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High Occupancy Vehicle/Toll Lanes: How Do They Operate and Where Do They Make Sense?

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Abstract

Motivated by the need for better utilization of existing high-occupancy vehicle (HOV) lanes and the potential of high-occupancy/toll (HOT) lanes to increase utilization, this paper examines the circumstances in which HOT lanes could provide a more desirable alternative to HOV lanes.

First, it compares the HOT and HOV lane concepts, showing how they are similar and how they differ. Both involve finding some type of equilibrium in which no additional people are motivated to use the lane. However, in the case of the HOT lane, the agency operating the facility can affect the equilibrium point by varying the toll, thus having the potential to increase lane utilization and reduce overall delay. HOT lanes also offer travelers the additional choice of a delay-free trip if they choose to pay a toll.

The question of how sensitive users are to the toll level was investigated using data from SR 91, in Orange County. During the first two years there was no toll for HOVs, so the toll lanes operated as true HOT lanes, even though they were officially named Variable Toll Express Lanes. Eastbound toll lane traffic showed a very low elasticity with respect to tolls—between $-.02$ and $-.16$. This means that if tolls are reduced by 50%, toll lane volumes will increase by between 1% to 8%. The westbound analysis found a positive relationship between tolls and toll lane traffic. This unreasonable result appears to be the result of a positive correlation between tolls freeway traffic volume that makes it impossible to separate the effects of the two variables. But this result also suggests that in the westbound direction toll lane volumes are not very sensitive to tolls.

The final section of the paper uses a model to compare the performance of an HOV, HOT, and mixed flow lane in various circumstances. In all cases modeled, a HOT lane results in substantially less delay than an HOV lane. It also performs better or almost as well as a mixed flow lane. HOV lanes perform better than mixed flow lanes under extreme congestion and when there is a fairly high proportion of HOVs before the HOV lane is established. These conditions result in good utilization of the lane and provide the motivation for travelers to shift to HOVs. Under less extreme congestion, and when the initial proportion of HOVs is low, mixed flow lanes perform better than HOV lanes. These findings suggest that in locations where tolling would be feasible and where HOV lane utilization is low, HOTs might be a preferable alternative to the HOV lane. However, before conversion of a particular lane there should be more detailed modeling to confirm that an HOT lane would perform as expected and to determine the effects of the conversion on the network in which the HOV lane is located.

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Occupancy Vehicle/Toll Lanes: How Do They Operate and Where Do They Make Sense?

1. Introduction

1.1 High-occupancy Vehicle (HOV) Lanes Under Attack

New Jersey's decommissioning of two of its HOV lanes and introduction of legislation requiring a study of the effects of HOV lanes on air quality and traffic safety have been followed by questioning of the effectiveness of HOV lanes in four other states including California. Assembly Bill 44, introduced in the California legislature in December 1998, called for the re-designation of all existing HOV lanes as mixed flow lanes and prohibited their conversion back to HOV lanes unless an HOV lane could be shown to be more cost-effective than a mixed-flow lane or a High Occupancy Toll (HOT) lane.¹ Although it does not appear that AB44 will be approved in the current legislative session, two other bills, AB 1647 and SB14 require regular assessment of the effectiveness of HOV lanes. AB 1647 provides for conversion to HOT or mixed flow lanes for HOV lanes that are not found to be effective.

1.2 HOV Lane Analyses

Although there have been reports on the usage of HOV lanes in California, a search of the literature found no evaluations of California freeways with operating HOV lanes that compared their performance to what it would have been if the HOV lanes were mixed-flow lanes. Unfortunately, project studies for most HOV lane projects that were planned between 1990 and 1998 did not include a comparison with a mixed-flow lane alternative; regional transportation agencies did not support construction of additional mixed-flow lanes. Of nine such studies, only one considered a mixed-flow lane alternative². That report

¹ This would likely result in the permanent elimination of most HOV lanes, since previous experience with conversion of mixed-flow lanes to HOV lanes suggests that it is not politically feasible on congested highways. Similarly, previous attempts to impose tolls on existing free roads suggest that converting a mixed-flow lane to a HOT lane would also be politically infeasible.

² This review, conducted by the author, included "Project Estimate of Highway Safety, Congestion, and Capacity" reports for the HOV lanes on:

- I-405 between I-5 and I-605 (1990)
- SR 57 between I-5 and the Los Angeles County Line (1990)
- SR 91 between the Orange Freeway and Magnolia Avenue in Riverside
- I-105 between Rte 405 and Rte 110 (1991)
- I-5 between SR 55 and SR 57 (1991)
- Rte 405 between Rte 101 and Rte 5 (1992)
- SR 405 between Rte 101 and I-5 (1992)
- SR 605 between Telegraph Road and Rte 10 (1993)

the "Project Study Report" for the HOV lane on I-605 between I-405 and the Los Angeles County Line (1997), and the "HOV Report" for the Carquinez Bridge Project (1998). These were supplied to Assemblyman McClintock in response to his request for studies of HOV lanes.

Only the 1993 study of SR 605 compared the HOV lane alternative to a mixed flow lane alternative. The others compared the HOV lane alternative only to the "do-nothing" alternative.

made the rather unlikely assumption that the number of vehicles per mixed-flow lane would be the same in both cases, suggesting that the mixed-flow lane alternative was not under serious consideration in this instance either. Although research suggests that in a wide range of situations, HOV lanes are less effective in reducing person-delay and emissions than mixed flow lanes³, there are probably sites in California where HOV lanes are more effective than mixed-flow lanes and sites where HOT lanes would provide greater benefits than either HOV or mixed-flow lanes. It would be good to identify these sites before the opportunity for maintaining them as HOV lanes or converting them to HOT lanes is lost.

1.3 HOT Lane Experience

The opening of the SR 91 Value-Priced Express Lanes in Orange County in December 1995 has offered a rare opportunity to study the benefits of HOT⁴ lanes. There are two such lanes in each direction for ten miles. They are separated from the adjacent mixed-flow lanes by a painted buffer and pylons. These lanes have been quite successful in reducing congestion for both users and non-users. Fears of public opposition and the lanes serving only high-income travelers have been dispelled by surveys of SR 91 travelers that show that the HOT lanes are used by people of all income levels and that time varying tolls, once instituted, are viewed favorably by most people⁵. The lanes have also generated substantial toll revenues, \$12.7 million in 1997 to offset the \$9.1 million operating cost in 1997 and the \$133 million capital costs⁶.

The Express Lanes on Interstate 15 in San Diego, where HOVs travel free and other vehicles pay tolls that vary with congestion level, began operation in early 1998. There are two reversible lanes in the peak direction only, which are separated from the other lanes by fixed barriers. These lanes will provide further opportunities to study the HOT lane concept, but data from this facility were not available at the time of this study.

There is a tendency in the transportation world to copy something that is successful without analyzing why it succeeded. But success is almost always context-dependent. The section of SR 91 where the express lanes are now located was very congested, and enough delay remained after the addition of the new lanes to motivate large numbers of people to pay the toll in order to save time. Will HOT lanes be the best option on a less congested route? Are they always a more cost-effective alternative to HOV lanes? In what circumstances do they provide greater net benefits than mixed-flow lanes?

³ Dahlgren, J.W. (1996) *An Analysis of the Effectiveness of High Occupancy Vehicle Lanes*, Ph.D. Dissertation, University of California at Berkeley, pp 100-135

⁴ Although the SR 91 lanes are not called HOT lanes, and HOVs on SR 91 now pay a toll, HOVs paid no toll during the time period analyzed in this study, and the road operated like those described in Fielding's and Klein's 1993 report, *High Occupancy / Toll Lanes: Phasing in Congestion Pricing a Lane at a Time*, published by the Reason Foundation.

⁵ Sullivan, E., (1998) *Evaluating the Impacts of the SR 91 Variable Toll Express Lane Facility* Cal Poly State University

⁶ 1997 CPTC Annual Report, pp fourteen through twenty

1.4 This Report

This report is a first step in trying to answer these questions and determine the circumstances in which HOT lanes are the most cost-effective alternative. Unfortunately, limited data has been available for analysis, so the results are necessarily preliminary. Further study of the variable toll express lanes on SR 91 and study of the express lanes on I-15 will be needed to confirm and expand the findings contained in this report.

However, even without extensive data, the HOT lane concept can be analyzed and some conclusions can be drawn. This paper first examines the HOT lane concept and lays out the factors that determine its performance in terms of congestion reduction and cost. It then analyzes the elasticity of demand for travel time savings exhibited on SR 91. Models for estimating travel time savings resulting from converting an existing HOV lane to a HOT lane and for constructing a new HOT lane are presented in the next section. This provides a basis for some generalizations regarding the circumstances in which construction of a new HOT lane or conversion of an existing HOV lane to an HOT lane would be cost-effective. The report concludes with policy recommendations and recommendations for further study of the HOT lane alternative.

2. The HOT Lane Concept

A HOT lane is an HOV lane that non-HOVs are allowed to use if they pay a toll. It must have excess capacity when used only by HOVs, and tolls are set at a level that keeps use below capacity. The tolls are collected electronically so that there is little or no delay in entering the lane. Such lanes are generally separated from the main lanes by pylons, striped zones, or fixed barriers, limiting the need for enforcement to locations where vehicles enter the lane. It might be possible to use some variant of the HOT lane concept in locations where there were not barriers between HOV lanes and mixed flow lanes, but this would require enforcement based not just on vehicle occupancy but also on some proof of toll payment. This might be a visual sticker that could be purchased that would allow HOV lane use during the congested period; this would be similar to the Express Pass used on I-15 before variable tolls were instituted. Minneapolis-St Paul has considered non-separated HOT lanes with tag reading gantries every $\frac{1}{2}$ mile that would light up if a toll tag were read. Officers observing non-HOVs in the lane would check to see that the gantry light lit up⁷.

2.1 Similarities Between HOT and HOV Lanes

The working of a HOT lane is similar to the working of an HOV lane. In both cases delay is reduced for those who use the lane during congested periods, and delay is also reduced for those remaining on the main lanes because of the shift of some of the traffic to the HOV or HOT lane. However, as more people shift to these lanes, the delay on the main lanes is reduced, and people are willing to pay less to use the lane and have less incentive to form a carpool in order to use the lane. Eventually, an equilibrium is reached at which no additional people are motivated to carpool or pay a toll in order to use the lane. Of

⁷ Posting to Congestion Pricing Forum (con-pric@tc.umn.edu) March 17, 1999 Kenneth Buckeye of the Minnesota DOT.

course, this equilibrium point depends on the carpool occupancy requirement and, in the case of a HOT lane, on the toll. In both cases people will be motivated to form car pools or pay tolls *only if congestion remains on the main lanes*. The addition of an HOV lane is more effective in reducing overall congestion than the addition of a mixed flow lane *only* if significant delay remains on the mixed-flow lanes and there is initially a relatively high proportion of HOVs⁸. The first condition provides the motivation to switch to a carpool, the second condition provides a high initial level of HOV lane utilization.

2.2 Differences Between HOV and HOT Lanes

2.2.1 Choice of Level of Service

The most significant difference between the two types of lanes is that a HOT lane provides all travelers with a choice of levels of service—they can travel in congested traffic or they can pay a toll and travel without delay. They are assured that they can buy a delay-free trip if they really need it. Tolls currently charged are not beyond the means of any automobile operator on an occasional basis, and many people who use the SR 91 HOT express lanes do so only occasionally⁹. Having different levels of service available puts the freeways more in line with other services for which consumers have a choice of price and quality. In some locations people pay a premium for express bus service. Airline travelers pay more for a flight at a more convenient, more congested time of day, or for a first class seat. Generally consumers are better off if they have more choices.

2.2.2 Better Utilization

HOV lanes typically carry fewer vehicles than could be accommodated at free flow speeds. On many HOV lanes originally requiring 3 occupants the occupancy requirement has been reduced to 2 in order to increase utilization¹⁰, and the great majority of HOV lanes now require only two HOV occupants. In some cases, for example the Katy HOV lane in Houston, utilization is low when 3 occupants are required, but is too high when only 2 occupants are required¹¹. This exposes a basic problem with HOV lanes, the utilization of the lane can not be optimized by requiring fractions of people. HOT lanes, on the other hand, can have adjustable tolls and thereby may be able to avoid under-utilization during congested periods. The toll is set at a level that is high enough to maintain free flow but low enough to achieve high utilization. Because HOT lanes can be more fully utilized than HOV lanes, they provide more delay reduction for main lane travelers than HOV lanes.

⁸ Dahlgren, J. W. (1998) “High Occupancy Vehicle Lanes: Not Always More Effective Than General Purpose Lanes”, *Transportation Research A*. Volume 32, No. 2. 106-110

⁹ Sullivan, E., (1998) *Evaluating the Impacts of the SR 91 Variable Toll Express Lane Facility* Cal Poly State University, p 58

¹⁰ Examples are US-101 in Marin County, I-5 North in Seattle, I-10 in Phoenix, I-279 in Pittsburgh, I-84 in Hartford, I-93 in Boston, I-66 inside the beltway in Virginia, and SR 509 in Seattle.

¹¹ Because of this problem 2-person carpools have been allowed to pay a toll to use the 3-person carpool lane on the Katy freeway.

2.2.3 Cost Contribution

Another advantage of HOT lanes over HOV lanes is that their users bear some of the cost of the HOT lane construction and operation. HOT lanes also establish the principle of price incentives to encourage more efficient use of the transportation system.

2.2.4 Benefits of HOT Lanes Compared to HOV Lanes

Unless an HOV lane is fully utilized, a HOT lane has the potential for higher utilization. By moving more vehicles from the main lanes, a HOT lane will reduce delay more for people remaining on the main lanes.

HOT lane tolls can be set to maintain the incentive to carpool, and if the numbers of HOVs should increase, the toll can be raised to lessen use in order to maintain free flow. Although it could be argued that some people who otherwise would have chosen to carpool will choose to not carpool and pay a toll instead, this is not borne out by the SR 91 experience¹².

2.2.5 Setting Tolls

Tolls serve three goals: to maximize the efficiency of the HOT lane, to maintain free-flow speeds for HOVs, and to generate revenue. To meet the first goal, tolls must not be set too high. To meet the second, they must not be set too low. To meet the third, they must be set so as to generate the maximum revenue. The ultimate selection of a toll level will involve a compromise that reflects decision-makers' weighting of these three goals.

2.2.6 HOT Lane Costs

Conversion of an HOV lane to a HOT lane will impose costs for:

- design and environmental assessment
- any additional right-of-way and paving required for the toll collection system
- installation and operation of the automated toll collection system
- transponders located in the vehicles
- any additional enforcement

HOV lanes typically cost more than conventional, mixed flow lanes, especially if they are separated by a physical barrier or additional pavement. They often include enforcement zones, which add to the cost. Enforcement costs range from \$20,000 to \$215,000 annually¹³.

¹² Sullivan, E., (1998) *Evaluating the Impacts of the SR 91 Variable Toll Express Lane Facility* Cal Poly State University, pp 95-105

¹³ Parsons Brinkerhoff (1994) *High Occupancy Vehicle Workshop Materials* Seventh International Conference on High-Occupancy Vehicle Systems, Transportation Research Board, Los Angeles, California

3. Elasticity of Demand for Travel Time Savings

3.1 Why Elasticity Matters

It is difficult to estimate the price elasticity of demand for anything because demand depends on so many things other than price, and these are likely to be unknown. The elasticity depends on the existence and cost of good substitutes for the good or service, on the income of the buyers, and on the prices and qualities of all other goods and services available to them. Furthermore, the elasticity is likely to vary at different levels of demand. When the service for which one is estimating elasticity is difficult to measure, the task becomes even more difficult. But knowing the price elasticity of demand enables estimates of the demand if prices are changed. Since correctly setting the toll for a HOT lane is key to achieving HOT lane demand that will provide high utilization and free flow, we are interested in estimating the elasticity of demand for travel time savings.

3.2 Elasticity Analysis

3.2.1 Analytical Approach

The theory behind the analysis is that the number of people using the toll lane depends primarily on the delay on the main line, the total travel demand on SR 91, and the toll. Although surveys of users of the SR 91 toll lanes showed that people used the toll lanes even when there was no delay on the main lane and for reasons other than simply saving time¹⁴, all of these are positively correlated with travel time savings or main line demand. Therefore, it is assumed that their effects are captured by the delay on the main line and the travel demand. Because the item of interest is the elasticity of demand for travel time savings, the analysis focuses on toll lane volume as a function of dollars per minute of travel time saved—the toll divided by the delay—and total travel demand.

3.2.2 Limitations of the Analysis

The analysis was hampered by both data limitations and statistical limitations.

3.2.2.1 Data Limitations

First, the analysis covers only two weeks in only one month for only two years. Second, although five-minute data were available for the main lanes, there was no way to utilize this data because only hourly data were available for the toll lanes.

A third limitation was that the actual main lane volumes do not reflect the total demand when the freeway is congested because not everyone wanting to use the freeway is able to reach the desired section of freeway. Volumes understate real demand as congestion is building and overstate demand as congestion is dissipating. To avoid the problem that large delay would result in a low volume even though the total demand was high, average flows rather than actual flows for a particular day, time, and year were used.

Finally, actual travel times were not available, so they were estimated from average hourly speeds at Imperial East and Gypsum Canyon East. Because the speeds were obtained from

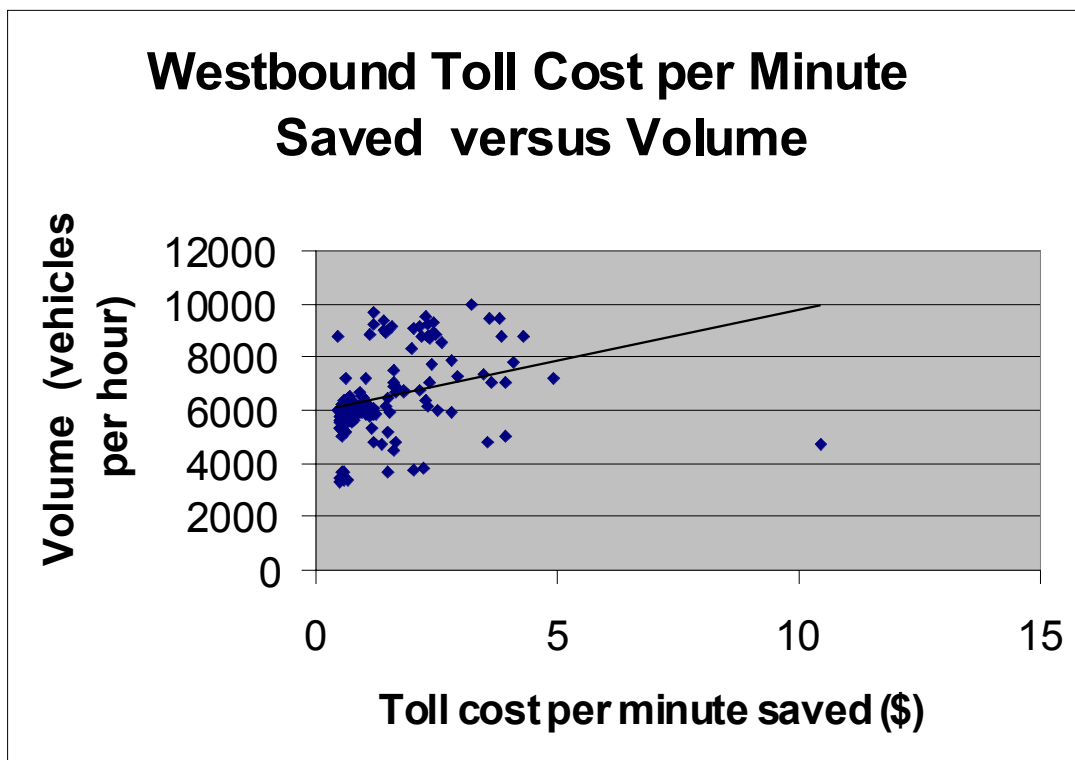
¹⁴ Almost half of the people surveyed gave no other reason for using the HOT lanes than travel time savings, but the rest cited additional reasons, such as easier, more comfortable driving, greater safety, and more predictable travel times. See page 61 of Sullivan, E., (1998) *Evaluating the Impacts of the SR 91 Variable Toll Express Lane Facility* Cal Poly State University.

loop detectors at only two locations and because they were hourly averages applied to the entire length of the main lane, they are not very accurate. In particular, they did not capture the variations in travel times within hours.

Actual travel time data and 5 minute toll lane volumes would provide much more reliable data and allow more credible results.¹⁵

3.2.2.2 Statistical Limitations

Figure 1



The research was undertaken in the hope that the toll would not be so well correlated with volume and delay that its effect would be obscured. This did not prove to be a problem. As can be seen from Figure 1, the plot of the westbound data, there is considerable range in the toll cost per minute saved (this is the toll divided by the delay). However, Figure 1 shows another problem. The trend line through the plot shows that there is a positive correlation between the toll cost per minute saved and the volume. This means that the effects of the volume may obscure the effects of the toll cost. Fortunately, there was no such correlation in the eastbound data.

¹⁵ Actual travel times for use in further analysis of the elasticity of demand for travel time savings could be collected if there were toll tag readers at the end of the toll lanes and on the main lanes at the beginning and end of the toll lanes. Over 100,000 toll tags have been issued but fewer than 20,000 vehicles use the toll lanes in each direction each day. This suggests that many of the vehicles in the main lanes have toll tags.

3.2.3 Eastbound Analysis

A number of regression formulations were investigated. These including linear models, log-log models, logit models, and translog models. Different variables were included in the models. The best fit for east bound travel is obtained with a linear regression of the logarithm of the toll lane volume on the logarithms of cost per minute saved, average volume at Imperial and average volume at Gypsum Canyon. This results in a constant elasticity of demand curve, which is often found in empirical studies of demand.

Table 1 shows the results of this regression. The high F with a low significance probability indicates a good fit for the model as a whole. The low P-values indicate a low probability that the variables included in the model are not significant. The numbers in the Lower and Upper 95% columns indicate that the probability is 90% that the actual value of the coefficient for the variable falls between the values in these columns. However, the range in values for the elasticity (the \$/minute saved) is quite large.

Table 1 Results of Linear Regression of the Natural Logarithm of Toll Lane Volumes on the Natural Logarithms of Average Hourly Volumes and Toll Cost per Minute of Travel Time Saved – Eastbound SR 91

SUMMARY OUTPUT

<i>Regression Statistics</i>						
Multiple R	0.95					
R Square	0.91					
Adjusted R Square	0.90					
Standard Error	0.30					
Observations	140					

ANOVA						
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>	
Regression	3	121.78	40.59	439.57	9.16E-70	
Residual	136	12.56	0.09			
Total	139	134.34				

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
Intercept	-32.09	1.29	-24.93	1.44E-52	-34.64	-29.55
ln (\$/min saved)	-0.09	0.04	-2.44	1.58E-02	-0.16	-0.02
ln (avg hourly volume at Imperial)	2.03	0.45	4.54	1.24E-05	1.15	2.92
ln (avg hourly volume at Gypsum Canyon)	2.32	0.37	6.36	2.83E-09	1.60	3.05

3.2.4 Westbound Analysis

A similar westbound analysis found significant *positive* coefficients for the logarithm of \$/minute saved, indicating that as the cost per minute saved goes up, so does the demand to use the toll lane. Since this does not seem reasonable, it appears that this positive coefficient is a spurious result caused the positive correlations between the independent variables making it impossible to separate their effects. It also suggests that the effect of the toll was small enough to be overwhelmed by the effects of these other factors.

In order to investigate the effect of delay alone on toll lane use, toll lane use was regressed on speed and year for the same hour of the day over a sample of 9 days. The demand was assumed to be the same during the same hour on all days during the same year. A dummy variable was used for the later year to capture the effect of growth and an increase in toll during some hours in the second year. The results of these analyses are shown in Table 2. For all congested hours the coefficient related to mainline speed was negative, confirming the expectation that toll lane use would be higher when speeds were lower on the main lanes. However, for some hours, the p-value was greater than .05 indicating that travel time was not statistically significant at the 5% level.

Table 2 Results of Analysis of Mainline Speed versus Toll Lane Use

Direction	Time	Speed Coefficient	P-value
West bound	5:00 AM	-18.79	0.029
	6:00 AM	-4.77	0.004
	7:00 AM	-11.17	0.175
	8:00 AM	-3.08	0.652
	9:00 AM	-3.81	0.797
	10:00 AM	-34.01	0.003
East bound	2:00 PM	-0.73	0.840
	3:00 PM	-23.70	0.012
	4:00 PM	-4.63	0.202
	5:00 PM	-6.93	0.050
	6:00 PM	-2.76	0.703

3.3 Initial Findings Show a Low Price Elasticity for Travel Time Savings

The eastbound analysis shows that there is a 90% probability that the price elasticity of demand for travel time savings in the eastbound direction is between -.02 and $-.16^{16}$. This

¹⁶ When the natural logarithm of the dependent variable has a linear relationship with the natural logarithms of the independent variables, the coefficients of the latter represent the elasticity of the dependent variable with respect to the independent variables.

means that eastbound toll lane use is not very sensitive to the toll—a 10% decrease in tolls would result in only a .2 to 1.6% increase in toll lane use. If the price elasticity is indeed this low, then rather large changes in tolls may be necessary to maintain the desired utilization of the lanes. Of course, it also means that tolls can be increased without causing a large decline in toll lane use. This latter result was found when tolls were increased on the SR 91 toll lanes¹⁷.

The westbound analysis supports the conclusion that there is a low price elasticity for travel time savings. It also suggests that toll lane use is not very sensitive to current travel time savings, as indicated by main lane speed.

These findings raise the possibility that the hypothesis that people's decisions to use the toll lane are based on the current delay and toll may be incorrect. The decisions may be based on past experience regarding delay or uncertainty regarding delay. A comparison of volumes, tolls, and travel times over a series of years may tell more about the price elasticity of demand for travel savings than an analysis over periods of days. Further research on SR 91 using shorter time intervals and different times of year is needed before any firm conclusions can be drawn about the elasticity of demand in the SR 91 corridor. Analyses based on other HOT lane sites, such as I-15 in San Diego, are needed to determine how transferable elasticities are from one location to another.

3.4 Determining Elasticity by Trial and Error

Good data are expensive to gather and time consuming to analyze. The results of an analysis in one location or time may not be transferable to another location or time because the populations of people making the choices may differ in ways that affect the elasticity of their demand for travel time savings. So, analysis is not a substitute for real world trial and error, although it is likely to provide a starting point that is closer to the real value. In fact, in the real world the profit-maximizing price is usually found through a process of trial and error. The advantage of analysis is that one can estimate in advance those situations where tolls are not likely to be effective in providing revenues or in eliminating congestion.

4. Modeling the Effects of HOT, HOV, and Mixed Flow Lanes

In order to examine the relative effects of adding an HOV lane, an HOT lane or a mixed flow lane to a freeway, a model was developed to calculate the average person-delay over the peak period¹⁸, the maximum delay, and the number of vehicle trips. Given these data, the relative emissions effects can be estimated on the basis of vehicle hours and vehicle trips. A model used to analyze the circumstances in which HOV lanes have greater benefits than mixed flow lanes has been expanded to include HOT lanes¹⁹. The original

¹⁷ Sullivan, E., (1998) *Evaluating the Impacts of the SR 91 Variable Toll Express Lane Facility* Cal Poly State University p 70.

¹⁸ This is average delay over the *initial* congested period. Therefore, because the delay is averaged over the same period in all cases, the average delay is in proportion to the total person delay.

¹⁹ Dahlgren, J.W. (1996) *An Analysis of the Effectiveness of High Occupancy Vehicle Lanes*, Ph.D. Dissertation, University of California at Berkeley

model and the modifications made to accommodate the HOT lane analysis are described in detail in the appendix.

4.1 Cases Modeled

Six cases were modeled to show how the relative performance of the three types of lane varies with circumstances. The variables that varied in these cases were:

Initial percent of HOVs: 5%, 10%, and 20%

Initial percent of people in HOVs: 10.2%, 20.3%, 45%

Initial maximum delay: 15 minutes and 45 minutes

4.2 Assumptions

HOT lane utilization was assumed to be 1800 vehicles per lane per hour, a volume that would provide good utilization but still allow free flow speeds. Maximum flow for mixed flow lanes was assumed to be 2000 vehicles per lane per hour. It was assumed in all cases that there were initially 3 lanes and a fourth lane was added as either an HOV, HOT, or mixed flow lane. The initial congested period, before the lane was added, was assumed to be three hours, with travel time building at a constant rate until the middle of the congested period, and then falling at a constant rate until the end of the congested period. It was assumed that the carpool occupancy requirement was 2, that all HOVs would use the HOV lane, and that no non-HOVs would use it. It was further assumed that the choice to carpool could be described by a logit choice model. In order to show the best possible performance for HOV lanes, a value for sensitivity to travel time of -.05 per minute of round trip in-vehicle travel time was used. This is at the high end of all such values found in the literature²⁰. Unfortunately, no studies of the sensitivity of car-pool formation to travel time savings as a result of HOV lanes were found.

4.3 Findings

Table 3 shows how sensitive the performances of the three types of lanes are to circumstances. When there are initially very long delays, such as 45 minutes, and at least 10% HOVs before the lane is added, then HOV lanes perform better than mixed flow lanes. But when the initial delay is only 15 minutes, HOV lanes do not perform as well as mixed flow lanes. In general, HOV lanes perform better relative to mixed flow lanes when the initial delay is high and when the initial proportion of HOVs is high—under these conditions the HOV lane will be well utilized and more effective in motivating people to shift to HOVs. Because HOT lanes generally can achieve higher utilization than HOV lanes, the overall delay with a HOT lane is less than with an HOV lane. Table 3 also shows that the delay differential between a HOT lane and HOV lane decreases as the initial proportion of HOVs increases. This occurs because the HOV lane becomes more fully utilized.

²⁰ A discussion of reasonable values of travel time coefficients is contained in pages 53 through 55 of Dahlgren, J.W. (1996) *An Analysis of the Effectiveness of High Occupancy Vehicle Lanes*, Ph.D. Dissertation, University of California at Berkeley

Table 3 Delay for Alternate Types of Lanes

	Initially 5% HOVs and 10.2% of people in HOVs		Initially 10% HOVs and 20.3% of people in HOVs		Initially 20% HOVs and 45% of people in HOVs	
	Initial Maximum Delay					
	15	45	15	45	15	45
Add HOV Lane						
Maximum Delay (minutes)	5.7	15.7	1.9	9.8	0	5.8
Avg Delay in Mixed Flow lane ²¹ (minutes)	2.3	8.1	.8	5.2	0	2.7
Add mixed flow lane						
Maximum Delay (minutes)	0	11.2	0	11.2	0	11.2
Avg Delay in Mixed Flow lane (minutes)	0	4.5	0	4.5	0	4.5
Add HOT lane						
Maximum Delay (minutes)	0	11.5	0	8.7	0	3.7
Avg Delay in Mixed Flow lane (minutes)	0	5.8	0	4.8	0	2.5

4.4 The Tradeoff Between Choice and Added Delay

The benefits of an HOT lane over a mixed flow lane come from providing consumers with more choice. However, the ability to choose comes at a price—people not opting to use the HOT lane have somewhat longer travel times than if all the lanes were mixed flow lanes. It is difficult to demonstrate the value of this choice because its value differs for different individuals at different times. At any particular time there will be some people who are in a hurry. They may be trying to catch a plane, pick up children from day care, or get to work or an important meeting on time. These people are willing to pay something to avoid delay. If they are willing to pay to avoid delay, they are at least as well off and most likely better off than if they were not able to avoid delay by paying a toll. The question is whether their gain exceeds the additional time burden this imposes on the other people—in essence the toll players are cutting ahead of them in line. If all of the travelers had equal resources and all of the toll revenues were used to benefit all travelers on the road, there would be no question. Those to whom time meant the most would use the toll lane and essentially pay the others for this use. Of course, not every traveler has equal resources. This means that those with more money can use the toll lane more frequently than those with less money, and this has been the case on SR 91. But this is true of all goods and

²¹ There may be cases where there is a little delay in the HOV lane. However, this table shows just the delay in the mixed flow lanes.

services—it is not peculiar to transportation, and on SR 91 people in all income groups used the HOT lane.

5 Some Preliminary Policy Recommendations

Table 3 shows that HOV lanes performed best when the freeway was very congested and there was already a large percentage HOVs. This should result in a high level of HOV lane utilization and a substantial shift of people to HOVs. But there are many HOV lanes that are not highly utilized. In such cases, HOT lanes generally reduce delay more than HOV lanes because they can be more fully utilized. If the additional delay reduction from converting such a lane to a HOT lane can be estimated, and if the value to travelers of this delay reduction exceeds the capital and operating costs of converting the lane to a HOT lane, then this should be done. Of course, this should not be done if these costs exceed the benefits from the conversion. In such cases, it makes more sense to convert the HOV lanes to mixed flow lanes.

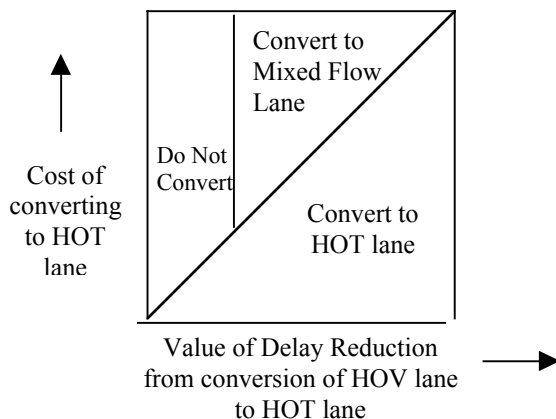


Figure 2 Circumstances in Which HOV Lanes should be converted to HOT or Mixed Flow Lanes

Figure 2 illustrates the circumstances in which conversion of an HOV lane to another type of lane is appropriate. In the section on the left, the value of the delay reduction from conversion to a HOT lane is low because the HOV lane is already well utilized, and the lane should not be converted. In the lower right half of the graph, the value of the delay reduction from converting to a HOT lane exceeds the cost of the conversion, so the public would benefit from such a conversion. In the smaller triangle on the upper right, the value of the delay reduction is relatively high, but not as high as the cost of converting to a HOT lane. In this situation, the public would benefit from a conversion of the HOV lane to a mixed flow lane. Of course, no HOV lane exists in isolation, it is part of a larger network, and the network characteristics will determine the value of the delay reduction from conversion to either a HOT lane of mixed flow lane.

It is recommended that a review of its HOV lanes in California be undertaken to determine which would be the best candidates for conversion to HOT lanes. These would be those

falling in the lower right corner in Figure 2. It might be possible to make this part of the federally sponsored Value Pricing Pilot Program.

5.1 Recommended Further Research

Further studies should be conducted, using I-15 data or improved SR 91 data, in order to learn more about the elasticity of demand for the HOT lanes. The analysis described in Section 4 assumed that tolls could be set to fully utilize the HOT lanes. This is certainly theoretically possible, but there may be practical constraints that would preclude this result. The more that is known about people's choice to use a HOT lane, the better the basis for decisions regarding which HOVs lane should be converted.

The experience on SR 91 and I-15 should be reviewed to determine how to effectively make the conversion, how best to set the tolls, how to manage payment, how to operate and enforce payment on the lanes, and what to do with the revenues.

Appendix

A Model for Comparing the Effects of HOT, HOV, and Mixed Flow Lanes

This appendix first describes the HOV model. It then describes modifications to accommodate modeling of HOT lanes.

A.1 HOV Lane Model

Current planning methods for HOV lanes generally use static transportation planning models which provide only peak hour travel times and volumes. Translation of these measures into vehicle-delay and vehicle-trips requires assumptions that are highly uncertain, such as the distribution of trips, mode shift, and vehicle occupancy over time. An exception is the FREQ model, which has been used in the evaluations of the Houston HOV lanes. It is dynamic and can model delay for freeways with or without HOV lanes or a parallel arterial. However, its extensive data requirements make it expensive to use. An HOV lane planning method developed by Dowling Associates (1996) is less data intensive and also accounts for the dynamic nature of travel demand but does not account for the continuous interaction between the proportion of HOVs and the travel time differential between mixed flow and HOV lanes.

The model used in this research bases estimates of the proportion of people using HOVs on the time differential between the HOV lane and other lanes, which is constantly changing. The model is easy to use and transparent so that the effects of uncertain inputs can be easily examined. Because it has limited data requirements, available resources for collecting and verifying data can be concentrated on less data. A key feature of the model is that, while it is very simple and does not include all of the effects of adding a lane, it can be shown that not including these effects does not change the performance ranking of the two types of lane. For example, neither this model nor FREQ include the effects of changes in trip start time. While such changes strongly affect travel patterns and delay, they are not important in comparing the effectiveness of an HOV lane versus a mixed flow lane because, as will be discussed later, whichever lane yielded the greatest benefits before the shift in trip starting times will also yield the greatest benefits after the shift. The same is true of the effects of route shifts and induced trips.

Estimating Delay

Consider an idealized freeway segment as shown in Figure A1. There is a bottleneck at the downstream end and the neck is long and uniform, contains no entry or exit points, and extends beyond the area subject to congestion. The queue builds up and dissipates during the peak period as shown in the lower section of Figure A1. Vehicles arrive at a constant rate until the time of the maximum queue and then arrive at a lower constant rate until the queue is dissipated. An idealized queue can be constructed from the following information:

- (a) the length of the congested period
- (b) the maximum delay (maximum travel time minus free flow travel time)
- (c) the time at which the maximum delay occurs

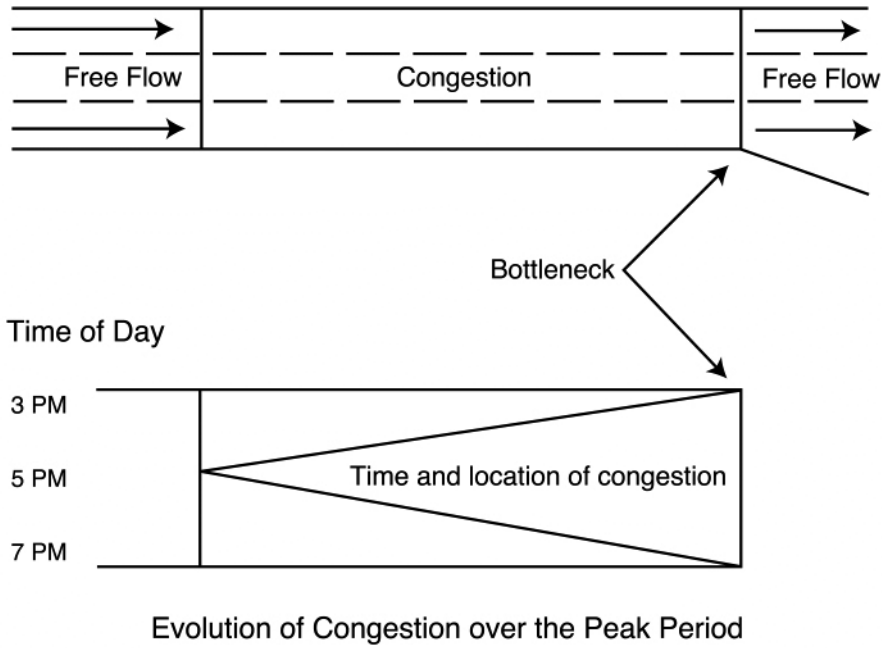


Figure A1 Idealized Freeway Segment

(d) the freeway capacity.

$$Q(t) = A(t) - D(t) \quad (1)$$

The queue can be represented as in Figure A2. The congested period extends from 0 to t_E , with the maximum delay occurring at t^{\max} . The cumulative number of vehicles attempting to pass through the bottleneck at time t is $A(t)$ and the number actually passing through is $D(t)=ct$, where c is the capacity of the bottleneck per unit of time. The number waiting to pass through at time t is

The delay for a vehicle arriving at time t is

$$w(t) = \frac{Q(t)}{c} = \frac{A(t) - D(t)}{c} = \frac{A(t)}{c} - t \quad (2)$$

The total delay to all travelers over the peak period is the area between $A(t)$ and $D(t)$, which equals

$$\int_0^{t_E} [A(t) - D(t)] dt \quad (3)$$

This idealized queue, combined with vehicle occupancies for HOVs and LOVs and the changes in freeway capacity for LOVs and HOVs, can be used to estimate the changes in person-delay and emissions from adding an HOV lane, adding an additional mixed flow lane, or converting an existing lane to an HOV lane.

Assumptions Regarding Delay

Recent research, some of it undertaken to inform revisions to the speed-flow relationships in the Highway Capacity Manual, has suggested that speed remains relatively constant until a freeway approaches capacity, at which point a queue forms and flow remains at capacity regardless of the queue length (Hurdle and Soloman, 1986; Hall and Hall, 1990; Banks, 1991; and Chin and May, 1991). The research supports a model in which 1) all delay is caused by queueing and none by increasing density per se, and 2) once the freeway reaches capacity, flow remains constant. In other words, the speed flow curve is a horizontal line at free flow speed until capacity is approached, at which point it begins to turn into a vertical line indicating constant capacity regardless of speed as shown in Figure A3.

Estimating the Shift to HOVs

The probability of making a trip via HOV is a function of the attributes of 1) the HOV trip, 2) the trip via non-HOV (a single occupant vehicle in most cases), and 3) the person making the trip. HOV attributes include: waiting time, travel time, time and inconvenience arranging the carpool, ambiance in the waiting area and the HOV, and cost. Single occupant vehicle attributes include travel time, parking availability and cost, vehicle ambiance, driving conditions, and vehicle operating cost. Traveler attributes include regularity and flexibility of working hours, work and home location, child care requirements, income, and availability of an automobile.

The probability that a particular individual will use an HOV can be represented by a logit model:

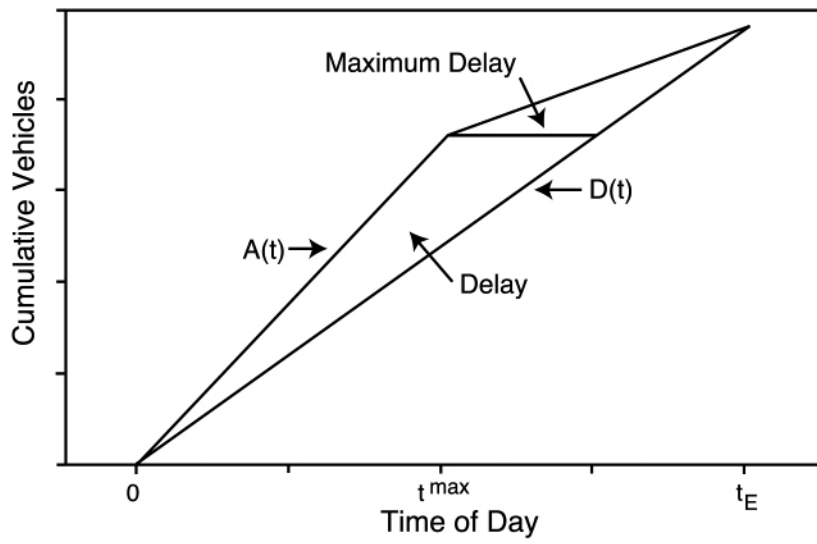


Figure 2A Idealized Queue

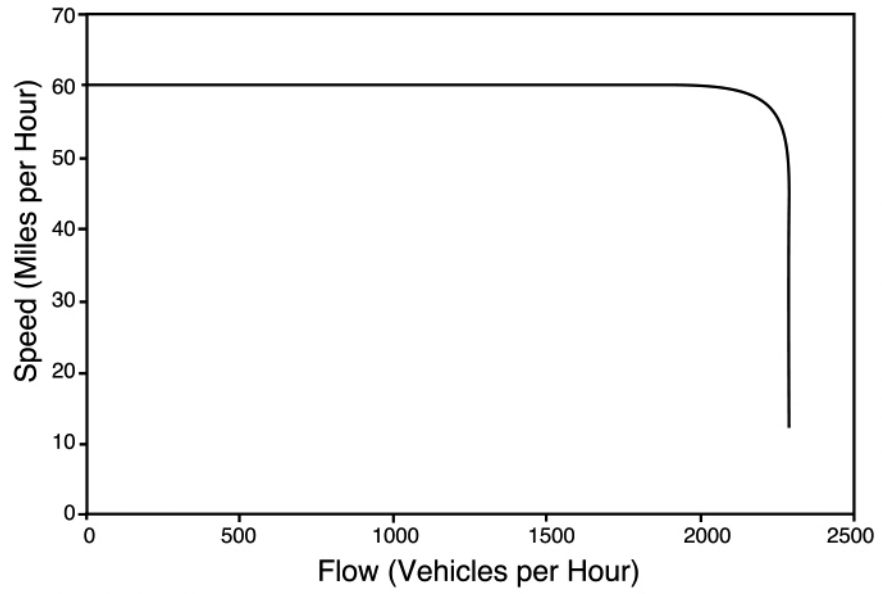


Figure A3 Freeway Speed versus Flow

$$P_{HOV} = \frac{e^{\beta_i H_i}}{e^{\beta_i H_i} + e^{\beta_i L_i}} = \frac{I}{I + e^{\beta_i L_i - \beta_i H_i}} = \frac{I}{I + \Gamma e^{\beta_i (L_i - H_i)}} \quad (4)$$

where the β_i are the coefficients of the attributes and the H_i and the L_i are the traveler and modal attributes related to the HOV and LOV trip, respectively. When an HOV or mixed flow lane is added, the only attributes that change are the travel times for the two modes. Therefore, all other attributes and their coefficients can be represented by a constant, Γ . As a result, the exponent of e is reduced to $\beta_i(L_i - H_i)$, where β_i is the coefficient of the travel time and L_i and H_i are the travel times via mixed flow lanes and the HOV lane respectively. The same coefficient for travel time is assumed for both HOVs and LOVs.

Each individual has different personal and modal attributes, and consequently different probabilities of using each mode, represented by a different Γ . Some people can not shift to an HOV. They may have irregular or unpredictable trip starting times, they may have an unusual trip origin or destination, they may need their vehicle at their destination, or they may need to transport equipment, materials, or children. Each region has different travel patterns and opportunities for HOV travel. The extent of the shift depends on these factors as well as the travel time advantage resulting from the HOV lane. Figure A4 shows three hypothetical distributions of the proportion of people using HOVs. The vertical axis shows the proportion using HOVs. The horizontal axis shows the *freeway* travel time differential between HOV and LOV (not the *total* travel time differential). Without an HOV lane, this differential is 0.

The highest curve represents the distribution in an area in which most of the people who could possibly travel via HOV are already doing so. This might be an area with a strong urban center, high congestion in the center, and good bus service. The middle curve might represent the distribution in an area where there are unutilized opportunities for ridesharing and transit. The lowest curve represents the distribution in an area where few people use HOVs, perhaps because transit service is poor or non-existent and opportunities for convenient carpooling are limited. In all three cases, some people are using HOVs when the travel time differential is zero. When the freeway travel time for HOVs is reduced, increasing the differential between LOV and HOV travel time, $L_i - H_i$, to V , the proportion of people using HOVs increases in all three cases. But the increases in the proportion of people using HOVs, S_1 , S_2 , and S_3 , are quite different. In deciding whether to build an HOV lane or not, the likely shape of this curve should be considered.

The distributions in Figure A4 were the sums of the probabilities of individuals using an HOV for each value of $L_i - H_i$. Despite differences in each person's probability of using an HOV, for simplicity, the model used in this research assumes that all travelers have the same probability of using an HOV. This gives the upper limit to the number of people who might shift mode.

Given this assumption, the expected proportion of people using HOVs is equal to the individual probability of using an HOV:

$$P_{HOV} = \frac{I}{I + \Gamma e^{\beta_i (L_i - H_i)}} \quad (5)$$

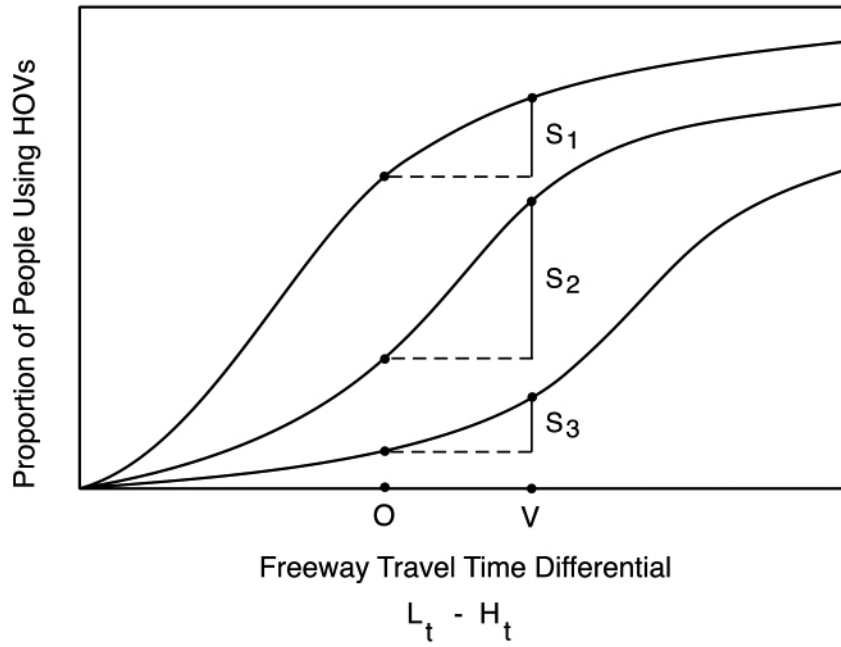


Figure 4A The proportion of People Shifting to HOVs Depends on the Propensity to Use HOVs

Because the travel time differential, $L_t - H_t$, is initially 0, Γ can be calculated from the proportion of people initially using HOVs. Estimation of β_t is another matter. Published HOV lane evaluations do not include data that link the proportion of people using HOVs to the changing travel time differential or to shifts from other times and routes, so it has not been possible to estimate travel time coefficients from experience with real HOV lanes. Therefore, a range of values based on the mode choice literature was used: -.02 per minute of round trip travel time (Small, 1977); -.002, -.003, -.004, -.006 (McFadden and Talvitie, 1977); -.0082 (Koppelman, 1983); and -.012 and -.016 (Kollo, 1986). Using this wide range of values increases the likelihood that the true value is considered and allows an examination of the effects of this coefficient on results.

Interaction of the Travel Time Differential and Mode Shift with an HOV Lane

Travel times on the mixed flow lane will change over the course of the peak period, as can be seen in Figure A5 from Wade, et. al (1992), which shows travel times for the mixed flow and HOV lanes on the Katy freeway in Houston during the peak period.

The proportion of people entering the freeway at a particular time who will use HOVs depends on the travel time differential, $L_t - H_t$, at that particular time, but the travel time differential, in turn, depends on the proportion of people who, up to that time, have used HOVs. This travel time differential is the difference between the delay for the HOVs and the delay for LOVs. To calculate these delays we modify Equation 2, letting $A(t)$ represent cumulative *person* arrivals at the freeway, $P(t)$ represent cumulative person arrivals in HOVs, L and H represent LOV and HOV average occupancies, and C_L and C_H represent capacities on the mixed flow and HOV lanes, respectively. The congested period begins at time $t=0$; congestion on the HOV lane begins at time t_H . Delay for the LOVs entering the freeway at time t is

$$w_L(t) = \max\left\{\frac{A(t) - P(t)}{C_L} - t, 0\right\} = \max\left\{\frac{A(t) - P(t)}{L C_L} - t, 0\right\} \quad (6)$$

and for the HOVs is

$$w_H(t) = \max\left\{\frac{P(t) - P(t_H)}{C_H} - (t - t_H), 0\right\} \quad (7)$$

$P(t)$, the cumulative person arrivals in HOVs by time t , in turn depends on the travel time differential $L_t - H_t$ at time t , which equals $w_L(t) - w_H(t)$.

$$P(t) = \int_0^t [a(x) P_{HOV}(x)] dx = \int_0^t a(x) \frac{1}{1 + \Gamma e^{\beta_t [w_L(x) - w_H(x)]}} dx \quad (8)$$

where

$$a(x) = \frac{dA(x)}{dx} \quad (9)$$

Equation 8 is not solved analytically, but is the basis for calculating $P(t)$ numerically over one minute intervals. Using this method, $P(t)$ equals the value of the expression inside the

integral evaluated at t plus the sum of this expression for all previous values of t . The travel time differential, $w_L(t) - w_H(t)$, is also calculated for each minute and used to calculate $P(t)$ for the subsequent minute interval. For people entering the freeway during each interval, total person-delay, vehicle-delay, and vehicle-trips are calculated. These are summed to obtain total person-delay, vehicle-delay, and vehicle-trips for the entire peak period. Any standard spreadsheet software can be used for the calculations. These calculations are made for 1) an added HOV lane and 2) an added mixed flow lane.

Effects of Model Assumptions

The model contains a number of assumptions. They are summarized in Table A1. The assumptions in the first group make an HOV lane appear to have greater individual benefits relative to a mixed flow lane than would actually be the case. The assumptions in the second group would not change the ranking of the alternatives in terms of individual benefits. The effects of the assumptions in the third group would depend upon the situation. The effects of these last two assumptions are not as strong as the overall effects of the assumptions that lead to an overstatement of the benefits of an HOV lane relative to a mixed flow lane. Therefore, it is assumed that on balance the model overstates the benefits of HOV lanes relative to those of mixed flow lanes.

A.2 Modifications to Accommodate Analysis of HOT Lanes

The analysis regarding the HOT lanes does not estimate the number of people in the HOT lanes—it assumes that tolls are set so that the lanes are optimally utilized. This makes the model much simpler but it means that the results must be interpreted as dependent upon being able to set a toll that fully utilizes the HOT lanes when the main lanes are congested. While this is theoretically possible, the feasibility of doing this in all situations remains to be demonstrated.

For each minute interval the model estimates the delay on the main lanes and uses it to estimate how many people will use HOVs. Clearly this is not a decision that is made on the spur of the moment each day, but is the result of experiences over past days. Then the model calculates the number of additional vehicles that can be accommodated without causing delay on the HOT lane. This number of vehicles is assigned to the HOT lane and the travel time for vehicles entering during the next one minute interval is calculated. The HOT lanes on I-15 are using variable tolls based on expected delay. Hopefully sufficient data will be available from these lanes to learn how people respond to shifting tolls.

EFFECTS OF ASSUMPTIONS

Assumptions That Lead to an Overstatement of the Benefits of an HOV Lane Relative to a General Purpose Lane	
Identical probabilities of using an HOV	The mode shift with identical probabilities is always greater than with different probabilities
No downstream entries	Downstream entries cause measured delay to be more than actual average delay--more delay favors an HOV lane
No reduction in convenience due to shift to HOV	Only the time saving beyond that necessary to induce a shift is a benefit
All HOVs use the HOV lane	Benefits of HOV lane are less if fewer vehicles use it
People do not drive to meet the carpool or bus	Driving to meet the carpool or bus would increase emissions substantially
Assumptions That Do Not Change the Ranking of an Added HOV Lane Versus an Added General Purpose Lane	
No route shifts	Benefits are larger with larger route shifts, and larger delay reductions result in larger route shifts
No shifts in trip start time	Larger delay reductions allow larger shifts in trip start times
No induced trips	Benefits from new trips are greater and costs of these trips are less with larger reductions in delay. Air quality benefits of reduced delay are likely to be greater than air quality costs of induced trips
No vehicles entering and exiting the queue before the bottleneck	Benefits to these vehicles are greater with larger reductions in delay
Assumptions Whose Effects Depend on the Situation	
Vehicles arrive at a constant rate until the time of maximum delay and at a lower constant rate thereafter	If the arrival rate is linearly increasing and the time of maximum delay is less than 2/3 through the peak period, the relative benefits of an HOV lane will be understated; otherwise they will be overstated
Only HOVs use the HOV lane	Allowing cheating increases utilization of the HOV lane but reduces the incentive to use an HOV

Table 1A

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