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Summary

A review of methods for static-field compression of an electron ring is shown to suggest advantages for a method in which there is no axial acceleration or deceleration of the ring. In the method proposed here the static magnetic field itself is of such a character that the electrons are neither focused nor defocused in the axial direction. The integrity and movement of the ring through the compressor is controlled by a small traveling magnetic well. The feasibility of creating such a traveling well is discussed, and an example is presented of a current distribution capable of producing the static magnetic field of the compressor.

Introduction

In the original proposal of Veksler et al.,[1] the electron ring of an electron-ring accelerator (ERA) is compressed by a pulsed field from a large to a small radius and with an associated increase of electron energy. As Christofilos[2] and others[3,4,5,6] have noted, compression can be achieved (without an energy gain) in a static magnetic field. With acceleration divorced from the compression function, the need for large supplies of pulsed power is avoided, and increased repetition rates become possible--at the expense of a higher-energy injector.

The method proposed (independently) in Refs. 3, 4, 5, 6 is essentially the reverse of a normal magnetic expansion process.[7] In contrast to the situation during normal expansion, the ring will not hold ions during the compression process and hence will not be self-focused; accordingly, there are critical questions concerning the feasibility of achieving rings of small minor dimensions in a static-field compressor--in the face of an inherent energy spread and transverse emittance of the electron beam from the injector.

The method proposed by Christofilos (Ref. 2) has one or more alternating sections of axial acceleration and deceleration and therefore can provide focusing except within a short region between adjacent decelerating and subsequent accelerating sections. Such defocusing regions may not be serious if special methods are employed,[8] or if the crossing is sufficiently rapid.[2,8] However, as in the other static compressor proposals, the requirement of final rings with small minor dimensions seems to impose an almost unattainable demand on the energy spread of the injected beam (cf. Ref. 5).

We propose a static-field compressor in which there is no axial ring deceleration (or acceleration) and hence no very stringent sensitivity to initial energy spread. Furthermore, the fields of the static compressor are neither focusing or defocusing in the axial direction so that, with the addition of a small traveling magnetic well, transverse focusing can be maintained throughout the compression process, and the integrity of the ring maintained. A traveling magnetic pulse, matched to the repetition rate of an electron injector, can easily be attained in practice,[9] and thus can preserve the high pulse-rate advantage conceived for a static-field compressor. In addition, the continuous control of the electron ring may prove advantageous for the proper phasing of a compressed and loaded ring into an accelerating section.

In this paper we first discuss general aspects of the compressor static field and the associated traveling well. Subsequently, we give an example of a possible field configuration and coil arrangement. An appendix is devoted to describing the computational procedures employed in seeking practical designs. A schematic general view of the proposed compressor is given in Fig. 1.

The Static Field

In the design of the static compressor we have at our disposal the choice of the surface $r = r(z)$ on which the electron ring moves. Because the fields are static, the energy of an electron does not change during the compression process, and with no axial acceleration, the orbital component of momentum remains substantially constant for an electron on a circular (equilibrium) orbit. With the requirement that this orbit lie on a specified surface $r = r(z)$, it is necessary that

$$r(z)\cdot B_z[r(z),z] = \text{constant.} \quad (1)$$

We also wish to impose the requirement that rings not be accelerated (or decelerated) in the z-direction. Since the force in the z-direction is proportional to B_r, we require--for all z included in the compression process--that

$$B_r[r(z),z] = 0. \quad (2)$$

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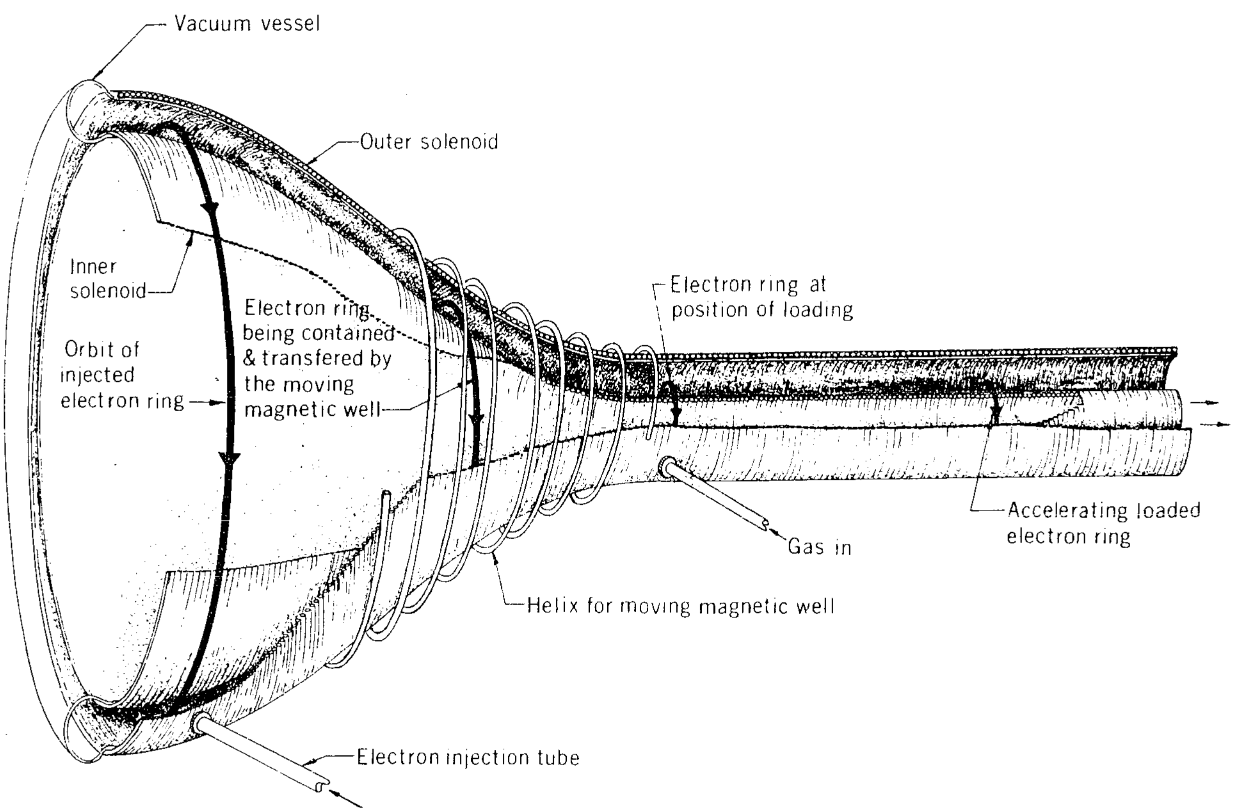


Fig. 1. Schematic view of the static compressor. Note the inner and outer coils. Injection is at the left, loading takes place just after compression, and a magnetic expansion unit is shown on the right. The traveling magnetic well is supplied by a current pulse on the (slow-wave) helix.

The specification of B_z and B_r on the surface $r = r(z)$ can be seen to be: (1) consistent with Maxwell's equations, and hence a permissible procedure; and (2) adequate to determine completely the field for points near the surface $r = r(z)$ (as an expansion in powers of the distance from the surface). Thus we find that, if R_0 is the (arbitrary) injection radius at which the field B_z takes the (arbitrary) value B_0, then

$$B_z(r,z) = \frac{B_0R_0}{r(z)}\left\{1 - \frac{[dr(z)/dz]^2[r - r(z)]}{r(z)\{1 + [dr(z)/dz]^2\}} + \cdots\right\} \quad (3)$$

and

$$B_r(r,z) = \frac{B_0R_0}{r^2(z)}\left\{\frac{[dr(z)/dz][r - r(z)]}{\{1 + [dr(z)/dz]^2\}} + \cdots\right\}, \quad (4)$$

through first order in $[r - r(z)]$. It is easy to verify that these first-order expressions satisfy (1) and (2), as well as the zero-order equations which follow from

$$\nabla \cdot \mathbf{B} = 0, \qquad \nabla \times \mathbf{B} = 0, \quad (5)$$

namely

$$\left[\frac{\partial B_r(r,z)}{\partial z} - \frac{\partial B_z(r,z)}{\partial r}\right]_{r=r(z)} = 0, \quad (6)$$

and

$$\left[\frac{B_r(r,z)}{r} + \frac{\partial B_r(r,z)}{\partial r} + \frac{\partial B_z(r,z)}{\partial z}\right]_{r=r(z)} = 0. \quad (7)$$

Focusing Properties of the Static Field

The focusing properties of the static field follow from the dynamical equations of motion for electrons in the field of Eqs. (3) and (4). At first thought one might argue that, since B_r is zero along the trajectory $r = r(z)$, the derivative of B_r in the z-direction is not zero, the

— can be large; say $|B_r| \sim 50$ gauss
what, then, are the "focusing" effects for radial deviations?

field index n is nonvanishing, and the static field would necessarily have some focusing or defocusing effect. One recalls, however, that the usual derivation of focusing is special to situations with median-plane symmetry, which is not present in the static compressor. We have undertaken a detailed calculation (outlined below) of small-amplitude motion in the region of the equilibrium orbit $r = r(z)$. The general result is that the focusing involves two field indices. When we employ the fields of Eqs. (3) and (4) to determine the indices, we find that the static field determined by Eqs. (1) and (2) has the same focusing frequencies as a uniform field (namely one mode in neutral equilibrium).

Proceeding as we have, this result is certainly nonobvious; we are aware that such a simple conclusion probably can be obtained by a general consideration of our original (simple) field specifications. In lieu of such an argument, we burden the reader, in the remainder of this section, with details of the straightforward argument.

Starting from the principle of least action,

$$\delta \int (\underset{\sim}{p}_{mech} - \underset{\sim}{A})\cdot d\underset{\sim}{s} = 0 \qquad (8)$$

(with the mechanical momentum measured in units of "magnetic rigidity"), one can conveniently derive the equations for a general particle trajectory. Keeping only first-order terms in the motion about the equilibrium orbit $r = r(z)$, one obtains

$$\frac{px''}{r(z) + x} - p + [r(z) + x]B_z(r,z) = 0 \qquad (9a)$$

and

$$\frac{pz''}{r(z) + x} - [r(z) + x]B_r(r,z) = 0, \qquad (9b)$$

where $p = r(z)B_z[r(z),z]$, $x = r - r(z)$, and the primes denote differentiation with respect to the azimuthal angle θ.

Expanding the fields about the equilibrium orbit, and employing Maxwell's equations to relate field derivatives, we may put the coupled equations in the form

$$x'' + (1 - n)x + \alpha z = 0 \qquad (10a)$$

and

$$z'' + nz + \alpha x = 0, \qquad (10b)$$

where the field indices n and α are defined by

$$n \equiv - \left[\frac{r}{B_z(r,z)} \frac{\partial B_z(r,z)}{\partial r}\right]\Bigg|_{r=r(z)}$$

$$n = - \left[\frac{r}{B_z(r,z)} \frac{\partial B_r(r,z)}{\partial z}\right]\Bigg|_{r=r(z)} \qquad (11a)$$

and

$$\alpha \equiv \left[\frac{r}{B_z(r,z)} \frac{\partial B_z(r,z)}{\partial z}\right]\Bigg|_{r=r(z)},$$

$$\alpha = - \left[\frac{r}{B_z(r,z)} \frac{\partial B_r(r,z)}{\partial r}\right]\Bigg|_{r=r(z)}, \qquad (11b)$$

with

$$B_r(r,z)\Big|_{r=r(z)} = 0 .$$

The characteristic frequencies of the system (10) are given by

$$\nu^2 = \frac{1}{2} \pm [(\tfrac{1}{2} - n)^2 + \alpha^2]^{1/2}, \qquad (12)$$

and correspond to eigenmodes in which the r and z motion is mixed. From (3) and (4), and the definition of n and α [Eqs. (11a,b)], we find

$$n = \frac{[dr(z)/dz]^2}{\{1 + [dr(z)/dz]^2\}}, \qquad (13a)$$

$$\alpha = - \frac{dr(z)/dz}{\{1 + [dr(z)/dz]^2\}}, \qquad (13b)$$

and consequently, from (12), $\nu^2 = 0, 1$. These frequencies are the same as would be obtained in a uniform field (but now with some coupling of r and z motion corresponding to the pure-r and pure-z modes of the uniform field). One mode is on an integral resonance, and the other is in neutral equilibrium.

The Moving Magnetic Well

In order to control ring position along the trajectory $r = r(z)$, and also to supply axial focusing, a moving magnetic well must be added to the static field of the compressor. Because of the neutral equilibrium of the axial mode in the static field, only a modest strength is required for the moving field.

A number of possibilities have been suggested for creating the moving well; a particularly interesting proposal is to send a current pulse down a slow-wave structure. Dombrowski has considered a design involving a helix with a surrounding dielectric layer and an outer conducting sheath.[9] He finds that the dispersion of a current pulse can be made acceptably small, while the rather slow decrease of impedance with frequency is advantageous for matching into a modulated power supply. Details may be found in Ref. 9, but the conclusion is that a helix appears to be a practical solution to the problem of ring focusing and control.

Numerical Example

A practical compressor design consists of specifying coil radii, positions, and associated currents. The expansion of fields about the trajectory $r = r(z)$ (which was described above) could be used to generate fields at distances away from the trajectory; these fields could then be "terminated" by suitable current distributions in such a way as to require no further currents at greater distances. This procedure is not easy to follow. Furthermore, it is not clear in advance at what point singularities will appear in the expansion and thus dictate the location of currents. If these singularities are too close to the trajectory, $r = r(z)$, they would preclude adequate room for particle oscillations or adequate vacuum chamber width for pumping, and might force the windings to be inconveniently thin in order that intolerable field ripples be avoided.

In order, then, to demonstrate the feasibility of the compressor in cases of practical interest, we have resorted to digital computation. The computational studies were undertaken by Steven Sackett, and the procedures employed are described in the Appendix.

In Fig. 2 we present one numerical example which should suffice to show the practicality of the device. The compressor has a length of one meter; it accepts electrons at a radius of 57 cm and compresses them, by a factor of 7.8, to a radius of 7.3 cm. The separation between inner and outer coil surfaces is 8.0 cm. It can be seen that the required coil currents are smooth (the oscillations near the ends presumably can be removed by slight lengthening of the solenoid) and not excessive in magnitude.

Acknowledgments

We are indebted to Steven Sackett for development of the computer program which made possible the practical design studies. We are also indebted to him for agreeing to let us include here the material which constitutes the Appendix of this paper. We are grateful to Professor George Dombrowski for his interest in the problem of the moving magnetic well, and for his contributions to this subject. Finally, we acknowledge that this work was a result of N. C. Christofilos' stimulating lectures and remarks on the advantages of static compressors.

Appendix: Determination of Coil Currents*

Practical designs have been investigated by choosing (1) a desired compression surface $r = r(z)$, and (2) a desired set of coil locations. The currents which must be supplied to the coils to give the necessary compressor fields are then computed. Because of the linearity of Maxwell's equations, the problem reduces to solving the system of (linear) equations:

$$\sum_{j=1}^{n} B_{ij}^{(z)} I_j = B_{zi} ; \quad i = 1, \cdots m, \qquad \text{(A1)}$$

$$\sum_{j=1}^{n} B_{ij}^{(r)} I_j = B_{ri} ; \quad i = 1, \cdots m , \qquad \text{(A2)}$$

where $B_{ij}^{(z)}$ and $B_{ij}^{(r)}$ denote the z and r components, respectively, of the field at point i due to unit current in coil j, and the field components desired at point i are denoted by B_{zi} and B_{ri}.

A practical solution for the currents I_j may be obtained by taking $2m \geq n$ and obtaining the best fit to (A1, A2) in a least-squared sense, with the possibility of a relative weighting of the B_r equations compared with the B_z equations. This process will, in general, lead to currents that are not smoothly varying, or that are large in value. Consequently we require that the quantity to be minimized be supplemented by

$$w_0 \sum_{j=1}^{n} I_j^2 + w_1 \sum_{j=2}^{n} (I_{j-1} - I_j)^2 + w_2 \sum_{j=2}^{n-1} (I_{j-1} - I_{j+1})^2 + w_3 \sum_{j=2}^{n-1} (I_{j-1} - 2I_j + I_{j+1})^2 , \qquad \text{(A3)}$$

where the w_i are weighting factors.

The computational procedure consists of a reduction of the matrix equations, by orthogonal Householder transformations, followed by iterations which successively improve the least-squares fit.[10] The computer program first generates the fields to be fitted, B_i, and the matrix of coefficients, B_{ij} (employing the fields of infinitely thin wire loops); it then (using input values of weights and a convergence criterion) determines I_j. Since machine language is used, the speed is high. Output is numerical and also graphical.

*This Appendix was prepared by Steven Sackett.

In the example cited in the text, the orbit trajectory was taken to be a cosine curve with 100 fitting points between $z = 5$ cm and 95 cm. There were 200 coils located between $z = 0$ and $z = 100$ cm on two cosine curves separated by 8 cm. The B_r weighting factor was 5, $w_0 = 10^{-10}$, $w_1 = 0$, $w_2 = 10^{-12}$, and $w_3 = 10^{-9}$. (These values were seen, in some survey studies, to be effective.) The time to solve the problem was 14.8 seconds on a CDC 6600, and the total sums of squares of the relative errors in B_z and in B_r were respectively 8.4×10^{-6} and 7.3×10^{-6}.

References

1. V. I. Veksler et al., Proc. VI International Conference on High Energy Accelerators, Cambridge Electron Accelerator, Cambridge, 1967, p. 289.

2. N. C. Christofilos, Static Compression of Relativistic Electron Rings, Lawrence Radiation Laboratory Technical Note UCID-15378, September 17, 1968 (submitted to Phys. Rev. Letters).

3. E. M. McMillan, private communication (1967).

4. E. Keil, private communication (1968).

5. R. E. Berg, H. Kim, M. P. Reiser, and G. T. Zorn, Possibilities of Forming a Compressed Electron Ring in a Static Magnetic Field, University of Maryland Department of Physics Technical Report 918, November 1968 (submitted to Phys. Rev. Letters).

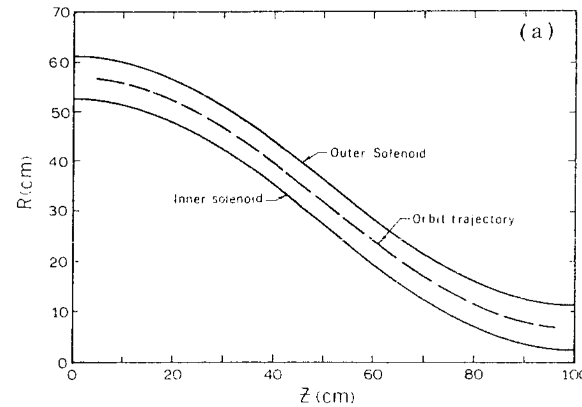
6. W. Hintz, private communication (1969).

7. See, for example, Symposium on Electron Ring Accelerators, Lawrence Radiation Laboratory Report UCRL-18103, 1968, p. 16.

8. N. C. Christofilos, private communication.

9. G. E. Dombrowski, Use of a Helix for Transport of Electron Rings Through Static Compression Fields, Lawrence Radiation Laboratory Technical Note ERAN-24, 1968 (unpublished).

10. G. Golup, "Numerical Methods for Solving Linear Least-Squares Problems," Numerische Math. 7, 3 (1965), p. 206-216.



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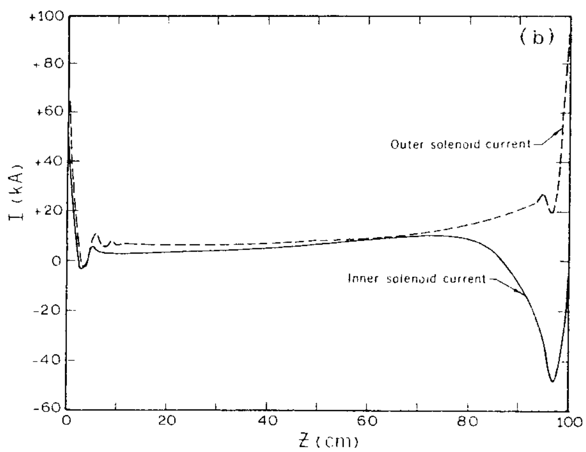

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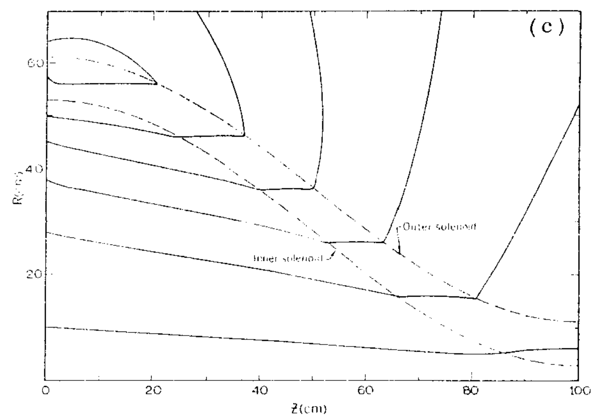

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Fig. 2. Static compressor having a compression ratio of 7.8:1. (a) Geometry of the coils and ring trajectory. (b) Currents required in each turn of two 100-turn solenoids (or, equivalently, the turn densities required for series-wound coils). The current values correspond to a field $B_z = 4.0$ kg at $r = 56.7$ cm; i.e. electron kinetic energy of 68.3 MeV. (c) "Flux plot" showing lines of force (the density of lines does not reflect field strength).