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Authors

Anholt, R.

Gould, H.

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R. Anholt and H. Gould

November 1985

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RELATIVISTIC HEAVY-ION-ATOM COLLISIONS

R. Anholt

Department of Physics

Stanford University

Stanford, California 94305

Harvey Gould

Materials and Molecular Research Division

Lawrence Berkeley Laboratory, University of California

Berkeley, California 94720

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I. INTRODUCTION

The coupling of Lawrence Berkeley Laboratory's heavy ion linear accelerator (Super-HILAC) with its relativistic synchrotron (Bevatron) to produce the Bevalac has opened up new frontiers in atomic-collision and atomic-structure physics at relativistic velocities. This upgrading of 1950's and 1960's accelerator technology has produced an accelerator capable of accelerating uranium ions to energies of ~ 1 -GeV/amu (total energy 238 GeV), which makes possible, for the first time, the study of collisions and interactions of the heaviest few-electron ions.

From the point of view of atomic-collision physics, the high relative velocities obtainable with relativistic projectiles is of foremost interest. Many fundamental processes in atomic-collision physics scale with the ratio of the ion velocity to the velocity of the active electron. With relativistic projectiles one can obtain velocities that are high with respect to the fastest electron in the heaviest atom, the uranium K electron. The ions can be stripped, so that one can study one- or two-electron, high-Z ions where both the atomic structure problem and the treatment of scattering processes are greatly simplified. Most theories of scattering processes are greatly simplified also. While scattering theories at low relative velocities are becoming increasingly refined only by employing numerically intensive coupled-channel calculations including large numbers of basis states, at relativistic velocities one can still make use of high-velocity theories such as the first Born approximation (Bethe 1930, 1932) for ionization and excitation processes and the second-Born (Drisko 1955), eikonal (Eichler 1985), strong potential Born (Macek and Alston 1982) or impulse approximations (Briggs 1977, Amundsen and Jakubassa 1980) for capture processes.

Several effects have been closely studied with relativistic heavy ions. For example, the binding and polarization effects on ionization and excitation processes at intermediate velocities (Basbas et al. 1978), target atomic screening and antiscreening effects on ionization and excitation (McGuire et al. 1981), and the importance of electron transfer from excited target states into excited projectile states (Meyerhof et al. 1985) have been examined.

What is most attractive, however, is that the atomic-structure and collision physics at high relative velocities are sufficiently simple that one can begin to address important problems long thought to be intractable. One example of this is the complex interactions occurring inside solid targets (Betz 1972). The influence of excited states of the projectile on producing higher charge states of ions in solid-target stripping foils over gas-targets can be studied experimentally in a regime that also can be approached theoretically. Discouraged by the difficulties posed by solid-target research, in many atomic-collision-physics studies experiments are done exclusively with gas targets, thus limiting experimental possibilities due to the paucity of monatomic gases in nature. However, in relativistic heavy-ion atomic physics one can confront these problems head on, solve them, and derive rules and formulas that are possibly applicable to both relativistic and nonrelativistic projectiles in gas and solid targets.

From the point of view of atomic-structure physics, the study of one- and two-electron uranium and other high-Z ions allows the examination of higher-order corrections in $Z\alpha$ (α is the fine-structure constant) to the quantum-electrodynamic (QED) self-energy contribution to the binding energy and to the anomalous magnetic moment of the electron bound in a Coulomb field. Some of these cannot be measured using low-Z ions. For example, in few-electron

uranium, the self-energy contribution to the binding energy comes predominantly from terms which are of high order in $Z\alpha$. These terms make almost no contribution in low-Z Lamb-shift measurements. Consequently, even a modest-precision Lamb-shift measurement in high-Z ions like uranium can provide a powerful test of QED in the high-Z, strong-field limit.

In this review, we will depart from the structure normally used in scientific articles, and will merge the theoretical discussion with the discussion of the experiments and results. We retain a discussion of the experimental methods in Sect. II, and discuss ionization processes (Sect. III), electron-capture processes (IV), collisions in solid targets (V), x-ray continuum production in Sect. VI, ultrarelativistic collisions in Sec. VII, atomic-structure experiments in Sec. VII, and the conclusions are in Sec. IX.

We define at the outset the projectile atomic number Z_p , the target atomic number Z_t , the ion velocity v , and $\beta=v/c$, where c is the speed of light. The ion kinetic energy is equal to $(\gamma-1)Mc^2$, where M is the nuclear mass and $\gamma=(1-\beta^2)^{-1/2}$. The K-shell electron binding energy is E_K , $\alpha = e^2/\hbar c = 1/137$ is the fine-structure constant, and, in general, in theories of ionization and capture where Z_p and Z_t are not specified, we denote Z_x by the perturbing charge and Z by the charge to which the active electron is initially attached. v_K is the initial velocity of the bound K electron and is equal to Z in atomic units.

II. EXPERIMENT

At the end of 1985 only first generation experiments using relativistic heavy ions have been completed (Crawford 1979, Salamon et al. 1981, Ahlen and Tarle' 1983, Waddington et al. 1983, Gould et al. 1984, 1985, Anholt et al. 1977, 1984a, 1984b, 1985, 1986, Heckman et al. 1984, Meyerhof et al. 1985, Thieberger et al. 1985.) The methods employed to study atomic collisions of very heavy ions at relativistic velocities have been limited by the 10^5 to 10^7 ions/pulse (one pulse every 4 to 6 seconds) which can be delivered by the presently only available relativistic heavy ion accelerator, the Bevalac. The combination of a low intensity and a fairly large emittance of 30π mm-mR has so far discouraged attempts to measure the impact-parameter or scattering-angle dependence of inner-shell ionization or capture. Channeling experiments may be possible. X-ray measurements with very thin solid targets or gas targets is possible, but requires techniques for suppressing background due target electron bremsstrahlung, secondary electrons (delta rays) and nuclear disintegration.

On the other hand, the use of relativistic heavy ions permits several experimental simplifications. For example, the main source of error in x-ray production cross section measurements at low velocities is the beam intensity normalization, determined either by beam current integration or Rutherford scattering normalization (Anholt 1985b). With relativistic heavy ion, the cross sections are often very large, so one can count every incident ions with $\approx 100\%$ efficiency, using surface barrier or scintillator detectors. The vacuum requirements for relativistic (heavy) ions are not stringent, and if one is not interested in the projectile charge state (or has already analyzed it) one can pass the beam through a thin window and count the particles in air.

In this section we discuss two kinds of first generation experiments: x-ray

cross section measurements and projectile charge changing cross section measurements.

A. X-Ray Cross Section Measurements

In measuring x-ray cross sections using relativistic heavy ions, which can pass through several grams/cm² of matter, an active collimation system has been used (Anholt et al. 1984b). The relativistic beam is arranged to pass through a ≈ 30 cm² thin metal foil target placed between a pair of about 1 cm² active-area surface-barrier or particle scintillator detectors. The number of projectile ions passing through both particle detectors is counted in coincidence with detected x rays coming from the target. The target is much larger than the particle detectors to insure that an ion passing through both particle detectors must also pass through the target. This method has ^{been} used by Anholt et al. (1984b) for relativistic heavy ions and by Bak et al. (1983, 1985) for a mixed beam of protons, pions, and electrons (where the particle detectors also identified the particle).

At high particle intensities surface barrier detectors and scintillator photo-multipliers begin to saturate, so ionization chambers have been used. For absolute measurements they are calibrated against single-particle detectors at lower particle fluxes or against known cross sections such as that for the $^{12}\text{C}(p,pn)^{11}\text{C}$ nuclear reaction (Anholt et al. 1976). The conversion of x-ray counts to x-ray production cross sections is discussed in Hoffmann et al. (1979), Anholt et al. (1976, 1984b), and Bak et al. (1985).

Figure 1 illustrates the kinds of x-ray emission processes observed at relativistic velocities. Prominent features in the x-ray spectrum produced by uranium colliding with a uranium target are the K x rays coming from the target uranium atom and K x rays coming from the projectile uranium ion. The

target x rays produce a sharp peak in the spectra since the target atom is at rest and does not recoil significantly when the projectile ion passes by and excites a K vacancy. The projectile x rays, viewed perpendicular to the beam, are shifted due to the transverse Doppler shift, and are Doppler broadened due to the range of angles subtended by the x-ray detector. The projectile $K\alpha$ x rays are clearly seen, but the less intense $K\beta$ x rays are not seen above the continuum x-ray background.

Also seen in the U + Be spectrum (Fig. 1) is a broad peak near 275 keV, due to the radiative capture of target electrons into the K-shell of the uranium projectile. Radiative electron capture is the inverse of the photoelectric effect. An electron, at rest in the laboratory frame, has a kinetic energy of 230 keV in the rest frame of the 422 MeV/amu projectile. When the electron is captured into the projectile K-shell, a photon is emitted whose energy is equal to the electron kinetic energy plus the K-shell binding energy. In the laboratory frame the photon is Doppler shifted. Viewed perpendicular to the beam direction, the photon energy in the laboratory is $E'_x = E_x/\gamma$ where E_x is the photon energy in the rest frame of the projectile and $\gamma = (1-\beta^2)^{-1/2}$ is the relativistic energy factor with $\beta = v/c$. A smaller peak due to the radiative capture into the L- and higher projectile shells is also present, but is difficult to separate from the continuum x-ray background.

Primary and secondary electron bremsstrahlung account for most of the background. Primary bremsstrahlung is produced by the target electron scattering from the projectile nucleus (Yamadera et al. 1981 Anholt et al. 1986). It is a fundamental process that cannot be removed unless one removes all of the target electrons. For a 422 MeV/amu projectile, the highest energy photon which one can produce is one in which the electron is left at rest after bremsstrahlung,

so that $E_{\text{max}} = (\gamma - 1)mc^2 = 230 \text{ keV}$. In the projectile frame, this photon energy is Doppler shifted and extends from 590 keV in the forward direction down to 92 keV in the backward direction and is 164 keV at 90° . Primary bremsstrahlung is readily seen in U + Be collisions, Fig 1. For high-Z targets, bremsstrahlung from secondary electrons produced by Rutherford scattering is the dominant process for producing radiation (Folkmann et al.1973, Ishii et al.1977). In this two-step process target electrons are first ejected by Rutherford scattering on the projectile, then ^{emit} bremsstrahlung in collisions with other target nuclei, or if they leave the target, with other near-by matter. Because the bremsstrahlung cross sections scale as Z_t^2 , secondary bremsstrahlung is the dominant source of continuum x rays in high-Z targets. A target electron, scattered by a 422 MeV/amu ion will have an energy of 1.1 MeV if scattered forward and zero if scattered perpendicular to the projectile. Bremsstrahlung of secondary electrons thus produces a photon continuum to a maximum energy in the laboratory frame of 1.1 MeV.

To first order, relativistic ion-induced target K-shell ionization cross sections are the K x-ray production cross sections divided by the K-shell fluorescence yield. This neglects two background effects, secondary excitation by ionized target electrons (delta rays) ^{or} photoelectric excitation by primary and secondary electron bremsstrahlung radiation, and target K-electron capture. The apparent cross sections for ion-induced K-shell ionization in thick targets are as much as 30% larger than for thin targets due to background effects. Their subtraction has been discussed by Jarvis et al.(1972) and Anholt et al.(1976, 1984b). These effects can be largely avoided by using very thin targets.

The other background effect is the capture of target K-electrons by the nearly bare projectiles (Gray et al.1976). The K-capture contribution to the total target K-vacancy yield is given by

$$Y_{Kc} = \int_0^T dT' [(\sigma_{KK} + \sigma_{KH})Y_0(T') + (\frac{1}{2}\sigma_{KK} + \sigma_{KH})Y_1(T') + \sigma_{KH}Y_2(T') + \dots], \quad (1)$$

where $Y_n(T)$ is the fractional yield of projectiles carrying n electrons, σ_{KK} is the cross section for the capture of a target K-electron into the K-shell of a bare projectile, σ_{KH} is the cross section for capture of a target K-electron into any higher shell of the projectile, and T is the target thickness (in units of atoms per unit area). For targets of equilibrium thickness, the contribution to the total K-vacancy production cross section is given by (Anholt et al. 1985a)

$$\sigma_{Kc} = (\sigma_{KK} + \sigma_{KH})F_0 + (\frac{1}{2}\sigma_{KK} + \sigma_{KH})F_1 + \sigma_{KH}F_2 + \dots, \quad (2)$$

where F_n is the equilibrium fraction of projectiles carrying n electrons. The evaluation of the capture contribution requires knowledge of the capture cross sections σ_{KK} and σ_{KH} and the projectile charge state fractions. The capture contributions to the total K-vacancy production cross sections can generally be neglected for highly asymmetric collisions. They are important for some measurements with relativistic heavy ions, e.g. 82 MeV/amu to 200 MeV/amu Xe ions incident on Ag ($Z=47$) and heavier targets (Anholt et al. 1984b, 1985).

Projectile K x-ray production cross sections, while less straight forward to interpret, provide information about the states of ions in matter. They are discussed in Sect. V.B.

B. Charge Changing Cross Section Measurements

Charge changing cross sections can be studied by passing a beam of known charge state composition through a target and measuring the fraction of ions that have either captured or lost an electron. Figure 2 shows the experimental arrangement used at the Lawrence Berkeley Laboratory's Bevalac (Greiner et al. 1974, Gould et al. 1984, 1985). A collimated beam of a single charge state passes through a chamber containing targets that can be remotely inserted into

the beam. The resulting charge states are analyzed by a pair of dipole magnets and are detected by a 50-cm long position sensitive proportional counter (Borkowski and Kopp 1975). Between the targets and the dipole magnets the beam passes through a pair of focusing quadrupole magnets. After the analyzing magnets, the beam passes through a thin window at the end of the vacuum chamber. The detector sits in air. A charge state distribution produced by passing 962 MeV/amu U^{68+} ions through 7.1 mg/cm² mylar is shown in Fig. 3.

Charge changing cross sections may be measured by the "thick target" method (Datz et al. 1970, Betz 1972) or the "thin target" method (Nikolaev 1965, Datz et al. 1970). In both cases one passes a beam of known charge states through targets of known thickness and measures the resulting charge state fractions. In the thin target method only the linear part of the charge state fraction versus target thickness curve is used. In the thick target method the target thickness is increased as far as possible often until the equilibrium thickness charge state distribution is approached.

The thick target method is useful in measuring cross sections involving charge states which are not present in the incident beam. Since many charge states may be studied with a single incident charge state, the technique is useful in survey experiments, where a large number of cross sections can be obtained without retuning of the accelerator. It is also used in experiments with limited beam intensity (Alonso and Gould 1982, Gould et al. 1984). To extract cross sections, one least-squares-fits calculated cross sections to the thickness dependent charge fractions (Datz et al. 1970, Betz 1972). Since many unknown cross sections are to be determined, one must choose the target thicknesses such that each charge state of interest is present for as many different target thicknesses as possible. If multiple capture and ionization needs to be included, finding the

"best fit" can be tedious.

In the "thin-target" method (Nikolaev 1965, Datz et al. 1970) the targets are sufficiently thin so that in principal the yield increases linearly with target thickness. The ionization or capture cross section is just the yield Y_n divided by the target thickness. In practice finite-thickness targets are used, so one fits Y_n/T to a straight line to obtain the $T=0$ intercept. This method is simple and can give unambiguous cross sections (Meyerhof et al. 1985). A disadvantage of the "thin-target" method is that it requires collecting data using target thicknesses for which only a small percentage of the ions undergo a charge changing collision. Consequently the counting statistics may be poor. Errors can occur both because of the low absolute count rates, limited by beam intensity, and because the relatively large count rate for the incident charge state can cause pile up and dead time in the detector leading to measurement error.

The thin-target charge state measurements generally provide a much cleaner way of studying ionization processes than x-ray measurements of target ionization. In target ionization experiments the projectile both ionizes the target and captures target electrons. In charge state measurements of projectile ionization the target is neutral and there is no possibility for it to capture projectile electrons. One also avoids the secondary processes (Anholt et al. 1986) and fluorescence-yield uncertainties. However in the theory of projectile ionization, one must account for the screening of the perturbing target nucleus by the target electrons.

III. IONIZATION PROCESSES

A. Overview

Inner-shell ionization with relativistic heavy ions can be classified according to three types: target K-vacancy production, projectile ionization in low-Z ions, and projectile ionization in high-Z ions (Anholt et al. 1985). It is important to distinguish the physical effects present for each type, which are summarized in Table I.

Since relativistic heavy ions travel at velocities that are high with respect to the electrons, a logical starting point in the analysis of electron ionization is the first (plane-wave) Born approximation (PWBA) which has been formulated by Bethe (1930, 1932) and Møller (1932), and has been widely applied to analyze inner-shell ionization at nonrelativistic velocities (Merzbacher and Lewis 1958, Basbas et al. 1973, 1978). These theories generally use hydrogenic electron wave functions. For target K vacancy production, one uses effective charges (Z_t is replaced by $Z^* = Z_t - 0.3$ for the K-shell electron), and one replaces the hydrogenic K-electron binding energy $\frac{1}{2}(Z^*)^2$ by the experimental value $E_K = 0.75 \frac{1}{2}(Z^*)^2$ for neutral atoms. Nonrelativistic hydrogenic wave functions and charges Z^* are valid for low-Z projectile ionization, but for high-Z projectile ionization, Dirac electronic wave functions and binding energies $E_K [= (1-s)mc^2$, where $s^2 = 1 - \alpha^2 Z^2$ and $\alpha = 1/137$], should be used.

An effect not present for target K-shell ionization by nearly bare projectiles is the screening of the perturbing nuclear charge by attached projectile electrons. The cross sections for ionizing low-Z projectiles by neutral target atoms are significantly reduced by the screening of the perturbing target nucleus by the target electrons (McGuire et al. 1981). The

screening has less effect on the cross sections for ionizing high- Z relativistic projectiles, but due to contributions to projectile ionization by the perturbations of the Z_t separate target electrons, the ionization cross sections vary as $Z_p^2 + Z_t$.

In the nonrelativistic PWBA, the interaction Hamiltonian consists of just the Coulomb interaction between the perturbing nucleus and electron. For relativistic ions, the PWBA also includes the current-current interaction between the perturbing nucleus and active electron (Møller 1932; Davidovic et al. 1978; Anholt 1979). Including this interaction gives additional incoherent contributions to inner-shell ionization and excitation called the transverse excitation cross sections to distinguish them from the longitudinal contributions coming mainly from the static Coulomb interaction. The transverse ionization cross sections have been calculated for target K-vacancy production using hydrogenic wave functions following similar methods used for nonrelativistic projectiles (Merzbacher and Lewis 1958). At 5-GeV/amu, the transverse and longitudinal ionization cross sections are about equal, but the transverse cross sections continue to increase as $\ln Y^2$ at high energies where the longitudinal ones approach a constant. The transverse contributions are also present for projectile ionization. For low- Z projectiles, target screening reduces their magnitude considerably (Gillespie et al. 1978; Anholt 1985a).

The PWBA is based on first-order perturbation theory, and tends to break down when the perturbing potential becomes very large. The nonlinear response of the amplitude for ionization to an increase of the perturbing charge is viewed as the effect of the distortion of the electronic wave functions (Basbas et al. 1978). This distortion reduces the ionization cross

sections at low ion velocities, but increases them at high velocities. At low velocities, the electrons form diatomic molecular orbitals around the projectile and target nuclei, thereby becoming more tightly bound and more difficult to excite, so smaller ionization cross sections are seen (Basbas et al 1973; Anholt and Meyerhof 1977). At high ion velocities, the polarization of the electron clouds decreases the electron-perturbing-nucleus distance, thereby increasing the strength of the perturbing potential, leading to higher ionization cross sections (Basbas et al. 1971a, 1971b, 1978). These distortion effects affect high-Z projectile ionization and target K-vacancy production by relativistic projectiles. However, low-Z projectile ionization is less affected because the relative velocity v/v_K is very high.

Finally, K-electron capture contributions [Eqs.(1)-(2)] to total K-vacancy production cross sections are only present when target K-vacancy production is studied. Fully occupied target atoms cannot capture projectile electrons efficiently, so projectile ionization is relatively unaffected by capture considerations.

Section III.B presents the formulation of the PWBA for ionization by relativistic projectiles, including transverse excitation. Sections C and D discuss the evaluation of the PWBA cross sections numerically and in the dipole approximation. The target-nucleus screening effect on projectile ionization is discussed in Sect.E, and wave-function distortion effects are discussed in Sect. F.

B. The Plane Wave Born Approximation

The theory of target electron ionization by incident ions is the basis of our knowledge of electronic stopping powers (Fano 1963; Bethe and Ashkin

1953), which are crucial to the design of apparatus and the interpretation of measurements in nuclear and high-energy physics. The theory was formulated by Møller (1932), Bethe (1930, 1932), and Bethe and Fermi (1932). The dipole approximation was used to calculate stopping powers (see e.g. Fano 1963). In the Lorentz gauge, the interaction between the perturbing nucleus and electron is given by (Moiseiwitsch 1980, 1985, Amundsen and Aashamar 1981):

$$H' = -eV(\mathbf{r},t) + e\vec{\alpha}\cdot\mathbf{A}(\mathbf{r},t), \quad (3)$$

where the Coulomb interaction is given by

$$V(\mathbf{r},t) = \int [\rho(\mathbf{R})]_{\text{ret}} \frac{d\mathbf{R}}{|\mathbf{r}-\mathbf{R}|}, \quad (4)$$

the A field is given by

$$\mathbf{A}(\mathbf{r},t) = \frac{e}{c} \int [\mathbf{j}(\mathbf{R})]_{\text{ret}} \frac{d\mathbf{R}}{|\mathbf{r}-\mathbf{R}|}, \quad (5)$$

$$\rho(\mathbf{R}) = Z_X e \phi_{\mathbf{K}_f}^\dagger(\mathbf{R}) \phi_{\mathbf{K}_i}(\mathbf{R}),$$

$$\mathbf{j}(\mathbf{R}) = Z_X e c \phi_{\mathbf{K}_f}^\dagger(\mathbf{R}) \vec{\alpha}_R \phi_{\mathbf{K}_i}(\mathbf{R}), \quad (6)$$

$\mathbf{r}$ denotes the electron coordinant, $\mathbf{R}$ the nuclear coordinant, Z_X is the charge of the perturbing nucleus, $\vec{\alpha}$ is the Dirac matrix, $[\rho]$ and $[\mathbf{j}]$ denote the charge and current densities at the retarded time, and $\mathbf{K}_i$ and $\mathbf{K}_f$ are initial and final projectile momenta. In nonrelativistic formulations, the current-current interaction $e\vec{\alpha}\cdot\mathbf{A}(\mathbf{r},t)$ in Eq. (3) is not included.

The cross section for the excitation of an electron from state i to f while the projectile goes from momentum states $\phi_{\mathbf{K}_i}(\mathbf{R})$ to $\phi_{\mathbf{K}_f}(\mathbf{R})$ is given by (Fano 1963)

$$d\sigma_{if} = \frac{1}{2\pi v^2} |\langle \phi_f(\mathbf{r}) | H' | \phi_i(\mathbf{r}) \rangle|^2 q dq, \quad (7)$$

where $\phi_i(\mathbf{r})$ denotes electronic wave functions and $q = |\mathbf{K}_f - \mathbf{K}_i|$ is the momentum transferred to the electron.

In the PWBA, one sets

$$\phi_{\mathbf{K}_i} \sim \exp(i\mathbf{K}_i \cdot \mathbf{R} - \omega_i t), \quad (8)$$

and by Fourier transforming the retarded potentials into momentum space

$$\frac{\exp[i\beta q_0 |\mathbf{R} - \mathbf{r}|]}{|\mathbf{R} - \mathbf{r}|} \rightarrow \int \frac{d\mathbf{k}}{k^2 - q_0^2 \beta^2} \exp[i\mathbf{k} \cdot (\mathbf{R} - \mathbf{r})], \quad (9)$$

the integral over nuclear coordinants gives delta functions, so the cross section becomes

$$d\sigma_{if} = 8\pi \left[\frac{Z_X e^2}{\hbar v} \right]^2 \frac{q dq}{(q^2 - q_0^2 \beta^2)^2} \left| \int \phi_f^*(\mathbf{r}) (1 - \vec{\beta} \cdot \vec{\alpha}_r) \exp(i\mathbf{q} \cdot \mathbf{r}) \phi_i(\mathbf{r}) d^3\mathbf{r} \right|^2, \quad (10)$$

where $q_0 = \Delta E_{if}/v$ is the minimum momentum transferred to the electron, ΔE_{if} is the difference in electronic energies of states i and f , and $\vec{\beta}$ is the ion velocity. The vector product $\vec{\beta} \cdot \vec{\alpha}$ can be decomposed as

$$\vec{\beta} \cdot \vec{\alpha} = \beta \cos \lambda \alpha_z + \beta \sin \lambda \alpha_x, \quad (11)$$

where $\cos \lambda = q_0/q$ and α_i is the i th component of the Dirac matrix. The cross section is then given by

$$d\sigma_{if} = 8\pi \left[\frac{Z_X e^2}{\hbar v} \right]^2 \frac{q dq}{(q^2 - q_0^2 \beta^2)^2} \left[|F_{if} - \beta \cos \lambda G_{zif}|^2 + |\beta \sin \lambda G_{xif}|^2 \right], \quad (12)$$

where

$$F_{if} = \int \phi_f^*(\mathbf{r}) \exp(i\mathbf{q} \cdot \mathbf{r}) \phi_i(\mathbf{r}) d^3\mathbf{r}, \quad (13)$$

$$G_{xif} = \int \phi_f^*(\mathbf{r}) \alpha_x \exp(i\mathbf{q} \cdot \mathbf{r}) \phi_i(\mathbf{r}) d^3\mathbf{r}, \quad (14)$$

and similiarly for G_{zif} . The first term in Eq.(12) is due to the longitudinal interaction between the perturbing nucleus and electron, and the second is the transverse interaction. One can write the cross section in terms of the

incoherent sum because the longitudinal and transverse interactions excite the electron to states with different azimuthal quantum numbers.

Commutation relations give (Anholt 1979)

$$G_{zif} = \frac{\Delta E_{if} F_{if}}{qc}, \quad (15)$$

so that the term containing $F_{if} - \beta \cos \lambda G_{zif}$ can be written as

$$|F_{if} - \beta \cos \lambda G_{zif}|^2 = \frac{(q^2 - q_0^2 \beta^2)^2}{q^4} |F_{if}|^2, \quad (16)$$

which cancels the retardation factor $(q^2 - q_0^2 \beta^2)^{-2}$ appearing in Eq. (12) for the longitudinal interaction. For electron ionization [$\phi_f = \phi_\epsilon(\mathbf{r})$], one obtains, after summing over final states (integrating over ionized electron kinetic energies ϵ):

$$\sigma_i = 8\pi \left[\frac{Z_X e^2}{\hbar v} \right]^2 \int_0^\infty d\epsilon \int_{q_0}^\infty \left[\frac{dq}{q^3} |F_{i\epsilon}(q)|^2 + \frac{qdq}{(q^2 - q_0^2 \beta^2)^2} |\beta \sin \lambda G_{xi\epsilon}|^2 \right]. \quad (17)$$

The first term in this expression is identical to that used in the nonrelativistic PWBA (Merzbacher and Lewis 1958). In nonrelativistic derivations of the PWBA, both the current-current interaction and retardation effects are neglected, but the only effect on the final formula is the absence of the transverse ionization term in Eq. (17). This allows us to use nonrelativistic theories (Merzbacher and Lewis 1958) to calculate the most important contribution to ionization by relativistic ions.

C. The Dipole Approximation

The dipole or Bethe (1930) approximation plays an important role in the calculation of electron stopping powers (Fano 1963). One sets

$$F_{i\epsilon} = \langle \phi_\epsilon^*(\mathbf{r}) | \exp(i\mathbf{q} \cdot \mathbf{r}) | \phi_i(\mathbf{r}) \rangle$$

$$\approx iq \cdot \langle \phi_\epsilon^*(\mathbf{r}) | \mathbf{r} | \phi_i(\mathbf{r}) \rangle, \quad (18)$$

and

$$G_{xi\epsilon} \approx iq_0\beta \langle \phi_\epsilon^*(\mathbf{r}) | \mathbf{r} | \phi_i(\mathbf{r}) \rangle, \quad (19)$$

thereby obtaining

$$\sigma_i = 8\pi \left[\frac{Z_X e^2}{\hbar v} \right]^2 \int_0^\infty d\epsilon \left| \langle \phi_\epsilon^*(\mathbf{r}) | \mathbf{r} | \phi_i(\mathbf{r}) \rangle \right|^2 \\ \times \int_{q_0}^{q_{\max}} dq \left[\frac{1}{q} + \frac{q\beta^4 q_0^2 (1 - q_0^2/q^2)}{(q^2 - q_0^2 \beta^2)^2} \right]. \quad (20)$$

For heavy projectiles the maximum momentum that can be transferred to the electron $q_{\max} \approx 2Mc\gamma\beta$, where M is the projectile mass, is essentially infinite, hence in numerical evaluations of Eq.(17) one often puts $q_{\max}=\infty$, but this would give $\ln\infty$ in the first term of Eq.(20). The application of the dipole approximation is only valid if q^{-1} is smaller than the spatial range of the electron wave functions; the expectation value $\langle \mathbf{q} \cdot \mathbf{r} \rangle$ should be much less than unity. This limits the maximum value of q to be less than $q_{\max} \sim \sqrt{2E_i}$, where E_i is the electron binding energy in atomic units. (For K-shell ionization, $q_{\max}$ is equal to Z in atomic units, since $\mathbf{r}$ is of the order of Z^{-1} .) For the transverse contribution, most of the integral over q comes from the region near $q=q_0$ (and no logarithmic divergences are present), so we can put $q_{\max}=\infty$ there. Using $q_0=E_i/v$ (which neglects the electron kinetic energy ϵ), we get

$$\sigma_i = 4\pi \frac{Z_X^2 \alpha^2}{\beta^2} \int_0^\infty d\epsilon \left| \langle \phi_\epsilon | \mathbf{r} | \phi_i \rangle \right|^2 \left[\ln\left(\frac{2mv^2}{E_i}\right) + \ln\gamma^2 - \beta^2 \right]. \quad (21)$$

Evaluations of electronic stopping powers $\langle \Delta E \sigma \rangle$, where ΔE is the energy transferred to the electron, make use of the fact that $(\epsilon + E_i) |\langle \phi_\epsilon | \mathbf{r} | \phi_i \rangle|^2$ is the oscillator strength, and the integral over final states is the sum over oscillator strengths, giving unity for each target electron. For inner-shell ionization, one could use a sum rule for the matrix elements $|\langle \phi_\epsilon | \mathbf{r} | \phi_i \rangle|^2$ (Bethe and Salpeter 1957). One can also relate the matrix elements $|\langle \phi_\epsilon | \mathbf{r} | \phi_i \rangle|^2$ to photoelectric ionization cross sections (Pratt et al. 1973) in the dipole approximation.

$$\sigma_{iPE}(\omega = E_i + \epsilon) = (2\pi)^2 \alpha \omega |\langle \phi_\epsilon | \mathbf{r} | \phi_i \rangle|^2. \quad (22)$$

Hence the ionization cross sections can be written as

$$\sigma_i = 4\pi \frac{Z_X^2 \alpha^2}{\beta^2} \int_0^\infty d\epsilon \frac{\sigma_{iPE}(E_i + \epsilon)}{(2\pi)^2 \alpha (E_i + \epsilon)} \left[\ln\left(\frac{2mv^2}{E_i}\right) + \ln\gamma^2 - \beta^2 \right]. \quad (23)$$

For a rapid estimation of the ionization cross section, one can assume that the photoelectric cross sections vary as

$$\sigma_{iPE}(E_i + \epsilon) \sim \sigma_{iPE}(E_i) \left[\frac{E_i}{E_i + \epsilon} \right]^n, \quad (24)$$

where n (~ 2 to 3) is a fitting constant. Then the integral over ϵ is easily done, and one obtains (Anholt et al. 1984b)

$$\sigma_i = 4\pi \frac{Z_X^2 \alpha^2}{\beta^2} \frac{\sigma_{iPE}(E_i)}{(2\pi)^2 \alpha n} \left[\ln\left(\frac{2mv^2}{E_i}\right) + \ln\gamma^2 - \beta^2 \right]. \quad (25)$$

The cross sections evaluated so far have been defined per electron, but if one uses atomic photoelectric ionization cross sections in Eq. (25) (Scofield 1973), the sum over initial states is automatically included. [In numerically evaluating this expression, all quantities are dimensionless except the photoelectric cross section, hence the ionization cross section takes the

dimension of the photoelectric one.]

Equation (25) indicates the general characteristics of ionization and excitation cross sections at relativistic velocities, which are similar to those seen for K-shell ionization of uranium by protons in Fig. 4. The first logarithmic term in Eq.(25) comes from the longitudinal interaction. At high relativistic projectile energies, the ion velocity approaches c and no longer varies with energy (or γ) so that the longitudinal cross section approaches a constant. The transverse contribution increases as the $\ln\gamma^2$, however. The additional spin-flip contribution shown in Fig.4, but not present in Eq.(25), is discussed in the following section.

D. Evaluation of Inner Shell Ionization Cross Sections

Although Scofield (1978) and Amundsen and Aashamar (1981) evaluated the longitudinal and transverse form factors using relativistic Hartree-Fock wave functions, inner-shell electronic wave functions are traditionally approximated by nonrelativistic hydrogenic wave functions (Merzbacher and Lewis 1958). To preserve the spin structure of the electronic wave functions (needed if one does not want to approximate the Dirac matrix α_x with its nonrelativistic ansatz), one uses Pauli wave functions (Davidovic et al. 1978, Anholt 1979). For electrons with spin quantum numbers $m_s = +1/2$ and $-1/2$, the 4-component Pauli wavefunctions are given by

$$\begin{aligned}\phi_{i,f} &= N_i \left[1, 0, -\frac{i}{K_2} \frac{\partial}{\partial z}, -\frac{i}{K_2} \left(\frac{\partial}{\partial x} + \frac{i\partial}{\partial y} \right) \right] \bar{\phi}_{i,f} \quad m_s = +1/2, \\ \phi_{i,f} &= N_i \left[0, 1, -\frac{i}{K_2} \left(\frac{\partial}{\partial x} - \frac{i\partial}{\partial y} \right), \frac{i}{K_2} \frac{\partial}{\partial z} \right] \bar{\phi}_{i,f} \quad m_s = -1/2,\end{aligned}\quad (26)$$

where N_i is a normalization factor, $K_2 \sim 2c$, and $\bar{\phi}_{i,f}$ is the nonrelativistic

hydrogenic wave function. For the K-shell, one could use

$$\phi_{1s} = (2Z)^{3/2} \exp(-Zr) Y_{00}(\theta, \phi), \quad N_1 = [1 + (Z\alpha/2)^2]^{-1/2}, \quad (27)$$

where Y_{00} is the spherical harmonic wave function. Following Bethe (1930), the ionization matrix elements are usually calculated in parabolic coordinants. Defining $\eta_K = (\beta/Z\alpha)^2$, $Q = q^2/Z^2$, $Q_{\min} = q_0^2/Z^2 = W^2/4\eta_K$, and $W = 1 + k^2 = 1 + 2\epsilon/Z^2$ one obtains for the K-shell ionization cross section per electron (Merzbacher and Lewis 1958, Khandelwal et al. 1969, Anholt 1979)

$$\sigma_K = 4\pi \frac{(Z_X a_0)^2}{\eta_K Z^4} \int_{W_{\min}}^{\infty} d^2 dW \int_{Q_{\min}}^{\infty} \frac{dQ}{Q^2} \times \left[F_K(Q, W) + \frac{(\beta Z\alpha)^2 (1 - Q_{\min}/Q)}{(1 - \beta^2 Q_{\min}/Q)^2} (G_K + \frac{1}{4} Q F_K) \right], \quad (28)$$

where

$$F_K = (3Q + W) Q A_3,$$

$$G_K = \frac{1}{4} W A_2,$$

$$A_n = \frac{2^7 \exp\{-2/k \tan^{-1}[2k/(Q+1-k^2)]\}}{3(1 - \exp(-2\pi/k))[(Q-k^2+1)^2 + 4k^2]^n}$$

and

$$d^2 = (1 + (Z\alpha/2)^2)^{-1} (1 + (W-1)(Z\alpha/2)^2)^{-1}. \quad (29)$$

The first two terms in Eq.(28) containing F_K and G_K come from the longitudinal and transverse contributions to the total K-shell ionization cross sections. In evaluating Eq. (28) for target vacancy production one uses an effective charge $Z = Z^* = Z_t - 0.3$ for K-shell ionization, dictated by Slater (1930) screening rules. The value of $W_{\min}$ is determined by the experimental K-shell binding energy E_K , and is equal to θ_K , $2E_K/Z^{*2}$. The two-fold numerical integration is carried out as described in Merzbacher and Lewis

(1958). Figure 5 compares calculations of target K- vacancy production by 4.88 GeV protons with measurements of Anholt et al.(1976). The presence of the transverse excitation term is clearly seen. The theoretical longitudinal cross sections alone are about one-half of the measured cross sections. Adding the transverse cross sections nearly brings the measured and calculated total K-shell ionization cross sections into agreement.

Equation (28) also contains an electron spin-flip term (the last term in F_K) which is unique to relativistic ion-induced ionization. This term is important only for high-Z atoms, e.g. p+U collisions. Since both the spin-flip and transverse cross sections increase as $\ln \gamma^2$ (Fig.4), the spin-flip term is never greater than about 0.25 times the transverse cross sections for K-shell ionization. The ratios of the spin-flip cross sections to the transverse cross sections scale as the square of the electron binding energy, hence for L- and outer-shell ionization, the spin-flip contributions should be negligibly small. Leung and Rustgi (1983) have explored the possibility of measuring the spin-flip contribution to K-shell ionization using polarized proton beams.

The 4.88-GeV proton measurements are the highest-velocity measurements of inner-shell ionization that have been done with heavy particles. Relativistic effects have been explored at greater velocities (or greater values of γ) using incident electrons. In applying this theory to calculate electron-induced K-shell ionization cross sections, we note that the maximum momentum transfer cannot exceed the incident electron momentum, and the maximum energy transfer cannot exceed the electron kinetic energy. Since the contributions to the ionization cross sections peak where the energy transfer is equal to the K-shell binding energy (which never exceeds 133-keV for $Z=92$),

electron-induced ionization cross sections for electron energies exceeding ~1 MeV can be calculated with Eq. (28) without modifications. Figure 6 compares electron-induced ionization cross sections for 2-MeV ($\gamma=5$) to 900-MeV ($\gamma=2000$) electrons with Eq. (28). The measured cross sections clearly are in agreement with theory, except perhaps for 2-MeV electrons incident on high-Z targets, where the electron binding energies are largest compared to the incident electron kinetic energy.

One need not evaluate the longitudinal part of the ionization cross sections numerically since tables of the reduced cross section

$$f(\eta_K, \theta_K) = \int_{\theta_K}^{\infty} dW \int_{Q_{\min}}^{\infty} \frac{dQ}{Q^2} F_K(Q, W)$$

or

$$F_K(\eta_K/\theta_K^2, \theta_K) = \theta_K f(\eta_K, \theta_K)/\eta_K, \quad (30)$$

are available (Khandelwal et al. 1969, Choi et al. 1973, Rice et al. 1977, Benke and Kropf 1978, Johnson et al. 1979) for the K, L, and M inner-shell electrons. One must calculate η_K using $\eta_K = (\beta/Z^*\alpha)^2$ instead of the formulas based on ion energy given in the tables, however. For the transverse contribution, the dipole approximation tends to be an accurate approximation down to the smallest η_K values where the transverse term is still significant. (This is less true for the longitudinal part, however, where the dipole approximation is only accurate to about a factor of two for $\eta_K \approx 1$.) The transverse contribution to the K-shell ionization cross section (per electron) in the dipole approximation is given by (Anholt 1979)

$$\sigma_{Kt} = 1.45 \times 10^4 \left(\frac{Z_x}{Z\beta\theta_K^2} \right)^2 \left(\frac{4}{9} \theta_K - \frac{1}{12} \right) (\ln\gamma^2 - \beta^2) \text{ barns}. \quad (31)$$

Although the transverse contribution has not been worked out for any other shell but the K-shell, one can estimate its magnitude using Eq. (25):

$$\sigma_{it} = 4\pi \frac{Z_X^2 \alpha^2}{\beta^2} \frac{\sigma_{iPE}(E_1)}{(2\pi)^2 \alpha n} [\ln \gamma^2 - \beta^2] . \quad (32)$$

Anholt (1979) gives a formula for calculating the spin-flip contribution to the K-shell ionization cross section. The spin-flip contributions should be negligible for L-, M-, and outer-shell ionization.

Scofield (1978) has calculated cross sections for electron-induced K- and L-shell ionization, and Amundsen and Aashamer (1981) have calculated cross sections for proton-induced ionization using fully relativistic, Hartree-Fock or Hartree-Fock-Slater electronic wave functions. An important feature missing when one uses nonrelativistic hydrogenic wave functions is the small- r divergence of the wavefunctions (Amundsen et al. 1976). For example, Dirac 1s wave functions vary as

$$\phi_{1s} \sim r^{s-1} \exp(-Zr) , \quad (33)$$

where $s^2 = 1 - \alpha^2 Z^2$ is less than unity. The increased electron density at small r increases the density of high-momentum components in the 1s momentum wave function. At low velocities v where the minimum momentum transfer $q_0 \sim E_K/v$ is very large, the larger density of high momentum components in the initial and final electronic wave functions gives increased ionization cross sections (Amundsen et al. 1976; Anholt 1978). For the present relativistic heavy ions where the momentum transfer q_0 is small, and where one can put

$$F_K(q) = iq \cdot \langle \phi_E | \mathbf{r} | \phi_K \rangle , \quad (34)$$

the divergence leads to a contraction of the radial wave functions which gives smaller values of the expectation value of r , hence one obtains smaller cross sections. Anholt et al. (1985) evaluated the longitudinal contributions

to projectile K-shell ionization in Xe, Au, and U projectiles using the relativistic PWBA formulation of Jamnik and Zupancic (1957). This calculation uses Dirac wave functions and makes a multipole expansion of the final-state wave functions. The calculated U K-shell ionization cross sections for $\beta=0.2$ to 1 are up to 20% smaller when Dirac wave functions and binding energies are used.

D. Target-Atom Screening

In calculating projectile ionization by neutral atoms, one must take into account the electronic screening of the perturbing target nucleus (Bates and Griffing 1953, 1954, 1955, McGuire et al. 1981, Briggs and Taulbjerg 1978). McGuire et al. (1981) formulated a theory of the screening of He atoms, which can be generalized to many-electron target atoms (Anholt 1985a). For projectile ionization, the perturbing charge $Z_X^2 = Z_t^2$ appearing in Eq. (17) is replaced with

$$S(q) = \left[Z_t - \sum_i \langle \psi_i | \exp(iq \cdot r) | \psi_i \rangle \right]^2 + Z_t - \sum_i \left| \langle \psi_i | \exp(iq \cdot r) | \psi_i \rangle \right|^2, \quad (35)$$

where ψ_i is the target atomic orbital for the i^{th} electron, and the sum includes all target electrons. In this expression, q is the momentum transferred to the electron, hence $S(q)$ must be included inside the integral over q (Eq. 17) or Q [$=q^2/Z^2$ in Eq. (28)]. The first term in Eq.(35) is the effective screened target charge; if q approaches zero, the charge vanishes. Then, ionization, which would normally occur at large impact parameters (of the order of q^{-1}), does not occur, because the target nucleus is completely screened; the projectile electron sees a neutral perturbing atom (Anholt 1985a). However, in cases where q is reasonably large, the matrix elements $\langle \psi_i | \exp(iq \cdot r) | \psi_i \rangle$ are small, so the effective charge is close to Z_t . The antiscreening term, given by the middle term in Z_t in Eq. (35), is the contribution to projectile ionization by the target electrons. If q is large, the ionization cross section is proportional to $Z_t^2 + Z_t$, where Z_t^2 comes from the Coulomb potential between the target nucleus and active electron, and the factor of Z_t comes from Z_t separate electron-electron Coulomb interactions.

Since at $q=0$, ionization by the neutral target atom cannot occur, the final term is the antiscreening correction (ASC) approaching Z_t at small q , thus cancelling the Z_t factor.

Target matrix elements, $\langle \psi_i | \exp(i\mathbf{q} \cdot \mathbf{r}) | \psi_i \rangle$, can be calculated using Hartree-Fock or other suitable many-electron wave functions. In the first term of Eq. (35), $\sum_i \langle \psi_i | \exp(i\mathbf{q} \cdot \mathbf{r}) | \psi_i \rangle$ is just the atomic form factor $F_t(q, Z_t)$ used in Compton scattering calculations, so these values can be taken from tabulations (e.g. Hubbel et al. 1975).

Tables of the ASC factor, $\sum_i |\langle \psi_i | \exp(i\mathbf{q} \cdot \mathbf{r}) | \psi_K \rangle|^2$, are not available, however. The ASC factor lies between $F_t(q, Z_t)$ and $|F_t(q, Z_t)|^2/Z_t$ (Anholt (1985a), but one obtains nearly identical cross sections independent of whether F_t or F_t^2/Z_t is used for the ASC. Figure 7 shows calculations of projectile 1s and 2s ionization in 400-MeV/amu Ne and 962-MeV/amu U atoms. Target-atom screening significantly reduces Ne 2s and Ne 1s ionization cross sections (where the momentum transfer is smallest), but the U 2s and 1s ionization cross sections are only slightly different from the unscreened Born results. For U, the momentum transfer is large, so $F_t(q, Z_t)$ and the ASC are nearly zero, and the cross sections vary as $Z_t^2 + Z_t$. The calculated uranium cross sections are therefore larger than unscreened Born results at low Z_t . Small screening effects reduce the uranium cross sections below the unscreened values at large Z_t .

Equation (35) neglects kinetic-energy constraints on the target electron-induced excitation contributions. These are negligible for relativistic low- Z ions, but can be important for high- Z ions. For a target electron to ionize a projectile electron, the target-electron kinetic energy in the projectile

frame must be greater than the binding energy of the projectile electron. Thus at 200 MeV/amu, the target electron (energy ~100 keV) cannot ionize U K electrons (binding energy 115 to 132 keV, depending on the degree of ionization). For high-Z projectiles where, according to Eq.(35), the cross sections vary as $(Z_t^2 + Z_t)\sigma_B$ (σ_B is the PWBA cross section for protons), Anholt et al. (1985) replaced the term $Z_t\sigma_B$ with $Z_t\sigma_e$, where σ_e is the calculated electron-induced cross section of Rudge and Schwartz (1966) for the relevant electron kinetic energy (100 keV in the example given above). The cross sections of Rudge and Schwartz agree with measurements for electron kinetic energies near the ionization threshold. The validity of this approximation for K-shell ionization in atomic H+H collisions, where the screening effects can be calculated exactly in the PWBA (Bates and Griffing 1953, 1954, 1955) has been confirmed by Anholt (unpublished).

The transverse ionization cross sections are more drastically reduced by target atom screening than are the longitudinal cross sections (Anholt 1985a; Gillespie et al. 1978). This occurs because most of the contributions to the integral over q for transverse excitation comes from the region $q \sim q_0$, where $S(q)$ is smallest. The longitudinal cross sections, however, fall off as q^{-1} for high velocities, hence larger contributions to the longitudinal cross sections comes regions of large q where $S(q)$ is larger. In the impact-parameter picture important contributions to transverse ionization occur at larger impact parameters than for longitudinal excitation. Here the projectile electron sees a more effectively screened target atom. For example, in 2.1-GeV/amu Ne+Ag collisions, the transverse Ne K-shell ionization cross section is reduced by a factor of 0.06 by Ag target-atom screening, but the longitudinal cross section is only reduced by a factor of 0.5. As a

result, transverse ionization is negligible in these collisions.

F. Distortion Effects

The PWBA is based on first-order perturbation theory. The ionization amplitudes increase linearly with the perturbing charge Z_X so the cross sections (in the absence of target screening) increase as Z_X^2 , where Z_X comes from the perturbing Hamiltonian, $Z_X e^2 / |R-r|$. In high-velocity collisions (characterized for K-shell ionization by $v/v_K \gg 1$) excitation can occur at very large impact parameters b where the internuclear distance R , on the average, is much greater than electron distance r . Hence, the potential is always small, and the linear PWBA or semiclassical approximation is always valid. However, at small velocities ($v < v_K$) or even intermediate velocities ($v \approx v_K$), excitation occurs at small impact parameters where the strong perturbing potential can significantly distort the inner-shell wave functions, giving a nonlinear dependence of the amplitude on Z_X (Basbas et al 1973, 1978).

Distortion effects on inner-shell ionization are often analyzed in terms of the binding and polarization effects (Basbas et al. 1978). One can understand these effects by considering the following schematic coupled-channel calculation in an atomic basis. We consider three closely coupled states: the 1s state (amplitude a_0) and the 3p (a_1) and 3d (a_2) states. We thus model ionization by 1s+3p (+3d) excitation. The current-current interaction, and the m-subshells are neglected, so we retain only three channels (instead of 9). Also back coupling to the initial state is neglected. The rate of change of the excitation amplitudes is given by (Skinner 1962; Kingston et al. 1960, Bates 1959)

$$\dot{a}_0 = -a_0 V_{00}$$

$$\begin{aligned}\dot{a}_1 &= a_0 V_{10} e^{i\omega t} - V_{11} a_1 + V_{12} a_2 \\ \dot{a}_2 &= a_0 V_{20} e^{i\omega t} + V_{21} a_1 - V_{22} a_2 ,\end{aligned}\quad (36)$$

where ω is the 1s-3p or 1s-3d binding frequency difference, and

$$V_{if} = \int \phi_f^*(\mathbf{r}) \frac{Zx e^2}{|\mathbf{R}(t) - \mathbf{r}|} \phi_i(\mathbf{r}) d\mathbf{r} . \quad (37)$$

These equations are solved for $a_i(\infty)$ with the initial condition $a_0(-\infty)=1$. First order perturbation theories calculate $a_i(+\infty)$ by setting $a_0(t)=1$ for all t , hence they integrate the equation

$$\dot{a}_1(t) = V_{10}(t) e^{i\omega t} \quad (38)$$

along an ion trajectory with impact-parameter b . It is important to realize though that the perturbing potential appears not only as the off diagonal term V_{10} , but as the diagonal part of the perturbation matrix and in the coupling between the p and d states. One usually solves such equations by making a phase transform (Skinner 1962)

$$d_j(t) = a_j(t) \exp \left[i \int_0^t V_{jj}(t') dt' \right] , \quad (39)$$

obtaining for the desired 1s-3p excitation amplitude

$$\dot{d}_1(t) = d_0 V_{10} \exp \left[i \int_0^t (\omega + V_{00} - V_{11}) dt' \right] + d_2 V_{12} \exp \left[i \int_0^t (V_{22} - V_{11}) dt' \right] . \quad (40)$$

As the perturbing charge increases, three effects on the excitation amplitude $d_1(\infty)$ can occur. By comparing $\dot{d}_1(t)$ with $\dot{a}_1(t)$, we identify these as binding effects [increase in the binding energy difference ω due to the addition of $V_{00}(t)-V_{11}(t)$], polarization effects [increase in the excitation amplitude d_1 due to the addition of the term in d_2], and unitarity effects [depletion of

the initial amplitude $d_0(t) < 1$]. At high velocities where $q \sim \omega/v$ is small and excitation occurs at large impact parameters where $R > b$ and $V_{00}(R)$ and $V_{11}(R)$ are small, the addition of these potentials does not affect d_1 significantly, so that if $d_0(t) \approx 1$, the amplitudes should vary as

$$d_1(\infty) \sim Z_X d_B + Z_X^2 d_{12}, \quad (41)$$

where d_B is the first-order amplitude, a_1 from Eq.(38), calculated for $Z_X=1$ and we call d_{12} the polarization amplitude. After integrating $|d_1(\infty)|^2$ over impact parameters, the ionization or excitation cross sections should vary as

$$\sigma \sim Z_X^2 \sigma_B + O(Z_X^3) + O(Z_X^4), \quad (42)$$

where $O(Z_X^n)$ denotes terms of the order of Z_X^n . This is called the polarization effect. In theories of electron stopping powers, one assumes that the attractive perturbing potential polarizes the electron clouds, decreasing the effective electron-perturbing nucleus distance, thus increasing the ionization cross section. This has been analyzed classically (Ashley et al 1973), quantum mechanically (Hill and Merzbacher 1974) using harmonic oscillator models of the atom, and quantum mechanically in coupled-channel calculations of K-shell ionization by Ford et al. (1977) and Becker et al (1980). The type of harmonic oscillator wave functions appearing in the calculations of Hill and Merzbacher (1974) suggest intermediate coupling to d states as in our schematic coupled-channel model, but other states are also important.

At low ion velocities, the momentum transfer $q=\omega/v$ is large, and the K-shell ionization probabilities fall off as $(v/\omega)^n$, where $n \sim 10$ (Basbas et al. 1973). In these collisions, increasing the energy transfer by adding $V_{00}-V_{11}$ will further reduce the cross sections. For the 1s level, the binding energy

at $R=0$ in this model increases from $\frac{1}{2} Z^2$ au to $\frac{3}{2} Z^2$ au for symmetric collisions, hence this binding effect significantly reduces ionization cross sections for large Z_x at small velocities (Brandt et al 1966; Anholt and Meyerhof 1977).

For the intermediate to high velocities relevant to K-shell ionization by high-Z relativistic ions, the theory of Basbas et al. (1978) which interpolates between theories of the binding effect at low velocity and theories of the polarization effect at high velocities have been applied. The results of this theory agree well with coupled-channel calculations of Becker et al. (1980) for p, He, and C induced Ar K-shell ionization. Figure 8 compares target K-vacancy production cross sections for a variety of high-Z projectiles with PWBA calculations, dipole approximation results, and the theory of Basbas et al. (1978; denoted Borr corr. in Figure 8). Remarkably, despite the large perturbing charges and large atomic charges, the measured cross sections are not too different from the first order PWBA calculations (or the dipole approximation ones which are here shown to illustrate the accuracy of Eq. (25) compared to the exact PWBA theory). The Ne and Kr induced K-shell ionization cross sections are negligibly affected by distortion effects, which is due to the relatively small perturbing charges there. For low- Z_t , (corresponding to high v/v_K), the theory of Basbas et al. gives larger ionization cross sections than the PWBA due to the dominance of polarization effects, but for high- Z_t and low-velocity 82-MeV/amu Xe collisions (corresponding to low v/v_K), the Basbas theory gives smaller cross sections due to the binding effect. However, the data shown in Figure 8 are not sufficiently precise to distinguish between the different theories. Also, at large Z_t , electron-capture contributions to K-vacancy production by the

nearly bare projectiles are present. (For further discussion of electron capture, see Sect.IV).

Electron-capture contributions to ionization are absent when one measures projectile ionization. Figure 9 shows reduced Xe 1s ionization cross sections versus Z_t for 82-, 140-, and 200-MeV/amu Xe projectiles. For high-Z projectiles, target-atom screening is expected to be negligible, hence the cross sections are expected to vary as $(Z_t^2 + Z_t)\sigma_B$ in the PWBA. The reduced cross sections are obtained by subtracting the estimated target-electron-induced ionization cross sections $Z_t\sigma_e$ from the measured reduced cross sections, and dividing by Z_t^2 . In the Born approximation, the cross sections should be independent of Z_t . These measurements are ideal for studying distortion effects: the velocities are sufficiently small, so that the transverse excitation contributions are only 4% (at the largest energy) of the total cross sections, electronic relativistic effects are small, and the momentum transfer is sufficiently large that the target nuclei are basically unshielded (Anholt et al. 1985).

The theory of Basbas et al. (1978; thin solid lines in Fig. 9) predicts that for these collisions, the reduced cross sections increase with Z_t , because the polarization effects are more important than binding effects. This theory disagrees with the bulk of the high- Z_t data. At the heart of this theory, a cut-off impact parameter $b_c = c_K a_K$ is present. The theory assumes that for impact parameters smaller than b_c , binding effects are important, and for larger impact parameters, polarization effects are dominant. To obtain better agreement with the data shown in Fig.9, Anholt et al (1985) semiempirically increased the c_K value from $c_K=1.5$ to $c_K=3$ to reduce the polarization effect and enhance the binding effect. Using $c_K=3$

brings the theory into better agreement with experiment (dashed lines in Fig. 9). It was found by trial and error that no improvement is obtained with other values of c_K ; the value $c_K=3$ gives the best overall compromise fit.

The theory of Basbas et al. (1978) is usually applied to calculate target inner-shell vacancy production where $Z_X \ll Z$ ($Z_p \ll Z_t$). For $Z_X = Z$, several theories which have been developed to calculate ionization in $H^+ + H$ collisions (Park 1983) may be applied. To compare these theories with measurements for relativistic heavy ions where $Z_p = Z_t$, Anholt et al (1985) obtained reduced cross sections for $Xe^{53+} + Xe$ collisions by interpolating the reduced cross sections in Fig.9 between measurements for $Z_t = 47$ and 79. By plotting $Z^2 \sigma$ ($Z = Z_t = Z_p$) versus the proton kinetic energy, they compare $Xe + Xe$ with $p + H$ ionization cross sections (Park 1983, Shah and Gilbody 1981) at the same value of v/v_K . The Xe energy scale in Fig.10 is related to the $p + H$ one using

$$E_{Xe} = 932 \left[\frac{1}{\sqrt{1-\beta^2}} - 1 \right]; \quad \beta = \frac{54}{137.037} \sqrt{0.04025 E_p}, \quad (43)$$

where E_{Xe} is in MeV/amu and E_p is the proton energy in keV. The Xe energy scale ends at $E_p = 160$ keV where β is equal to unity. This type of scaling is exact for symmetric collisions ($Z=Z_X$) in the PWBA (Khandelwal et al 1969) and in molecular perturbed stationary-state calculations for one-electron systems (Thorson 1975). Since the target electron antiscreening effects have been removed in deriving the reduced cross sections, and the electronic relativistic effects and transverse excitation are negligible, this scaling should be nearly exact in the $Xe + Xe$ collisions. The measured $Xe + Xe$ points in Fig.10 are clearly in good agreement with the measurements of Shah and Gilbody (1981) and Park (1983 and private communication). We also show in

Fig.10 the calculations of Basbas et al. (1978) using $c_K = 1.5$ (thin-solid line) and $c_K = 3$ (dashed line). Those using $c_K = 3$ are in reasonable agreement with the measured p+H and Xe+Xe cross sections, but the original theory with $c_K = 1.5$ overestimates the p+H cross sections.

Theories of ionization and excitation of hydrogen by protons based on the Glauber (1955) approximation have given some of the best agreement with experiment. (See Park 1983 and McGuire 1982 for comparisons.) An important feature of the Glauber approximation is that it preserves the unitarity of the ionization amplitude for 1s ionization. For symmetric collisions near $v \sim v_K$, the first-order semiclassical approximation (Hansteen et al. 1975) predicts ionization probabilities that are greater than 1/2 in small-impact-parameter collisions. Such large ionization and equally large excitation probabilities deplete the initial 1s occupation amplitudes $d_o(t)$ in Eq.(40), assumed to be unity for all time t in first-order theories like the PWBA. Therefore, smaller ionization probabilities and cross sections can be expected.

It is interesting that the Glauber approximation, which accounts for unitarity effects, but neglects binding and polarization effects, agrees well with experiment and with the semiempirical modification of the Basbas theory accounting for (mainly) binding effects, but neglecting unitarity ones. Figure 9 compares Glauber calculations (chain curve) of the reduced ionization cross sections with experiment. Disagreement between the two occurs for $Z_t < 20$, where the data points are higher than the Born calculations, and the Glauber cross sections always lie below the Born ones. The low- Z_t data points in Fig. 9 do not agree with the original theory of Basbas et al. either. These data points are most affected by target antiscreening. If the full Born

electron-electron contribution $Z_t\sigma_B$ were subtracted from the measured cross sections instead of $Z_t\sigma_e$ (σ_e incorporating threshold effects), the reduced cross sections would be in better agreement with the Born and Glauber calculations at low Z_t (e.g. the 82-MeV/amu Xe+Be reduced cross sections are reduced by 25%). These considerations suggest that the discrepancy at low Z_t may be due to our lack of a complete theory of target screening and antiscreening near the electron ionization threshold velocity.

Since for ions of a given velocity, v/v_L , the velocity relative to the projectile L shell, is greater than v/v_K , the binding and polarization effects should have a negligible effect on target L x-ray production. Also, target L electron capture in Xe+Pb collisions is likely to be as negligible as target K electron capture in Xe+Zr collisions (see Figure 8). In Fig. 11 we compare L x-ray cross sections with the first-order PWBA (including transverse excitation, but neglecting distortion and projectile screening effects). The measured cross sections are in good agreement with calculations using theoretical ionization cross sections and measured or semiempirical L-shell fluorescence yields and Koster-Kronig transition probabilities (Salem et al. 1971, Anholt et al. 1984b).

IV. ELECTRON CAPTURE PROCESSES

A. Radiative Electron Capture

Radiative electron capture is the inverse of the photoelectric process. An electron incident on the projectile ion is captured with the emission of a photon whose energy is the electron kinetic energy $(\gamma-1)mc^2$ in the projectile frame plus the final electron binding energy E_i . Raisbeck and Yiou (1971) noted that REC was needed to explain total electron capture cross sections for ~300-MeV protons incident on low-Z target atoms. The REC photon was observed at nonrelativistic velocities by Schnopper et al. (1972), Kienle et al. (1973), and others (e.g. Spindler et al. 1979), and at relativistic velocities by Anholt et al. (1984a).

The cross section for REC into the projectile shell i can be calculated from the photoelectric cross section σ_{iPE} using

$$\sigma_{RECI} = Z_t \left(\frac{k}{\gamma\beta}\right)^2 \sigma_{iPE}(k), \quad (44)$$

where $k=E_i/mc^2 + \gamma-1$. Under the condition that the binding energy of the target electron can be neglected, there are the Z_t electrons per target atom that can be captured with equal probability. One uses atomic photoelectric cross sections (per filled atom) hence the REC cross sections are for bare projectiles.

The advantage of writing the REC cross section in terms of the photoelectric cross section is that much is known about photoelectric processes in atomic physics (Pratt et al. 1973). For numerical calculations, we have the choice of many possible photoelectric cross-section formulas. For low-Z, relativistic ions, the Sauter formula (Sauter 1931, Pratt et al. 1973) provides reliable cross sections (Anholt 1985a). For high-Z,

relativistic ions at velocities greater than 82-MeV/amu for Xe and greater than 400 MeV/amu U, the Sauter formula with higher order relativistic corrections (Pratt et al. 1973; Eq. 6.1.8) tends to break down at low velocities, so photoelectric cross sections calculated with exact Dirac wave functions (Hultberg et al. 1967) are used there. Tabulations of photoelectric cross sections for all shells calculated using many-electron wave functions are available (Scofield 1973). Meyerhof et al. (1985) used the bound-state normalization theory of Pratt (1960; Schmickley 1966) to approximate Dirac L and M-shell photoelectric cross sections using many-electron ones.

Figure 1 shows a measured photon spectrum for 422-MeV/amu U+Be collisions (Anholt et al. 1986). The photon peak at ~270 keV is the most prominent feature in this spectrum, and is due to REC into the projectile K-shell. A second peak at ~190 keV is due to the REC into L and higher shells of U. The 190-keV peak is nearly inseparable from the continuum primary electron bremsstrahlung spectrum ending at ~ 160 keV. The REC photon peak energies in the laboratory frame are given by

$$E_{x'} = \frac{E_i + (\gamma-1)mc^2}{\gamma(1-\beta\cos\theta')}, \quad (45)$$

so, although the rest frame energy of the K-shell REC photon is ~380 keV, the laboratory frame energy at $\theta'=85^\circ$ in Figure 1 is ~270 keV. The width of the REC lines is determined by Doppler broadening (the spread in angles $\Delta\theta'$ subtended by the x-ray detector giving a spread in x-ray energies $\Delta E_{x'}$), and the Fermi momentum of the target electrons. In Fig.1 the REC width for U+Be is determined entirely by the angular acceptance of the x-ray detector. Equation (45) assumes that the target electrons are at rest. For target electrons with momentum p_t , the width of the REC peak is of the order of

$\gamma c \vec{\beta} \cdot \vec{p}_t$ (Kleber and Jakubassa 1975), which, except for the inner shell electrons of high- Z_t atoms, is smaller than the Doppler width. The REC line shapes shown in Fig.1 were calculated by Anholt et al. (1986).

One interesting feature of REC is that although the peak energy is Doppler shifted with laboratory angle, the angular distribution does not have the forward peaking characteristic of a Doppler shifted photon, but varies as $\sin^2\theta'$ in the laboratory. This is due to the cancellation of electron-retardation and Lorentz-transform effects (Spindler et al. 1979). In the projectile frame, the incident electron is captured and the emitted photon angular distribution is peaked in the direction of the incident electron, which is backwards in the laboratory. The photon angular distribution varies as

$$\frac{d\sigma}{d\Omega} \sim \frac{\sin^2\theta}{(1-\beta\cos\theta)^4}, \quad (46)$$

where θ is the angle between the direction of the incident electron and the photon. The Lorentz transform

$$\cos\theta = \frac{\beta - \cos\theta'}{1 - \beta\cos\theta'}; \quad \frac{d\Omega}{d\Omega'} = \frac{1 - \beta^2}{(1 - \beta\cos\theta')^2}, \quad (47)$$

where θ' is the laboratory angle, shifts the angular distribution forward in the laboratory, so that the resulting laboratory angular distribution is nearly exactly $\sin^2\theta'$. Equation (46) is not exact, however, and small deviations from $\sin^2\theta'$ are seen in Fig.12 (Anholt et al. 1984b), and expected when Dirac (Hultberg et al. 1967) or Sauter (1931) calculations of the photoelectric angular distribution are used (Figure 12). Although the measured points are not sufficiently accurate to test photoelectric angular distributions, they do clearly demonstrate the electron-retardation-Lorentz-transform-cancellation effect.

The total REC photon cross sections for thick targets are given by the REC capture cross sections for bare projectiles multiplied by the equilibrium number of projectile K vacancies N_{Kv}

$$\sigma_{\text{RECK}\gamma} = \frac{1}{2} N_{Kv} \sigma_{\text{RECK}} . \quad (48)$$

This equation can be used to measure either the REC cross section or the number of projectile K vacancies inside the solid targets. If the number of K vacancies inside the solid targets is equal to that measured downstream using a magnetic spectrometer,

$$N_{Kv} = 2F_0 + F_1 , \quad (49)$$

where F_0 and F_1 are the measured fractions (≤ 1) of projectiles carrying 0 and 1 electrons respectively, one can obtain the REC cross section. Figure 13 compares measured and calculated REC cross sections and derived and measured K-vacancy numbers for a variety of collisions. For the cases thus far studied (82- and 200-MeV/amu Xe and 422 MeV/amu U), consistent K-vacancy numbers are obtained using REC and charge-state measurements (Anholt et al. 1984a). The equivalence of the in-target and downstream charge-state fractions is consistent with the discussion in Sect. V.

B. Nonradiative Electron Capture

Unlike REC where the difference between the energy of the initial and final states of the captured electron is released by the photon, NRC involves the transfer of energy or momentum between the electron and the projectile motion, mediated by the electromagnetic perturbation potential. For this reason, NRC is a process like ionization. Unlike for ionization, however, electron capture involves a transfer of the electron from the target frame

to the projectile frame. It is convenient to use instead of binding energies E_i and E_f , total electron energies

$$\tilde{E}_{i,f} = mc^2 - E_{i,f} , \quad (50)$$

which for K-shell electrons in hydrogenic atoms is simply

$$\tilde{E}_f = mc^2 s_p , \quad (51)$$

where $s_p = (1 - \alpha^2 Z_p^2)^{1/2}$. The longitudinal momentum transfer to the target electron then is given by (Moiseiwitsch and Stockman 1980, Eichler 1985)

$$q_0 = \frac{1}{v} \frac{\tilde{E}_f - \gamma \tilde{E}_i}{\gamma} , \quad (52)$$

the absolute magnitude of which approaches the ratio of the energy transfer to the ion velocity, $1/v (E_i - E_f + \frac{1}{2}mv^2)$, at low velocities.

In the impact-parameter picture, the cross section for the capture of an electron from an initial state ($j\mu$) to a final state ($j'\mu'$) is given by (Eichler 1985; see also Moiseiwitsch 1985, Moiseiwitsch and Stockman 1980, Shakeshaft 1979)

$$\sigma_{fi} = \frac{1}{2j+1} \sum_{\mu\mu'} |A_{fi}(b,v)|^2 d^2b , \quad (53)$$

where the capture amplitude is given by

$$A_{fi} = i \int dt dr \psi_f^\dagger(r_p', t') S \frac{Z_p}{r_p} \psi_i(r_t, t) , \quad (54)$$

r_p' is the electron-projectile nucleus distance, r_t is the electron-target nucleus distance, S transforms the wave function from the target frame (unprimed coordinates) to the projectile frame (primed coordinates),

$$S = \left(\frac{\gamma+1}{2}\right)^{1/2} \left(1 - \frac{\gamma}{\gamma+1} \vec{\alpha} \cdot \vec{\beta}\right) , \quad (55)$$

and the first-order projectile and target wave functions in their respective

frames are written as

$$\begin{aligned}\psi_i(r_t, t) &= \phi_i(r_t) \exp(-i\tilde{E}_i t) \\ \psi_f(r_p', t') &= \phi_f(r_p') \exp(-i\tilde{E}_f t') .\end{aligned}\tag{56}$$

Equations (53)-(56) have been evaluated by Oppenheimer (1928) and Brinkman and Kramers (1930) for nonrelativistic projectile velocities where $\gamma=1$ and $S=1$. Shakeshaft (1979), and Moiseiwitsch and Stockman (1980) calculated NRC cross sections for relativistic projectile velocities. These first-order OBK theories disagree with experiment at both relativistic and nonrelativistic velocities, exceeding measurements by significant factors (2 to 10), as shown for 1050-MeV/amu Ne collisions in Figure 14.

In recent years, much progress has been made in the theory of nonrelativistic NRC. At relatively high, but nonrelativistic velocities, the impulse (Briggs 1977, Amundsen and Jakubassa 1980), eikonal (Eichler and Chan 1975, Chan and Eichler 1979), second-Born (Drisko 1955, Shakeshaft and Spruch 1979), and strong-potential Born approximations (Macek and Alston 1982, Briggs et al. 1982), give better agreement with experiment than do the first-order theories. For relativistic projectiles, calculations have only been made using the second Born and eikonal approximations.

Humphries and Moiseiwitsch (1984,1985) used the second-Born approximation to calculate NRC for low-Z relativistic projectiles. They found that for both relativistic and nonrelativistic p+H collisions, the second Born cross sections are about 0.3 times the OBK cross sections. However, this reduction is still not sufficient to bring the second Born calculations into agreement with experiment (Figures 14 and 15).

Anholt and Eichler (1985) found good agreement between relativistic eikonal-approximation calculations of NRC cross sections and experiment. In

the prior form of the eikonal approximation, the interaction between the electron and target nucleus is approximately treated to all orders of perturbation theory by replacing the projectile wave functions with

$$\psi_f(r'_p, t') = \phi(r'_p) \exp(-i\tilde{E}_f t') \exp\left[-iZ_t' \int_t^\infty \frac{d\tau}{r_t}\right], \quad (57)$$

where r_t is the electron coordinant relative to the target nucleus, and $Z_t' = Z_t$. The interaction between the electron and projectile is treated only in first order. This version physically describes a hard collision of the electron with the projectile nucleus followed by multiple soft collisions with the target nucleus. Eichler (1985) has shown that the calculation of eikonal cross sections for relativistic projectiles can be simplified considerably using a density matrix formalism. The numerical evaluation of the cross section consists of integrating over the momentum transfer (which is also done in the OBK approximation) and another variable λ stemming from the integration over $d\tau$ in the eikonal phase factor in Equation (57).

Since the eikonal approximation is a high-energy approximation, one expects to find good agreement between theory and experiment if the ion velocity is much larger than the velocity of the active electron. For highly asymmetric collisions, the electron velocity in atomic units is equal to the value of the largest atomic charge. In Fig. 15, this implies that v should be much greater than Z_t . Hence the eikonal results are shown as dashed lines in the region where $v < 2Z_t$. The solid lines are in good agreement with experiment, but where $v < 2Z_t$, the cross sections calculated using the eikonal approximation are below experiment.

Impulse-approximation and other second order Born approximation formulations exist for relativistic projectiles (Jakubassa-Amundsen and Amundsen 1980, 1985), but no calculations have been done. Attention has focussed on the asymptotic dependence of NRC (Shakeshaft and Spruch 1979). The nonrelativistic OBK cross sections fall off as E^{-6} or v^{-12} at high velocities. At relativistic velocities, v approaches a constant c , hence most of the velocity factors drop out, and the relativistic OBK cross sections fall off as E^{-1} or γ^{-1} (Moiseiwitsch and Stockman 1980). (One can also see from Eq. (52) that the momentum transfer approaches a constant at large γ). The eikonal (Eichler 1985) and second-Born calculations of Humphries and Moiseiwitsch (1984) also find an asymptotic γ^{-1} dependence for capture of 1s target electrons into 1s orbitals of the projectile, but Jakubassa-Amundsen and Amundsen (1980, 1985) find a $(\ln\gamma)^2/\gamma$ asymptotic dependence in both the impulse and exact second-Born approximations. (Humphries and Moiseiwitsch make a peaking approximation which the Amundsens avoid). Shakeshaft (1979) has suggested that the question of the asymptotic fall off of the NRC cross sections may be academic, since REC is dominant at high relativistic velocities. However, this is not true for high-Z projectiles and target atoms. For U+U collisions, the REC and eikonal NRC cross sections are approximately equal above ~ 15 -GeV/amu, where both fall off as γ^{-1} (see also Sect. VI).

C. Electron Capture in High-Z Ions

Calculations of NRC cross sections often consider only transitions from 1s orbitals of the target to 1s orbitals of the projectile because these transitions are dominant at velocities that are large compared to the orbital

velocities. For very heavy ions, this condition may not be reached even at relativistic velocities, and there may be appreciable capture into outer shells of the projectile or from outer shells of the target.

At high velocities, the momentum transferred to the electron for electron capture $q_0 \sim (\gamma-1)mc/\gamma\beta$ becomes large, making the small- r (high momentum) part of the projectile- and target-electron wavefunctions important. The dominant contribution near the origin is from s orbitals whose electron density scales as $(Z/n)^3$. Thus, the total capture cross section from any filled target shell to any unfilled projectile shell should vary as (Meyerhof et al. 1985)

$$\sigma_{\text{capt}}(n_t \rightarrow n_p) \sim \left[\frac{Z_p}{n_p} \right]^3 \left[\frac{Z_t^*}{n_t} \right]^3 Z_p^2 (Z_t^*)^2. \quad (58)$$

For bare projectiles, one can use the full projectile charge Z_p , but for many-electron target wave functions, the electron density at the origin is smaller, and a reduced target charge Z_t^* is used. Calculations of NRC use relativistic or nonrelativistic hydrogenic target wave functions with reduced effective charges $Z_t^* = Z_t - \Delta Z$, where $\Delta Z = 0.3$ and 4.15 for the K and L shells (Slater 1930). Anholt and Eichler (1985) examined the validity of using effective charges to simulate many-electron wave functions for electron capture in highly asymmetric relativistic heavy-ion-atom collisions.

Figure 16 shows relative state-to-state NRC cross sections for 1050-MeV/amu Ne+Ag, 197-MeV/amu Xe+Ag, and 82-MeV/amu Xe+Au collisions. For asymmetric collisions such as Ne+Ag, Eq.(58) is nearly valid. For any given n_t , the cross sections decrease as n_p^{-3} . For low Z_t and constant n_p , the cross sections fall off faster than n_t^{-3} , due to the reduction of the target

charge Z_t^* by an n -dependent ΔZ value. At high Z_t , the relative target L capture cross sections are larger than Eq.(58) predicts (see also Fig. 14), due to the smaller momentum transfer needed for target L-electron capture.

For 82 MeV/amu Xe projectiles incident on Au targets, 1s-1s transitions are no longer dominant (Fig.16). In these collisions, capture of target L electrons into the projectile K shell gives the largest cross sections, but other state-to-state cross sections are large also. This occurs because the momentum transfer for transitions such as 2s,2p+1s is significantly smaller than for 1s-1s transitions. Since the cross sections fall off very steeply with increasing momentum transfer, the smaller momentum transfer overcompensates the n_t^{-3} or n_p^{-3} fall offs.

The dominance of capture into excited states of the projectile is directly seen in measurements of electron capture by Xe^{54+} ions (bare nuclei) and Xe^{52+} ions (filled K-shell) incident on high- Z_t targets (Figs. 17, 18, and 19; Meyerhof et al. 1985). For 82-MeV/amu Xe+Au and Ag collisions, approximately equal capture cross sections for Xe^{52+} and Xe^{54+} ions are found, indicating that capture into the K shell of Xe is not dominant. The REC cross sections vary approximately as n_p^{-3} . For Xe+Be collisions, REC dominates, and the Xe^{52+} capture cross sections are much smaller than the Xe^{54+} ones. Figures 17 and 18 also compare measured cross sections for electron capture by Xe^{52+} and Xe^{54+} with eikonal calculations. The eikonal approximation treats the interaction between the electron and one of the nuclei to all orders in perturbation theory and the interaction with the other nucleus to only first order. Depending on which is treated perturbatively, one obtains different cross-section results. This is referred to as the post-prior discrepancy. The usual choice for 1s-1s transitions is to use the prior form if Z_t is greater

than Z_p , and otherwise the post form. Meyerhof et al. (1985) proposed a criterion based on the largest interaction potential $\langle Z/r \rangle$. Since the expectation value of $\langle Z/r \rangle$ for hydrogenic atoms varies as Z^2/n^2 , independent of the electron subshell, one should use the post form if $Z_p/n_p > Z_t/n_t$ or the prior form if $Z_p/n_p < Z_t/n_t$. The post form is obtained from the prior form, Eqs. (54)-(57), by interchanging Z_p and Z_t everywhere, putting $Z_t' = Z_p$, and interchanging initial and final states. The solid lines in Figs. 17 and 18 were calculated with the higher potential criterion, and the dashed lines were calculated using the higher-charge one. In most cases the Z/n criterion (solid lines) for choosing the post or prior form agrees better with the data. Since the Z/n criterion is based on an estimate of the interaction strength using a diagonal matrix element, the effect of the shell size may be overestimated. Therefore, in some cases, the conventional higher-charge criterion (dashed lines) gives better agreement with the data.

V. Collisions in Solid Targets

Although atomic-collisions studies are usually done using solid targets, understanding the manifold interactions of ions in matter is one of the least tractable problems in atomic physics. At nonrelativistic velocities, one finds higher projectile charge states emerging from solid targets than from gas targets (Betz 1972). This is a useful effect in accelerator operation where higher ion charge states produce higher energy beams. The Bohr-Lindhard (1954) interpretation of gas-solid charge-state differences emphasizes the role of electronic excitation. Electrons are more easily ionized from excited states, therefore if the time between collisions is shorter than the decay times of the excited states, higher charge states will be obtained. For low-Z ions, higher charge states are produced in solid targets than in gases. The effect is expected to diminish for few-electron, very high-Z ions where the lifetimes of excited states are generally very short. Betz and Grodzins (1970) proposed that post-target Auger decay of excited states also gives higher charge states.

Relativistic heavy ions allow the detailed study of the role of excited states on the charge states and collisional interactions of ions in matter (Anholt 1985a; Anholt and Meyerhof 1986). Low-Z projectiles have small radiative decay rates, so that high excited-state populations are present in solid targets. Calculations are simple because only zero- or one-electron projectiles are present. Because the relative velocities are very high, the high-velocity approximations for ionization, excitation, and capture are valid. For high-Z, few-electron ions, the allowed electric-dipole decay rates are very large compared to the rate of charge-changing collisions, so that the relative excited-state populations are lower.

Nevertheless, a few metastable states decay slowly. Their influence on the charge states of high-Z ions in solid targets is a matter of investigation.

Experimental information about ions in matter is obtained from (1) the equilibrium charge states of ions measured downstream of the target using magnetic spectrometers, (2) the target-thickness dependence of the charge-state yields, (3) projectile $K\alpha$ (and $K\beta$) x-ray production solid targets, (4) metastable state populations (observed by delayed photon decay), and (5) REC photon production, which was discussed in Sect. IV. A.

For relatively thick targets, experimental cross sections are obtained by least-squares fitting the target-thickness dependence of measured charge state yields. Our models of ions in matter indicate how those cross sections should be interpreted.

A. Low-Z ions

For low-Z ions a four-state model consisting of two charge states, zero- and one-electron, and two excited states, 2s and 2p, was formulated (Anholt 1985a). This model was used to obtain equilibrium charge-state distributions and target-thickness dependences for relativistic ions with $Z \leq 18$ incident on solid targets between Be and Au. The model is schematically illustrated in Fig. 20. Electron capture cross sections into the 1s, 2s, and 2p states are designated as a_1 , a_2 , and a_3 . The 1s, 2s, or 2p electrons can be ionized with cross sections s_1 , s_2 , or s_3 . Monopole 1s-2s (x_1), dipole 1s-2p (x_2), and 2s-2p (x_3) excitation and de-excitation can occur. Besides collisional de-excitation, the states can decay by radiative decay, of which the 2p→1s decay rate is the largest. The 2s→1s and 2p→2s radiative decay rates can be neglected compared to the collisional de-excitation cross sections. In order to compare decay rates

with collisional cross sections, we define a decay cross section as

$$d_2 = \frac{\lambda_{2p+1s}}{n_2 \beta c \gamma} , \quad (59)$$

where λ_{2p+1s} is the radiative decay rate (Bethe and Salpeter 1957), and n_2 is the target-atom density. (We could also make calculations on a rate scale by converting the collisional cross sections to collision rates by multiplying by $n_2 \beta c$.)

From diagrams like that in Fig. 20a, one can immediately write down the rate equations governing the populations N_i of the various states.

$$\begin{aligned} \dot{N}_0 &= -(a_1 + a_2 + a_3)N_0 + s_1 N_1 + s_2 N_2 + s_3 N_3 , \\ \dot{N}_1 &= a_1 N_0 - (s_1 + x_1 + x_2)N_1 + x_1 N_2 + \left(\frac{x_2}{3} + d_2\right)N_3 , \\ \dot{N}_2 &= a_2 N_0 + x_1 N_1 - (s_2 + x_1 + x_3)N_2 + \frac{x_3}{3} N_3 , \\ \dot{N}_3 &= a_3 N_0 + x_2 N_1 + x_3 N_2 - \left(s_3 + d_2 + \frac{x_2 + x_3}{3}\right)N_3 , \end{aligned} \quad (60)$$

where $\dot{N}_i = dN_i/dT$, T is the target thickness in atoms/cm², and $\sum_i N_i = 1$. These equations are solved with the initial condition $N_i(0) = \delta_{i0}$ or δ_{i1} , depending on the incident ion charge states. The ratio of ions having an electron to fully stripped projectiles was measured, which is given by

$$R(T) = \frac{N_1(T) + N_2(T) + N_3(T)}{N_0(T)} . \quad (61)$$

The equilibrium ratio R_{eq} is obtained by setting all $\dot{N}_i = 0$, and solving the resulting set of linear equations.

With relativistic, low- Z ions, one can calculate accurately most of the required cross sections in Eq.(60), some of which are shown in Fig.21. The equilibrium ratios can then be calculated numerically or using approximations to the model. In the model of Anholt (1985a), it was

assumed that, because of the large magnitudes of the 2s-2p excitation cross sections, the 2s and 2p populations equilibrate according to the level multiplicity $N(2p)/N(2s) = 3$. Combining the equations for the 2s and 2p populations, as depicted in Fig.20b, yields three sets of linear equations, which can be solved analytically for the equilibrium ratio:

$$R_{eq} = \frac{\frac{3}{4}d_2(a_1+a_2+a_3) + \bar{S}_2 a_1}{\frac{3}{4}d_2 s_1 + (s_1+x_1+x_2)\bar{S}_2} \left[1 + \frac{x_1+x_2}{\frac{3}{4}d_2 + \bar{S}_2} \right] + \frac{a_2+a_3}{\frac{3}{4}d_2 + \bar{S}_2}, \quad (62)$$

where $\bar{S}_2 = \frac{1}{4}(3s_2 + s_2)$. This equation illuminates the origins of gas-solid charge-state differences. In a gas target, the decay cross sections d_2 are dominant because $n_2=0$, hence one has

$$R_{eq} = (a_1+a_2+a_3)/s_1. \quad (63)$$

This equation states that all captured electrons decay to the ground state, so the ratio of projectiles with one electron to zero electrons is the ratio of the total capture cross section to the 1s ionization one. However, for low-Z, relativistic projectiles in solid targets, one has $\bar{S}_2 \gg \frac{3}{4}d_2$ [Fig.21], hence we get

$$R_{eq} = \frac{a_1}{s_1+x_1+x_2} \left[1 + \frac{x_1+x_2}{\bar{S}_2} \right] + \frac{a_2+a_3}{\bar{S}_2}. \quad (64)$$

In this case, electrons captured into excited states (with cross sections a_2+a_3) do not decay, but are ionized with cross sections $\bar{S}_2$, so the fraction of projectiles having one electron in the 2s or 2p state is just $(a_2+a_3)/\bar{S}_2$. An electron captured into the 1s state can be excited into the n=2 state, where, if the L-shell ionization cross section is very large, it is ionized. Therefore, the fraction of projectiles with 1s electrons is approximately the 1s capture cross section divided by the 1s ionization plus excitation cross sections $s_1+x_1+x_2$. However, the L ionization cross sections are not

infinitely large, so the term $1+(x_1+x_2)/\bar{s}_2$ represents a correction; a fractional number of excited 1s electrons given by $a_1(x_1+x_2)/\bar{s}_2(s_1+x_1+x_2)$ remain in the $n=2$ states, and contribute to the total number of electron-bearing ions.

Comparison of Eqs.(63) and (64) predicts differences between relativistic ion equilibrium ratios measured in gas and solid targets. For the case of 400-MeV/amu Ne + Cu collisions, Eq.(63) gives $R_{eq} = 0.3 \times 10^{-6}$ for gas targets and Eq.(64) gives $R_{eq} = 0.23 \times 10^{-6}$ for solid targets, a 30% difference. For other Z_t values, the differences between gas- and solid-target charge states are shown by the dashed and solid lines for R_{eq} in Fig.21b. The solid-target fractions of one-electron projectiles are smaller, implying higher charge states, in agreement with nonrelativistic-velocity results (Betz 1972). Approximating R_{eq} for solid targets by the first term in Eq. (64), one can write

$$\frac{R_{gas}}{R_{solid}} = \frac{a_1+a_2+a_3}{a_1} \frac{s_1+x_1+x_2}{s_1} > 1 . \quad (65)$$

The solid-target charge state is higher than the gas-target value (or the one-electron fraction is lower, as expressed in Eq.(65)), because (1) in a gas, capture into all states leads to attachment, but only 1s capture leads to attachment in solids, and (2) in solids, excitation into excited states has a high probability of subsequent ionization, hence the effective ionization cross sections, $s_1+x_1+x_2$, are larger. This model is similar to the Bohr-Lindhard model since both emphasize the higher likelihood of ionizing excited electrons.

In the measurements of Crawford (1979), bare incident projectiles on thick targets were used ($N_i(0) = \delta_{i0}$), and $R(T)$ was fit to

$$R(T) = R_{eq} [1 - \exp(-\sigma_S T)], \quad (66)$$

where σ_S is an effective stripping cross section. Crawford (1979) interpreted σ_S as the 1s ionization cross section s_1 . If one interprets the equilibrium charge-state fraction in Eq. (64) as an effective capture cross section divided by an effective ionization cross section, neglecting the terms in $a_2 + a_3$, the effective ionization cross section is given by

$$\sigma_S = (s_1 + x_1 + x_2) / (1 + \frac{x_1 + x_2}{\bar{s}_2}). \quad (67)$$

Because excitation to the 2s and 2p levels with cross sections $x_1 + x_2$ enhances ionization, the effective ionization cross section is larger than s_1 , but is smaller than $s_1 + x_1 + x_2$ by the factor $1/[1 + (x_1 + x_2)/\bar{s}_2]$. Effective ionization cross sections may also be obtained by fitting the numerically calculated solutions to Eqs.(60)-(61) for $R(T)$ to Eq. (66).

Figure 22 shows results for 250- and 2100-MeV/amu Ne ions. At low- Z_t , REC dominates, and R_{eq} decreases with Z_t because the ionization cross sections increase as Z_t^2 and the REC ones increase linearly with Z_t . Eventually, R_{eq} increases with increasing Z_t due to NRC contributions to the total electron capture cross sections. Agreement between the measured and calculated R_{eq} values at low Z_t indicates that the model is complete, and the necessary cross sections and decay rates can be accurately calculated. Therefore, at higher Z_t , one can extract NRC cross sections from the measured equilibrium ratios shown in Figure 22. This is the origin of the NRC cross sections, which were compared with theory in Sec. IV (Figures 14 and 15).

In addition, where the target-thickness dependence of the one-electron charge-state ratio $R(T)$ was measured by Crawford (1979), the extracted

effective ionization cross sections agree with Eq.(67), and with fits to the numerically calculated target-thickness dependence of $R(T)$. Equation (67) therefore indicates how one should interpret thick-target stripping cross sections. For low-Z ions in solid targets, one measures a sum of the stripping and excitation cross sections. Larger effective solid-target stripping cross sections are consistent with the Bohr-Lindhard (1954) model. However, this is only for thick targets where the approach to equilibrium is measured. We have verified numerically that if one-electron ions are incident on thin targets, the yield of zero-electron ions increases as $s_1 T$ for $s_1 T \ll 1$.

B. High-Z ions

In high-Z, few-electron ions, the lifetimes of most states that decay directly to the ground state are much shorter than the mean time between charge-changing collisions or collisions where excitation occurs. For 197-MeV/amu Xe+Ag collisions, the cross section for the radiative decay of the 2p state in H-like ions is 4×10^6 barns, while the cross section for 2p ionization is only 3×10^5 barns. For low-Z ions, the predicted difference between charge states of ions in gas and solid targets occurs because the radiative decay cross sections [$\frac{1}{4} d_2$ in Eq.(62)] are smaller than the L-shell ionization cross sections $\bar{s}_2$ in solid targets (Fig.21) and are much larger than $\bar{s}_2$ in gas targets. Since the radiative decay cross sections are larger than L-shell ionization cross sections in both gas and solid targets in high-Z collisions, this would suggest that differences between gas and solid target charge-state distributions should vanish. Furthermore, one should be able to predict the charge-state distributions with a ground-state model.

The ground-state model of equilibrium charge states assumes that independent of whether electrons are captured into the ground state of a bare ion (with cross section a_1) or into excited states (with cross sections into the 2s, a_2 , and into the 2p, a_3), they always decay to ground state so that the charge-state distribution can be calculated with just the total electron capture cross sections and the ground-state 1s single-electron ionization s_1 and double-electron $1s^2$ ionization cross sections c_1 . The rate of change of the charge-state fractions F_j with $j=0, 1$, or 2 electrons are given by

$$\begin{aligned}\dot{F}_0 &= -(a_1+a_2+a_3) F_0 + s_1 F_1 + c_1 F_2 , \\ \dot{F}_1 &= (a_1+a_2+a_3) F_0 - (s_1+\frac{a_1}{2} +a_2+a_3) F_1 + 2s_1 F_2 , \\ \dot{F}_2 &= (\frac{a_1}{2}+a_2+a_3) F_1 - (c_1 + 2s_1) F_2 ,\end{aligned}\tag{68}$$

where $\dot{F}_i = dF_i/dT$, and T is the target thickness in atoms/cm². The equilibrium charge-state fractions are found by setting the derivatives equal to zero, and solving the resulting set of linear equations (Allison 1958)

$$\begin{aligned}F_0 &= (1 + A(1+B))^{-1} \\ F_1 &= AF_0 , \quad F_2 = BF_1 \\ A &= \frac{a_1+a_2+a_3}{s_1+Bc_1} \\ B &= \frac{\frac{a_1}{2} +a_2 +a_3}{2s_1+c_1} .\end{aligned}\tag{69}$$

The measured ionization and electron capture cross sections for 197-MeV/amu Xe ions, discussed in Sects. III and IV, are all that is needed to calculate the equilibrium charge states of 197-MeV/amu Xe ions in

matter. The ionization and capture cross sections are compiled in Fig. 23. The cross sections a_1 and a_2+a_3 are related to the Xe^{5+} and Xe^{5++} electron-capture cross sections using

$$\begin{aligned} a_1 &= \sigma(\text{Xe}^{5++}) - \sigma(\text{Xe}^{5+}) \\ a_2+a_3 &= \sigma(\text{Xe}^{5+}) . \end{aligned} \quad (70)$$

Double-electron ionization cross sections were discussed by Anholt et al. (1985). No evidence for simultaneous two-electron capture was observed in 197-MeV/amu Xe collisions, so double-electron capture has been omitted in the present calculations. Figure 24 compares measured equilibrium charge-state fractions for 0-, 1- and 2-electron ions with the ground-state model calculations (dashed lines). Although the measured and calculated values of F_0 and F_1 agree reasonably well, the ground-state model predicts fractions of 2-electron ions that are slightly higher than experiment. The shapes of the charge-state curves in Fig. 24 are due to competition between ionization and two capture processes, similar to that seen in Figs. 21 and 22 for low-Z ions. At low- Z_t , REC is dominant. The REC cross section increases linearly with Z_t , and the ionization cross sections increase as Z_t^2 , hence the fractions of one- and two-electron projectiles fall off with Z_t . Near $Z_t=15$, NRC becomes significant. The NRC cross sections increase faster with Z_t than the ionization cross sections up to $Z_t \sim 50$, hence the one- and two-electron fractions increase. Above $Z_t=50$, the NRC cross sections do not increase much faster than the ionization ones, hence the charge-state fractions level off.

The differences between the measured charge-state fractions and the ground-state model predictions can be traced to the influence of metastable excited states. For instance, in H-like ions, 2p electrons

decay quickly to the ground state, but 2s electrons decay more slowly, usually by excitation to the 2p state followed by 2p→1s radiative transitions. In He-like ions, the 1s2p 1P_1 and 3P_1 states decay quickly, but the 1s2s 1S_0 and 3S_1 and 1s2p 3P_0 and 3P_2 states decay by 1s2s→1s2p de-excitation and/or excitation to the rapidly decaying 1P_1 and 3P_1 states. Anholt and Meyerhof (1986) describe an 11-state model of the states of relativistic Xe ions in matter that includes the H-like 1s, 2s, and 2p states and the He-like 1s², 1s2s, and 1s2p states. The same measured ionization and electron capture cross sections were used as in the ground-state model calculations, but in addition, 1s-2s, 1s-2p, 2s-2p and 1s2s-1s2p excitation and 2s and 2p ionization cross sections were calculated absolutely or relative to the 1s ionization cross sections using the PWBA. The results of the 11-state model calculations are shown by solid lines in Fig. 24. The uncertainties in the measured ionization and capture cross sections give uncertainties in the calculated charge-state fractions. The upper solid lines were calculated using the largest capture and smallest ionization cross sections, and the lower solid lines were calculated using the highest ionization and lowest capture cross sections within the experimental uncertainties. The charge-state fractions calculated using the most likely capture and ionization cross sections are in good agreement with experiment.

Anholt and Meyerhof (1986) developed an analytical quasiground-state model that illuminates the origin of the difference between the calculated ground-state and 11-state charge-state fractions. This model assumes that the ions are usually in their ground 1s or 1s² states, hence the small populations of excited states can be calculated from the charge-state

fractions and ratios of excitation cross sections to excitation plus decay plus excited-state ionization cross sections. In the quasiground-state model, the ratio F_2/F_1 is given by

$$\frac{F_2}{F_1} = \frac{\frac{a_1}{2} + a_2 r_2 + a_3 r_3}{(2s_1 + c_1) r_s}, \quad (71)$$

which should be compared with the ground-state model value, B in Eq.(69), where r_2 , r_3 , and r_s are all unity. In 197-MeV/amu Xe+Ag collisions, one obtains $r_2 \approx 0.53$, $r_3 \approx 0.63$, and $r_s = 1.08$, so that $F_2/F_1 = 0.144$, instead of $B = 0.197$ (the numerical 11-state model gives $F_2/F_1 = 0.140$). Formulas for r_2 , r_3 , and r_s are given by Anholt and Meyerhof (1986). For other targets, the r values are not significantly different from the Xe+Ag ones.

For low- Z ions, higher charge states are seen in solid targets than in gas targets principally because the effective ionization cross sections are larger in solid targets. For high- Z ions, higher charge states are seen because the effective cross sections for capture into excited states, $r_2 a_2 + r_3 a_3$, are lower than in gas targets. Equation (67) showed that the effective ionization cross sections for low- Z ions in solid targets are larger than the 1s ionization cross sections by the addition of the excitation cross sections $x_1 + x_2$. This increases the effective ionization cross sections in 400-MeV/amu Ne+Ag collisions by a factor of 1.4. However, the factor $[1 + (x_1 + x_2)/\bar{s}_2]^{-1}$ in Eq.(67) limits the enhancement to only a factor of ~ 1.2 . Electrons excited into the 2s and 2p states of low- Z ions are easily ionized, so that excitation and ionization both lead to electron loss. For high- Z ions, however, dipole excitation populates states that decay rapidly back to the ground state, so that the effective

ionization cross sections are not enhanced significantly by dipole excitation. Most of the factor of $r_s=1.08$ increase in the effective ionization cross sections for He-like ions, $r_s(2s_1+c_1)$ in Eq.(71), comes from monopole excitation to $1s2s$ states. For high-Z ions, the excited-state capture cross sections a_2 and a_3 are reduced by about 50% in thick-solid targets. Electrons captured into the metastable $1s2s$ and $1s2p$ states decay slowly by $1s2s-1s2p$ excitation, so that there is a high probability that the electrons will be ionized before decaying to the ground state, hence will contribute less to the fraction of He-like ions. For low-Z ions where electrons captured into even the $2p$ states do not decay rapidly, excited-state capture contributes negligibly to the one-electron charge-state fractions. However, in low-Z ions, the excited-state capture cross sections are only 20% of the total electron capture cross sections into bare ions. For high-Z ions, the relative probability of electron capture into excited states of the projectile is much larger, as discussed in Sec. IV, so the 50% reduction in the excited-state capture cross sections for high-Z ions in thick targets, gives a significant change in the 2-electron charge-state fractions.

Information about the states of high-Z ions in matter can be obtained from the projectile K x-ray production cross sections. For example, in the 11-state model, the equilibrium $K\alpha$ x-ray production cross sections are given by

$$\sigma_{K\alpha} = d_1 N(2p) + d_2 N(^1P_1) + d_3 N(^3P_1), \quad (72)$$

where d_i are decay cross sections [Eq.(59)] for the $2p$, 1P_1 , and 3P_1 states, and N_i are the calculated equilibrium populations of the $2p$, 1P_1 , and 3P_1 states. Figure 25 compares calculations of $K\alpha$ x-ray production in

197-MeV/amu Xe collisions with experiment. The major theoretical uncertainty in the model is in the fraction of electrons captured by Xe^{5+} ions that go into the 2p state. If we assume that a_3 is equal to the total Xe^{5+} capture cross section, we obtain the upper dashed curves in Fig. 25; if we assume that $a_3=0$, we get the lower curves. All other cross sections in these calculations are identical to those giving the charge-state fractions agreeing well with experiment in Fig. 24. The fraction of the Xe^{5+} electron capture cross section to the 2p state, calculated using the eikonal approximation for NRC and photoelectric cross sections for REC, gives good agreement with the measured Xe $K\alpha$ x-ray production cross sections.

The quasiground-state model may be applied to analyse the origin of projectile $K\alpha$ x-ray production in thick targets (Anholt and Meyerhof 1986). Three mechanisms are present in 197-MeV/amu Xe collisions: 1s-2p excitation followed by radiative decay, 2p capture followed by radiative decay, and 1s-2s excitation or 2s capture followed by 2s-2p transfer and radiative decay. In 197-MeV/amu Xe collisions, 1s-2p excitation is dominant, and 2p capture and the 2s contributions each give about 10% of the total $K\alpha$ x-ray cross section. The quasiground-state-model formulas for projectile $K\alpha$ x-ray production were extended to 3- and 4-electron ions, so that cross sections for 82-MeV/amu Xe collisions could be calculated from measured charge-state fractions where significant 3- and 4-electron fractions are present (Fig.25). The uncertainties in the 2p capture cross sections are also present for 82-MeV/amu Xe ions, and give uncertainties in the calculated $K\alpha$ x-ray cross sections. The calculations using the relative eikonal and REC capture cross sections are in good agreement with experiment.

VI. X-RAY CONTINUUM PROCESSES

A. Primary Bremsstrahlung

Radiative electron capture is the capture of target electrons into bound states of the projectile. An analogous process is the capture into continuum states of the projectile. Viewed differently: in the projectile frame, the projectile nucleus is bombarded by target electrons with mean kinetic energy $(\gamma-1)mc^2$. These may emit bremsstrahlung photons with energy E_x as large as the electron kinetic energy in the projectile frame, 109 keV for 200-MeV/amu projectiles. In the laboratory frame, the end-point x-ray energy is given by

$$E_x' = (\gamma-1)mc^2 \gamma^{-1} (1-\beta\cos\theta')^{-1}, \quad (73)$$

which varies in 200-MeV/amu collisions from approximately 56 keV at backward angles to 200 keV at forward ones.

As a first approximation, the primary-bremsstrahlung (PB) cross section is just the bremsstrahlung cross section for Z_t electrons with kinetic energy $T = (\gamma-1)mc^2$ bombarding the projectile nucleus Lorentz transformed into the laboratory (Schnopper et al. 1974, Spindler et al. 1979, Yamadera et al 1981, Anholt et al. 1986):

$$\frac{d^2\sigma_{PB}(E_x', \theta')}{dE_x' d\Omega'} = Z_t \frac{d^2\sigma_{brem}(E_x, \theta, Z_p, T)}{dE_x d\Omega} \frac{dE_x}{dE_x'} \frac{d\Omega}{d\Omega'}, \quad (74)$$

where the relation between θ and θ' and $d\Omega$ and $d\Omega'$ are given by Eq.(47), and

$$\frac{dE_x}{dE_x'} = \frac{1-\beta\cos\theta'}{(1-\beta^2)^{1/2}}. \quad (75)$$

Here the unprimed quantities are projectile-frame (center-of-mass for projectile-electron collisions) quantities, and the primed ones are laboratory ones. For the bremsstrahlung cross section, the Bethe-Heitler (1934) formula [Eq.(3BN) of Koch and Motz 1959] including the Elwert (1939) correction factor

was used.

Equations (73) to (75) assume the target electrons are free and have no intrinsic momentum, therefore the theory gives a sharp cut off in the continuum spectrum where the x-ray center-of-mass energy is equal to the electron kinetic energy. When one adds the Fermi momentum of the target electrons p_s to the translational momentum $p = \gamma\beta mc$, larger electron kinetic energies and therefore higher bremsstrahlung end-point energies are obtained. The continuum shape can be calculated in the impulse approximation using (Jakubassa and Kleber 1975, Anholt et al. 1986)

$$\frac{d^2\sigma_{PB}}{dE_x' d\Omega'} = \int_{-\infty}^{+\infty} dp_z J(p_z) \frac{d^2\sigma_{PB}(T, E_x', \theta')}{dE_x' d\Omega'} , \quad (76)$$

where the PB cross section on the right-hand side is given by Eq.(74), $J(p_z)$ is the Compton profile (Biggs et al. 1975), and the electron kinetic energy is given by

$$T = \left[[(\gamma\beta + p_z/mc)^2 + 1]^{1/2} - 1 \right] mc^2. \quad (77)$$

Finally, one averages over the laboratory angles subtended by the x-ray detector. These folding procedures have no effect on the PB cross section for x-ray energies well below the end point (Fig.1). The electron momentum folding causes the spectrum to drop off more slowly above the end point; the Doppler folding shifts the position of the end point to slightly higher energies, due to the inclusion of smaller laboratory angles, and rounds off the continuum shape near the end point.

The calculated PB cross sections are generally lower than the measurements of Anholt et al. (1986), as shown in Fig.1 and discussed in more detail below. However, the shape of the continua agree reasonably well with

experiment, at least where PB is not obscured by REC lines and bremsstrahlung from secondary electrons (see below), as in Be-target collisions.

The electron bremsstrahlung cross section has an approximate angular distribution in the center-of-mass frame of the form (Tseng et al. 1979)

$$\frac{d^2\sigma_{\text{brem}}}{dE_X d\Omega} = \frac{\sin^2\theta + \frac{1}{2} - \frac{3\beta}{1+\beta^2} \cos\theta}{(1-\beta\cos\theta)^4}, \quad (78)$$

where θ here is the angle between the electron direction and photon direction, so the angular distribution in the projectile frame is peaked in the direction opposite to the beam direction. The first term in this equation is identical in form to the photoelectron angular distribution determining REC, hence gives an angular distribution of PB in the laboratory of the form:

$$\frac{d^2\sigma_{\text{PB}}(E_X', \theta')}{dE_X' d\Omega'} \sim \sin^2\theta' (1-\beta\cos\theta'). \quad (79)$$

Unlike for REC, the PB cross section is differential in laboratory x-ray energy, so instead of obtaining an angular distribution proportional to $\sin^2\theta'$, an additional term $1-\beta\cos\theta'$ is present. The angular distribution is determined keeping the center-of-mass x-ray energy E_X constant. For $E_X=45$ keV in 197-MeV/amu Xe + Be collisions, the continuum cross sections are taken at $E_X'=62$ keV at 45° , 37 keV at 90° , 27 keV at 135° , etc. Figure 26 shows that the measured angular distributions are indeed peaked at backward angles, as suggested by Eq.(79). The solid lines were calculated with the Bethe-Heitler (1934) formula for the bremsstrahlung angular distribution, but were normalized to the data near $\theta'=90^\circ$ (Anholt et al. 1986). The agreement between the measured and calculated PB angular distribution shapes is very good.

B. Secondary Electron Bremsstrahlung

In a binary collision between a 197-MeV/amu heavy ion and a nearly free electron, electrons with kinetic energies up to 450 keV can be produced. These electrons can collide with other nearby target nuclei in solid targets, emitting secondary-electron bremsstrahlung (SEB) photons with energies up to ~450 keV. To calculate the cross section for SEB, we assume (i) that Z_t target electrons are free and scatter elastically from the projectile nucleus, and (ii) the ionized electrons follow a tortuous path inside the solid target so electrons never leave the target material, and the angle between the bremsstrahlung photon direction and the projectile direction is random. The latter assumption follows from a calculation (Jackson 1962) of the mean multiple scattering angle for 50- to 200-keV electrons in the 5 to 50-mg/cm² targets used. The mean multiple-scattering angle is of the order of several radians, so one cannot assume the electrons travel in a straight line after being emitted. With these assumptions, the cross section is given by (Anholt et al. 1986, Folkmann et al. 1973, Ishii et al. 1977)

$$\frac{d\sigma_{\text{SEB}}(E_x')}{dE_x'} = Z_t \int_{E_x'}^{E_{\text{max}}} dE_e' \frac{d\sigma_{\text{elas}}}{dE_e'} \int_{E_e}^{E_x'} \frac{dE_e'}{S(E_e')} n_2 \frac{d\sigma_{\text{brem}}(E_x')}{dE_x'}, \quad (80)$$

where the elastic electron scattering cross section is given by the McKinley-Feshbach (1948) equation, $E_{\text{max}} = 2\gamma^2\beta^2 mc^2$, $S(E_e)$ is the electron stopping power in the target material (Ahlen 1980), $\gamma_e = 1 + E_e'/mc^2$, $\beta_e^2 = 1 - \gamma_e^{-2}$, and $d\sigma_{\text{brem}}/dE_x$ is the angle-integrated bremsstrahlung cross section calculated using the Bethe-Heitler-Elwert formula (Koch and Motz 1959). SEB is dominant for high- Z_t targets where the agreement between theory and experiment is very good [Fig.1 for U+U collisions; Anholt et al 1986)].

C. The Z_t -dependence of continuum x-ray production

For SEB, the bremsstrahlung cross section in Eq.(80) varies as Z_t^2 , the stopping power $S(E)$ varies roughly as Z_t/A_t , where A_t is the target atomic number, n_2 varies as A_t^{-1} , the elastic cross section varies as Z_p^2 , and Z_t electrons can participate in SEB, so the SEB cross section for infinite target thickness varies as $Z_p^2 Z_t^2$. For PB, the bremsstrahlung cross section varies as Z_p^2 and Z_t electrons participate, so the PB cross section varies as $Z_p^2 Z_t$.

Figure 27 shows the Z_t -dependence of continuum x-ray production in 82-, 197-, and 422-MeV/amu Xe and U collisions (Anholt et al. 1986). The x-ray energy chosen falls within the region where both PB and SEB are present. The x-ray energies were chosen to avoid as much as possible interference with characteristic x rays and REC (though cross sections for continua falling beneath target x-ray lines could not be obtained at some Z_t values).

Good agreement between the SEB calculations and experiment is found at high- Z_t , but there is a systematic discrepancy at low- Z_t where PB dominates. Since the shape of the calculated and measured x-ray spectra agree reasonably well, identical qualitative results are expected at other x-ray energies. If one subtracts the calculated SEB contributions from the measured cross sections, the resulting cross sections increase linearly with Z_t , as predicted by the PB theory, but are factors of 1.7 (197-MeV/amu Xe) to 2.9 (422-MeV/amu U) too high.

The reason for the discrepancy between calculations of PB and experiment is not known. The calculated PB cross sections are probably accurate to within $\pm 30\%$. Tseng et.al.(1979) and Lee et.al.(1976) compared double- and single-differential electron bremsstrahlung cross sections calculated with the Bethe-Heitler-Elwert formula, with numerical calculations using fully

screened atomic electronic wavefunctions or Dirac wave functions. For the present nearly bare projectiles, Dirac electronic wave functions should be used, but there is generally less than a $\pm 30\%$ difference between the three different calculations in the relevant electron energy, Z , and angular range. The reasonably good agreement between measured atomic electron bremsstrahlung cross sections (Quarles et al. 1981) for similar electron energy and Z ranges and the same calculations (Kissel et al. 1983), suggests that it is unlikely that the origin of the disagreement between the PB calculations and experiment lies in the electron bremsstrahlung cross sections.

The SEB calculations are less certain than the PB ones, but we have not hypothesized a realistic mechanism that would quantitatively account for the discrepancy between theory and experiment for the Be targets. For example, one could assume that a significant number of electrons escape from the Be target and collide with the higher- Z Al ($Z_{\text{wall}} = 13$) or mylar ($Z_{\text{wall}} \sim 6.6$) chamber walls. The relative contribution from the chamber wall increases with Z_{wall} , not as Z_{wall}^2 , because the electron stopping in the wall cancels one power of Z . A simple model which assumes that the electrons travel in a straight line, then upon leaving the target collide with an Al wall in the same position as the target and make SEB along the remaining part of their range, increases the U+Be SEB contributions by less than a factor of 1.5, which cannot explain the entire discrepancy between theory and experiment there.

In conclusion, the shape of the PB spectrum and the angular distribution of the radiation agree with theory, but the magnitude of the measured cross sections differ by factors of 1.7 to 2.9. The measurements are not

sufficient to ascertain whether the discrepancy between theory and experiment is a Z_p -dependent or velocity-dependent effect. The two measurements for Xe ions at 82- and 197-MeV/amu where the ratio of experiment to theory varies from 1.7 ± 0.2 to 1.8 ± 0.2 are not sufficient to tell. Measurements with Xe and U ions up to 962-MeV/amu or U ions down to 100-MeV/amu would be useful.

VII. ULTRARELATIVISTIC COLLISIONS

Ultrarelativistic heavy ion accelerators have been proposed for the study of quark matter. The energies of these proposed accelerators range from about 10 GeV/amu for fixed-target machines (Lawrence Berkeley Laboratory 1982, Brookhaven National Laboratory 1983) up to 100 GeV/amu per beam for colliders (Lawrence Berkeley Laboratory 1979, Brookhaven National Laboratory 1984). A 100 GeV/amu collider is equivalent to a fixed-target energy of 20 TeV/amu. An interesting question is the type and significance of atomic collisions in this energy range.

Radiative and nonradiative capture cross sections decrease at relativistic energies, but ionization cross sections increase logarithmically, leading, it would appear, to a desert of bare ions. This may not always be the case because electrons from electron - positron pair production may be captured by the ions. The cross section for electron - positron pair production reaches several thousand barns for 15 GeV/amu uranium on fixed-target uranium and an astonishing 10^5 barns for 100 GeV/amu colliding uranium beams. If even a small fraction of the electrons are captured by the uranium ions, this would result in a significant charge capture cross section compared to the usual radiative and nonradiative capture cross sections.

A detailed theory of electron-positron charge -changing processes at relativistic energies has not yet been worked out, but it is possible to make a crude estimate of the charge changing cross section from a simple formula for electron - positron pair production and from a classical model for the binding of the electron. The cross section for producing electron - positron pairs from the Coulomb field of two (bare) nuclei was one of the early quantum-mechanics collisions problems (Landau and Lifshitz 1934, v. Weizsacker 1934, Williams 1935,

Bhabha 1935, 1935a, Nishina, Tomonaga and Kobayasi 1935, Oppenheimer 1935, Nordheim 1935, Racah 1937). The mechanism for producing these pairs may be thought of as pair-producing absorption of the virtual photons from the motional Coulomb field scattering from a static Coulomb field (Weitsäcker 1934, Williams 1935). A simple formula for the total cross section for large γ is (Bhabha 1935a, 1935b):

$$\sigma_{\text{pair}} = (28/27\pi) \alpha^2 Z_p^2 Z_t^2 r_0^2 (\log \gamma)^3, \quad (81)$$

where $r_0 = e^2/mc^2 = 2.8 \times 10^{-13}$ is the classical electron radius. For uranium on uranium $\sigma_{\text{pair}} = 98 (\log \gamma)^3$ barns.

Cross sections for electron - positron pair production for uranium on uranium at ultra-relativistic energies, computed from Eq. (81) are given in Table II. In a 100 GeV/amu uranium collider with a luminosity of $10^{27} \text{ cm}^{-2}\text{s}^{-1}$ 10^8 electron - positron pairs per second would be produced in each interaction region. Such a machine has been proposed (Brookhaven National Laboratory 1984). Heavier leptons will also be produced in smaller numbers. Upper limits, obtained by neglecting the nuclear size, are, for muons m_e/m_μ times the electron pair rate, or about 2000/sec. Tau pairs and other heavy particles can also be produced. Appreciable fractions of all of these particles would be bound to the uranium nuclei. As the pair production rate is luminosity dependent rather than beam current dependent, pair production may be used as a real-time non-destructive luminometer. Additional discussion of atomic physics effects in relativistic heavy ion collisions may be found in Gould (1984, 1985b) and in Brookhaven National Laboratory (1984).

The electrons most likely to be captured after pair production are those having momenta which overlap with the momenta of the final bound K electron. This means that electrons with kinetic energy less than the uranium K-shell

binding energy may be captured. The energy distribution of the pairs produced by colliding 15 GeV/amu uranium ions with a fixed uranium target extends to about 10 MeV, but is peaked at the lower energies. The fraction of electrons within the K shell with kinetic energies of less than 130 keV would then be roughly $130 \text{ keV} / 10 \text{ MeV} \approx 0.01$, hence approximately 1 percent of the electrons could be captured into the K-shell. For muons, which have larger binding energies, the capture fraction could be larger, but the production cross sections are probably much lower.

A synchrotron booster has been proposed to inject heavy ions into the Brookhaven Alternate Gradient Synchrotron (Brookhaven National Laboratory - 1984) allowing acceleration to 15 GeV/amu. The calculated equilibrium ratio of one-electron Au^{79+} to bare Au^{79+} at 15 GeV/amu is shown in Fig. 28. Neglecting pair production, the radiative electron capture (REC), nonradiative electron capture (NRC), and projectile ionization cross sections produce equilibrium ratios as a function of Z_t resembling those of lower-energy relativistic ions (Figs. 22 and 24). For low Z_t , REC, which scales as Z_t , is dominant. Since the ionization cross sections scale as Z_t^2 , the equilibrium ratios decrease with increasing Z_t in this region. At high Z_t ($Z_t=80$), NRC becomes important so that the equilibrium ratios increase rapidly with Z_t . The NRC cross sections shown in Fig. 28 were calculated using the eikonal approximation (Eichler 1985), and the REC cross sections were calculated using the Sauter formula for photoelectric cross sections (Sauter 1931, Pratt et al. 1973). If one percent of the electrons made by electron-positron pair production are captured, the equilibrium ratios increase by 2.5×10^{-4} . The increase is independent of Z_t because both the pair production and ionization cross sections scale as Z_t^2 .

VIII. RELATIVISTIC FEW-ELECTRON IONS IN QUANTUM ELECTRODYNAMICS EXPERIMENTS

Relativistic, very high-Z atoms present special opportunities for atomic structure experiments because any atom in the periodic table can be stripped to few- electron or even bare ions. Relativistic and quantum-electrodynamics (QED) effects can then be studied in systems where they are very large, can be experimentally isolated, and where precise and unambiguous calculations can be performed.

Tests of QED in the highest-Z, few- electron atoms are of particular interest because strong field tests of the theory can not be matched in low Z experiments of any conceivable precision. At $Z=92$, the contributions to the Lamb shift are the self-energy, -57 eV (Mohr 1982), the vacuum polarization, +14 eV (Mohr 1983), and the finite nuclear size correction, -33 eV (Mohr 1983). The vacuum polarization, but not the self energy, is well tested in muonic atom experiments. High-Z Lamb shift experiments are primarily a test of the self-energy in a strong Coulomb field.

The self-energy contribution comes almost entirely from terms which are of very high order in $Z\alpha$ (where α is the fine structure constant). Because these terms are large only at very high Z (strong Coulomb field), they are not tested in present lower-Z Lamb shift and fine structure experiments. The contribution of the higher order terms in the self-energy can be seen by comparing the series expansion for the self energy of the $1^2S_{1/2}$ state with a numerical evaluation to all orders in $Z\alpha$ (Mohr 1982, Desiderio and Johnson 1971, Erickson 1971, Cheng and Johnson 1976, Johnson and Soff 1985, Mohr 1974, Sapirstein 1981). If we write the self energy Σ_n in a power series in α (the fine structure constant) and $Z\alpha$, we have:

$$\begin{aligned} \Sigma_n = n^{-3} (\alpha/\pi) m_e c^2 & \left[[A_{40} + A_{41} \ln(Z\alpha)^{-2}] (Z\alpha)^4 + A_{50} (Z\alpha)^5 \right. \\ & + [A_{60} + A_{61} \ln(Z\alpha)^{-2} + A_{62} \ln^2(Z\alpha)^{-2}] (Z\alpha)^6 + A_{70} (Z\alpha)^7 \\ & \left. + \text{higher order terms} \right] \end{aligned} \quad (82)$$

Where n is the principal quantum number and m_e is the electron mass. Values of the coefficients A_{40} to A_{70} can be found in Mohr (1974) and Sapirstein (1981). Figure 29 shows the ratio of the higher-order terms in the self-energy (terms of higher order than $A_{70}[Z\alpha]^7$) to the total self-energy. In neutral hydrogen the higher-order terms in the self-energy contribute about 0.1 parts per million to the Lamb shift, which is nearly 100 times smaller than the uncertainty due to proton structure (see for example Lundeen and Pipkin 1981). At $Z=18$ the contribution is only about 1% of the Lamb shift, which is equal to or smaller than the typical experimental uncertainties (Gould and Marrus 1983, O.R. Wood *et al.* 1982, H.D. Strater *et al.* 1984, P. Pellegrin *et al.* 1982). At $Z=92$ however, the higher-order terms are essentially the entire self-energy contribution, and make up over half of the total Lamb shift.

Previous Lamb shift experiments are compatible with both the series expansion and numerical calculations to all orders in $Z\alpha$. Again, this is because the higher-order terms in the self-energy are large only at very high Z , hence they are not measured in present QED experiments. A consequence is that a significant deviation from QED at high Z is unobservable in previous experiments, and the difference between the numerical calculation and the series expansion is not well tested.

The presently available flux of 10^5 uranium ions per second at the Lawrence Berkeley Laboratory's Bevalac and a planned increase of a factor of ten makes several atomic spectroscopy experiments which test high- Z QED possible. These experiments include: a) Measurement of the 2^3P_0 lifetime in a heliumlike very

heavy atom which, with the $2^3P_0 - 2^3S_1$ electric dipole matrix element, and the $2^3P_0 - 1^1S_0$ two-photon decay rate determines the $2^3P_0 - 2^3S_1$ splitting; b) Direct spectroscopic measurement of the UV transitions $2^3P_0 - 2^3S_1$ in heliumlike atoms and the $2^2P_{1/2} - 2^2S_{1/2}$ transition in lithiumlike atoms (about 254 eV and 283 eV in heliumlike uranium and lithiumlike uranium respectively); c) Measurements of the $2^2P_{3/2} - 1^2S_{1/2}$ transition in a hydrogenlike atom (roughly 102 keV in hydrogenlike uranium); d) Measurements of the $2^3P_2 - 2^3S_1$ transition in a heliumlike atom (about 4.5 keV in heliumlike uranium); e) Measurements of the hyperfine splitting in hydrogenlike thallium (about $3800 \text{ \AA}$); and f) Measurements of the g-factor of the $1^2S_{1/2}$ state of a hydrogenlike atom. Measurements e) and f) are tests of the QED contribution to the anomalous magnetic moment of an electron bound in a Coulomb field. A more complete description of very high-Z QED experiments can be found in Gould (1985a, 1985b).

VII. CONCLUSIONS

Studies of ionization and capture processes using relativistic heavy ions can probe theories of ionization and electron capture in a regime where the ion velocity equals or exceeds the K-electron velocity. For ionization, this has the consequence that we can, for the first time, connect studies of inner-shell ionization (which have traditionally been made using light ions incident on heavy atoms at low to medium velocities where $v/v_K \lesssim 3$) with studies of $p+H$, $p+He$, and $He+He$ collisions where $v/v_K > 1$. By studying projectile ionization as a function of the perturbing nuclear charge, one can continuously connect measurements in asymmetric collisions where there are small perturbing charges with studies of symmetric collisions in the region of $v/v_K \sim 1$.

The study of NRC at relativistic velocities has significantly advanced the theory of electron capture. NRC into heavy ions is dominated by capture of outer-shell electrons into outer-shell projectile orbitals. The eikonal theory has evolved to take into account outer-shell capture.

The study of relativistic ions allows, for the first time, the formulation of an ab initio theory of ions in matter incorporating excited-state effects. Although minimal models have been used until now, the theories are in good agreement with measurements of projectile charge states emerging from solid targets and projectile x-ray production. The questions we can answer with these models concern the differences between gas- and solid-target charge states, the origin of projectile $K\alpha$ x-ray production in solid targets, and what is measured when one obtains cross sections from fits of the target-thickness dependence of projectile charge states in thin and thick targets.

Atomic collisions at ultrarelativistic energies have not yet been explored experimentally. Electron capture from pair production is expected, but reliable estimates of the pair-capture cross sections have yet to be calculated. For high-Z ions and targets, REC and NRC are expected to be of comparable magnitude, hence it may be possible to test asymptotic theories of NRC.

Atomic structure experiments will grow more sophisticated as facilities improve and experience is developed. Detailed understanding of relativistic atomic collisions enables the spectroscopists to populate desired states efficiently, minimizing interfering transitions and background.

The measurements that have been made until now have barely begun to explore the myriad of possible studies of relativistic heavy ions. This review has concentrated on studies of cross sections for atomic-collision processes since such measurements are the simplest to do under the present experimental conditions. We look forward to the application of relativistic heavy ion storage rings, and new relativistic heavy-ion accelerators to atomic physics.

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Figure Captions

Figure 1: Double differential x-ray production cross sections seen in 422-MeV/amu U+Be and U+U collisions at $\theta'=85^\circ$. The curves show calculated contributions due to secondary-electron bremsstrahlung (short-dashed lines), radiative electron capture (long-dashed lines), and primary bremsstrahlung (chain curves). The solid curves show the total intensity. From Anholt et al (1986).

Figure 2: Schematic diagram of the experimental apparatus used to measure charge-changing cross sections at the Bevalac. Target foils were inserted into the beam in the box marked "target area". The magnets B40Q2A and B are quadrupole, and B40M2 and 3 are dipole magnets. The dipole magnets dispersed the charge state beams into a large vacuum chamber. The position sensitive detector could be moved along the exit window of the vacuum chamber. Beams of ions in a selected charge state were prepared by stripping and magnetic analysis upstream of the apparatus. From Gould et al. (1985) and Meyerhof et al. (1985).

Figure 3: Charge states of incident 960-MeV/amu U^{6+} after passing through 7.1-mg/cm² mylar. The spatial separation between adjacent peaks is approximately 1 cm. The horizontal focus of the beam and position resolution of the position-sensitive detector are about 0.2 cm. From Crawford et al. (1983).

Figure 4: Very-high-energy behavior of proton-induced uranium K-shell ionization cross sections calculated using the plane-wave-Born approximation.

From Anholt (1979).

Figure 5: Measured and calculated K-vacancy production by 4.88 GeV protons versus target atomic number. Solid line: longitudinal contribution; dashed line: longitudinal and transverse contributions; chain curve: total cross section, including spin-flip contributions. From Anholt (1979).

Figure 6: K-vacancy production by 2-MeV (Li-Scholz et al. 1973), 50-MeV (Hoffmann et al. 1978, 1980), 300-MeV and 900-MeV (Middlemann et al. 1970) electrons. The solid lines are plane-wave-Born approximations calculations using Eq.(28). From Anholt (1979).

Figure 7: Calculated cross sections for ionizing 2s and 1s projectile electrons for 400-MeV/amu Ne and 962-MeV/amu U ions versus Z_t . If screening is neglected, σ/Z_t^2 is independent of Z_t (solid curves). Screening without antiscreening gives the chain curve. The short- and long-dashed curves were calculated using F and $|F|^2/Z_t$ respectively for the antiscreening correction. The difference between the two calculations is negligible. For U, screening effects are almost negligible, and the cross sections vary as $Z_t^2 + Z_t$ due to the antiscreening term. From Anholt (1985a).

Figure 8: Measured and calculated K-shell vacancy production cross sections. The numbers beside each curve denote the ion energy in MeV/amu. The ionization cross sections were calculated with Eq.(28) (solid line), with the Bethe (dipole) approximation (chain curve), and with the Born approximation

with corrections for binding and polarization effects (long dashed curve; Basbas et al. 1978). Fully stripped projectiles can also capture target K electrons, giving the estimated contributions shown by the short dashed curves. From Anholt et al. (1984b).

Figure 9: Reduced projectile ionization cross sections σ/Z_t^2 for 81.5-, 140-, and 200-MeV/amu Xe ions plotted against target atomic number. The reduced cross sections σ_{red} were obtained by subtracting the electron induced cross sections $Z_t\sigma_e$ from the measured ones and dividing by Z_t^2 . The thick solid lines are PWBA calculations, and the thin solid lines were calculated with the theory of Basbas et al. (1978) for binding and polarization effects using $c_K = 1.5$ and $c_K = 3$ (dashed lines). The chain-curve are Glauber-approximation calculations. From Anholt et al. (1985).

Figure 10: Scaled Xe+Xe ($\square$) and p+H 1s ionization cross sections plotted against proton kinetic energy. The PWBA (thick solid line), Basbas theory using $c_K = 1.5$ (thin solid line) and $c_K = 3$ (dashed line), and Glauber theory (chain curve) results are shown. The p+H data points are from Park (1983 and private communication; triangles) and Shah and Gilbody (1981; closed circles). Some of Park's points for $E_p > 50$ keV have been omitted. From Anholt et al. (1985).

Figure 11: Target $L\alpha$, $L\beta$, and LY x-ray production for several projectiles (Anholt et al. 1984b). The solid line shows the plane-wave-Born approximation ionization cross sections converted to x-ray cross sections using single-vacancy fluorescence yields and Koster-Kronig transition

probabilities.

Figure 12: Angular distribution of K REC photons seen in 197-MeV/a.m.u. Xe+Be collisions and the ratio to $\sin^2\theta_{lab}$ normalized at $\theta_{lab}=90^\circ$ (Anholt et al. 1984a). For the photoelectron angular distributions, from which the REC angular distributions are derived, results from Sauter (1931; dashed lines), Hultberg et al. (1967; chain curve), and neglecting electron retardation (solid line) were used. The Hultberg angular distributions were calculated using Dirac wave functions, but are for 150-keV protons incident on Sn, corresponding to similiar 210-MeV/amu Sn+Be collisions.

Figure 13: Measured (points) and calculated (solid line) K REC cross sections and the deduced average number of projectile K vacancies. Crosses from post-target charge fraction measurements of Gould et.al. (1984), and the solid lines for N_{Kv} are charge-fraction measurements from Meyerhof et al. (1985).

Figure 14: Projectile K electron capture cross sections (per bare ion) for 1050-MeV/amu Ne ions. The OBK results (Moiseiwitsch and Stockman 1980) are shown by the dashed lines and the $\square$ point, and the eikonal calculations with shielding for target K, L, and the sum of K and L capture are shown by the solid lines. A second-Born approximation calculation (Humphries and Moiseiwitsch 1985) is shown by the ∇ point. Data ($\bullet$) from Crawford (1979) as analysed by Anholt (1985a). From Anholt and Eichler (1985).

Figure 15: Projectile K-electron capture cross sections calculated using the eikonal approximation (lines) compared with measurements of Crawford (1979)

as analysed by Anholt (1985a) and second-Born calculations (Humphries and Moiseiwitsch 1985; 7 points, always lying immediately above the data points shown by closed circles). The dashed part of the curves show the region where the eikonal approximation, including target K and L capture, may not be valid (see text).

Figure 16: Calculated relative cross sections for the capture of electrons from target shells with $n_t = 1, 2, 3$ (A is the sum of all target shells) into bare projectile shells with $n_p = 1, 2, 3$, and all shells with $n_p \geq 4$ for 1050-MeV/amu Ne + Ag, 197-MeV/amu Xe + Ag, and 82-MeV/amu Xe + Au. The numbers above each bar represent n_t quantum numbers, and below the axis represent n_p quantum numbers. From Meyerhof et al. (1985).

Figure 17: Electron capture cross sections for Xe^{54+} ions incident on Be, Mylar (My), Al, Cu, Ag, and Au targets. The solid line gives the total REC and NRC cross sections where the NRC cross sections were calculated with the eikonal approximation according to the higher-potential post-prior prescription. In the dashed lines, the higher-charge criterion was used. The chain curve shows REC cross sections, which are dominant Be target. From Meyerhof et al. (1985).

Figure 18: Same as Fig. 17 for Xe^{52+} ions, where capture into Xe K shell is not allowed, hence only capture into the L, M, and higher orbitals of Xe occurs.

Figure 19: Measured Xe^{52+} (triangles) and Xe^{54+} (circles) electron capture

cross sections. For low-energy Xe+Au collisions, the Xe^{54+} and Xe^{52+} cross sections are approximately equal, indicating that capture into excited states of the projectile is dominant. For Xe+Be collisions, REC is dominant, and the Xe^{52+} cross sections are factors of about 1/5 smaller than the Xe^{54+} ones, as expected for REC into the projectile L and higher shells.

Figure 20: Schematic level diagram showing transitions leading to attachment (a_1, a_2, a_3), ionization (s_1, s_2, s_3), excitation (x_1, x_2, x_3), and decay d_2 . a) A four-state model including the fully stripped ions N_0 and those with one electron in the 1s, 2s, or 2p state. (b) A simplified three-state model.

Figure 21: Bottom: Calculated cross sections for 1s+2s, 1s+2p, and 2s+2p excitation (solid lines), 1s and 2s ionization (dashed lines), 2p+1s radiative decay (chain curve), and REC into the projectile K shell (dotted line) for 400-MeV/amu Ne ions. Top: The solid line shows calculated equilibrium ratios R_{eq} of electron-bearing projectiles to bare ions in solid targets, and the dashed line in gas targets. Data points from Crawford (1979).

Figure 22: Top: Equilibrium ratios calculated using only the REC cross sections (solid line) and including the derived NRC cross sections (dashed line) for 250-MeV/amu and 2100-MeV/amu Ne ions. Bottom: the radiative 1s capture cross section (solid line), the effective ionization cross section σ_s calculated using Eq.(67) (chain curve) and by fitting the numerically calculated values of $R(T)$ [Eq.(66);dashed line]. The derived 1s nonradiative capture cross sections are shown by long dashed lines. The data is from Crawford (1979).

Figure 23: Calculated cross sections for $2p \rightarrow 1s$ radiative decay (dotted line), $1s$ and $2s$ ionization (solid lines), $2s \rightarrow 2p$ excitation (chain curve), and REC (dashed lines) for 197-MeV/amu Xe collisions. The measured $1s$ ionization cross sections are shown by closed circles, and measured Xe^{54+} and Xe^{52+} electron capture cross sections are shown by triangles with connecting solid lines to guide the eye. From Anholt and Meyerhof (1986).

Figure 24: Measured and calculated charge-state fractions for 197-MeV/amu Xe collisions. The experimental uncertainties in the ionization and capture cross sections shown in Fig.23 give uncertainties in the calculated charge-state fractions extending from the lower to upper solid lines. The ground-state model (dashed lines) should be compared with the middle solid lines, based on the most likely electron capture and ionization cross sections. From Anholt and Meyerhof (1986).

Figure 25: Measured and calculated Xe $K\alpha$ x-ray production cross sections in 82- and 197-MeV/amu Xe collisions. The uncertainties in the fraction of the Xe^{52+} electron capture into the $2p$ states give theoretical uncertainties extending from the lower to upper dashed curves. From Anholt and Meyerhof (1986).

Figure 26: The angular distribution of radiation in 197-MeV/amu Xe+Be collisions compared with the Bethe-Heitler calculations of primary bremsstrahlung, normalized at $\theta' = 90^\circ$. The numbers give the center-of-mass (projectile frame) photon energy in keV.

Figure 27: Cross sections for continuum x-ray emission in 82-MeV/amu Xe collisions at laboratory x-ray energies of 40 keV, 197-MeV/amu Xe collisions at 60 keV, and 422-MeV/amu U collisions at 140 keV versus target atomic number. Dashed line: secondary electron bremsstrahlung; chain curve: primary bremsstrahlung; solid line: total intensity.

Figure 28: The calculated equilibrium fraction of electron bearing 15-GeV/amu Au ions for various targets Z_t . The equilibrium fraction is the ratio of the total electron capture cross section into bare Au ions to the 1s ionization cross section. The solid line was calculated including just REC, the dashed line was calculated including NRC and REC, and the chain curve including capture of 1% of the electrons produced by pair creation.

Figure 29: Ratio of the higher-order terms in the self-energy of the $1s_{1/2}$ state to the total self energy obtained by comparing the series expansion values through terms of the order of $A_{70} (Z\alpha)^7$ with numerical calculations to all orders in $Z\alpha$. The series expansion changes sign near $Z=60$ allowing the ratio of the higher-order self energy to the total self energy to exceed unity for very high Z . From Gould (1985a).

TABLE 1
EFFECTS ON K-SHELL IONIZATION

Effect	On Target	On Projectile Ionization	
	Ionization	Low-Z	High-Z
-----	-----	-----	-----
First order theory	PWBA	PWBA	PWBA
Electronic wavefunction	$\psi_0 \sim e^{-Z^*r}$; use $Z^* = Z_t - 0.3$	$\psi_0 \sim e^{-Zr}$; $Z = Z_p$	Dirac; $Z = Z_p$ $\psi_0 \sim r^{s-1} e^{-Zr}$
Binding energy	$E_K = \theta_K \frac{1}{2} Z^{*2}$	$E_K = \frac{1}{2} Z^2$	$E_K = (1-s)mc^2$, $s^2 = 1 - \alpha^2 Z^2$
Perturbing nucleus screening	bare projectiles no screening	cross-section reductions	$Z_t^2 \rightarrow Z_t^2 + Z_t$
Relativistic velocity $\vec{\beta} \cdot \vec{\alpha}$ interaction	calculate with Z^*, θ_K	negligible	calculate with Dirac wavefunctions
Wavefunction distortion	Polarization+ binding effect	$v/v_K > 10$ no correction	$v/v_K \sim 1$ to 2 corrections needed
Experimental effects	Target K- electron capture		

Table II. Pair production cross sections for U^{92+} on U^{92+}		
energy/beam GeV/amu	equivalent fixed target energy GeV/amu	cross section kilo-barns
2	15	2
4	48	6
20	880	31
100	20400	98

Fig. 1

Fig. 2

XBL 835-9639

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

XBL 8410-4589

Fig. 13

Fig. 14

Fig. 15

Fig. 16

Fig. 17

Xe^{52+}

Fig. 18

Fig. 19

Fig. 20

Fig. 21

Fig. 22

Fig. 23

Fig. 24

Fig. 25

Fig. 26

Fig. 27

Fig. 28

XCG 838-7228

Fig. 29

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*LAWRENCE BERKELEY LABORATORY
TECHNICAL INFORMATION DEPARTMENT
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720*