DISPLACEMENTS (CONSTANT  OR VARIABLE THICKNESS)
C     ******************************************************************
C
      COMMON /STFARG /  ST(12,12), B(3), A(3), ET, NU, THICK,
     1 BETA(6), SGP(6,3), SIGMA(6,3), RTH(6), EFLAG, SMLDIS
      DIMENSION U(3,6), V(3,6), UV(3,6,2), CX(3,3), CY(3,3),
     1 EP(3,3), E(3,3,6), F(3,3,3,3), IPERM(3), BA(3,2), S(3),
     2 HEL(3,3), HELARG(6,3,3), HPL(6,3,3), HPLARG(6,6,3,3), DUM(1) ,
     3 JXX(3,3), JYY(3,3), JXY(3,3), HG(3,3,6), AX(3), AY(3),
     4 SIGXX(6), SIGYY(6), SIGXY(6)
      EQUIVALENCE (BA,B), (E,ST), (EP,E(55)), (F,DUM), (U,UV),(V,UV(19))
      EQUIVALENCE (SIGXX,SIGMA(1)),(SIGYY,SIGMA(7)),(SIGXY,SIGMA(13))
      DATA IPERM /2,3,1/
      REAL JXX, JYY, JXY, NU, NUT, NUH
      LOGICAL EFLAG, SMLDIS
C
      DATA  HEL / 30., 15., 15.,  15., 30., 15.,  15., 15., 30. /
      DATA HELARG /
     1    6.0,  -2.0,  -2.0,  12.0,   4.0,  12.0,
     2    0.0,   0.0,  -1.0,   8.0,   4.0,   4.0,
     3    0.0,  -1.0,   0.0,   4.0,   4.0,   8.0,
     4    0.0,   0.0,  -1.0,   8.0,   4.0,   4.0,
     5   -2.0,   6.0,  -2.0,  12.0,  12.0,   4.0,
     6   -1.0,   0.0,   0.0,   4.0,   8.0,   4.0,
     7    0.0,  -1.0,   0.0,   4.0,   4.0,   8.0,
     8   -1.0,   0.0,   0.0,   4.0,   8.0,   4.0,
     9   -2.0,  -2.0,   6.0,   4.0,  12.0,  12.0 /
      DATA  HPL  /
     1   46.0,   2.0,   2.0,  50.0,  10.0,  50.0,
     2    7.0,   7.0,   1.0,  35.0,  15.0,  15.0,
     3    7.0,   1.0,   7.0,  15.0,  15.0,  35.0,
     4    7.0,   7.0,   1.0,  35.0,  15.0,  15.0,
     5    2.0,  46.0,   2.0,  50.0,  50.0,  10.0,
     6    1.0,   7.0,   7.0,  15.0,  35.0,  15.0,
     7    7.0,   1.0,   7.0,  15.0,  15.0,  35.0,
     8    1.0,   7.0,   7.0,  15.0,  35.0,  15.0,
     9    2.0,   2.0,  46.0,  10.0,  50.0,  50.0  /
      DATA  (((HPLARG(L,K,I,1),L=1,6),K=1,6),I=1,3)   /
     1  450.0, -67.0, -67.0, 306.0,  38.0, 306.0,
     2   -4.5,  10.0,  -2.5,  27.0,   9.0,   3.0,
     3   -4.5,  -2.5,  10.0,   3.0,   9.0,  27.0,
     4  126.0, -73.5, -80.5, 630.0, 154.0, 294.0,
     5  -21.0, -10.5, -10.5,  84.0,  84.0,  84.0,
     6  126.0, -80.5, -73.5, 294.0, 154.0, 630.0,
     7   48.0, -14.5,  -9.5,  76.0,   9.0,  38.0,
     8  -14.5,  48.0,  -9.5,  76.0,  38.0,   9.0,
     9   -2.0,  -2.0,   5.0,   2.0,   9.0,   9.0,
     1   -3.5,  -3.5, -56.0, 490.0, 154.0, 154.0,
     2  -31.5,   3.5, -21.0, 126.0, 154.0,  84.0,
     3    3.5, -31.5, -21.0, 126.0,  84.0, 154.0,
     4   48.0,  -9.5, -14.5,  38.0,   9.0,  76.0,
```

```
     5   -2.0,    5.0,   -2.0,    9.0,    9.0,    2.0,
     6  -14.5,   -9.5,   48.0,    9.0,   38.0,   76.0,
     7    3.5,  -21.0,  -31.5,  154.0,   84.0,  126.0,
     8  -31.5,  -21.0,    3.5,   84.0,  154.0,  126.0,
     9   -3.5,  -56.0,   -3.5,  154.0,  154.0,  490.0 /
      DATA  (((HPLARG(L,K,I,2),L=1,6),K=1,6),I=1,3)   /
     1   48.0,  -14.5,   -9.5,   76.0,    9.0,   38.0,
     2  -14.5,   48.0,   -9.5,   76.0,   38.0,    9.0,
     3   -2.0,   -2.0,    5.0,    2.0,    9.0,    9.0,
     4   -3.5,   -3.5,  -56.0,  490.0,  154.0,  154.0,
     5  -31.5,    3.5,  -21.0,  126.0,  154.0,   84.0,
     6    3.5,  -31.5,  -21.0,  126.0,   84.0,  154.0,
     7   10.0,   -4.5,   -2.5,   27.0,    3.0,    9.0,
     8  -67.0,  450.0,  -67.0,  306.0,  306.0,   38.0,
     9   -2.5,   -4.5,   10.0,    3.0,   27.0,    9.0,
     1  -73.5,  126.0,  -80.5,  630.0,  294.0,  154.0,
     2  -80.5,  126.0,  -73.5,  294.0,  630.0,  154.0,
     3  -10.5,  -21.0,  -10.5,   84.0,   84.0,   84.0,
     4    5.0,   -2.0,   -2.0,    9.0,    2.0,    9.0,
     5   -9.5,   48.0,  -14.5,   38.0,   76.0,    9.0,
     6   -9.5,  -14.5,   48.0,    9.0,   76.0,   38.0,
     7  -21.0,    3.5,  -31.5,  154.0,  126.0,   84.0,
     8  -56.0,   -3.5,   -3.5,  154.0,  490.0,  154.0,
     9  -21.0,  -31.5,    3.5,   84.0,  126.0,  154.0 /
      DATA  (((HPLARG(L,K,I,3),L=1,6),K=1,6),I=1,3)   /
     1   48.0,   -9.5,  -14.5,   38.0,    9.0,   76.0,
     2   -2.0,    5.0,   -2.0,    9.0,    9.0,    2.0,
     3  -14.5,   -9.5,   48.0,    9.0,   38.0,   76.0,
     4    3.5,  -21.0,  -31.5,  154.0,   84.0,  126.0,
     5  -31.5,  -21.0,    3.5,   84.0,  154.0,  126.0,
     6   -3.5,  -56.0,   -3.5,  154.0,  154.0,  490.0,
     7    5.0,   -2.0,   -2.0,    9.0,    2.0,    9.0,
     8   -9.5,   48.0,  -14.5,   38.0,   76.0,    9.0,
     9   -9.5,  -14.5,   48.0,    9.0,   76.0,   38.0,
     1  -21.0,    3.5,  -31.5,  154.0,  126.0,   84.0,
     2  -56.0,   -3.5,   -3.5,  154.0,  490.0,  154.0,
     3  -21.0,  -31.5,    3.5,   84.0,  126.0,  154.0,
     4   10.0,   -2.5,   -4.5,    9.0,    3.0,   27.0,
     5   -2.5,   10.0,   -4.5,    9.0,   27.0,    3.0,
     6  -67.0,  -67.0,  450.0,   38.0,  306.0,  306.0,
     7  -10.5,  -10.5,  -21.0,   84.0,   84.0,   84.0,
     8  -80.5,  -73.5,  126.0,  154.0,  630.0,  294.0,
     9  -73.5,  -80.5,  126.0,  154.0,  294.0,  630.0 /
      DATA  HG / 6., 0., 0., 0.,-2.,-1., 0.,-1.,-2.,
     1           -2., 0.,-1., 0., 6., 0.,-1., 0.,-2.,
     2           -2.,-1., 0.,-1.,-2., 0., 0., 0., 6.,
     3           12., 8., 4., 8.,12., 4., 4., 4., 4.,
     4            4., 4., 4., 4.,12., 8., 4., 8.,12.,
     5           12., 4., 8., 4., 4., 4., 8., 4.,12. /
C
      NUT = 1. - NU
      NUH = NUT/2.
      ER = ET/(1.-NU**2)
      DO 100  I = 1,81
  100 DUM(I) = 0.
      IF (EFLAG)  GO TO 170
```

```
C
C     PLASTIC OR PARTIALLY PLASTIC ELEMENT
C
      DO 140  K = 1,6
      S(1) = SGP(K,1) + NU*SGP(K,2)
      S(2) = SGP(K,2) + NU*SGP(K,1)
      S(3) = NUH*SGP(K,3)
      DO 130  I = 1,3
      DO 120  J = 1,3
  120 EP(I,J)  =  -BETA(K)*SGP(K,I)*S(J)
  130 EP(I,I) = 1. + EP(I,I)
      DO 140  J = 1,3
      E(1,J,K) = ER*(EP(1,J) + NU*EP(2,J))
      E(2,J,K) = ER*(EP(2,J) + NU*EP(1,J))
  140 E(3,J,K) = ER*NUH*EP(3,J)
      SC = 1920.
      IF (SMLDIS)  GO TO 150
      SC = 40320.
      DO 145  K = 1,6
      DO 145  I = 1,3
      DO 145  J = I,3
      HPL(K,I,J) = 0.
      DO 145  L = 1,6
  145 HPL(K,I,J) = HPL(K,I,J) + HPLARG(L,K,I,J)*RTH(L)
  150 DO 160  L = 1,3
      DO 160  M = 1,3
      DO 160  I = 1,3
      DO 160  J = I,3
      DO 155  K = 1,6
  155 F(I,J,L,M) = F(I,J,L,M) + HPL(K,I,J)*E (L,M,K)
  160 F(J,I,L,M) = F(I,J,L,M)
      GO TO 200
C
C     ELASTIC ELEMENT
C
  170 SC = 360.
      IF (SMLDIS)  GO TO 185
      DO 180  I = 1,3
      DO 180  J = I,3
      HEL(I,J) = 0.
      DO 175  L = 1,6
  175 HEL(I,J) = HEL(I,J) + HELARG(L,I,J)*RTH(L)
  180 HEL(J,I) = HEL(I,J)
  185 DO 190  I = 1,3
      DO 190  J = 1,3
      F(I,J,1,1) = ER*HEL(I,J)
      F(I,J,2,2) = F(I,J,1,1)
      F(I,J,1,2) = F(I,J,1,1)*NU
      F(I,J,2,1) = F(I,J,1,2)
  190 F(I,J,3,3) = F(I,J,1,1)*NUH
C
C     STRAINS FROM DISPLACEMENTS
C
  200 DO 210  L = 1,3
      L1 = IPERM(L)
      L2 = IPERM(L1)
```

```
      L3 = L + 3
      DO 210  N = 1,2
      D0 = BA(L,N)
      D1 = BA(L1,N)
      UV(L ,L,N) = 3.*D0
      UV(L1,L,N) =  - D0
      UV(L2,L,N) =  - D0
      UV(L ,L3,N) = 4.*D1
      UV(L1,L3,N) = 4.*D0
  210 UV(L2,L3,N) = 0.
      AREA = A(3)*B(2) - A(2)*B(3)
      COMM = THICK/(SC*AREA)
C
C     NODAL FORCES FROM STRAINS
C
      DO 300  I = 1,6
      DO 240  L = 1,3
      DO 240  N = 1,3
      X1 = 0.
      X2 = 0.
      DO 230  K = 1,3
      X = U(K,I)
      Y = V(K,I)
      X1 = X1 + X*F(K,L,1,N) + Y*F(K,L,3,N)
  230 X2 = X2 + Y*F(K,L,2,N) + X*F(K,L,3,N)
      CX(L,N) = X1*COMM
  240 CY(L,N) = X2*COMM
C
C     ELEMENT STIFFNESS
C
      K2 = 2*I
      K1 = K2 - 1
      DO 300  J = I,6
      L2 = 2*J
      L1 = L2 - 1
      X1 = 0.
      X2 = 0.
      X3 = 0.
      X4 = 0.
      DO 280  K = 1,3
      X = U(K,J)
      Y = V(K,J)
      X1 = X1 + CX(K,1)*X + CX(K,3)*Y
      X2 = X2 + CX(K,2)*Y + CX(K,3)*X
      X3 = X3 + CY(K,1)*X + CY(K,3)*Y
  280 X4 = X4 + CY(K,2)*Y + CY(K,3)*X
      ST(K1,L1) = X1
      ST(L1,K1) = X1
      ST(K1,L2) = X2
      ST(L2,K1) = X2
      ST(K2,L1) = X3
      ST(L1,K2) = X3
      ST(K2,L2) = X4
  300 ST(L2,K2) = X4
```

```
      IF (SMLDIS) RETURN
C
C     GEOMETRIC STIFFNESS FOR FINITE DISPLACEMENT ANALYSIS
C
      DO 350  I = 1,3
      DO 350  J = 1,3
      JXX(I,J) = 0.
      JYY(I,J) = 0.
      JXY(I,J) = 0.
      DO 350  K = 1,6
      COMM = FG(I,J,K)
      JXX(I,J) = JXX(I,J) + COMM*SIGXX(K)
      JYY(I,J) = JYY(I,J) + COMM*SIGYY(K)
  350 JXY(I,J) = JXY(I,J) + COMM*SIGXY(K)
      AVTH = (RTH(1)+RTH(2)+RTH(3)+RTH(4)+RTH(5)+RTH(6))/6. *THICK
      COMM = AVTH /(360.*AREA)
      DO 400  J = 1,6
      L2 = 2*J
      L1 = L2 - 1
      DO 360  I = 1,3
      AX(I) = 0.
      AY(I) = 0.
      DO 360  K = 1,3
      X = U(K,J)
      Y = V(K,J)
      AX(I) = AX(I) + JXX(I,K)*X + JXY(I,K)*Y
  360 AY(I) = AY(I) + JXY(K,I)*X + JYY(I,K)*Y
      DO 400  I = 1,6
      K2 = 2*I
      K1 = K2 - 1
      X = 0.
      DO 380  K = 1,3
  380 X = X + U(K,I)*AX(K) + V(K,I)*AY(K)
      X = X*COMM
      ST(K1,L1) = ST(K1,L1) + X
  400 ST(K2,L2) = ST(K2,L2) + X
      RETURN
      END
```