Title
Some Pitfalls in Testing the Law of One Price in Commodity Markets

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Publication Date
2005-08-02

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ABSTRACT

Several articles find no support for the law of one price (LOP) in commodity markets. Only a few articles find some support. A rejection of the LOP would strike at the heart of economic theory. A rejection would suggest that firms do not maximize wealth and households do not maximize utility. Our objective here is to show how four common pitfalls can cause tests of the LOP to fail when in fact the LOP holds. All tests of the LOP that fail to support the theory fall in to at least one pitfall and many fall in to three or four. All of these pitfalls are the result of ignoring important practical implications of arbitrage.

JEL: D41, F15, F3, G1.

21 August 2005

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† We would like to thank Doug Steigerwald, Charles Engel and participants at various seminars for their comments.

Starting with Isard (1977) and Richardson (1978), several studies have failed to find any support for the law of one price (LOP) in commodity markets. A few studies such as Michael, Nobay and Peel (1994) and Sarno, Taylor and Chowdhury (2004) have found some support. This failure to find clear support for the LOP strikes at the heart of economic theory. The failure of the law of one price, as that law is generally understood, would suggest that individuals and firms ignore clear opportunities to increase wealth. Such behavior raises serious questions about wealth and utility maximization, the cornerstones of economic theory. Our primary purpose is to show how four common pitfalls can cause tests of the LOP to fail when in fact it holds.

Section I briefly discusses the law of one price. The major objective of that section is to demonstrate that the law of one price, as it is generally understood, involves arbitrage. All of the pitfalls described later are the result of ignoring practical implications of arbitrage for testing the law of one price in commodity markets. Section II describes the data. Section III uses that data to show that the LOP worked in the wheat market between the United States and Japan. Section IV describes and illustrates the pitfalls into which almost all tests of the LOP have fallen. The final section is a brief summary.

I. Law of One Price

Empirical research on the law of one price has not been very supportive.[1] Early tests of the LOP almost unanimously rejected the LOP. In addition to Isard (1977) and Richardson (1987), see for example Ardeni (1989), Fraser, Taylor and Webster (1991) and Ceglowski (1994).[2]

Later empirical work is mixed. Michael, Nobay and Peel (1994), Obstfeld and Taylor (1997), Vataja (2000), Lo and Zivot (2001) and Sarno, Taylor and Chowdhury (2004) find some support for the LOP. But Asplund and Friberg (2001), Engel and Rogers (2001) and Parsley and Wei (2001) find little or no support for the LOP. The primary reason for this lack of support is that all of these tests of the LOP fall into at least two of the pitfalls described later.

[1] We ignore the earlier empirical work that excludes unit roots and cointegration.

[2] Not all the early work rejected the LOP. See for example Protopapadakis and Stoll (1986), Baffes (1991) and Goodwin (1992).

A. *Arbitrage and the LOP*

The "law of one price", when applied to commodity markets, does not mean the same thing to everyone. A few articles do not appeal or refer to arbitrage in discussing the law of one price. In those articles, apparently the law of one price is simply a tendency for prices of similar commodities to converge. See for example Engle and Rogers (2001), McChesney, Shughart and Haddock (2004) and Eckard (2004).

However the vast majority of the literature on the LOP appeals or refers to arbitrage. Officer (1986, 160) puts it this way:

> For the law of one price of tradables to be valid, a sufficient condition is that the markets involved be purely and perfectly competitive (in the Chamberlinian sense). This would assure the existence of perfect arbitrage.

Other published research that appeals or refers to arbitrage includes the following: Asplund and Friberg (2001), Ardeni (1989), Baffes (1991), Ceglowski (1994), Fraser, Taylor and Webster (1991), Goodwin (1990) and (1992), Goodwin, Grennes and Wohlgenant (1990a) and (1990b), Isard (1977), Lo and Zivot (2001), Michael, Nobay and Peel (1994), Obstfield and Taylor (1997), Parsley and Wei (1996), Protopapadakis and Stoll (1983) and (1986), Richardson (1978), Sarno, Taylor and Chowdhury (2004), and Vataja (2000).

Definitions of the law of one price in dictionaries and encyclopedias for economics also appeal or refer to arbitrage. For example the *Penguin Dictionary of Economics* (1998) defines the LOP as follows: "The law, articulated by Jevons, stating that 'In the same open market, at any moment, there cannot be two prices for the same article.' The reason is that, if they did exist, arbitrage should occur until the prices converge." More extensive discussions of the LOP in *The New Palgrave: A Dictionary of Economics*, (1987) and *The New Palgrave Dictionary of Money and Finance (1992)* also appeal to arbitrage.

Many encyclopedias for economics do not have a separate entry for the LOP. Instead the LOP often appears under the heading of *Arbitrage*. For example, under the heading of *Arbitrage* in the *International Encyclopedia of Economics* (1997) there is a subheading *Principal Terms*. Under that subheading the encyclopedia describes the LOP as follows: "a term referring to prices of commodities that, in different markets, will differ only by the cost of transportation from one market to the other." Using the term "Law of the Single Price", *The Fortune Encyclopedia of Economics* does something similar. In our perusals of dictionaries and encyclopedias, we did not find a single reference to the law of one price that did not appeal or refer to arbitrage.

When we refer to the "law of one price", we mean this dominant interpretation where arbitrage is the mechanism that produces the LOP.

Effective arbitrage imposes at least 3 conditions on the transactions used in a valid test of the law of one price: (1) Products must be identical. (2) Resale must be possible. (3) There is no risk. All tests of the LOP of which we are aware violate at least one of these conditions and many violate all three.

B. *Our Model*

Our model of the law of one price begins with a standard statement for the LOP. Let P_t^J be the price in dollars of a metric ton of a particular variety of wheat in Japan in month t. Let P_t^P (P_t^G) be the price of that same wheat in dollars at Pacific (Gulf) ports in month t. Let F_t represent the freight rates and C_t represent all the other relevant transaction costs.

$$P_t^J/(P_t^P + F_t + C_t) = 1.0 + U_t \tag{1}$$

However our interpretation of equation 1 is not standard. As pointed out later, because commodity arbitrage takes time, both prices and the relevant transaction costs should be from forward, not spot, contracts in month t. Under this interpretation, the error term U_t should be relatively small and not highly correlated. Large and persistent errors in equation 1 would be inconsistent with effective arbitrage.

Like all previous tests of the LOP, we are forced to use spot prices. With spot prices, freight rates and other transaction costs, equation 1 becomes equation 2.

$$p_t^J/(p_t^P + f_t + c_t) = 1.0 + U_t + z_t \tag{2}$$

In equation 2, lower case indicates spot rather than forward and z_t is the additional error due to using spot prices and transaction costs. While U_t should be relatively small and not highly autocorrelated, z_t can be large and highly autocorrelated.

II. Data

We use the same data for Japan as Goodwin, Grennes and Wohlgenant (1990a) and (1990b), and Michael, Nobay and Peel (1994). One advantage of these data is that they avoid exchange rates because the prices in Japan are quoted in dollars. To check for any possible errors, and because it was not obvious to us how Michael, Nobay and Peel, or MNP, accounted for a few missing observations, we went back to their original data source *World Wheat Statistics* and recollected all the data. When there was a missing observation we replaced it with the previous observation. When there were two missing observations in a row, we replaced the first with the previous observation and the second with the following observation. During the interval used here, there are never more than two missing observations in a row.

Wheat prices are per metric ton and the monthly freight rates are per metric ton. Japanese prices are in dollars and include certificates and freight. U.S. prices are free on board. Table 1 describes the three varieties of wheat, the time periods and the various acronyms used later. For purposes of comparison, we use the same time periods used by Michael, Nobay and Peel (1994).[3]

Table 1
Varieties of Wheat

Variety	Acronym/U.S. Ports	Period
No. 2 Dark Northern Spring 14% Protein	DNS Pacific and Gulf	1975:09 to 1981:10
No. 2 Western White	WW Pacific	1975:09 to 1981:10
No. 2 Hard Winter 13% Protein	HW Gulf	1976:01 to 1981:07

[3] The error structure in our cointegration tests can be sensitive to the time period.

Adding a 'J' as in DNSJ indicates a Japanese price. A 'P' indicates a price at a Pacific port. A 'G' indicates a price at a Gulf port. A 'T' indicates that the price includes the freight rate. For example, DNSGT is the FOB price for Dark Northern Spring wheat at Gulf ports plus the freight rate from Gulf ports to Japan.

Freight rates vary with the size of the ship and at times more than one rate is published. We use the same freight rates as MNP. Unlike the prices, freight rates are not spot. Footnotes in various issues of *World Wheat Statistics* describe the freight rates as follows: "Estimated mid-month rates based on current chartering practices for vessels to load six weeks ahead." In other words, the freight rates are forward, not spot, prices.

Table 2
Unit Root Tests for Freight Rates to Japan†

Series	Gulf Ports	Pacific Ports
ADF (Lag) [Probability]	-1.35 (4) [0.600]	-0.73 (0) [0.831]
PP (Lag) [Probability]	-1.10 (4) [0.712]	-0.74 (1) [0.828]
Q (Lag) [Probability]	0.23 (1) [0.630]	0.05 (1) [0.827]
LM (Lag) [Probability]	1.31 (1) [0.252]	0.05 (1) [0.828]
Arch (Lag) [Probability]	0.54 (2) [0.762]	0.07 (1) [0.791]
Squared Residuals		
Q (Lag) [Probability]	0.06 (1) [0.815]	0.07 (1) [0.785]

† Here and in all other tables, LM is the Breusch-Godfrey obs*R^2 serial correlation LM test.

MNP show that the wheat prices have a unit root. Because they work with freight rates relative to wheat prices, they do not look for a unit root in the freight rates. Table 2 shows that, like the prices, one cannot reject the hypothesis that freight rates have a unit root. Neither the augmented Dicky-Fuller (ADF) nor Phillips Perron t tests (PP) in Table 2 reject the null of a unit root. Here and in all other tables, we consider lags up to 12 months and we use the Akaike Information Criteria for the ADF test to choose the optimal lag. In Table 2 and all following tables, we use a variety of tests for structure in the errors from the ADF test. The LM, Arch and both Q tests here and in the following tables report the test for the lag with the lowest probability. If the test for that lag is not significant, the test is not significant at any other lag. As an example, Table 8 in the Appendix shows all the estimates for freight rates from Gulf ports to Japan. In Table 2, none of these tests are even close to being significant.[4]

Our data are probably the best available. Only a few studies have included actual shipping costs. Our import and export prices are for "identical" varieties of wheat.[5] Export prices are free on board and import prices include certificates and freight, which avoids potentially serious problems created by intra-national distribution costs, tariffs and most other trade impediments.

III. The LOP Worked in the U.S.-Japanese Wheat Market

When we use the Japanese data and time periods used by Michael, Nobay and Peel (1990), and avoid most of the pitfalls discussed later, there is strong support for the law of one price. Because we have data on freight rates, we test the law of one price by looking for evidence of cointegration between import prices and export prices plus the freight rate. To account for the fact that commodity arbitrage takes time, we lag export prices one month.[6] To account for the fact that the freight rates are forward rates, we lag freight rates an additional month. Equation 3 describes our implicit test equation.[7]

$$\text{Log}(p_t^J) - \text{Log}(p_{t-1}^P + F_{t-2}) = U_t + z_t + [c_{t-1}/(p_{t-1}^P + F_{t-2})] \tag{3}$$

Because we have spot prices and do not have all the relevant transaction costs, we cannot test for the stationarity of U_t. We can only test for the stationarity of $U_t + z_t + [c_{t-1}/(p_{t-1}^P+F_{t-2})]$.

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[4] These tests and all others use EView4.

[5] Even though the prices are for identical varieties of wheat, identical varieties can differ in chemical content and other characteristics such as whether or not the grain has been washed. These additional characteristics would be spelled out in the relevant forward contracts.

[6] No attempt was made to find an 'optimal' lag.

[7] Since $\text{Log}(p_{t-1}^P+F_{t-2}+c_{t-1})$ may be written as $\log(p_{t-1}^P+F_{t-2})+\log\{1.0+[c_{t-1}/(p_{t-1}^P+ F_{t-2})]\}$ and, when x is small, $\log(1.0+x) \approx x$, $\log[p_t^J/(p_{t-1}^P+F_{t-2}+c_{t-1})] = \log[1.0+U_t+z_t]$ can be written as $\log(p_t^J)-\log(p_{t-1}^P+F_{t-2}) = U_t+z_t+[c_{t-1}/(p_{t-1}^P+ F_{t-2})]$.

For each of the four pairs of wheat prices, Table 3 shows the results for three different tests for cointegration: Johansen, ADF and PP. The ADF and PP are residual based. We have included the PP test because that test appears to be less sensitive to lag length than the Johansen or ADF. We choose the lag length as we did for the earlier tests for a unit root in freight rates. We also apply the same tests to the residuals from the ADF test.

The results in Table 3 provide strong support for the LOP. Of the 12 tests in Table 3, one rejects the null of no cointegration at the 5 percent level and 10 reject that null at the 1 percent level. Only one test fails to reject the null of no cointegration and that one is almost significant at the 5 percent level.[8] This is far stronger support than most tests of the LOP have found.

Table 3
Cointegration Tests for Three Varieties of Wheat

Series	DNSGJT	DNSPJT	WWPJT	HWGJT
ADF (Lag)	-4.29** (0)	-5.17** (0)	-5.93** (0)	-3.38* (7)
PP (Lag)	-4.35** (0)	-5.22** (1)	-5.98** (1)	-5.27** (7)
Johansen (Lag)	33.95** (0)	143.43** (0)	146.83** (0)	109.73 (7)
Q (Lag) [Probability]	15.71 (12) [0.205]	0.16 (1) [0.693]	1.11 (2) [0.575]	0.02 (1) [0.884]
LM (Lag) [Probability]	9.86 (8) [0.275]	0.70 (1) [0.403]	0.96 (1) [0.326]	0.48 (1) [0.486]
Arch (Lag) [Probability]	15.53 (11) [0.159]	3.18 (1) [0.074]	5.52 (2) [0.063]	7.39 (4) [0.12]
Squared Residuals				
Q (Lag) [Probability]	18.15 (11) [0.078]	3.34 (1) [0.68]	6.75 (2) [0.034]	03.56 (2) [0.169]

* Significant at 5%. ** Significant at 1%. Critical values for residual based tests (3.17) and (3.77) from Engle and Granger (1987).

Our tests in Table 3 differ from similar tests in MNP in three ways. First, MNP look for three way cointegration between Japanese prices, U.S. prices, and freight rates as a proportion of the import price. Because of the complex interdependence between price differentials and freight rates, their procedure seems to complicate the error structure. Second, the maximum lag length that MNP consider is only 6. Extending the

[8] The Johansen trace test for HWGJT is 19.73 and the 5 percent significance level is 19.95.

lag to 12 months weakens their results. Third, MNP attempt to account for the effects of time in commodity arbitrage by lagging the freight rates one month while using current export prices. Apparently MNP did not realize that the freight rates were forward prices. Our procedure of lagging export prices by one month and freight rates by an additional month appears to do a better job of accounting for the fact that it takes time for wheat to travel from the United States to Japan.

IV. Potential Pitfalls in Testing the LOP

A. *Transaction Costs*

As far as we are aware, every test that fails to support the LOP also fails to account for the effects of transaction costs. Table 4 illustrates how omitting transaction costs can weaken cointegration tests.

Table 4
Cointegration Tests without Freight Rates

Series	DNSGJ	DNSPJ	WWPJ	HWGJ
ADF (Lag)	-3.62* (0)	-2.39 (3)	-3.17* (2)	-2.79 (7)
PP (Lag)	-3.67* (1)	-3.85** (3)	-4.32** (2)	-4.52** (7)
Johansen (Lag)	32.27** (0)	10.79 (3)	7.82 (2)	18.05 (7)
Q (Lag) [Probability]	15.00 (12) [0.241]	1.57 (5) [0.905]	0.96 (6) [0.814]	4.74 (8) [0.785]
LM (Lag) [Probability]	14.16 (12) [0.291]	4.75 (5) [0.447]	0.25 (1) [0.619]	7.50 (7) [0.379]
Arch (Lag) [Probability]	17.95 (11) [0.082]	2.18 (1) [0.139]	11.20 (12) [0.511]	6.47 (5) [0.263]
Squared Residuals				
Q (Lag) [Probability]	22.24 (12) [0.031]	2.30 (1) [0.13]	3.15 (4) [0.534]	7.44 (5) [0.190]

- Significant at 5%. ** Significant at 1%.
- Critical values for residual based tests (3.17) and (3.77) from Engle and Granger (1987).

Table 4 is the same as Table 3 except that the freight rates have been dropped from the export price. That is, these tests for cointegration uses just import prices and the export price lagged one month.

Table 4 shows less support for the LOP than Table 3. In Table 3 only one of the 12 tests fails to support the LOP. In Table 4, five of the 12 tests fail to support the LOP. In Table 3, 10 of the tests are significant at the 1 percent level. In Table 4, only four of the 12 tests are significant at the 1 percent level.

B. *Time*

Protopapadakis and Stoll (1986) and Benninga and Protopapadakis (1988) develop formal models describing the role of time in commodity arbitrage. Unfortunately most of the empirical literature on testing the law of one price has failed to recognize the practical problems created by the fact that commodity arbitrage takes time. Some exceptions include Goodwin (1990), Goodwin, Grennes and Wohlgenant (1990a) and (1990b), and Michael, Nobay and Peel (1994).

When arbitrage takes time, as it does for most commodities, it is impossible to test the law of one price directly with spot prices.[9] Any test using spot prices tests a combination of two propositions: (1) the law of one price holds for forward prices. (2) market forces other than arbitrage provide at least a long-run link between forward prices and spot prices.[10]

Consider the case of wheat in Pacific ports and Tokyo. Suppose on May15, 2010, a vicious and unexpected storm destroys half the wheat crop in Japan. On May 16, the spot price in Tokyo for wheat from the United States rises far above the spot price in Pacific ports plus spot freight rates and other relevant transaction costs. Because it is impossible to move wheat from Pacific ports to Japan within a couple of days, this differential in spot prices does not create an opportunity for arbitrage. As a result, this differential, which is much larger than the conventional transaction costs, is consistent with the LOP. [11]

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[9] A few products with very high prices relative to their transportation costs such as diamonds might be exceptions.
[10] Futures prices are not appropriate because futures are financial contracts where the delivery or receipt of the product is rare. However futures markets are one part of the market mechanism that links forward and spot prices
[11] Using a Boeing 747, it might be physically possible to move wheat from Seattle to Tokyo in a couple of days, but the cost would be prohibitive.

A differential between *forward* prices does generate an opportunity for profitable arbitrage. Forward prices in Tokyo also rise. Higher forward prices in Tokyo than at Pacific ports create an opportunity for wheat dealers to engage in profitable arbitrage. To eliminate risk, within as short a period of time as is possible, arbitrageurs enter into at least three forward contracts:[12] One contract is for wheat to be delivered on board ship in say six weeks. Another contract secures the ship that will load the wheat in six weeks and fixes the freight rate. A third contract sells the wheat in Tokyo when the ship is scheduled to arrive. All of these contracts are forward contracts and all prices are forward prices. This combination of forward contracts meets all the conditions for arbitrage: identical products, resale, and no risk. The only risk in this transaction is that the other side of a contract may not fulfill the contract, but that risk is present in all arbitrage. If any of these contracts are omitted, the transaction is no longer arbitrage because it involves risk.

Arbitrage raises forward prices in Pacific ports relative to what they would have been without arbitrage. Arbitrage lowers forward prices in Tokyo relative to what they would have been. Arbitrage also bids up forward freight rates relative to what they would have been. Arbitrage does not directly reduce the differential in current spot prices, but it indirectly reduces that differential. Higher forward prices in Pacific ports encourage grain elevators there to hold over wheat for future delivery. The reduced supply of wheat for sale in the spot market raises spot prices in Pacific ports. Lower forward prices in Tokyo encourage grain elevators there to offer more wheat in the spot market, reducing current spot prices in Tokyo.

The LOP does not provide a *direct* link between spot wheat prices in Pacific ports and Tokyo. Instead the LOP is part of an indirect link between those spot prices. With effective arbitrage, differentials between the relevant forward prices should be relatively small and not highly autocorrelated. Because there is no direct market mechanism to reduce differentials in spot prices, spot differentials will be relatively large and highly autocorrelated. That is why all tests of the LOP that use spot prices will find highly autocorrelated errors.

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[12] At the same time arbitragers must also arrange for financing, insurance and drawing up the appropriate documents.

In their empirical work, Protopapadakis and Stoll (1983) and (1986) try to deal with the issue of time by using futures prices to test arbitrage. Their results suggest that time is important for commodity arbitrage. They find little support for the law of one price using spot prices for commodities like copper, but they find substantial support using futures prices. Their work is an important step in the right direction, but at best it only provides an indirect test of commodity arbitrage. Futures contracts are financial contracts. Arbitrage between foreign exchange markets and commodity futures markets in the United States and abroad is a form of financial, not commodity, arbitrage. Futures contracts normally do not involve delivery of the product. Using futures prices to test the law of one price in commodity markets implicitly assumes that the futures prices are acceptable proxies for the appropriate forward prices.

Protopapadakis and Stoll (1986), Goodwin, Grennes and Wohlgenant (1990a, 1990b), Goodwin (1990) and Michael, Nobay and Peel (1994) subtly alter the law of one price by replacing at least one price with its expectation. Unlike the LOP, this expectations modified law of one price involves risk because at least one price is an expected price.

Without the appropriate forward prices and transaction costs, there is no way to test the LOP directly. But we can improve our tests with spot prices by recognizing the importance of time for commodity arbitrage. A comparison of Tables 3 and 5 illustrates how failing to recognize the effects of time can weaken tests of the LOP.

Table 5 is the same as Table 3 except that export prices are not lagged. Equation 3 describes the implicit test equation used in Table 3. Equation 4 describes the implicit test equation used in Table 5.

$$\mathrm{Log}(p_t^J) - \mathrm{Log}(p_{t-1}^P + F_{t-2}) = U_t + z_t + [c_{t-1}/(p_{t-1}^P + F_{t-2})] \quad (3)$$

$$\mathrm{Log}(p_t^J) - \mathrm{Log}(p_t^P + F_{t-1}) = U_t + z_t + [c_{t-1}/(p_{t-1}^P + F_{t-1})] \quad (4)$$

Like Michael, Nobay and Peel (1994), Table 5 lags the freight rate one month but uses current prices. Unlike Table 3, Table 5 ignores the fact that it takes time for wheat to move from the United States to Japan.

Ignoring time has only a slight effect for the estimates between Pacific ports and Japan. For example, from Table 3 to Table 5 the ADF statistic for WWPJT only changes from -5.93 to -4.00. Both statistics remain significant at the 1 percent level. Ignoring time has more effect on the estimates between Gulf ports and Japan. In Table 3, for DNSGJT and WHGJT, all three tests reject the null of no cointegration. In Table 5 none of the three tests reject the null for DNSGJT. Only one of the three tests rejects the null for WHGJT. A likely explanation for this difference in the effect of time is that it takes longer for ships to go from Gulf ports to Japan than from Pacific ports to Japan.

Table 5

Testing for Cointegration with Current Prices

Series	DNSGJT	DNSPJT	WWPJT	HWGJT
ADF (Lag)	-2.28 (8)	-5.06** (2)	-4.00** (7)	-1.76 (10)
PP (Lag)	-2.94 (8)	-6.15** (2)	-5.35** (7)	-5.42** (10)
Johansen (Lag)	16.79 (8)	25.02** (2)	20.99* (7)	17.14 (10)
Q (Lag) [Probability]	0.02 (1) [0.898]	9.70 (12) [0.642]	0.05 (1) [0.830]	3.66 (7) [0.818]
LM (Lag) [Probability]	4.66 (5) [0.459]	5.83 (4) [0.212]	0.61 (1) [0.436]	9.47 (8) [0.304]
Arch (Lag) [Probability]	10.18 (8) [0.252]	0.69 (2) [0.710]	8.83 (9) [0.453]	11.96 (7) [0.102]
Squared Residuals				
Q (Lag) [Probability]	5.34 (4) [0.254]	0.74 (2) [0.691]	13.64 (9) [0.136]	13.67 (7) [0.057]

* Significant at 5%. ** Significant at 1%. Critical values for residual based tests (3.17) and (3.77) from Engle and Granger (1987).

C. *Products not Identical*

As far as we are aware, no article that uses identical wholesale prices fails to find some support for the LOP. (The pitfall associated with using retail prices is discussed later.) In many tests of the LOP such as in Ardeni (1989) and Fraser, Taylor and Webster (1991), the problem of prices for different goods is compounded by the use of price indexes rather than individual prices.

Even when a wholesale index is as refined as ‘vacuum cleaners’ or ‘tyres’ as in Fraser, Taylor and Webster (1991), the commodities in different indexes can differ substantially, particularly when the indexes are from different countries. A ‘tyre’ for a Morris Minor is hardly comparable to a tire for a Cadillac. Even within a country, prices for different makes and models of tires can vary by 100 percent. In addition, weights and product mixes can change over time differently in different indexes. Even for an individual type of commodity such as wheat or wool as in Ardeni (1989) or valves as in Ceglowski (1994), wholesale prices in different locations, particularly international prices, are often not for identical commodities.[13] Canadian wheat and U.S. wheat are similar. They are not identical. As is shown below, even the different varieties of wheat grown in the United States cannot be treated as though they were identical.

Unit values, which are also widely used, create even bigger problems.[14] When prices are unit values as some are in Ardeni (1989), the product mix is continually changing over time as the mix of exports (imports) changes. All of these problems can introduce spurious and highly autocorrelated errors that can cause cointegration tests to reject the law of one price when in fact it holds.

Although the problems associated with price indexes, unit values and dissimilar products are widely understood, their seriousness does not seem to be widely appreciated. Price differentials between very similar, but not identical, products are unlikely to have unit roots. It seems unlikely that relative prices for similar products would drift off to infinity. But those differentials are usually highly persistent. Even when prices for similar but not identical prices are cointegrated, that persistence can make it difficult to reject the null of no cointegration.

Table 6 uses our data to illustrate the potential effects of using prices for similar but not identical products. Table 6 probably understates the effect of using prices for different products because our varieties of wheat are

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[13] Unlike most tests of the LOP, Ceglowski tests for cointegration between FOB prices for the U.S., Germany and Japan for similar categories such as valves and electric motors.
[14] Unit values are calculated by dividing the value of exports (imports) by the volume of exports (imports).

probably more similar than the products in broad categories such as tires that have been widely used to test the LOP.

In Table 6 the columns labeled DNSJWWPT and WWJDNSGT show tests for cointegration between Dark Northern Spring and Winter White wheat. Both export prices include freight rates and are lagged as in Table 3. In the first column, the pairs are DNS in Japan versus WW at Pacific ports. In the second column it is WW in Japan versus DNS at Gulf ports.

Table 6
Testing for Cointegration between Different Varieties of Wheat

Series	DNSJWWPT	WWJDNSGT	DNSJWWJ	DNSPWWP
ADF (Lag)	-2.56 (3)	-2.66 (0)	-2.05 (3)	-2.28 (2)
PP (Lag)	-3.48* (3)	-2.72 (1)	-2.13 (3)	-2.05 (2)
Johansen (Lag)	12.60 (3)	23.96* (0)	13.56 (3)	11.83 (2)
Q (Lag) [Probability]	1.83 (5) [0.872]	0.26 (1) [0.608]	6.36 (8) [0.607]	0.29 (4) [0.510]
LM (Lag) [Probability]	5.30 (5) [0.381]	0.38 (1) [0.535]	5.65 (5) [0.342]	4.23 (4) [0.375]
Arch (Lag) [Probability]	8.81 (8) [0.358]	8.94 (7) [0.257]	1.18 (1) [0.278]	1.00 (1) [0.316]
Squared Residuals				
Q (Lag) [Probability]	9.87 (8) [0.451]	11.18 (7) [0.131]	1.24 (1) [0.265]	1.06 (1) [0.303]

* Significant at 5%. ** Significant at 1%.
Critical values for residual based tests (3.17) and (3.77) from Engle and Granger (1987).

In Table 3, when each variety is tested against its counterpart, tests for cointegration strongly reject the null of no cointegration. Only one test is insignificant and most are significant at the 1 percent level. In Table 6, none of the 6 tests is significant at the 1 percent level. Only two tests are significant at the 5 percent level. Even though these two varieties of wheat are very similar, they are different enough so that using them to test the LOP produces little support.

These tests fail to support the LOP because the price differential between the two varieties is very persistent. The last two columns in Table 6 labeled DNSJWWJ and DNSPWWP test for cointegration between the two varieties of wheat, first in Japan and then at Pacific ports. None of the six tests reject the null of no cointegration. Price differentials between similar products are probably stationary. But the differentials can be so persistent that it can be misleading to use them to test the law of one price with the relatively short intervals typically used in the literature.

D. *All Three Pitfalls: Transaction Costs, Products not Identical and Time*

Most tests of the LOP, particularly those that find little or no support, suffer from all three pitfalls. They do not account for the effects of transaction costs. The products are not identical. They do not make any adjustments to account for the fact that commodity arbitrage takes time. Table 7 illustrates how the combination of those three pitfalls can affect cointegration tests of the LOP.

Table 7

Testing for Cointegration with All Three Pitfalls

Series	DNSJWWP	WWJDNSG
ADF (Lag)	-1.95 (0)	-2.79 (3)
PP (Lag)	-2.07 (1)	-2.56 (3)
Johansen (Lag)	8.24 (0)	17.21 (3)
Q (Lag) [Probability]	1.35 (1) [0.245]	10.86 (12) [0.541]
LM (Lag) [Probability]	1.64 (1) [0.200]	14.38 (12) [0.277]
Arch (Lag) [Probability]	0.36 (1) [0.546]	5.18 (7) [0.638]
Squared Residuals		
Q (Lag) [Probability]	0.38 (1) [0.535]	2.36 (7) [0.607]

* Significant at 5%. ** Significant at 1%. Critical values for residual based tests (3.17) and (3.77) from Engle and Granger (1987).

As in Table 4, in Table 7 export prices do not include freight rates. As in Table 5, in Table 7 export prices are not lagged by one month. As in Table 6, the products in Table 7 are not identical. Unlike Table 3, in Table 7 there is no support at all for the law of one price.

E. *No Resale*

Recent rejections of the LOP are based almost exclusively on retail prices or retail price indexes. These recent rejections include Asplund and Friberg (2001), Engel and Rogers (2001), and Parsley and Wei (2001). Because resale is normally impossible or impractical at the retail level, the failure of retail prices or indexes to converge does not constitute a valid rejection of the LOP as that law is generally understood.

Consider the following mental experiment: Suppose retail prices for Bostonian shoes are 30 percent lower in Chicago than in New York. While visiting Chicago, tourists from New York will load up on shoes at Marshall Fields. Those purchases can work to equate retail shoe prices in New York and Chicago. But there is no arbitrage in Bostonian shoes at the retail level. No one buys Bostonian shoes at Marshall Fields in Chicago, takes them to New York in a suit case, and sells them to Macy's. The quantity of shoes available for each size and style in Marshall Fields is too small. More importantly, Macy's can buy the shoes even more cheaply at wholesale. Because arbitrage is normally impossible between retail markets, research that uses retail prices normally cannot reject the law of one price. All such research can tell us is how price differentials behave when there is no opportunity for arbitrage.[15]

Because their data is so impressive, Asplund and Friberg (2001) provides an excellent example of this problem. The unique aspect to their article is that they look at prices in different currencies for identical products at the same duty free outlets in Scandinavia. The outlets are stores on Birka Line ferries, Viking Line ferries and on the airline SAS. Using prices for identical products such as Smirnoff Vodka and the relevant exchange rates to test the law of one price, they find little support for the LOP.

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15 Retail prices are relevant for the Borders literature. That literature asks a very different question. One that is relevant for purchasing power parity, but not for the LOP.

Asplund and Friberg appear to accept the usual interpretation of the law of one price. One major heading in their article is *Arbitrage*. In addition, in their references they cite Goldberg and Knetter (1997) who include resale in their list of assumptions required for the LOP to hold. However resale is impossible in all three outlets.

Consider their example of Smirnoff Vodka at stores on Birka Line ferries. The vodka is priced in both Swedish kronor and Finnish markka. These prices remain unchanged for extended periods of time. For many months, even after adjusting for transaction costs in the foreign exchange market, the kronor price of the vodka divided by the markka price is higher than the kronor price of markka. If one could buy the vodka using markka, sell it back to the same store for kronor, and then use the kronor to buy markka, there would be an arbitrage profit. But such arbitrage is impossible. Even if the stores were willing to consider it, the kronor price offered to them by their own passengers would be higher than the wholesale price. If not, the stores would be loosing kronor on every sale. Although Asplund and Friberg may be saying something interesting about how firms price products, their article tells us nothing about the law of one price as that law is generally understood.

V. *Summary*

There is some confusion in the literature about what the law of one price means for commodity markets. A few articles interpret the LOP as simply a tendency for prices for similar commodities in two different locations to tend toward equality in the long run. However most of the literature either implicitly or explicitly assumes that arbitrage is the mechanism that produces the LOP. Definitions of the LOP in dictionaries and encyclopedias for economics do the same. We follow that standard interpretation. Other market forces may tend to equate prices in different locations, but arbitrage is the mechanism that produces the law of one price.

Although there is some evidence supporting the LOP, there is a large body of evidence that fails to support the law of one price. Rejecting the law of one price would strike at the heart of economic theory. Rejecting the LOP would suggest that firms do not maximize wealth and households do not maximize utility.

Our primary objective is to point out four pitfalls that plague this literature: *(1) Transaction Costs*. With only a few exceptions, most tests of the LOP fail to account for the effects of transaction costs. A comparison of Tables 3 and 4 illustrates how excluding freight rates can weaken cointegration tests of the LOP. *(2) Time*. Because commodity arbitrage takes time, a direct test of the LOP requires forward prices. Because forward prices have not been available, all tests of the LOP have used spot prices. Such tests actually test two propositions: (A) the LOP holds for forward prices and (B) other market mechanisms tend to equate forward and spot prices. Not only does the empirical literature fail to recognize this problem with spot prices, most articles do not even make an effort to compensate for the fact that commodity arbitrage takes time. A comparison of Tables 3 and 5 illustrates how ignoring time with spot prices can weaken cointegration tests of the LOP. *(3) Identical Products*. Only a few tests of the LOP use identical products. Most use categories of products such as 'wool' or 'tyres' or price indexes for such categories. A comparison of Tables 3 and 6 illustrates how using similar, but not identical, products can weaken cointegration tests of the LOP.

Table 7 combines all three of these pitfalls. Unlike Table 3, which shows strong support for the LOP, Table 7 shows no support.

Pitfall number *(4)* is *Retail Prices*. Several of the more recent attempts to test the LOP have used retail prices. If one accepts the standard interpretation of the LOP adopted here, namely that arbitrage is the mechanism that produces the LOP, then one cannot normally use retail prices to test the LOP. Arbitrage is normally impractical or impossible at the retail level because resale is impractical or impossible.

Because of the prevalence of these four pitfalls in the literature, we know of no reliable empirical evidence that would lead us to reject the law of one price in commodity markets.

APPENDIX: Example of Tests Applied to Residuals

Table 8
Tests Applied to Residuals for Gulf Ports in Table 2†

Lag	Q Statistic (Probability)	LM Statistic (Probability)	Arch Statistic (Probability)	Q Statistic for Squared Residuals (Probability)
1	***0.23*** ***(0.630)***	***1.31*** ***(0.252)***	0.05 (0.820)	***0.06*** ***(0.815)***
2	0.32 (0.854)	1.93 (0.380)	***0.54*** ***(0.762)***	0.31 (0.856)
3	0.33 (0.954)	2.25 (0.521)	0.82d (0.845)	0.49 (0.920)
4	0.34 (0.987)	2.27 (0.686)	0.98 (0.913)	0.58 (0.965)
5	0.36 (0.996)	2.28 (0.809)	1.73 (0.885)	0.935 (0.968)
6	0.72 (0.994)	2.44 (0.875)	1.99 (0.920)	1.03 (0.984)
7	1.18 (0.991)	3.24 (0.862)	3.17 (0.868)	1.51 (0.982)
8	1.36 (0.995)	3.52 (0.898)	4.35 (0.825)	1.85 (0.985)
9	2.43 (0.983)	5.26 (0.811)	4.44 (0.880)	1.89 (0.993)
10	4.65 (0.913)	8.11 (0.618)	4.87 (0.900)	1.90 (0.997)
11	7.95 (0.718)	11.48 (0.404)	5.70 (0.892)	1.99 (0.999)
12	7.96 (0.788)	11.62 (0.476)	6.99 (0.858)	2.56 (0.998)

† Results reported in Table 2 in bold italics.

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