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Conceptual Combination and Fuzzy Set Theory

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Conceptual combination is the process by which people combine existent simple concepts (e.g., brown and apple) into novel combinations (e.g., brown-apple). As a possible formalism for conceptual combination, most proponents of prototype concepts endorse fuzzy-set theory (e.g., Zadeh, 1965). Osherson and Smith (1981), however, argue that the amalgamation of fuzzy-set-theory and prototype concepts is fraught with problems.

Some fuzzy-set theory. A key notion in fuzzy-set theory is that of a characteristic function, which maps entities into numbers in a way that indicates the degree to which the entity is a member of some set or concept. To illustrate, consider the characteristic function, c_F, which measures degree of membership in the concept fish (F). When applied to any creature x, $c_F(x)$ yields a number between 0 and 1, where the larger $c_F(x)$, the more x belongs to F. Thus, our pet guppy may not be very typical of fish, so it gets a characteristic-function value of .80. Our pet dog will get a very low value, say .05. If we now consider pets (P), and its characteristic function c_P, then our guppy and dog might be assigned the values .70 and .90.

The issue of conceptual combination has often been reduced to a question about characteristic functions: namely, given that concepts P and F are combined to form the complex concept P&F, how do we specify P&F's characteristic function ($c_{P\&F}(x)$) on the basis of those of P and F ($c_P(x)$ and $c_F(x)$)? The answer from fuzzy set theory is that $c_{P\&F}(x)$ is the minimum of $c_P(x)$ and $c_F(x)$.

Applying this min rule to our pet guppy, g, yields

$$c_{P\&F}(g) = \min(c_P(g), c_F(g))$$

$$= \min(.70, .80) = .70$$

This says that our guppy is less typical of pet fish than it is of fish. And therein lies the problem. For as Osherson and Smith (1981) point out, intuition suggests that a guppy will be more typical of the conjunction pet fish than of either constituent, pet or fish. Osherson and Smith argue that this pet-fish example is just one of an indefinite number of counterexamples to the min rule.

Rationale for the present work. There are two problems with the Osherson and Smith (1981) counterexamples. First, they rest only on Osherson's and Smith's intuitions; such claims need to be tested against typicality ratings of naive subjects. Second, there is no indication of the generality of the failure of fuzzy-set theory; perhaps Osherson and Smith's counterexamples are of a few types in some underlying taxonomy of conjunctions, where other types might conform to the theory. To deal with these problems, we first present a taxonomy of adjective-noun conjunctions, and then describe some relevant experimental work.

An initial taxonomy of adjective-noun conjunctions. All counterexamples of the Osherson-Smith variety, such as pet-fish and brown-apple, have the following characteristics: the adjective concept (i.e., the property denoted by the adjective) is relevant to the noun concept (i.e., the object denoted by the noun) and negatively diagnostic of it; e.g., being brown is relevant to whether an object is an apple, and counts against it. More precisely, an adjective is negatively diagnostic of a noun to the extent that knowing that the adjective is a true description of some object increases the probability that the noun is a false description of that object, and knowing that the adjective is false of some object increases the probability that the noun is true of that object. An adjective is positively diagnostic of a noun to the extent that knowing that the adjective is true (false) of some object increases the probability that the noun is true (false) of that object. And an adjective is nondiagnostic of a noun to the extent that knowing that the adjective is true (false) of some object has no bearing on whether the noun is true or false of that object. Thus, in sliced-apple the adjective is largely nondiagnostic; in red apple the adjective is positively diagnostic; and in brown apple the adjective is negatively diagnostic.

In addition to the relation between the constituents, we also considered the degree to which the conjunction provides a true description of an object that is to be categorized. To keep things simple, we consider only the degree to which the to-be-categorized object manifests the property denoted by the adjective in the conjunction, and we let the object take either a high or low value on this property. This gives a total of six cases, presented in Table 1.

Table 1

Initial Taxonomy of Adjective-Noun Conjunctions

		Degree to Which Object Manifests Property	
		High	Low
Relation of Adjective Concept to Noun Concept	Nondiagnostic	(1) unsliced apple object is unsliced	(2) unsliced apple object is sliced
	Positively Diagnostic	(3) red-apple object is red	(4) red-apple object is brown
	Negatively Diagnostic	(5) brown-apple object is brown	(6) brown-apple object is red

Consider now how people might judge the typicality of various objects vis a via the different kinds of conjunctions in Table 1. In Case 1, since the constituent concepts are relatively independent of one another, people might separately judge the extent to which an object is an instance of the adjective concept and of the noun concept, and then combine the outcomes of these two distinct judgements into an overall typicality rating. Since this is the key idea behind fuzzy-set theory, some variant of the theory might prove adequate for Case 1. In contrast, Case 5, where the adjective is negatively diagnostic of the noun, captures the counterexamples used by Osherson and Smith (1981). Here, intuition suggests that an object with a high value on the property (e.g., an apple that is indeed brown) will be rated more typical of the conjunction (brown-apple) than of either constituent (brown or apple). The outcomes for the remaining Cases (2, 3, 4, and 6) might fall somewhere inbetween these extremes.

An experiment to test the taxonomy. For each of 48 pictured objects, one group of 20 subjects rated the object's typicality with respect to an adjective concept (e.g., red, brown, sliced), a second group of 20 subjects rated its typicality vis a vis a noun concept (e.g., apple), and a third group of 20 rated its typicality with respect to an adjective-noun conjunction (e.g., red apple, brown apple, sliced apple). The adjective-noun conjunctions were such that all six cases of our taxonomy were tested.

In the Noun group, on each trial the experimenter spoke the name of a noun, then a pictured object appeared and subjects rated how good an example it was of the noun concept. Each picture was presented once. In the Adjective group, on each trial the experimenter spoke the name of an adjective, then a pictured object appeared and subjects rated how good an example the pictured property was of the adjective concept. Now, each picture was presented twice, once with an adjective denoting a property that the pictured object had a high value on, and once with an adjective denoting a property that the picture had a low value on.; e.g., the picture of a red apple and that of a brown apple were presented once with "red" and once with "brown." In the Adj-Noun group, on each trial the experimenter spoke the names of an adjective and noun, then a picture was presented and subjects rated how good an example the pictured object was of the conjunctive concept. Each picture was presented twice, once with a conjunction whose adjective denoted a property the picture had a high value on, and once with a conjunction whose adjective denoted a property that the picture had a low value on; e.g., the picture of a red apple was presented once with "red apple" and once with "brown apple." All subjects had ten seconds to make a judgement, the judgements being made on a 10-point scale, where higher numbers indicated better examples.

The top half of Table 2 contains the data for the three cases of the taxonomy where the object has a high value on the property denoted by the adjective. For Case 1, we expected the minimum rule to work. The results are otherwise: the conjunction's typicality clearly exceed the minimum of its constituents. A comparable deviation from the min rule also occurred in Case 3. For Case 5, where we expected the largest violations of the min rule, the conjunctions' typicality exceeds the minimum value of the constituents by virtually half the scale! For all three cases, the deviation from the min rule is significant by a sign test.

Table 2

Typicality Ratings for Three Groups, Separately for Each Case

a. Object Has High Value on Property

Cases	Adjective Rating	Noun Rating	Adj-Noun Rating	Adj-Noun Minus Minimum
1: Nondiagnostic	8.71	7.25	8.65	1.40
3: Positively Diagnostic	8.50	7.81	8.87	1.06
5: Negatively Diagnostic	6.93	3.54	8.52	4.98

b. Object Has Low Value on Property

Cases	Adjective Rating	Noun Rating	Adj-Noun Rating	Adj-Noun Minus Minimum
2: Nondiagnostic	.45	7.25	.52	.07
4: Positively Diagnostic	.02	3.54	.10	.08
6: Negatively Diagnostic	.81	7.81	.39	-.42

As for alternatives rules within fuzzy-set theory, none seem to do a better job. Gougin's (1969) multiplicative rule suggests that the conjunction's typicality rating should be less than the minimum value of the constituents, which is even wronger than the min rule. Another alternative is that the conjunction's typicality value be the average of its constituents, but this too is violated by the data (see Table 2). The best-fitting post hoc rule is that the conjunction's typicality is the maximum of its constituents. The max rule works well for Cases 1 and 3 but fails for Case 5; and it is not really a serious possibility in fuzzy-set theory for if conjunctive concepts are represented by a maximum then there is no obvious way to represent disjunctive concepts.

The bottom half of Table 2 contains the results for cases where the pictured object had a low value on the property denoted by the adjective. For all three cases the min rule works well, but only because subjects in the Adjective and Adj-Noun groups judged the pictured objects to be nonmembers of the relevant concepts. Thus, when presented a picture of a brown apple and asked to judge its typicality of red or of red-apple, most subjects gave it 0 ratings. This floor-effect, which prevents us from taking the data in the bottom of Table 2 as a sensitive test of the min rule, reflects a poor choice of how to experimentally implement the extent to which an object instantiates the property denoted by the adjective. Thus, for the concept red, had we used pictures of red apples and reddish-brown apples, we might not have obtained so many 0 ratings for the concepts red and red-apple. This change has been made in our subsequent experiments.

In conclusion, for cases where an object "fits" a concept well, fuzzy set theory fails to provide an adequate account of conceptual combination.

References

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