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# HEAVY ION INDUCED TRANSFER REACTIONS LEADING TO WEAKLY BOUND FINAL STATES†

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A simple model is suggested for the study of heavy ion induced transfer reactions leading to weakly bound (or unbound) final states. The finite range and recoil effects are considered and a simple expression is obtained for the differential cross section at high energies.

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The theory of nucleon transfer leading to unbound states in light ion induced reactions has been formulated by Huby et al. [1] and by others[2]. The method that has been used is either to describe the unbound state as a quasibound state or, if the unbound state is in the vicinity of a resonance, to describe it as a Gamow state. Both of the methods lead to an expression for the transition amplitude which resembles the one for transfer to bound states. In light ion reactions, one further assumes a zero range approximation which simplifies the evaluation of the cross-section considerably.

In the case of heavy ion induced transfer reactions, the zero range approximation is not applicable. One thus has to face the problem of evaluating integrals which are two dimensional. In the early application of the distorted wave Born approximation (D.W.B.A.) to heavy ion induced reactions, Buttle and

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Goldfarb [3] invoked a "no-recoil" approximation, which involves eliminating all terms of the order of the ratio of masses of the transferred nucleon to either of the cores. This approximation allows one to evaluate the two dimensional integral in two parts, firstly, the evaluation of the form factor and secondly the evaluation of the distorted wave integral. The no-recoil approximation was equivalent to assuming the transfer of the nucleon to occur when the two nuclei were at rest relative to one another. Recently the problem of neutron transfer to unbound states has been studied by Baur and Trautmann [4], who explicitly calculated the features of Sub Couloub transfer. During recent years, experiments [5] have indicated the nonadequacy of the no-recoil approximation, and approximate [6] and exact [7] calculations of recoil corrections have been made. These calculations exhibit the importance of the translational motion of the transferred nucleon.

In the present note, we wish to extend the theory of transfer to weakly bound final states incorporating the finite range and the recoil effects.

For the sake of simplicity, we shall use the notation of Buttle and Goldfarb [3] and represent the reaction as

$$\underbrace{(c_1+n)}_{a_1} + c_2 \rightarrow \underbrace{(c_2+n)}_{a_2} + c_1$$

The co-ordinate system will be identical to that of ref. 3. The DWBA transition amplitude of the transfer is given by

$$T_{fi}(\vec{k}_f,\vec{k}_i) = \langle \Psi_{a_2\alpha_2}(\xi_1 n)\Phi_{c_1\gamma_1}(\zeta)\chi^{(-)}(\vec{k}_f,\vec{r}_f)|V_{c_1n}(r_1)|\chi^{(+)}(\vec{k}_i,\vec{r}_i)\Phi_{c_2\gamma_2}(\xi) \times \Psi_{a_1\alpha_1}(\zeta_1 n)\rangle \quad (1)$$

One can integrate over the internal co-ordinates of the cores introducing the spectroscopic factors as follows:

$$\langle \Psi_{a_2\alpha_2}(\xi_1 n)|\Phi_{c_2\gamma_2}(\xi)\rangle \equiv \int d\xi \Psi_{a_2\alpha_2}{}^*(\xi_1 n)\Phi_{c_2\gamma_2}(\xi)$$

$$= \Theta^{1/2}_{j_2\ell_2} \sum_{\lambda_2\sigma_2} \langle c_2\gamma_2 j_2\zeta_2|a_2\alpha_2\rangle\langle \ell_2\lambda_2 s_2\sigma_2|j_2\zeta_2\rangle \quad (2)$$

$$\times \quad U_{\ell_2}(r_2)Y_{\ell_2\lambda_2}{}^*(\hat{r}_2)\chi^+_{s_2\sigma_2}(s)$$

and a similar expression for the parentage expansion of the projectile wave function $\Psi_{a_1\alpha_1}(\zeta_1 n)$. In eq. (2), the function $\chi^+_{s_2\sigma_2}(s)$ is the spin wave function of the nucleon in the residual nucleus, and $\Theta^{1/2}_{j_2\ell_2}$ is the spectroscopic factor. The transition amplitude becomes

$$T_{fi}(\vec{k}_f,\vec{k}_i) = \Theta^{1/2}_{j_1\ell_1}\Theta^{1/2}_{j_2\ell_2} \sum_{\substack{\lambda_1\lambda_2\\ LM}} (-1)^{\ell_1-s-\zeta_1} \frac{\hat{j}_1\hat{j}_2}{\hat{s}\ \hat{\ell}_2} U(\ell_1 j_1 \ell_2 j_2;SL)$$

$$\times \langle j_1-\zeta_1 j_2\zeta_2|LM\rangle\langle \ell_1\lambda_1 LM|\ell_2\lambda_2\rangle\langle c_1\gamma_1 j_1\zeta_1|a_1\alpha_1\rangle \quad (3)$$

$$\times \langle c_2\gamma_2 j_2\zeta_2|a_2\alpha_2\rangle$$

$$\times \int d^3r \int d^3r_1\, \chi^{(-)*}(\vec{k}_f,\vec{r}_f)\, U_{\ell_2}(r_2)Y_{\ell_2\lambda_2}{}^*(\hat{r}_2)V(r_1)\, U_{\ell_1}(r_1)Y_{\ell_1\lambda_1}(\hat{r}_1)$$

$$\times \chi^{(+)'}(\vec{k}_i,\vec{r}_i)$$

To derive eq. (3), we have assumed that the interaction $V(r_1)$ is spin independent.

If the nucleon is very weakly bound in the residual nucleus, one would expect that, in view of the strong absorption in the elastic channels, the transfer would occur in the asymptotic region of the nucleon wave function in the final nucleus. One could therefore approximate the function $U_{\ell_2}(r_2)$ by a spherical Hankel function, ie:

$$U_{\ell_2}(r_2) \cong N_{\ell_2} h_{\ell_2}^{(1)*}(i\chi_2 r_2) \tag{4}$$

where the decay constant $\chi_2$ is defined by

$$\frac{\hbar^2 \chi_2^2}{2m} = \mathcal{E}_2 \tag{5}$$

$\mathcal{E}_2$ being the separation energy. One can use the addition theorem

$$h_{\ell_2}^{(1)*}(i\chi_2 r_2) Y_{\ell_2\lambda_2}^{*}(\hat{r}_2) = \sqrt{4\pi} \sum_{\substack{\ell\lambda \\ \ell'\lambda'}} i^{\ell'-\ell-\ell_2} \frac{\hat{\ell}\,\hat{\ell}_2}{\hat{\ell}_1} \langle \ell\lambda\ell_2\lambda_2|\ell'\lambda'\rangle \tag{6}$$

$$\times \langle \ell o \ell_2|\ell' o\rangle j_{\ell'}^{*}(i\chi_2 r_1)\; Y_{\ell'\lambda'}^{*}(\hat{r}_1) h_{\ell}^{(1)*}(i\chi_2 r)$$

$$\times\; Y_{\ell\lambda}(r)$$

We now define what we refer to as weak binding. The integration over the $r_1$ is restricted by the range of the interaction $V(r_1)$. One would expect it to be of the order of the radius of the projectile. We use the criterion

$$\chi_2 R_1 < 1, \tag{7}$$

where $R_1$ is the radius of the projectile, to define weak binding. If eq. (7) is satisfied, one can verify that in eq. (8), the spherical Bessel functions satisfy the condition

$$j_{\ell'}^{*}(i\chi_2 r_1) \simeq \delta_{\ell' o} \tag{8}$$

$$h_{\ell_2}^{(1)*}(i\chi_2 r_2)\, Y_{\ell_2\lambda_2}^{*}(\hat{r}_2) = (-1)^{\ell_2}\, h_{\ell_2}^{(1)*}(i\chi_2 r) Y_{\ell_2\lambda_2}^{*}(\hat{r}) \tag{9}$$

eq. (9) implies that under the weak binding condition, eq. (7), the wave function of the nucleon in the final nucleus has no component in the direction of $r_1$, and that in the no-recoil approximation the transfer amplitude would identically vanish or be extremely small*. This is a particular case where the reaction proceeds entirely through the effect of recoil.

At high energies, where one could expect the diffraction model to be valid, the integrals considerably simplify. We use the model of Dodd and Greider [8] for simplicity. The elastic scattering wave function is described by

$$\chi^{(+)}(\vec{k}_i,\vec{r}_i) = \exp(i\vec{k}_i,\vec{r}_i)\ \Theta\ (r_i) \tag{10}$$

Where $\Theta(r_i)$ vanishes in the region of overlap of the ions and in the shadow region. With the use of eq. (9) and (10) the integral in eq. (3) becomes

$$(-1)^{\ell_2} N_{\ell_2}^2 \int d\vec{r}_1\, e^{-ik_R\cdot\vec{r}_1}\, V(r_1) U_{\ell_1}(r_1) \int d\vec{r}\, e^{i\vec{q}\cdot\vec{r}}\, h_{\ell_2}^{(1)*}(i\chi_2 r) Y_{\ell_2\lambda_2}^{*}(r) \times\ \Theta\ (r) \tag{11}$$

Where

$$\vec{q} = \vec{k}_i - \frac{M_{c_2}}{M_{a_2}} \vec{k}_f \tag{12a}$$

and

$$\vec{k}_R = m\left(\frac{\vec{k}_i}{M_{a_1}} + \frac{\vec{k}_f}{M_{a_2}}\right) \tag{12b}$$

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*The integral over $\vec{r}_1$ will identically vanish if the nucleon is not in a s-state in the projectile.

It can be verified that $\vec{k}_R$ is the recoil momentum, and the first integral in eq. (11) is the Fourier transform of the product of the potential and the projectile wave function. The differential cross section becomes

$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_t}{(2\pi\hbar^2)^2} \frac{k_t}{k_i} \frac{(2a_1+1)}{(2c_2+1)} \times \frac{(4\pi)}{(2S+1)(2\ell_2+1)} \Theta_{\ell_1 j_1} \Theta_{\ell_2 j_2} (N_{\ell_2})^2$$
$$\times |G_{\ell_1}(k_R)|^2 \sum_{\lambda_2} |\beta_{\ell_2\lambda_2}|^2 \qquad (13)$$

Where

$$G_{\ell_1}(k_R) = \int_0^\infty r_1^2 dr_1 j_{\ell_1}(k_R r_1)\, V(r_1)\, U_{\ell_1}(r_1) \qquad (14a)$$

and

$$\beta_{\ell_2\lambda_2} = \int d^3r\, e^{i\vec{q}\cdot\vec{r}}\, h_{\ell_2}^{(1)*}(i\chi_2 r) Y_{\ell_2\lambda_2}(\hat{r})\, \theta\,(r) \qquad (14b)$$

Eq. (13) is valid for transfer of particles with intrinsic spin of 1/2 or 0. The factorization of the cross-section into the two terms $G_{\ell_1}(k_R)$ and $\beta_{\ell_2\lambda_2}$ is characteristic of a reaction of the type (p,2p). If the final binding energy is small, the final channel behaves like a three body channel and the result in eq. (13) is not surprising.

The above treatment can also be applied to reactions where the transferred particle is in a resonant state in the final system. An example of this type would be one where one the two ions in the final system is $^8$Be, which is composed of two alpha particles in a s-wave resonance at about 90 kev above the threshold. The wave function of the particle will then be of the form

$$U_{\ell_2}(r_2) \simeq N_{\ell_2} \frac{\sin(k_2 r_2 + \delta)}{k_2 r_2} \tag{15}$$

which satisfies an addition theorem similar to eq. (6), and the final result would be identical.

In order to obtain the simple result of Eq. (13), we had ignored the dependendence of the distorted wave integral, $\beta_{\ell_2\lambda_2}$ on $\chi_2$. If the transfer process is assumed to be peripheral, the distorted wave integral is dependant upon $\chi_2$ approximately as $\frac{1}{(\chi_2 R)^{\ell+1}}$ where $\ell$ is the angular momentum transfer and R is the sum of radii of the ions. The dependence of the nuclear overlap integral on $\chi_2$ on the other hand can be approximated as $(\chi_2 R_1)^{\ell'}$. Hence, the contribution from the higher order term will be of the order of $\frac{R_1}{R}$ of the leading term calculated in Eq. (11). If the target is heavy in comparison with the projectile, the ratio $\frac{R_1}{R}$ is likely to be small. The feature of factorization of the differential cross section expressed by Eq. (13) would result if the masses of the projectile and target are very different and if the Q of the reaction is close to the optimum value. The latter condition is necessary if one assumes the reaction to be peripheral.

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