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Exploratory Dynamic Models of Alternative Fuel Vehicle Adoption

DISSERTATION

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ABSTRACT OF THE DISSERTATION

Exploratory Dynamic Models of Alternative Fuel Vehicle Adoption

By

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Doctor of Philosophy in Civil and Environmental Engineering

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Identifying socioeconomic characteristics and vehicle characteristics, including a market share of a specific vehicle, influencing on a choice of a vehicle is important for forecasting demands for alternative fuel vehicles (AFVs). Over the time, how changes in these characteristics will affect on the demands is also important. And by connecting with supply, how changes in demands for AFVs will make an effect on the supplies becomes important. This paper forecasts market shares of AFVs in demands and supplies.

First, in a demand part, a dataset of National Household Travel Survey in 2009 is used to identify factors which influence on a choice of AFVs by logit models. And then by using coefficients from the logit models, a dynamic normative model is proposed to forecast demands for Toyota Prius, a sort of hybrid vehicles, with respect to changes in characteristics such as a gas price and a vehicle price. Because a dynamic normative model is a simulation model with unknown values of parameters, these values are randomly defined to track the changes in market shares of Prius based on an annual vehicle market share data.

Next, in a supply part, proportions of hydrogen fuel cell vehicles (HFCVs) with respect to the

density of hydrogen refueling stations are estimated by logit models. And then by using these results, a competition model is proposed to forecast supplies for HFCVs. Forecasting supplies for HFCVs is based on demands which is forecasted from a dynamic normative model.

Last, it is found that supplies of HFCVs from the competition model exceed affordable numbers of themselves for the market, because the demands for HFCVs from a dynamic normative model don’t consider affordable numbers of HFCVs for the market. Therefore, to connect results from two models, feedback methods are used.

The results indicate that the market share of AFVs will exceed that of ICEs when: 1) a gasoline price is increased, 2) a vehicle price of AFVs is decreased, 3) the initial market share of AFVs is large, and 4) the density of refueling stations is increased.

1. Introduction

1.1 Alternative fuel vehicles

Alternative fuel vehicles (AFVs) are vehicles that use fuels other than gasoline to operate. AFVs include electric vehicles (EVs), hybrid electric vehicles (HEVs), plug-in hybrid electric vehicles (PHEVs) and hydrogen fuel cell vehicles (HFCVs). Because AFVs don't use gasoline to operate, there are minimal, if any, harmful emissions associated with their use. And, compared to vehicles powered by internal combustion engines (ICEVs) using gasoline, AFVs typically produce less noise during driving. However, as yet, they have not penetrated vehicle markets significantly, principally because of the following: 1) AFVs are generally more expensive than comparable ICEVs, 2) compared to gasoline vehicles, AFVs typically have shorter range and poorer performance than comparable ICEVs, and 3) the number of refueling/recharging stations for AFVs is far fewer than that of gas stations and, for many AFVs, virtually nonexistent.

The introduction of AFVs to the market has been a relatively recent occurrence—one that has generally preceded the deployment of the infrastructure necessary for their widespread adoption. In an effort to encourage people to purchase them despite the marginal infrastructure, several incentives, such as High Occupancy Vehicle (HOV) lane eligibility, have been enacted. However, compared to typical ICEVs, ownership of AFVs (beyond that of the Toyota Prius Hybrid, which currently is the best-selling car in California) is not widespread, mainly because of the relatively high prices of vehicles and the hesitancy of buyers to invest such substantial resources in technologies that have not yet been widely adopted. These concerns are heightened by limited range, in the case of BEVs, and the availability of refueling stations, in the case of HFCVs. Moreover, although the Toyota Prius electric hybrid is currently widespread, the presence of full AFVs is virtually non-existent in today's market, making the choice of owning an AFV an individual choice with no peer reinforcement. Of course, there have been efforts in reducing vehicle price, as well as in improving range, increasing fuel availability, and providing incentives. And with these efforts, it is expected that AFVs ultimately will penetrate the market in sufficient numbers to be a competitive alternative to ICEVs. Then, a critical question of interest that should be asked is “when?” The time for sufficient penetration of AFVs may be short or long, depending on the conditions (e.g., changes in vehicle price, increased range, increased number of peers selecting AFVs, and so on) under which they operate. Then, changes in which factors, and by what amount, will produce an environment in which AFVs penetrate more easily in a short time? This is the focus of this research.

1.2 Objectives

The purpose of this research is to identify factors that are conducive to significant market penetration of AFVs. Possible factors include both intrinsic (e.g., vehicle price, range, operating cost, and so on) and extrinsic (e.g., household income, gender, education level, etc.) influences, as well as the influences of societal norms and trends.

The objectives in achieving this purpose are the following:

- Identify influential factors related to vehicles, households, and society
- Propose a simulation model that can forecast the initial stages of dynamic market penetration of AFVs
- Propose and set several scenarios for simulation
- Use the model to identify the influences of the various factors on market penetrations of AFVs

By achieving the objectives above, the influence on market penetration of AFVs relative to changes in each factor will be observed for a number of different scenarios. From the results of these scenarios, factors that are conducive to sustainable levels of AFV penetration will be identified.

1.3 Expected contributions

This research focuses on the impact of changes in various factors on the potential for AFVs' market penetration. Rather than estimating coefficients and evaluating goodness of fit of the dynamic models proposed here, the emphasis is on identifying conditions for which AFVs may be a sustainable alternative to conventional ICEVs. It is expected that the results of the research will provide guidelines for determining the infrastructure necessary for significant numbers of AFVs to be supported. More generally, it is also expected that the models introduced in this study will provide a point of departure for forecasting demands for hypothetical alternatives.

1.4 Outline

In Chapter 2, literatures that focus on available datasets related to AFVs are reviewed, and the factors that may affect the choice of AFVs are discussed. After that, the literature that has focused on models for forecasting demands for AFVs (i.e., discrete choice and simulation models) is reviewed. In Chapter 3, coefficients of several variables are estimated by logit models to observe influences on the choice of AFVs. In Chapter 4, with these coefficients and a model using stimuli and response from changing variables, changes in market share of Toyota Prius over time are demonstrated. In Chapter 5, expected demands for HFCVs are observed with respect to the density of hydrogen refueling stations. In Chapter 6, with demands obtained from Chapter 5 and a model assuming competitions between ICEVs and HFCVs, changes in market share of HFCVs from a supply standpoint are observed. The results obtained from combining from these two different models are shown in Chapter 6. And in Chapter 7, the conclusions and the expected contributions of this study are discussed.

2. Literature Review

2.1 Datasets for forecasting the use of hypothetical vehicles

Although some HEVs (e.g., Prius, Civic Hybrid, and so on) and BEVs (e.g., Honda Insight) are already on the road, AFVs remain largely hypothetical vehicles to the mass market, principally because of their own scarcity as well as the scarcity of the infrastructure needed to support their widespread adoption. Typically, a survey of a household's travel is used to forecast the vehicle used for travel. However, survey data, or revealed preference (RP) data, typically addresses only the vehicles that are currently operated on the road. As noted, AFVs are far outnumbered by ICEVs, and so travel survey data does not contain enough data for forecasting the use of these vehicles. Consequently, under current conditions, RP data cannot be used for forecasting the potential demand for AFVs.

In order to overcome this drawback of RP data, stated preference (SP) methods have been used in the majority of work related to AFVs. While RP survey methods deal with existing alternatives, SP methods deal with hypothetical alternatives, usually by characterizing the alternatives in terms of specific attributes that can then be used to "construct" the utility or value of a hypothetical alternative. In this way, SP methods can be used to predict people's preferences for AFVs. For example, Golob et al. (1993) attempted to predict demands for electric and clean-fuel vehicles by using SP survey data drawn from the South Coast Air Basin of California.

Similar to RP data, SP data also has significant drawbacks in application to AFVs. Chief among these is that people's choices as indicated on the SP survey are not necessarily coincident with real choices made in the future when the alternative will be introduced. So, it is important to make data obtained from SP questions similar to real world situations, or at least to minimize the gap between survey data and real world choices. Considering this, Brownstone et al. (1996) tried to predict choices of AFVs by using SP choices conditioned on current RP data. And Brownstone et al. (2000) tried to estimate the demand for AFVs by jointing California households' SP data with RP data for automobiles.

Because both the structure and the focus of SP surveys are different from those of RP surveys, jointing these two datasets raises certain accuracy issues. Bhat and Castelar (2002) addressed four issues derived from jointing RP and SP data: inter-alternative error structure, scale difference, unobserved heterogeneity effects, and state-dependence and heterogeneity in state-dependence. They argue that for accurate results from joint RP-SP data, these issues should be considered simultaneously, since they seem to have significant interactions. They also conclude that the flexible, or open, form rather than closed form of estimation models is necessary for accurate results using joint datasets. Although calculations using closed form models are relatively easy, for joint datasets they can result in unacceptable estimations because they ignore the issues indicated above.

2.2 Factors that may influence the use of AFVs

As already mentioned, for the large part, AFVs are hypothetical vehicles. Therefore, specifying factors that are likely to influence the adoption and use of these vehicles is important. In order to forecast vehicle use in the future, Train (1986) developed simulation models for one base year scenario and six future scenarios, in which some variables were changed. Through the simulations, he found that gasoline price and vehicle price would have the greatest influence on the use of AFVs, while the influence of employment status was small. Bunch et al. (1993) conducted an SP survey to analyze demand for AFVs in California, by providing respondents the information about fuel price, vehicle price, range, emissions, performances and fuel availability for AFVs and gasoline vehicles. Golob et al. (1994) specified endogenous variables (i.e., VMT, driver's age, gender and employment status) and exogenous variables (vehicle characteristics and household characteristics) for their structural equation model of vehicle use. Golob et al. [1](1997) extended this work to forecasting the use of AFVs by including such SP variables as range between refueling. Brownstone et al. (1994) and Potoglou and Kanaroglou (2007) found that reduced monetary costs, relieved purchase tax and low emissions rates would increase the number of households choosing AFVs, while such incentives as free parking and HOV lane eligibility had insignificant effects. They also found that fuel availability was a concern when households considered choosing AFVs. Yeh (2007) found that the adoption of AFVs and alternative fuels is influenced by: 1) technologies and fuel choices (cost, range, fuel availability, reliability, and safety), 2) context (social, economic, cultural, and spatial characteristics), and 3) impacts (economic, health, environmental, energy, and land-use changes).

Although some AFVs (specifically, HEVs) are already on the road, most people do not know much about these vehicles at this point in time. Because of this, when choosing vehicles, people may be expected to rely more heavily on the opinions and experiences of other people who already own AFVs than they would in a similar situation involving ICEVs. The "neighbor" effect, introduced by Mau et al. (2008), can be one of factors that may influence vehicle choice. Mau et al. (2008) and Axsen et al. (2009) tried to forecast the use of AFVs by performing two steps: 1) provide information about HEVs (e.g., articles, vehicle brochures and market conditions) to potential respondents, and then 2) perform a vehicle choice survey with these respondents.

Another factor that can be considered for forecasting the use of AFVs is their symbolism to vehicle owners. Heffner et al. (2007) focused on the symbolism of HEVs and then speculated on the role that symbolism may play in the future markets for Fuel Cell Vehicles (FCVs). According to them, people sometimes use automobiles to predict owners' intelligence, life satisfaction, and behavior toward others. And for people, HEVs not only symbolize the idea of saving fuel costs, but also describe their owners as intelligent, sensible people who made smart choices in their lives. The work provided guidance on how FCVs can penetrate markets by using symbolism, based on the results from HEVs. However, it did not address how these symbolisms could be included in forecasting choice and the use of AFVs.

2.3 Models for forecasting demands of AFVs

2.3.1 Discrete choice models

Logit models have been used frequently for forecasting vehicle choice. However, standard logit models based on revealed preferences are valid only for alternatives that are in common use. Because AFVs are vehicles that are not in common use yet, probabilities of choosing AFVs cannot be estimated through standard logit models. To address this shortcoming, several studies have developed methods that can forecast the choice probability of AFVs. One way is to use standard logit models that include additional data. Bunch et al. (1993), Golob et al. (1993), Golob et al. [2](1997) and Potoglou and Kanaroglou (2007) tried to forecast demand for AFVs with logit models using SP data. Greene (2001) developed nested logit models using estimates other than those from SP data (i.e., purchase price elasticity, fuel cost per mile, maintenance cost, battery replacement cost, the value of range, value of home-based refueling/recharging, acceleration performance, and so on).

Others have based studies on different versions of the basic logit model. Beggs et al. (1981) developed an ordered logit model that uses individual data of ranked choice. It used a property of the conditional distribution of extreme random variates to extend the basic logit formulation to the ranked case. The property is that the utility distribution of the most favored choice is independent of the ordering of the less favored choices even when the exact ordering is used to condition on. With this property, Beggs et al. rewrote the formulation of probability as that of conditional probability of choice. Brownstone (2000) developed a mixed logit model using an error term (not ε), which has zero mean and is independently and identically distributed. This error term is interpreted as an error component that induces heteroskedasticity and correlation over alternatives in the unobserved portion of utility. The choice probabilities obtained through a mixed logit depend on parameters; the coefficients of variables and the fixed parameter of the distribution for error term, while those through the standard logit depend only on the former. This leads to a problem when calibrating derivatives of the log likelihood function. The inclusion of the latter removes the guarantee of global concavity and so the Hessian matrix is not guaranteed to be positive definite. This results in computationally slower estimation.

Dagsvik et al. (2002) developed a stochastic model for ranking as a means to find potential demand for AFVs. They also provided random utility models with serially dependent utilities. According to them, the utilities of an alternative may be correlated across experiments even if the corresponding attributes differ. The reason for this may be that an individual's state of mind and perception capacities vary more or less slowly over time, and as a result, preference evaluations in the last and current experiments may tend to be more strongly correlated than are preference evaluations in experiments that are more remote in time. By reflecting this, they presented an extension of IIA to the case where choices take place over time, which was proposed in Dagsvik (2000). They interpreted the formulation as follows: in the particular case that the past choice sets are constant, but where the choice set in the current period is expanded to include new alternatives that were never feasible before, the probability of choosing an alternative among the new alternatives that enter the choice set is independent of the choice history. In other words, even if previous choices provide information about the preferences over the alternatives in the "old" choice set, these choices provide no information about the utility of the "new" alternatives.

Ahn et al. (2008) tried to forecast household choice of AFVs by using the multiple discrete-continuous extreme value model (MDCEV), which had been developed by Bhat (2005). The MDCEV model defines utility over vehicle types as following:

$$U = \sum_{j=1}^{K} \psi\left(x_j\right)\left(m_j + \gamma_j\right)^{\alpha_j}$$

where K is the number of vehicle types in an alternative set, $\psi\left(x_j\right)$ is the baseline utility explained by the attributes x_j of alternative j, m_j is the respondent's stated expected usage of alternative j, α_j influences the rate of diminishing marginal utility and γ determines the translation. This function is valid if $\psi\left(x_j\right) > 0$ and $0 < \alpha_j < 1$—this condition should be held to ensure diminishing marginal utility—for all *j*. According to Bhat and Sen (2006), the term γ determines if corner solutions are allowed (i.e., a household doesn't own one or more vehicle types) or if only interior solutions are allowed (i.e., a household is constrained by formulation to own all vehicle types). From the utility structure above, a statistical model can be developed by adopting a random utility specification. A multiplicative random element is introduced to the baseline utility as follows:

$$\psi\left(x_j, \varepsilon_j\right) = \psi\left(x_j\right) \cdot e^{\varepsilon_j} = \exp\left(\beta' x_j + \varepsilon_j\right)$$

where ε_j captures unobserved characteristics that impact the baseline utility for vehicle type j.

With this baseline utility function, the MDCEV model becomes:

$$U = \sum_{j=1}^{K} \left[\exp\left(\beta' x_j + \varepsilon_j \right) \right] \left(m_j + \gamma_j \right)^{\alpha_j}$$

To ensure the condition that $0 < \alpha_j < 1$, Bhat and Sen (2006) parameterized α_j as $\frac{1}{\left[1 + \exp\left(-\delta_j \right) \right]}$. And, to allow the satiation parameters to vary across households, δ_j is specified as $\delta_j = \theta_j^{'} y_j$, where y_j is a vector of household characteristics and θ_j is a corresponding vector of parameters. Using the utility above, the probability that the household owns *i* of the K vehicle types is calculated as follows:

$$P\left(m_i > 0 \text{ and } m_s = 0;\ i = 1, 2, ..., I \text{ and } s = I + 1, ..., K \right)$$

$$= \left[\prod_{i=1}^{I} c_i \right] \left[\sum_{i=1}^{I} \frac{1}{c_i} \right] \left[\frac{\prod_{i=1}^{I} e^{V_i}}{\left(\sum_{j=1}^{K} e^{V_j} \right)^{I}} \right] (I - 1)!$$

where $c_i = \left(\frac{1 - \alpha_i}{m_i + \gamma_i} \right)$ and $V_j = \beta' x_j + \ln \alpha_j + \left(\alpha_j - 1 \right) \cdot \ln\left(m_j + \gamma_j \right)$.

When *i*=1 (i.e., only one vehicle type is chosen by the household), the model above becomes the standard MNL model.

2.3.2 Effect of refueling/recharging stations

Keles et al. (2008) proposed a different approach to forecasting market penetration of HFCVs using an analysis based on agent behavior. They postulated that an important variable influencing the attractiveness of HFCVs is the refueling effect, representing the impact of the available hydrogen infrastructure, i.e., the share of filling stations offering hydrogen fuel. In their model, its evaluation depends on the development of hydrogen filling stations modeled in the filling stations module. They focused on an analysis of infrastructure extension based on the number of available urban and highway filling stations, where the dominating element is the share of urban fueling station providing hydrogen fuel. Expectedly, they found that the new number of refueling stations depends on the demand for hydrogen fuel—if the existing stations cannot satisfy demand, new stations will be constructed to match the number needed to serve the excess demand. They further introduced the system element "social attractiveness" as an indicator for the influence of the social environment on the decisions made by private consumers in order to calculate the private demand caused by urban as well as interurban traffic. They postulated that the decision of an individual to buy a HFCV depends on the pressure or pull exerted by his/her social environment, especially during the introduction phase.

Schwoon (2006) proposed a series of agent-based and evolutionary models to examine the possible transition to HFCVs. In the model, the total utility a consumer k obtains from buying car $c_{i,t}$ is:

$$U_{k,t}^{tot}\left(c_{i,t}\right)=\frac{\left[\beta_k U_{k,t}\left(c_{i,t}\right)+\left(1-\beta_k\right)\cdot SN_{k,t}\left(c_{i,t}\right)\right]\cdot RFE_{k,t}\left(c_{i,t}\right)}{\left[p\left(c_{i,t}\right)\cdot\left(1+tax_t\left(1-FCV\right)\right)\right]^{\left|\varepsilon_{own}\right|}}$$

In the model form, because price is a crucial determinant of the buying decision, government is assumed to impose a value added tax (tax_t) on ICEVs to stimulate the diffusion of HFCVs. The effectiveness of such a tax depends on the responsiveness of utility toward (after tax) price changes, which is defined by the elasticity ε_{own}. If the absolute value of ε_{own} is high, the impact of the tax on utility and, therefore, on technology choice is also high. The numerator evaluates the utility that the consumer can derive from the features of a specific car. The utility is assumed to be a weighted average of the direct utility $U_{k,t}$ associated with the characteristics of the car and the social need $SN_{k,t}$ (i.e., the impact of neighbors on decisions), jointly scaled by a variable called the refueling effect ($RFE_{k,t}$). The "social need" effect reflects the presumption that the emotional decision of whether or not to buy a futuristic and unfamiliar HFCV might be guided by decisions of neighbors. Such a social need $SN_{k,t}$ is defined by the share of the product type in the neighborhood (including the deciding consumer), i.e., in the case of a FCV, it is the number of neighbors driving a FCV plus 1, divided by the total size of the neighborhood, including the deciding consumer. The weight β_k varies over individuals and is taken from a random draw from a normal distribution within the boundaries 0.4 and 1 in the central case. Refueling, i.e., the sufficient availability of hydrogen—a major concern for every consumer considering a HFCV—is captured by the variable RFE, as being essential to total utility in the case of a HFCV. The refueling effect changes over time if a considerable hydrogen infrastructure gets installed. Furthermore, people are assumed to differ in their individual refueling needs. Put together, RFE is constructed as a function of fuel availability at time *t*, represented by the share of filling stations that provide hydrogen ($s_{H2,t}$) and individual driving patterns (DP_K)

$$RFE_{k,t}\left(c_{i,FCV,t}\right) = 1 - FCV \cdot DP_k \cdot \exp\left(\gamma s_{H2,t}\right)$$

where $\gamma \leq 0$ is a parameter determining the importance of fuel availability. Refueling is assumed irrelevant for ICEVs. Individual driving patterns are assumed to vary between 0 (only short trips in familiar areas) and 1 (many long distant trips in unknown areas) and are fixed over time. On the supply side, it is assumed that fuel station companies increase the share of stations as HFCVs enter the market. If the share of newly registered HFCVs is larger than the share of stations, infrastructure grows by the highest amount that is technologically feasible ($g_{H2}^{\max}$). Otherwise the share of stations develops as

$$s_{H2,t+1} = s_{H2,t} + \min\left[g_{H2}^{\max}, v\left(s_{FCV,t}^{\max} - s_{FCV,t-1}^{\max}\right) + g_{H2}^{exog} \right]$$

where $s_{FCV,t}^{\max}$ is the maximum share of newly registered HFCVs up to time t and g_{H2}^{exog} is a demand-independent increase in the share, which is greater than zero in the "exogenous hydrogen" scenarios. In general, this equation states that the build-up of stations accelerates if, in the current period, the share of newly registered HFCVs reached a new maximum. Then, the difference in maximum shares affects the share of stations by the factor v.

Miyagawa (2013) tried to find a relationship between the density of refueling stations and refueling availability. His work is based on the random probability of refueling within a specified distance. In order to find refueling availability related to the density of refueling stations, he developed models using the Poisson distribution for three cases: a driver can refuel at 1) both origin (O) and destination (D), 2) either O or D, and 3) neither O nor D.

2.3.3 Dynamic normative model

Landahl (1938) introduced a neural stimuli-response model to explain the basic neural mechanism of a human. He assumed the case in which two stimuli of the same object are simultaneously presented for a given time. Then, the following equations are proposed:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{dt}} = \mathrm{AE}\text{-}\alpha\varepsilon \tag{1}$$

where,

E: the intensity of excitation from a stimulus

ε: the excitation factor

With equation (1), Landahl proposed that the excitation factor ε increases with time at a rate A which is proportional to the excitation intensity E and decreases at a rate α which is proportional to its own concentration. The solution of the equation for constant E is:

$$\varepsilon = \frac{AE}{\alpha}\left(1 - e^{-\alpha t}\right) \tag{2}$$

where we take $\varepsilon = 0$ for t=0 as the initial condition, and where t is the time required for stimulus to take an effect.

Based on Landahl's work, Recker (2012) developed a dynamic normative model for vehicle choice. The basic formulation of the model is as follows:

$$\alpha \frac{d\omega}{dt} + \omega(t) = \Omega(t) \tag{3}$$

where ω is the level of excitement in the neural center in reaction to a particular stimulus of strength Ω, and α is a growth rate parameter that is inversely proportional to the rate at which the final level of neural excitement is achieved. For constant Ω, the equation has the negative exponential growth rate as a solution.

In his application of the model to vehicle choice, Recker assumed that stimuli consist of intrinsic and extrinsic components. Intrinsic stimuli address the perception of changes in a vehicle's characteristics including annual gas cost, and extrinsic stimuli address the perception of changes in proportions of people who will choose a specific vehicle.

In the case of the intrinsic utilities of the alternatives, the dynamics are dictated by exogenous changes to the existing attribute levels $\mathbf{X}_k^{o}$ at $t = t_0$, bringing them to new levels $\mathbf{X}_k^{*}$ for $t > t_0$. Although such changes typically occur instantaneously (e.g., introduction of new models, addition of refueling stations, fuel cost increases, improvements in battery technology), the concordance of an individual's perception of the changes with the actual changes may take some time to develop as a result of a learning process. Denoting by ω_{kj} an individual's perception of the change in the level of the utility of intrinsic attribute j of alternative k, the appropriate dynamic relationship is given by Equation (4).

$$\begin{aligned} a_{kj}\frac{d\omega_{kj}}{dt} + \omega_{kj}(t) = \beta_j \cdot (x_{kj}^{*} - x_{kj}^{\circ}) \cdot H(t - t_{\circ}) \ \ , \ \ \forall x_{kj} \in \mathbf{X}_k \ ,\forall k \in \mathbf{A} \\ \omega_{kj}(t_{\circ}) = 0 \end{aligned} \tag{4}$$

where $H(\cdot)$ is the heaviside function, the β_j are the unit partwise utility weights associated with attribute j, and where a_{kj} is an unknown parameter that is inversely proportional to the rate at which the individual's perception finds concordance with the true value of x_{kj}. The solution to this equation is

$$\omega_{kj}(t) = \beta_j \cdot (x_{kj}^{*} - x_{kj}^{\circ}) \cdot [1 - e^{-(t - t_{\circ})/a_{kj}}] \ , \ \ t > t_{\circ}. \tag{5}$$

It is assumed that the normative component, or extrinsic component of utility can be represented as $V_k^e(\rho_k)$, where ρ_k represents the proportion of the individual's peer group choosing alternative k. Define by $f_k(\mathbf{\Phi}_k^\circ)$ the joint density function for $\mathbf{\Phi}_k^\circ$ at time $t = t_\circ$. Then, the expected proportion choosing alternative k at some time $t > t_\circ$ becomes:

$$\rho_k(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} L[\mathbf{\Phi}_k^\circ + \boldsymbol{\delta}\mathbf{\Phi}_k(t)] \cdot f_k(\mathbf{\Phi}_k^\circ) \cdot d\mathbf{\Phi}_k^\circ . \qquad (6)$$
$$\Phi_k = V_k - V_l \ , k \neq l$$

As a first-order approximation, and consistent with assumptions regarding the intrinsic components of utility, the strength of the stimulus to respond to normative influences is assumed simply to be proportional to ρ_k, the proportion of persons in an individual's peer group who can be expected to choose alternative k at any time t. Then, under similar assumptions regarding the dynamics governing response to these external stimuli,

$$b_k \frac{d\upsilon_k}{dt} + \upsilon_k(t) = \lambda[\rho_k(t) - \rho_k(t_\circ)] \cdot H(t - t_\circ) \ , \ \forall k \in \mathbf{A} \qquad (7)$$
$$\upsilon_k(t_\circ) = 0$$

where λ is an unknown proportionality constant whose magnitude and sense are direct functions of an individual's motivation to comply to social "norms," and where b_k has a similar definition to its counterpart in Equation (4).

With a utility at the initial time, intrinsic stimuli ω_{kj}, and extrinsic stimuli υ_k, the final utility of alternative k is defined as Equation (8).

$$V_k = V_k^0 + \sum_j \omega_{kj} + \upsilon_k \quad (8)$$

In this research, we apply the dynamic normative model above to a dataset of market share for vehicles in order to determine the values of parameters a_{kj}, b_k, and λ, that best track changes in demands for each vehicle.

2.3.4 Competing species model

Models discussed in the previous sections focus on demand and are based on travel survey data obtained at a fixed specific time. These models are generally unsuited in forecasting changes in demand for AFVs and ICEVs over time when the supply side conditions react to demand. As the purpose of this research is to identify those factors, including those associated with changing supply-side conditions, that influence the market penetrations of AFVs over time, it is necessary to take another modeling approach to fulfill this purpose—dynamic simulation models are particularly useful for this purpose.

Although they have not been used in the vehicle demand field, predator-prey models and competing species models—drawn from mathematical biology—may provide a useful platform. Predator-prey models are based on the relationships between predator and prey in natural environments (e.g. birth, death, and so on). The basic structure of a predator-prey system is shown in Figure 1.

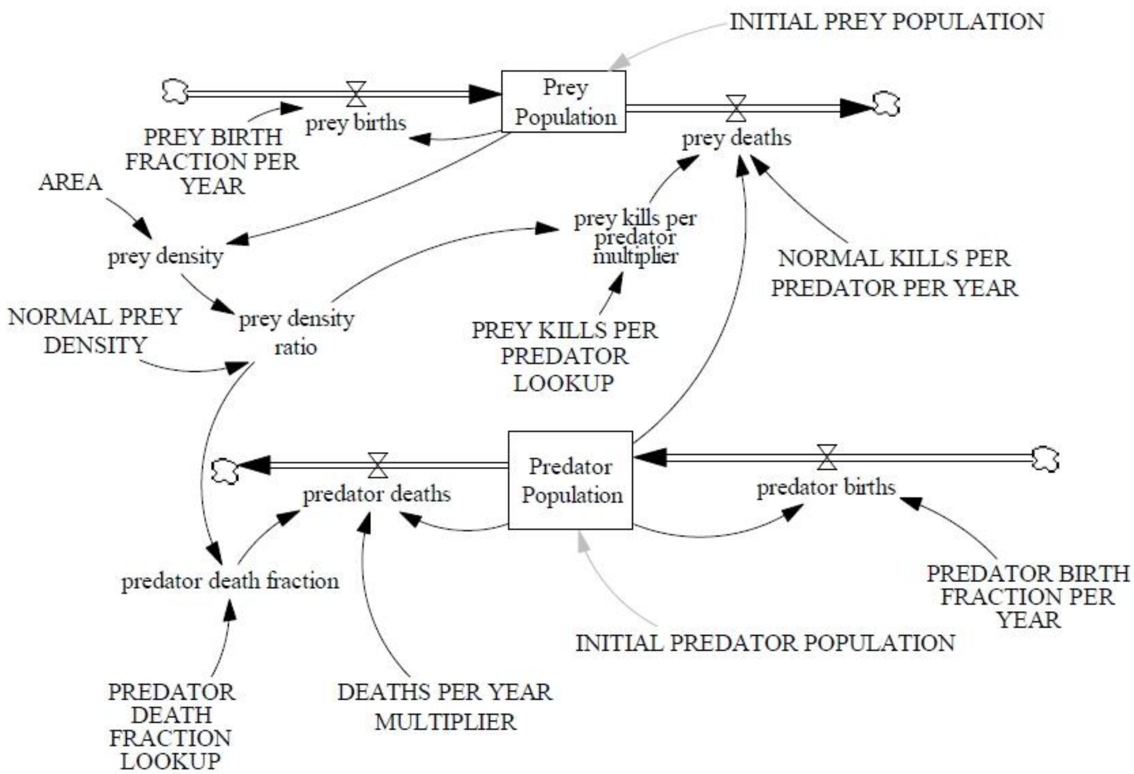

Figure 1. Predator-prey system
Source: Whelan (1994)

In the system shown in Figure 1, if the birth rate of a prey is more than its death rate, the number of prey increases infinitely. However, because a predator kills the prey for survival, the number of the prey never infinitely increases and may even decrease. And, if the number of this prey is too small, the numbers of predators decrease because they don't have enough food to maintain the current number. With repetition of this interaction, the numbers of predator and prey can achieve a balance, or equilibrium (which may be extinction of one, or both). In order to formulate predator-prey system, the following factors need to be considered; number of predator and prey, birth and death rate, and interaction between predator and prey.

According to Smith (1974), in the predator-prey system, there are oscillations in the numbers of predator and prey because of birth and death rates. However, when there are competitive interactions between two species, no oscillation is produced. The concept of competing species is based on competitions among species for survival. Within the predator-prey system, there are competitions among predators for a limited amount of prey. In these competitions, the predator fulfilling certain conditions can take more prey and so can survive. The Lotka-Volterra equation is one of the most well-known competing species models. The basic formula is as follows (May and Leonard (1975)):

$$\frac{dN_i(t)}{dt} = r_i N_i(t)\left[1 - \sum_{j=1}^{n} \alpha_{ij} N_j(t)\right] \tag{9}$$

where $N_i(t)$ is the number of individuals in the *i*th species at time *t*, r_i is the intrinsic growth rate of *i*th species, and α_{ij} is the competition coefficient measuring the extent to which the *j*th species affects the growth rate of the *i*th.

With equation (9), May and Leonard specified the three-competitor system by making assumptions that 1) $r_1 = r_2 = r_3 = r$; 2) with respect to competition, competitor 2 affects competitor 1 as competitor 3 affects 2 as 1 affects 3, (i.e., $\alpha_{12} = \alpha_{23} = \alpha_{31} = \alpha$); 3) similarly $\alpha_{21} = \alpha_{32} = \alpha_{13} = \beta$; and 4) rescale the populations N_i so that $\alpha_{ii} = 1$ and rescale *t* so that *r*=1.

$$\frac{dN_1(t)}{dt} = N_1\left[1 - N_1 - \alpha N_2 - \beta N_3\right] \tag{10}$$

$$\frac{dN_2(t)}{dt} = N_2\left[1 - \beta N_1 - N_2 - \alpha N_3\right] \tag{11}$$

$$\frac{dN_3(t)}{dt} = N_3\left[1 - \alpha N_1 - \beta N_2 - N_3\right] \tag{12}$$

In an analogy to competing species models, in vehicle markets, ICEVs and AFVs become predators, and customers become preys. And, in order to be chosen by a sustainable number of customers, ICEVs and AFVs should compete by innovating themselves. However, because vehicles are not life forms, they cannot control their number in isolation—maximum numbers of vehicles supportable need to be defined. By using these concepts, Redmond (2011) introduced a modified Lotka-Volterra equation, in order to forecast the growth of AFVs in the immediate future.

$$\frac{dN_i(t)}{dt} = r_i N_i(t)\left[1 - \frac{\sum_{j=1}^{n} \alpha_{ij} N_j(t)}{K_i}\right] \tag{13}$$

where N_i is the number of alternative *i*, K_i is the maximum number of alternative *i* that can be supported, r_i is the increase rate of alternative *i*, and α_{ij} are the values of interactions between *i* and *j* $(\alpha_{ii} = 1)$.

The degree of market penetration for vehicles will also be influenced by such external factors as number of customers (considering "birth" and "death"), and the number of refueling or recharging stations. Because AFVs are a new technology, the number of customers and refueling stations are crucial for their market penetration. In particular, the number of refueling stations depends on the demand for AFVs themselves, and so does the number of AFVs. If the number of refueling stations increases, the demand for AFVs will likely also increase. And then the number of stations increases again as a response to this increasing demand. In short, AFVs and stations are in a "chicken and egg" problem. This can be explained with the concept of mutualism. Because AFVs and refueling stations cannot survive without each other, this relationship can be defined as an obligate mutualism, which is classified in Boucher (1985). Focusing on this, Redmond (2011) tried to forecast the growth of AFVs, ICEVs and refueling stations; in the following equations, three alternatives are considered; Hydrogen Fuel Cell Vehicles (HFCVs) (*i*=1), Battery Electric Vehicles (BEVs) (*i*=2), and Internal Combustion Engines (ICEs) (*i*=3).

$$\frac{dN_i(t)}{dt} = r_i N_i(t)\left[1 - \frac{\sum_{j=1}^{n=2}\left(\alpha_{ij} - \alpha_{i3}\right) N_j(t) + \gamma \alpha_{i3} P(t)}{K_i(t)}\right];\ i = 1, 2 \tag{14}$$

$$\frac{dN_i(t)}{dt} = r_i N_i(t)\left[\frac{K_i(t) - N_i(t) + \theta_i R_i(t)}{K_i(t)}\right];\ i = 1, 2 \tag{15}$$

$$\frac{dR_i(t)}{dt} = \rho_i R_i(t)\left[\frac{\Gamma_{i0} + \sigma_i K_i(t) - R_i(t) + \beta_i N_i(t)}{\Gamma_{i0} + \sigma_i K_i(t)}\right];\ i = 1, 2 \tag{16}$$

where N_i is the number of alternative *i*, P is the population of the study area, R_i is the number of refueling stations for alternative *i*, K_i is the maximum number of alternative *i* that can be supported, Γ_{i0} is the number of refueling stations in the initial condition, r_i is the increase rate of alternative *i*, ρ_i is the increase rate of refueling stations, α_{ij} are the values of interactions between *i* and *j* $\left(\alpha_{ii} = 1\right)$, β_i are the values of interactions for refueling stations, γ is the proportionality constant (vehicles per person), θ_i is the influence of alternative *i* on others, and σ_i is a parameter whose value should be given.

In the predator-prey model described above, changes in one or more factors of the competing vehicles will alter the ratios of their respective choices by customers. Then, the principal question is: which factors sway customers to choose certain vehicles, or to change to AFVs? Here, we propose that the answer to question can be guided by a stimuli-response model.

2.4 Summary

There are a number of factors that are presumed to influence the demand for, and the market penetrations of, AFVs. These factors include those related to the vehicles themselves and those related to households and their positions in society. Using these factors, the demand for AFVs can be forecast by using discrete choice models based on SP data. However, discrete choice models are static models, and so dynamic models are necessary to forecast the change of market penetrations over time. Focusing on this, a dynamic normative model based on an individual's stimuli-response was reviewed.. After that, a predator-prey or competing species model, which is from mathematical biology, was also reviewed. Based on these literature reviews, the model for this research will be provided in the next section.

3. Dynamic normative model

3.1 Formulation

Based on the work by Landahl (1938), Recker (2012) developed a dynamic normative model to capture the influences of the so-called “bandwagon effect” as it applies to the growth in the demand for popularized choices. The basic formulation of the model is in the form expressed by Equation (17):

$$\alpha \frac{d\omega}{dt} + \omega(t) = \Omega(t) \tag{17}$$

where ω is the level of excitement in the neural center in reaction to a particular stimulus of strength Ω, and α is a growth rate parameter that is inversely proportional to the rate at which the final level of neural excitement is achieved. For constant Ω, equation (17) has the negative exponential growth rate as a solution.

Recker assumed that stimuli consist of intrinsic and extrinsic components. Intrinsic stimuli address the perception of changes in a vehicle’s characteristics including annual gas cost, and extrinsic stimuli address the perception of changes in proportions of people who will choose a specific vehicle.

In the case of the intrinsic utilities of the alternatives, the dynamics are dictated by exogenous changes to the existing attribute levels $\mathbf{X}_k^{o}$ at $t = t_0$, bringing them to new levels $\mathbf{X}_k^{*}$ for $t > t_0$. Although such changes typically occur instantaneously (e.g., introduction of new models, addition of refueling stations, fuel cost increases, improvements in battery technology), the concordance of an individual’s perception of the changes with the actual changes may take some time to develop as a result of a learning process. Denoting by ω_{kj} an individual’s perception of the change in the level of the utility of intrinsic attribute j of alternative k, the appropriate dynamic relationship is given by Equation (18).

$$a_{kj}\frac{d\omega_{kj}}{dt}+\omega_{kj}(t)=\beta_j\cdot(x_{kj}^{*}-x_{kj}^{\circ})\cdot H(t\text{-}t_{\circ}) \ , \ \forall x_{kj}\in\mathbf{X}_k \ ,\forall k\in\mathbf{A} \qquad (18)$$
$$\omega_{kj}(t_{\circ})=0$$

where $H(\cdot)$ is the heaviside function, the β_j are the unit partwise utility weights associated with attribute j, and where a_{kj} is an unknown parameter that is inversely proportional to the rate at which the individual's perception finds concordance with the true value of x_{kj}. The solution to this equation is

$$\omega_{kj}(t)=\beta_j\cdot(x_{kj}^{*}-x_{kj}^{\circ})\cdot[1-e^{-(t-t_{\circ})/a_{kj}}] \ , \ t>t_{\circ}. \qquad (19)$$

It is assumed that the normative, or extrinsic, component of utility can be represented as $V_k^e(\rho_k)$, where ρ_k represents the proportion of the individual's peer group choosing alternative k. Define by $f_k(\mathbf{\Phi}_k^{\circ})$ the joint density function for $\mathbf{\Phi}_k^{\circ}$ at time $t=t_{\circ}$. Then, the expected proportion choosing alternative k at some time $t>t_{\circ}$ can be represented as:

$$\rho_k(t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}L[\mathbf{\Phi}_k^{\circ}+\boldsymbol{\delta}\mathbf{\Phi}_k(t)]\cdot f_k(\mathbf{\Phi}_k^{\circ})\cdot d\mathbf{\Phi}_k^{\circ}. \qquad (20)$$

$$\Phi_k=V_k-V_l \ , k\neq l$$

As a first-order approximation, and consistent with assumptions regarding the intrinsic components of utility, consider that the strength of the stimulus to respond to normative influences is simply proportional to ρ_k, the proportion of persons in an individual's peer group who can be expected to choose alternative k at any time t. Then, under similar assumptions regarding the dynamics governing response to these external stimuli,

$$b_k \frac{d\upsilon_k}{dt} + \upsilon_k(t) = \lambda[\rho_k(t) - \rho_k(t_\circ)] \cdot H(t - t_\circ) \ , \ \forall k \in \mathbf{A} \qquad (21)$$
$$\upsilon_k(t_\circ) = 0$$

where λ is an unknown proportionality constant whose magnitude and sense are direct functions of an individual's motivation to comply, and where b_k has a similar definition to its counterpart in Equation (18).

With a utility at the initial time, intrinsic stimuli ω_{kj}, and extrinsic stimuli υ_k, the final utility of alternative k is defined as Equation (22).

$$U_k = U_k^0 + \sum_j \omega_{kj} + \upsilon_k \qquad (22)$$

From data for market share for vehicles and the dynamic normative model above, the values of parameters a_{kj}, b_k, and λ, that can track the changes in demands for each vehicle can be found.

3.2 Dataset

Data from the National Household Travel Survey (NHTS) 2009 is used to help establish: 1) relative importance of various factors in choice of vehicles among comparable ICEVs and market-ready AFVs (HEVs), and 2) whether or not the hypothesis of the "bandwagon" effect for

AFVs is plausibly valid. Our analysis focuses on consumer choice among four vehicles: Honda Civic, Toyota Corolla, Honda Civic Hybrid and Toyota Prius—the latter two being the predominant examples of AFVs present in the marketplace. Our key hypothesis is that the data will support that the choice of Toyota Prius (which is readily identifiable as an AFV) has been significantly affected by a dynamic "bandwagon" effect that is not present in the choices among other comparable vehicles and, in particular, not present in the case of the choice of Honda Civic Hybrid which is virtually indistinguishable from its Honda Civic ICE counterpart.

A sample of households in the NHTS 2009 that purchased Honda Civic, Toyota Corolla, Honda Civic Hybrid or Toyota Prius anytime during the years from 2003 through 2008 was chosen for this phase of the analysis. This time period was taken to coincide with the introduction of the Honda Civic Hybrid to the market as a 2003 year model, although Toyota Prius had been introduced prior to this. Although the NHTS data does not have the vehicle purchase year, the period that a vehicle has been owned and the date that the diary was completed are given. In our analysis, the vehicle purchase year is calculated by subtracting the period of owning a vehicle from the date of the diary. Because of lack of precise information, it was assumed that vehicle characteristics (e.g., price, fuel economy) of each vehicle reported pertained to those of a new vehicle of the model year of the year of purchase; i.e., equivalent to assuming that all vehicles were purchased new. The distribution of vehicle purchases contained in our analysis dataset is shown in Table 1; Table 2 provides corresponding data for the whole US population.

TABLE 1 Sample Market Share of Vehicles in Year 2003 Through 2008

Year	Honda Civic	Toyota Corolla	Civic Hybrid	Toyota Prius	Total
2003	118	166	28	32	344
2004	174	220	42	93	529
2005	213	283	41	177	714
2006	305	368	69	221	963
2007	362	411	70	360	1,203
2008	365	355	71	270	1,061
Total	1,537	1,803	321	1,153	4,814

TABLE 2 US Market Share of Vehicles in Year 2003 Through 2012

Year	Honda Civic	Toyota Corolla	Civic Hybrid	Toyota Prius	Total
2003	277,872	325,477	21,800	24,600	649,749
2004	283,625	333,161	25,571	53,991	696,348
2005	282,551	341,290	25,864	107,897	757,602
2006	285,387	387,388	31,251	106,971	810,997
2007	298,520	371,390	32,575	181,221	883,706
2008	307,992	351,007	31,297	158,574	848,870
2009	244,603	296,874	15,119	139,682	696,278
2010	252,882	266,082	7,336	140,928	667,228
2011	216,532	240,259	4,703	136,463	597,957
2012	310,753	290,947	7,156	147,503	756,359
Total	2,760,717	3,203,875	202,672	1,197,830	7,365,094

Source: www.goodcarbadcar.net, www.afdc.energy.gov/data/tab/vehicles/data_set/10301

There are total 4,814 sample observations available for use in the analysis, each of which contains (either explicitly, or via calculation): vehicle mpg, gas price ($/gal.), vehicle price (MSRP-$1,500 rebated price of state incentive for HEVs), drivers' education level and gender. For the extrinsic part of utility (i.e., the potential "bandwagon" influence), the dataset of market share for these vehicles in the whole U.S. area is used as a variable in the model. The concept used here is that the visual presence of the vehicle in the market place can be used as a surrogate for the stimulus associated with the effects of imitative behavior. Comparative analyses are performed on the following pairs of vehicles: Honda Civic vs. Honda Civic Hybrid, Honda Civic vs. Toyota Corolla, Toyota Corolla vs. Toyota Prius, and Honda Civic Hybrid vs. Toyota Prius. Prior to the choice analysis used to confirm/reject our prime hypothesis and to identify relative weights of the components of utility (intrinsic as well as extrinsic) affecting vehicle choice, we present some summary statistics.

1) Honda Civic vs. Honda Civic Hybrid

The relative market shares of Honda Civic and Honda Civic Hybrid from 2003 to 2012 are displayed in Table 3 (for the sample) and in Table 4 (for the US). From these tables, it can be seen that, compared to Honda Civic, the Honda Civic Hybrid has had a substantially lower market share both in the sample, as well as in the population—we note that the market share in the sample is almost double that in the population; a factor that may be attributable to owners of hybrid vehicles being more internally motivated to participate in the travel survey. Although sales of the Honda Civic Hybrid have remained relatively constant (with respect to Honda Civic) during the years for which we have sample data (2003-2008), we note a rather precipitous drop in its market share post 2008 (Table 4, Figure 2).

TABLE 3 Sample of Civic and Civic Hybrid in NHTS 2009

Purchase Year	Civic	Civic Hybrid	Total	Rate (%)	
				Civic	Civic Hybrid
2003	118	28	146	80.8	19.2
2004	174	42	216	80.6	19.4
2005	213	41	254	83.9	16.1
2006	305	69	374	81.6	18.4
2007	362	70	432	83.8	16.2
2008	365	71	436	83.7	16.3

TABLE 4 Market Share of Civic and Civic Hybrid in Year 2003 Through 2012

Year	Civic	Civic Hybrid	Total	Market share (%)	
				Civic	Civic Hybrid
2003	277,872	21,800	299,672	92.7	7.3
2004	283,625	25,571	309,196	91.7	8.3
2005	282,551	25,864	308,415	91.6	8.4
2006	285,387	31,251	316,638	90.1	9.9
2007	298,520	32,575	331,095	90.2	9.8
2008	307,992	31,297	339,289	90.8	9.2
2009	244,603	15,119	259,722	94.2	5.8
2010	252,882	7,336	260,218	97.2	2.8
2011	216,532	4,703	221,235	97.9	2.1
2012	310,753	7,156	317,909	97.7	2.3

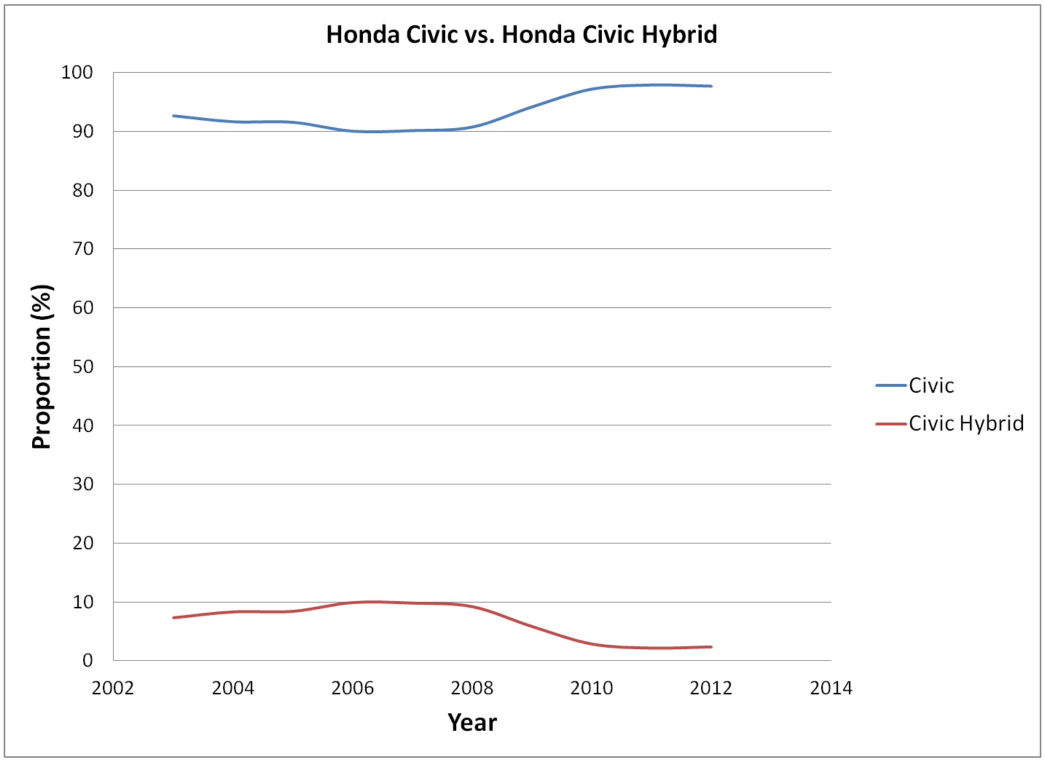

Figure 2. Honda Civic vs. Honda Civic Hybrid Market Trend

2) Honda Civic vs. Toyota Corolla

The sample from NHTS 2009 and the market share of Honda Civic and Toyota Corolla are shown in Table 5 and Table 6. As seen in Table 5, the relative proportion of Civic in our sample has steadily increased since 2003, surpassing that of Corolla in 2008. In contrast in the market (Table 6 and Figure 3), except for year 2012, Corolla has possessed a steady slightly larger market share than Civic.

TABLE 5 Sample of Civic and Corolla in NHTS 2009

Purchase Year	Civic	Corolla	Total	Rate (%)	
				Civic	Corolla
2003	118	166	284	41.5	58.5
2004	174	220	394	44.2	55.8
2005	213	283	496	42.9	57.1
2006	305	368	673	45.3	54.7
2007	362	411	773	46.8	53.2
2008	365	355	720	50.7	49.3

TABLE 6 Market Share of Civic and Corolla in Year 2003 Through 2012

Year	Civic	Corolla	Total	Market share (%)	
				Civic	Corolla
2003	277,872	325,477	603,349	46.1	53.9
2004	283,625	333,161	616,786	46.0	54.0
2005	282,551	341,290	623,841	45.3	54.7
2006	285,387	387,388	672,775	42.4	57.6
2007	298,520	371,390	669,910	44.6	55.4
2008	307,992	351,007	658,999	46.7	53.3
2009	244,603	296,874	541,477	45.2	54.8
2010	252,882	266,082	518,964	48.7	51.3
2011	216,532	240,259	456,791	47.4	52.6
2012	310,753	290,947	601,700	51.6	48.4

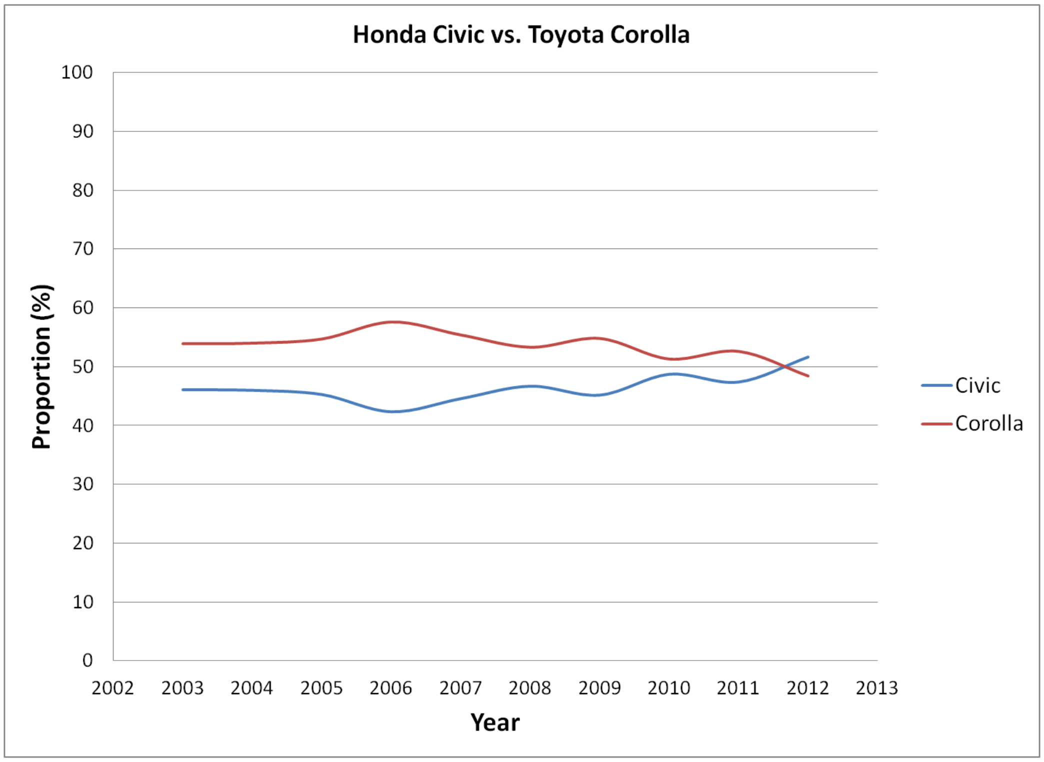

Figure 3. Toyota Corolla vs. Toyota Prius Market Trend

3) Toyota Corolla vs. Toyota Prius

The sample from NHTS 2009 and the market share of Corolla and Prius are in Table 7 and Table 8. It can be seen that the market share of Prius has steadily increased over the years (Figure 4), although it has been smaller than that of Corolla.

TABLE 7 Sample of Corolla and Prius in NHTS 2009

Purchase Year	Corolla	Prius	Total	Rate (%)	
				Corolla	Prius
2003	166	32	198	83.8	16.2
2004	220	93	313	70.3	29.7
2005	283	177	460	61.5	38.5
2006	368	221	589	62.5	37.5
2007	411	360	771	53.3	46.7
2008	355	270	625	56.8	43.2

TABLE 8 Market Share of Corolla and Prius in Year 2003 Through 2012

Year	Corolla	Prius	Total	Market share (%)	
				Corolla	Prius
2003	325,477	24,600	350,077	93.0	7.0
2004	333,161	53,991	387,152	86.1	13.9
2005	341,290	107,897	449,187	76.0	24.0
2006	387,388	106,971	494,359	78.4	21.6
2007	371,390	181,221	552,611	67.2	32.8
2008	351,007	158,574	509,581	68.9	31.1
2009	296,874	139,682	436,556	68.0	32.0
2010	266,082	140,928	407,010	65.4	34.6
2011	240,259	136,463	376,722	63.8	36.2
2012	290,947	147,503	438,450	66.4	33.6

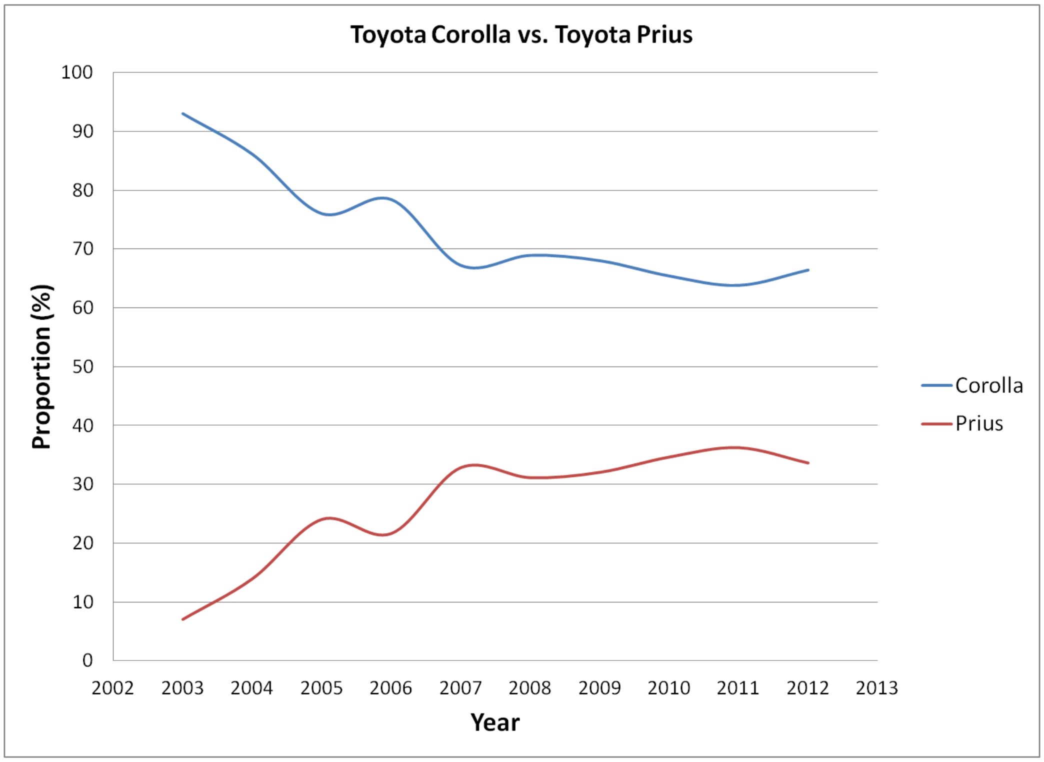

Figure 4. Toyota Corolla vs. Toyota Prius Market Trend

4) Honda Civic Hybrid vs. Toyota Prius

The sample from NHTS 2009 and the market share of Civic Hybrid and Prius are in Table 9 and Table 10, respectively. The relatively dramatic fate of these two HEVs is demonstrated clearly in these tables; the trend shown in Figure 5 offer encouragement that our hypothesis regarding the bandwagon effect is plausible.

TABLE 9 Sample of Civic Hybrid and Prius in NHTS 2009

Purchase Year	Civic Hybrid	Prius	Total	Rate (%)	
				Civic Hybrid	Prius
2003	28	32	60	46.7	53.3
2004	42	93	135	31.1	68.9
2005	41	177	218	18.8	81.2
2006	69	221	290	23.8	76.2
2007	70	360	431	16.3	83.7
2008	71	270	341	20.8	79.2

TABLE 10 Market Share of Civic Hybrid and Prius in Year 2003 Through 2012

Year	Civic Hybrid	Prius	Total	Market share (%)	
				Civic Hybrid	Prius
2003	21,800	24,600	46,400	47.0	53.0
2004	25,571	53,991	79,562	32.1	67.9
2005	25,864	107,897	133,761	19.3	80.7
2006	31,251	106,971	138,222	22.6	77.4
2007	32,575	181,221	213,796	15.2	84.8
2008	31,297	158,574	189,871	16.5	83.5
2009	15,119	139,682	154,801	10.0	90.0
2010	7,336	140,928	148,264	4.9	95.1
2011	4,703	136,463	141,166	3.3	96.7
2012	7,156	147,503	154,659	4.6	95.4

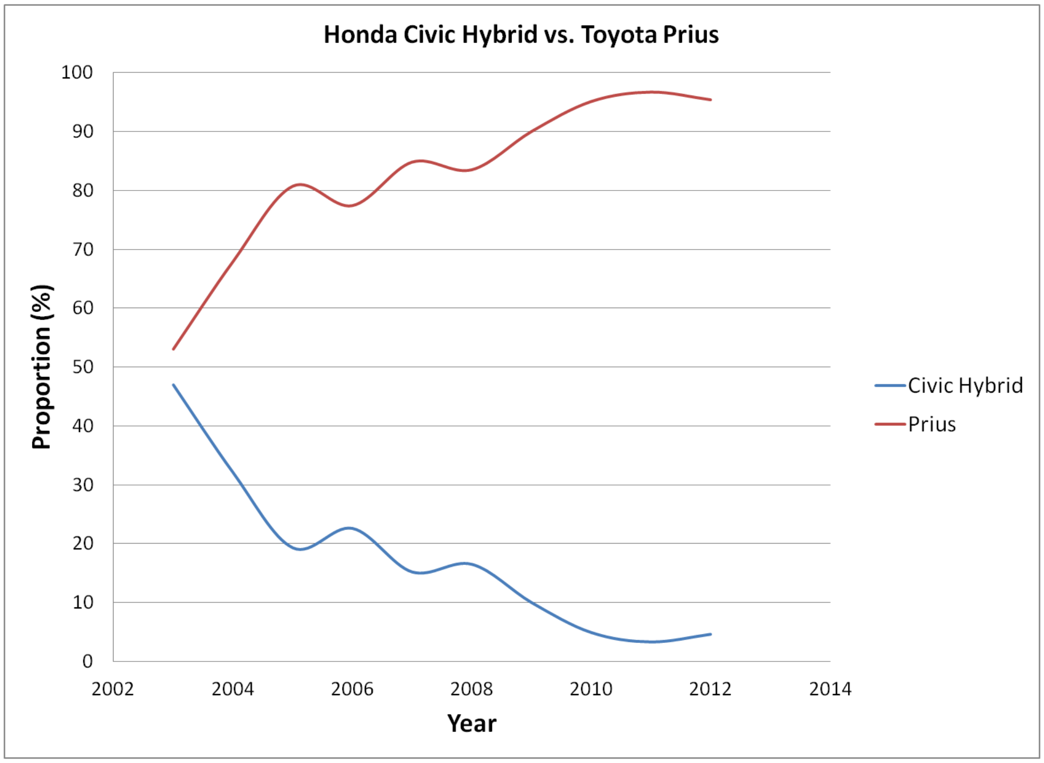

Figure 5. Honda Civic Hybrid vs. Toyota Prius Market Trend

3.3 Discrete Choice Analysis

As stated in previous sections, data involving choice between general AFVs (beyond HEVs) are simply nonexistent. Here, we perform an analysis using discrete choice models to try to gain "order of magnitude" estimates of the roles of various intrinsic characteristics of the vehicles (e.g., fuel efficiency), socio-demographic characteristics (e.g., income), and extrinsic factors (i.e., market share) on purchase choices involving AFVs. Specifically, we use data from NHTS 2009 to estimate binary choice models for three cases: Honda Civic vs. Honda Civic Hybrid, Toyota Corolla vs. Toyota Prius, and Honda Civic Hybrid vs. Toyota Prius. We use these models to: 1) infer the relative magnitudes of various characteristics in determining choice outcomes, and 2) test verification of our hypotheses regarding the dynamic effects of the "bandwagon" phenomenon and to gauge its relative magnitude compared to other, intrinsic, characteristics in the formulation of utility. We test this latter effect by including a "market share" variable which is cast as a surrogate for the impact of popularity on the choice—the idea being that the greater the vehicle's percentage in the population of vehicles the greater the likelihood of an individual encountering (and identifying) the vehicle as being "popular."

3.3.1 Honda Civic vs. Honda Civic Hybrid

Although visually virtually indistinguishable, Honda Civic and Honda Civic Hybrid are different vehicles; Civic Hybrid runs with electricity as well as gasoline, although its performance is same as that of a gasoline vehicle. In this case, one of decision factors for choosing vehicles will be how many "in the neighborhood" already choose Civic Hybrid, for which we use the market share of Civic Hybrid (by year of purchase) as an explanatory variable. Here, we would expect to reject the hypothesis that market share, or popularity, is a significant determinant of choice since the outward distinction between the two is negligible.

In the following, we use the shorthand notations: $\mathbf{A}=\{\text{Civic},\text{Civic Hybrid}\}=\{C, CH\}$, with descriptive attributes, $\mathbf{X}=\{C, G, P, E, S\}$, where $C=$constant $G=$annual gas cost/household income, $P=$vehicle price/household income, $E=$education level, and $S=$gender (1 for male and 0 for female); M_k is used to designate an alternative specific constant.

Assuming an annual travel distance of 15,000 miles, using mpg and gas price, annual gas cost can be calculated with Equation (23).

$$\text{Annual gas cost (\$)}=\frac{15,000\text{miles}}{\text{mpg}}\times\text{gas price (\$/gal.)} \tag{23}$$

The corresponding static deterministic utilities are then

$$V_C = \beta_C C_C + \lambda \rho_{CH} + \beta_{\mathrm{G}} G_{\mathrm{C}} + \beta_{\mathrm{P}} P_{\mathrm{C}} + \beta_E E_C + \beta_S S_{\mathrm{C}}$$
$$V_{CH} = \beta_{\mathrm{G}} G_{CH} + \beta_{\mathrm{P}} P_{CH} \tag{24}$$

where $C_C \equiv 1$ and where ρ_{CH} is the market share of Civic Hybrid.

The results of the estimation are displayed in Table 11; variable coefficient t scores are shown in parentheses.

TABLE 11 Coefficients of Discrete Choice Model: Civic vs. Civic Hybrid

Model: Civic vs. Civic Hybrid	Value of Coefficient	Coefficient Symbol	M_k	
			Civic	Civic Hybrid
Constant	0.243748 (0.1179)	β_C	1	0
Market share of Civic Hybrid	-43.541694 (-0.7830)	λ	1	0
Annual gas cost/household income	-175.749725 (-15.1699)	β_G	N.A.	N.A.
Vehicle price/household income	-80.821102 (-17.3419)	β_P	N.A.	N.A.
Education level	-0.125358 (-1.3661)	β_E	1	0
Gender	-0.064565 (-0.3519)	β_S	1	0
Likelihood	-400.73			
R squared	0.53139			
Likelihood Ratio test	908.82			

Then, based on the estimation shown in Table 11,

$$V_C = 0.244 - 43.542\rho_{CH} - 175.750G_{\mathrm{C}} - 80.821P_{\mathrm{C}} - 0.125E_C - 0.065S_{\mathrm{C}}$$
$$V_{CH} = -175.750G_{CH} - 80.821\rho_{CH} \tag{25}$$

As expected, the results indicate that the effect of imitative behavior (bandwagon), as represented in the surrogate variable ρ_{CH} tests insignificant for this choice situation.

3.3.2 Toyota Corolla vs. Toyota Prius

This case is the same as Civic vs. Civic Hybrid, with the market share of Prius used as a variable. Utilities for these vehicles are as Equation (26).

$$\begin{aligned} V_{CO} &= \beta_C C_{CO} + \lambda \rho_P + \beta_{\mathrm{G}} G_{\mathrm{CO}} + \beta_{\mathrm{P}} P_{\mathrm{CO}} + \beta_E E_{CO} + \beta_S S_{\mathrm{CO}} \\ V_P &= \beta_{\mathrm{G}} G_P + \beta_{\mathrm{P}} P_P \end{aligned} \tag{26}$$

where $C_{CO} \equiv 1$ and where ρ_P is the market share of Prius. The estimation results are displayed in Table 12.

TABLE 12 Coefficients of Discrete Choice Model: Corolla vs. Prius

Model: Corolla vs. Prius	Value of Coefficient	Coefficient Symbol	M_k	
			Corolla	Prius
Constant	1.269303 (4.2429)	β_C	1	0
Market share of Prius	-10.139152 (-13.9814)	λ	1	0
Annual gas cost/household income	-153.448539 (-15.2407)	β_G	N.A.	N.A.
Vehicle price/household income	-50.508068 (-23.2628)	β_P	N.A.	N.A.
Education level	-0.269733 (-5.2190)	β_E	1	0
Gender	-0.065641 (-0.6072)	β_S	1	0
Likelihood	-1116.7			
R squared	0.43513			
Likelihood Ratio test	1720.4			

Then, based on the estimation from Table 12,

$$\begin{aligned} V_{CO} &= 1.269 - 10.139\rho_P - 153.449G_{\mathrm{CO}} - 50.508P_{\mathrm{CO}} - 0.270E_{CO} - 0.066S_{\mathrm{CO}} \\ V_P &= -153.449G_P - 50.508P_P \end{aligned} \tag{27}$$

Here, we see that the market share of the Prius has a relative positive impact on the probability of choice of Prius (equivalently decreasing the utility of Corolla), confirming our hypothesis of the bandwagon effect.

3.3.3 Honda Civic Hybrid vs. Toyota Prius

The Honda Civic Hybrid and Toyota Prius are HEVs. Similar to their conventional counterparts, Civic and Corolla, they also are similar in performance and price. Moreover, both have less gas cost than do ICEs. In this case, we propose that the market share of one of these HEV's will be an influential factor on the choice between these two vehicles; the market share of Prius is used as a variable. Utilities for these vehicles are as Equation (28).

$$\begin{aligned} V_{CH} &= \beta_C C_{CH} + \lambda\rho_P + \beta_{\mathrm{G}} G_{\mathrm{CH}} + \beta_{\mathrm{P}} P_{\mathrm{CH}} + \beta_E E_{CH} + \beta_S S_{\mathrm{CH}} \\ V_P &= \beta_{\mathrm{G}} G_P \end{aligned} \tag{28}$$

where $C_{CH} \equiv 1$ and where ρ_P is the market share of Prius.

TABLE 13 Coefficients of Discrete Choice Model: Civic Hybrid vs. Prius

Model: Civic Hybrid vs. Prius	Value of Coefficient	Coefficient Symbol	M_k	
			Civic Hybrid	Prius
Constant	3.090622 (4.4606)	β_C	1	0
Market share of Prius	-4.741397 (-6.0791)	λ	1	0
Annual gas cost/household income	14.507632 (0.8894)	β_G	N.A.	N.A.
Vehicle price/household income	N.A.	N.A.	N.A.	N.A.
Education level	-0.189312 (-2.9367)	β_E	1	0
Gender	0.242450 (1.8827)	β_S	1	0
Likelihood	-749.52			
R squared	0.029729			
Likelihood Ratio test	45.931			

Then, based on the estimation from Table 13,

$$\begin{aligned} V_{CH} &= 3.091 - 4.741\rho_P + 14.508G_{\mathrm{CH}} - 0.189E_{CH} + 0.242S_{\mathrm{CH}} \\ V_P &= 14.508G_P \end{aligned} \tag{29}$$

Once again, we see that the market share of the Prius has a relative positive impact on the probability of choice of Prius (equivalently decreasing the utility of Civic Hybrid), confirming our hypothesis of the bandwagon effect. However, as expected the influence of the bandwagon effect for these two HEVs is less than in the case of Prius vs. the Corolla ICE.

3.3.4 Summary

The results above generally support the hypothesis that there is a “bandwagon” effect on an individual’s choice of a distinctively different HEV; it is found that the market share of a vehicle has a significant effect on a choice of the opposite vehicle, except for the case of Civic vs. Civic

Hybrid—two vehicles that are virtually indistinguishable to the motoring public. In the next section, we present a potential mechanism for capturing such effects, focused on using the results obtained for the case of Toyota Corolla and Toyota Prius as a base for our comparison of dynamic, imitative behavior.

3.4 Fitting the dynamic normative model

Consider the case of two alternatives, $\mathbf{A} = \{\text{Corolla}, \text{Prius}\} = \{C, P\}$, with five descriptive attributes, $\mathbf{X} = \{C, G, P, E, S\}$, where $C =$ constant $G =$ annual gas cost/household income, $P =$ vehicle price/household income, $E =$ education level, and $S =$ gender. Define $\{C_C^0, G_C^0, P_C^0, E_C^0, S_C^0\}$ and $\{C_C^*, G_C^*, P_C^*, E_C^*, S_C^*\}$ as the values of the intrinsic descriptive attributes of the Toyota Corolla at time $t = 0$ and $t = t^*$, respectively; similarly, define $\{C_P^0, G_P^0, P_P^0, E_P^0, S_P^0\}$ and $\{C_P^*, G_P^*, P_P^*, E_P^*, S_P^*\}$ as the values of the intrinsic descriptive attributes of the Toyota Prius at time $t = 0$ and $t = t^*$, respectively. (NOTE: $C_k^0 \equiv C_k^*, E_k^0 \equiv E_k^*, S_k^0 \equiv S_k^*;\ k = C, P$.) Also denote $\rho_C(0), \rho_P(0)$ as the initial values of the market shares of Corolla and Prius, respectively. Then,

$$\mathbf{A}=\left[\begin{array}{c|c}\mathbf{A}_1 & \mathbf{0}\\ \hline \mathbf{0} & \mathbf{A}_2\end{array}\right]=\left[\begin{array}{ccccc|ccccc} a_{C,C} & & & & & & & & & \\ & a_{C,G} & & & & & & & & \\ & & a_{C,P} & & & & & \mathbf{0} & & \\ & & & a_{C,E} & & & & & & \\ & & & & a_{C,S} & & & & & \\ \hline & & & & & a_{P,C} & & & & \\ & & & & & & a_{P,G} & & & \\ & & \mathbf{0} & & & & & a_{P,P} & & \\ & & & & & & & & a_{P,E} & \\ & & & & & & & & & a_{P,S} \end{array}\right],\ \boldsymbol{\Omega}=\left[\begin{array}{c}\boldsymbol{\Omega}_1\\ \hline \boldsymbol{\Omega}_2\end{array}\right]=\left[\begin{array}{c}\omega_{C,C}\\ \omega_{C,G}\\ \omega_{C,P}\\ \omega_{C,E}\\ \omega_{C,S}\\ \hline \omega_{P,C}\\ \omega_{P,G}\\ \omega_{P,P}\\ \omega_{P,E}\\ \omega_{P,S}\end{array}\right],\ \mathbf{X}=\left[\begin{array}{c}\mathbf{X}_1\\ \hline \mathbf{X}_2\end{array}\right]=\left[\begin{array}{c}C_C\\ G_C\\ P_C\\ E_C\\ S_C\\ \hline C_P\\ G_P\\ P_P\\ E_P\\ S_P\end{array}\right]$$

$$\boldsymbol{\delta}\mathbf{Z}=\left[\begin{array}{c}\boldsymbol{\delta}\mathbf{Z}_1\\ \hline \boldsymbol{\delta}\mathbf{Z}_2\end{array}\right]=\left[\begin{array}{c}\beta_C\left(C_C^*-C_C^0\right)\\ \beta_G\left(G_C^*-G_C^0\right)\\ \beta_P\left(P_C^*-P_C^0\right)\\ \beta_E\left(E_C^*-E_C^0\right)\\ \beta_S\left(S_C^*-S_C^0\right)\\ \hline \beta_C\left(C_P^*-C_P^0\right)\\ \beta_G\left(G_P^*-G_P^0\right)\\ \beta_P\left(P_P^*-P_P^0\right)\\ \beta_E\left(E_P^*-E_P^0\right)\\ \beta_S\left(S_P^*-S_P^0\right)\end{array}\right] \tag{30}$$

$$\boldsymbol{\Omega}(t)=\left(\mathbf{I}-e^{-(t-t_\circ)\cdot\mathbf{A}^{-1}}\right)\cdot\boldsymbol{\delta}\mathbf{Z}\ ,\ \ t>t_\circ \tag{31}$$

$$
\mathbf{A}^{-1} = \left[\begin{array}{ccccc:ccccc}
a_{C,C}^{-1} & & & & & & & & & \\
 & a_{C,G}^{-1} & & & & & & & & \\
 & & a_{C,P}^{-1} & & & & & \mathbf{0} & & \\
 & & & a_{C,E}^{-1} & & & & & & \\
 & & & & a_{C,S}^{-1} & & & & & \\
\hdashline
 & & & & & a_{P,C}^{-1} & & & & \\
 & & & & & & a_{P,G}^{-1} & & & \\
 & & \mathbf{0} & & & & & a_{P,P}^{-1} & & \\
 & & & & & & & & a_{P,E}^{-1} & \\
 & & & & & & & & & a_{P,S}^{-1}
\end{array}\right]
$$

$$
e^{-(t-t_0)\mathbf{A}^{-1}} = \left[\begin{array}{ccccc:ccccc}
e^{-(t-t_0)a_{C,C}^{-1}} & & & & & & & & & \\
 & e^{-(t-t_0)a_{C,G}^{-1}} & & & & & & & & \\
 & & e^{-(t-t_0)a_{C,P}^{-1}} & & & & & \mathbf{0} & & \\
 & & & e^{-(t-t_0)a_{C,E}^{-1}} & & & & & & \\
 & & & & e^{-(t-t_0)a_{C,S}^{-1}} & & & & & \\
\hdashline
 & & & & & e^{-(t-t_0)a_{P,C}^{-1}} & & & & \\
 & & & & & & e^{-(t-t_0)a_{P,G}^{-1}} & & & \\
 & & \mathbf{0} & & & & & e^{-(t-t_0)a_{P,P}^{-1}} & & \\
 & & & & & & & & e^{-(t-t_0)a_{P,E}^{-1}} & \\
 & & & & & & & & & e^{-(t-t_0)a_{P,S}^{-1}}
\end{array}\right]
$$

$$
\mathbf{\Omega}(t) = \left[\begin{array}{ccccc:ccccc}
1-e^{-(t-t_0)a_{C,C}^{-1}} & & & & & & & & & \\
 & 1-e^{-(t-t_0)a_{C,G}^{-1}} & & & & & & & & \\
 & & 1-e^{-(t-t_0)a_{C,P}^{-1}} & & & & & \mathbf{0} & & \\
 & & & 1-e^{-(t-t_0)a_{C,E}^{-1}} & & & & & & \\
 & & & & 1-e^{-(t-t_0)a_{C,S}^{-1}} & & & & & \\
\hdashline
 & & & & & 1-e^{-(t-t_0)a_{P,C}^{-1}} & & & & \\
 & & & & & & 1-e^{-(t-t_0)a_{P,G}^{-1}} & & & \\
 & & \mathbf{0} & & & & & 1-e^{-(t-t_0)a_{P,P}^{-1}} & & \\
 & & & & & & & & 1-e^{-(t-t_0)a_{P,E}^{-1}} & \\
 & & & & & & & & & 1-e^{-(t-t_0)a_{P,S}^{-1}}
\end{array}\right] \cdot \delta\mathbf{Z}\ , \quad t > t_\circ
$$

$$
\mathbf{\Omega}(t)=\left[\begin{array}{ccccc|ccccc}
1-e^{-(t-t_0)a_{C,C}^{-1}} & & & & & & & & & \\
 & 1-e^{-(t-t_0)a_{C,G}^{-1}} & & & & & & & & \\
 & & 1-e^{-(t-t_0)a_{C,P}^{-1}} & & & & & \mathbf{0} & & \\
 & & & 1-e^{-(t-t_0)a_{C,E}^{-1}} & & & & & & \\
 & & & & 1-e^{-(t-t_0)a_{C,S}^{-1}} & & & & & \\
\hline
 & & & & & 1-e^{-(t-t_0)a_{P,C}^{-1}} & & & & \\
 & & & & & & 1-e^{-(t-t_0)a_{P,G}^{-1}} & & & \\
 & & \mathbf{0} & & & & & 1-e^{-(t-t_0)a_{P,P}^{-1}} & & \\
 & & & & & & & & 1-e^{-(t-t_0)a_{P,E}^{-1}} & \\
 & & & & & & & & & 1-e^{-(t-t_0)a_{P,S}^{-1}}
\end{array}\right]\cdot\left[\begin{array}{c}
\beta_C\left(C_C^{*}-C_C^{0}\right)\\
\beta_G\left(G_C^{*}-G_C^{0}\right)\\
\beta_P\left(P_C^{*}-P_C^{0}\right)\\
\beta_E\left(E_C^{*}-E_C^{0}\right)\\
\beta_S\left(S_C^{*}-S_C^{0}\right)\\
\hline
\beta_C\left(C_P^{*}-C_P^{0}\right)\\
\beta_G\left(G_P^{*}-G_P^{0}\right)\\
\beta_P\left(P_P^{*}-P_P^{0}\right)\\
\beta_E\left(E_P^{*}-E_P^{0}\right)\\
\beta_S\left(S_P^{*}-S_P^{0}\right)
\end{array}\right],\quad t>t_{\circ}
$$

$$
\mathbf{\Omega}(t)=\left[\begin{array}{c}
\beta_C\left(C_C^{*}-C_C^{0}\right)\left(1-e^{-(t-t_0)a_{C,C}^{-1}}\right)\\
\beta_G\left(G_C^{*}-G_C^{0}\right)\left(1-e^{-(t-t_0)a_{C,G}^{-1}}\right)\\
\beta_P\left(P_C^{*}-P_C^{0}\right)\left(1-e^{-(t-t_0)a_{C,P}^{-1}}\right)\\
\beta_E\left(E_C^{*}-E_C^{0}\right)\left(1-e^{-(t-t_0)a_{C,E}^{-1}}\right)\\
\beta_S\left(S_C^{*}-S_C^{0}\right)\left(1-e^{-(t-t_0)a_{C,S}^{-1}}\right)\\
\hline
\beta_C\left(C_P^{*}-C_P^{0}\right)\left(1-e^{-(t-t_0)a_{P,C}^{-1}}\right)\\
\beta_G\left(G_P^{*}-G_P^{0}\right)\left(1-e^{-(t-t_0)a_{P,G}^{-1}}\right)\\
\beta_P\left(P_P^{*}-P_P^{0}\right)\left(1-e^{-(t-t_0)a_{P,P}^{-1}}\right)\\
\beta_E\left(E_P^{*}-E_P^{0}\right)\left(1-e^{-(t-t_0)a_{P,E}^{-1}}\right)\\
\beta_S\left(S_P^{*}-S_P^{0}\right)\left(1-e^{-(t-t_0)a_{P,S}^{-1}}\right)
\end{array}\right],\quad t>t_{\circ} \tag{32}
$$

$$
\mathbf{\Phi}_k(t)=\mathbf{\Delta}_k\cdot\left(\tilde{\mathbf{D}}'\cdot\mathbf{\Omega}(t)+\mathbf{\Psi}(t)+\mathbf{V}^{\mathrm{o}}\right)
$$

$$
\underset{nq\times n}{\tilde{\mathbf{D}}}=\left[\begin{array}{c|c|c|c}
\underset{q\times 1}{\mathbf{1}} & \underset{q\times 1}{\mathbf{0}} & & \underset{q\times 1}{\mathbf{0}}\\
\hline
\underset{q\times 1}{\mathbf{0}} & \underset{q\times 1}{\mathbf{1}} & & \\
\hline
 & & \ddots & \underset{q\times 1}{\mathbf{0}}\\
\hline
\underset{q\times 1}{\mathbf{0}} & & \underset{q\times 1}{\mathbf{0}} & \underset{q\times 1}{\mathbf{1}}
\end{array}\right]=\left[\begin{array}{c|c}
1 & 0\\
1 & 0\\
1 & 0\\
1 & 0\\
1 & 0\\
\hline
0 & 1\\
0 & 1\\
0 & 1\\
0 & 1\\
0 & 1
\end{array}\right];\ \tilde{\mathbf{D}}'=\left[\begin{array}{ccccc|ccccc}
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0\\
\hline
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1
\end{array}\right]
$$

$$
\mathbf{\Delta}_k = \begin{bmatrix}
-\delta_{11} & -\delta_{12} & \cdots & -\delta_{1,k-1} & \delta_{kk} & -\delta_{1,k+1} & \cdots & -\delta_{1n} \\
-\delta_{21} & -\delta_{22} & \cdots & -\delta_{2,k-1} & \delta_{kk} & -\delta_{2,k+1} & \cdots & -\delta_{2n} \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \\
-\delta_{k-1,1} & -\delta_{k-1,2} & \cdots & -\delta_{k-1,k-1} & \delta_{kk} & -\delta_{k-1,k+1} & \cdots & -\delta_{k-1,n} \\
-\delta_{k+1,1} & -\delta_{k+1,2} & \cdots & -\delta_{k+1,k-1} & \delta_{kk} & -\delta_{k+1,k+1} & \cdots & -\delta_{k+1,n} \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
-\delta_{n1} & -\delta_{n2} & \cdots & -\delta_{n,k-1} & \delta_{kk} & -\delta_{n,k+1} & \cdots & -\delta_{n,n}
\end{bmatrix}
$$

$$
\mathbf{\Delta}_C = \begin{bmatrix} 1 & -1 \end{bmatrix}
$$

$$
\tilde{\mathbf{D}}' \cdot \mathbf{\Omega}(t) = \left[\begin{array}{ccccc:ccccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{array}\right] \cdot \left[\begin{array}{c}
\beta_C\left(C_C^* - C_C^0\right)\left(1 - e^{-(t-t_0)a_{C,C}^{-1}}\right) \\
\beta_G\left(G_C^* - G_C^0\right)\left(1 - e^{-(t-t_0)a_{C,G}^{-1}}\right) \\
\beta_P\left(P_C^* - P_C^0\right)\left(1 - e^{-(t-t_0)a_{C,P}^{-1}}\right) \\
\beta_E\left(E_C^* - E_C^0\right)\left(1 - e^{-(t-t_0)a_{C,E}^{-1}}\right) \\
\beta_S\left(S_C^* - S_C^0\right)\left(1 - e^{-(t-t_0)a_{C,S}^{-1}}\right) \\
\hdashline
\beta_C\left(C_P^* - C_P^0\right)\left(1 - e^{-(t-t_0)a_{P,C}^{-1}}\right) \\
\beta_G\left(G_P^* - G_P^0\right)\left(1 - e^{-(t-t_0)a_{P,G}^{-1}}\right) \\
\beta_P\left(P_P^* - P_P^0\right)\left(1 - e^{-(t-t_0)a_{P,P}^{-1}}\right) \\
\beta_E\left(E_P^* - E_P^0\right)\left(1 - e^{-(t-t_0)a_{P,E}^{-1}}\right) \\
\beta_S\left(S_P^* - S_P^0\right)\left(1 - e^{-(t-t_0)a_{P,S}^{-1}}\right)
\end{array}\right], \quad t > t_\circ
$$

$$
= \begin{bmatrix}
\beta_C\left(C_C^* - C_C^0\right)\left(1 - e^{-(t-t_0)a_{C,C}^{-1}}\right) + \beta_G\left(G_C^* - G_C^0\right)\left(1 - e^{-(t-t_0)a_{C,G}^{-1}}\right) + \beta_P\left(P_C^* - P_C^0\right)\left(1 - e^{-(t-t_0)a_{C,P}^{-1}}\right) + \beta_E\left(E_C^* - E_C^0\right)\left(1 - e^{-(t-t_0)a_{C,E}^{-1}}\right) + \beta_S\left(S_C^* - S_C^0\right)\left(1 - e^{-(t-t_0)a_{C,S}^{-1}}\right) \\
\beta_C\left(C_P^* - C_P^0\right)\left(1 - e^{-(t-t_0)a_{P,C}^{-1}}\right) + \beta_G\left(G_P^* - G_P^0\right)\left(1 - e^{-(t-t_0)a_{P,G}^{-1}}\right) + \beta_P\left(P_P^* - P_P^0\right)\left(1 - e^{-(t-t_0)a_{P,P}^{-1}}\right) + \beta_E\left(E_P^* - E_P^0\right)\left(1 - e^{-(t-t_0)a_{P,E}^{-1}}\right) + \beta_S\left(S_P^* - S_P^0\right)\left(1 - e^{-(t-t_0)a_{P,S}^{-1}}\right)
\end{bmatrix}
$$

$$
\mathbf{\Delta}_C \cdot \tilde{\mathbf{D}}' \cdot \mathbf{\Omega}(t) = \begin{bmatrix} 1 & -1 \end{bmatrix} \cdot \begin{bmatrix}
\beta_C\left(C_C^* - C_C^0\right)\left(1 - e^{-(t-t_0)a_{C,C}^{-1}}\right) + \beta_G\left(G_C^* - G_C^0\right)\left(1 - e^{-(t-t_0)a_{C,G}^{-1}}\right) + \beta_P\left(P_C^* - P_C^0\right)\left(1 - e^{-(t-t_0)a_{C,P}^{-1}}\right) + \beta_E\left(E_C^* - E_C^0\right)\left(1 - e^{-(t-t_0)a_{C,E}^{-1}}\right) + \beta_S\left(S_C^* - S_C^0\right)\left(1 - e^{-(t-t_0)a_{C,S}^{-1}}\right) \\
\beta_C\left(C_P^* - C_P^0\right)\left(1 - e^{-(t-t_0)a_{P,C}^{-1}}\right) + \beta_G\left(G_P^* - G_P^0\right)\left(1 - e^{-(t-t_0)a_{P,G}^{-1}}\right) + \beta_P\left(P_P^* - P_P^0\right)\left(1 - e^{-(t-t_0)a_{P,P}^{-1}}\right) + \beta_E\left(E_P^* - E_P^0\right)\left(1 - e^{-(t-t_0)a_{P,E}^{-1}}\right) + \beta_S\left(S_P^* - S_P^0\right)\left(1 - e^{-(t-t_0)a_{P,S}^{-1}}\right)
\end{bmatrix}
$$

$$
\begin{aligned}
&= \beta_C\left(C_C^* - C_C^0\right)\left(1 - e^{-(t-t_0)a_{C,C}^{-1}}\right) + \beta_G\left(G_C^* - G_C^0\right)\left(1 - e^{-(t-t_0)a_{C,G}^{-1}}\right) + \beta_P\left(P_C^* - P_C^0\right)\left(1 - e^{-(t-t_0)a_{C,P}^{-1}}\right) + \beta_E\left(E_C^* - E_C^0\right)\left(1 - e^{-(t-t_0)a_{C,E}^{-1}}\right) + \beta_S\left(S_C^* - S_C^0\right)\left(1 - e^{-(t-t_0)a_{C,S}^{-1}}\right) \\
&\quad - \beta_C\left(C_P^* - C_P^0\right)\left(1 - e^{-(t-t_0)a_{P,C}^{-1}}\right) - \beta_G\left(G_P^* - G_P^0\right)\left(1 - e^{-(t-t_0)a_{P,G}^{-1}}\right) - \beta_P\left(P_P^* - P_P^0\right)\left(1 - e^{-(t-t_0)a_{P,P}^{-1}}\right) - \beta_E\left(E_P^* - E_P^0\right)\left(1 - e^{-(t-t_0)a_{P,E}^{-1}}\right) - \beta_S\left(S_P^* - S_P^0\right)\left(1 - e^{-(t-t_0)a_{P,S}^{-1}}\right)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{\Phi}_C &= \phi_C - \phi_P = \mathbf{\Delta}_C \cdot \left(\tilde{\mathbf{D}}' \cdot \mathbf{\Omega}(t) + \mathbf{\Psi}(t) + \mathbf{V}^o\right) \\
&= \beta_C\left(C_C^* - C_C^0\right)\left(1 - e^{-(t-t_0)a_{C,C}^{-1}}\right) + \beta_G\left(G_C^* - G_C^0\right)\left(1 - e^{-(t-t_0)a_{C,G}^{-1}}\right) + \beta_P\left(P_C^* - P_C^0\right)\left(1 - e^{-(t-t_0)a_{C,P}^{-1}}\right) + \beta_E\left(E_C^* - E_C^0\right)\left(1 - e^{-(t-t_0)a_{C,E}^{-1}}\right) + \beta_S\left(S_C^* - S_C^0\right)\left(1 - e^{-(t-t_0)a_{C,S}^{-1}}\right) \\
&\quad - \beta_C\left(C_P^* - C_P^0\right)\left(1 - e^{-(t-t_0)a_{P,C}^{-1}}\right) - \beta_G\left(G_P^* - G_P^0\right)\left(1 - e^{-(t-t_0)a_{P,G}^{-1}}\right) - \beta_P\left(P_P^* - P_P^0\right)\left(1 - e^{-(t-t_0)a_{P,P}^{-1}}\right) - \beta_E\left(E_P^* - E_P^0\right)\left(1 - e^{-(t-t_0)a_{P,E}^{-1}}\right) - \beta_S\left(S_P^* - S_P^0\right)\left(1 - e^{-(t-t_0)a_{P,S}^{-1}}\right) + \left(\upsilon_C - \upsilon_P\right) + \phi_C^o
\end{aligned}
$$

$$
\mathbf{\Phi}_C = \tau(t) + \left(\upsilon_C(t) - \upsilon_P(t)\right) + \Phi_C^o \qquad (33)
$$

where

$$
\begin{aligned}
\tau(t) = {} & \beta_C\left(C_C^* - C_C^0\right)\left(1-e^{-(t-t_0)a_{C,C}^{-1}}\right) + \beta_G\left(G_C^* - G_C^0\right)\left(1-e^{-(t-t_0)a_{C,G}^{-1}}\right) + \beta_P\left(P_C^* - P_C^0\right)\left(1-e^{-(t-t_0)a_{C,P}^{-1}}\right) + \beta_E\left(E_C^* - E_C^0\right)\left(1-e^{-(t-t_0)a_{C,E}^{-1}}\right) + \beta_S\left(S_C^* - S_C^0\right)\left(1-e^{-(t-t_0)a_{C,S}^{-1}}\right) \\
& -\beta_C\left(C_P^* - C_P^0\right)\left(1-e^{-(t-t_0)a_{P,C}^{-1}}\right) - \beta_G\left(G_P^* - G_P^0\right)\left(1-e^{-(t-t_0)a_{P,G}^{-1}}\right) - \beta_P\left(P_P^* - P_P^0\right)\left(1-e^{-(t-t_0)a_{P,P}^{-1}}\right) - \beta_E\left(E_P^* - E_P^0\right)\left(1-e^{-(t-t_0)a_{P,E}^{-1}}\right) - \beta_S\left(S_P^* - S_P^0\right)\left(1-e^{-(t-t_0)a_{P,S}^{-1}}\right)
\end{aligned}
$$

$$
\mathbf{B} = \begin{bmatrix} b_1 & 0 & & 0 \\ 0 & b_2 & & \\ & & \ddots & 0 \\ 0 & & 0 & b_n \end{bmatrix} , \quad \mathbf{\Psi}(t) = \begin{bmatrix} \upsilon_1(t) \\ \upsilon_2(t) \\ \vdots \\ \upsilon_n(t) \end{bmatrix} , \quad \mathbf{\Theta}(t) = \begin{bmatrix} \rho_1(t) \\ \rho_2(t) \\ \vdots \\ \rho_n(t) \end{bmatrix} = \begin{bmatrix} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} L(\mathbf{\Phi}_1)\cdot f_1(\mathbf{\Phi}_1^\circ)\cdot d\mathbf{\Phi}_1^\circ \\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} L(\mathbf{\Phi}_2)\cdot f_2(\mathbf{\Phi}_2^\circ)\cdot d\mathbf{\Phi}_2^\circ \\ \vdots \\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} L(\mathbf{\Phi}_n)\cdot f_n(\mathbf{\Phi}_n^\circ)\cdot d\mathbf{\Phi}_n^\circ \end{bmatrix}
$$

$$
\mathbf{\Phi}_C = [\phi_{C,P}]
$$

$$
\phi_{C,P} = \mathrm{V}_C - \mathrm{V}_P
$$

$$
\mathbf{B} = \begin{bmatrix} b_C & 0 \\ 0 & b_P \end{bmatrix} , \quad \mathbf{\Psi}(t) = \begin{bmatrix} \upsilon_C(t) \\ \upsilon_P(t) \end{bmatrix},
$$

$$
\mathbf{\Theta}(t) = \begin{bmatrix} \rho_C(t) \\ \rho_P(t) \end{bmatrix} = \begin{bmatrix} \int_{-\infty}^{\infty} L(\mathbf{\Phi}_C)\cdot f(\mathbf{\Phi}_C^o) d\mathbf{\Phi}_C^o \\ 1-\rho_C(t) \end{bmatrix} \tag{34}
$$

$$
L[\mathbf{\Phi}_C(t)] = \left(1 + \sum_{\substack{\forall l \in \mathbf{A} \\ l \neq k}} e^{-\phi_{kl}(t)}\right)^{-1} = \left(1+e^{-\mathbf{\Phi}_C(t)}\right)^{-1} \tag{35}
$$

Assume $V_C^o \sim N\left(\bar{V}_C^o, \sigma_C\right)$, $V_P^o \sim N\left(\bar{V}_P^o, \sigma_P\right)$. Then

$$\Phi_C^o = \left(V_C^o - V_P^o\right) \sim N\left(\bar{\Phi}_C^o, \sigma_{C,P}\right)$$

where

$$\bar{\Phi}_C^o = \left(\bar{V}_C^o - \bar{V}_P^o\right),\ \sigma_{C,P}^2 = \sigma_C^2 + \sigma_P^2$$

Then

$$f_C(\boldsymbol{\Phi}_C^\circ) = (2\pi)^{-1/2}\sigma_{C,P}^{-1} \cdot e^{-(\Phi_C^\circ - \bar{\Phi}_C^\circ)^2/2\sigma_{C,P}^2}$$

$$\mathbf{B} = \begin{bmatrix} b_C & 0 \\ 0 & b_P \end{bmatrix}, \quad \mathbf{\Psi} = \begin{bmatrix} \upsilon_C \\ \upsilon_P \end{bmatrix},$$

$$\mathbf{\Theta}(t) = \begin{bmatrix} \rho_C(t) \\ \rho_P(t) \end{bmatrix} = \begin{bmatrix} (2\pi)^{-1/2}\sigma_{C,P}^{-1} \cdot \int_{-\infty}^{\infty} \left(1 + e^{-\mathbf{\Phi}_C(t)}\right)^{-1} \cdot e^{-(\Phi_C^\circ - \bar{\Phi}_C^\circ)^2/2\sigma_{C,P}^2} d\Phi_C^o \\ 1 - \rho_C(t) \end{bmatrix} \tag{36}$$

Or, substituting Equation (23),

$$\mathbf{\Theta}(t) = \begin{bmatrix} \rho_C(t) \\ \rho_P(t) \end{bmatrix} = \begin{bmatrix} (2\pi)^{-1/2}\sigma_{C,P}^{-1} \cdot \int_{-\infty}^{\infty} \left(1 + e^{-\left(\tau(t) + \left(\upsilon_C(t) - \upsilon_P(t)\right) + \Phi_C^o\right)}\right)^{-1} \cdot e^{-(\Phi_C^\circ - \bar{\Phi}_C^\circ)^2/2\sigma_{C,P}^2} d\Phi_C^o \\ 1 - \rho_C(t) \end{bmatrix}$$

Also, recall

$$\mathbf{B}\frac{d\mathbf{\Psi}}{dt} + \mathbf{\Psi} = \lambda \cdot \boldsymbol{\delta\theta}\left(\mathbf{\Psi}\right) \cdot H(t - t_\circ)$$
$$\mathbf{\Psi}(t_\circ) = 0$$

$$\begin{bmatrix} b_C & 0 \\ 0 & b_P \end{bmatrix} \cdot \begin{bmatrix} \dfrac{d\upsilon_C}{dt} \\ \dfrac{d\upsilon_P}{dt} \end{bmatrix} + \begin{bmatrix} \upsilon_C \\ \upsilon_P \end{bmatrix} = \lambda \cdot \begin{bmatrix} \rho_C(t) - \rho_C(t_o) \\ \rho_P(t) - \rho_P(t_o) \end{bmatrix} \cdot H(t - t_\circ)$$

$$\begin{bmatrix} \upsilon_C\left(t_o\right) \\ \upsilon_P\left(t_o\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Finally,

$$\begin{bmatrix}\rho_C(t)\\ \rho_P(t)\end{bmatrix} = \begin{bmatrix}(2\pi)^{-1/2}\sigma_{C,P}^{-1}\cdot\int_{-\infty}^{\infty}\left(1+e^{-\left(\tau(t)+\left(\upsilon_C(t)-\upsilon_P(t)\right)+\Phi_C^o\right)}\right)^{-1}\cdot e^{-(\Phi_C^\circ-\bar{\Phi}_C^\circ)^2/2\sigma_{C,P}^2}d\Phi_C^o\\ 1-\rho_C(t)\end{bmatrix} \quad (37a)$$

$$\begin{bmatrix}b_C & 0\\ 0 & b_P\end{bmatrix}\cdot\begin{bmatrix}\dfrac{d\upsilon_C}{dt}\\ \dfrac{d\upsilon_P}{dt}\end{bmatrix} + \begin{bmatrix}\upsilon_C\\ \upsilon_P\end{bmatrix} = \lambda\cdot\begin{bmatrix}\rho_C(t)-\rho_C(t_o)\\ \rho_P(t)-\rho_P(t_o)\end{bmatrix}\cdot H(t-t_\circ)$$

$$\begin{bmatrix}\upsilon_C\left(t_o\right)\\ \upsilon_P\left(t_o\right)\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} \quad (37b)$$

Or, since from Equations (37b)

$$\begin{bmatrix}\rho_C(t)\\ \rho_P(t)\end{bmatrix} = \lambda^{-1}\cdot\begin{bmatrix}b_C & 0\\ 0 & b_P\end{bmatrix}\cdot\begin{bmatrix}\dfrac{d\upsilon_C}{dt}\\ \dfrac{d\upsilon_P}{dt}\end{bmatrix} + \lambda^{-1}\cdot\begin{bmatrix}\upsilon_C\\ \upsilon_P\end{bmatrix} + \begin{bmatrix}\rho_C(t_o)\\ \rho_P(t_o)\end{bmatrix}$$

we obtain the following integro-differential equations for $\upsilon_{ICE}(t)$, $\upsilon_{BEV}(t)$

$$\begin{bmatrix}b_C & 0\\ 0 & b_P\end{bmatrix}\cdot\begin{bmatrix}\dfrac{d\upsilon_C}{dt}\\ \dfrac{d\upsilon_P}{dt}\end{bmatrix} + \begin{bmatrix}\upsilon_C(t)\\ \upsilon_P(t)\end{bmatrix} = \lambda\begin{bmatrix}(2\pi)^{-1/2}\sigma_{C,P}^{-1}\cdot\int_{-\infty}^{\infty}\left(1+e^{-\left(\tau(t)+\left(\upsilon_C(t)-\upsilon_P(t)\right)+\Phi_C^o\right)}\right)^{-1}\cdot e^{-(\Phi_C^\circ-\bar{\Phi}_C^\circ)^2/2\sigma_{C,P}^2}d\Phi_C^o\\ 1-\rho_C(t)\end{bmatrix} - \lambda\begin{bmatrix}\rho_C(t_o)\\ 1-\rho_C(t_o)\end{bmatrix}$$

Or,

$$\begin{bmatrix}b_C & 0\\ 0 & b_P\end{bmatrix}\cdot\begin{bmatrix}\dfrac{d\upsilon_C}{dt}\\ \dfrac{d\upsilon_P}{dt}\end{bmatrix} + \begin{bmatrix}\upsilon_C(t)\\ \upsilon_P(t)\end{bmatrix} = \lambda\begin{bmatrix}\mathcal{I}\left(\upsilon_C,\upsilon_P\right)\\ 1-\mathcal{I}\left(\upsilon_C,\upsilon_P\right)\end{bmatrix} - \lambda\begin{bmatrix}\rho_C(t_o)\\ 1-\rho_C(t_o)\end{bmatrix} \quad (38)$$

where

$$\mathcal{J}\left(\upsilon_C,\upsilon_P\right)=(2\pi)^{-1/2}\sigma_{C,P}^{-1}\cdot\int_{-\infty}^{\infty}\left(1+e^{-\left(\tau(t)+\left(\upsilon_C(t)-\upsilon_P(t)\right)+\Phi_C^{o}\right)}\right)^{-1}\cdot e^{-(\Phi_C^{\circ}-\bar{\Phi}_C^{\circ})^2/2\sigma_{C,P}^2}d\Phi_C^{o}$$

with given parameters:

β_C , β_G, β_P, β_E, β_S, C_C^* ,C_C^{o} ,C_P^* ,C_P^{o} ,G_C^* ,G_C^{o} ,G_P^* ,G_P^{o} ,P_C^* ,P_C^{o} ,P_P^* ,P_P^{o} ,E_C^* ,E_C^{o} ,E_P^* ,E_P^{o} ,
S_C^* ,S_C^{o} ,S_P^* ,S_P^{o} ,$a_{C,C}$,$a_{C,G}$,$a_{C,P}$,$a_{C,E}$,$a_{C,S}$,$a_{P,C}$,$a_{P,G}$,$a_{P,P}$,$a_{P,E}$,$a_{P,S}$,b_C ,b_P ,λ ,
σ_C, σ_P, $\bar{V}_C^{\text{o}}$, $\bar{V}_P^{\text{o}}$, $\rho_C(t_{\text{o}})$, $\rho_P(t_{\text{o}})$

3.4.1 Numerical solutions

As discussed previously, exact solutions to equations (38) are difficult, if not impossible to obtain. Here we employ a simple numerical finite difference discretization scheme to obtain approximate solutions to the demonstration example. Specifically, equations (38) are represented by their difference approximations:

$$\begin{bmatrix} b_C & 0 \\ 0 & b_P \end{bmatrix}\cdot\begin{bmatrix} \dfrac{\upsilon_C(t)-\upsilon_C(t-\Delta t)}{\Delta t} \\ \dfrac{\upsilon_P(t)-\upsilon_P(t-\Delta t)}{\Delta t} \end{bmatrix}+\begin{bmatrix} \upsilon_C(t) \\ \upsilon_P(t) \end{bmatrix}=\lambda\begin{bmatrix} \mathcal{J}\left(\upsilon_C,\upsilon_P\right) \\ 1-\mathcal{J}\left(\upsilon_C,\upsilon_P\right) \end{bmatrix}-\lambda\begin{bmatrix} \rho_C(t_o) \\ 1-\rho_C(t_o) \end{bmatrix}$$

$$\begin{bmatrix} b_C & 0 \\ 0 & b_P \end{bmatrix}\cdot\begin{bmatrix} \upsilon_C(t)-\upsilon_C(t-\Delta t) \\ \upsilon_P(t)-\upsilon_P(t-\Delta t) \end{bmatrix}+\Delta t\begin{bmatrix} \upsilon_C(t) \\ \upsilon_P(t) \end{bmatrix}=\lambda\Delta t\begin{bmatrix} \mathcal{J}\left(\upsilon_C,\upsilon_P\right) \\ 1-\mathcal{J}\left(\upsilon_C,\upsilon_P\right) \end{bmatrix}-\lambda\Delta t\begin{bmatrix} \rho_C(t_o) \\ 1-\rho_C(t_o) \end{bmatrix}$$

$$\begin{bmatrix} b_C+\Delta t & 0 \\ 0 & b_P+\Delta t \end{bmatrix}\cdot\begin{bmatrix} \upsilon_C(t) \\ \upsilon_P(t) \end{bmatrix}-\begin{bmatrix} b_C & 0 \\ 0 & b_P \end{bmatrix}\cdot\begin{bmatrix} \upsilon_C(t-\Delta t) \\ \upsilon_P(t-\Delta t) \end{bmatrix}\approx\lambda\Delta t\begin{bmatrix} \mathcal{J}\left(\upsilon_C,\upsilon_P\right) \\ 1-\mathcal{J}\left(\upsilon_C,\upsilon_P\right) \end{bmatrix}-\lambda\Delta t\begin{bmatrix} \rho_C(t_o) \\ 1-\rho_C(t_o) \end{bmatrix}$$

$$\begin{bmatrix} \upsilon_C(t) \\ \upsilon_P(t) \end{bmatrix}\approx\begin{bmatrix} b_C+\Delta t & 0 \\ 0 & b_P+\Delta t \end{bmatrix}^{-1}\cdot\left\{\lambda\Delta t\begin{bmatrix} \mathcal{J}\left(\upsilon_C,\upsilon_P\right) \\ 1-\mathcal{J}\left(\upsilon_C,\upsilon_P\right) \end{bmatrix}-\lambda\Delta t\begin{bmatrix} \rho_C(t_o) \\ 1-\rho_C(t_o) \end{bmatrix}+\begin{bmatrix} b_C & 0 \\ 0 & b_P \end{bmatrix}\cdot\begin{bmatrix} \upsilon_C(t-\Delta t) \\ \upsilon_P(t-\Delta t) \end{bmatrix}\right\}$$

$$\mathcal{J}\left(\upsilon_C,\upsilon_P\right)\approx(2\pi)^{-1/2}\sigma_{C,P}^{-1}\cdot\sum_{n=-\infty}^{n=\infty}\left(1+e^{-\left(\tau(t)+\left(\upsilon_C(t)-\upsilon_P(t)\right)+n\Delta x\right)}\right)^{-1}\cdot e^{-(n\Delta x-\bar{\Phi}_C^{\circ})^2/2\sigma_{C,P}^2}\cdot\Delta x$$

where

$$\begin{aligned}\tau(t)=&\beta_C\left(C_C^*-C_C^0\right)\left(1-e^{-(t-t_0)a_{C,C}^{-1}}\right)+\beta_G\left(G_C^*-G_C^0\right)\left(1-e^{-(t-t_0)a_{C,G}^{-1}}\right)+\beta_P\left(P_C^*-P_C^0\right)\left(1-e^{-(t-t_0)a_{C,P}^{-1}}\right)+\beta_E\left(E_C^*-E_C^0\right)\left(1-e^{-(t-t_0)a_{C,E}^{-1}}\right)+\beta_S\left(S_C^*-S_C^0\right)\left(1-e^{-(t-t_0)a_{C,S}^{-1}}\right)\\&-\beta_C\left(C_P^*-C_P^0\right)\left(1-e^{-(t-t_0)a_{P,C}^{-1}}\right)-\beta_G\left(G_P^*-G_P^0\right)\left(1-e^{-(t-t_0)a_{P,G}^{-1}}\right)-\beta_P\left(P_P^*-P_P^0\right)\left(1-e^{-(t-t_0)a_{P,P}^{-1}}\right)-\beta_E\left(E_P^*-E_P^0\right)\left(1-e^{-(t-t_0)a_{P,E}^{-1}}\right)-\beta_S\left(S_P^*-S_P^0\right)\left(1-e^{-(t-t_0)a_{P,S}^{-1}}\right)\end{aligned}$$

For this particular case, since $C_k^0\equiv C_k^*, E_k^0\equiv E_k^*, S_k^0\equiv S_k^*;\ k=C,P$:

$$\begin{aligned}\tau(t)=&\beta_G\left(G_C^*-G_C^0\right)\left(1-e^{-(t-t_0)a_{C,G}^{-1}}\right)+\beta_P\left(P_C^*-P_C^0\right)\left(1-e^{-(t-t_0)a_{C,P}^{-1}}\right)\\&-\beta_G\left(G_P^*-G_P^0\right)\left(1-e^{-(t-t_0)a_{P,G}^{-1}}\right)-\beta_P\left(P_P^*-P_P^0\right)\left(1-e^{-(t-t_0)a_{P,P}^{-1}}\right)\end{aligned}$$

With this formulation, the change in demand by stimuli from several factors (e.g. gas price, and market share of vehicles) will be examined for several scenarios.

3.4.2 Basic Scenario Parameters

As a basic scenario, the following is assumed: gas price at the initial time step (2003) is \$1.36/gal., and that at the final time step (2012) is \$3.64/gal.; a \$1,500 rebate is applied to Prius and Civic Hybrid at the final time step. Here, and in other scenarios presented under Section 3.4, for demonstration purposes, the values for Education, E_k, and Gender, S_k, are held fixed at values 5 (Graduate or Professional degree) and 0 (=female) , respectively. And, the value of *dt* is assumed as 0.01. Other initial conditions are displayed in Table 14.

TABLE 14 Values of Variables for Civic Hybrid, Corolla, and Prius

	Civic Hybrid	Corolla	Prius
MPG	45.35	32.05	49.65
Price	\$26,135 \$24,635 with rebate	\$18,515	\$25,700 \$24,200 with rebate

With these basic assumptions, the parameters of the dynamic normative model are adjusted to "best" match the observed dynamic changes in demand for the respective vehicles. In the model, the values for the various relative importance weights in the utility functions and the value of the dynamic parameter, λ, are those determined by the estimation of the corresponding discrete choice model.

3.4.3 Corolla (CO) vs. Prius (P) Dynamics

The values of parameters that provide the best match to the empirical evidence are in Table 15. And, the results are shown in Figure 6.

TABLE 15 Values of Parameters for Corolla vs. Prius

	Corolla	Prius
a_G	3.5	100
a_P	N.A.	1
b	2500	2500
s	0.4	0.3
Φ_{CO}^{0}	2.7	
λ	-10.1	

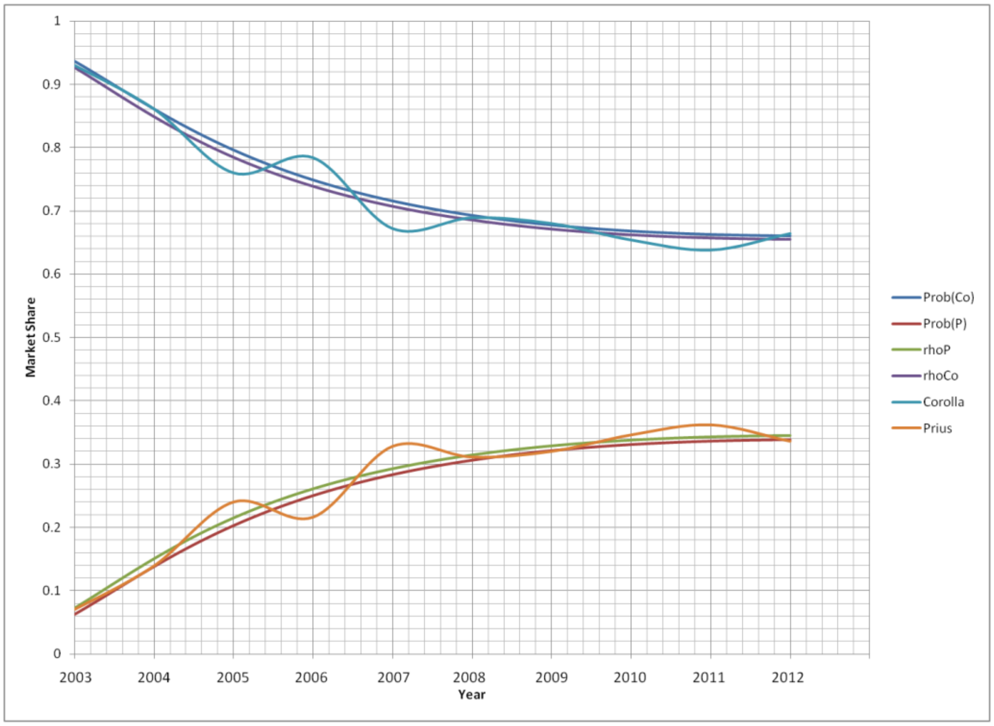

Figure 6. Results for Corolla vs. Prius

The results indicate that the dynamic normative model, in conjunction with the discrete choice assumptions, track the growth in the market share of the Toyota Prius Hybrid with reasonable accuracy.

3.4.4 Civic Hybrid (CH) vs. Prius (P)

The values of parameters are in Table 16. And, the results are shown in Figure 7.

TABLE 16 Values of Parameters for Civic Hybrid vs. Prius

	Civic Hybrid	Prius
a_G	-2.7	1
b	200	200
s	0.1	0.5
Φ^0_{CH}	-0.1	
λ	-4.7	

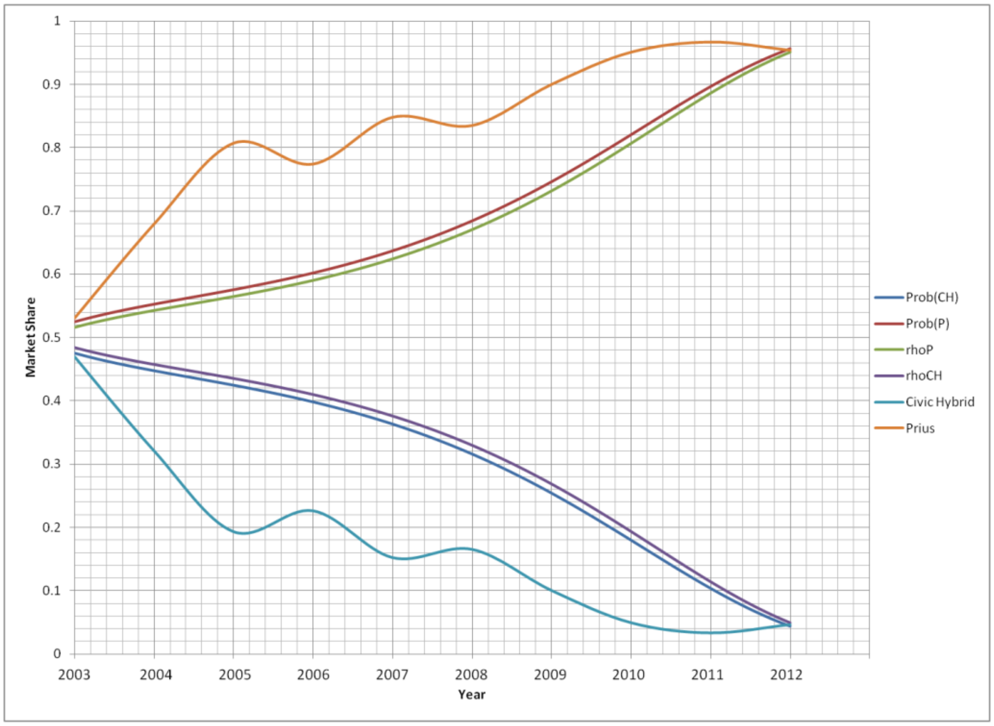

Figure 7. Results for Civic Hybrid vs. Prius

Here again, although not quite as conclusively as in the case of Prius vs. Corolla example, we see that the dynamic model is able to capture the general trend of the relative market shares of Prius vs. Civic Hybrid over the study period. The results above show that dynamic normative models for each case are able to track the market trends from year 2003 through 2008. To test sensitivity, these models are applied to several scenarios in the next section.

3.5 Sensitivity analysis

In the following sections, we examine the sensitivity of the dynamic normative model relative to key factors.

3.5.1 Changing the initial points for market share

In this section, we test the sensitivities of the dynamic trends relative to the initial state of the market. Specifically, the initial values for market share of each vehicle are changed to find influences on the changes in market trend over the years. Except for s and Φ^0, the values of parameters for the dynamic normative models remain equal to those in the previous section.

i) $\rho_P^0 = 0.2$

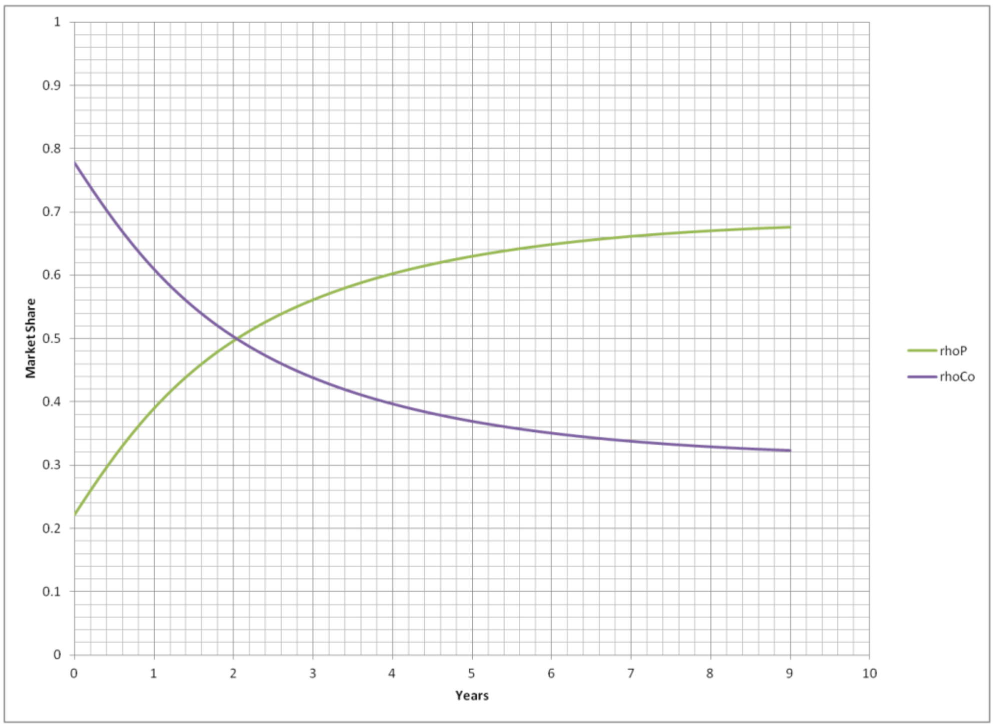

Figure 8. Market Trends Corolla (CO) vs. Prius (P) with $\Phi_{CO}^0 = 1.4$, $\rho_{CO}^0 = 0.8, \rho_P^0 = 0.2$

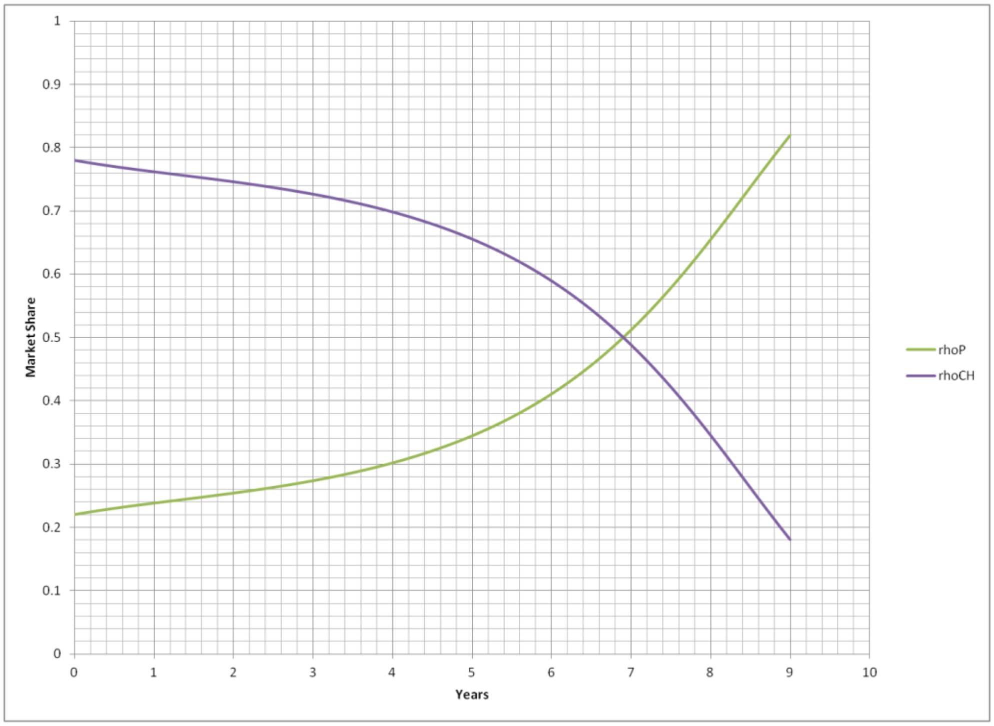

Figure 9. Market Trends Civic Hybrid (CH) vs. Prius (P) with $\Phi_{CH}^{0} = 1.4\ \rho_{CH}^{0} = 0.8, \rho_{P}^{0} = 0.2$

ii) $\rho_{P}^{0} = 0.4$

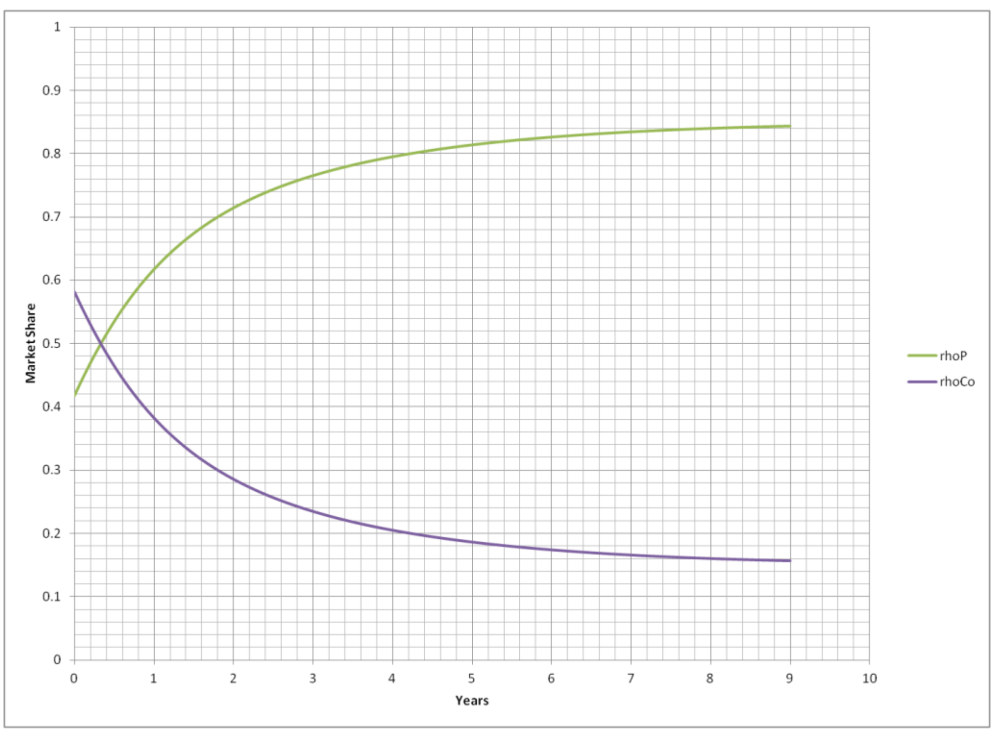

Figure 10. Market Trends Corolla (CO) vs. Prius (P) with $\Phi_{CO}^{0} = 0.4, \rho_{CO}^{0} = 0.6, \rho_{P}^{0} = 0.4$

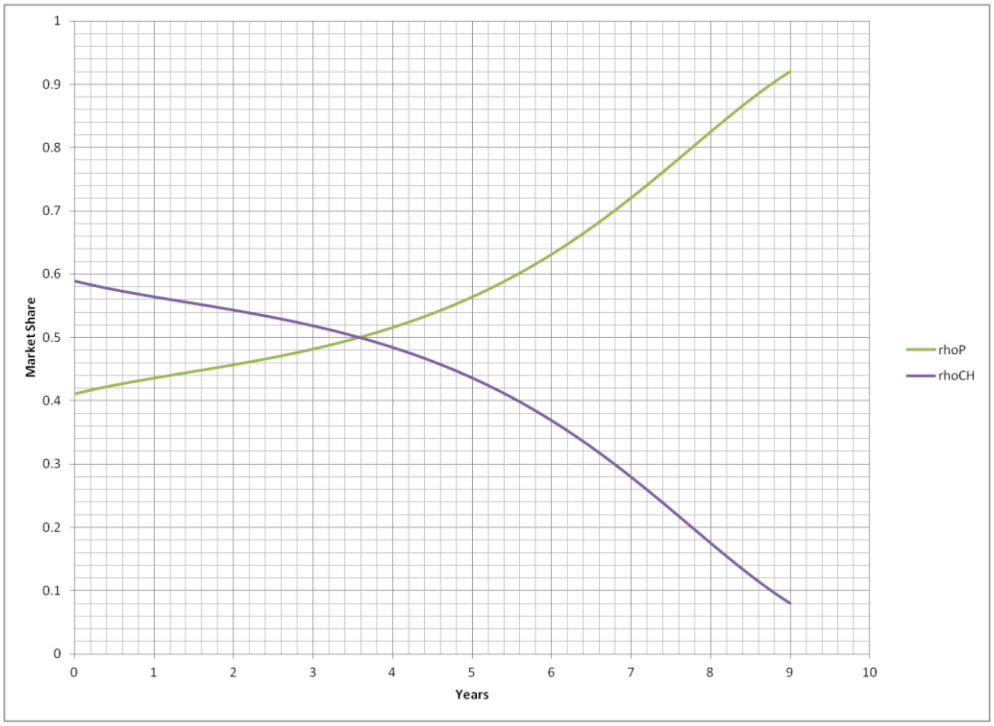

Figure 11. Market Trends Civic Hybrid (CH) vs. Prius (P) with $\Phi_{CH}^{0} = 0.4, \rho_{CH}^{0} = 0.6, \rho_{P}^{0} = 0.4$

iii) $\rho_P^0 = 0.8$

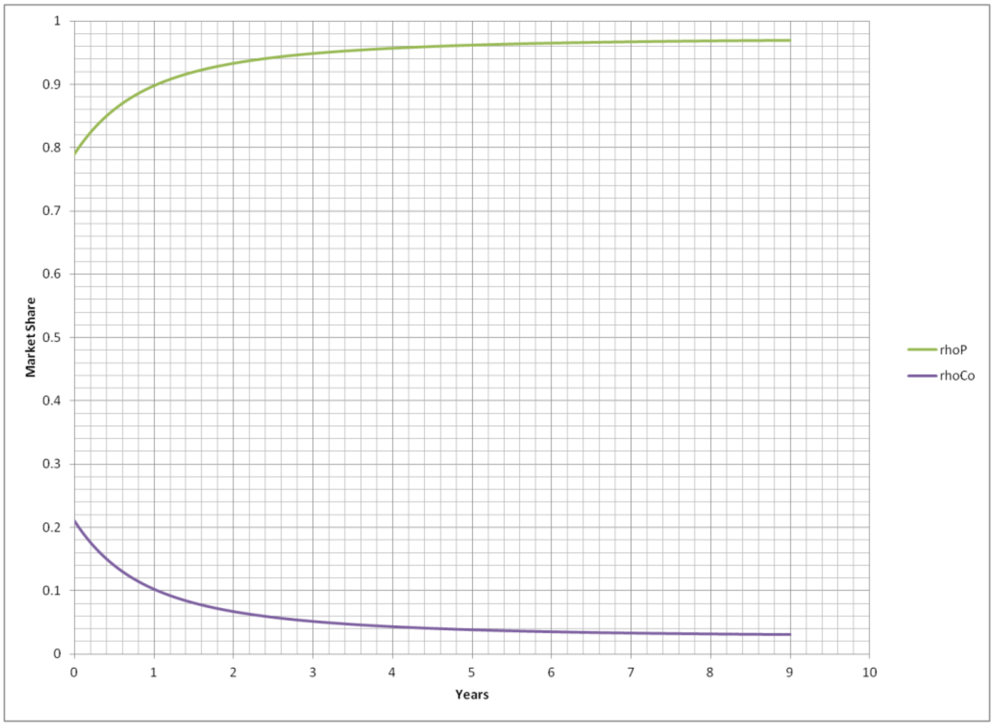

Figure 12. Market Trends Corolla (CO) vs. Prius (P) with $\Phi_{CO}^0 = -1.4$, $\rho_{CO}^0 = 0.2, \rho_P^0 = 0.8$

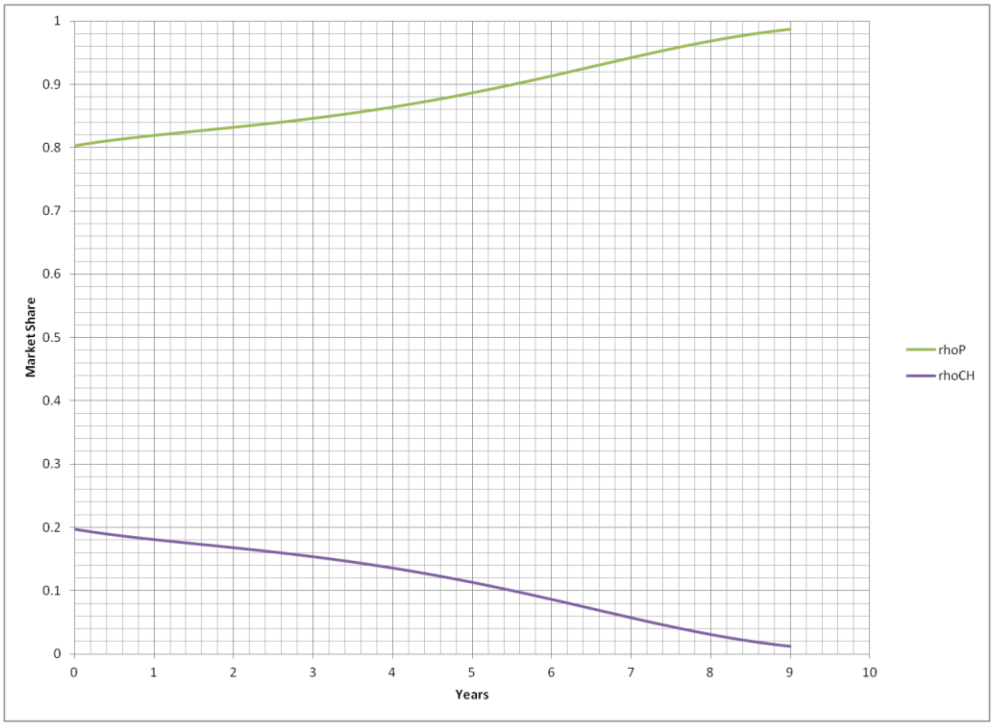

Figure 13. Market Trends Civic Hybrid (CH) vs. Prius (P) with
$\Phi_{CH}^{0} = -1.4, \rho_{CH}^{0} = 0.2, \rho_{P}^{0} = 0.8$

From the results above, the dynamic market trends for both Corolla vs. Prius and Civic Hybrid vs. Prius shows sensitivity to changes in their initial positions in the market. In more detail, the changes in market share of Prius show a faster crossing to a dominant market share with respect to changes in the initial market share of Corolla than with those of Civic Hybrid. The reason of this result can be explained as: although the vehicle price of Prius is higher than that of Corolla, the benefits of saving operating cost with choosing Prius is larger than the benefit obtained from choosing Corolla. On the other hand, Civic Hybrid and Prius have similar advantages in saving operating cost. Therefore, the changes in market share between Prius and Corolla occur faster than those between Prius and Civic Hybrid. And the greater the initial market share of Prius, the larger this effect would be on changes in market share due to the bandwagon effect. In conclusion, the larger initial market share of Prius, the faster the pathway to having a greater market share than that of the other vehicle.

3.5.2 Changing gas prices

In the base (default) case, gas price ($/gal.) in the initial time step is $1.36, and that in the final time step is $3.64. In the case considered here, the gas price in the final time step is changed to find influence on market trend over the years. The initial market shares of each vehicle and values of parameters for the dynamic normative model remain equal to the base case.

i) $Gas^{*} = \$4.09 / gal$.

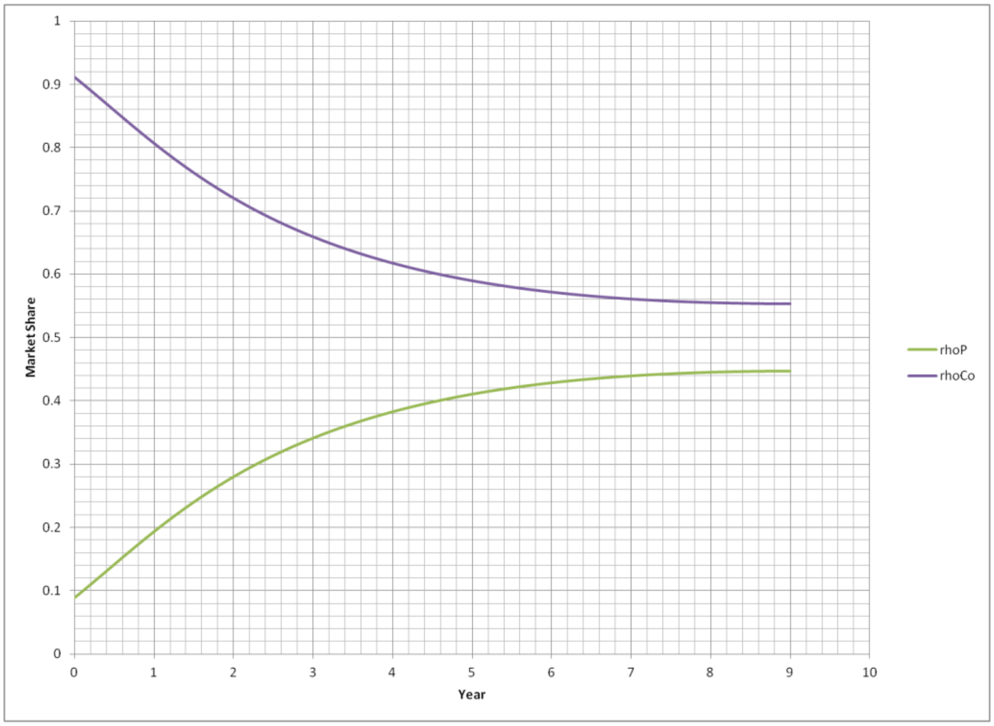

Figure 14. Market Trends Corolla vs. Prius with Gas Price of $4.09/ gal.

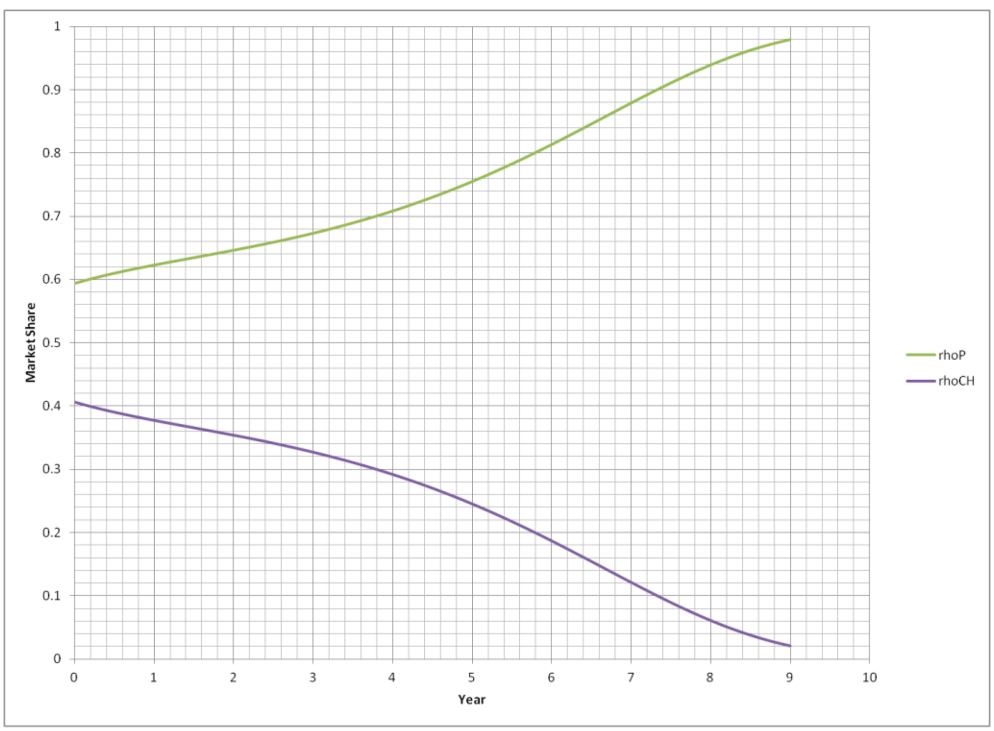

Figure 15. Market Trends Civic Hybrid vs. Prius with Gas Price of $4.09/ gal.

ii) $Gas^* = \$4.49 / gal$.

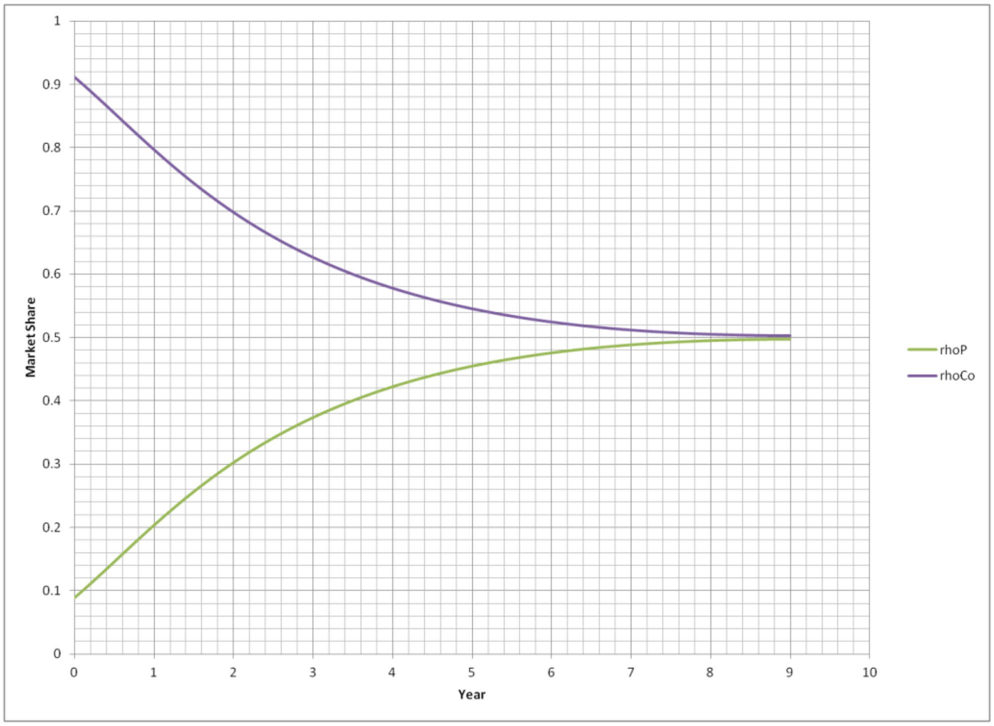

Figure 16. Market Trends Corolla vs. Prius with Gas Price of $4.49/ gal.

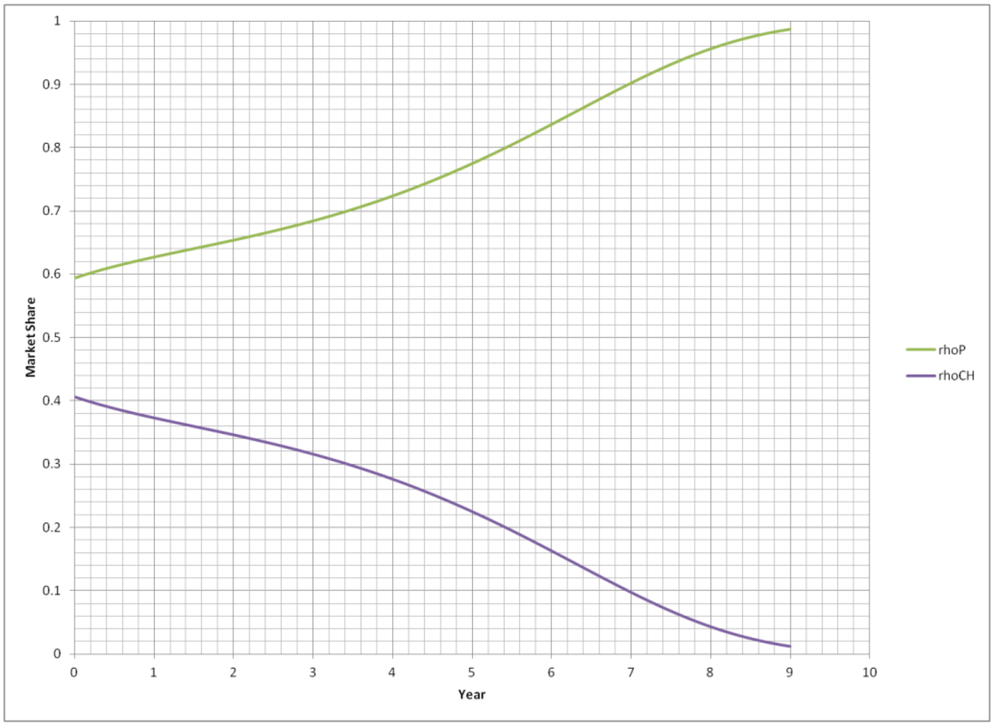

Figure 17. Market Trends Civic Hybrid vs. Prius with Gas Price of $4.49/ gal.

As expected, from the results above, the market trends between Corolla vs. Prius show a sensitivity to rising gas price—a price of gas of at least about $4.50 per gallon appears necessary for Prius to overtake Corolla in terms of market share. On the other hand, the market trends between Civic Hybrid vs. Prius show almost no change with respect to raising gas price. As mentioned in the previous section related to initial market shares, the reason for this result can be explained by the difference in the benefit of saving operation cost between the two vehicles. In other words, Prius and Corolla have different characteristics in mpg which has a direct effect on operating cost, and so changes in gas price would likely produce a change in market shares between these two vehicles. However, Prius and Civic Hybrid have similar characteristics in mpg, which means that these two vehicles have similar benefits of saving operation cost. Therefore, changes in gas price would not be expected to make a significant effect on market shares between Prius and Civic Hybrid.

3.5.3 Changing vehicle prices

In the base (default) case, a $1,500 rebate is applied to Civic Hybrid and Prius; the effect is one in which the prices of these vehicles become those in which $1,500 is subtracted from MSRP. Here, two scenarios are presented; i) No rebate for Civic Hybrid and Prius, and ii) $5,000 rebate. The results are presented with the base scenario for comparison purposes.

i) No rebate

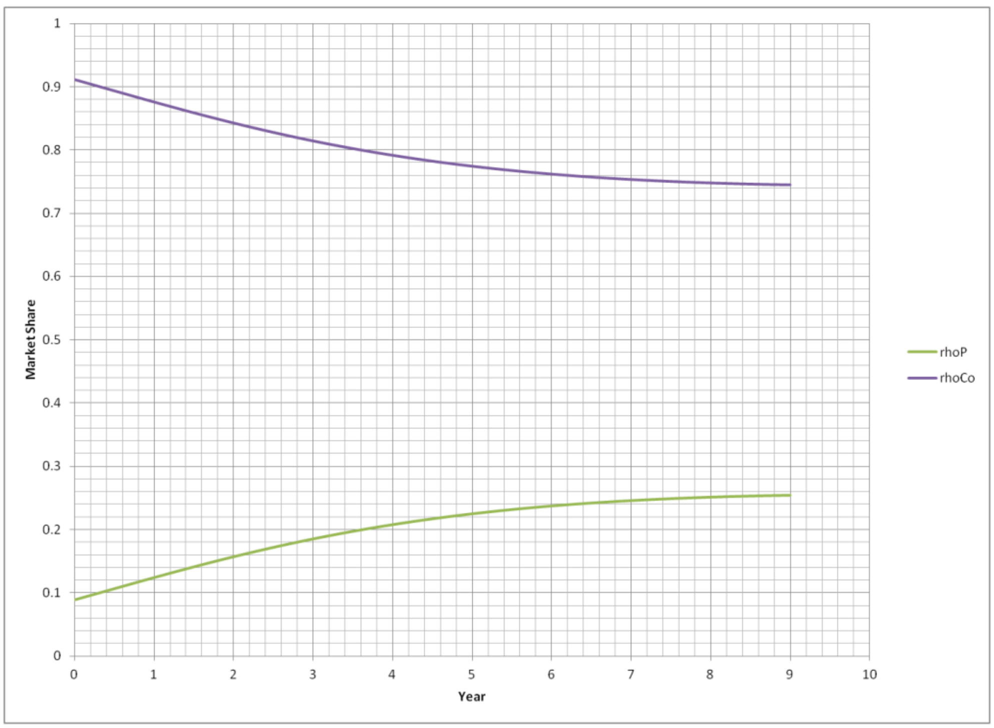

Figure 18. Market Trends without Rebates

ii) Base scenario

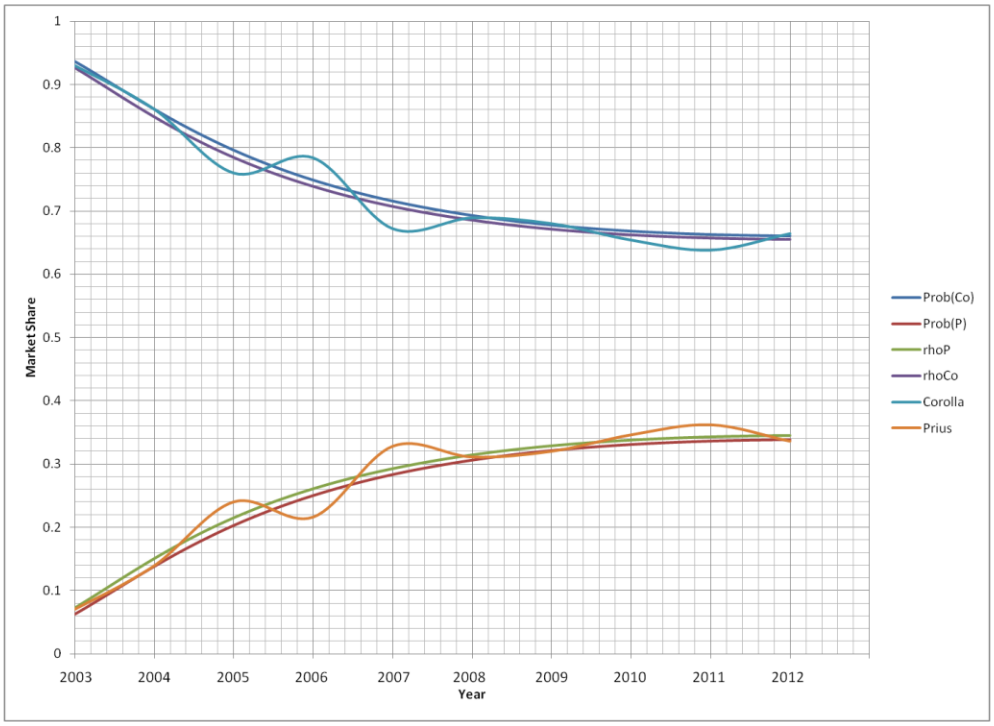

Figure 19. Market Trends with Basic Scenario

iii) $5,000 rebate

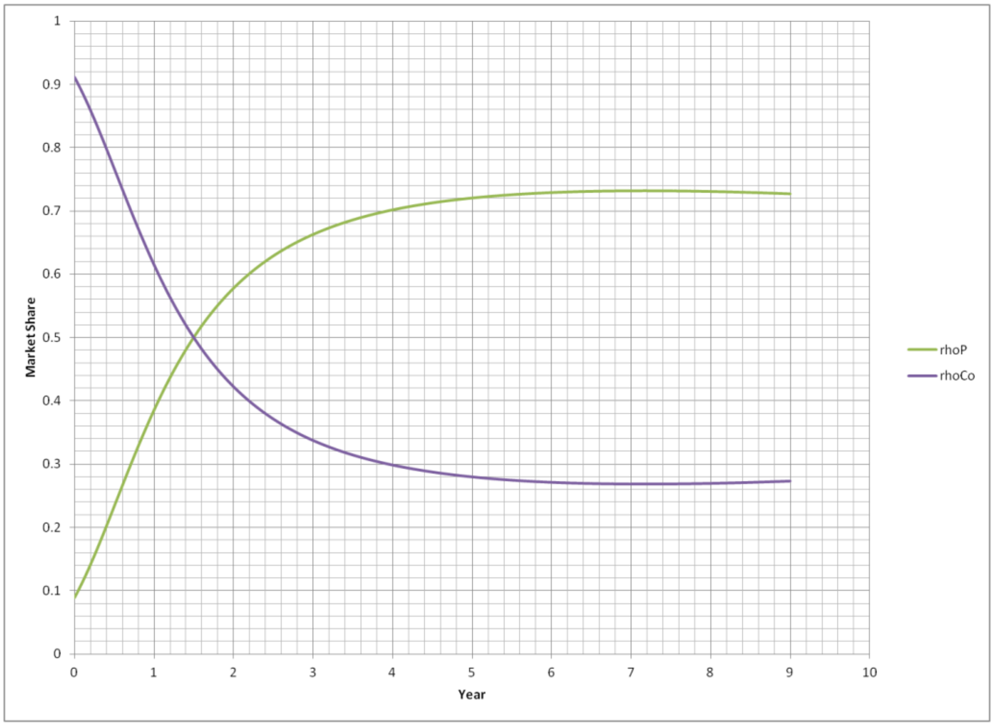

Figure 20. Market Trends with Rebates of $5,000

The results show that the higher the rebate in vehicle price, the larger and more rapid the change in the market trend of Prius.

3.5.4 Gender effects

To compare the sensitivity of choice of vehicle between males and females, changes in the market share of vehicles with respect to gender are analyzed by separating the dataset of people who own Toyota Corolla and Toyota Prius (total 2,959 samples) by gender. In the dataset, the distribution by gender is shown in Table 17.

TABLE 17 Distribution of Respondents by Gender

Year	Male				Female			
	Number		Percentage (%)		Number		Percentage (%)	
	Corolla	Prius	Corolla	Prius	Corolla	Prius	Corolla	Prius
2003	54	16	77.1	22.9	112	16	87.5	12.5
2004	55	43	56.1	43.9	165	50	76.7	23.3
2005	111	83	57.2	42.8	172	94	64.7	35.3
2006	130	89	59.4	40.6	238	133	64.2	35.8
2007	142	144	49.7	50.3	270	217	55.4	44.6
2008	127	106	54.5	45.5	228	164	58.2	41.8
Total	619	481			1,185	674		

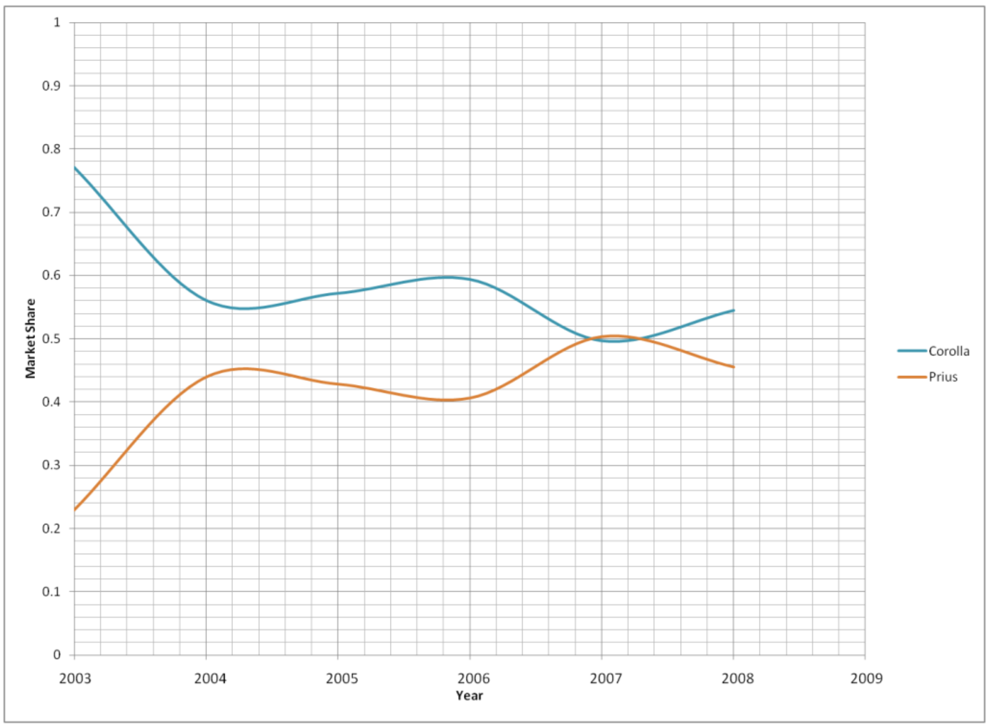

Figure 21. Market Trends Between Corolla and Prius in Males

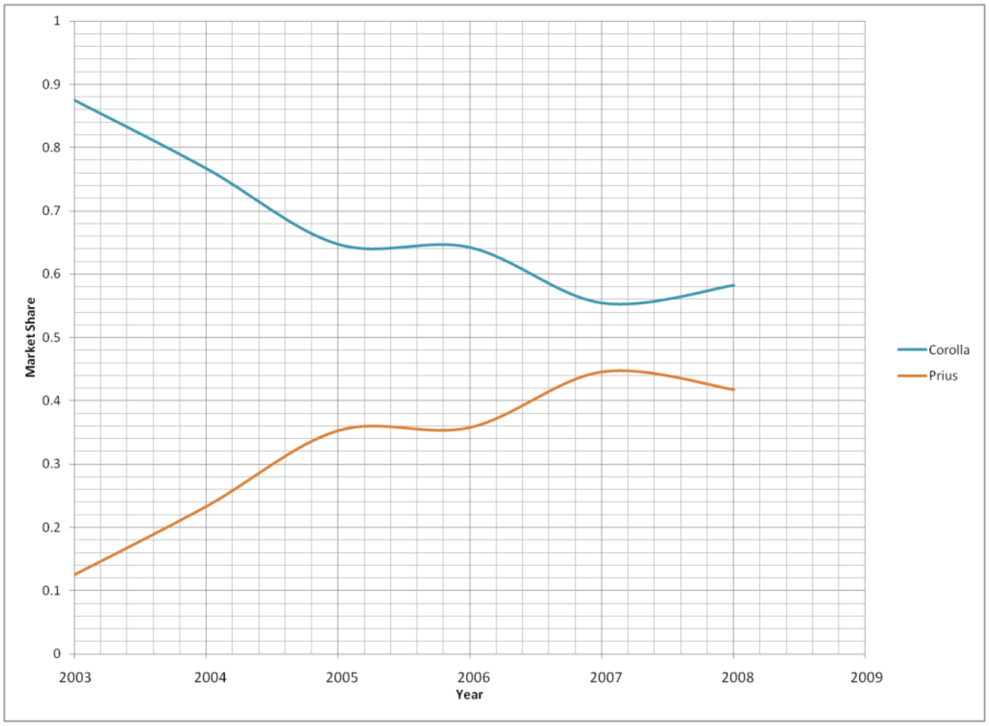

Figure 22. Market Trends Between Corolla and Prius in Females

a) Male

In order to best track the changes in the graph of male respondents, the values of parameters for the dynamic normative model are as in Table 18, and the results are shown as Figure 23. In Table 18, the values of a_G for Corolla and Prius are 10 and 100, with an assumption that male drivers of Corolla are more sensitive to the change in gas price than those of Prius. In the case of a_P, it is not applied to Corolla because the vehicle price of Corolla is not reduced by rebates. The values of s are defined to reduce the gap between results from a logit model and those from a dynamic normative model. And the value of Φ_{CO}^{0} is defined to reduce a gap between the initial market share (Year 2003) from the vehicle sales data and the initial market share which is derived from a dynamic normative model.

TABLE 18 Values of Parameters for Males

	Corolla	Prius
a_G	10	100
a_P	N.A.	1
b	800	800
s	0.5	0.3
Φ^0_{CO}	1.3	
λ	-10.1	

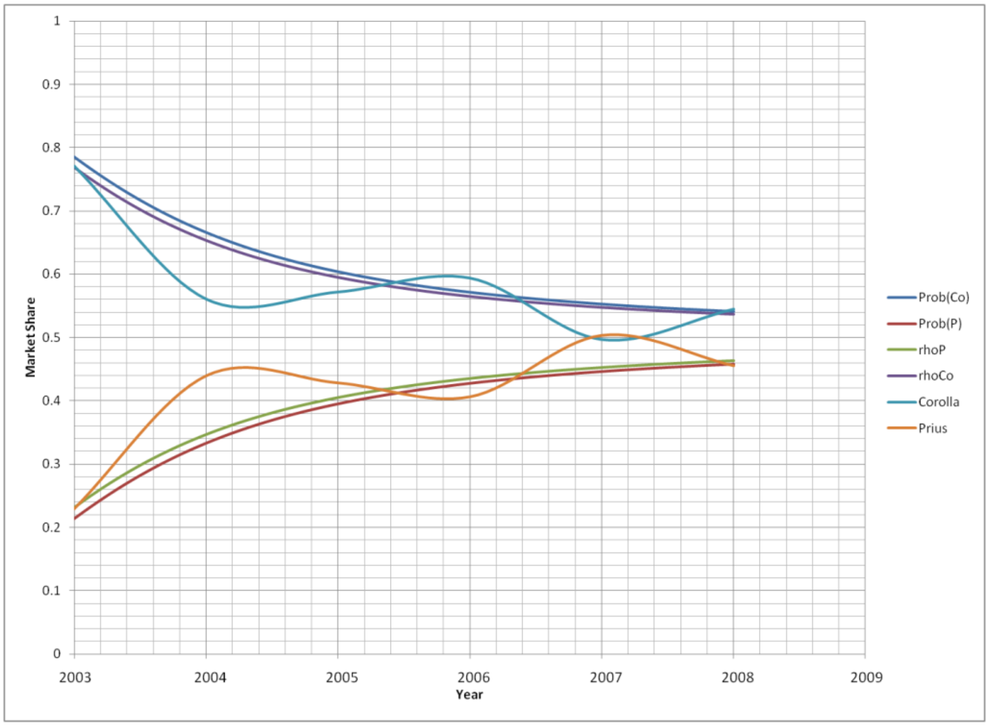

Figure 23. Change in the Market Share of Corolla and Prius in Males

b) Female

Similarly, in order to track the changes in a graph of females, the best values of parameters for the dynamic normative model are as in Table 19, and the results are shown in Figure 24. In Table 19, the values of a_G for Corolla and Prius are 5 and 100, with an assumption that female drivers of Corolla are more sensitive to the change in gas price than those of Prius. In the case of a_P, it is not applied to Corolla because the vehicle price of Corolla is not reduced by rebates. The values of s are defined to reduce the gap between results from a logit model and those from a dynamic normative model. And the value of Φ^0_{CO} is defined to reduce a gap between the initial market share (Year 2003) from the vehicle sales data and the initial market share which is derived from a dynamic normative model.

TABLE 19 Values of Parameters for Females

	Corolla	Prius
a_G	5	100
a_P	N.A.	1
b	1000	1000
s	0.5	0.3
Φ^0_{CO}	1.9	
λ	-10.1	

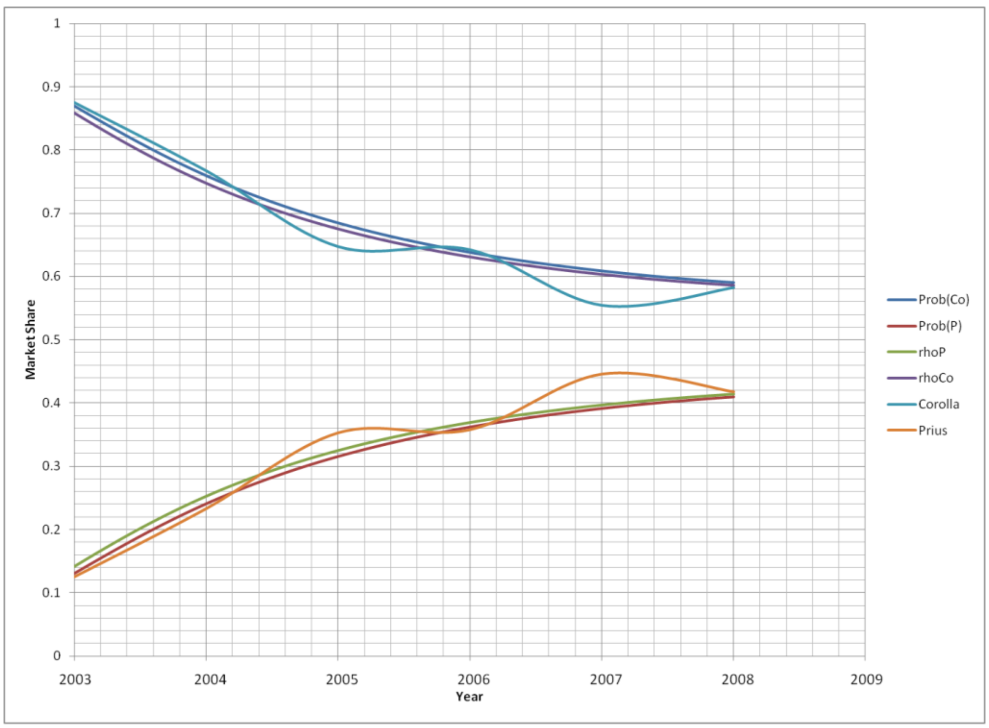

Figure 24. Change in the Market Share of Corolla and Prius in Females

The results above show that there are only minor differences in the sensitivity of the “best” model parameters to fit the dynamics of choice of Prius between the male group and the female group.

Table 20 shows the comparison of model parameters for males and females. In Table 20, by comparing a value of a_G for males who choose Corolla to that for females who choose the same vehicle, it is found that the value for males is larger than that for females. From this, it could be said that females who choose Corolla are more sensitive with a change in gas price than males who choose the same vehicle.

TABLE 20 Comparison of Fitted Model Parameters for Males vs. Females

	Corolla		Prius	
	Males	Females	Males	Females
a_G	10	5	100	100
a_P	N.A.	N.A.	1	1
b	800	1000	800	1000
s	0.5	0.5	0.3	0.3
Φ_{CO}^{0}	1.3	1.9		
λ	-10.1	-10.1	-10.1	-10.1

3.5.5 Education effects

To examine the influence of education level on the sensitivity to choice of vehicle, the change in the market share of vehicles with respect education level is analyzed by first separating the dataset of people who drive Toyota Corolla and Toyota Prius by education level. According to NHTS, the education level is defined as Table 21.

TABLE 21 Codes of Education Levels and Their Descriptions

Code	Description
1	Less than high school graduate
2	High school graduate
3	Some college or Associate's degree
4	Bachelor's degree
5	Graduate or Professional degree

a) Less than high school graduate (code 1)

In the dataset, the distribution of people in code 1 is in Table 22.

TABLE 22 Distribution of People (Code 1)

Year	Number		Percentage (%)		Total
	Corolla	Prius	Corolla	Prius	
2003	5	0	100	0	5
2004	3	1	75	25	4
2005	7	3	70	30	10
2006	9	0	100	0	9
2007	9	2	81.8	18.2	11
2008	8	1	88.9	11.1	9
Total	41	7			48

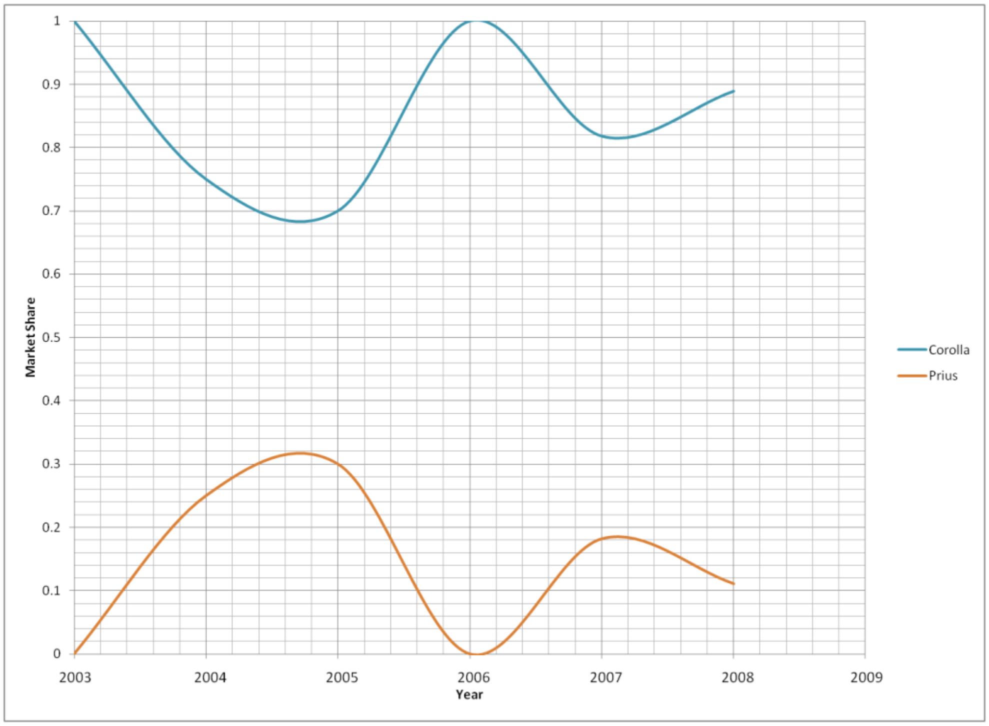

Figure 25. Market Trends between Corolla and Prius in Code 1 Group

The values of parameters for the dynamic normative model are in Table 23, and the results are shown in Figure 26.

TABLE 23 Values of Parameters (Code 1)

	Corolla	Prius
a_G	3	1000
a_P	N.A.	1
b	2500	2500
s	0.4	0.3
Φ_{CO}^{0}	4.1	

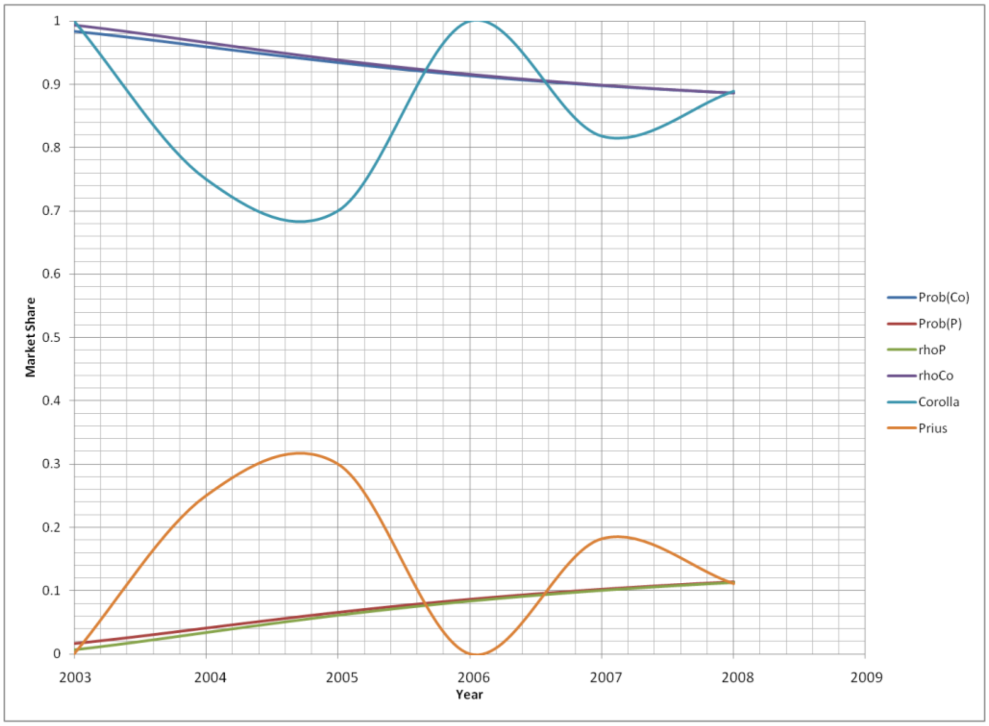

Figure 26. Change in the Market Share of Corolla and Prius (Code 1)

b) High school graduate (code 2)

In the dataset, the distribution of people in code 2 is in Table 24.

TABLE 24 Distribution of People (Code 2)

Year	Number		Percentage (%)		Total
	Corolla	Prius	Corolla	Prius	
2003	36	1	97.3	2.7	37
2004	50	4	92.6	7.4	54
2005	62	14	81.6	18.4	76
2006	68	17	80	20	85
2007	85	28	75.2	24.8	113
2008	96	17	85	15	113
Total	397	81			478

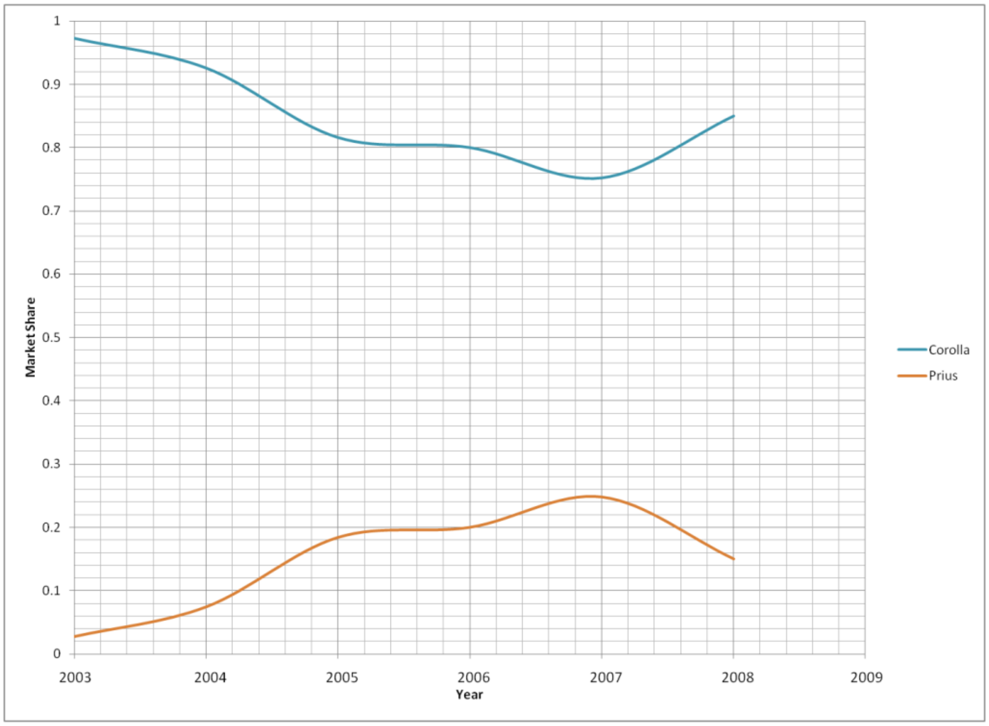

Figure 27. Market Trends between Corolla and Prius in Code 2 Group

The values of parameters for the dynamic normative model are in Table 25. And the result is shown in Figure 28.

TABLE 25 Values of Parameters (Code 2)

	Corolla	Prius
a_G	1	1000
a_P	N.A.	10
b	2500	2500
s	0.5	0.3
Φ_{CO}^{0}	3.6	

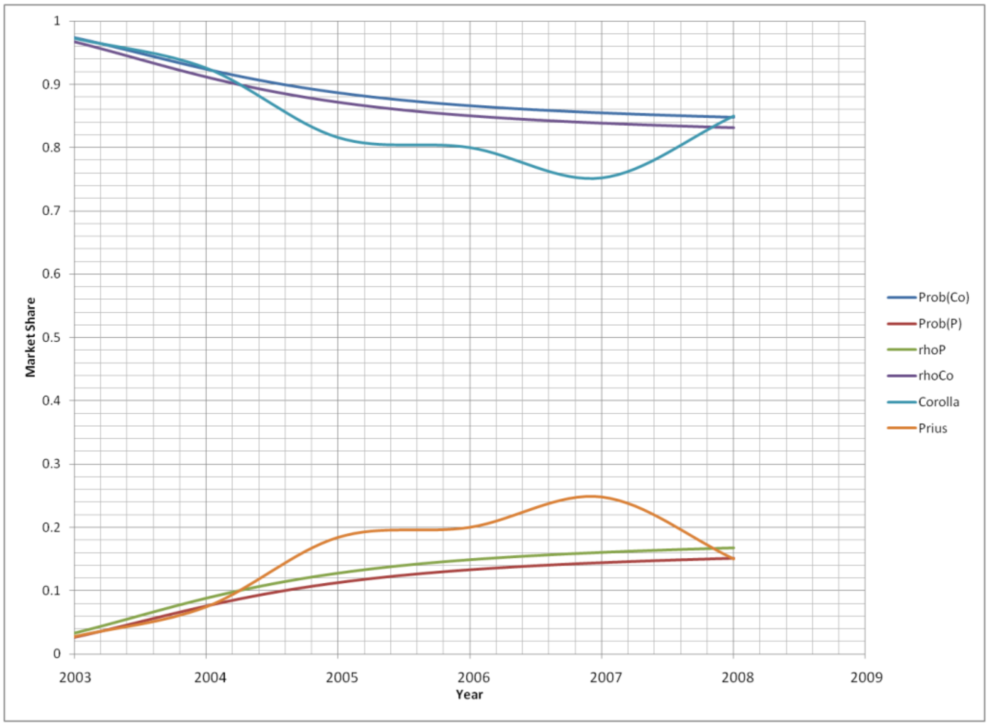

Figure 28. Change in the Market Share of Corolla and Prius (Code 2)

c) Some college or Associate's degree (code 3)

In the dataset, the distribution of people in code 3 is in Table 26.

TABLE 26 Distribution of People (Code 3)

Year	Number		Percentage (%)		Total
	Corolla	Prius	Corolla	Prius	
2003	54	2	96.4	3.6	56
2004	56	14	80	20	70
2005	80	25	76.2	23.8	105
2006	122	31	79.7	20.3	153
2007	125	67	65.1	34.9	192
2008	104	74	58.4	41.6	178
Total	541	213			754

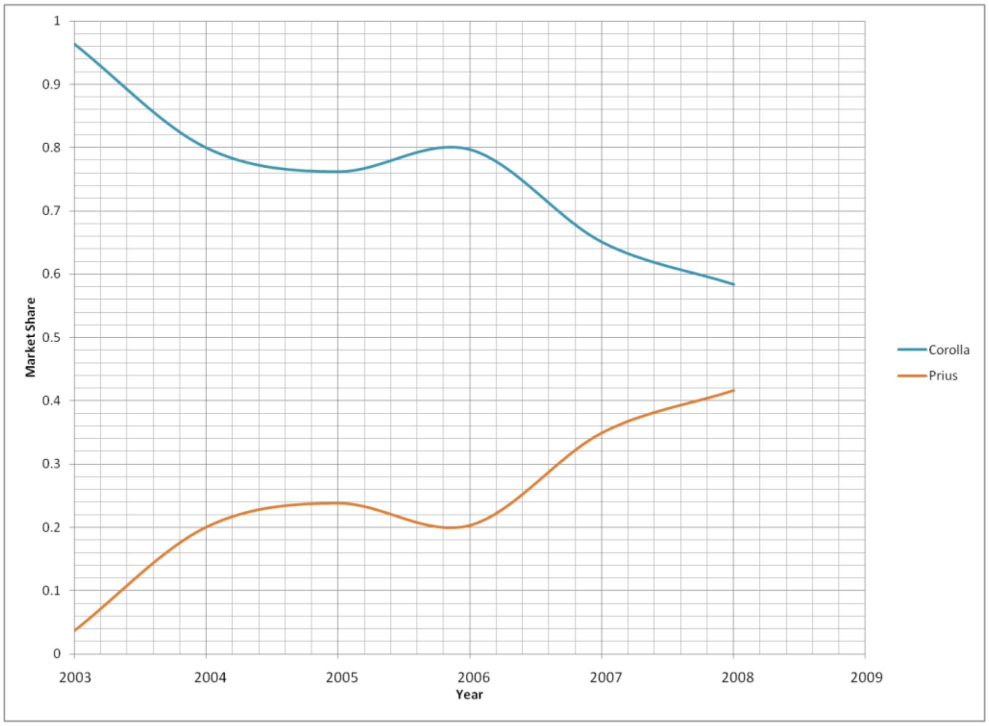

Figure 29. Market Trends between Corolla and Prius in Code 3 Group

The values of parameters for the dynamic normative model are in Table 27. And the result is shown in Figure 30.

TABLE 27 Values of Parameters (Code 3)

	Corolla	Prius
a_G	0.5	1000
a_P	N.A.	1
b	5000	5000
s	0.5	0.3
Φ_{CO}^{0}	3.3	

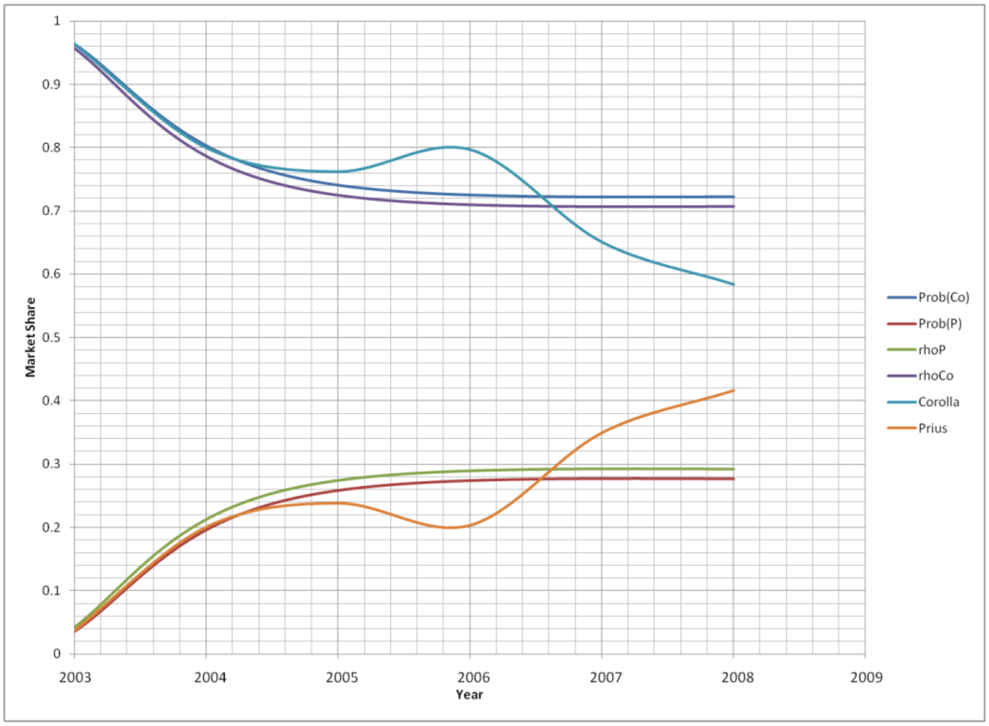

Figure 30. Change in the Market Share of Corolla and Prius (Code 3)

d) Bachelor's degree (code 4)

In the dataset, the distribution of people in code 4 is in Table 28.

TABLE 28 Distribution of People (Code 4)

Year	Number		Percentage (%)		Total
	Corolla	Prius	Corolla	Prius	
2003	34	12	73.9	26.1	46
2004	61	21	74.4	25.6	82
2005	70	56	55.6	44.4	126
2006	83	74	52.9	47.1	157
2007	99	105	48.5	51.5	204
2008	78	78	50	50	156
Total	425	346			771

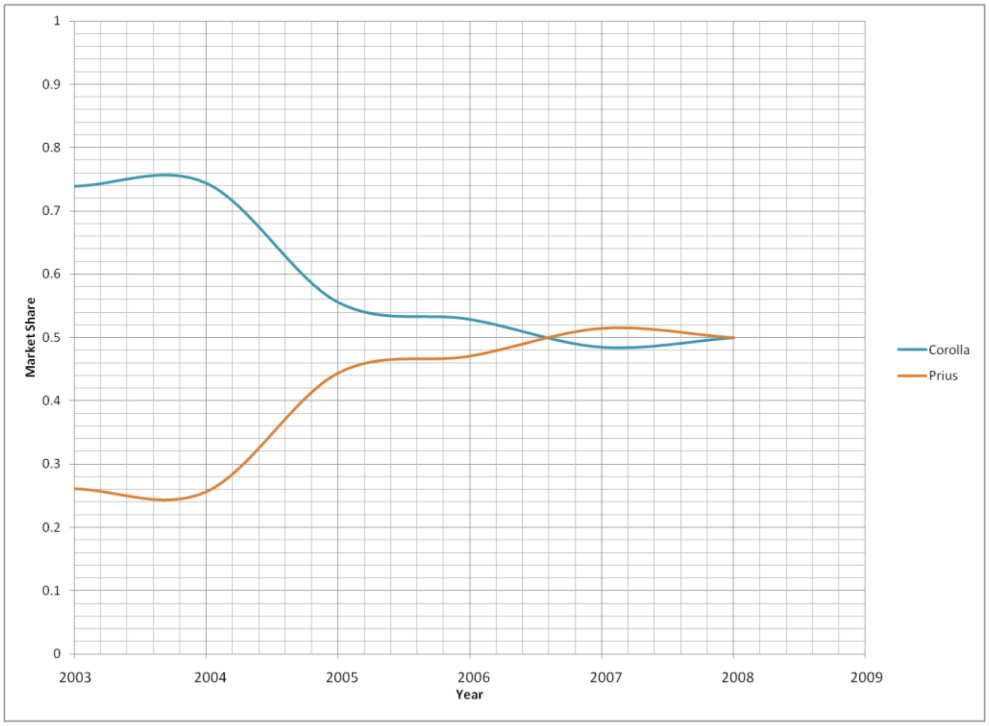

Figure 31. Market Trends between Corolla and Prius in Code 4 Group

The values of parameters for the dynamic normative model are in Table 29. And the result is shown in Figure 32.

TABLE 29 Values of Parameters (Code 4)

	Corolla	Prius
a_G	11	100
a_P	N.A.	1
b	1000	500
s	0.4	0.3
Φ^0_{CO}	1	

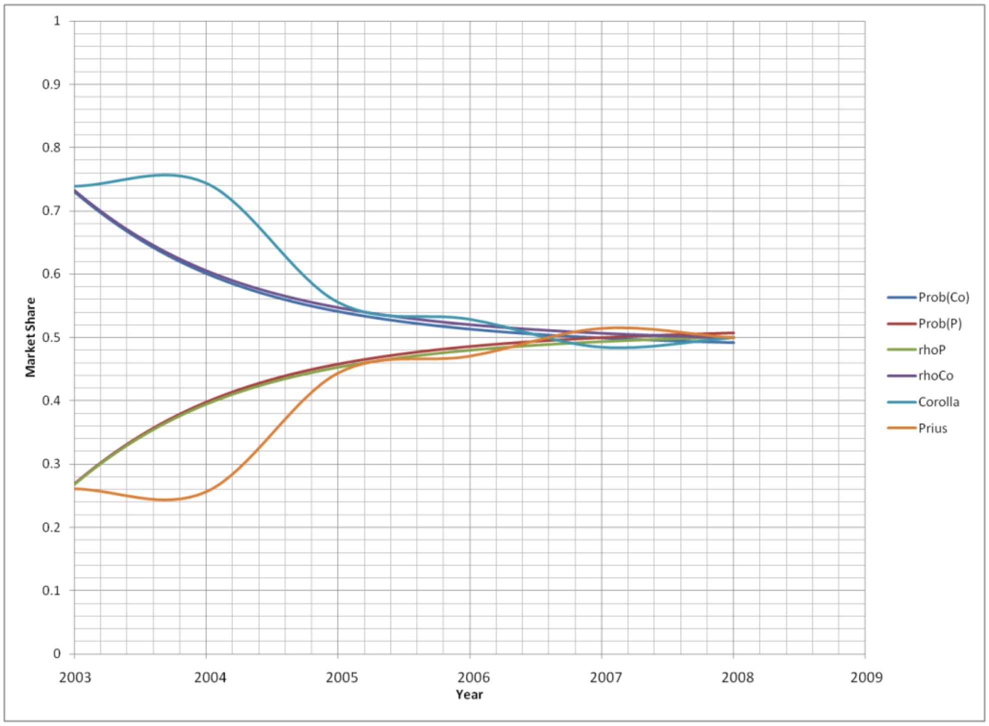

Figure 32. Change in the Market Share of Corolla and Prius (Code 4)

e) Graduate or Professional degree (code 5)

In the dataset, the distribution of people in code 5 is in Table 30.

TABLE 30 Distribution of People (Code 5)

Year	Number		Percentage (%)		Total
	Corolla	Prius	Corolla	Prius	
2003	37	17	68.5	31.5	54
2004	50	53	48.5	51.5	103
2005	64	79	44.8	55.2	143
2006	86	100	46.2	53.8	186
2007	94	159	37.2	62.8	253
2008	69	100	40.8	59.2	169
Total	400	508			908

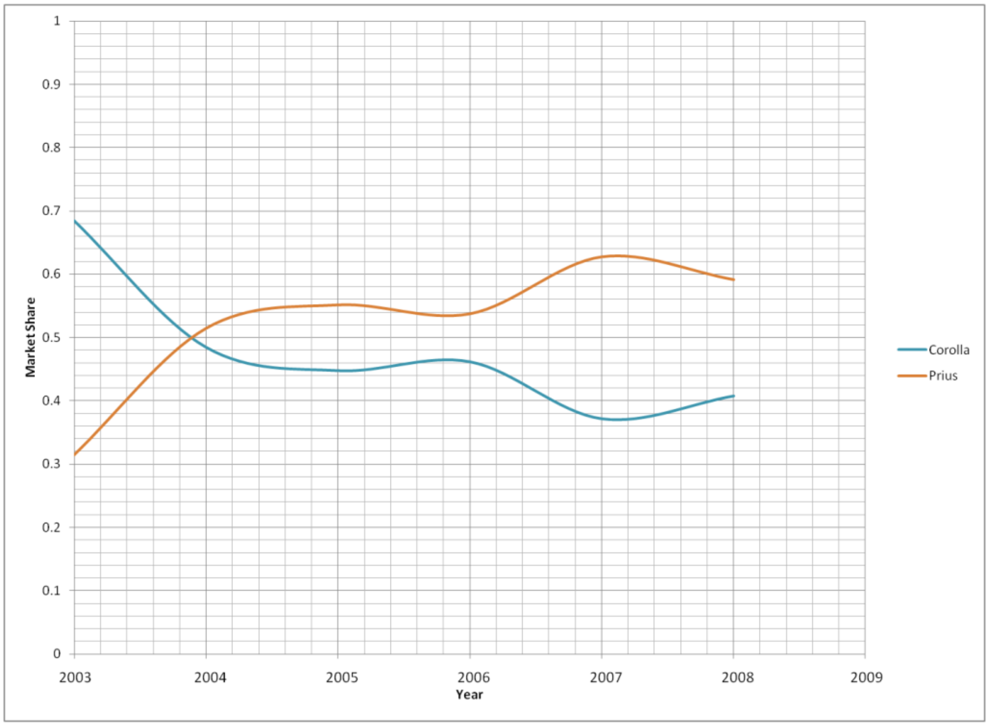

Figure 33. Market Trends between Corolla and Prius in Code 5 Group

The values of parameters for the dynamic normative model are in Table 31. And the result is shown in Figure 34.

TABLE 31 Values of Parameters (Code 5)

	Corolla	Prius
a_G	11	100
a_P	N.A.	0.3
b	2500	2500
s	0.4	0.3
Φ_{CO}^{0}	0.8	

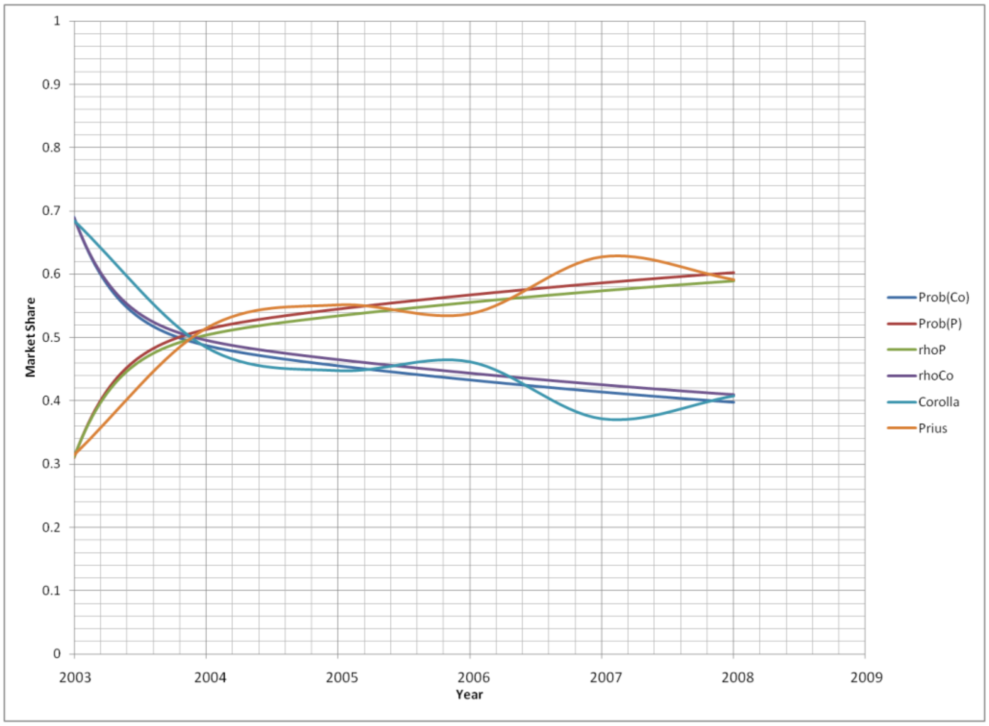

Figure 34. Change in the Market Share of Corolla and Prius (Code 5)

The results above show that, in the case of education level, the higher the education level, the more positive toward a choice of Prius—people associated with codes 4 (Bachelor degree) and 5 (Graduate degree), in particular, appear to be early adopters of the Prius. This result conforms to the expectation that, generally, people with higher educate levels are more aware of new technologies and more predisposed to choose alternatives that are socially responsible than are less educated. Under an assumption of rational economic behavior, these people are aware of the benefits of driving them (e.g., saving operating cost). So, higher educated people adapt to AFVs faster than do less educated counterparts. The comparison shown in Table 32 also indicates that different models for each market segment (by education level) may be warranted.

TABLE 32 Comparison of Fitted Model Parameters by Education Level

	Corolla					Prius				
	Code 1	Code 2	Code 3	Code 4	Code 5	Code 1	Code 2	Code 3	Code 4	Code 5
a_G	3	1	0.5	11	11	1000	1000	1000	100	100
a_P	N.A.	N.A.	N.A.	N.A.	N.A.	1	10	1	1	0.3
b	2500	2500	5000	1000	2500	2500	2500	5000	500	2500
s	0.4	0.5	0.5	0.4	0.4	0.3	0.3	0.3	0.3	0.3
Φ^0_{CO}	4.1	3.6	3.3	1	0.8					
λ	-10.1	-10.1	-10.1	-10.1	-10.1	-10.1	-10.1	-10.1	-10.1	-10.1

3.6 Summary

In this chapter, a dynamic normative model is developed and then applied to a number of different cases to forecast changes in market shares of vehicles over the time. The results show that a sensitivity of change in market share for a specific vehicle is influenced by several factors; the initial market share for the vehicle, gas price, a price of the vehicle, a gender of a driver, and an education level of a driver. In other words, a) the greater the initial market share for a specific vehicle, the greater the sensitivity to choosing this vehicle, b) the higher the gas price, the more sensitivity toward choosing a vehicle, c) the lower the vehicle price (which is affected by a rebate for AFVs), the more sensitivity toward choosing a vehicle, d) in the case of gender, there is no significant difference in the sensitivity to a choice of vehicle between a male group and a female group, but females driving ICEs are more sensitive to the change in gas price than males driving ICEs, and e) the higher the education level, the more sensitive in choosing a vehicle.

4. Demand for AFVs and the density of refueling stations

In this chapter, the change in the market trends of AFVs vehicles will be examined with respect to the change in the density of refueling stations. In the analysis, we assume a hypothetical HFCV and, using the models developed previously in which the parameters for Corolla and Prius are applied to generic ICEs and HFCVs, respectively, apply value of time associated with refueling as an additional cost to HFCVs. Specifically, we find the density for which market trends become stable.

4.1 Refueling availability vs. the density of refueling stations

A simplified relationship between the density of refueling/recharging stations and refueling availability can be explained with a Poisson distribution. Assume that there is a region S with distance of r as Figure 35:

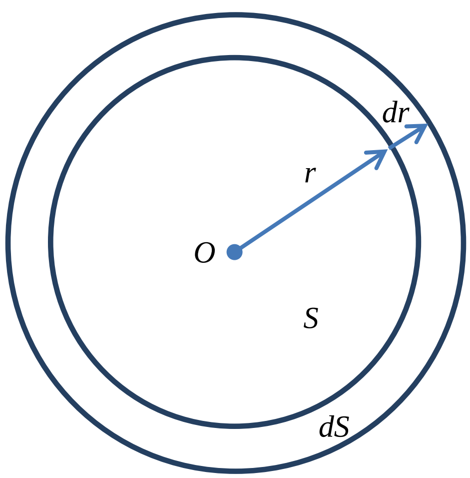

Figure 35. A Simple Region of S

And let δ denote the density of refueling stations, assumed to be uniform random. Then the probability that region S contains exactly x stations, $\Pr(x,S)$, is given as:

$$\Pr(x,S)=\frac{(\delta S)^x}{x!}\exp(-\delta S) \tag{39}$$

Next, consider that there is an additional region dS . Then the probability that region dS contains exactly x stations, $\Pr(x,dS)$, is given as:

$$\Pr(x,dS)=\frac{(\delta dS)^x}{x!}\exp(-\delta dS)=\frac{(2\pi\delta rdr)^x}{x!}\exp(-2\pi\delta rdr) \tag{41}$$

The probability that region S contains exactly 0 stations, $\Pr(0,dS)$, is given as:

$$\Pr(0,dS)=\frac{(\delta dS)^0}{0!}\exp(-\delta dS)=\frac{(2\pi\delta rdr)^0}{0!}\exp(-2\pi\delta rdr)=\exp(-2\pi\delta rdr) \tag{42}$$

The probability that region S contains at least 1 station, $\Pr(\geq 1,dS)$, is given as:

$$\Pr(\geq 1,dS)=1-\exp(-2\pi\delta rdr)\approx 1-(1-2\pi\delta rdr)=2\pi\delta rdr \tag{43}$$

Then the probability Pr(r) that the distance between a point selected randomly and the closest station is r is given by the joint probability that there are 0 stations up to a distance r from the point and that there is at least 1 station in the annulus defined by dr.

$$\Pr(r)=\exp(-\pi\delta r^2)\cdot 2\pi\delta rdr=2\pi\delta r\exp(-\pi\delta r^2)dr \tag{44}$$

And, the expected value of r, $E(r)$, is given as:

$$E(r) = \int_0^\infty r \Pr(r) = 2\pi\delta \int_0^\infty r^2 \exp(-\pi\delta r^2) dr = \frac{1}{2\sqrt{\delta}} \tag{45}$$

So, the expected roundtrip distance traveled for refueling is

$$E(\text{Refueling Distance}) = \frac{1}{\sqrt{\delta}} \tag{46}$$

4.1.1 Operating cost for vehicles with respect to the density of refueling stations

The operating cost for a vehicle consists of a fuel cost and the time cost for searching a refueling station. If a vehicle is assumed to run D_A miles in a year, the annual fuel cost C_{AF} becomes:

$$C_{AF} = \frac{D_A \text{ miles}}{\text{mpg}} \times \text{fuel price (\$/gal.)} \tag{47}$$

The annual distance traveled includes both the distance needed to access activities, D_{act}, and the distance needed for refueling, D_R, i.e.,

$$D_A = D_{act} + D_R \tag{48}$$

Then the annual fuel cost can be rewritten as:

$$C_{AF} = \frac{D_A \cdot C_F}{F} = \left(D_{act} + D_R\right)\frac{C_F}{F} \tag{49}$$

where F is the mpg for a vehicle, and C_F is the price of a fuel.

Let R^* denote the range of a vehicle. Then, the range available for trips (other than refueling), R, is given as:

$$R = R^* - \frac{1}{\sqrt{\delta}}, \delta \geq \frac{1}{\left(R^*\right)^2} \tag{50}$$

Let d denote the average length of a trip. Then, the latest trip, $k_{\max}$, before refueling is given as:

$$k_{\max} = INT\left(\frac{R}{d}\right) = INT\left(\frac{R^* - \frac{1}{\sqrt{\delta}}}{d}\right) \tag{51}$$

Since the latest trip for refueling must take place no earlier than the first trip,

$$INT\left(\frac{R^* - \frac{1}{\sqrt{\delta}}}{d}\right) > 0 \Rightarrow \frac{R^* - \frac{1}{\sqrt{\delta}}}{d} \geq 1 \Rightarrow R^* - \frac{1}{\sqrt{\delta}} \geq d \Rightarrow \sqrt{\delta}R^* - \sqrt{\delta}d \geq 1$$

$$\Rightarrow \sqrt{\delta} \geq \frac{1}{R^* - d} \Rightarrow \delta \geq \frac{1}{\left(R^* - d\right)^2}$$

And the residual range, $R_{k_{\max}}$, available to seek refueling is given as:

$$R_{k_{\max}} = R - k_{\max} d \tag{52}$$

From the equation above, the term $k_{\max} d$ becomes miles traveled before refueling. Then the number of refueling activities in a year becomes:

$$N_{\text{Refueling}} = \frac{D_{act}}{k_{\max} d} = \frac{D_{act}}{INT\left(\frac{R}{d}\right) \cdot d}, \frac{R}{d} \geq 1 \tag{53}$$

And, the expected annual travel distance for refueling activities, D_R , becomes:

$$\mathrm{D}_R = N_{\text{Refueling}} \cdot E(\text{Refueling Distance}) = \frac{D_{act}}{INT\left(\frac{R}{d}\right) \cdot d} \cdot \frac{1}{\sqrt{\delta}} \tag{54}$$

With an assumed speed of V_{avg} mph, an annual travel time for refueling activity, T_R , becomes:

$$T_R = \frac{\mathrm{D}_R}{V_{avg}} = \frac{D_{act}}{INT\left(\frac{R}{d}\right) \cdot d} \cdot \frac{1}{\sqrt{\delta}} \cdot \frac{1}{V_{avg}} \tag{55}$$

And, with an assumption that the average travel time cost is C_{TT} (\$/hr), an annual travel time cost for refueling activity, C_{TA}, becomes:

$$C_{TA} = C_{TT} \cdot T_R = C_{TT} \cdot \frac{D_{act}}{INT\left(\frac{R}{d}\right) d} \cdot \frac{1}{\sqrt{\delta}} \cdot \frac{1}{V_{avg}} \tag{56}$$

Then the total annual operating cost, C_{OA} , which is the sum of the annual fuel cost plus the annual refueling travel time cost, can be expressed as:

$$\text{Annual fuel cost} = C_{AF} = \left(D_{act} + D_R\right) \frac{C_H}{F_H} = \left(1 + \frac{1}{INT\left(\frac{R}{d}\right) d} \cdot \frac{1}{\sqrt{\delta}}\right) \cdot D_{act} \cdot \frac{C_H}{F_H} \tag{57}$$

Annual refueling travel time cost= $C_{TA} = C_{TT} \cdot \frac{1}{INT\left(\frac{R}{d}\right)d} \cdot \frac{1}{\sqrt{\delta}} \cdot \frac{1}{V_{avg}} \cdot D_{act}$ (58)

$$C_{OA} = \left(1 + \frac{1}{INT\left(\frac{R}{d}\right)d} \cdot \frac{1}{\sqrt{\delta}}\right) \cdot D_{act} \cdot \frac{C_H}{F_H} + C_{TT} \cdot \frac{D_{act}}{INT\left(\frac{R}{d}\right)d} \cdot \frac{1}{\sqrt{\delta}} \cdot \frac{1}{V_{avg}} \tag{59}$$

$$C_{OA} = \left[\left(1 + \frac{1}{INT\left(\frac{R}{d}\right)d} \cdot \frac{1}{\sqrt{\delta}}\right) \cdot \frac{C_H}{F_H} + C_{TT} \cdot \frac{1}{INT\left(\frac{R}{d}\right)d} \cdot \frac{1}{\sqrt{\delta}} \cdot \frac{1}{V_{avg}}\right] \cdot D_{act} \tag{60}$$

$$C_{OA} = \frac{C_H}{F_H} \cdot D_{act} + \left(\frac{1}{INT\left(\frac{R}{d}\right)d} \cdot \frac{1}{\sqrt{\delta}}\right)\left[\frac{C_H}{F_H} + C_{TT} \cdot \frac{1}{V_{avg}}\right] \cdot D_{act} \tag{61}$$

So, finally, the total operating cost as a function of refueling station density is given as:

$$C_{OA} = \left(\frac{C_H}{F_H} + R_{HF}\right) \cdot D_{act} \tag{62}$$

where

$$R_{HF} = \left(\frac{1}{INT\left(\frac{R}{d}\right)d} \cdot \frac{1}{\sqrt{\delta}}\right)\left[\frac{C_H}{F_H} + C_{TT} \cdot \frac{1}{V_{avg}}\right] = \text{Hydrogen Refueling Factor}$$

$$R = R^* - \frac{1}{\sqrt{\delta}} \ ; \ \frac{R}{d} \geq 1$$

$$\delta \geq \frac{1}{\left(R^* - d\right)^2}$$

Equation (62) represents the operating cost as a function of refueling station density and other factors related to the AFV characteristics (e.g., range, fuel economy).

4.2 Saturation refueling station densities for ICE vs. HFCV

The formulation of operating cost for a vehicle described in the previous section is applied to the case between ICEs and HFCVs. In this application, it is assumed that the range and mpg for HFCV are defined as 300 mi. and 61mpg, respectively. The price for HFCV is originally assumed as $30,000, and an initial tax rebate of $5,000 is applied to that price—the effective price becomes $25,000. The price of hydrogen fuel is assumed as $1/1.8kg (1.8kg=1gal.). (http://heshydrogen.com/hydrogen-fuel-cost-vs-gasoline/). In contrast to the HFCV, an ICE essentially can be refueled anywhere because of the vast number of refueling stations. Therefore, the cost incurred by the density of refueling stations is applied to HFCV only. Market shares for HFCVs with respect to the density of refueling stations are examined for several cases. The probability of choosing HFCVs and that of choosing ICEs are calculated using the choice probabilities prescribed by a binary logit model with coefficients estimated for Prius and Corolla, respectively; dynamics are those prescribed by the dynamic normative model.

4.2.1 Base Case

As shown in the previous section, the density of refueling stations for HFCVs must be larger than $2.31\times10^{-5} stations/mi^2$. Using this as a limit, we ran the dynamic model for various densities (and, corresponding expected costs of refueling) in order to determine refueling station densities for which HFCVs become a practical alternative.

The base case scenario reported in this section entails that in which a $5,000 rebate is applied and the bandwagon effects are included. Values of variables for the model are assumed as shown in Table 33 and Table 34.

TABLE 33 Values of Characteristic Variables for ICE and HFCV

	ICE	HFCV
Fuel price ($/gal.)	4.09, 4.49, and 5.99	1
Average annual distance driven (mi.)	15,000 (D_A)	15,000 (D_{act})
mpg	32	61
Vehicle price ($)	18,515	25,000 ($5,000 of rebate is applied)
Range (mi.)	N.A.	300

TABLE 34 Values of Other Variables

Variable	Value
Initial market share of HFCV	0.072
d (mi.)	32
V_{avg} (mph)	30
C_{TT} ($/hr.)	25
Household income ($)	100,000
Education level	5
Gender	0

In Table 34, the initial market share of HFCV is assumed as 0.072 because HFCVs have not been released on the market yet. And an average length of trip, d, is randomly assumed.

The results are shown in Figure 36. In Figure 36, the density saturation points where the market trend for HFCVs effectively don’t increase are approximately 0.0734 ($4.09/gal.), 0.0704 ($4.49/gal.), and 0.067 ($5.99/gal.).

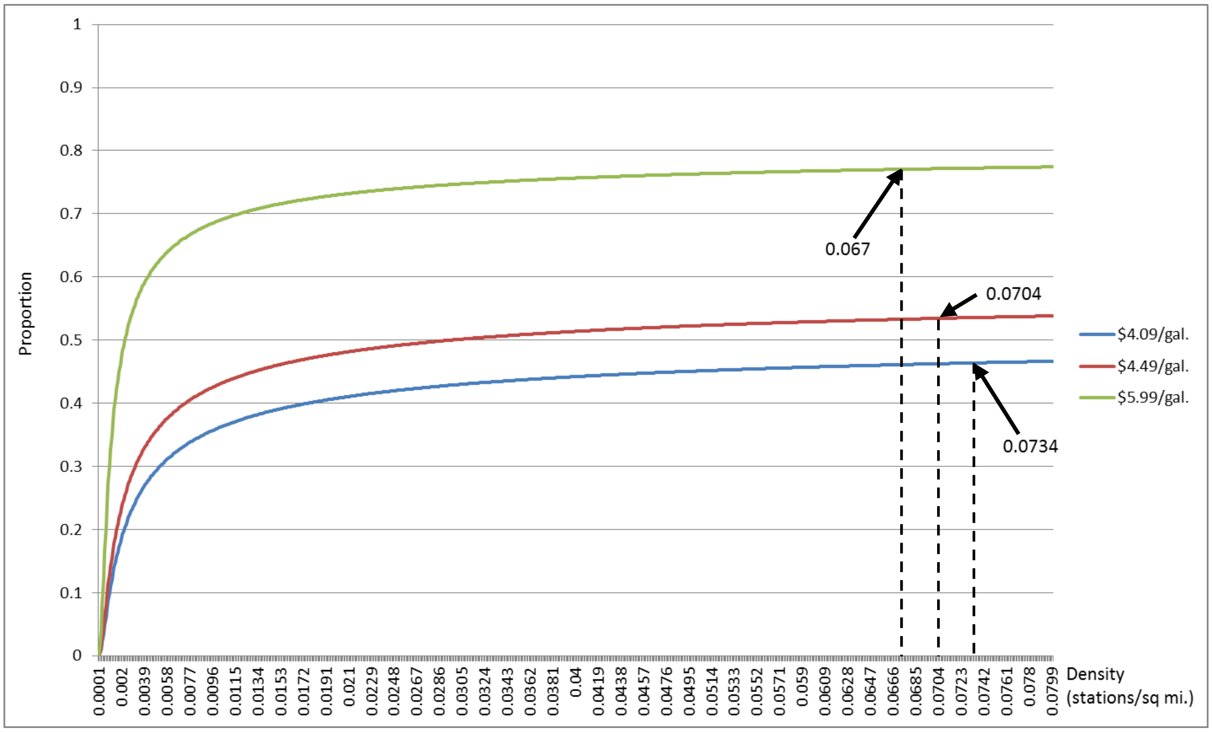

Figure 36. Market Shares for HFCV with $5,000 Rebate

4.2.2 No bandwagon effect is applied

In this case, the values of parameters are same with those in the base case, except for the initial market share of HFCV, which is set at 0. Additionally, the bandwagon effect is removed from the dynamic model. The results are shown in Figure 37.

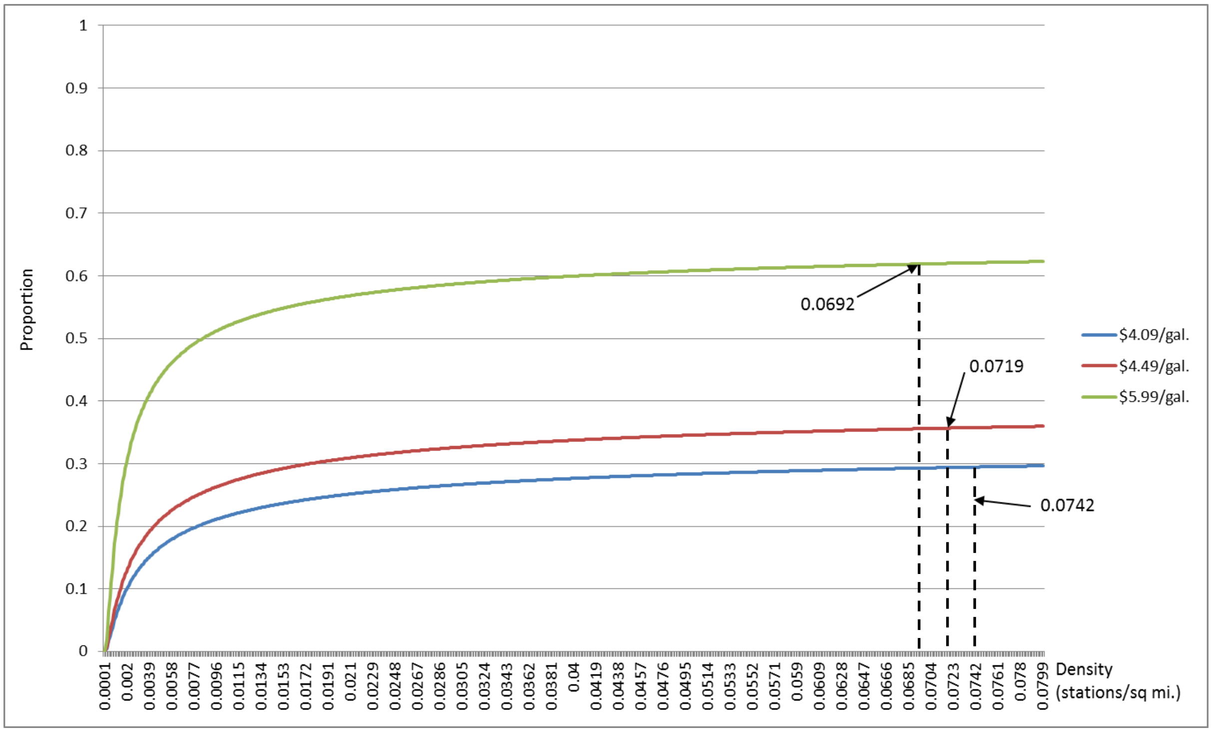

Figure 37. Market Trends for HFCV without Bandwagon Effect (100% Saturation Values)

In Figure 37, the saturation points where the market trends don't increase is observed to be about 0.0742 ($4.09/gal.), 0.0719 ($4.49/gal.), and 0.0692 ($5.99/gal.). These results contrast to those in Figure 35 (scenario including bandwagon), where the density saturation points where the market trend for HFCVs effectively don't increase are 0.0734 ($4.09/gal.), 0.0704 ($4.49/gal.), and 0.067 ($5.99/gal.). From these results, the bandwagon effect on the ultimate saturation density is seen to be about 1%, 2% and 3% for the three gasoline prices, respectively. However, although the saturation densities are not affected much by the bandwagon effect, the proportions at saturation are significantly higher with the bandwagon effect than without.

Other applications were run to find densities for reaching 25%, 50%, and 75% of saturation values. The results of these analyses are shown in Figures 38, 39 and 40.

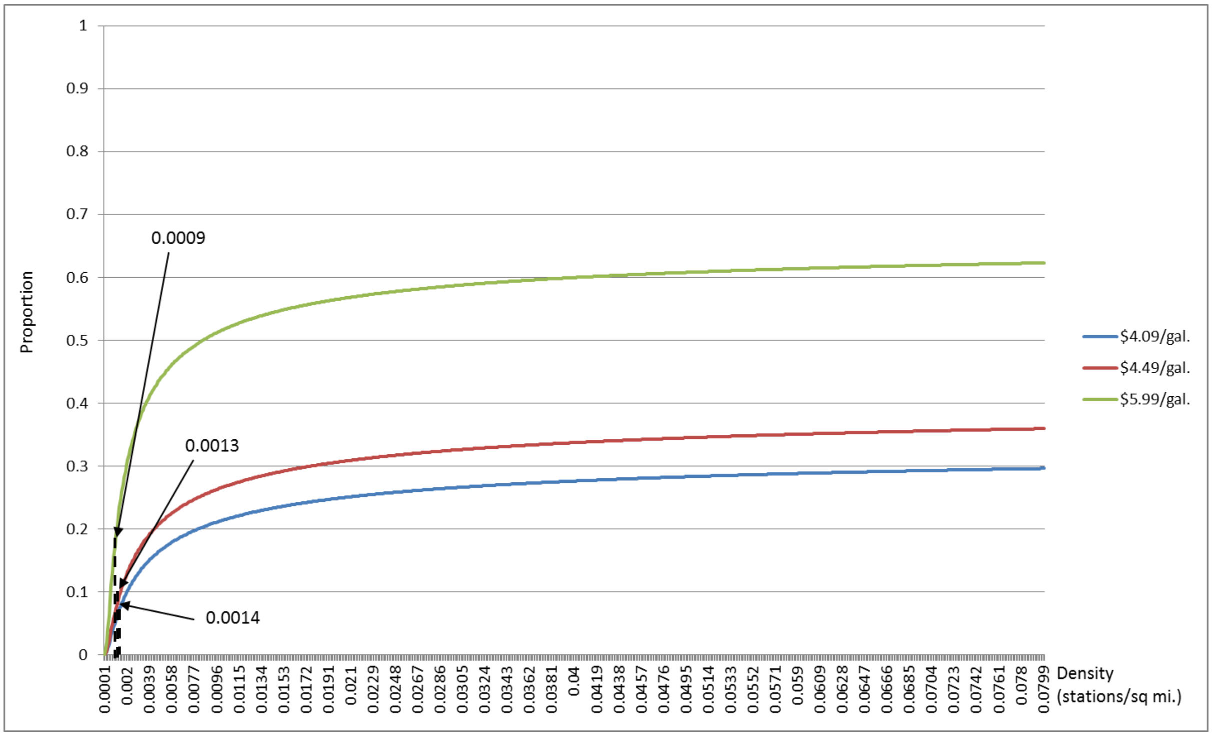

Figure 38. Market Trends for HFCV without Bandwagon Effect (25% Saturation Values)

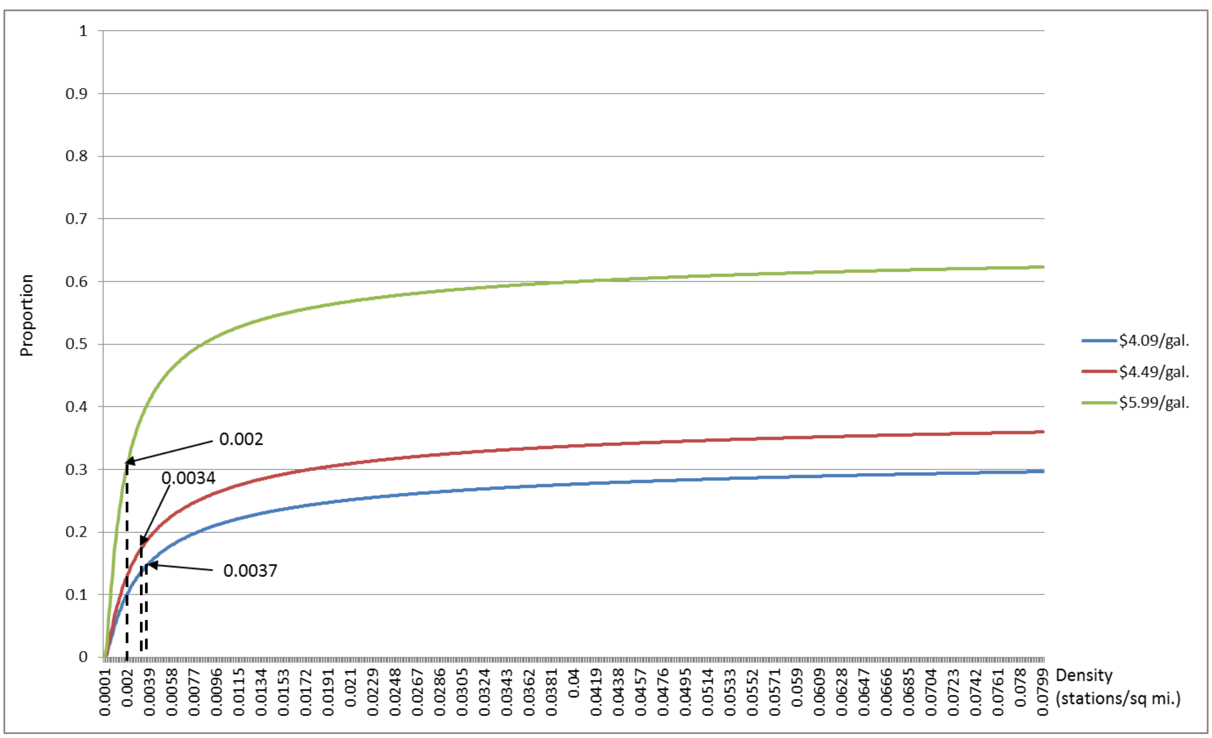

Figure 39. Market Trends for HFCV without Bandwagon Effect (50% Saturation Values)

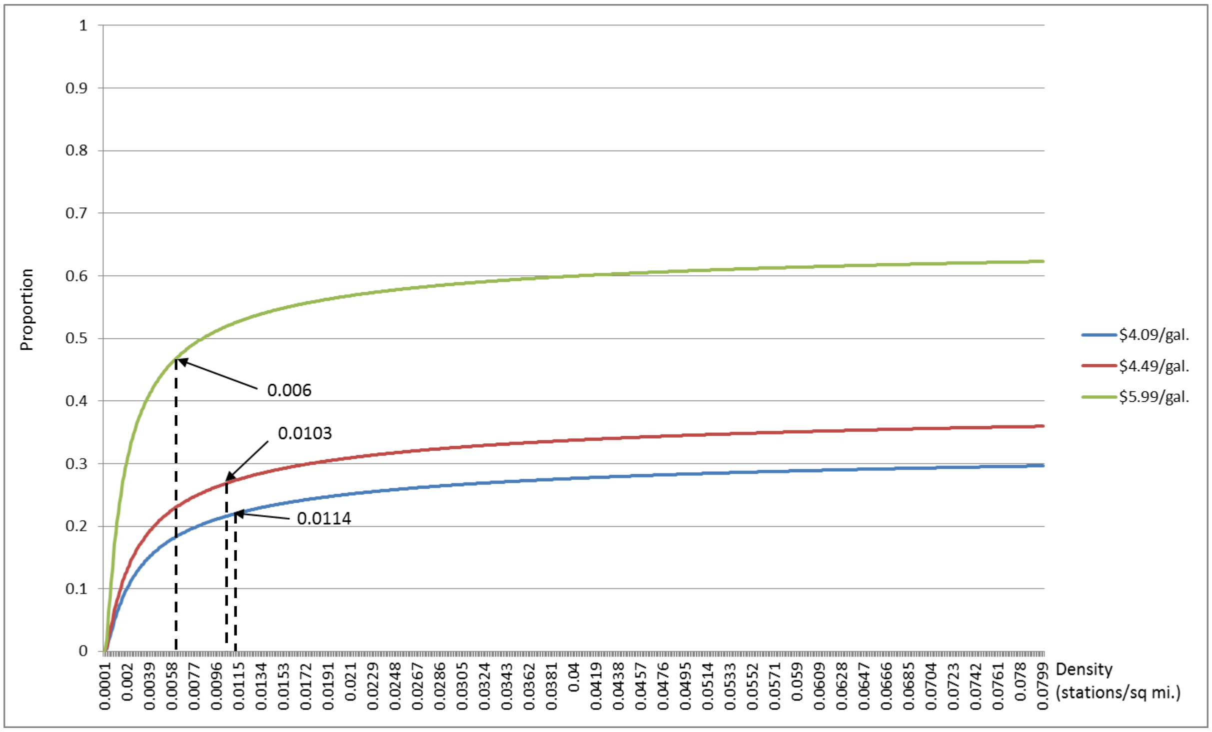

Figure 40. Market Trends for HFCV without Bandwagon Effect (75% Saturation Values)

As shown above, all three cases with different gas price show similar density of refueling stations for reaching 25% of saturation values. However, as the graphs reach 50% and 75% of saturation values, densities of refueling stations begin to show differences among themselves.

4.2.3 No Rebate is applied

In this case, the values of parameters are same with those in the base case, except for the vehicle price of HFCV which is defined as $ 30,000, i.e., no rebate is applied to HFCV.

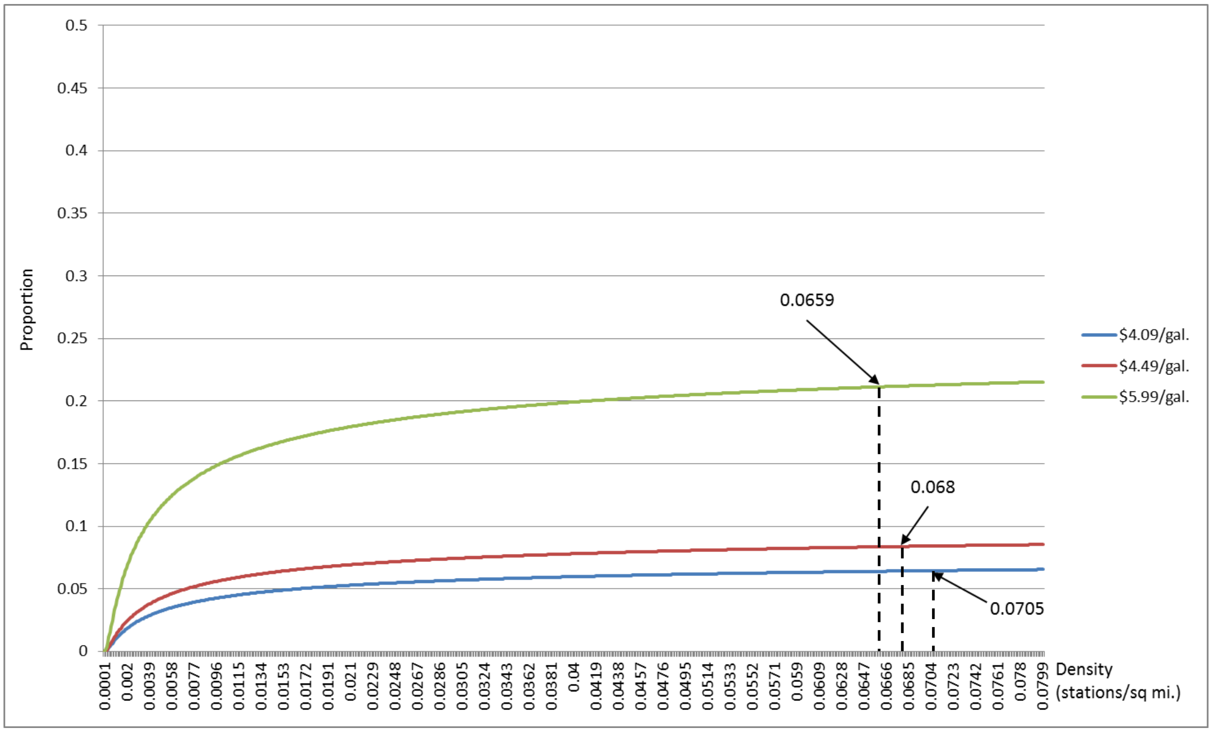

Figure 41. Market Trends for HFCV without Rebate

The results, shown in Figure 41, indicate that the saturation point where the market trends for HFCVs don't increase are about 0.0705 ($4.09/gal.), 0.068 ($4.49/gal.), and 0.0659 ($5.99/gal.). Because the price of the HFCV is higher than the price of ICE, the effect without rebate on market share of HFCV is lower than a base scenario. The lower the price of HFCVs, the faster increase in market share of HFCVs with respect to density of refueling stations. This means, for example, if a target market share of HFCVs is defined as 20%, the density of refueling stations required for this target in the base scenario is 0.0007 with gas price $5.99/gal., and the density of refueling stations in no rebate scenario is 0.0434 with $5.99/gal. gas price.

4.2.4 Effect of the initial market share of HFCVs on saturation density

In this section, the influence of the initial market share of HFCVs on market trends for HFCVs as a function of density of refueling stations and on the corresponding saturation densities of refueling stations are analyzed. Variables for the models are assumed as:

TABLE 35 Values of Characteristic Variables for ICE and HFCV

	ICE	HFCV
Fuel price ($/gal.)	4.09	1
Average annual distance driven (mi.)	15,000 (D_A)	15,000 (D_{act})
mpg	32	61
Vehicle price ($)	18,515	25,000 ($5,000 rebate)
Range (mi.)	N.A.	300

TABLE 36 Values of Other Variables

Variable	Value
Initial market share of HFCV	0, 0.05, 0.1, 0.2, and 0.3
d (mi.)	32
V_{avg} (mph)	30
C_{TT} ($/hr.)	25
Household income ($)	100,000

The results are shown in Figure 42. In Figure 42, it can be seen that the greater the initial market share of HFCVs, the smaller the saturation density of refueling stations. The reason can be explained by the bandwagon effect; i.e., when the initial market share of HFCVs becomes larger, more people react to the popularity of HFCVs. Therefore, the larger initial market share of HFCVs, the more people buy HFCVs, providing an increasing density of home locations with HFCVs, and a corresponding lower density of stations required for saturation.

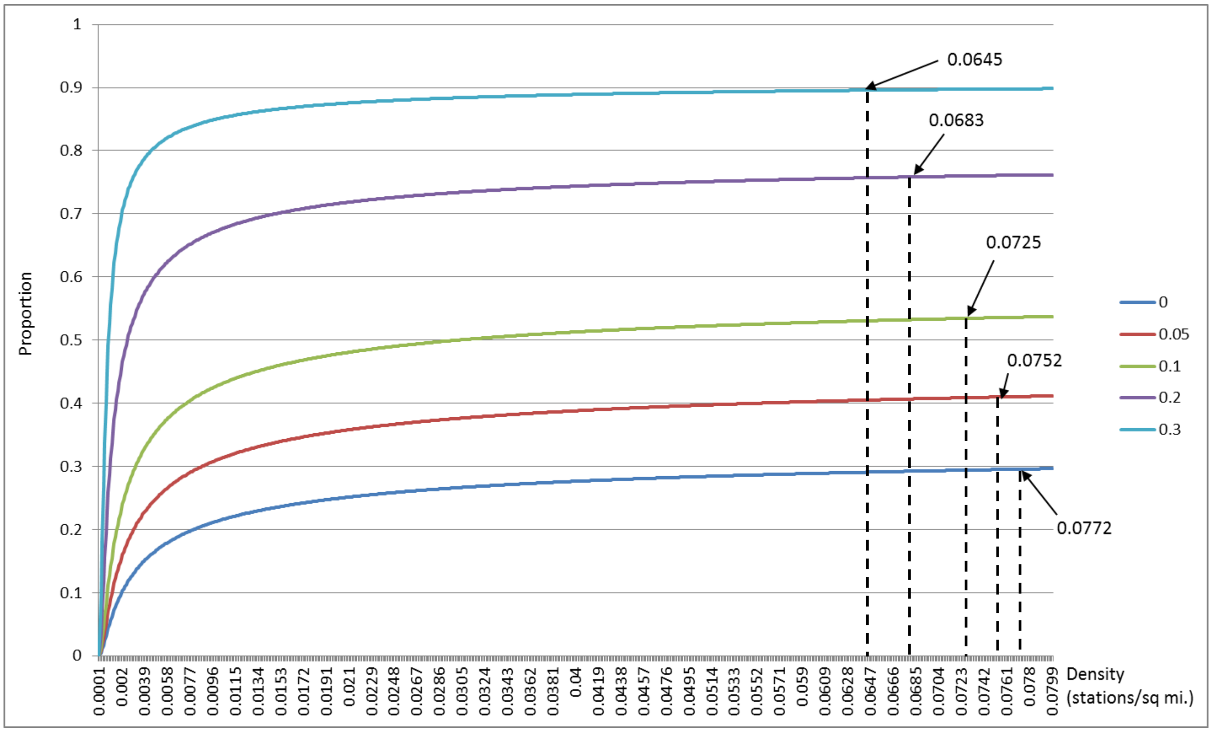

Figure 42. Market Trends for HFCV with Initial Market Share

4.3 Operating cost with respect to cost of gasoline and refueling station density

In this section, we analyze expected operating costs as a function of refueling station density and make a comparison to the monetary break-even point with regular gasoline. We consider two scenarios: 1) \$4.09/gal. cost of gasoline and 2) \$4.49/gal. cost of gasoline. Each scenario is examined based on the following values of variables:

TABLE 37 Values of Characteristic Variables for ICE and HFCV

	ICE	HFCV
Fuel price (\$/gal.)	4.09 and 4.49	1
Average annual distance driven (mi.)	15,000 (D_A)	15,000 (D_{act})
mpg	32	61
Vehicle price (\$)	18,515	25,000 (\$5,000 of rebate is applied)
Range (mi.)	N.A.	300

TABLE 38 Values of Other Variables

Variable	Value
Initial market share of HFCV	0.072
d (mi.)	32
V_{avg} (mph)	30
C_{TT} ($/hr.)	25
Household income ($)	100,000

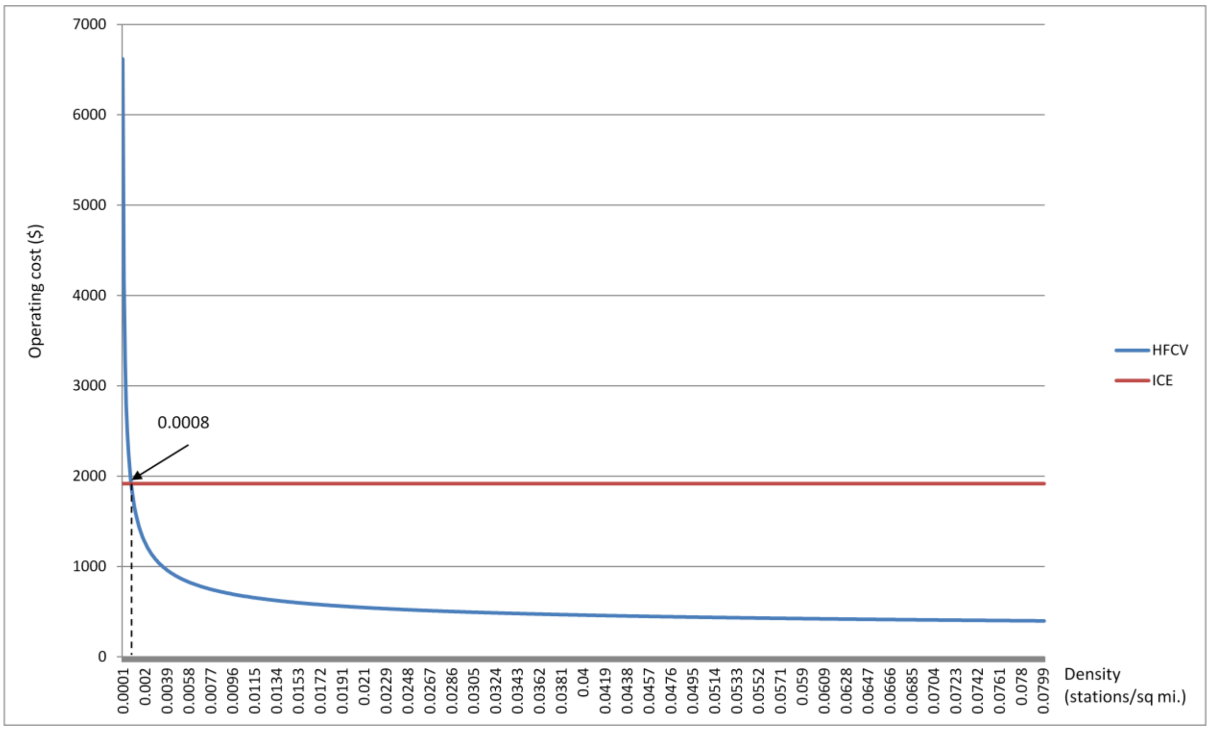

Figure 43. Operating Cost for HFCV against $4.09/gal. of Gas Price

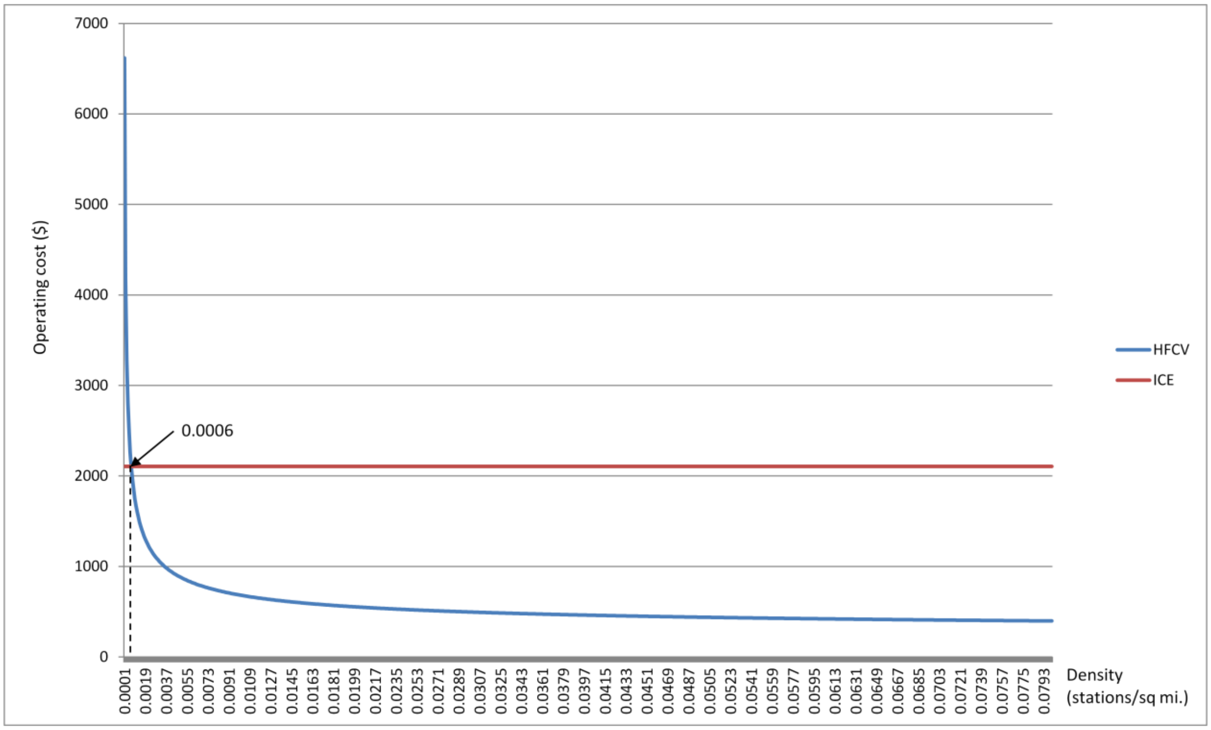

Figure 44. Operating Cost for HFCV against $4.49/gal. of Gas Price

In Figure 43, the operation cost for HFCV becomes lower than that for ICE after a density of 0.0008. And in Figure 44, the operation cost for HFCV becomes lower than that for ICE after a density of 0.0006.

4.4 Market share and operating cost with respect to refueling station density and the cost of hydrogen

In this section, we analyze expected operating costs as a function of the price of hydrogen and refueling station density and make a comparison to the monetary break-even point with regular gasoline, priced at $4.09/gal., and based on the following values of variables:

TABLE 39 Values of Characteristic Variables for ICE and HFCV

	ICE	HFCV
Fuel price ($/gal.)	4.09	1, 2, 4, and 6
Average annual distance driven (mi.)	15,000 (D_A)	15,000 (D_{act})
mpg	32	61
Vehicle price ($)	18,515	25,000 ($5,000 of rebate is applied)
Range (mi.)	N.A.	300

TABLE 40 Values of Other Variables

Variable	Value
Initial market share of HFCV	0.072
d (mi.)	32
V_{avg} (mph)	30
C_{TT} ($/hr.)	25
Household income ($)	100,000

The market trends for HFCV are shown in Figure 45. In Figure 45, the saturation point where the market trend effectively doesn't increase further is 0.0734 ($1.00/gal.), 0.0747 ($2.00/gal.), 0.0755 ($4.00/gal.) and 0.0776 ($6.00/gal.).

And the operating costs for HFCV and ICE ($4.09/gal. of gas price) are shown in Figure 46. In Figure 46, the operation cost for HFCV becomes lower than that for ICE after the density of 0.0008 ($1.00/gal.), 0.0011 ($2.00/gal.), 0.0026 ($4.00/gal.), and 0.0118 ($6.00/gal.).

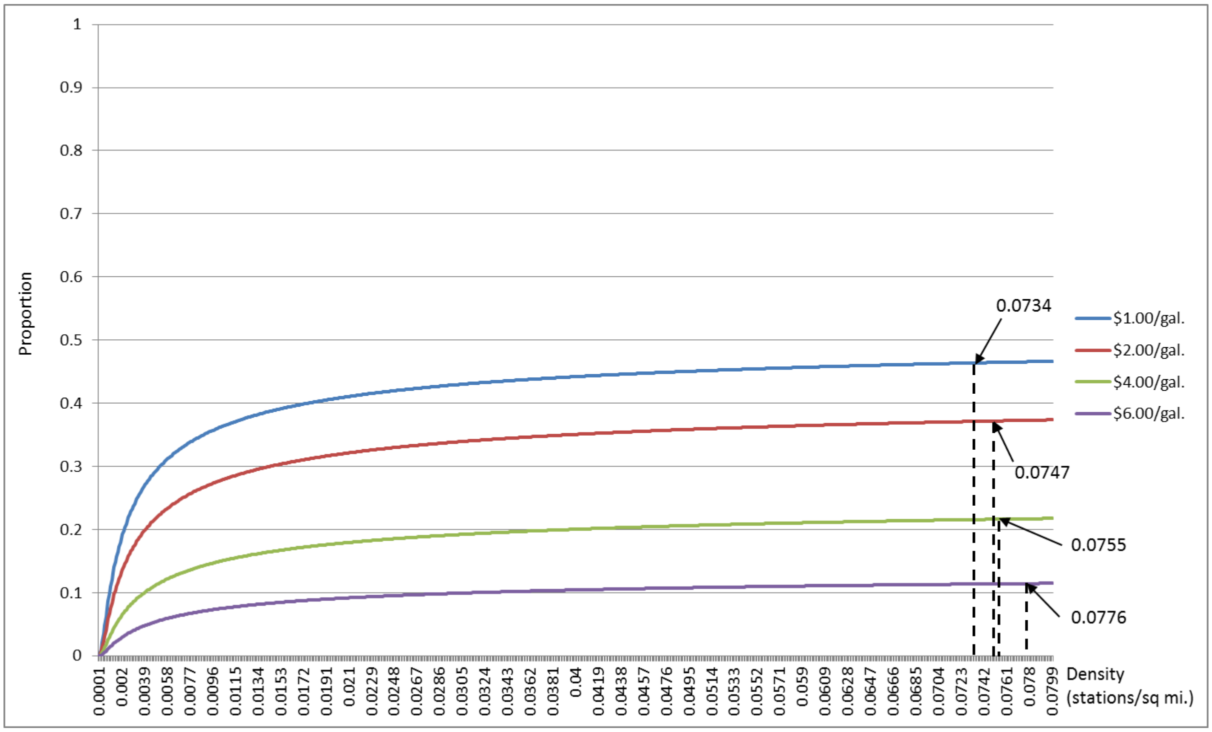

Figure 45. Market Trends for HFCV with Several Prices of Hydrogen Fuel

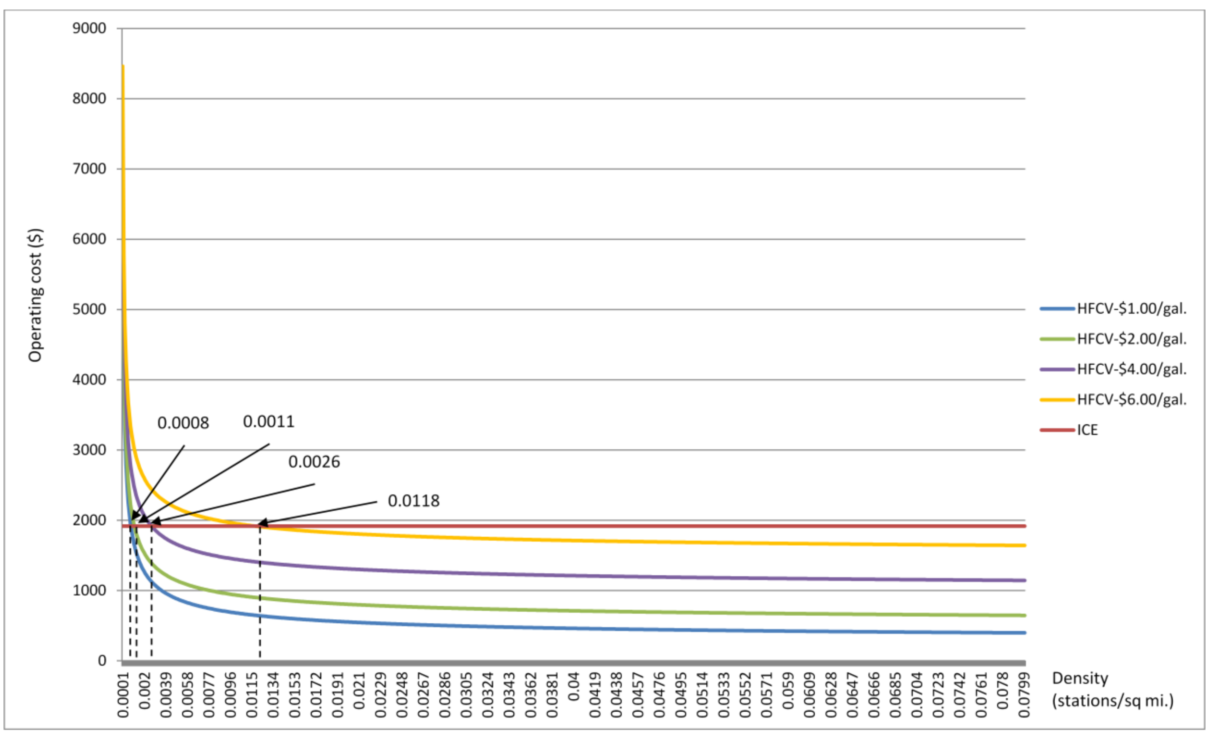

Figure 46. Operating Costs for HFCV against That of ICE with $4.09/gal.

4.5 Summary

In this chapter, several scenarios with respect to the density of refueling stations have been analyzed. The results show that: a) the higher gas price, the more proportion of HFCVs with the same density of refueling stations, b) without bandwagon effect, the proportion of HFCVs with a specific density is lower than that with bandwagon effect, c) in the case of no bandwagon effect, the densities of refueling stations with several gas prices are almost same at 25% of saturation value, and as reaching 50%, 75% and 100% of saturation values, the densities begin to be different, d) the lower vehicle price of HFCVs, the more proportions of HFCVs with same density of refueling stations, and e) the higher price of hydrogen fuel, the smaller the proportion of HFCVs with same density of refueling stations. And they also show that as a proportion of HFCVs with a specific density of refueling stations becomes greater, the density of refueling stations required in a saturation status becomes smaller. In conclusion, with a specific density of hydrogen refueling stations, a proportion of HFCVs is influenced by gas price, bandwagon effect, vehicle price of HFCVs, the initial market share of HFCVs, and the price of hydrogen fuel.

5. A competition model to account for changes in the supply of refueling stations

Because HFCVs are still on the precipice of introduction, the forecasted demands for these vehicles, which are found in the previous chapter, are subject to: a) supply of vehicles by manufacturers, and b) supply of refueling stations. In this chapter, a competition model is introduced as a model forecasting supply of vehicles, which makes outputs based on demands for vehicles and supply of refueling stations. Demands of vehicles, which are derived from a dynamic normative model, are used in this model as inputs. And also affordable proportions of vehicles which are found in Chapter 4 are used in the model.

5.1 A competition model

A formulation of a competition model is based on the work by Redmond (2011).

$$\frac{dN_i(t)}{dt} = r_i(t)N_i(t)\left[1 - \frac{\sum_{j=1}^{n} \alpha_{ij} N_j(t)}{K_i}\right] \qquad (63)$$

$$Auto = \{ICE, HFCV\}$$
$$i, j \in Auto$$

where

N_i = A market share of vehicle *i*, which is estimated from a dynamic normative model

K_i = An affordable market share of vehicle *i* with respect to the density of refueling/recharging stations, which is calculated from a logit model in Chapter 4. For ICE, the value is 1.

r_i = A natural increase rate of vehicle i

α_{ij} = A value of interaction between i and j $(\alpha_{ii} = 1)$

For HFCVs, K is the probability of choosing these vehicles with respect to the density of refueling stations; its value can be calculated directly from the logit choice model.

The increase rate of vehicle i over time t, $r_i(t)$, can be obtained as:

$$r_i(t) = \frac{\rho_i(t) - \rho_i(t-1)}{\rho_i(t-1)} \tag{64}$$

where $\rho_i(t)$ is a forecasted demand for a vehicle, which in this research is calculated from the dynamic normative model. For HFCVs, the value of $\rho_i(t)$ represents an approximation of what the "natural" rate of increase would be for these vehicles.

In the case of refueling stations, HFCVs and refueling stations are in an obligate mutualistic relationship. As Boucher (1985) said, an obligate mutualist can only survive by association with the other species. In other words, a survival of HFCVs depends on refueling stations for HFCVs, and vice versa. Therefore, a model for the interdependency of HFCVs and refueling stations can come from the mutualism model. Redmond (2011) derived this model from a modified competition model:

$$\begin{aligned} \frac{dN_1(t)}{dt} &= r_1 N_1(t)\left[\frac{K_1 - N_1(t) + \theta N_2(t)}{K_1}\right] \\ \frac{dN_2(t)}{dt} &= r_2 N_2(t)\left[\frac{K_2 - N_2(t) + \beta N_1(t)}{K_2}\right] \end{aligned} \tag{65}$$

By graphing these equations, the zero growth lines can be found. And, by setting the equation (65) to zero (for zero growth) and solving, the following equilibrium values, N_1^* and N_2^*, can be found:

$$
\begin{aligned}
N_1^* &= K_1 + \theta N_2^* \\
N_2^* &= K_2 + \beta N_1^*
\end{aligned}
\tag{66}
$$

By adapting these models to HFCVs and refueling stations for HFCVs, the following equations can be found:

$$
\begin{aligned}
\frac{dN_{i,R}(t)}{dt} &= r_i(t)N_i(t)\left[\frac{K_i(t) - N_i(t) + \theta_i R_i(t)}{K_i(t)}\right] \\
\frac{dR_i(t)}{dt} &= \eta_i(t)R_i(t)\left[\frac{\Gamma_i(t) - R_i(t) + \beta_i N_i(t)}{\Gamma_i(t)}\right]
\end{aligned}
\tag{67}
$$

where

$N_{i,R}$ = A market share of vehicle i with respect to a density of refueling stations

R_i = Density of refueling stations for alternative i

Γ_i = Affordable density of refueling stations for alternative i

η_i = Increase rate of refueling/recharging stations

θ_i = Influence of alternative i on others in Auto={ICE, HFCV}, which increases N_i at equilibrium

β_i = Values of interactions for refueling stations, which makes a delay to the growth of R_i

The increase rate of refueling stations, η_i, is:

$$\eta_i(t) = \frac{R_i(t) - R_i(t-1)}{R_i(t-1)} \tag{68}$$

The density of stations required to support $K_i(t)$ HFCVs, Γ_i, is given by:

$$\Gamma_i = \Gamma_{i0} + \sigma_i K_i(t) \tag{69}$$

where

Γ_{i0} = initial density of refueling stations

σ_i = parameter with specified value

Then, equation (67) is rewritten as:

$$\begin{aligned} \frac{dN_{i,R}(t)}{dt} &= r_i(t) N_i(t) \left[\frac{K_i(t) - N_i(t) + \theta_i R_i(t)}{K_i(t)} \right] \\ \frac{dR_i(t)}{dt} &= \eta_i(t) R_i(t) \left[\frac{\Gamma_{i0} + \sigma_i K_i(t) - R_i(t) + \beta_i N_i(t)}{\Gamma_{i0} + \sigma_i K_i(t)} \right] \end{aligned} \tag{70}$$

With equation (70), the market share of vehicle *i* in the time step *t*+1 can be obtained as:

$$N_i(t+1) = N_i(t) + \max\left(\frac{dN_i(t)}{dt}, \frac{dN_{i,R}(t)}{dt} \right) \tag{71}$$

5.2 Change in market trends with a competition model: ICE vs. HFCV

5.2.1 Base scenario

In this section, the market trends between Toyota Corolla and Honda Clarity FCX (an HFCV that, while developed, has yet to be released) are forecast considering both supply and demand. Because Corolla (Gasoline) and Clarity FCX (Hydrogen fuel) use different fuels, the density of refueling stations for HFCVs is considered in the competition model. Since HFCVs have not yet been released into the market, it can be expected that when finally released, the market share for them will be obviously small compared to that for ICEs. So, in the analyses presented, the initial market shares of ICEs and HFCVs are assumed as 0.928 and 0.072, respectively. In this first set of examples, we do not couple the supply of refueling stations to the demand; rather, the initial density of refueling stations, ρ_i for HFCV is assumed as 0.0006, which can be calculated from:

$$\frac{\text{Number of recharging stations}}{\text{Area of California State}} = \frac{110\ \textit{stations}}{163,696\ \textit{mi.}}$$

The conditions analyzed are in Table 41 and Table 42.

TABLE 41 Values of Characteristic Variables for ICE and HFCV

	ICE	HFCV
Fuel price ($/gal.)	Initial: 1.36 Final: 3.64	1
Average annual distance driven (mi.)	15,000 (D_A)	15,000 (D_{act})
mpg	32	61
Vehicle price ($)	Initial: 18,515 Final: 18,515	Initial: 30,000 Final: 25,000
Range (mi.)	N.A.	300

TABLE 42 Values of Other Variables

Variable	Value
Initial market share of HFCV	0.072
d (mi.)	32
V_{avg} (mph)	30
C_{TT} (\$/hr.)	25
Household income (\$)	100,000
Education level	5
Gender	0
Density of refueling station for HFCV	Initial: 0.0006 Final: 0.0788

Further, the increase rate for the density of refueling stations, η_i, is assumed as 0.01 at the initial time step.

Parameters for a dynamic normative model and a competition model are in Table 43 and Table 44.

TABLE 43 Values of Parameters for Dynamic Normative Model (ICE vs. HFCV)

	ICE	HFCV
a_G	3.5	100
a_P	N.A.	1
b	2500	2500
s	0.5	0.3
Φ^0_{ICE}	2.7	

TABLE 44 Values of Parameters for Competition Model (ICE vs. HFCV)

$\alpha_{ICE,ICE}$	$\alpha_{HFCV,HFCV}$	$\alpha_{ICE,HFCV}$	$\alpha_{HFCV,ICE}$	β_{HFCV}	θ_{HFCV}	σ_{HFCV}
1	1	0.1	0.1	0.1	2.8	0.39

The market trends between ICE and HFCV with the conditions and parameters above are shown in Figure 47. In Figure 47, it is observed that demands (rho) for ICEs and HFCVs change differently from supplies (Sup) for ICEs and HFCVs over the years. In the case of ICEs, although demand for ICEs is decreasing, there continues to be a large number of these vehicles in the market. In the case of HFCVs, on the other hand, demand for HFCVs is increasing; however, the level of supply for HFCVs is limited by an affordable market share for them based on the density of hydrogen fueling stations. Therefore, although there is a high demand on HFCVs, the supply level of these vehicles is low because of their affordable market share.

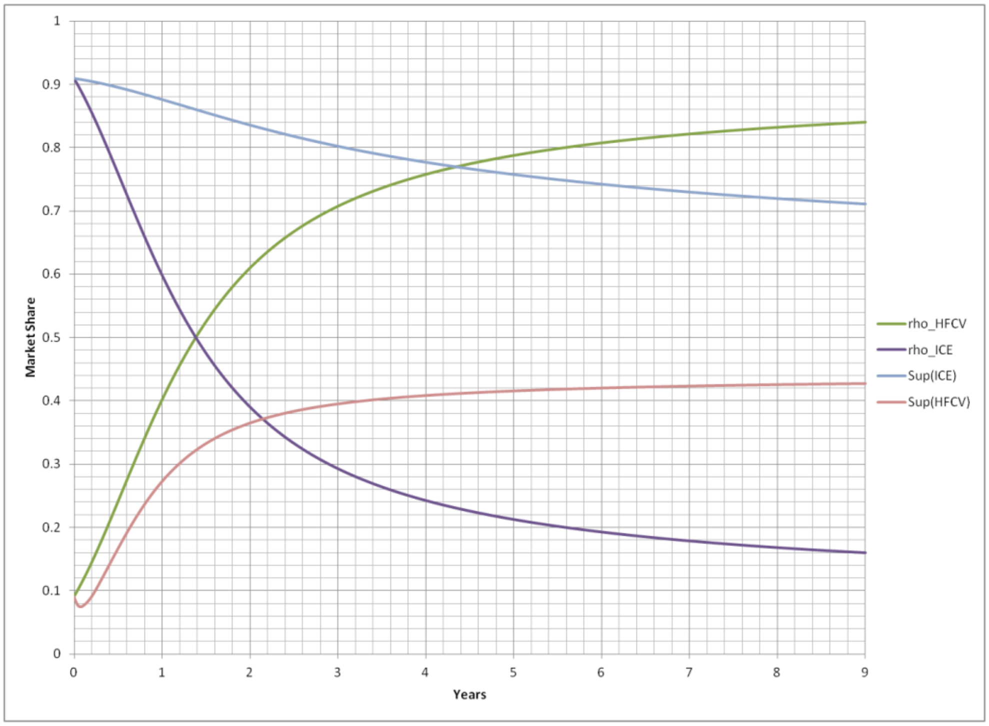

Figure 47. Demand and Supply between ICE and HFCV

5.2.2 Effect of gas prices on supply and demand

In this scenario, values of all variables are fixed as those in a basic scenario except for gas prices at the final time step. The results are shown in Figure 48 and Figure 49.

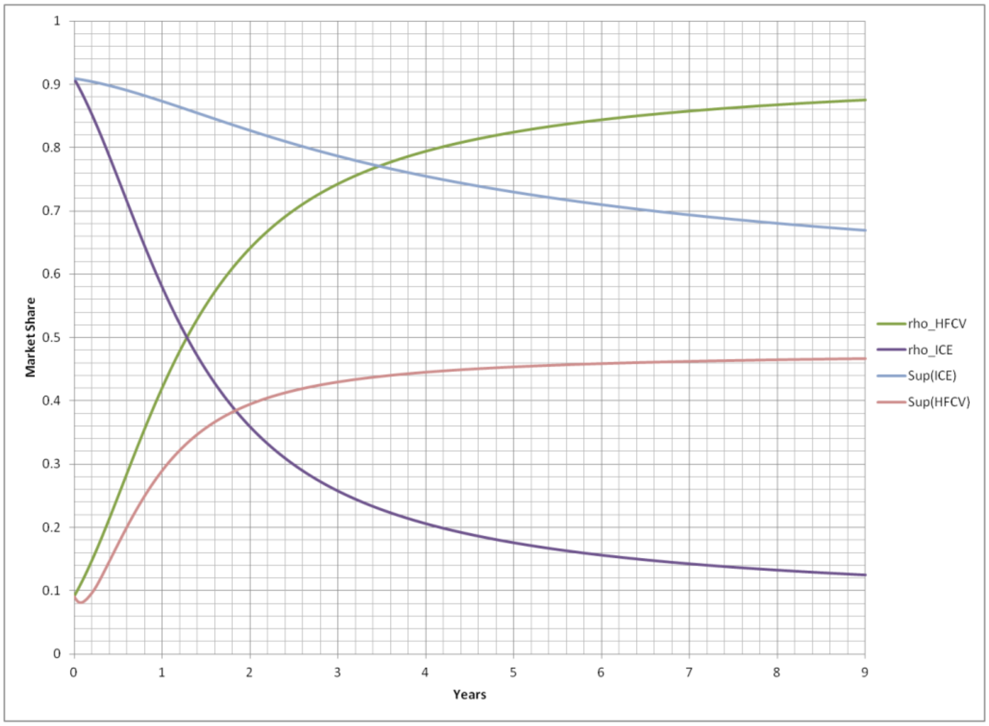

Figure 48. Demand and Supply between ICE and HFCV with $4.09/gal. of Gas Price

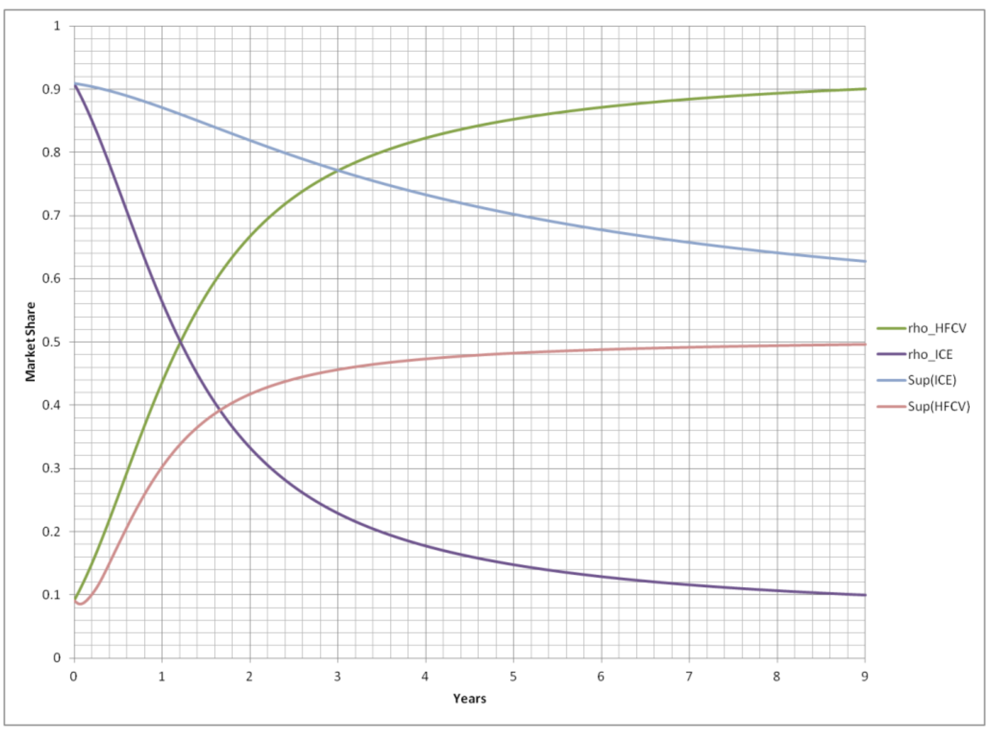

Figure 49. Demand and Supply between ICE and HFCV with $4.49/gal. of Gas Price

Figure 48 and Figure 49 show that the higher a gas price becomes, the greater the increase in both supply and demand for HFCVs (as well as a corresponding decrease in supply and demand for ICEs.)

5.2.3 Effect of final density of refueling stations on supply and demand

In this scenario, values of all variables are fixed as those in a basic scenario except for densities of refueling stations for HFCVs at the final time step. The densities of refueling stations for HFCVs are increased up to $\mathbf{0.01}/mi^2$ and $\mathbf{0.48}/mi^2$ at the final time step. The results are shown in Figure 50 and Figure 51.

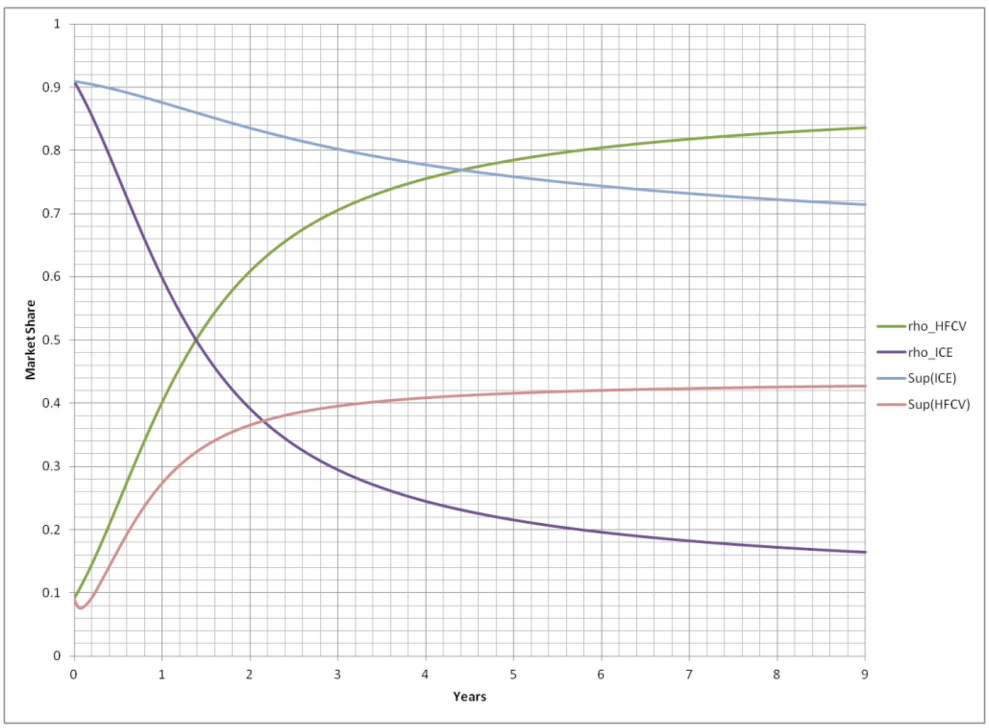

Figure 50. Demand and Supply for ICE and HFCV with Final Density of $\mathbf{0.01/mi^2}$

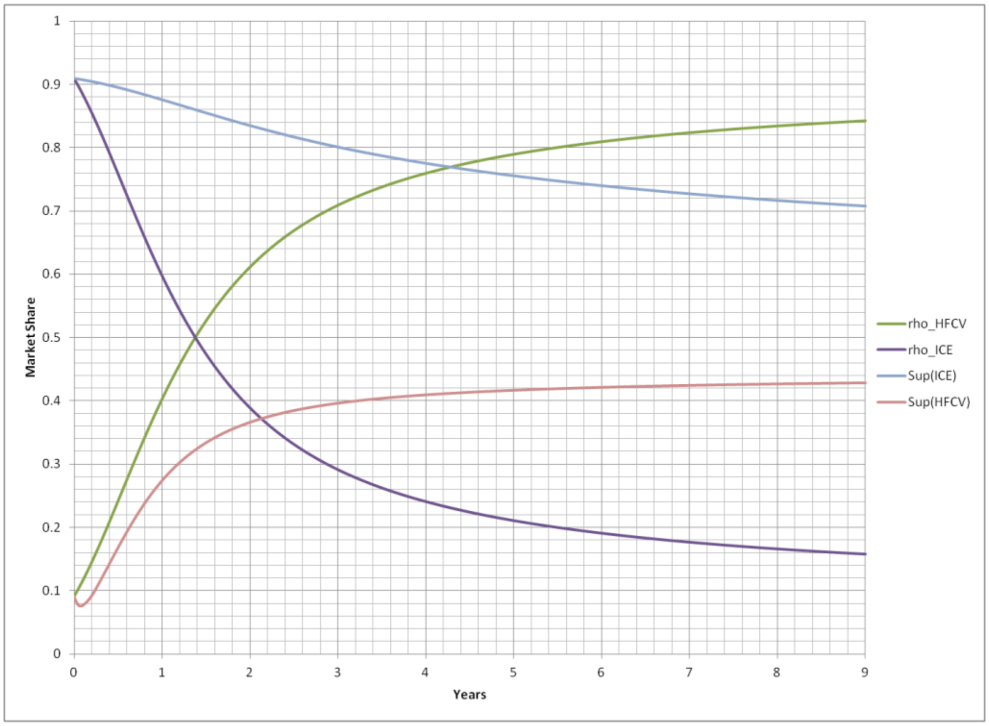

Figure 51. Demand and Supply for ICE and HFCV with Final Density of 0.48 /mi^2

Figure 50 and Figure 51 show that a change in the density of refueling stations at the final time step has no measurable effect on a demand and a supply for HFCVs and ICEs.

5.2.4 Effect of final prices of hydrogen fuel on supply and demand

In this scenario, values of all variables are fixed as those in the base scenario except for prices of hydrogen fuel at the final time step. The results are shown in Figure 52, Figure 53, and Figure 54.

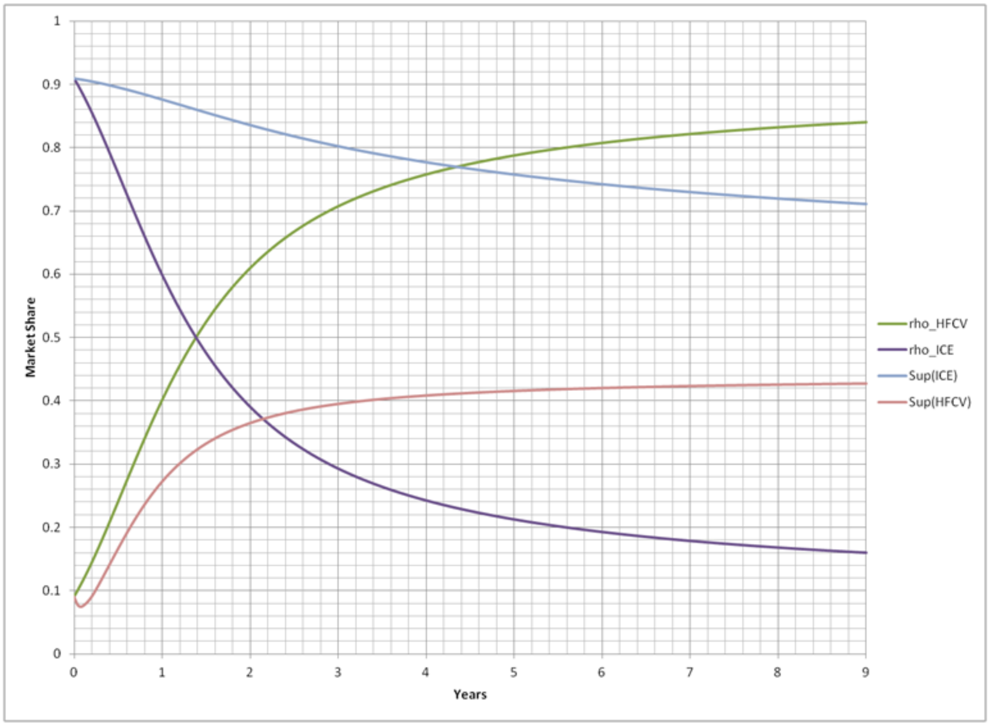

Figure 52. Demand and Supply for ICE and HFCV with $1/gal. of Hydrogen Fuel Price

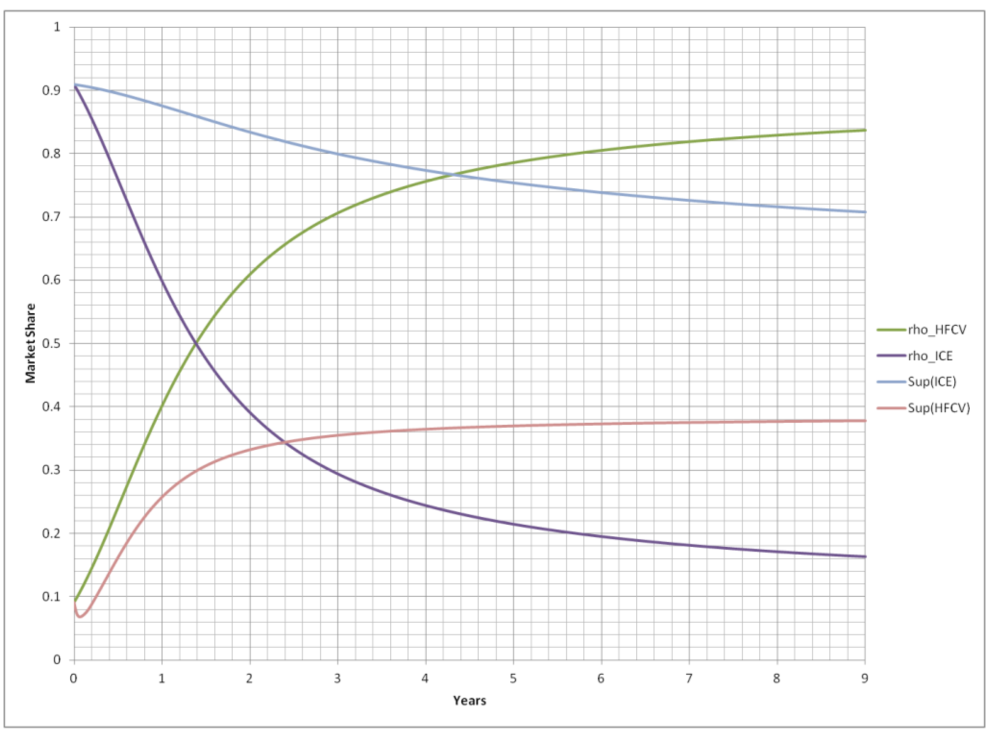

Figure 53. Demand and Supply for ICE and HFCV with $2/gal. of Hydrogen Fuel Price

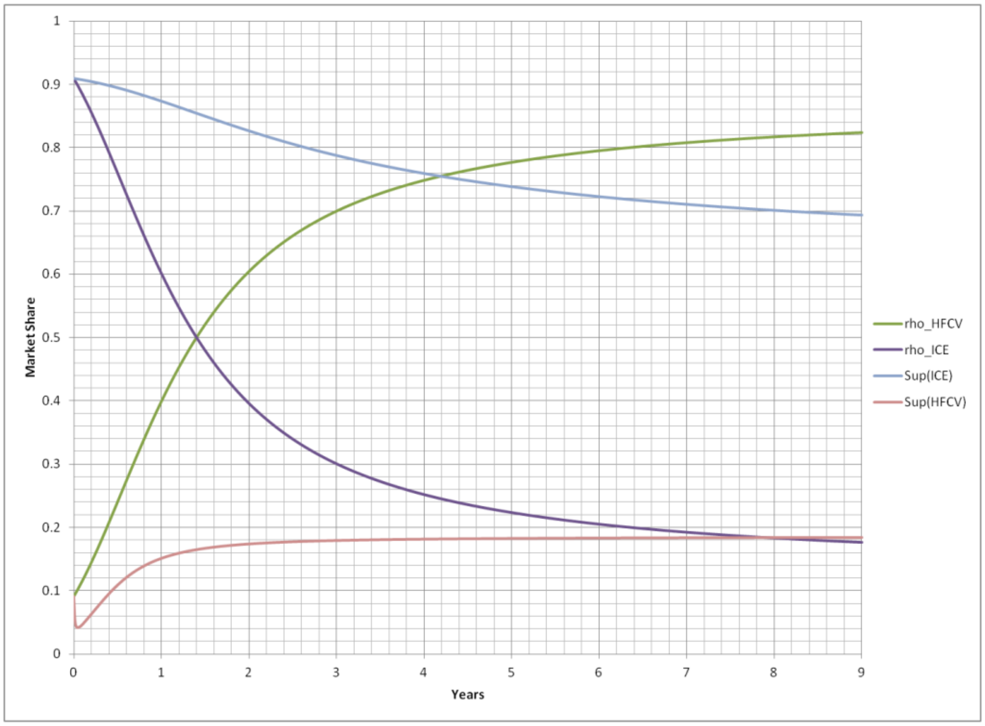

Figure 54. Demand and Supply for ICE and HFCV with $5/gal. of Hydrogen Fuel Price

Figure 52, Figure 53, and Figure 54 show that a higher price of hydrogen fuel at the final time step has only a slightly negative influence on demand for HFCVs. But, in the case of supply for HFCVs, a higher price of hydrogen fuel at the final times step has a large negative effect, because supplies of HFCVs are limited by their affordable market shares.

5.2.5 Effect of the change in price of HFCVs on supply and demand

In this scenario, values of all variables are fixed as those in the base scenario except for prices of HFCVs at the final time step. Analyses are performed for three prices of HFCVs: $30,000, $25,000 and $20,000. In the case of $30,000, it is assumed that no rebate ($5,000) is applied to a HFCV's original price. And so there is no change in a vehicle price for a HFCV over the time.

For $25,000, a rebate of $5,000 is applied to a HFCV's original price, which is same as a base case. And for $20,000, it is assumed that a rebate of $10,000 is applied to a HFCV's original price. Figure 55, Figure 56 and Figure 57 show the results.

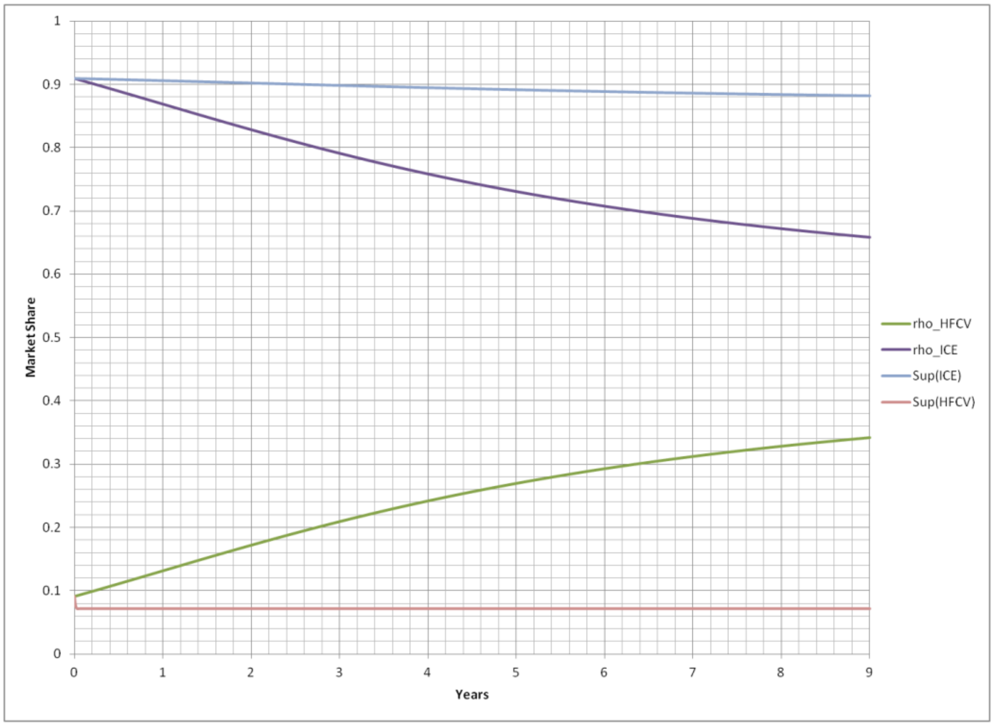

Figure 55. Demand and Supply for ICE and HFCV with $ 30,000 of HFCV

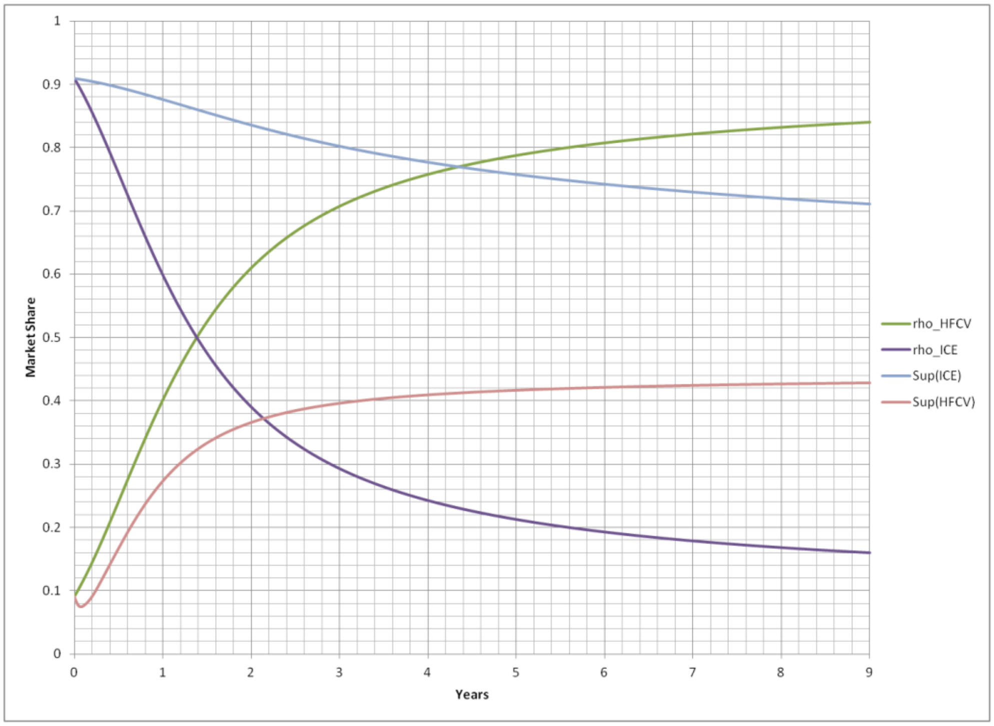

Figure 56. Demand and Supply for ICE and HFCV with $ 25,000 of HFCV

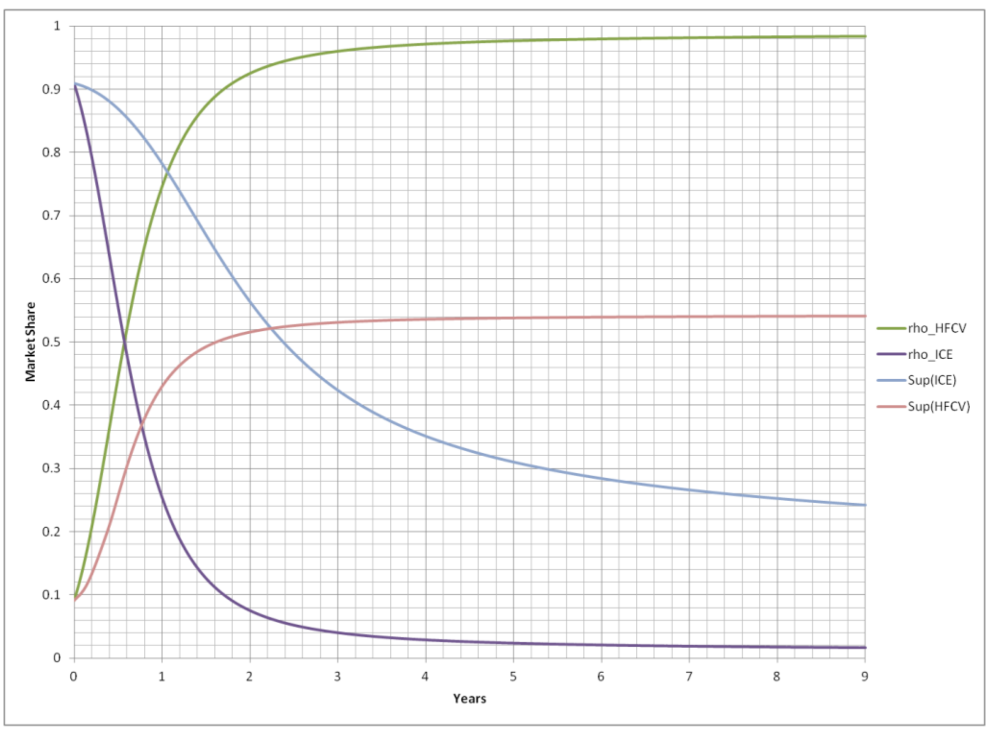

Figure 57. Demand and Supply for ICE and HFCV with $ 20,000 of HFCV

In Figure 55, the demand of HFCVs, rho_HFCV, increases up to about 0.34, but the supply of HFCVs, Sup (HFCV), is fixed at 0.07 which is lower than rho_HFCV. In Figure 56, rho_HFCV increases up to 0.84, and Sup (HFCV) increases up to 0.43. And in Figure 57, rho_HFCV increases up to 0.98, and Sup (HFCV) increases up to 0.54. These results show a dramatic change in the dynamic trends for both the demand and supply of HFCVs as the price of HFCVs increases; at a price of $30,000 HFCVs never achieve a dominant position in the market and supply remains relatively constant. This is in stark contrast to the case in which the price of the HFCV is $20,000, where the results indicate that HFCVs are projected to achieve a stable dominant share of the market within a period of five years.

5.3. Summary

In this chapter, competition models are applied to several scenarios related to ICE vs. HFCV. The results show that: a) the higher the gas price for ICEs, the more positive effect on both the demand for, and supply of, HFCVs, b) a change in density of refueling stations for HFCVs at the final time step has no measurable effect on the demand and supply trends for HFCVs, c) in the case of hydrogen fuel price, as the price goes higher, there is a slightly negative effect on the demand for HFCVs while there is a large negative effect on the supply of HFCVs, and d) as the vehicle price of HFCVs becomes lower, there are positive effects on both supply of, and demand for, HFCVs. Among these results, it is found that changes in demands or supplies for HFCVs are largely influenced by the prices of gasoline fuel and hydrogen fuel, and a vehicle price of HFCVs. This implies; although a performance of HFCVs (i.e., mpg and range) is better than ICEs, changes in fuel prices or a vehicle price of HFCVs should be considered to make HFCVs to be competitive.

6. Connecting demand and supply

In the previous chapters, we propose a dynamic normative model to forecast demand for a vehicle, and a competition model that forecasts the supply conditions for the vehicle. One of factors influencing on a supply for a specific vehicle i is an affordable demand of the vehicle, $K_i(t)$. However, although a dynamic normative model estimates a demand for a vehicle i, $\rho_i(t)$, which doesn't consider $K_i(t)$. And in a competition model, an increase rate of vehicle i relies on $\rho_i(t)$. So sometimes a supply of vehicle i exceeds $K_i(t)$, as shown in Figure 58.

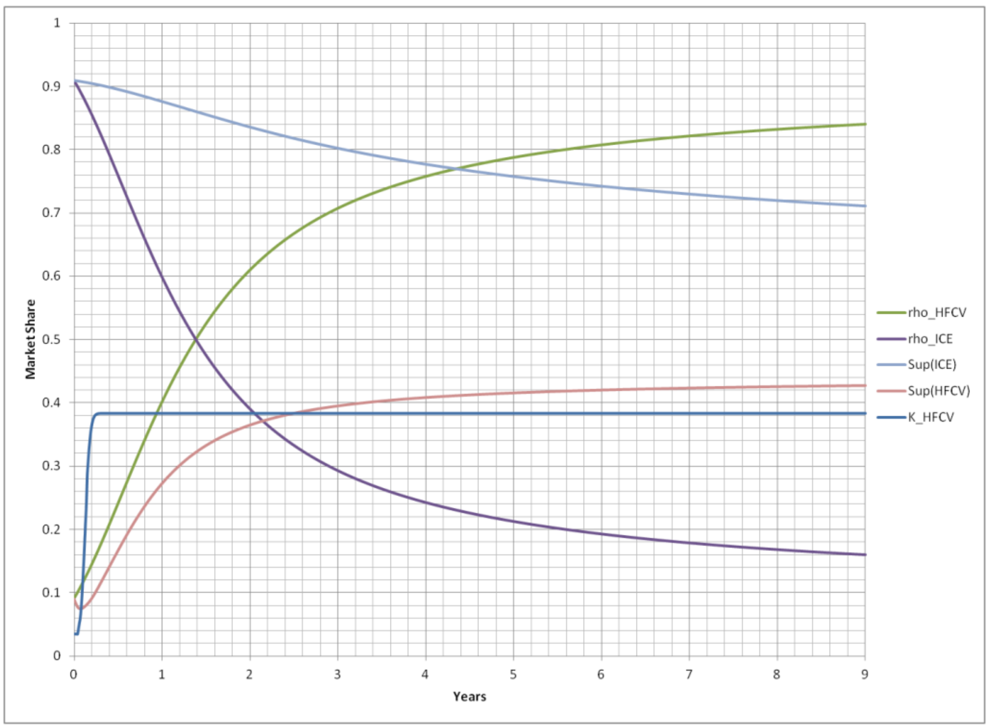

Figure 58. Demand and Supply for ICE and HFCV with K_HFCV

A graph in Figure 58 is same as that in Figure 47, and a graph of an affordable demand for HFCVs, K_HFCV, is added. In Figure 58, a graph of Sup (HFCV) exceeds that of K_HFCV at about 2.5 years, although it isn't supposed to do actually.

To forecast demand for a vehicle based on its supply (and vice versa), two models need to be connected. In this chapter, we propose a feedback model that connects demand for a vehicle, $\rho_i(t)$, with a feedback of $K_i(t)$.

6.1 Feedback: PID controller

In order to apply a feedback, we first define a variable $F_i(t)$ as the feedback for demand of vehicle *i* received from $K_i(t)$. A plausible formulation for $F_i(t)$ can be written as:

$$F_i(t) = \rho_i(t) + Feedback \tag{72}$$

We then assume that the increase rate of vehicle *i*, r_i, can be written as:

$$r_i(t) = \frac{F_i(t) - F_i(t-1)}{F_i(t-1)} \tag{73}$$

To find $F_i(t)$, a PID controller method is used for a feedback from an affordable market share of vehicle *i*, $K_i(t)$. The formulation of the PID controller is as equation (74).

$$\begin{aligned} Feedback &= Pe_i(t) + I\int_0^t e_i(\tau)d\tau + D\frac{d}{dt}e_i(t) \\ e_i(t) &= K_i(t) - \rho_i(t) \end{aligned} \tag{74}$$

where *P* is a tuning parameter for proportional gain, *I* is a tuning parameter for integral gain, and *D* is a tuning parameter for derivative gain. The several values for these parameters will be examined in this work.

The output from a PID controller is added to $\rho_i(t)$ to adjust the difference between $K_i(t)$ and

$\rho_i(t)$, and make an output of $F_i(t)$.

$$F_i(t) = \rho_i(t) + Pe_i(t) + I\int_0^t e_i(\tau)d\tau + D\frac{d}{dt}e_i(t)$$
$$e_i(t) = K_i(t) - \rho_i(t) \tag{75}$$

In the following sections, several subtypes of PID controllers are applied to the case of ICE vs. HFCV, and results are presented for different values of their respective parameters.

6.2 P controller

The P controller is defined as

$$F_i(t) = \rho_i(t) + Pe_i(t)$$
$$e_i(t) = K_i(t) - \rho_i(t) \tag{76}$$

In the following, results are presented for different values of the tuning parameter, P.

a) P=0.5

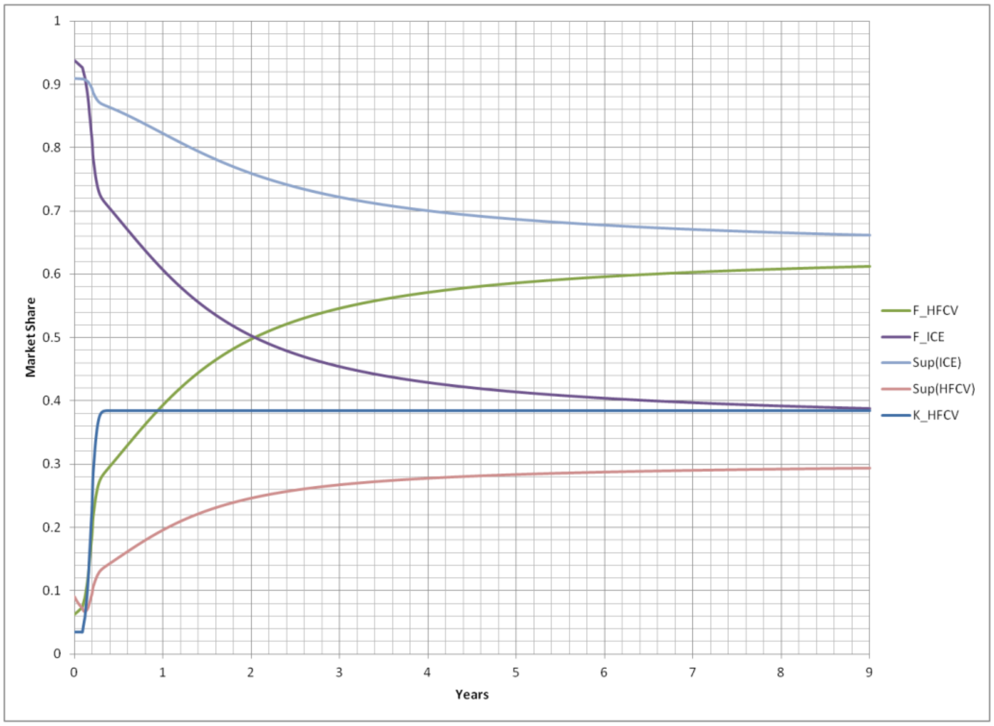

Figure 59. Demand and Supply for ICE and HFCV with P=0.5

b) P=0.2

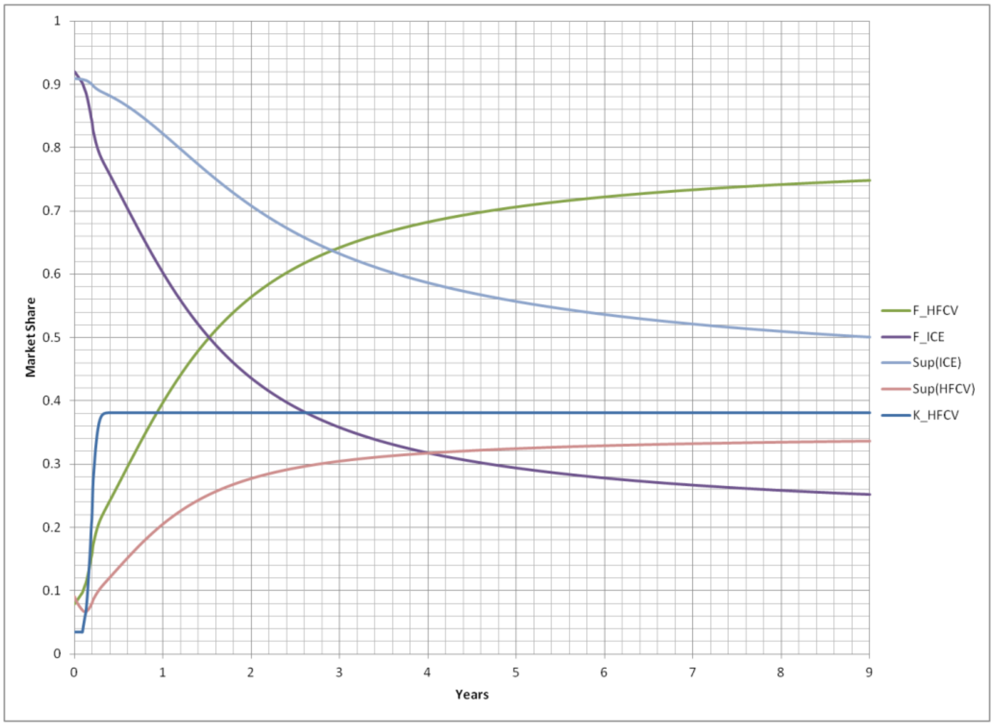

Figure 60. Demand and Supply for ICE and HFCV with P=0.2

c) P=0.1

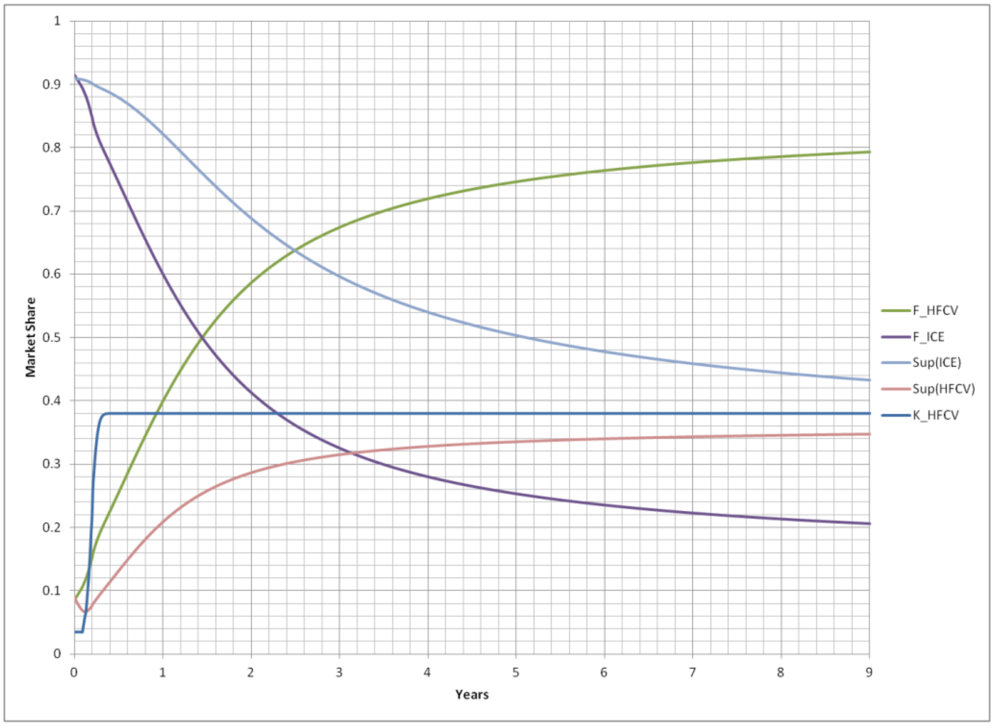

Figure 61. Demand and Supply for ICE and HFCV with P=0.1

d) P=0.01

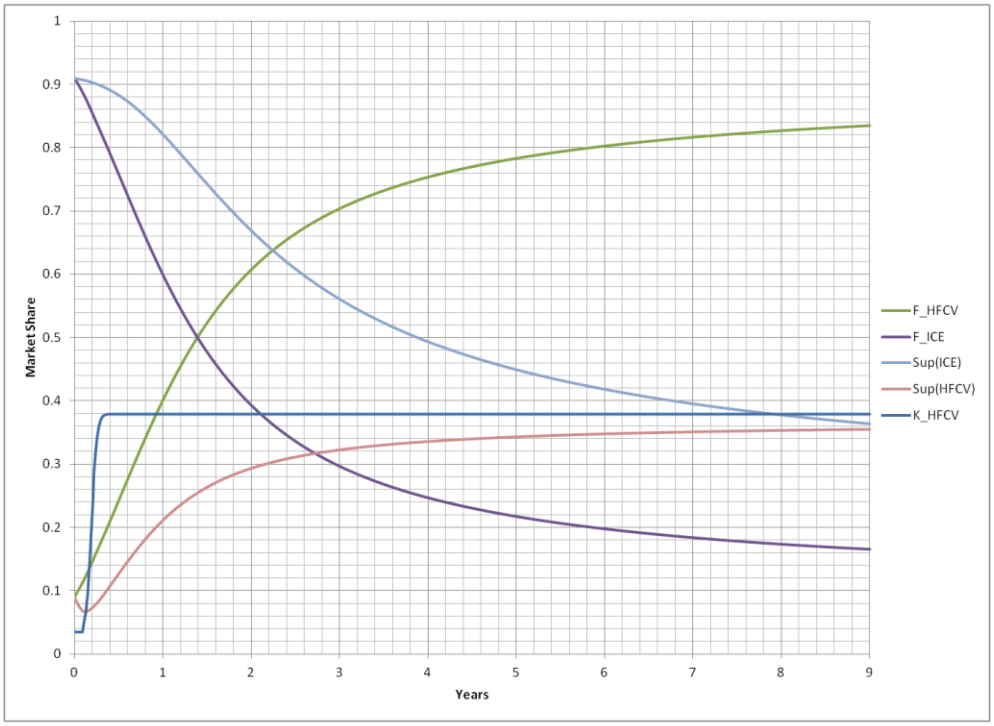

Figure 62. Demand and Supply for ICE and HFCV with P=0.01

e) P=0.001

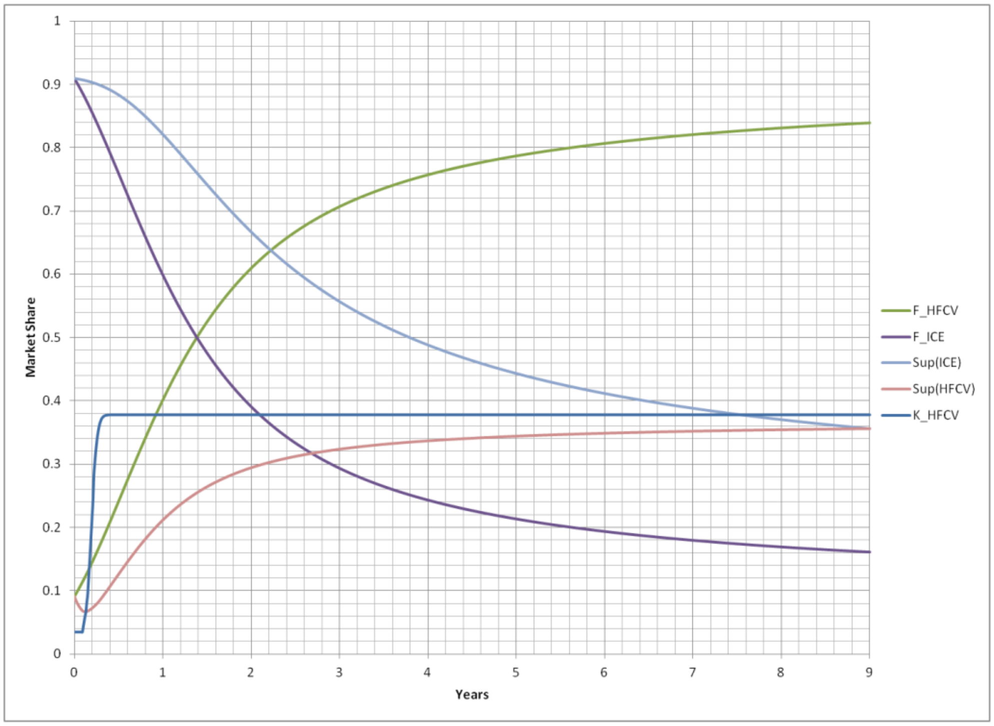

Figure 63. Demand and Supply for ICE and HFCV with P=0.001

From Figure 59 to 63, the supplies of HFCV, Sup (HFCV), are smaller than affordable demands for HFCV, K_HFCV, and the graphs of Sup (HFCV) go near to K_HFCV by the final time step. In a simple P controller, as a value of P becomes smaller, the gap between a modified demand for HFCVs, F_HFCV, and Sup (HFCV) becomes larger. And, by 0.001, the size of that gap stabilizes. In the case of choice situation, as a value of P becomes smaller, F_HFCV and Sup (HFCV) increase and a gap between these graphs also becomes higher. And by a value 0.001, the graphs of them become stabilized. Comparing to Figure 47, a graph of Sup (HFCV) is decreased from 0.42 to 0.36 at the final time step while a graph of F_HFCV shows no difference. However, a graph Sup (ICE) is also decreased and almost tracks a graph of F_ICE. The result implies that applying P controller makes a feedback effect on supplies and demands on ICEs and HFCVs, and as a P value becomes smaller, the effect becomes smaller and then stabilizes after a specific value.

6.3 I controller

The I controller is defined as

$$
\begin{aligned}
F_i(t) &= \rho_i(t) + I\int_0^t e_i(\tau)d\tau \\
e_i(t) &= K_i(t) - \rho_i(t)
\end{aligned}
\tag{77}
$$

In the following, results are presented for different values of the tuning parameter, I.

a) I=0.01

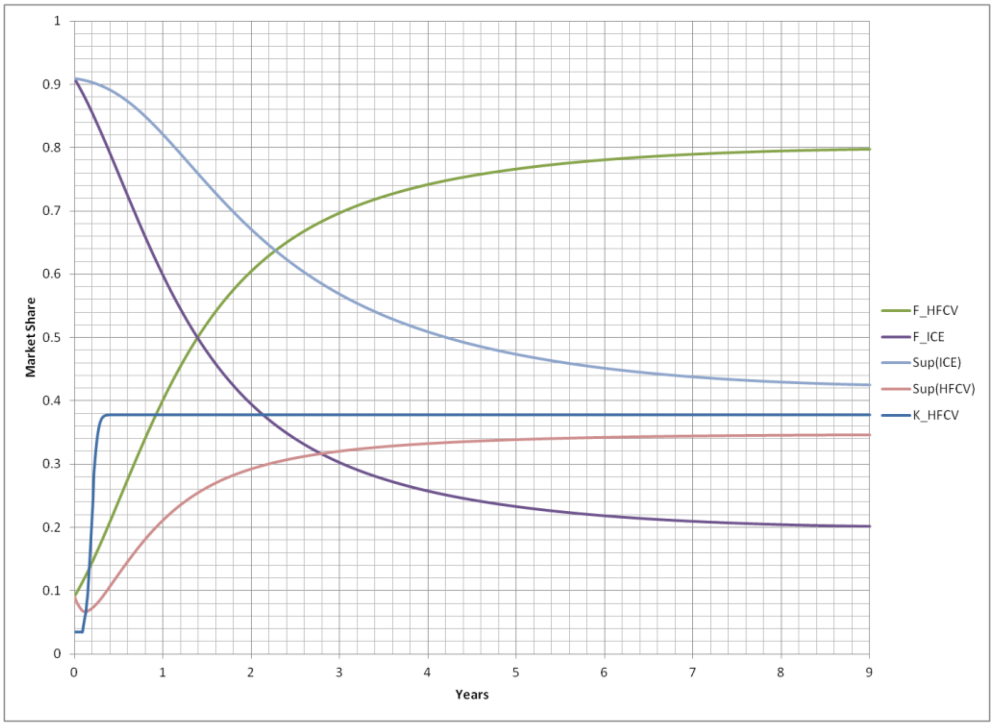

Figure 64. Demand and Supply for ICE and HFCV with I=0.01

b) I=0.005

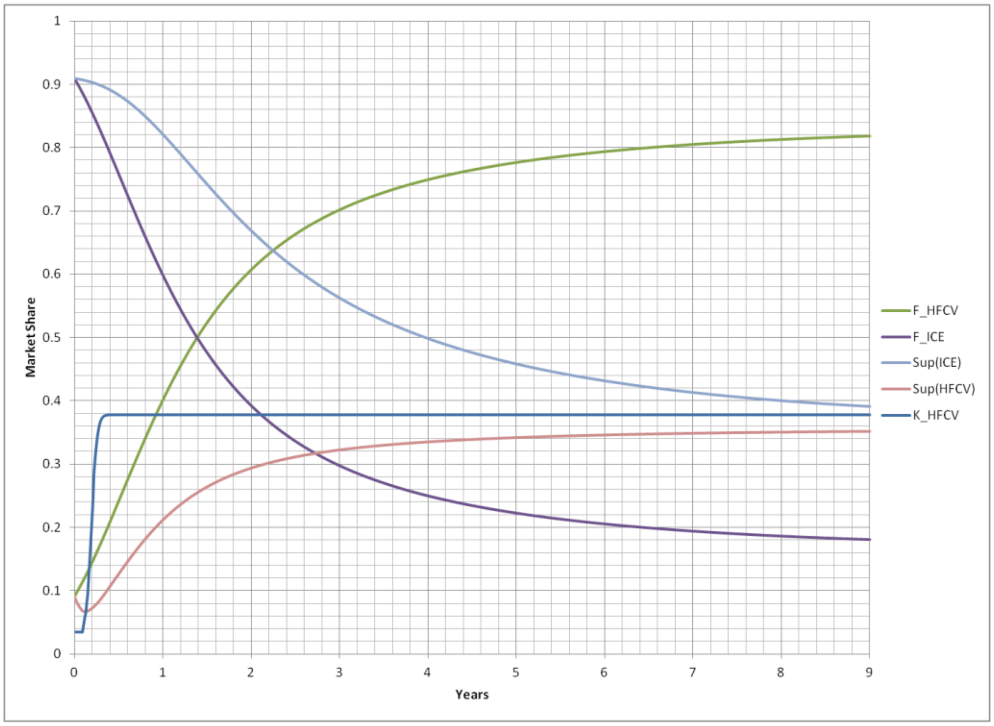

Figure 65. Demand and Supply for ICE and HFCV with I=0.005

c) I=0.001

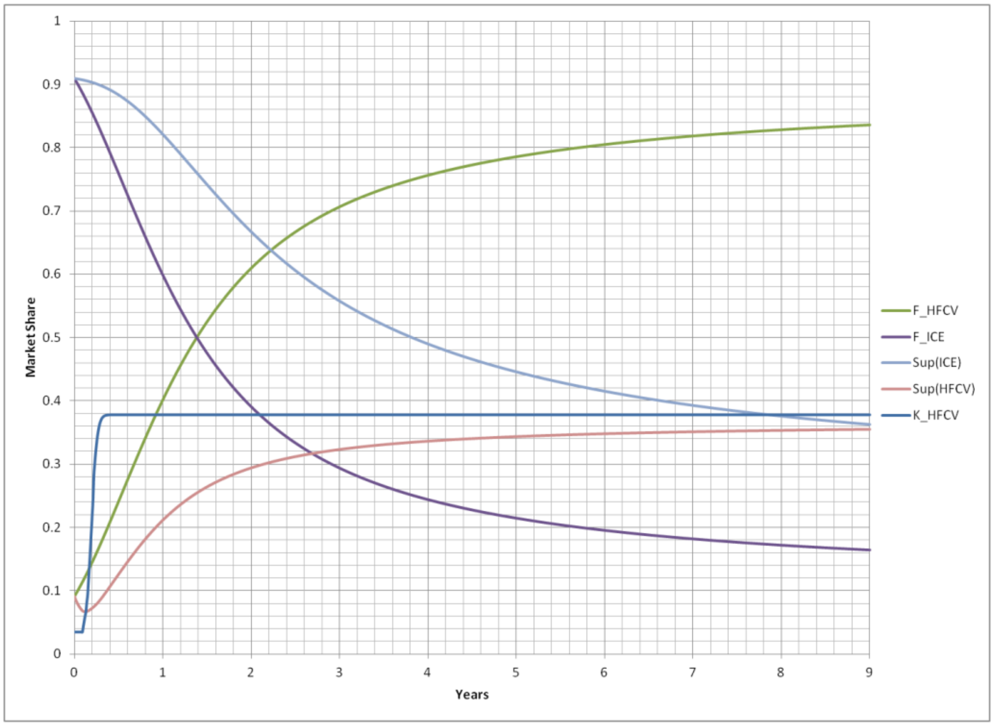

Figure 66. Demand and Supply for ICE and HFCV with I=0.001

d) I=0.0001

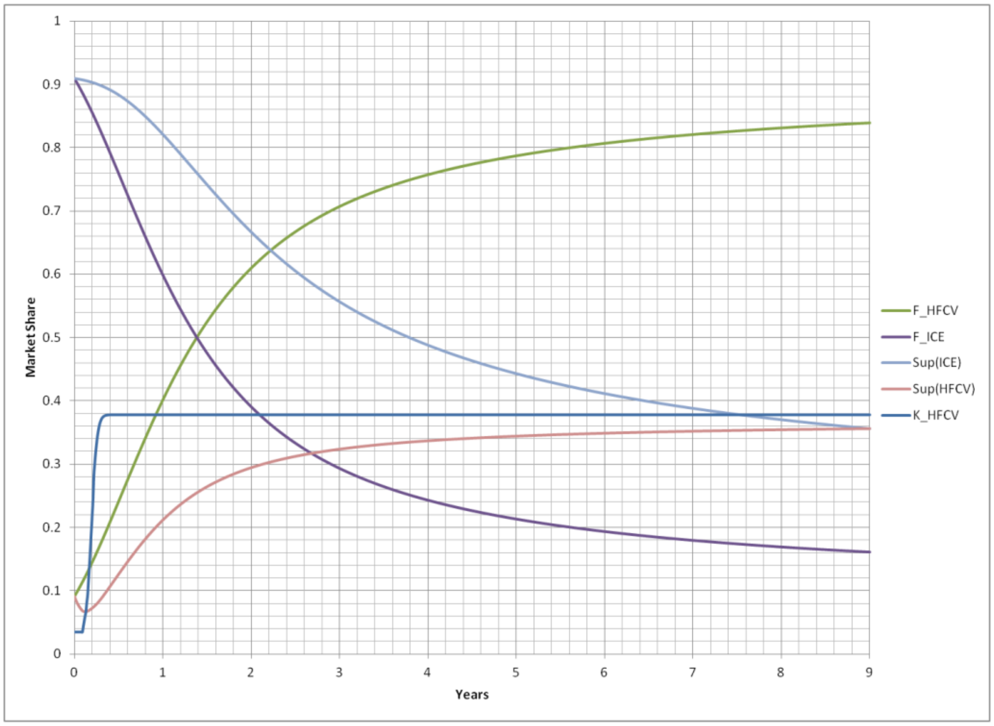

Figure 67. Demand and Supply for ICE and HFCV with I=0.0001

From Figure 64 to 67, the supplies of HFCV, Sup (HFCV), are smaller than affordable demands for HFCV, K_HFCV, and the graphs of Sup (HFCV) go near to K_HFCV by the final time step. In a simple I controller, as the value of I becomes smaller, the gap between F_HFCV and Sup (HFCV) becomes larger. And, by a value 0.0001, there is no more change in the gap. In the case of choice situation, as a value of P becomes smaller, F_HFCV and Sup (HFCV) increase and a gap between these graphs becomes higher. And by a value 0.0001, the graphs of them become stabilized. Comparing to P controller, I controller makes smaller change on supplies and demands for ICEs and HFCVs. And as an I value becomes smaller, the feedback effect also becomes smaller and then the graph converges at a specific value.

6.4 D controller:

The D controller is defined as

$$F_i(t) = \rho_i(t) + D\frac{d}{dt}e_i(t)$$
$$e_i(t) = K_i(t) - \rho_i(t) \qquad (78)$$

In the following, results are presented for different values of the tuning parameter, D.

a) D=0.01

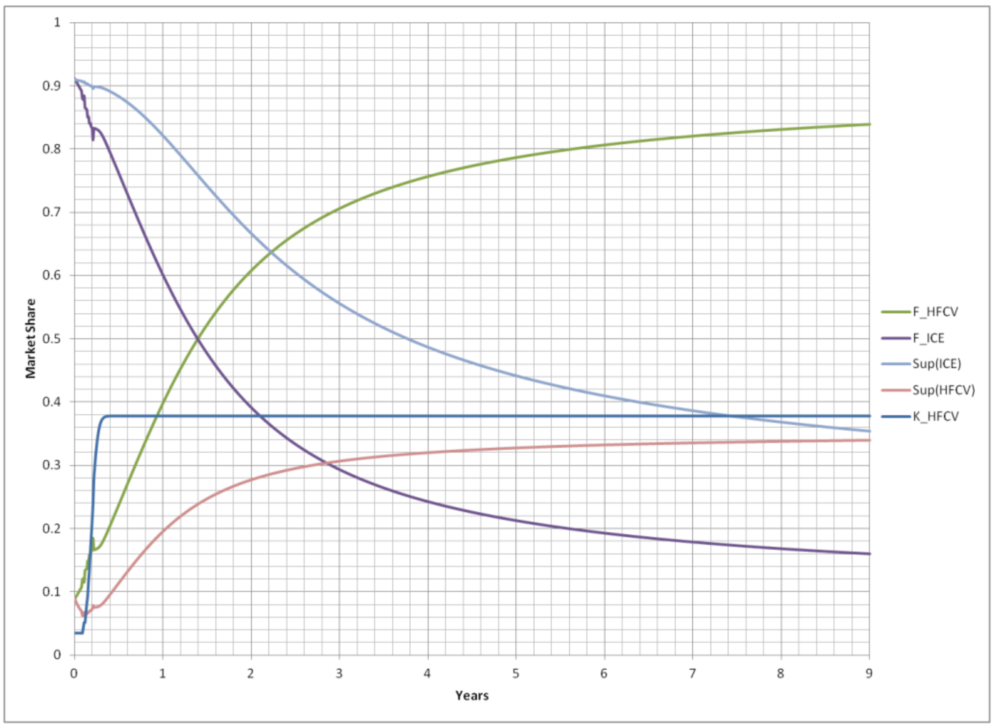

Figure 68. Demand and Supply for ICE and HFCV with D=0.01

b) D=0.005

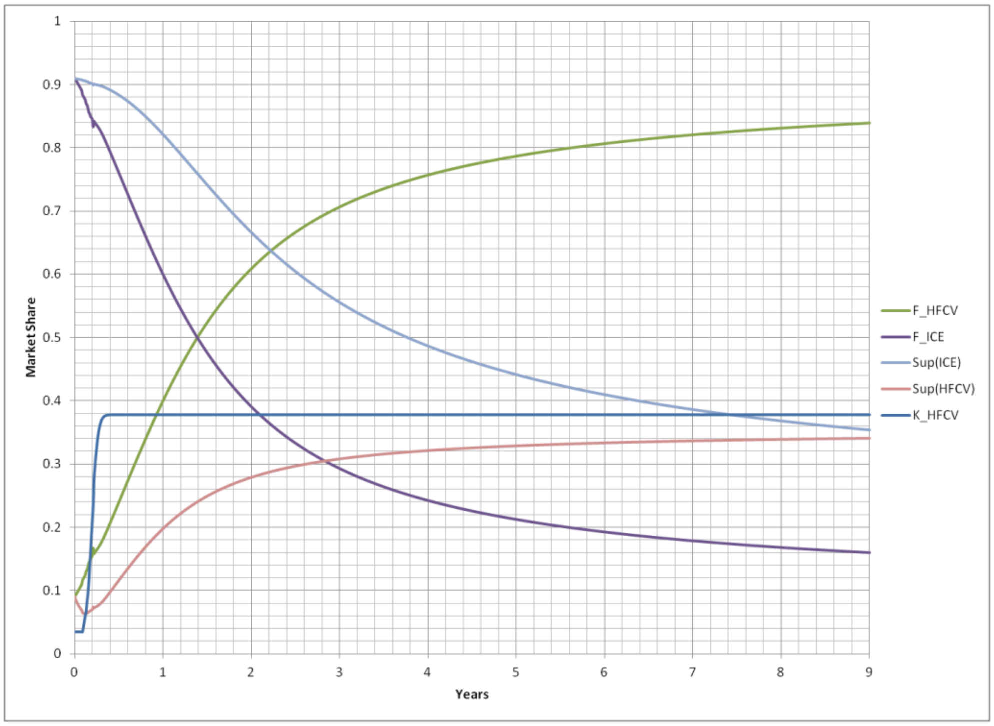

Figure 69. Demand and Supply for ICE and HFCV with D=0.005

c) D=0.001

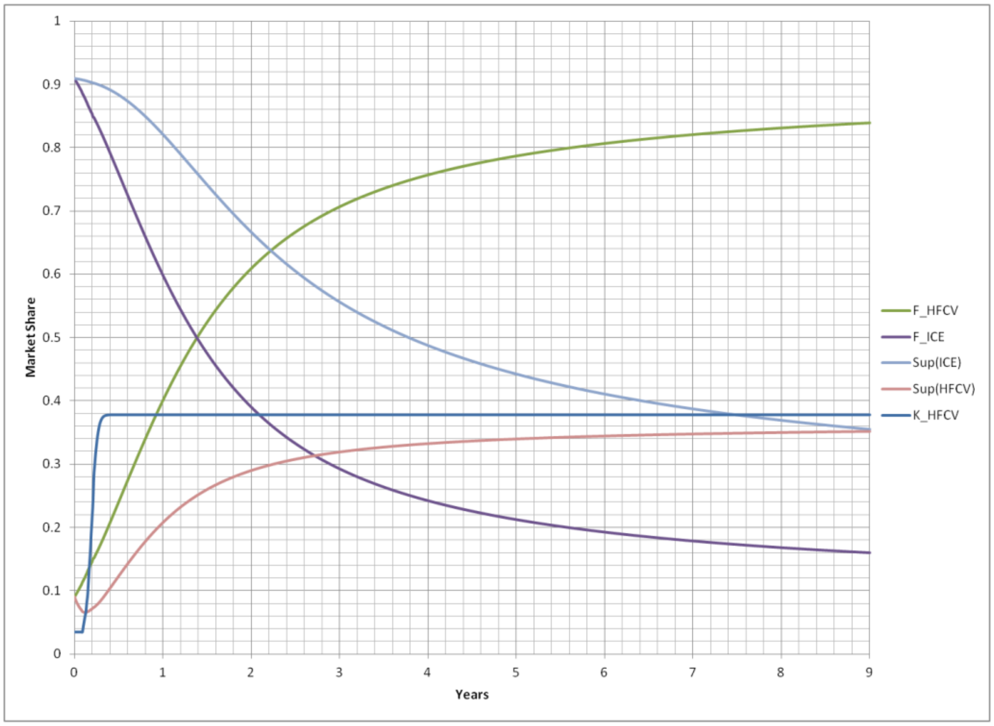

Figure 70. Demand and Supply for ICE and HFCV with D=0.001

d) D=0.0001

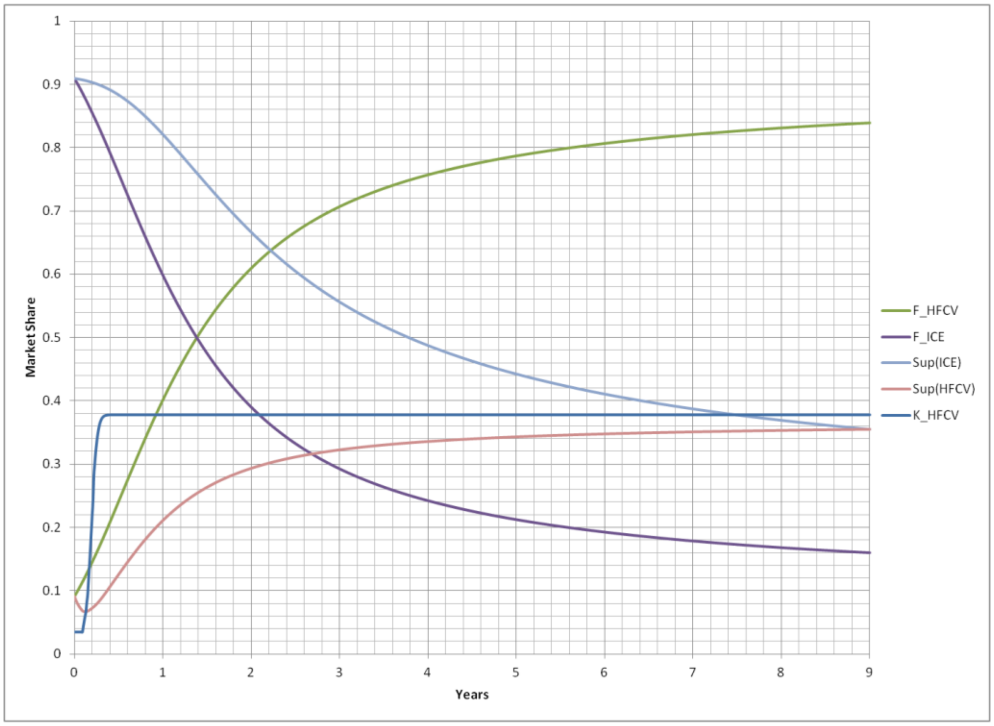

Figure 71. Demand and Supply for ICE and HFCV with D=0.0001

From Figure 68 to 71, the supplies of HFCV, Sup (HFCV), are smaller than affordable demands for HFCV, K_HFCV, and the graphs of Sup (HFCV) go near to K_HFCV by the final time step. For values of D in the range 0.01 and 0.005, there is an initial "fuzziness" in the dynamics, as shown in Figure 68 and 69; this is due to reactions to rates of change at small proportions—a characteristic of D controllers. And, as shown in Figure 70 and 71, there is relatively no change in the trends when values of D get smaller than about 0.001 to 0.0001. In the case of choice situation, there is no significant change in F_HFCV, Sup (HFCV), F_ICE or Sup (ICE). Applying D controller with variable D values makes no significant change in supplies and demands for ICEs and HFCVs. And with D values larger than 0.001, there are fuzziness in graphs. Therefore, it should be considered to apply D controller as a feedback.

6.5 PI controller

The PI controller is defined as

$$F_i(t) = \rho_i(t) + Pe_i(t) + I\int_0^t e_i(\tau)d\tau$$
$$e_i(t) = K_i(t) - \rho_i(t) \tag{79}$$

In the following, results are presented for different values of the tuning parameters, P and I.

a) P=0.1, I=0.01

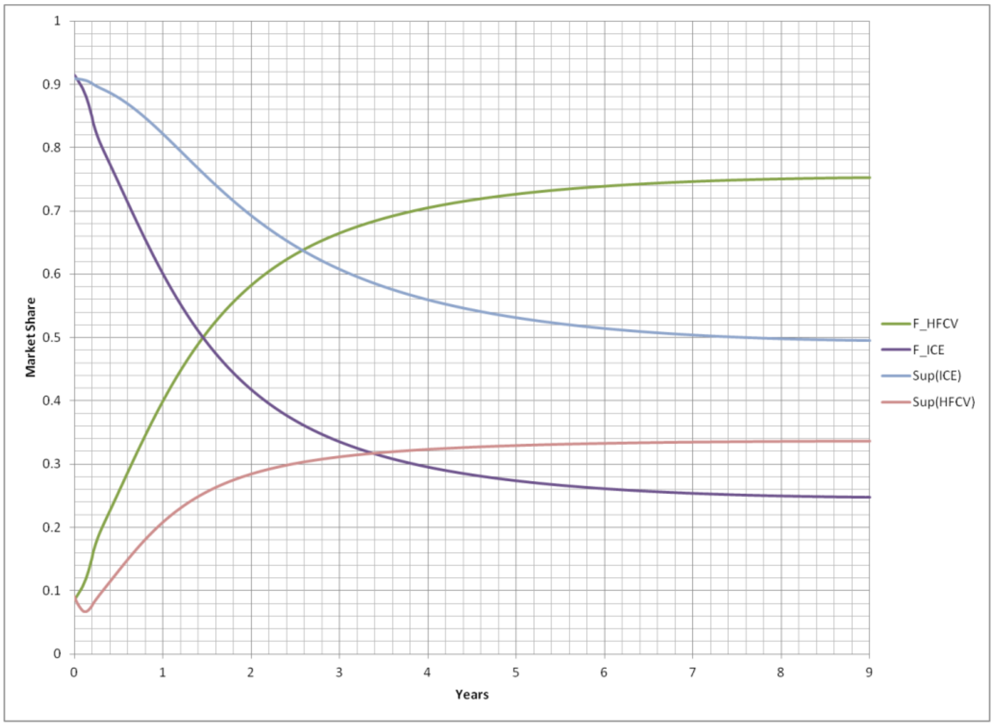

Figure 72. Demand and Supply for ICE and HFCV with P=0.1 and I=0.01

b) P=0.1, I=0.0001

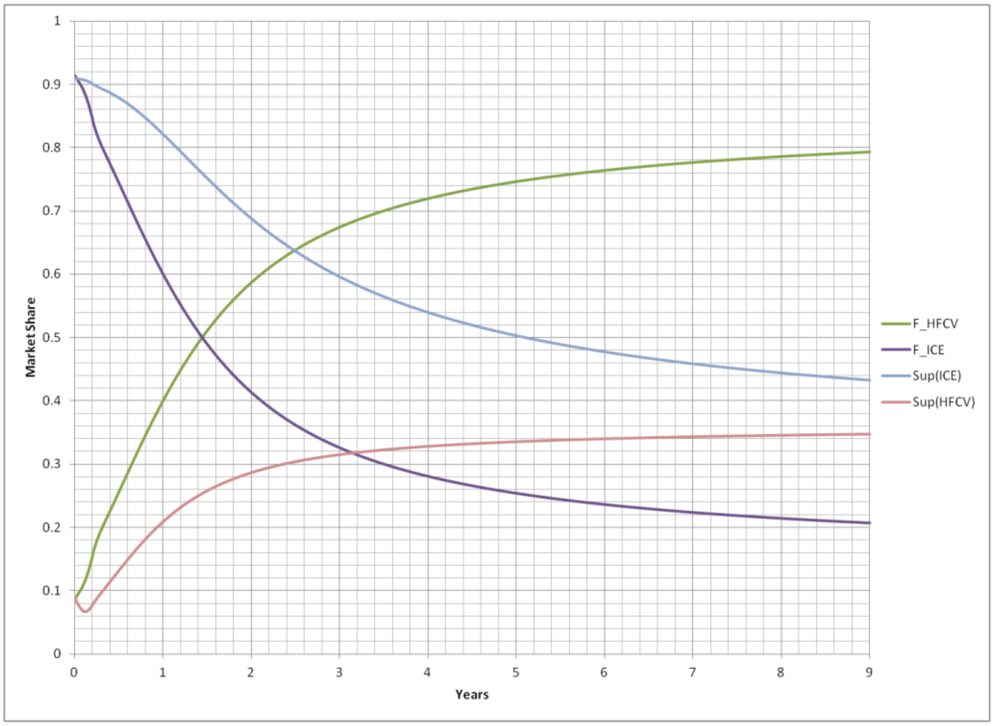

Figure 73. Demand and Supply for ICE and HFCV with P=0.1 and I=0.0001

c) P=0.001, I=0.0001

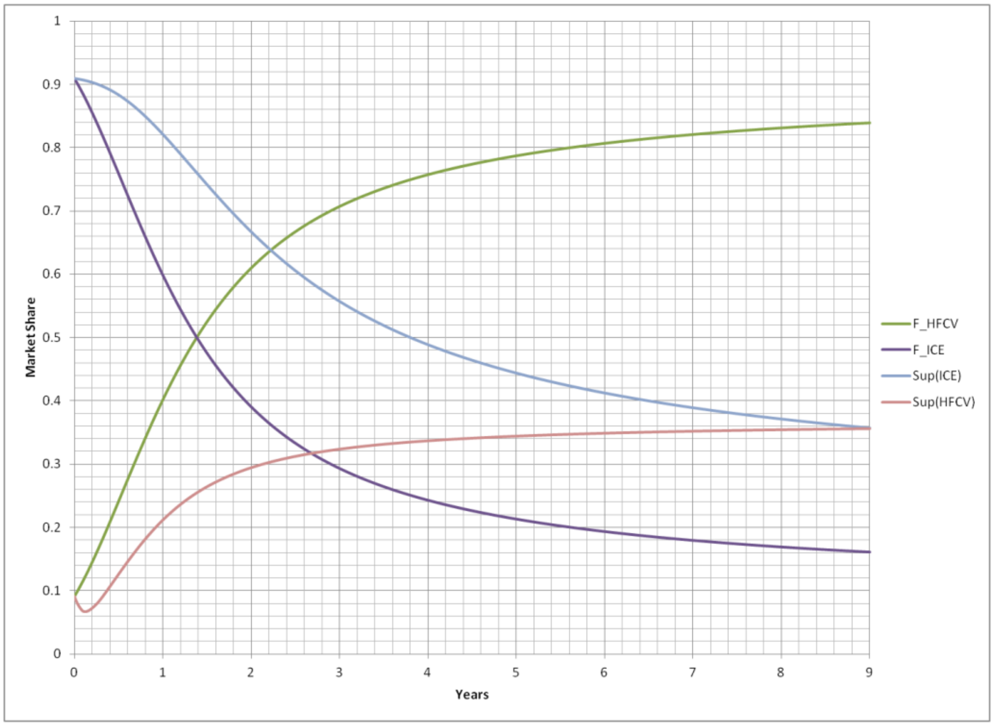

Figure 74. Demand and Supply for ICE and HFCV with P=0.001 and I=0.0001

In the case of choice situation, as values of P and I become smaller, F_HFCV and Sup (HFCV) increase. And a feedback effect on these graphs becomes smaller as values of P and I become smaller.

6.6 PD controller

The PD controller is defined as

$$F_i(t) = \rho_i(t) + Pe_i(t) + D\frac{d}{dt}e_i(t)$$
$$e_i(t) = K_i(t) - \rho_i(t) \quad (80)$$

In the following, results are presented for different values of the tuning parameters, P and D.

a) P=0.1, D=0.001

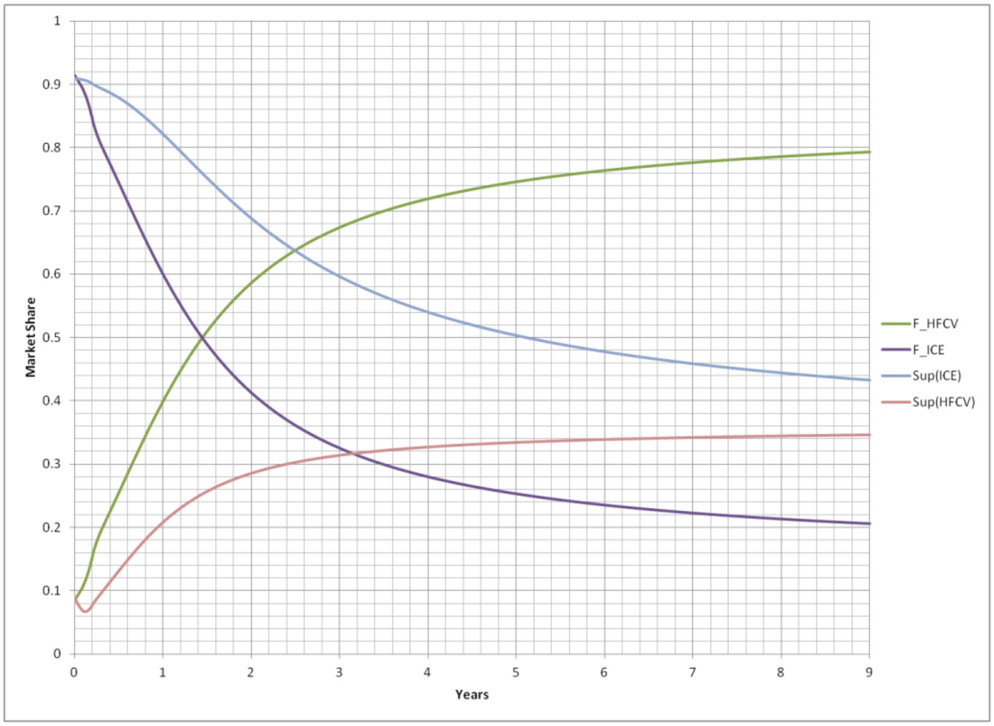

Figure 75. Demand and Supply for ICE and HFCV with P=0.1 and D=0.001

b) P=0.001, D=0.001

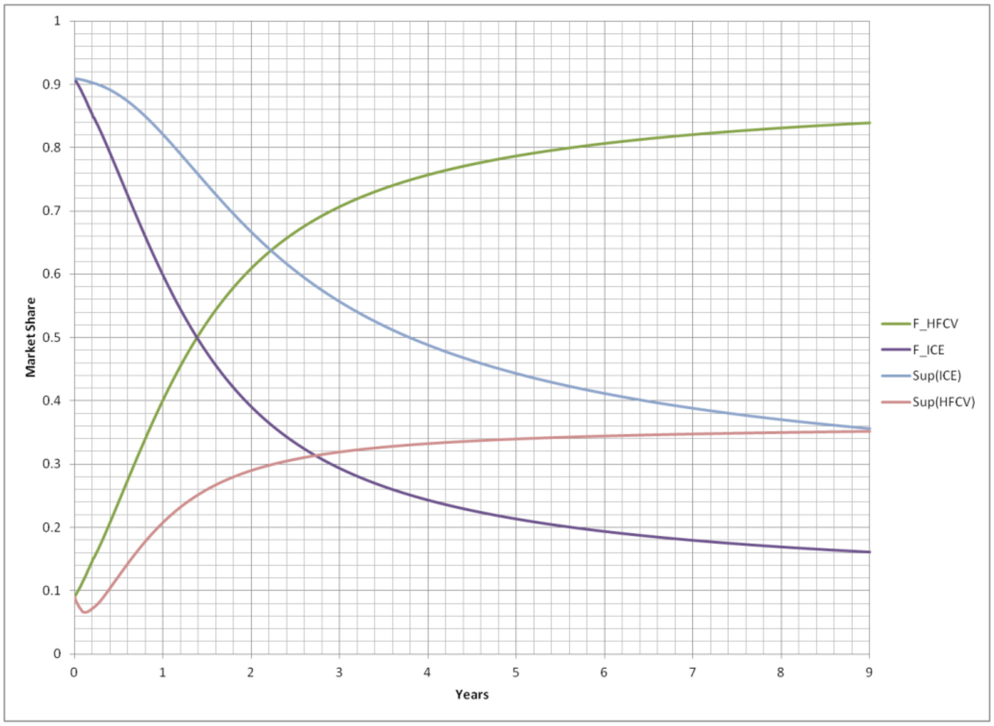

Figure 76. Demand and Supply for ICE and HFCV with P=0.001 and D=0.001

In the case of choice situation, as values of P and D become smaller, F_HFCV and Sup (HFCV) increase. And a feedback effect on these graphs becomes smaller as values of P and D become smaller.

6.7 ID controller

The ID controller is defined as

$$F_i(t) = \rho_i(t) + I\int_0^t e_i(\tau)d\tau + D\frac{d}{dt}e_i(t)$$
$$e_i(t) = K_i(t) - \rho_i(t) \qquad (81)$$

In the following, results are presented for different values of the tuning parameters, I and D.

a) I=0.01, D=0.001

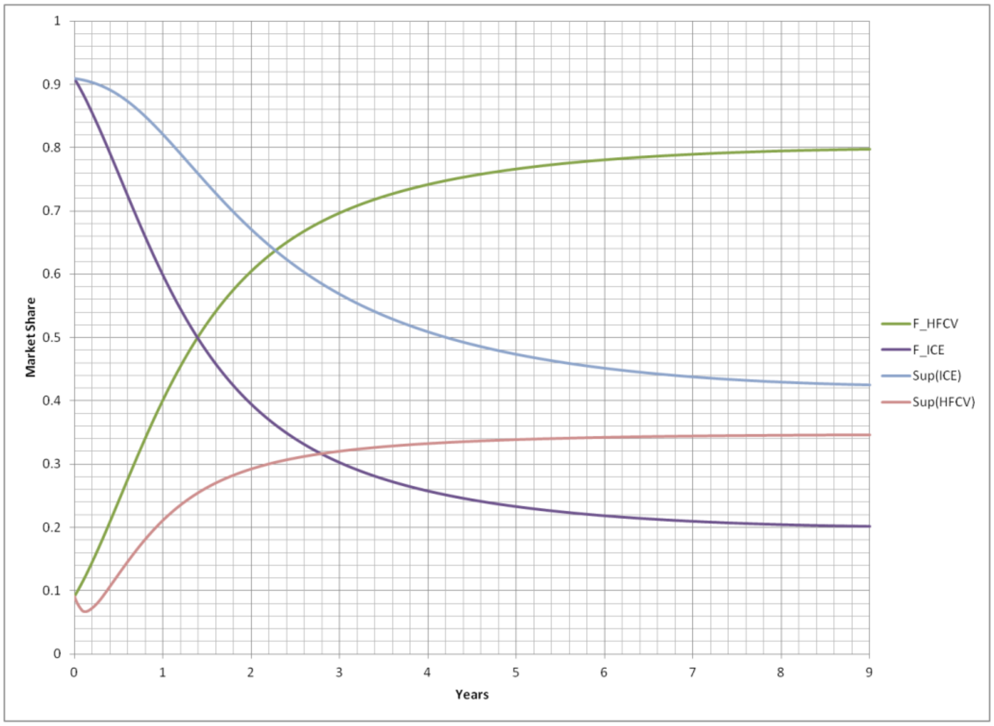

Figure 77. Demand and Supply for ICE and HFCV with I=0.01 and D=0.001

b) I=0.0001, D=0.001

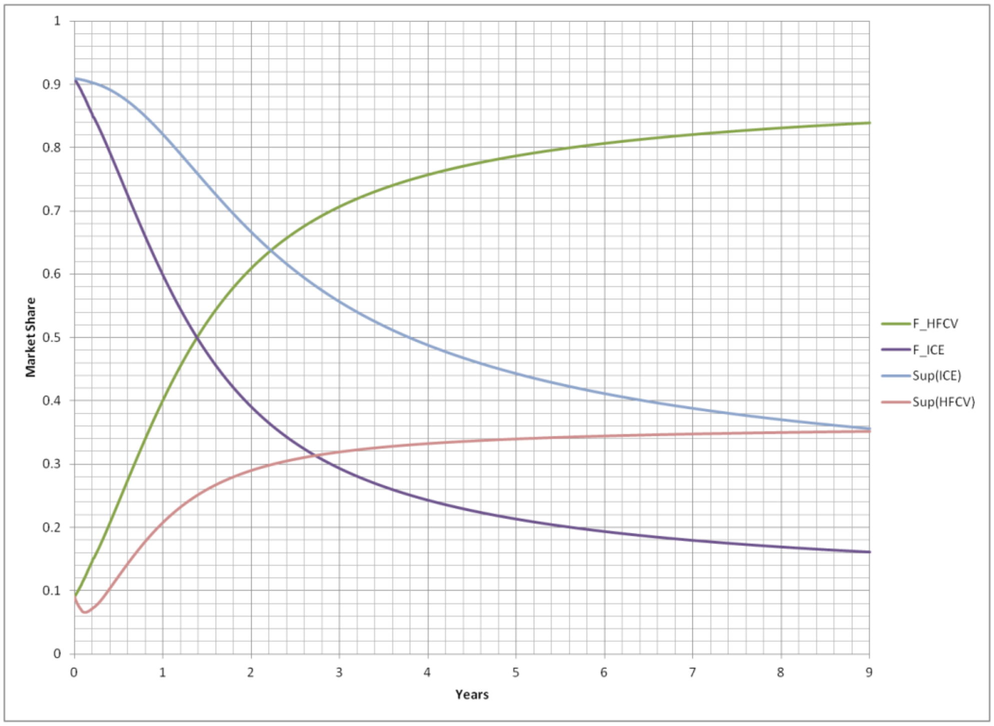

Figure 78. Demand and Supply for ICE and HFCV with I=0.0001 and D=0.001

In the case of choice situation, as values of I and D become smaller, F_HFCV and Sup (HFCV) increase. And a feedback effect on these graphs becomes smaller as values of I and D become smaller.

6.8 PID controller

The PID controller is defined as

$$F_i(t) = \rho_i(t) + Pe_i(t) + I\int_0^t e_i(\tau)d\tau + D\frac{d}{dt}e_i(t)$$
$$e_i(t) = K_i(t) - \rho_i(t) \tag{82}$$

In the following, results are presented for different values of the tuning parameters, P, I and D.

a) P=0.1, I=0.01, D=0.001

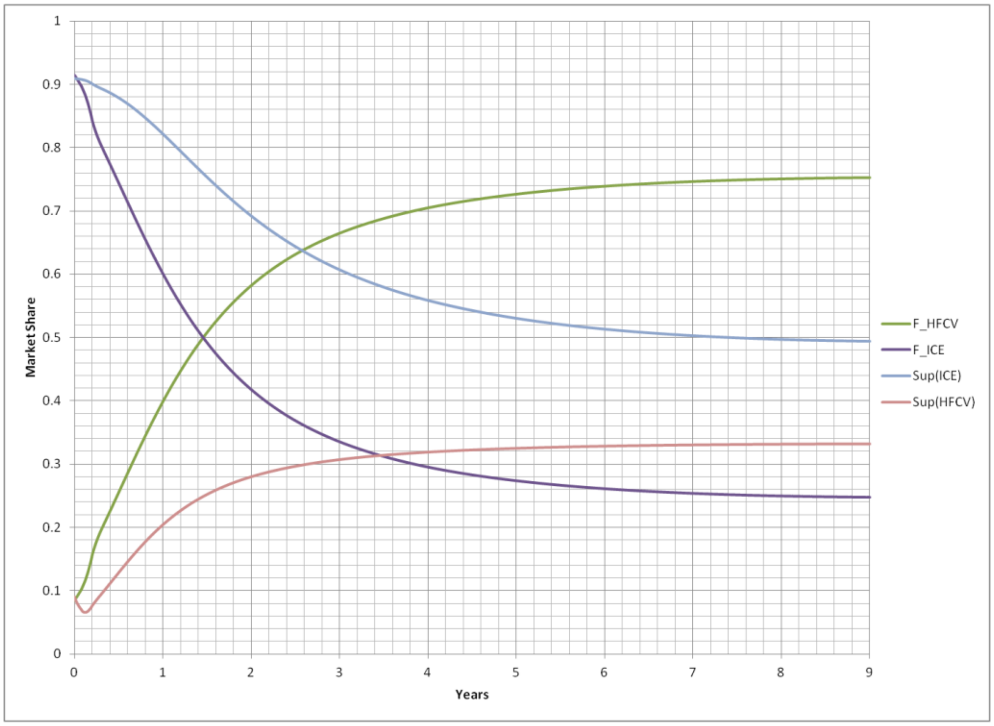

Figure 79. Demand and Supply for ICE and HFCV with P=0.1, I=0.01 and D=0.001

b) P=0.1, I=0.0001, D=0.001

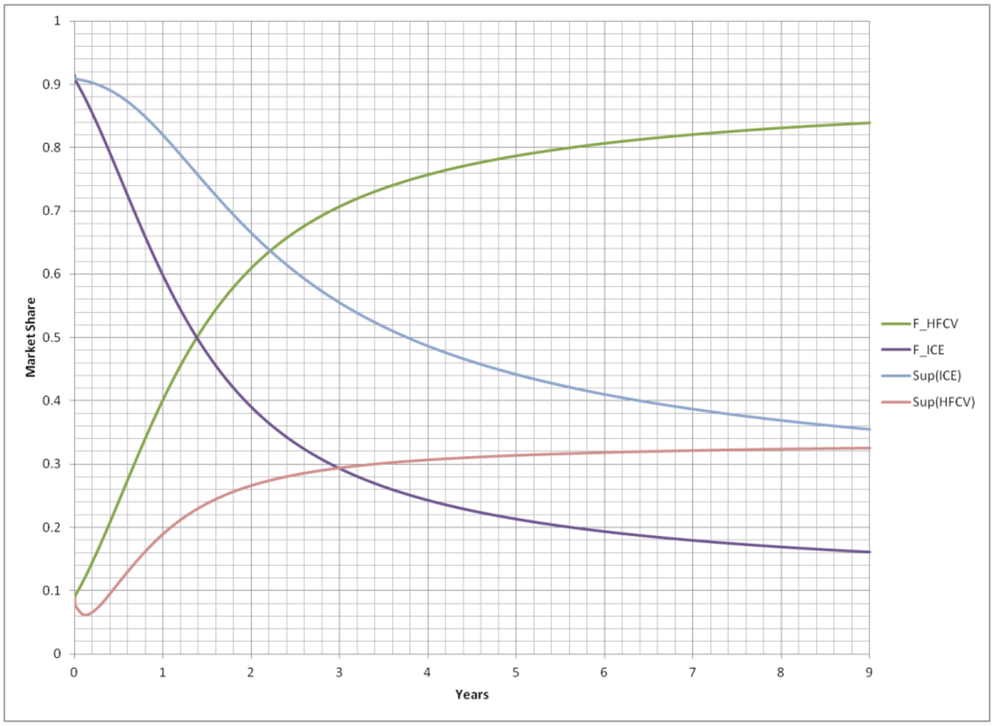

Figure 80. Demand and Supply for ICE and HFCV with P=0.1, I=0.0001 and D=0.001

c) P=0.001, I=0.0001, D=0.001

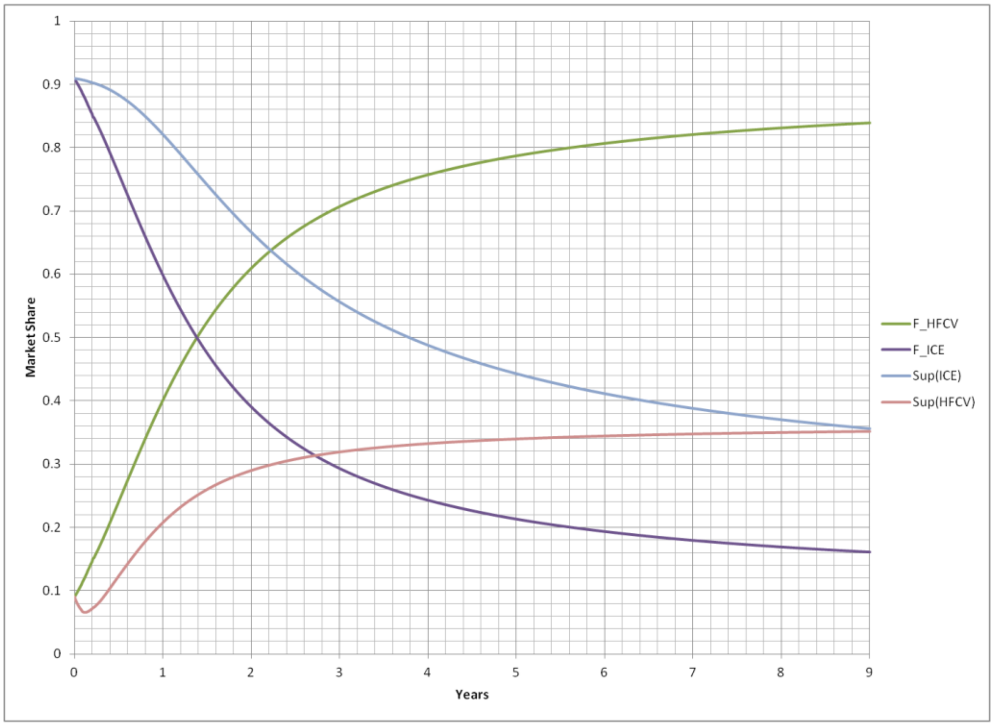

Figure 81. Demand and Supply for ICE and HFCV with P=0.001, I=0.0001 and D=0.001

In the case of choice situation, as values of P, I, and D become smaller, F_HFCV and Sup (HFCV) increase. And a feedback effect on these graphs becomes smaller as values of P, I, and D become smaller.

Figure 72 through Figure 81 show the results of applying various combinations of PID controller to connect a dynamic normative model and a competition model. The results show that modified demands and supplies for HFCVs vary with various values of P, I, and D. In general, as the values of P, I, and D become lower, gaps between demand and supply becomes higher and feedback effects on them become smaller. And when the values become; P=0.001, I=0.0001 and D=0.001, the graphs converge to same results, as shown in Figure 74, 76, 78, and 81, and there are no more changes in the gaps.

6.9 Summary

In this chapter, several types of feedback are applied, connecting the demand (an output from a dynamic normative model) and the supply (an output from a competition model) of HFCVs. This work is intended to attribute an affordable market share for HFCVs, $K_{HFCV}(t)$, to a demand for HFCVs which is calculated from a dynamic normative model and apply the modified demand to a competition model. The results show that the feedback adjusts a supply of HFCV not to exceed an affordable demand for HFCV. They also show that when the values of P, I, and D are high (lower than 1), the gaps between demands and supplies for HFCVs are small. However, as these values become lower, the gaps become larger and the feedback effects on demands and supplies for HFCVs become smaller. And at specific values for each P, I and D, there are no more changes in gaps.

7. Conclusions

In this research, factors that are expected to make viable the penetration of AFVs into the market have been examined. First, logit models are estimated to find the effect of various factors on demand for vehicles, including fuel price, vehicle price, and bandwagon effects. Then, by using the coefficients from logit models in conjunction with a dynamic normative model, the changes in market shares of AFVs over time have been projected for several different scenarios. Next, operating costs and expected demand for HFCVs are derived from calculated density of hydrogen refueling stations in order to examine refueling effects on demand for HFCVs, and to find saturation densities of stations for making HFCVs competitive. Then, a competition model is applied to the dynamic normative model to observe the interactions between the demand and supply sides of AFVs. Finally, feedback methods are applied to connect results from the two models.

The results indicate that the market share of AFVs will exceed that of ICEVs when: 1) a gasoline price is increased, 2) a vehicle price of AFVs is decreased, 3) the initial market share of AFVs is large, and 4) the density of refueling stations is increased.

Because the choices of individuals to own either Prius, or Corolla or Civic Hybrid are revealed in the dataset used in this study, the parameters for the choice models can be found to track the changes in market trends for these vehicles up to now; these results are used as a surrogate for the choice between AFVs and ICEVs. Specifically, the model results are applied to the hypothetical choice between a HFCV and a conventional ICEV. However, since HFCVs haven't been introduced in the market sufficiently yet to ascertain the revealed preferences of travelers toward their ownership and use, the results from the model certainly can, and should, be questioned for accuracy of assumptions; it is intended as a first step in trying to build more accurate forecasts of the supply conditions that would be first necessary for HFCVs to gain a foothold in the arena now dominated by ICEVs, at least partly because of the ubiquitous presence of gasoline stations. In the future, it is expected that data related to people who own these vehicles will be collected in sufficient numbers to obtain much more accurate forecasts of behavior and use.

The competition model assumes a competition between two vehicles for more customers. While the dynamic normative model tracks changes in market trends for vehicles on the demand side, the competition model tracks it in a supply side. In this study, the results show that supply of vehicles, which result from the model, trace demands for vehicles well. However, the competition model also is a simulation model. And like the dynamic normative model, there are many parameters that need to be defined or given. So, at least for reliability, the standard values for these parameters will have to be defined, for both the dynamic normative model and the competition model.

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