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John Clarke

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FUNDAMENTAL LIMITS ON SQUID TECHNOLOGY

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I. INTRODUCTION

In the last few years, the flux noise energy of the dc Superconducting Quantum Interference Device (SQUID) (Jaklevic et al., 1964) has been lowered by about four orders of magnitude. The most sensitive SQUID yet made is close to the limits set by fundamental quantum processes. The purpose of this article is to review these developments, and, in particular, to discuss at some length the limiting quantum processes in both single junctions and SQUIDs.

The article begins with a very brief review of flux quantization and the Josephson effects (Josephson, 1962) which includes a discussion of the resistively shunted junction (RSJ) in the presence of thermal noise. Section III outlines the theory of quantum noise in a single resistively shunted junction, and describes experiments to test the theory. Section IV is concerned with the dc SQUID, and starts out with a general overview of its properties and operation. This is followed by discussions of thermal and quantum noise, including both theory and experiment. The next section describes the use of a SQUID as an amplifier, and, in particular, the quantum noise limit. The importance of magnetic coupling is outlined. The following section deals with the subject of $1/f$ noise, and the restrictions that it places on the design of SQUIDs for low frequency applications. Section IV concludes with a brief discussion of two potential applications of quantum-limited SQUID amplifiers. Section V contains a summary.

In this article, we shall focus rather narrowly on the intrinsic noise properties of SQUIDS, and not address at all the role of the flux-locked loop that imposes limitations on such important parameters as slewing rate, frequency response, and dynamic range. We shall also ignore the problem of long term drift in SQUIDS, which is a little-understood problem of considerable importance in certain applications such as gradiometers.

II. SUPERCONDUCTIVITY AND THE JOSEPHSON EFFECT

According to the microscopic theory of Bardeen, Cooper, and Schrieffer (1957), the zero resistance of superconductors arises from the existence of Cooper pairs of electrons. These pairs are in a macroscopic quantum state that can be described by a single wave function $\Psi(\vec{r}, t) = |\Psi(\vec{r}, t)| e^{i\phi(\vec{r}, t)}$, where $\phi(\vec{r}, t)$ is the phase. In a closed loop of superconductor, the requirement that $\Psi(\vec{r}, t)$ be single-valued leads to the concept of flux quantization (London, 1951): The magnetic flux contained in the loop can take only the values

$$\Phi = n\Phi_0 \quad (n = 0, 1, 2 \dots), \quad (2.1)$$

where

$$\Phi_0 = h/2e \approx 2.07 \times 10^{-15} \text{ Wb} \quad (2.2)$$

is the flux quantum.

The Josephson effect (Josephson, 1962) involves the tunneling of Cooper pairs through an insulating barrier separating two superconductors. The supercurrent, I_s , flows through the barrier according to Josephson's current-phase relation

$$I_s = I_0 \sin \delta, \quad (2.3)$$

where $\delta = \phi_1 - \phi_2$ is the difference between the phases ϕ_1 and ϕ_2 of the two superconductors. The critical current of the junction, that is, the maximum supercurrent it can sustain, is I_0 . For currents less than I_0 , the phase difference is time independent, and the voltage across the junction is zero. On the other hand, when the applied current exceeds I_0 , there is a non-zero voltage, V , across the junction, and ϕ evolves with time according to Josephson's voltage-frequency relation

$$\dot{\delta} \equiv \omega = 2eV/\hbar = 2\pi V/\Phi_0. \quad (2.4)$$

($\dot{\delta} \equiv d\delta/dt$, where t is time.) Thus, the supercurrent oscillates at a frequency $\nu = \omega/2\pi$.

The current-voltage ($I - V$) characteristic of a Josephson tunnel junction is, in general, hysteretic, a property that is most undesirable for SQUIDS. The hysteresis can be eliminated by shunting the junction with an appropriate conductance (McCumber, 1968; Stewart, 1968). Hence, it is useful to review the properties of the resistively shunted junction (RSJ) model at this point.

The model, shown in Fig. 1, consists of a tunneling element with critical current I_0 in parallel with the self-capacitance of the junction, C , and shunted with an external resistance, R . A current noise source $I_N(t)$ is associated with R , and the applied current is I . The equation of motion is

$$V/R + I_0 \sin \delta + C \dot{V} = I + I_N(t). \quad (2.5)$$

Neglecting the noise term for the moment and setting $V = \hbar \dot{\delta}/2e$, we obtain

$$\frac{\hbar C}{2e} \ddot{\delta} + \frac{\hbar}{2eR} \dot{\delta} = I - I_0 \sin \delta = - \frac{2e}{\hbar} \frac{\partial U}{\partial \delta}, \quad (2.6)$$

where

$$U = - \frac{\Phi_0}{2\pi} (I\delta + I_0 \cos \delta). \quad (2.7)$$

The potential, U , is that of a "tilted washboard", and Eq. (2.6) describes the motion of a ball moving on the washboard. The term $\hbar C/2e$ represents the mass of the particle, while $\hbar/2eR$ represents the damping. Figure 2(a) shows the particle in the free-running mode $I > I_0$.

It is sometime convenient to rewrite Eq. (2.6) in the dimensionless form

$$\beta_C \ddot{\delta} + \dot{\delta} = i - \sin \delta, \quad (2.8)$$

Fig. 1. Resistively shunted junction model.

Fig. 2. The washboard model: (a) particle running down noise-free potential for the case $I > I_0$; (b) the case $I < I_0$ in the presence of noise — solid curve represents the mean potential, dashed curve represents instantaneous potential due to fluctuation.

where $\beta_c \equiv 2\pi I_0 R^2 C / \phi_0$, $i = I / I_0$, and time is in units of $\phi_0 / 2\pi I_0 R$. We can now consider two limiting cases. When $\beta_c \gg 1$ (heavy mass) the $I - V$ characteristic is hysteretic. As one increases the current from zero, the tilt increases until at $I = I_0$ (in the noise-free case) the ball rolls out of its initial potential minimum and starts to roll down the washboard. If one now reduces the current to below I_0 , the kinetic energy of the ball is sufficient to carry it over successive potential maxima: Thus, the junction remains at a non-zero voltage. One must reduce I to a value (depending on β_c) below I_0 to induce a transition to the zero voltage state. This mechanism is the source of hysteresis. On the other hand, in the overdamped limit $\beta_c \ll 1$, the term in δ in Eq. (2.8) is relatively small. When one increases I to above I_0 and then reduces it to just below I_0 , the damping is sufficient to bring the particle to rest, so that the junction immediately returns to zero voltage. Thus, there is no hysteresis. The cross-over from hysteretic to non-hysteretic behavior occurs at $\beta_c \sim 1$, the exact value depending on the level of noise. For the case $C = 0$, Eq. (2.6) can be solved exactly for the average voltage V :

$$V = R(I^2 - I_0^2)^{1/2}. \quad (\beta_c = 0) \quad (2.9)$$

The washboard model enables us to understand the influence of noise on the junction. We rewrite Eq. (2.5) as

$$\frac{\hbar C}{2e} \ddot{\delta} + \frac{\hbar}{2eR} \dot{\delta} + I_0 \sin \delta = I + I_N(t); \quad (2.10)$$

this is an example of a Langevin equation. In the thermal noise limit, the Nyquist noise current has the spectral density

$$S_I(\nu) = 4k_B T / R. \quad (2.11)$$

It is evident that $I_N(t)$ causes the tilt in the washboard to fluctuate.

ate with time. This fluctuation has two effects on the junction. First, when I is less than I_0 , from time to time fluctuations cause the ball to roll out of one potential minimum into the next. For the underdamped junction, this process induces a series of voltage pulses randomly spaced in time. Thus, the time average of the voltage is non-zero even though $I < I_0$, and the $I - V$ characteristic is "noise rounded" (Ambegaokar and Halperin, 1969). The second effect of the noise is to induce a voltage noise when the junction is biased with a fixed current. In the limit $\beta_c \ll 1$, for $I > I_0$, and when the frequency, ν_m , at which the noise is measured is much less than the Josephson frequency, ν_J , the spectral density of the voltage noise is (Likharev and Semenov, 1972; Vystavkin et al., 1974)

$$S_V(\nu_m) = \left[\frac{4k_B T}{R} + \frac{1}{2} \left(\frac{I_0}{I} \right)^2 \frac{4k_B T}{R} \right] R_D^2. \quad \begin{cases} \beta_c \ll 1 \\ I > I_0 \\ \nu_m \ll \nu_J \end{cases} \quad (2.12)$$

In Eq. (2.12), $R_D \equiv \partial V / \partial I$ is the dynamic resistance. The two terms on the right hand side of Eq. (2.12) are illustrated in Fig. 3. The first term, $(4k_B T / R) R_D^2$, represents Nyquist current noise generated at the measurement frequency ν_m flowing through a dynamic resistance R_D to produce a voltage noise. The second term, $\frac{1}{2} (I_0 / I)^2 (4k_B T / R) R_D^2$, represents Nyquist noise at frequencies $\nu_J \pm \nu_m$ mixed down to the measurement frequency by the Josephson oscillations. The factor $\frac{1}{2} (I_0 / I)^2$ is the mixing coefficient, which vanishes for sufficiently large bias currents. The mixing coefficients for noise near harmonics of the Josephson frequency, $2\nu_J$, $3\nu_J$..., are negligible in the limit $\nu_m / \nu_J \ll 1$.

This concludes our introductory review of the Josephson effect. We now turn to a discussion of quantum noise in a single junction.

Fig. 3. Schematic representation of the terms in Eq. (2.12). The Nyquist noise generated in the resistor at ν_m contributes at ν_m ; that generated at $\nu_J \pm \nu_m$ is mixed down to ν_m ; the mixing coefficients for the noise generated near $2\nu_J$ and higher harmonics are negligible for $\nu_m / \nu_J \ll 1$.

III. QUANTUM NOISE IN SINGLE JUNCTIONS

A. Theory

Quantum effects become important when $h\nu_J \gtrsim k_B T$, because Eq. (2.11), the Nyquist formula, then breaks down. Koch et al. (1980) suggested that quantum effects could be taken into account by retaining Eq. (2.10), but using for the spectral density of $I_N(t)$ the Callen-Welton (1951) formula

$$S_I(\nu) = \frac{4h\nu}{R} \left[\frac{1}{e^{h\nu/k_B T} - 1} + \frac{1}{2} \right] \equiv \frac{2h\nu}{R} \coth \left(\frac{h\nu}{2k_B T} \right). \quad (3.1)$$

Equation (3.1) has the limiting forms

$$S_I(\nu) \rightarrow \begin{cases} 4k_B T/R & (h\nu \ll k_B T) \\ 2h\nu/R & (h\nu \gg k_B T) \end{cases} \quad (3.2)$$

$$S_I(\nu) \rightarrow \begin{cases} 4k_B T/R & (h\nu \ll k_B T) \\ 2h\nu/R & (h\nu \gg k_B T) \end{cases} \quad (3.3)$$

In the case of Eq. (3.3), the noise arises from zero point fluctuations. When Eq. (3.1) describes the noise current in Eq. (2.10), the latter is an example of a quantum Langevin equation.

Following the procedure of Likharev and Semenov (1972), Koch et al. showed that in the limits $\beta_c \ll 1$, $I > I_0$, and $\nu_m \ll \nu_J$, the spectral density of the voltage noise for a current biased junction is

$$S_V(\nu_m) = \left[\frac{4k_B T}{R} + \frac{2eV}{R} \left(\frac{I_0}{I} \right)^2 \coth \left(\frac{eV}{k_B T} \right) \right] R_D^2. \quad \left\{ \begin{array}{l} \beta_c \ll 1 \\ I > I_0 \\ \nu_m \ll \nu_J \end{array} \right\} \quad (3.4)$$

Equation (3.4) is illustrated in Fig. 4. It has been assumed that

Fig. 4. Schematic representation of the terms in Eq. (3.4). The Nyquist noise generated in the resistor at ν_m contributes at ν_m ; the noise (including the contribution from the zero point term) generated at $\nu_J \pm \nu_m$ is mixed down to ν_m ; mixing coefficients for the noise generated near $2\nu_J$ and higher harmonics are negligible for $\nu_m/\nu_J \ll 1$.

$h\nu_m \ll k_B T$ so that the first term on the right hand side is in the thermal limit. Even though $S_I(\nu)$ increases with frequency at high frequencies, the contribution from noise near $2\nu_J, 3\nu_J \dots$ is of order $(\nu_m/\nu_J)^2$, and hence negligible in the limit $\nu_m \ll \nu_J$. In the limit $eV = h\nu_J/2 \ll k_B T$, we can set $\coth(eV/k_B T) = k_B T/eV$, and Eq. (3.4) reverts to Eq. (2.12).

It is of interest to consider the conditions under which quantum corrections may be observable. Clearly, we require $eV \gtrsim k_B T$; however, this condition is not sufficient because if one simply increases $eV/k_B T$ by increasing the bias current, the mixing coefficient will become very small, and the second term on the right hand side of Eq. (3.4) will be negligible compared with the first. To ensure that the second term is both in the quantum limit ($eV/k_B T \gg 1$) and larger than the first term, we require $eV \gg k_B T(I/I_0)^2$. With the aid of Eq. (2.9), it is easy to show that this condition is satisfied for some intermediate range of voltages provided

$$\kappa \equiv eI_0 R/k_B T \gg 1. \quad (3.5)$$

In the extreme quantum limit, Eq. (3.4) reduces to

$$S_V(\nu_m) = \frac{2eV}{R} \left(\frac{I_0}{I} \right)^2 R_D^2, \quad (3.6)$$

and the noise arises solely from zero point fluctuations mixed down from near the Josephson frequency.

Figure 5 shows $S_V(\nu_m)/4k_B TR$ vs I/I_0 . For $\kappa = 0.1$, the curve is almost identical with that predicted by the Likharev-Semenov result, Eq. (2.12). As κ is increased, the spectral density of the noise at intermediate currents progressively increases: This is the region in which quantum effects are observable. At high currents, all the curves tend asymptotically to unity, since the mixing coefficient tends to zero. At low currents, as $I \rightarrow I_0$ the curves also tend to the value predicted by Eq. (2.12). Notice, however, that the apparent divergence of the noise in this limit arises from the neglect of noise rounding in the theory, which implies that $R_D \rightarrow \infty$ as $I \rightarrow I_0$. In reality, noise rounding ensures that R_D remains finite so that as the current is lowered towards I_0 , the noise goes through a maximum and then decreases.

We conclude this section with some comments on the validity of the quantum Langevin equation. First, we note that although we have put in quantum corrections to the noise in Eq. (2.10), the terms in ϕ remain firmly classical; thus, the treatment is "semi-classical". It is well known that such a treatment is correct for a simple harmonic oscillator (for example, Louisell, 1973), but one must use caution in assuming that it is valid for any non-linear equation.

Another prediction of the quantum Langevin equation is "quantum

Fig. 5. Calculated low frequency spectral density of the voltage noise across a single resistively shunted junction vs. bias current for 5 values of $\kappa \equiv eI_0 R/k_B T$ and with $\beta_c \ll 1$ (Koch et al., 1980).

noise rounding". Thus, even at $T = 0$, for $I < I_0$ zero point fluctuations induce fluctuations in the tilt of the washboard, thereby enabling the particle to roll from one potential minimum to the next. By analogy with the thermal case, one might call this process "quantum activation". However, there has recently been much interest in an alternative process by which the junction may decay out of the zero voltage state, namely "macroscopic quantum tunneling" (MQT) (for example, Ivanchenko and Zil'berman, 1968; Leggett, 1978; Caldeira and Leggett, 1981, 1982; Amgegaokar et al., 1982; Voss and Webb, 1981, 1981a, Jackel et al., 1981). In this approach, one allows for the fact that the "particle" must actually be described by a quantum mechanical wave packet with a non-zero width, so that the particle can tunnel between potential wells when $I < I_0$. Thus, as the bias current is increased from zero, the junction will make a transition out of the zero voltage state before I reaches the critical current. If the junction is overdamped, this process will produce rounding of the $I - V$ characteristic at low voltages. Caldeira and Leggett (1981, 1982) have shown that the tunneling probability decreases exponentially from the WKB value as the damping of the junction is increased from zero (i.e. as β_c is reduced from infinity). The increase in damping causes the wave packet to narrow, thereby decreasing the tunneling probability. In the limit where the tunneling probability is small, and where the damping is not too large, the quantum Langevin

equation predicts a substantially higher escape rate than MQT (Koch, 1981; Voss and Webb, 1981a). Furthermore, for relatively underdamped junctions, Voss and Webb (1981) found experimentally that the probability of decay out of the zero voltage state appears to be much lower than that predicted by Caldeira and Leggett. Thus, in this limit, the activation rate predicted by the quantum Langevin equation is in serious disagreement with the experimental data. This discrepancy is hardly surprising: In the limit in which quantum mechanical broadening of the wave packet is substantial, the picture of a point particle moving on a washboard cannot be correct. Nevertheless, the quantum Langevin approach may still be appropriate for overdamped junctions ($\beta_c < 1$) when I is not too much less than I_0 , so that the probability of transitions from the zero voltage state is relatively high. It is clear that a more detailed investigation of the relationship between macroscopic quantum tunneling and quantum activation in the region $I \lesssim I_0$ is required. In this context, Schmid (1982) has shown that the quantum Langevin approach should be valid for $R \ll \hbar/4e^2 \approx 10^3 \Omega$. One would also like to have a clear-cut criterion for the limit in which one can neglect the quantum mechanical broadening of the wave packet.

For the remainder of this article we shall be concerned only with the limit $I > I_0$ in which the junction is free running, and MQT is irrelevant. Thus, our next task is to investigate the validity of Eq. (3.4) experimentally.

B. Experiment

Experiments to observe quantum corrections to the noise in resistively shunted junctions have been carried out by Koch et al. (1981a, 1982). To observe quantum noise effects, one requires junctions with $\kappa \gtrsim 1$. Writing $\kappa = (e/k_B T)(\beta_c \Phi_0 j_1 / 2\pi c)^{1/2}$, where j_1 is the critical current density and c is the capacitance per unit area, we see that junctions with high critical current densities are necessary to observe these effects in the liquid He^4 temperature range. At 4.2K, with $\beta_c = 0.2$, $j_1 = 10^4 \text{ A cm}^{-2}$ and $c = 0.04 \text{ pF } \mu\text{m}^{-2}$ we find $\kappa \approx 1.1$. This is a convenient value of κ , since, as the temperature is lowered to near 1K, κ increases so that quantum effects become dominant.

Koch et al. fabricated $\text{PbIn-In}_2\text{O}_3\text{-Pb}$ tunnel junctions, resistively shunted with CuAl films, on glass substrates using the photolithographic lift-off techniques described by workers at IBM (Greiner et al., 1980). The configuration is shown in Fig. 6. First, a 10 μm -wide Cu (0-3 wt.% Al) film 40 to 100 nm thick was deposited, followed by a 10 μm -wide, 250 nm-thick Pb (20 wt.% In) film at right angles to the CuAl strip. After another resist patterning, a SiO_2 oxide layer, 100 nm thick, was deposited and two windows were opened by lifting off the SiO_2 to expose the PbIn and CuAl films. After patterning the resist for the upper electrode, the exposed metal surfaces were cleaned by rf sputter-etching in Ar , the In_2O_3 oxide was

Fig. 6. Configuration of resistively shunted tunnel junction used in quantum noise measurements (Koch et al., 1982).

grown thermally in a low pressure of oxygen, and the 400 nm-thick Pb counter-electrode was deposited and lifted off. A final protective layer of SiO₂ was then evaporated. The diameter of the junction was about 2.5 μm, and the critical current ranged from 0.1 to 2 mA (0.2 to 4×10^4 A cm⁻²) at 4.2K, depending on the oxidation parameters. The capacitance of the junction was estimated to be 0.5 pF. The resistive shunt was about 5 μm long and ranged in resistance from 0.05 to 0.7Ω, depending on the thickness and composition of the CuAl. The Pb counterelectrode formed a ground plane for the shunt, reducing its inductance, L_s , to about 0.2 pH. Leads were attached to the junctions with pressed In pellets. Junctions fabricated with these techniques omitting the resistive shunts displayed excellent tunneling characteristics with little excess current at voltages below the sum of the gaps.

The apparatus for measuring the noise is shown schematically in Fig. 7. The junction was connected across two cooled LC-resonant tank circuits, either of which could be coupled to a low-noise pre-amplifier. By connecting together the tank circuit leads at the top of the cryostat a third, intermediate frequency could be obtained;

Fig. 7. Schematic of apparatus for measuring noise across a junction; the components enclosed in the dashed line are immersed in liquid helium (Koch et al., 1982).

the three resonant frequencies were 183 kHz, 106 kHz, and 70 kHz. The output from the preamplifier was mixed down to frequencies below 500 Hz, and the spectral density was then measured with a computer. The junction was enclosed in a superconducting can, and the cryostat was surrounded by a mu-metal shield. The cryostat, bias supply and preamplifier were enclosed in a shielded room. The gain of the preamplifier-mixer-computer chain was calibrated frequently against the Nyquist noise in a resistor (R_C) at room temperature. The accuracy of the gain was estimated to be $\pm 2\%$.

The spectral density of the voltage noise produced by the junction across the tank circuit was $Q^2 S_V(\nu_m) = \omega_m^2 L_T^2 [S_V(\nu_m)/R_D^2]$, where L_T is the inductance. Thus the quantity

$$\frac{S_V(\nu_m)}{R_D^2} = \frac{4k_B T}{R} + \frac{2eV}{R} \left(\frac{I_o}{I} \right)^2 \coth\left(\frac{eV}{k_B T} \right) \begin{cases} \beta_c \ll 1 \\ I > I_o \\ \nu_m \ll \nu_J \end{cases} \quad (3.7)$$

can be measured without knowing the Q of the tank circuit. In fact, there is a considerable advantage in comparing Eq. (3.7) rather than Eq. (3.4) with experiment. Real junctions may deviate from the simple RSJ model with $\beta_c = 0$ because the capacitance of junction, and, in some cases, the inductance of the shunt are non-negligible. Computer simulations, however indicate that the major effect of the deviations is to modify R_D , leaving the current noise terms on the right hand side of Eq. (3.7) relatively unchanged (Voss, 1981; Koch, 1981). Thus, for a real junction, the measured value of $S_V(\nu_m)$ may differ significantly from Eq. (3.4) while $S_V(\nu_m)/R_D^2$ is in good agreement with Eq. (3.7).

It was necessary to consider the following sources of error:

(i) The contributions of the measured voltage and current noises of the preamplifier were subtracted. The errors introduced in the spectral density of the junction noise ranged from about 5% at 4.2K to 15% at 1.6K.

(ii) Losses in the tank circuit (for example, due to the presence of stray resistance) were important only at the intermediate frequency. In this case, parts of the leads were at room temperature. The error after this noise contribution had been subtracted was 5%.

(iii) From the measurements at the three frequencies it was found that some junctions had a small $1/f$ noise component. The spectral density of this voltage noise was proportional to $(\partial V/\partial I_o)^2$, suggesting that the noise arose from fluctuations in the critical current (Clarke and Hawkins, 1976). The error introduced after the $1/f$ noise was subtracted was no more than 3%.

(iv) The noise measurements were made at voltages well below the

sum of the gaps of the superconductors. The small noise contribution from the quasiparticle current in this region was at most 1% of the current noise from the shunt resistor, and was neglected.

(v) The power dissipation in the resistive shunt raised its temperature above the bath temperature at higher bias voltages. The heating effect was determined as a function of temperature by reducing the critical current to zero with a magnetic field and measuring the Nyquist noise of the shunt for various bias currents. For the junction described in this article, the heating was significant only at high voltages ($V \gg k_B T/e$) where the term $\coth(eV/k_B T)$ in Eq. (3.7) is nearly independent of temperature. Thus, it was sufficient to correct only the term $4k_B T/R$ in Eq. (3.7) by subtracting $4k_B \Delta T/R$ from the measured noise, where ΔT is the temperature rise. The error was no more than 3%.

(vi) Considerable care was taken to shield the experiment from extraneous noise sources, such as 60 Hz pick-up, radio and television stations, and interference from nearby computers and lasers. To demonstrate that this shielding was sufficient, it was shown that the noise measured in resistors in the range 1.5 to 4.2K was within 3% of the predicted value. Furthermore, as we shall see, measurements on junctions in the classical limit $eV \ll k_B T$ were in excellent agreement with theory, and showed the correct temperature dependence.

We turn now to a discussion of the experimental results. We shall confine our discussion to a single junction: Results from other junctions can be found in the paper by Koch et al. (1982). The relevant parameters are listed in the caption to Fig. 8; in addition, there was an inductance associated with the shunt resistor

Fig. 8. I and R_D vs. V for resistively shunted junction at 4.2K with $I_0 = 0.51$ mA, $R = 0.70 \Omega$ (at $100 \mu V$ — R was somewhat voltage-dependent), $C = 0.5$ pF, $\beta_c = 0.38$, and $\kappa = 0.99$ (Koch et al., 1982).

of about 0.2pH.

Figure 8 shows I and R_D vs. V at 4.2K for this junction. There is a small dip in R_D at 800 μ V that is probably due to a self-resonance of the junction capacitance and shunt inductance, and some fine structure around 200 μ V of unknown origin. However, as mentioned earlier, small deviations in R_D from the prediction of the RSJ model are not expected to affect $S_V(v_m)/R_D^2$. In Fig. 9 we plot measured values of $S_V(v_m)/R_D^2$ vs. voltage (open circles) after the preamplifier noise has been subtracted. The solid circles are the noise after the 1/f noise subtraction and the heating correction have been made. At low voltages the correction is entirely due to 1/f noise, while at high voltages, the correction is largely due to heating. The solid line through the solid circles is the prediction of Eq. (3.7) using the measured values of R , I_0 , I , V , and T . The upper dashed line is the predicted noise in the absence of zero point fluctuations, that is

$$\frac{S'_V(v_m)}{R_D^2} = \frac{4k_B T}{R} + \frac{4eV}{R} \left(\frac{I_0}{I} \right)^2 \frac{1}{\exp(2eV/k_B T) - 1}. \quad (3.8)$$

The triangles in Fig. 9 represent the measured mixed-down noise, which was computed by subtracting $4k_B T/R$ from the solid circles. The solid line through the triangles is the mixed-down noise predicted by Eq. (3.7), $(2eV/R)(I_0/I)^2 \coth(eV/k_B T)$, while the lower dashed line is the mixed-down noise predicted by Eq. (3.8) in the absence of zero point fluctuations, $(4eV/R)(I_0/I)^2 [\exp(2eV/k_B T) - 1]^{-1}$.

Fig. 9. $S_V(v_m)/R_D^2$ vs. V at 4.2K for junction shown in Fig. 8. The open circles show the total measured noise across the junction; solid circles below show the noise remaining after correction for 1/f noise and heating. Upper solid and dashed lines are predictions of Eqs. (3.7) and (3.8). Solid triangles are measured mixed-down noise, lower solid and dashed lines are mixed-down noise predicted by Eqs. (3.7) and (3.8) (Koch et al., 1982).

It is evident that both the total measured noise across the junction and the measured mixed-down noise are in excellent agreement with the theory that includes a contribution from the mixed-down zero point fluctuations, and are substantially higher than the predictions of a theory that does not include this contribution.

Figure 10 shows the temperature dependence of the noise for twelve bias voltages ranging from 50 μV to 550 μV . The temperature $T = 2eV/k_B$ is indicated for the six lowest voltages; mixed-down noise at temperatures well above $2eV/k_B$ is in the classical limit, while that at temperatures well below $2eV/k_B$ is in the quantum limit. The mixed-down noise at the six highest voltages is in the quantum limit at all temperatures measured. For all twelve voltages, the total junction noise is in good agreement with the predictions of Eq. (3.7), and substantially greater than the predictions of Eq. (3.8). (The

Fig. 10. $S_V(v_m)/R_D^2$ vs. T for junction at 12 bias voltages. Notation is as for Fig. 9. Arrows indicate $2eV = k_B T$ (Koch et al., 1982).

discrepancy for 300 μV probably arises from the structure shown in Fig. 8.) The mixed-down noise at 350 μV and above is independent of temperature, and in excellent agreement with the value of Eq. (3.6), $S_V(0)/R_D^2 = (2eV/R)(I_0/I)^2$. As the voltage is lowered the mixed-down noise becomes increasingly temperature dependent, and remains in good agreement with the predictions of Eq. (3.7). At 50 μV , the mixed-down noise is in the classical limit for the whole temperature range, and proportional to T , as expected. This temperature dependence demonstrates that the contribution of any extraneous noise was negligible.

We can extract from our data the measured spectral density of the current noise $S_I(\nu)$ generated by the shunt resistance R at the Josephson frequency $\nu = 2eV/h$. We divide each value of the mixed-down noise by the mixing coefficient $(I_0/I)^2/2$, a procedure that converts the mixed-down noise in Eq. (3.7) into Eq. (3.1). The results are plotted in Fig. 11 for two temperatures. The solid lines are the corresponding predictions of Eq. (3.1) using measured values of $\nu = 2eV/h$, R , and T . The slight increase of the data above the theory at the highest voltages may reflect the presence of a resonance on the I-V characteristic. The agreement between the data and the predic-

Fig. 11. Measured spectral density of current noise in shunt resistor of junction at 4.2K (solid circles) and 1.6K (open circles). Solid lines are prediction of Eq. (3.1), while dashed lines are $(4h\nu/R)[\exp(h\nu/k_B T) - 1]^{-1}$ (Koch et al., 1982).

tions is rather good, bearing in mind that no fitting parameters are used. By contrast, the dashed lines represent the theoretical prediction in the absence of the zero point term, $(4\hbar\nu/R)[\exp(\hbar\nu/k_B T) - 1]^{-1}$, and fall far below the data at the higher frequencies. The existence of zero point fluctuations in the measured spectral density of the current noise is rather convincingly demonstrated.

In experiments on other junctions Koch et al. (1982) found that the noise in a junction at a fixed temperature exhibited the correct dependence on $\kappa \equiv eI_0 R/k_B T$ when I_0 was changed by means of a magnetic field. Furthermore, they studied one junction in which the shunt inductance was anomalously large, and the dynamic resistance showed substantial structure at voltages for which the Josephson frequency was a subharmonic of the LC-resonant frequency. The measured noise, however, was in good agreement with a computer prediction in which these resonances were included. Because of the higher non-linearities of this junction, the mixing coefficients for noise at harmonics of the Josephson frequency ($2\nu_J$, $3\nu_J$...) are no longer negligible, and it is necessary to include these contributions in the computer simulation.

In concluding this section, it should be emphasized again that only measured parameters were used in comparing the data with theory in Figs. 9 to 11; there were no fitting parameters. Thus, the quantum Langevin equation accurately predicts the limiting voltage noise in a current-biased resistively shunted junction for $\beta_C < 1$ and $I > I_0$. The noise arises from zero point fluctuations mixed down from near the Josephson frequency. At the moment, there are no measurements of the noise in the noise rounded region $I < I_0$ — quantum effects are quite negligible in this region for the junctions studied in the He^4 temperature range.

Finally, the fact that zero point fluctuations can be observed at frequencies as high as 500 GHz (Fig. 11) implies that a Josephson mixer using its self-oscillations as the local oscillator is an ideal quantum-limited device at these frequencies. Unfortunately, a mixer in this mode is of limited practical use because the frequency of the oscillator has a rather large linewidth induced by the voltage fluctuations. Likharev and Semenov (1972) have shown that the rms linewidth, $\Delta\nu_J$, is related to the spectral density of the low frequency voltage noise by $\Delta\nu_J = \pi S_V(\nu_m)/\phi_0^2$. Using the quantum limited expression for $S_V(\nu_m)$, Eq. (3.6), we find $\Delta\nu_J = 2e\pi V(I_0/I)^2(R_D^2/R)/\phi_0^2$. With the aid of Eq. (2.9), we can write this in the form $\Delta\nu_J = 2e\pi R^3 I_0^2/\phi_0^2 V$. Taking $R = 1\Omega$, $I_0 = 1\text{mA}$, and $V = 1\text{mV}$, we find $\Delta\nu_J \approx 250\text{ MHz}$ for a frequency of 500 GHz, a linewidth that is undesirably large for many applications.

IV. DC SQUIDS

A. Introduction

The principal objective of this section on SQUIDS is to apply the ideas of quantum noise developed in Sec. III to the dc SQUID, and then to compare these results with experiment. Before becoming involved in the quantum limited case, however, it is convenient first to outline the general principles of the dc SQUID, and to discuss the thermal noise limit.

The dc SQUID consists of two Josephson junctions connected in parallel on a superconducting loop of inductance L as shown in Fig. 12(a). We assume that the junctions are identical and overdamped ($\beta_c < 1$) so that the I-V characteristic is non-hysteretic. When the flux, Φ , threading the SQUID loop is changed, the critical current and hence the I-V characteristic oscillate with period Φ_0 , as indicated in Fig. 12(b). If the SQUID is biased with a current greater than the maximum critical current, the voltage across the SQUID is periodic in the flux, as shown in Fig. 12(c). Thus, the SQUID is simply a flux-to-voltage transducer, with a transfer function $V_\Phi \equiv (\partial V / \partial \Phi)_I$ that is a maximum near $(2n + 1)\Phi_0/4$. In most applications, at least for low frequency measurements, the SQUID is operated in a flux-locked loop. An oscillating flux is applied to the SQUID and the resulting voltage across the SQUID is amplified by a cooled LC-resonant circuit or transformer and then by a low noise amplifier. The signal from the amplifier is lock-in detected, and the output from the lock-in is fed back into a coil coupled to the SQUID. Thus, feedback maintains a constant flux in the SQUID, and the dynamic range can be as high as $10^3 \Phi_0$ or more. Details of flux modulation schemes have appeared elsewhere (Clarke et al., 1976; Clarke, 1977, 1980).

The simplest characterization of the sensitivity of a SQUID is in terms of its equivalent flux noise, which has a spectral density

Fig. 12. (a) Configuration of dc SQUID; (b) current-voltage (I-V) characteristic with $\Phi = n\Phi_0$ and $(n + 1/2)\Phi_0$, where n is an integer; (c) V vs. Φ at constant bias current.

$$S_{\phi}(f) = S_V(f)/V_{\phi}^2. \quad (4.1)$$

In Eq. (4.1), $S_V(f)$ is the spectral density of the voltage noise across the SQUID. However, in order to compare SQUIDs with different inductances, it is more useful to define a flux noise energy per unit bandwidth

$$\frac{\epsilon(f)}{1\text{Hz}} = \frac{S_{\phi}(f)}{2L}. \quad (4.2)$$

As we shall see later, $\epsilon/1\text{Hz}$ is not a complete specification of the SQUID noise sources; however, it is a convenient measure for comparing different devices.

The first dc SQUID for which detailed noise measurements were made and compared with theory was the cylindrical device shown in Fig. 13(a) (Clarke et al., 1976). The spectral density of the equivalent flux noise measured in a flux-locked loop is shown in Fig. 13(b). The noise is nearly white at frequencies above 2×10^{-2} Hz; below this frequency the noise scales approximately as $1/f$. The roll-off at higher frequencies is due to a low-pass filter in the measurement circuit. In the white noise region, the equivalent flux noise is approximately $3.5 \times 10^{-5} \phi_0 \text{ Hz}^{-1/2}$; with an area of 7 mm^2 , this corresponds to a magnetic field sensitivity of about $10^{-14} \text{ THz}^{-1/2}$. The corresponding noise energy, with an estimated SQUID inductance of 1 nH , is about $3 \times 10^{-30} \text{ JHz}^{-1}$.

Fig. 13. (a) Configuration of cylindrical dc SQUID; (b) spectral density of SQUID flux noise, $S_{\phi}(f)$, for typical cylindrical SQUID (Clarke et al., 1976).

In the next section we compare these results with theoretical predictions, and see how subsequent devices have been fabricated with greatly improved noise energies.

B. Thermal Noise in the dc SQUID

A model for noise calculations in the dc SQUID is shown in Fig. 14. The current in the SQUID loop is $J(t)$. There are two independent noise currents, $I_{N1}(t)$ and $I_{N2}(t)$, associated with the two shunt resistors. The phase differences across the two tunnel junctions, δ_1 and δ_2 , obey the following equations (Tesche and Clarke, 1977; Koch et al., 1981)

$$V = \frac{\hbar}{2e} (\dot{\delta}_1 + \dot{\delta}_2), \quad (4.3)$$

$$J = \frac{\Phi_0}{2\pi L} \left(\delta_1 - \delta_2 - \frac{2\pi\Phi}{\Phi_0} \right), \quad (4.4)$$

$$\frac{\hbar C}{2e} \ddot{\delta}_1 + \frac{\hbar}{2eR} \dot{\delta}_1 = \frac{I}{2} - J - I_0 \sin \delta_1 + I_{N1}, \quad (4.5)$$

and

$$\frac{\hbar C}{2e} \ddot{\delta}_2 + \frac{\hbar}{2eR} \dot{\delta}_2 = \frac{I}{2} + J - I_0 \sin \delta_2 + I_{N2}. \quad (4.6)$$

Equation (4.3) relates the voltage to the average rate of change of phase, Eq. (4.4) relates the circulating current to $\delta_1 - \delta_2$ and to the applied flux Φ , while Eqs. (4.5) and (4.6) are two Langevin equations coupled via J . Tesche and Clarke (1977) solved the equations in the limit $C = 0$ with $\Gamma \equiv 2\pi k_B T / I_0 \Phi_0 = 0.05$ and with a spectral density $4k_B T / R$ for I_{N1} and I_{N2} . They computed V_ϕ and S_ϕ vs. I for different values of applied flux and for various values of $\beta = 2LI_0 / \Phi_0$. As in the case of the single junction, the noise was computed for frequencies much less than the Josephson frequency, and was found to have a white power spectrum in this region. To optimize the SQUID, they assumed the maximum value of β_c was unity, thereby deter-

Fig. 14. Model for noise calculations in the dc SQUID.

mining an upper limit on R for given values of C and I_0 . It should be noted, however, that the introduction of a non-zero capacitance in Eqs. (4.5) and (4.6) undoubtedly changes the computed values of V_ϕ and S_ϕ somewhat, so that the results must be regarded as approximate.

The conclusions reached were as follows. The SQUID is optimized when $\beta = 1$ and (somewhat arbitrarily) $\beta_c = 1$, and has its best performance when ϕ is near $(2n + 1)\phi_0/4$. The optimized value of V_ϕ is*

$$V_\phi \approx R/L \quad (4.7)$$

while the spectral density of the optimized voltage noise is

$$S_v \approx 16k_B TR. \quad (4.8)$$

The corresponding optimized flux noise energy is

$$\epsilon/1\text{Hz} \approx 9k_B TL/R. \quad (4.9)$$

It is convenient to eliminate R from Eq. (4.9) by setting $\beta_c = \beta = 1$ to obtain

$$\epsilon/1\text{Hz} \approx 16k_B T(LC)^{1/2}. \quad \left\{ \begin{array}{l} \beta = 1 \\ \beta_c = 1 \\ \Gamma = 0.05 \end{array} \right\} \quad (4.10)$$

The numerical coefficients in Eqs. (4.9) and (4.10) should be treated with suspicion because of the neglect of the capacitance in the calculations. Nevertheless, the result gives a clear prescription for reducing the flux noise energy of SQUIDs: One should reduce T , L , and/or C .

The performance of the cylindrical SQUID can be compared with Eq. (4.10). Taking $T = 4.2\text{K}$, $L \approx 1\text{nH}$, and $C \approx 1\text{nF}^+$, we find $\epsilon/1\text{Hz} \approx 10^{-30}\text{JHz}^{-1}$, a factor of 3 smaller than the observed value. We note, however, that we have neglected the effects of flux modulation, which increases the noise somewhat, by a factor that depends on the details of the modulation scheme. Thus, the agreement between theory and experiment is not unreasonable.

Since the prediction given in Eq. (4.10) appeared, many SQUIDs have been fabricated to test its validity. These have all been in a planar configuration; two examples appear in Secs. IV D and

*Bruines et al. (1982) have pointed out a numerical error of about 1.5 in the value of V_ϕ obtained by Tesche and Clarke (1977). The corrected values have been used here.

+The estimation of C is difficult. We shall rather arbitrarily take $0.04\text{pF } \mu\text{m}^{-2}$ for Pb-based junctions and $0.08\text{pF } \mu\text{m}^{-2}$ for Nb-based junctions in this article.

IV F. The main emphasis has been to reduce the inductance, and, particularly, the capacitance of the SQUIDS by means of photolithography or electron beam lithography. The performance of a selection of devices is summarized in Fig. 15, where the measured

Fig. 15. Measured values of $\epsilon/1\text{Hz}$ vs. $16k_B T(LC)^{1/2} = 9k_B T L/R$ for a number of dc SQUIDS involving thin film tunnel junctions (except f). The SQUIDS referred to are:

- a Clarke et al., 1976 (4.2K)
- b Koch and Clarke, 1979 (4.2K)
- c Voss et al., 1980 (4.2K, 1.6K)
- d Ketchen and Voss, 1979 (4.2K, 1.8K)
- e Voss et al., 1981 (4.2K, 1.5K)
- f Voss et al., 1981 (4.2K) - nanobridge
- g Cromar and Carelli, 1981 (4.2K)
- h Cromar and Carelli, 1981 (4.2K)
- i Ketchen and Jaycox, 1982 (4.2K, 1.5K)
- j Carelli and Foglietti, 1982 (4.2K)
- k Van Harlingen et al., 1982 (4.2K, 1.4K) - 1/f noise subtracted
- l Martinis and Clarke, 1982 (4.2K)
- m De Waal et al., 1981 (4.2K)

values of $\epsilon/1\text{Hz}$ are plotted against $16k_B T(LC)^{1/2} = 9k_B TL/R$. No distinction has been made among devices that were operated in a flux-locked loop and those that were operated as small-signal amplifiers with no feedback. It should also be noted that in some cases the value of Γ was considerably less than 0.05, so that Eq. (4.10) cannot be expected to hold exactly. Nevertheless, although there is a general scatter of the results, it is evident that the measured noise energies follow the predicted values rather well.

It is also apparent that the performance of the most sensitive devices approaches $\epsilon/1\text{Hz} \sim 10^{-34} \text{JHz}^{-1}$ or $\hbar$. One might expect quantum effects to be important in this range, and the following section is devoted to a discussion of quantum noise in SQUIDS.

C. Quantum Noise in the dc SQUID: Theory

Koch et al. (1981) have extended their calculations on quantum noise in single junctions to the case of the dc SQUID. They again solved Eqs. (4.3) to (4.6) on a computer, this time retaining the terms in C and using Eq. (3.1), $S_I(\nu) = (2\hbar\nu/R)\coth(\hbar\nu/2k_B T)$, for the spectral density of the noise currents. The computed values of V_ϕ , S_V , and $\epsilon/1\text{Hz}$ at frequencies much less than ν_J for a particular SQUID with low values of L and C are shown in Fig. 16(a), (b) and

Fig. 16. Values of $\partial V/\partial \Phi$, S_V , $\epsilon/1\text{Hz}$, S_J , S_{VJ} , and $n(s)$ vs. Φ/Φ_0 computed at frequencies much less than the Josephson frequency for a SQUID at $T = 0$ with $\kappa\Gamma = 2e^2 R/\hbar = 0.02$, $\beta = 1$, $I/I_0 = 1.63$, and for $\beta_c = 0.25, 0.5$, and 1 (Koch et al., 1981).

(c). The optimum value of $\epsilon/1\text{Hz}$ is

$$\epsilon/1\text{Hz} \approx \hbar, \quad (4.11)$$

and is determined by zero point fluctuations. It cannot be emphasized too strongly, however, that there is no exact minimum value of $\epsilon/1\text{Hz}$ determined by quantum mechanics, and that the values computed for SQUIDS with different parameters are likely to be scattered around $\hbar$ rather than exactly equal to $\hbar$.

Koch et al. (1981) extended their calculations to non-zero temperatures, and found that $\epsilon/1\text{Hz}$ increased to about $3\hbar$ at 4.2K for the parameters given in the caption of Fig. 16.

D. Quantum Noise in the dc SQUID: Experiment

To my knowledge, there have been three types of SQUIDS with noise energies within a factor of six of $\hbar$ (Cromar and Carelli, 1981; Voss et al., 1981; Van Harlingen et al., 1982). We briefly describe the most sensitive, that of Van Harlingen et al. (1982). The configuration of their SQUID is shown in Fig. 17 (inset), and is very similar to that of Cromar and Carelli (1981). Two Pb (20 wt.% In) - In_2O_3 - Pb tunnel junctions with a nominal diameter of $2\text{ }\mu\text{m}$ and a separation of $30\text{ }\mu\text{m}$ were defined by lifting off windows in a SiO_2 insulating layer. Two additional windows determined the length of a $10\text{ }\mu\text{m}$ -wide CuAl shunt resistance for each junction. The capacitance of each junction was estimated to be about 0.3pF , while the SQUID loop had an inductance of about 2pH . Flux was applied to the SQUID

Fig. 17. Measured optimized values of $\epsilon/1\text{Hz}$ vs. T at three frequencies for most sensitive SQUID of Van Harlingen et al. (1982). Inset shows configuration of SQUID.

by means of a current along the counter electrode.

The spectral density of the voltage noise, $S_V(f)$, was measured using the apparatus shown in Fig. 7. Values of V_ϕ were measured by applying a 1kHz-flux to the SQUID with an amplitude of typically $10^{-4} \phi_0$, and measuring the voltage across the SQUID with a lock-in detector. At each temperature, the flux noise $S_\phi(f) = S_V(f)/V_\phi^2$ was measured with the flux near $(2n+1)\phi_0/4$ and with V_ϕ maximized with respect to the bias current. The optimized values of $\epsilon/1\text{Hz}$ vs. T for the most sensitive SQUID are shown in Fig. 17 for three measurement frequencies. At each frequency, the noise energy is roughly linear in the temperature. The best measured value of $\epsilon/1\text{Hz}$ at 1.4K and 202kHz was $3.2\hbar \pm 0.2\hbar$, corresponding to an equivalent flux noise of $(1.7 \pm 0.1) \times 10^{-8} \phi_0 \text{Hz}^{-1/2}$. Clearly, there is a substantial frequency-dependent contribution to the noise. From measurements at the three frequencies, it was found that the spectral density of the total noise could be expressed as the sum of a white noise component and a $1/f$ noise component. The values of these components are listed in Table I. Unfortunately, the rather large relative magnitudes of the $1/f$ noise give rise to substantial error bars on the estimates of the white noise, and make it impossible to determine whether or not the noise tends to an asymptote at the lowest temperatures. The lowest extrapolated white noise energy (at 1.4K) was $(0.8 \pm 0.7)\hbar$. The best estimates of the white noise energy at 4.2K and 1.4K have been plotted in Fig. 15. Given the substantial errors, the measured values of the white noise energy are in reasonable agreement with the predictions (Koch et al., 1981): For a SQUID with $L \approx 2\text{pH}$, $C \approx 0.3\text{pF}$, $\beta_c \approx 1$, and $\beta \approx 1$, we expect $\epsilon/1\text{Hz}$ to be about $3\hbar$ at 4.2K, decreasing to a little more than $\hbar$ at 1K.

From the measurements, one can conclude that it is possible to operate SQUIDS with white noise energies approaching $\hbar$, but the theory has clearly not been tested as rigorously as in the case of

Table I. Performance of Ultra Low-Noise SQUID^a (at 4.2K, $L = 1.9 \text{ pH}$, $C \approx 0.3 \text{ pF}$, $I_0 = 0.55 \text{ mA}$, $R \approx 1.3\Omega$, $\beta = 1.0$, $\beta_c \approx 0.9$)

T	V_ϕ	R_D	$\epsilon/1\text{Hz}$	$\epsilon/1\text{Hz}$	$\epsilon/1\text{Hz}$
			202kHz	white	1/f (100kHz)
(K)	(mV/ ϕ_0)	(Ω)	($\hbar$)	($\hbar$)	($\hbar$)
4.2	6.8	4.7	14.4	3.6 ± 3.0	21.6 ± 4.7
2.8	9.4	6.4	8.6	2.8 ± 1.8	11.6 ± 2.8
1.8	12.5	8.9	4.5	1.5 ± 1.0	6.0 ± 1.5
1.4	13.9	11.1	3.2	0.8 ± 0.7	4.8 ± 1.0

^aVan Harlingen et al., 1982.

the single junctions reported in Sec. III. More accurate measurements with this type of SQUID would necessitate higher frequencies to avoid the errors introduced by subtracting the $1/f$ noise. Furthermore, the type of SQUID described here is of very low inductance so that it is difficult to couple it efficiently to an input coil of, say, $0.1 - 1 \mu\text{H}$ as is often required in typical applications. Other SQUIDS that have achieved noise energies below 20h suffer from the same disadvantage. A discussion of techniques for coupling to planar SQUIDS appears in Sec. IV F.

E. SQUID Amplifiers

Within the reasonably near future, it is likely that SQUIDS will be fabricated in which the white flux noise is determined by zero point fluctuations. However, it does not automatically follow that such a SQUID can be used as an amplifier in which the measurement of a signal applied to the input is quantum noise limited. The purpose of this section is to discuss the design of such amplifiers, and to show that quantum limited performance should in fact, be possible if the input circuit is optimized appropriately.

When a SQUID is inductively coupled to an input circuit, the current noise circulating in the SQUID loop inevitably induces a voltage noise in this circuit. This additional noise source implies that $\epsilon/1\text{Hz}$ is not a complete specification of the SQUID noise, as mentioned in Sec. IV A. The current noise has a spectral density, $S_J(f)$, that has been calculated by Tesche and Clarke (1979) in the thermal limit, and by Koch et al. (1981) in the quantum limit. The current noise is partially correlated with the voltage noise across the SQUID, some of which arises from the flux noise generated by the current noise. The resulting cross spectral density, $S_{VJ}(f)$, must also be taken into account. The matter is further complicated because the calculated values of V_ϕ , S_V , S_J , and S_{VJ} are for a bare SQUID, whereas the values for a SQUID coupled to an input circuit will be modified by the presence of that circuit. Unfortunately, corrections for this effect are by no means straightforward. At the signal frequency ($\ll \nu_J$) the model of an inductance coupled to the SQUID is an excellent approximation, but at the Josephson frequency (at least a few GHz) the model breaks down because of the presence of stray capacitance between the coil and the SQUID, and between the turns of the coil.* To my knowledge, there has not yet been a successful attempt to take these capacitances into account in model calculations. For the present purpose, we shall assume that the coupling between the coil and the SQUID is weak, so that we can use parameters computed for the bare SQUID. The calculations follow those

* Hilbert and Clarke (1982) have found that for a planar SQUID tightly coupled to a thin film spiral coil (Ketchen and Jaycox, 1982) the frequency response extends to about 200MHz; see Sec. IV H.

Fig. 18. Schematic of dc SQUID coupled to a tuned input circuit; V_i and V_o are the input and output voltages.

of Clarke et al. (1979) and Koch et al. (1981).*

We consider only the case of the tuned amplifier shown in Fig. 18. Here, L_i is the inductance coupled to the SQUID, with mutual inductance M_i , C_i and R_i are the series capacitance and resistance, and V_i and V_o are the input and output voltages. The total impedance of the input circuit is

$$Z_i \approx R_i + \omega L_i + \frac{1}{\omega C_i} + \frac{\omega^2 M_i^2}{Z_s}, \quad (4.12)$$

where Z_s is the SQUID impedance which consists of a resistive and inductive component in parallel (Koch, 1982). Since $M_i^2 = \alpha^2 L L_i$, and α^2 is small, we shall neglect the term $\omega^2 M_i^2 / Z_s$ for low signal frequencies, ω . The current noise, $J_N(t)$, in the SQUID induces a voltage source $-M_i J_N$ in the input circuit that generates a current $-M_i J_N / Z_i$ and hence a flux $-M_i^2 J_N / Z_i$ in the SQUID. Thus, the total voltage noise at the output of the SQUID is†

$$V'_N(t) = V_N(t) - M_i^2 J_N V_\phi / Z_i. \quad (4.13)$$

Computing the spectral densities from Eq. (4.13) we find the spectral density of the noise at the SQUID output:

$$S'_V(f) = S_V(f) + \frac{M_i^4 \omega^2 S_J(f) V_\phi^2}{|Z_i|^2} - \frac{2 M_i^2 \omega S_V(f) V_\phi (\omega L_i - 1/\omega C_i)}{|Z_i|^2}. \quad (4.14)$$

*Koch (1982) and Tesche (1982) have each calculated the modifications to the SQUID parameters assuming that the simple model of a capacitance-free inductance coupled to a SQUID is correct. Koch demonstrates explicitly that his results reduce to those given here in the weak-coupling limit.

†In Clarke et al. (1979), the sign of the term $M_i^2 J_N V_\phi / Z_i$ was incorrectly given as positive: This error was pointed out by Koch (1982). Fortunately, this error does not affect the results for the tuned amplifier described here.

The corresponding mean square signal is given by

$$\langle V_o^2 \rangle = M_i^2 V_\phi^2 \langle V_i^2 \rangle / |Z_i|^2, \quad (4.15)$$

and the signal-to-noise ratio in a bandwidth B is

$$\langle V_o^2 \rangle / S_v'(f) B. \quad (4.16)$$

In the thermal limit, we introduce a noise temperature, T_N , by setting $\langle V_o^2 \rangle / S_v'(f) B = 1$ with $\langle V_i^2 \rangle = 4k_B T_N R_i B$.

One can then optimize the noise temperature with respect to C_i and R_i , to find

$$T_N^{(opt)} = (\pi f / k_B V_\phi) (S_v S_J - S_{vJ}^2)^{1/2}. \quad (4.17)$$

We note that $T_N^{(opt)}$ scales with frequency, and otherwise depends only on the SQUID parameters. Also, $T_N^{(opt)}$ is independent of α^2 (although the optimum values of R_i and C_i are not), implying that tight coupling between the input circuit and the SQUID is not essential for optimum noise performance.

A comment on the relationship between $T_N^{(opt)}$ and $\epsilon/1\text{Hz}$ in the thermal limit is in order. In their simulation for an optimized SQUID with $\Gamma = 0.05$, Tesche and Clarke (1977, 1979) found $S_v \approx 16 k_B T R$, $S_J \approx 11 k_B T / R$, and $S_{vJ} \approx 12 k_B T$. Inserting these values into Eq. (4.17), one finds $T_N^{(opt)} \approx 18 f T / V_\phi$, or, with Eqs. (4.7) and (4.9), $T_N^{(opt)} \approx (2 f \epsilon / 1\text{Hz}) / k_B$. Thus, although it is true that $\epsilon/1\text{Hz}$ does not completely specify the noise sources in the SQUID, nevertheless, to the extent that one has faith in the computer model, it is clear that as one improves $\epsilon/1\text{Hz}$ one produces a corresponding improvement in $T_N^{(opt)}$.

We turn now to a consideration of the noise temperature in the quantum limit. The values of $S_v(f)$, $S_J(f)$, and $S_{vJ}(f)$ vs. flux for an optimized SQUID at $T = 0$ are shown in Fig. 16(b), (d), and (e) (Koch et al., 1981). These spectral densities are computed at frequencies much below the Josephson frequency, where the noise is white. Since the classical definition of T_N no longer holds, it is convenient to rewrite Eq. (4.17) in terms of $\langle V_i^2 \rangle$. We then define the quantity $n^{(s)} h f = \langle V_i^2 \rangle / 4 R_i B$ as the mean photon power per unit bandwidth in the input circuit due to SQUID noise. The quantity $n^{(s)}$ is the mean number of photons induced in the input circuit by the SQUID.* We thus find

$$n^{(s)} = \pi (S_v S_J - S_{vJ}^2)^{1/2} / h V_\phi. \quad (4.18)$$

*The quantity $n^{(s)}$ is identical to the quantity A ("added noise number") used subsequently by Caves (1982).

The variation of $n^{(s)}$ with flux according to Eq. (4.18) is shown in Fig. 16(f). The minimum value (Koch et al., 1981) is

$$n^{(s)} \approx \frac{1}{2}. \quad (T = 0) \quad (4.19)$$

The approximate equality arises from uncertainties in the computations. The minimum value for any linear phase preserving amplifier is exactly $\frac{1}{2}$. Inspection of Eq. (4.18) reveals that the establishment of a lower limit on $n^{(s)}$ does not impose a lower limit on S_V or, consequently, on $\epsilon/l\text{Hz}$. Thus, the exact value of S_V (and $\epsilon/l\text{Hz}$) when $n^{(s)} = \frac{1}{2}$ may depend on the parameters of the SQUID, so that there is no precise lower limit on $\epsilon/l\text{Hz}$. Nevertheless, it is believed that the values of $\epsilon/l\text{Hz}$ for a quantum limited SQUID are in the vicinity of $\hbar$.

Equation (4.19) can be rewritten as a noise temperature (Louisell et al., 1961) by adding the zero point energy in the resistance R_1 , $\hbar f/2$, to the contribution from the SQUID, $\hbar f/2$, to find a total energy $\hbar f$. One then equates $\hbar f$ to the available noise energy available from R_1 in the quantum limit, $\hbar f / [\exp(\hbar f/k_B T) - 1]$, to obtain the result

$$T_N = \hbar f / k_B \ln 2. \quad (4.20)$$

Thus, Eqs. (4.3) to (4.6) using Eq. (3.1) for the noise driving term predict that a SQUID at $T = 0$ weakly coupled to a tuned input circuit also at $T = 0$ should behave as an ideal quantum-limited amplifier. As yet, no practical SQUID amplifier has come within orders of magnitude of this limit. At a temperature of 1K, $n^{(s)}$ will increase to about 2 for the SQUID discussed here (Koch et al., 1981); more important, the noise in the input circuit will be very much in the thermal limit except at unrealistically high frequencies. Two ways in which the quantum limit may perhaps be achieved are discussed in Sec. IV H.

F. Coupling to SQUIDS

Although the efficiency with which an input circuit is coupled to a SQUID does not affect the noise temperature for a tuned circuit, this is not the case for the untuned input circuit. Furthermore, even for the tuned case, very small values of α^2 may lead to unreasonable values of $R_1^{(\text{opt})}$ (Clarke et al., 1979). Thus, from a practical standpoint, it is usually necessary to achieve values of α^2 that are not too small.

For the cylindrical SQUID [Fig. 13(a)], one could achieve values of α^2 of about 0.5 simply by winding a solenoid around the SQUID. The advent of the planar SQUID, however, a configuration that is virtually mandatory if one is to use lithography, has made good coupling a more subtle problem. This difficulty has been overcome by Jaycox

and Ketchen (1981) who used a spiral coil deposited over the body of the SQUID, and by Cromar and Carelli (1981) and Carelli and Foglietti (1982) who used a SQUID with many large-inductance loops in parallel. We briefly describe the former device.

The essential idea (Jaycox and Ketchen, 1981; Ketchen and Jaycox, 1982) is shown in Fig. 19. The SQUID loop is square, and the width of the film, w , is somewhat greater than the width of the hole, d . In the limit $w \gg d$, the loop-inductance is given by $L(\text{loop}) = 1.25 \mu_0 d$. The superconducting input coil is deposited as a spiral overlaying the SQUID loop, and separated from it by an insulating layer. Jaycox and Ketchen give the following expressions for the inductances and α^2 :

$$L = L(\text{loop}) + L_j, \quad (4.21)$$

$$M_i = n(L - L_j), \quad (4.22)$$

$$L_i = n^2(L - L_j) + L_s, \quad (4.23)$$

and

$$\alpha^2 = (1 - L_j/L)[1 + L_s/n^2(L - L_j)]^{-1}, \quad (4.24)$$

where L_j is the parasitic inductance associated with the junctions, n is the number of turns in the input coil, and L_s is the stripline inductance of the coil. Among several versions of this SQUID, Ketchen and Jaycox (1982) tested one with measured values of $d \approx 50 \mu\text{m}$, $n = 50$, $L \approx 90\text{pH}$, $L_i \approx 190 \text{ nH}$, and $\alpha^2 = 0.86$. These values were in excellent agreement with the predictions of Eqs. (4.21) to (4.24). Using a second SQUID to detect the voltage, they found a noise energy

Fig. 19. Planar SQUID with spiral input coil (only 2 turns are shown) and quarter-turn modulation coil (Ketchen and Jaycox, 1982).

referred to the input coil, $S_\phi/2\alpha^2L$, of about $120\hbar$ at 4.2K. At 1.5K, $S_\phi/2\alpha^2L$ decreased to about $50\hbar$, the lowest noise energy referred to an input coil yet reported, although it was not obtained in a flux-locked configuration. Martinis and Clarke (1982) have fabricated a series of planar Nb-NbOx-Pb SQUIDS using an adaption of the configuration of Ketchen and Jaycox, and have achieved noise energies of about $200\hbar$ at 4.2K in a flux-locked loop.

G. Flicker (1/f) Noise in SQUIDS

No discussion of the noise limitations of SQUIDS can be complete without some reference to the question of 1/f noise. As is evident from Figs. 13(b) and 17, the spectral density of the flux noise at low frequencies scales as 1/f; all SQUIDS on which careful measurements have been made show similar behavior.

The 1/f noise in single resistively shunted Josephson junctions arises from fluctuations in the critical current which in turn are believed to arise from temperature fluctuations (Clarke and Hawkins, 1976). However, while the present model predicts the noise level in large area ($\sim 10^4\mu\text{m}^2$) junctions quite well, the 1/f noise in the small area ($\sim 6\mu\text{m}^2$) junctions studied by Koch et al. (1982) is substantially lower than that predicted by the model. This observation does not necessarily invalidate the temperature fluctuation model altogether, but certainly suggests that it is incomplete. Given the level of 1/f noise in single junctions, one then can estimate the noise that would be generated in a SQUID. In the simplest models (Clarke et al., 1976; Ketchen and Tsui, 1980), one simply assumes that the spectral density of the flux noise is $2L^2S_{I_0}(f)$, where $S_{I_0}(f)$ is the spectral density of the critical current fluctuations in each junction, which contribute independently. (One also assumes that the SQUID is flux modulated, so that the 1/f voltage noise does not contribute.) Detailed computer models (Koch et al., 1982a; Tesche, 1982a) produce more exact numerical coefficients linking the critical current noise to the flux noise. The essential conclusion from these models is that the 1/f noise observed in tunnel junction SQUIDS is much greater than that predicted from the critical current fluctuation model; in the case of the cylindrical SQUID by two orders of magnitude. Thus, it appears unlikely that critical current fluctuations are the dominant source of 1/f noise. In fact, Koch et al. (1982a) used two different schemes in which the bias current and flux were modulated to demonstrate explicitly that the predominant source of 1/f noise could be regarded as a flux noise. At present, the origin of this noise is not known.

Koch et al. (1982a) examined the 1/f flux noise in 5 different types of tunnel junction SQUIDS made at Berkeley with inductances and capacitances that each ranged over 3 orders of magnitude, and areas that ranged over almost six orders of magnitude. They found that the spectral density of the 1/f flux noise at 4.2K was remarkably consis-

tent, varying from $[(0.4 \text{ to } 3) \times 10^{-10}/f] \phi_0^2 \text{Hz}^{-1}$, with an average of about $(10^{-10}/f) \phi_0^2 \text{Hz}^{-1}$. The $1/f$ noise in the SQUID of Ketchen and Jaycox (1982) at 4.2K was about $(3 \times 10^{-10}/f) \phi_0^2 \text{Hz}^{-1}$, while that of Carelli and Foglietti (1982) was lower than any of these values, about $(10^{-11}/f) \phi_0^2 \text{Hz}^{-1}$.

It is important to note that the lack of dependence of the $1/f$ flux noise on the SQUID parameters gives a quite different criterion for optimizing the noise energy at low frequencies compared with that derived on the assumptions that the noise arises from critical current fluctuations and that the temperature fluctuation model is valid (Ketchen, 1981; Ketchen and Tsui, 1981). As an example, suppose that we have a SQUID that is quantum limited in the white noise region, and we require the $1/f$ "knee frequency" (at which the white and $1/f$ noise energies are equal) to be as low as possible given that $S_{\phi}^{1/f}$ remains constant at $(10^{-10}/f) \phi_0^2 \text{Hz}^{-1}$. Now $\epsilon^{1/f}/1\text{Hz} = S_{\phi}^{1/f}/2L = [2 \times 10^{-31}/(L/\ln H)f] \text{JHz}^{-1} \approx 2000\hbar/(L/\ln H)f$. Equating this result to the white noise energy, $\hbar$, we find a knee frequency of $2\text{kHz}/(L/\ln H)$. Thus, one should make L as large as possible, consistent with the requirement that the thermal or quantum flux fluctuations must not be so large as to destroy the periodicity of the SQUID. Thus, to obtain a knee frequency of 1kHz (for example, for a gravity wave antenna, see Sec. IV H), one must choose $L \gtrsim 2\text{nH}$. Unhappily, for the SQUID to be quantum noise limited in the white noise region at 1K , this choice of L requires a junction capacitance of no greater than 10^{-4}pF [from Eq. (4.10)]. This value is likely to present a significant challenge, since it implies a junction area of less than $1/400 \mu\text{m}^2$. Of course, this discussion assumes that the $1/f$ noise arises from flux noise rather than critical current fluctuations even for such tiny junctions, and should be regarded with some suspicion until appropriate measurements can be made. Nevertheless, the argument illustrates the importance of establishing an appropriate model for $1/f$ noise, even if it is entirely empirical.

The problem of $1/f$ noise in SQUIDS remains an outstanding one. It now appears that "flux noise" is responsible for the predominant contribution, at least in a certain family of devices involving thin film tunnel junctions, but a good deal of light has yet to be shed on the origin of this noise.

H. Applications of Quantum-Limited SQUID Amplifiers

It should be realized that at the low signal frequencies at which SQUIDS are usually operated, the optimized noise temperature in the thermal limit, $T_N^{(\text{opt})}$, is exceedingly small. For example, if we take $R/L = 20 \mu\text{V}$ (corresponding to a frequency of 10^{10}Hz), $T = 4\text{K}$, and $f = 100\text{Hz}$, we find $T_N^{(\text{opt})} \approx 18\hbar T/V_{\phi} \approx 0.75 \mu\text{K}$. Thus, the thermal noise power in the input resistor overwhelms the SQUID noise by more than 6 orders of magnitude, implying that the SQUID noise is totally negligible even for very badly matched input circuits, and rendering

any discussion of the quantum limit redundant. In order to have any chance of observing quantum limited performance one clearly requires $k_B T \lesssim \hbar f$. If $T = 1\text{K}$, we require $f \gtrsim 25\text{GHz}$, a frequency that is well above the capability of any presently-designed input circuit. On the other hand, if one were to cool the input circuit to 10 mK , one would reduce the minimum required frequency to about 250 MHz . Hilbert and Clarke (1982) have operated SQUIDS with a configuration similar to that of Ketchen and Jaycox (1982) at signal frequencies of up to 200 MHz , and it seems likely that this frequency range can be extended — perhaps to about 1GHz . Although this type of SQUID is far from the quantum limit of 4.2K , it would be relatively close at 10 mK if the noise decreases as predicted by Eq. (4.10). Thus it may be possible to achieve quantum limited amplifiers in a dilution refrigerator for signal frequencies of a few hundred megahertz.

A second very exotic potential application of quantum limited dc SQUIDS is as the transducer for gravity wave antennas. A number of groups around the world are designing antennas consisting of massive cylindrical bars cooled to liquid He^4 temperatures. A gravity wave excites a longitudinal oscillation in the bar, and one means of detecting the motion is with a SQUID amplifier. For example, the detector at Stanford (Boughn et al., 1977), which has been operational for some years, consists of a 4800kg Al bar cooled to 2K and with a mechanical Q of order 10^6 at a resonant frequency of $\nu_a = 840\text{Hz}$. A transducer is attached to one end of the bar, and consists of a superconducting niobium diaphragm with two superconducting pancake coils facing its two sides. The coils carry a persistent supercurrent and are connected in parallel with the input coil of a SQUID. The diaphragm is a high- Q mechanical resonator the motion of which modulates the inductance of the two pick-up coils. The current fed to the SQUID is proportional to the displacement of the diaphragm from its equilibrium position.

In order to achieve the quantum limit in the detector, at first sight one might expect that the experiment should be cooled to temperatures below $840(\hbar/k_B)\text{K} \approx 40\text{ nK}$! Fortunately, however, one can take advantage of the very high Q of the system together with the relatively short length of the gravity wave signal, $\tau_s \sim 1\text{ ms}$, to achieve the quantum limit, at least in principle, in a detector that is at a realizable temperature (Giffard, 1976). The decay time of the antenna is very long, on the order of $Q/\nu_a \sim 10^3\text{ s}$. Thus, one is required to detect a short pulse, of length τ_s , on a background that is changing very slowly with time. It can be shown that under these circumstances, the effective antenna temperature is

$$T_{\text{eff}} \sim \frac{T_s}{Q/\nu_a}, \quad (4.25)$$

about $10^{-6}T$ for the values given above. Thus, if the bar is cooled to 10 mK in a dilution refrigerator, $T_{\text{eff}} \sim 10\text{ nK}$, and the bar is quantum limited. Needless to say, the achievement of this goal will

require a very dedicated effort, including the development of a suitable quantum limited SQUID.

V. CONCLUDING SUMMARY

In this chapter, I have tried to show how one can introduce quantum noise effects into overdamped resistively shunted Josephson junctions. The central idea is to replace the classical current noise term in the resistor with the quantum term which has a spectral density $(2\hbar\nu/R)\coth(\hbar\nu/2k_B T)$. The equation of motion, however, remains the classical description of a particle moving on a tilted washboard [Eq. (2.10)]. This so-called quantum Langevin equation can be solved exactly for the case $\beta_c \ll 1$ and for $I > I_0$; at $T = 0$ it predicts that the voltage noise at low frequencies, $\nu_m \ll \nu_J$, has a spectral density $(2eV/R)(I_0/I)^2$, and arises from zero point fluctuations near the Josephson frequency, ν_J , mixed down to the measurement frequency, ν_m . Measurements on junctions designed to exhibit substantial quantum corrections to the noise in the He^4 temperature range were in excellent agreement with the theory (Figs. 9 and 10). In particular, it was demonstrated that the noise driving term did indeed have a spectral density $(2\hbar\nu/R)\coth(\hbar\nu/2k_B T)$ (Fig. 11). These results indicate rather strongly that the quantum Langevin equation is a valid description of a resistively shunted junction in the free running mode $I > I_0$. The measurements also demonstrate that the Josephson junction operated as a self-mixer is an ideal quantum-limited device at frequencies up to 500GHz. The formalism developed in Sec. III A could also be extended fairly readily to calculate the effects of quantum noise in a Josephson mixer with an external local oscillator.

The quantum Langevin equation fails for underdamped junctions biased well below the critical current. In this regime, quantum mechanical broadening of the wave packet describing the "particle" becomes important, and the picture of a point particle moving on a tilted washboard is no longer valid. Instead, one should use the macroscopic quantum tunneling description. However, the quantum Langevin equation may still be appropriate for overdamped junctions biased just below the critical current, and may give an accurate description of quantum noise rounding of the $I - V$ characteristic. A clarification of the relationship between the quantum Langevin equation and MQT in this regime is very much needed.

The theory of dc SQUIDS in the thermal limit (Sec. V B) predicts $\epsilon/1\text{Hz} \approx 16k_B T(LC)^{1/2}$. A large variety of SQUIDS are in reasonable agreement with this formula, as shown in Fig. 15. The quantum Langevin approach has also been extended to the dc SQUID (Sec. IV C), but, unfortunately, only computer solutions are possible. For an optimized SQUID at $T = 0$, the theory predicts $\epsilon/1\text{Hz} \approx \hbar$. It was emphasized that although the noise energy per Hz is a convenient measure

with which to compare SQUIDS, it is not a complete description of the noise in SQUIDS, and there is no exact fundamental lower limit on its value. The best practical SQUID had a noise energy of about $3\hbar$ at 1.4K; much of this was $1/f$ noise, and the extrapolated value of the white noise was $(0.8 \pm 0.7)\hbar$. It should be remarked that it is more difficult to observe quantum noise in a SQUID than in a single junction using the same junction technologies. The SQUID imposes a further constraint, namely that it must be operated at a relatively low voltage where V_ϕ is a maximum. Thus, one must achieve a higher value of $\kappa \equiv eI_Q R/k_B T$ (for example, by operating at a lower temperature) than would be necessary to observe quantum effects in a single junction.

Section IV E described the coupling of a SQUID to a tuned input circuit to make an amplifier. In this situation, the current noise in the SQUID, which is partially correlated with the voltage noise across the SQUID and which induces a voltage noise into the input circuit, must be taken into account. For weak coupling between the SQUID and the input circuit and for a signal frequency, f , much less than ν_J , the theory predicts that the optimized amplifier at $T = 0$ should have a noise temperature $T_N \approx \hbar f/k_B \ln 2$. Alternatively, one can say that the SQUID induces an average energy of $\hbar f/2$ into the input circuit.

Although quantum limited amplifiers do not in principle require very efficient coupling between the SQUID and the input circuit [Eq. (4.17)], in practice it is undesirable for the coupling to become too weak. At least two efficient coupling schemes for planar SQUIDS have been described in the literature (Sec. IV F), and it would appear that this problem is largely solved.

The problem of $1/f$ noise in SQUIDS at low frequencies is perennial (Sec. IV G). At least for SQUIDS based on tunnel junctions (as opposed to artificial barriers, microbridges, or point contacts), it now appears that the $1/f$ noise does not arise predominantly from critical current fluctuations, but from some unknown source of "flux noise". The magnitude of the $1/f$ flux noise, $S_\phi^{1/f}$, is remarkably constant (say, within one order of magnitude of $(10^{-10}/f)\phi_0^2 \text{Hz}^{-1}$) for a wide range of SQUID inductance, capacitance, and area. To the extent that this conclusion holds true, it implies that to obtain the lowest possible noise energy at low frequencies one must make the SQUID inductance as large as possible.

What are likely future developments? To incorporate a SQUID into an amplifier capable of making a quantum limited measurement is not going to be easy. In my opinion, it will be necessary first to go to low temperatures, to make the ratio of signal frequency to temperature as large as possible. Next, one must either operate at high frequencies, so that $\hbar f/k_B T > 1$, or look for short pulses with high Q circuits, as is the case for gravity wave detectors (Sec. IV G). In either case the problems are formidable. Needless to say, the possi-

bility of observing gravity waves is an extremely exciting one, and likely to stimulate strenuous efforts to build quantum limited detectors in the coming years.

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