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An overview of Phase-cut, synchronization, augmented-projections using power iterations.

I. INTRODUCTION

For background and notation, see[1,2]. In brief, we have an unkown object ψ a known "illumination matrix" $\mathbf{Q}$ (or support mask for classical CDI[2]), a 2D FFT operator $\boldsymbol{F}$ and set of frames $\mathbf{z}$, which are related to measured Fourier amplitudes a by:

$$\mathbf{z} = \boldsymbol{F}\mathbf{Q}\psi, \qquad |\mathbf{z}| = a.$$

A popular approach is to find a vector z such that:

$$\|[I - P_Q]\mathbf{z}\| = 0, \tag{1}$$
$$\|[I - P_a]\mathbf{z}\| = \|\,|\mathbf{z}| - a\| = 0, \tag{2}$$

are satisfied simultaneoulsy. $P_{\mathbf{Q}}$ projects a vector onto the range of $\boldsymbol{F}\mathbf{Q}$:

$$P_Q = \boldsymbol{F}\mathbf{Q}(\mathbf{Q}^*\mathbf{Q})^{-1}\mathbf{Q}^*\boldsymbol{F}^*. \tag{3}$$

The unkown vector ψ is obtaned from $\mathbf{z}$ by $\psi = (\mathbf{Q}^*\mathbf{Q})^{-1}\mathbf{Q}^*\boldsymbol{F}^*\mathbf{z}$.

The *Fourier magnitude* projection operator, when applied to a vector $\mathbf{z}$, yields:

$$P_{\mathbf{a}}\mathbf{z} = \frac{\mathbf{z}}{|\mathbf{z}|} \cdot \mathbf{a}, \tag{4}$$

where division and product are intended as element-wise operations.

A. Phase optimization

Here we want to minimize Eq. 1 w.r.t. a phase factor ϕ. That is, we want to find:

$$\arg\min_{\phi} \quad \|[I - P_{\mathbf{Q}}]\mathrm{Diag}(|a|)\phi\|^2,$$
$$\arg\min_{\phi} \quad \phi\left[\mathrm{Diag}\left(a^2\right) - H\right]\phi, \tag{5}$$
$$H = \mathrm{Diag}\left(a\right)P_Q\mathrm{Diag}\left(a\right)$$

where $(\phi_i^*\phi_i = 1, \forall i)$. I discuss three aproaches.

 a. Synchronization. Since $\|\mathrm{Diag}\left(a\right)\phi\| = \|a\|$ is independend on the choice of ϕ, we rewrite Eq, (5) as:

$$\arg\max_{\phi} \quad \phi^*H\phi \tag{6}$$

if we relax $(|\phi_i| = 1, \forall i)$ and use $\|\phi\| = k$ instead[3], all we need to do is to find the eigenvector with largest eigenvalue, which we can do by the power iteration method:

$$\phi^{\ell+1} = \frac{H\phi^\ell}{|H\phi^\ell|}, \qquad \mathbf{z} = |a|\phi. \tag{7}$$

It is easy to show that (Eq. 7) is equivalent to the classical alternating projection method:

$$z^{(\ell+1)} = P_a P_{\mathbf{Q}} z^{(\ell)}$$

this is known to stagnate with classical CDI but to converge (slowly) in ptychographic imaging.

 b. Phase-cut[4]. Since the diagonal term $\phi^*\mathrm{Diag}\left(H\right)\phi$ is also independend on the choice of ϕ (for $|\phi_i| = 1$), we can remove it when we compute the largest eigenvalue by power iterations[4]:

$$\phi^{\ell+1} = \frac{[H - \mathrm{Diag}\left(H\right)]\phi^\ell}{|[H - \mathrm{Diag}\left(H\right)]\phi^\ell|} \tag{8}$$

This is equivalent to the following update:

$$z^{\ell+1} = P_a\left(P_{\mathbf{Q}} - \mathrm{Diag}\left(P_Q\right)\right)z^\ell$$

Efficient implementation can be performed by pre-computing $\mathrm{Diag}\left(P_{\mathbf{Q}}\right)$. In classical CDI this is simply the sum of the support volume (or area) normalized by the oversampled volume, in ptychographic imaging is the ratio of the power of the illumination function for a single frame over the sum of all illuminations, $P_{Q_{ii}} = \frac{\|Q_i\|^2}{\|\mathbf{Q}\|^2}$.

 c. Augmented projections[1]. If we solve the minimization problem (Eq. (5) s.t. $\|\phi\| = k$) by inverse iteration method we obtain:

$$\phi^{\ell+1} = \frac{[\mathrm{Diag}\left(a^2\right) - H]^{-1}\phi}{|[\mathrm{Diag}\left(a^2\right) - H]^{-1}\phi|} \tag{9}$$

In[1] it was observed that computing Eq. (7) and a coarse solution to Eq. (9) in an alternating fashon improved convergence rate in large scale ptychographic imaging. More work is needed to determine the optimal combination of Eqs. (7,8,9) and augmented lagrangian alternating direction methods (ALADM or ADM)[5] in large scale phase retrieval problems.

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