Submitted to Physics Letters

UCRL-18914
Preprint

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May 1969

AEC Contract No. W-7405-eng-48

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DISCLAIMER

NON-RIGIDITY IN ALPHA SCATTERING FROM DEFORMED NUCLEI*

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Phenomenological vibrational wavefunctions have been used to determine the effect of nuclear non-rigidity on inelastic scattering within the ground band and to calculate the cross-section of vibrational levels in strongly deformed nuclei.

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To determine the importance of departures from strict rigidity of the shape of strongly deformed nuclei on previous determinations[1)] of nuclear deformation parameters, we have carried out calculations of inelastic scattering of 50 MeV α-particles in which some representation of the non-rigidity is included. The non-rigidity manifests itself through the presence of β and γ bands. We have used the phenomenological wavefunctions of Faessler, Greiner, and Sheline[2)] (FGS) in which the ground band is mixed by the vibration rotation interaction (VRI) with the β and γ vibrational bands, as well as various bands of higher excitation which we have ignored. We have also used FGS wavefunctions to calculate certain gamma-band cross-sections.

The scattering cross-sections were calculated by the coupled channel method (see ref. 3; we shall follow the notation therein) using an optical

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*Work performed under the auspices of the U. S. Atomic Energy Commission.

model coupling with the surface expanded to second order about the <u>non-spherical</u> equilibrium. The nuclear surface is written as

$$R = R_0 + \delta R \tag{1}$$

$$\frac{\delta R}{R_0} = \sum_\lambda \beta_\lambda Y_{\lambda 0} + (a_0' Y_{20} + a_2' [Y_{2,2}+Y_{2,-2}]) \tag{2}$$

The β_λ are constant deformation parameters but a_0' and a_2' are dynamical variables being, the vibrational collective coordinates as defined by FGS.

Define $\delta^{(n)}$ and $\Delta^{(n)}$ by

$$\left(\sum \beta_\lambda Y_{\lambda 0}\right)^n = \sum_{LK} \delta_{LK}^{(n)} Y_{LK} \tag{3}$$

$$\left[a_0' Y_{20} + a_2' (Y_{22}+Y_{2-2})\right]^n \equiv \sum_{LK} \Delta_{LK}^{(n)} Y_{LK} \tag{4}$$

(A recursion relation for $\delta^{(n)}$ was given in ref. 3.) Using these results we can write the Taylor expansion of the nuclear optical potential as

$$V(r-R) = V(r-R_0) + \sum_{LK} Y_{LK} \sum_{n=1}^{\infty} \frac{(-R_0)^n}{n!} \bar{\delta}_{LK}^{(n)} \frac{\delta^n V(r)}{\delta r^n} \tag{5}$$

where

$$\bar{\delta}_{LK}^{(n)} = \delta_{LK}^{(n)} + \sum A \begin{pmatrix} L' & K' \\ L'' & K'' \\ L & K \end{pmatrix} \left\{ n\, \delta_{L'K'}^{(n-1)} \Delta_{L''K''}^{(1)} + \frac{n(n-1)}{2} \delta_{L'K'}^{(n-2)} \Delta_{L''K''}^{(2)} \right\} \tag{6}$$

$$A \begin{pmatrix} L' & K' \\ L'' & K'' \\ L & K \end{pmatrix} = \left(\frac{\hat{L}'\hat{L}''\hat{L}}{4}\right)^{1/2} \begin{pmatrix} L' & L'' & L \\ K' & K'' & -K \end{pmatrix} \begin{pmatrix} L' & L'' & L \\ 0 & 0 & 0 \end{pmatrix} \tag{7}$$

We note that $\Delta^{(2)}$, which is usually ignored, couples states within a band, or states differing by 2 phonons; the most marked effect is to increase the elastic scattering at large angles. The matrix elements of $\Delta^{(1)}$ and $\Delta^{(2)}$ are calculated from the FGS wave functions with B, C_0, C_2 which fit the energy levels, and β_2 which predicts the ground band (α,α') angular distributions for the nucleus in question. Vibrations will effect the inelastic scattering within the ground band in two ways. 1) Strongly coupled vibrational bands may provide, by virtue of higher order transitions, alternative ways of exciting the upper members of a ground band in a way that cannot be taken account of through the optical potential. We have found this effect to be small. 2) The vibration-rotation interaction (VRI) mixes the ground band with the β and γ bands; for the 6^+ states of certain nuclei around N = 90, the admixture of β-vibration (the stretching mode vibration) is quite large (the amplitude of β vibration in ground band 6^+ level of ^{152}Sm is 0.288 according to the FGS model). These components are excited coherently in inelastic scattering and might be expected to modify the cross section.

The admixture of the β-band by the VRI into the ground band corresponds to centrifugal stretching; in fact, we find that β-band admixture of the FGS wavefunctions allows us to calculate $\langle \beta_2 + a_0' \rangle_{FGS}$ for the 4^+ and 6^+ levels of ^{152}Sm which correspond very closely to the stretching required to fit the spectra predicted by FGS with fixed inertial parameter (i.e., varying β in $E_I = \frac{I(I+1)}{6B\ \beta^2}$). However, the prescription for the explicit inclusion of stretching by simply increasing the deformation for the higher states of a rigid rotor calculation is ambiguous and the cross-section calculated is found not to correspond closely to that obtained by an admixture of β-vibration.

Of the rare earth nuclei for which β_4 and β_6 deformations were extracted in the earlier work[1], the least rigid is ^{152}Sm, as indicated by its low lying β-band. We show in fig. 1 a calculation of the cross-sections to the ground band of this nucleus in which the non-rigidity is accounted for in the way described above, and compare the results with a calculation based on a rigid rotor nucleus. The cross-section of the 6^+ level is increased by as much as 30% over the rigid rotor cross-section. The results depend on the elastic constants and the optical model coupling, and correspond to an upper bound on the effect for reasons discussed below. We note that a direct excitation through a Y_6 deformation enhances the forward angle 6^+ cross section, whereas stretching results in the more uniform enhancement of cross-section characteristic of multiple excitations. The impurity of the 6^+ experimental data (it contains the 0^+ β band head) made it impossible to use this work to obtain better values of β_4 or β_6. The nucleus ^{154}Sm is stiffer to β-vibrations, and, as shown in fig. 2, the effect of vibrations on the ground band scattering is less. For most strongly deformed nuclei we conclude that the nuclear non-rigidity does not invalidate the earlier extraction of deformation parameters. In ^{166}Er for example, the effect is quite negligible.

We have also attempted to fit existing data[4] for the 2^+ γ band level in ^{154}Sm (fig. 2) and the 2^+ and 4^+ γ-band levels in ^{166}Er (fig. 3). In each case, in order to fit the 2^+ level, it was found necessary to reduce the optical model coupling by a factor of about 0.7. Whether this is a property of the optical model when used to couple states less collective than rotational, or a property of the harmonic phenomenological wavefunctions of FGS is not known. It is for this reason that we stated above that the effect on the ground band is an upper bound. The higher order terms should be more strongly attenuated. It was not possible to fit the 4^+ level of ^{166}Er by any manipulation (ad hoc mixing, anharmonicity, static Y_4 deformations, etc.) of a quadrupole vibration method. The slope and anomalously large magnitude of this angular distribution suggest a direct excitation[5]. We discuss a method of fitting this level elsewhere[6]. We conclude however that the Bohr-Mottelson picture of γ-vibrations is inadequate for ^{166}Er.

We comment that the angular distribution for the $\gamma 2^+$ level of ^{166}Er can be fitted[6] with a fixed Y_{22} asymmetry (Davydov model). The asymmetry needed is about twice[6] that needed to fit the γ-band levels using the Davydov Chaban model[7].

Footnotes and References

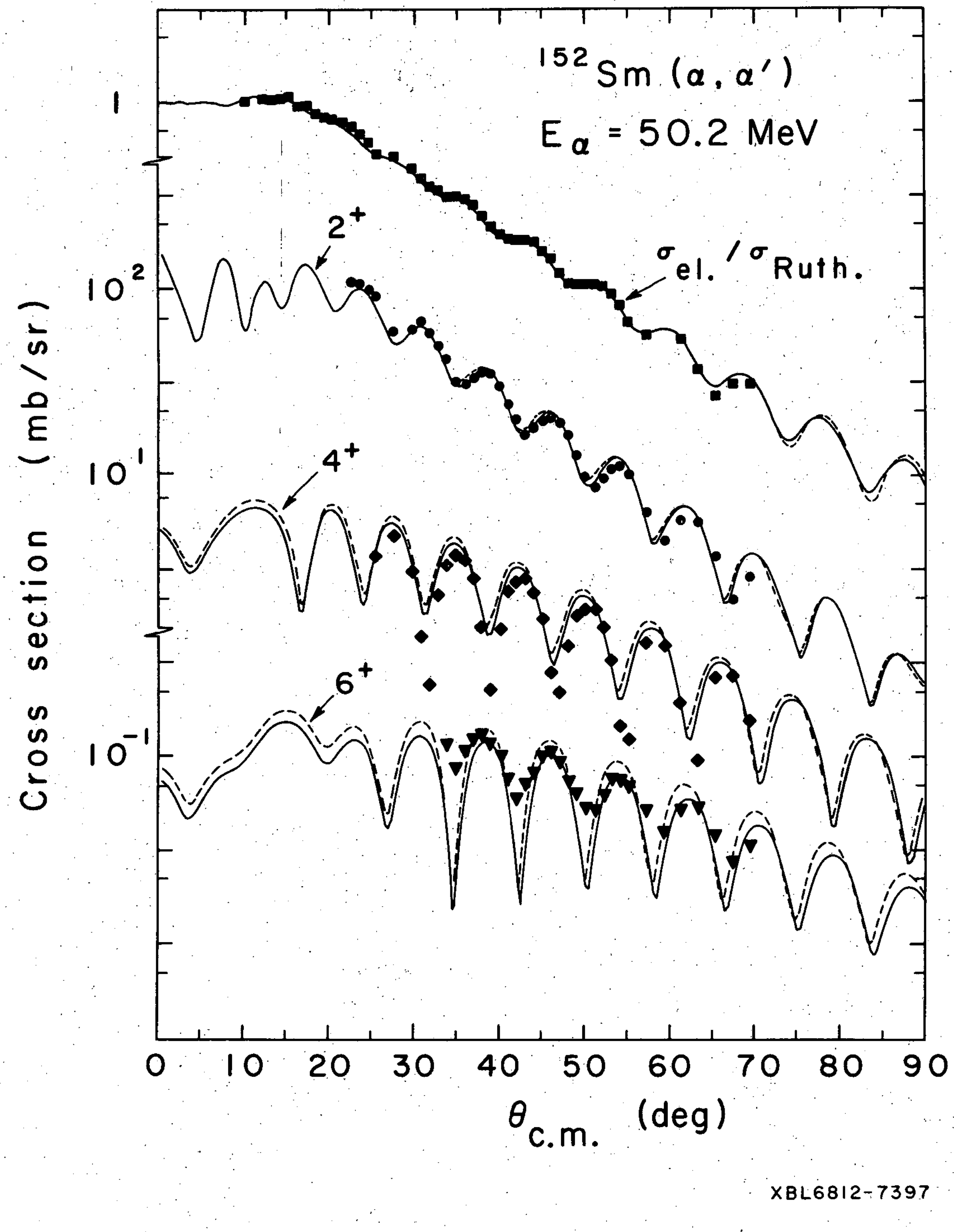
1. D. L. Hendrie et al., Phys. Letters 26B (1968) 127.
2. A. Faessler, W. Greiner, and R. K. Sheline, Nucl. Phys. 70 (1965) 33. Note that β_0 of this paper becomes β_2 in ours.
3. See for example, N. K. Glendenning, UCRL-17503 and Proc. Int. Sch. of Phys. "Enrico Fermi" Course XL (1967) ed. by M. Jean (Academic Press, New York, to be published).
4. The Sm data was measured by B. G. Harvey, D. L. Hendrie, and O. N. Jarvis. The Er data, which is preliminary, was measured by B. G. Harvey, D. L. Hendrie, and Jeannette Mahoney.
5. P. O. Tjøm and B. Elbek, Nucl. Phys. A107 (1968) 385.
6. Raymond S. Mackintosh, UCRL-18919, submitted for publication.
7. Davydov-Chaban and Faessler-Greiner programs transmitted to us by D. L. Hendrie.

Figure Captions

Fig. 1. Angular distributions for the ground band of ^{152}Sm of 50 MeV alpha particles comparing the excitation of a rigid rotor and a "soft" rotor which is coupled to various vibrational modes. The data shown for the 6^+ contains the unresolved 0^+ β bandhead. The solid curve shows the scattering when ^{152}Sm is treated as a rigid rotor, the dashed curve shows the effect of introducing vibrational degrees of freedom.

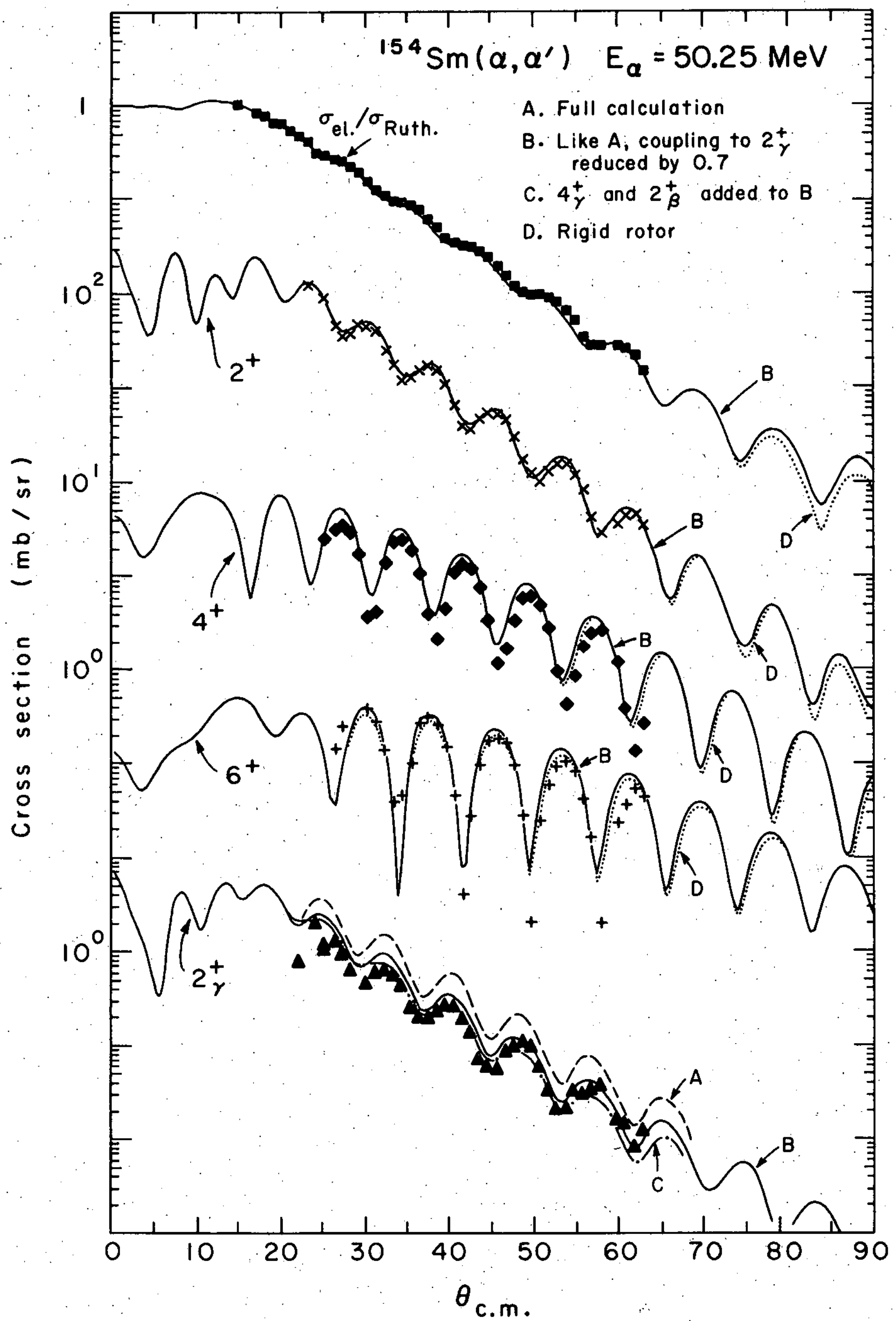
Fig. 2. Angular distribution for the ground band and gamma bandhead for ^{154}Sm. The necessity of reducing the coupling to the γ-band is shown together with the influence in the system of coupled channels of the $\beta 2^+$ and $\gamma 4^+$ states for which there is no data. Angular distributions for these states are not shown in the figure.

Fig. 3. Typical angular distribution to the ground band and gamma band of ^{166}Er. There is no data for the 3^+ level.



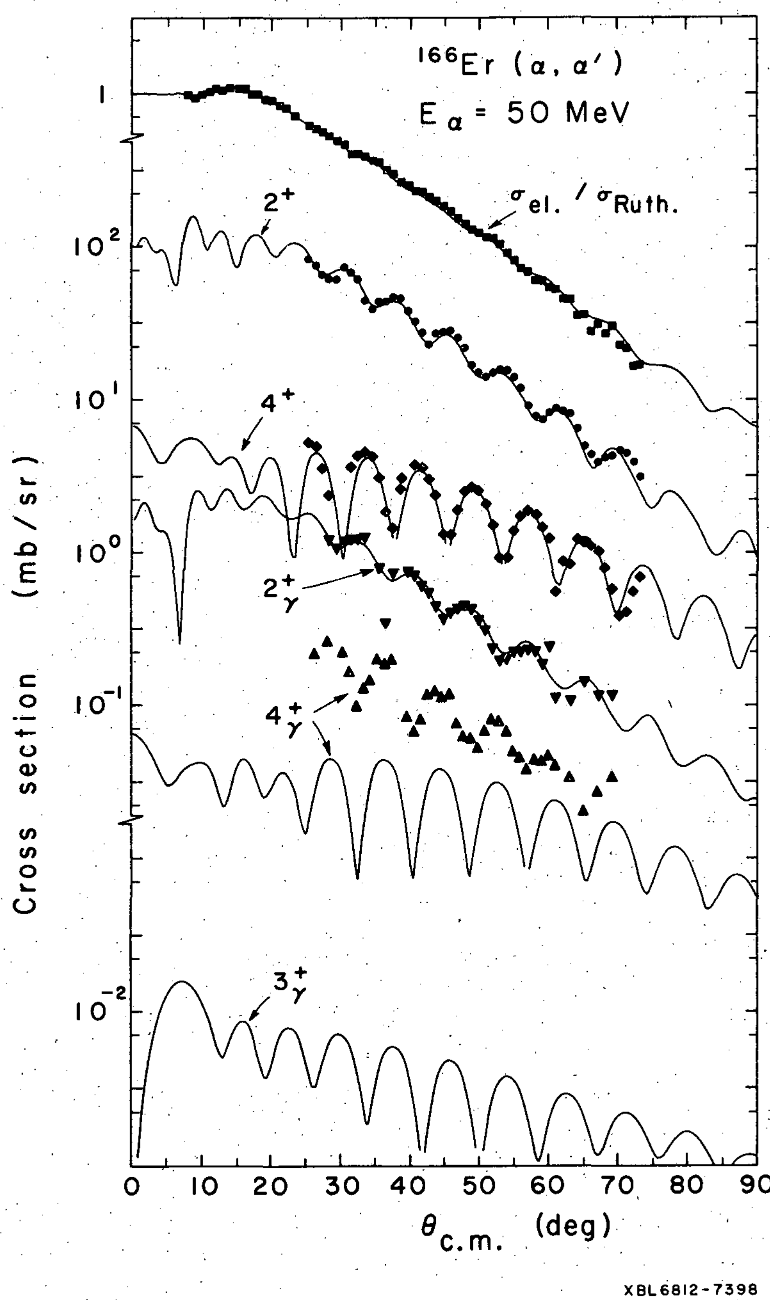
^{152}Sm (α, α′)
E$_α$ = 50.2 MeV
2$^+$
4$^+$
6$^+$
σ$_{el.}$ / σ$_{Ruth.}$
1
10^2
10^1
10^{-1}
Cross section (mb/sr)
0 10 20 30 40 50 60 70 80 90
θ$_{c.m.}$ (deg)
XBL6812-7397

Fig. 1



XBL695-2812

Fig. 2



^{166}Er (α, α')
E_α = 50 MeV
$\sigma_{el.}/\sigma_{Ruth.}$
2^+
4^+
2^+_γ
4^+_γ
3^+_γ
1
10^2
10^1
10^0
10^{-1}
10^{-2}
Cross section (mb/sr)
0 10 20 30 40 50 60 70 80 90
$\theta_{c.m.}$ (deg)
XBL6812-7398

Fig. 3

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