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The effect of stimulus presentation time on bias: A diffusion-model based analysis

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Abstract

There are two main types of bias in simple decision tasks, response bias and stimulus bias. Response bias is a starting level of evidence in favor of a biased response, whereas stimulus bias is the evaluation of stimuli in favor of a biased response. Previous research typically dissociates between these two types of bias. Some studies suggest that it can be difficult to induce response bias without stimulus bias (Ratcliff & McKoon, 2008; van Ravenzwaaij, Mulder, Tuerlinckx, & Wagenmakers, 2012). We used a two-alternative forced-choice brightness discrimination task in which we manipulated the presentation length of the stimuli. We analyzed the data with a hierarchical diffusion model. The results show an overall response bias, as well as stimulus bias that increases as stimulus presentation time decreases. We argue that the results suggest a need to revise how stimulus bias is conceptualized through the drift rate parameter of the diffusion model.

Keywords: diffusion model; response bias; stimulus bias; prior bias; dynamic bias; drift criterion

Introduction

Decision bias is an important area of research because it reveals information about the underlying processes that drive decision making, highlighting how different contexts and goals can influence decision-making behaviour in different ways (White & Poldrack, 2014). Simple decision tasks, where individuals are asked multiple choice questions with only two possible responses, are fairly common in the field of decision making. Research has suggested that there are two distinct types of bias in simple decision tasks: response bias and stimulus bias, also known as prior and dynamic bias, respectively (van Ravenzwaaij et al., 2012; White & Poldrack, 2014). Response bias is a preparedness to make a certain response, whereas stimulus bias is an asymmetry in how two stimuli of equal value/magnitude but opposing valences are processed as evidence for their respective responses.

Decision making can be thought of as sampling information from your environment to build support for a response over time. There have been a number of response time models that have been proposed to formalize this concept. One popular model is the diffusion model (Ratcliff, 1978; Ratcliff & Rouder, 1998; Ratcliff, 2002). To exemplify this model, suppose an observer is tasked with categorizing the stimulus presented in Figure 1a as dark or bright, depending on whether it contains more black or white circles. The diffusion model keeps track of a single quantity of evidence, which reflects the relative amount of accumulated evidence for one choice over the other. This means that in this example, evidence for a 'dark' response counts as evidence against a 'bright' response, illustrated in Figure 1b. Once evidence for one response reaches a boundary, a decision is made.

The basic diffusion model is defined by 4 main parameters that are attributed to different cognitive components that make up the speed and accuracy involved in decision making. These parameters consist of the drift rate, starting point of evidence accumulation, response boundaries, and non-decision time parameters. The non-decision time represents the time taken to perform the processes not directly associated with the evidence accumulation process e.g. motor response to press a button associated with a response. The boundary refers to the amount of evidence required to make a response, and is often characterized as the level of caution the observer has chosen. The starting point of evidence accumulation and drift rate are the two parameters associated with response and stimulus bias respectively.

The starting point parameter is used to represent a baseline level of evidence towards a specific response before stimulus information is accumulated as evidence. Response bias is essentially a shift in the start point parameter, meaning less evidence is required to reach one response boundary compared to the other, as illustrated in Figure 1c. A start point halfway between the two response boundaries indicates no response bias. The drift rate describes the average rate at which evidence is accumulated in favour of one response over the other. Stimulus bias is when one type of stimulus elicits a stronger or weaker drift rate compared to the other type of stimulus. Stimulus bias is illustrated in Figure 1d.

Response bias and stimulus bias both play a large role in decision making, however they are typically presented as independent of each other and dissociable i.e., the preparedness to make a response does not affect the evidence accumulation process. The characterization of these processes as independent confers two main advantages. Firstly, it makes the model more parsimonious. Secondly, it gives a way to account for the different effects that different manipulations have on response times and accuracies.

A number of studies have contributed to this dissociation of response bias and stimulus bias and their associated parameters. The start point can be influenced by the relative frequencies of the presented stimuli and the relative reward rates associated with the stimuli, with limited effects on other parameters (Bogacz, Brown, Moehlis, Holmes, & Cohen, 2006; Diederich & Busemeyer, 2006; Ratcliff & McKoon, 2008; White & Poldrack, 2014).

On the other hand, studies have illustrated that the drift rate is influenced by the quality and discriminability of information presented during a trial (Palmer, Huk, & Shadlen, 2005; Ratcliff & McKoon, 2008; Voss, Rothermund, & Voss, 2004). The standard interpretation for a bias in the drift rate param-

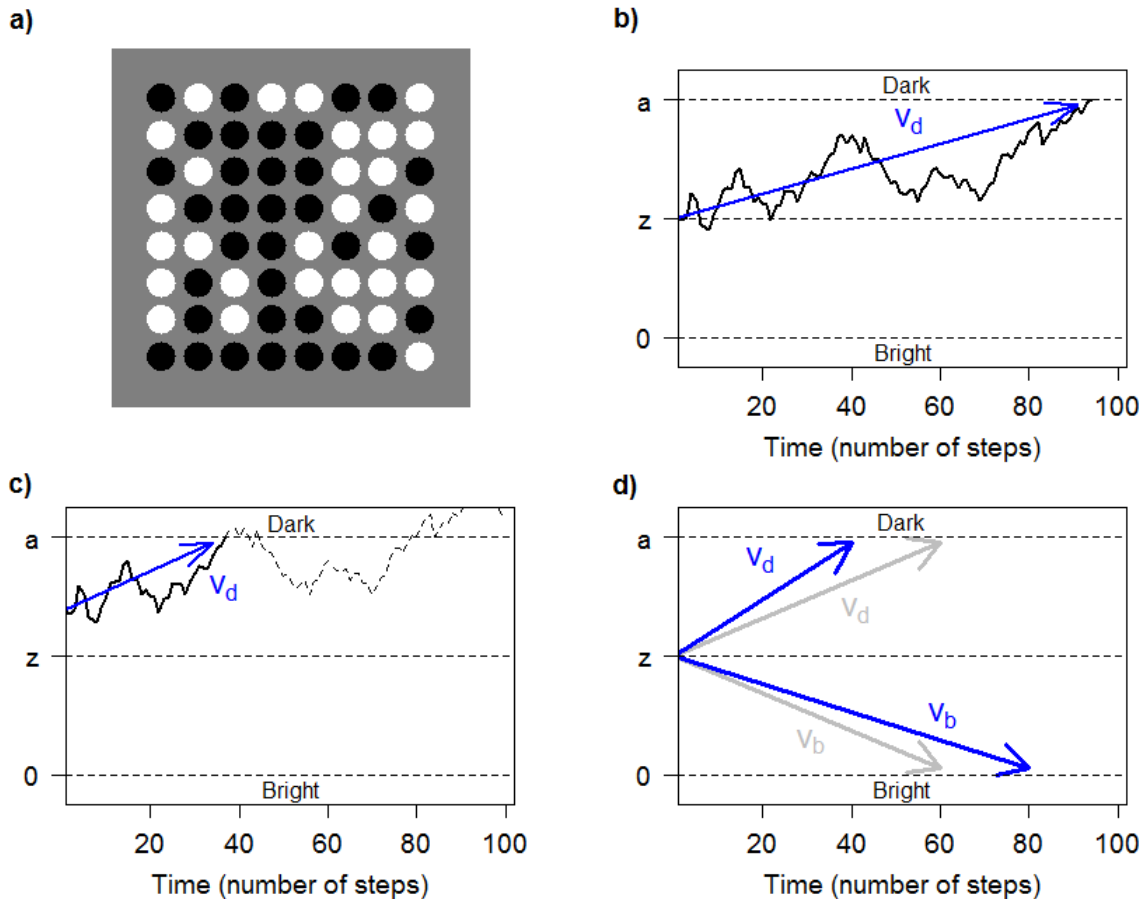


Figure 1: (a) Example of a stimulus where the participant had to decide the stimulus was dark or bright based on the proportions of black and white circles. (b) Diagram of the basic diffusion process. For this example, the top boundary represents the threshold for a ‘dark’ response, and the bottom boundary is the threshold for a ‘bright’ response. (c) Effect of a start point bias towards ‘dark’ responses. There is a shift in the starting point of evidence accumulation such that, given the same course of evidence accumulation observed in Figure 1b, the dark threshold is reached earlier. The dotted line represents the further evidence accumulation if the threshold was not reached. (d) Example of stimulus bias, with V_d and V_b representing drift rates for ‘dark’ and ‘bright’ stimuli respectively. Evidence is collected more quickly for the ‘dark’ response than it is for the ‘bright’ response. The grey arrows represent drift rates where there is no stimulus bias.

ter is one of criterion setting (Mulder, Wagenmakers, Ratcliff, Boekel, & Forstmann, 2012; Ratcliff, 1985; van Ravenzwaaij et al., 2012). The information observed from a stimulus is compared to a criterion, and the difference between the stimulus information and the criterion yields the evidence value that is to be accumulated in the model. Changing the criterion to permit more evidence for a particular response produces a bias. White, Mumford and Poldrack (2012) demonstrates this in a size discrimination task by showing participants a standard against which upcoming lines should be compared in order to determine what constitutes a ‘long’ or ‘short’ response. Their manipulation of this standard selectively influenced a drift criterion parameter in a diffusion model.

There have been studies that used the diffusion model to focus on specifically dissociating these two types of bias.

Leite and Ratcliff (2011) examined the effects of stimulus frequency, response payoff, and decision criterion manipulations on start points and drift criterion parameters through a numerosity discrimination task where participants had to decide whether volume of asterisks contained within a 10 by 10 grid could be categorised as a ‘low’ amount or a ‘high’ amount, based on some given criteria. They found that changes in the start point parameter alone were able to account for changes in the RT and accuracy data when they manipulated stimulus frequency and payoff. When they manipulated the decision criterion for what was considered ‘low’ and ‘high’, they found the data was best fit by shifts in the drift criterion parameter. Similarly, White and Poldrack (2014) used a perceptual discrimination task and a recognition memory task and found that response bias and stimulus bias can be inde-

pendently induced in the diffusion model through the use of stimulus frequency and decision criterion manipulations respectively.

Other research in this field however, has proposed that these biases and the underlying parameters may not be necessarily be independently manipulated. Ratcliff and McKoon (2008) examined the effect of relative frequency and stimulus difficulty manipulations on model parameters using a motion discrimination paradigm. When stimulus difficulty was manipulated, they found that only drift rate varied, however when relative frequency of the stimuli varied, they found a bias in the start point as well as a modest effect on the drift criterion.

Additionally, van Ravenzwaaij et al. (2012) proposed that, theoretically, response bias is sufficient to account for optimal performance in a variable or fixed difficulty task when relative frequency of stimuli is manipulated, but only under certain conditions (cf. Moran, 2015). However, they found that the model fits of empirical data from individuals performing a motion discrimination task show that the relative frequency manipulations had effects on start points and drift criterion in both fixed and variable difficulty tasks.

Rather than being independent of each other, it is possible that base rate information plays a role in moderating how individuals evaluate information under certain circumstances. This provides a potential explanation for why both response bias and stimulus bias were found in studies which manipulated relative frequencies of stimuli. The current experiment aimed to test how response bias and stimulus bias may be expressed under conditions of limited information and differing stimulus frequencies. In doing so, we wanted to observe if this dissociation of these biases and their related parameters holds true. Our experiment empirically evaluated the effect of the relative frequency of stimuli and the duration of stimulus presentation on the parameters of the diffusion model through the use of a hierarchical Bayesian version of the simple diffusion model.

Method

Design

The stimuli used were various combinations of 64 black and white circles in a 8 by 8 grid. In each stimulus, there were 35 circles of one color, and 29 of the other. Participants were instructed to make 'black' or 'white' responses for each stimulus they were presented, indicating which color circle of which there were more. For clarity, stimuli with more black circles will be referred to as 'dark' stimuli and the associated response will be 'dark' responses. Similarly, stimuli with more white circles will be referred to as 'bright' stimuli and the associated response will be 'bright'. An example of a dark stimulus is shown in Figure 1a.

The independent variables manipulated were relative frequencies of stimuli and the presentation length of the stimuli. Relative frequencies of stimuli were manipulated across blocks. This manipulation had three levels, dark biased (two

thirds of block were dark stimuli), bright biased (two thirds of block were bright stimuli), and unbiased (even proportions of dark and bright stimuli). There were 13 presentation lengths of stimuli, ranging from 0ms (where no stimulus is shown) to 200ms in 16.7ms (1 frame on a 60 Hz monitor) intervals. The presentation length varied from trial to trial within each block. The experiment consisted of 9 blocks of 80 trials each.

Procedure

At the start of the experiment, participants received instructions on the aim of the task and what they should expect to see on each trial. It was stated that the presentation time of the stimuli will vary within each block. Participants were also told the relative frequencies of each type of stimulus (dark and bright) will differ across blocks and received information about the proportions of dark and bright stimuli at the start of each block. At the end of each block, participants are given an opportunity to take a self-paced break before continuing onto the next block.

At the start of each trial, participants were required to press and hold the spacebar with the index finger of their dominant hand. Once spacebar was held, a fixation cross was presented for 500ms, followed by a mask presented for 100ms, followed by the stimulus. The stimulus is presented for a random duration from 0-200ms, followed by a backward mask of 100ms. There was 16.7ms before the disappearance of the mask and the appearance of the stimulus. Once they were prompted for a response, they had to release the spacebar and indicate a response using the 'F' or 'J' key to indicate whether they thought stimulus was 'dark' or 'bright' using the same finger the held down spacebar with. If they released their finger too early, they received a warning and the experiment would progress to the next trial. These instructions were given to discourage preemptive responses.

After each trial, participants received feedback on screen based on the accuracy of their response. For the 0ms trials where there is no 'correct' response, the feedback for their response was probabilistically determined based on the bias condition for the current block. 'CORRECT' was presented in green if their response was accurate and 'INCORRECT' was presented in red if their response was inaccurate. This feedback was on screen for 750ms before they were allowed to continue to the next trial.

Before the task began, participants were given 12 practice trials consisting of equal numbers of dark and bright stimuli. Each of the presentation lengths, excluding the 0ms presentation length, were used for one of the practice trials.

Model specification

Data were fit using a hierarchical Bayesian version of the simple diffusion model (for more information on hierarchical diffusion models see Vandekerckhove, Tuerlinckx, & Lee, 2011). MCMC estimation was performed through the JAGS Wiener module (Wabersich & Vandekerckhove, 2014) to estimate the parameters by running 3 chains with 5000 iterations each.

Individual-participant level parameters of the diffusion model were assumed to come from Gaussian distributions at the population level. For example, a drift rate for participant i in condition j was modelled as $v_{ij} \sim N(\mu^j, \lambda^j)$, where μ^j is the population-level mean drift rate parameter for condition j , and λ^j is the precision of the population-level drift rate parameter for condition j . The priors for the population-level mean parameters were set to be vague and relatively uninformative. For non-decision time, we used a normal distribution of mean 0 and a precision of 100, truncated to be above zero. For the boundary parameter, we used a normal distribution with mean of 3 and a precision of 2, truncated to be above zero. For start-point parameters, we used a uniform distribution from 0 to 1. For drift rate, we used a normal distribution with mean 0 and precision of 1. For the population-level precision parameters, λ , we used gamma distributions with shape and rate parameters of 0.001.

The distance between the boundaries (a), the mean distance of the starting point (z), the average rate of evidence accumulation (v), and the non-decision time parameter (T) were estimated for each individual while also estimated on a population level. The results discussed are the population level parameters estimated by the model. For the purpose of model fitting, dark responses were made when evidence passed the boundary at a and bright responses were made when evidence passed the boundary at 0 . This means that higher start points and positive drift rates represent more starting evidence and evidence accumulation for dark responses and lower start points and negative drift rates represent more starting evidence and evidence accumulation for bright responses.

We allowed start points and drift rates to vary freely across all conditions in the experiment, but constrained boundaries and non-decision times to be equal across the three levels of relative frequencies of stimuli conditions. This results in the estimation of 13 boundary parameters and 13 non-decision time parameters (for the each of the trial types), 39 start point parameters (for each trial type across the 3 levels of relative frequencies of stimuli) and 78 drift rate parameters (same as the start point parameters, but estimated separately for the dark stimuli and the bright stimuli). This results in a total of 143 population level parameters. In the following section, we discuss the posterior distributions of the population-level mean parameters.

Results

Figure 2 illustrates posterior distributions of population level start point parameters for each presentation time. A start point closer to 1 and 0 indicates higher starting evidence for dark and bright responses, respectively. When no bias is expected in the start points, a start point of 0.5 is expected. This is what we observed for the unbiased blocks - start points for the unbiased stimulus frequency blocks are distributed around 0.5 across all presentation time conditions, as shown in the green violin plots in Figure 2. From the results of previous exper-

iments that manipulated relative frequencies of stimuli, we expect a bias in the start point in both the dark and bright biased conditions (Leite & Ratcliff, 2011; Ratcliff & McKoon, 2008; van Ravenzwaaij et al., 2012; White & Poldrack, 2014). In the dark biased conditions, we expect start points to be above 0.5 and in bright biased conditions, start points are expected to be below 0.5. Our results are in line with this expectation and are fairly consistent across the different presentation times.

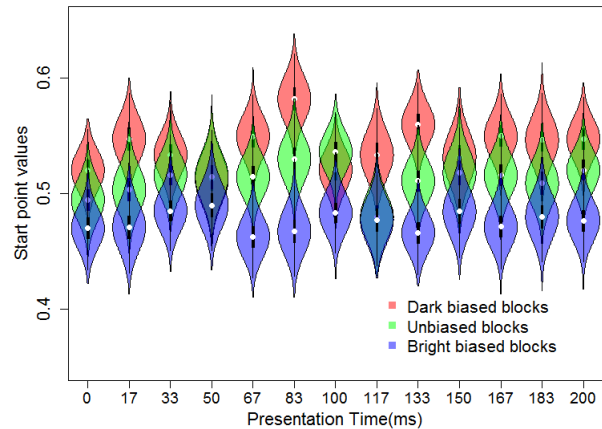


Figure 2: Violin plots of posterior distributions of population level start point parameters for each presentation time.

Regarding the estimates of the drift rate parameters, Since bright and dark stimuli carry the same amount of information (i.e. same proportion of dominant-color circles) we expect the drift rates to have the same magnitude, but in opposite directions. Stimulus bias is calculated as the average drift rate across dark and bright stimuli for a presentation time in a type of block. Since the drift rates for dark and bright stimuli should be equal but with opposite valences, if there is no bias, we expect the average drift rate to be 0.

For the unbiased blocks, the longer a stimulus was presented, the higher the drift rate in the direction of the response associated with that stimulus, with drift rates for short presentation times being distributed around 0, as shown in Figure 3a. This matches our expectations of a higher drift rate when more information (longer presentation time of stimuli) is presented, resulting in limited observed stimulus bias (as shown in the green plots in Figure 3d). For the biased blocks, we observe an overall shift in the drift rates for both bright and dark stimuli, away from 0 and towards the response for the biased stimuli. This is particularly prevalent for the shorter presentation time conditions i.e., for the bright biased blocks, drift rates are in the direction of a bright response when the stimulus is presented for a limited amount of time (0-67ms), regardless of what stimulus was presented (illustrated in Figure 3b). A similar effect is present for the dark biased blocks, illustrated in Figure 3c. Consequently, we found that the stimulus bias observed in the shorter presentation time conditions

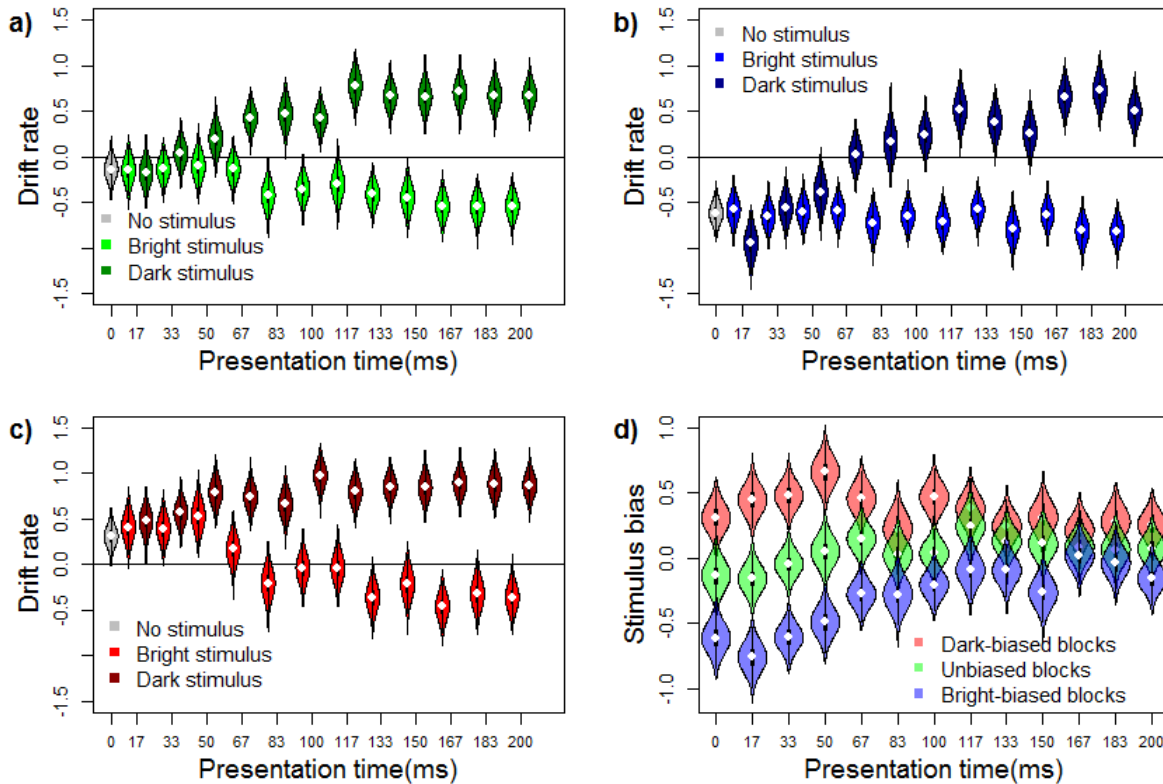


Figure 3: Violin plots of posterior distributions of drift rates for dark and bright stimuli across presentation times for the (a) unbiased, (b) dark biased and (c) bright biased blocks. (d) Average of the dark and bright stimulus drift rates for each presentation time for each block bias type.

was in favor of the biased stimuli for the biased blocks. This also changed as a function of presentation time; we observed a clear trend of stimulus bias decreasing as presentation time increased, summarized in Figure 3d.

Discussion

There are two main findings to take away from the experiment. Firstly, it replicated the response bias effect produced by relative frequencies of stimuli manipulations demonstrated in previous research (Bogacz et al., 2006; van Ravenzwaaij et al., 2012; White & Poldrack, 2014). Secondly, the results from the experiment suggest that stimulus bias has an inverse relationship with presentation time in relative stimulus frequency manipulations. The second finding is particularly interesting because does not coincide with typical interpretations of drift rate and drift rate bias in the diffusion model. If the drift rate reflects the accumulation of information, as stimulus information approaches 0, so too should the drift rate. The results of our current experiment contradict this, continuing to show modest drift rates towards the biased response when there is limited stimulus information.

One possible deviation from this perspective that could explain these results is a model which allows the drift rate to vary across the length of a trial. Its possible that initially,

drift rate is driven by biases or sequential effects but is updated as the information from a presented stimulus becomes apparent. Diederich and Busemeyer (2006) discuss a similar concept of a two stage processing model for data from a perceptual discrimination task in which payoffs and deadlines were manipulated. They proposed a model that suggests there are two stages within a trial in which different aspects of the task inform the drift rate. This model suggests that during the first stage, payoff information determined the drift rate but after some period of time, stimulus information takes over. They found that this model was best able to account for the data when compared to two other models, one that allowed boundaries to vary over time, and another that allowed drift rates to vary across time.

On the other hand, a study by Ratcliff and Rouder (2000) manipulated stimulus presentation time in order to examine the concept of non-stationary drift rate in a two choice identification task. They found that a model with a non-stationary drift rate, where the drift rate rose during the onset of a stimulus and then fell to 0 once it was masked, was unable to satisfactorily explain the data. However, a model that used a constant drift rate over time fit the data well, suggesting that there is a constant accumulation of evidence over time even when the stimuli are shown then masked during a trial. In

light of their findings, Ratcliff and Rouder clarify that these findings may not necessarily extend to other domains such as perceptual stimuli (such as the one used in the current experiment) because a cognitive representation may not necessarily be the output of perceptual processing as it is in a letter identification task. Where previous studies focused on purely the onset of a stimulus, none have addressed how response bias may interact with stimulus onset asynchrony.

Another possible explanation is that expectancies or subjective values of responses, more typically reflected in the start point parameter, may moderate how the drift rate is set. The distinction between stimulus and response biases in the diffusion model is analogous to the Bayesian distinction between prior and likelihood. Bogacz, Brown, Moehlis, Holmes and Cohen (2006) argued that the diffusion model is a special case of Wald's (1945) sequential probability ratio test, which is an optimal procedure for deciding between two hypotheses (Wald & Wolfowitz, 1948). Under this equivalence, the start point of evidence accumulation corresponds to the prior probability of the two competing hypotheses (responses). The transformation of information into evidence is carried out by a likelihood function. The posterior probability of the hypotheses are then used as prior probabilities as the next piece of information is to be evaluated. Once the posterior probability of any one hypothesis is large enough, then a response is triggered.

Under the Bayesian framework, a drift rate bias is an adaptation of the likelihood function that is used to transform information from the stimulus into the evidence for competing responses. Our results suggest that the typical interpretation of drift rate bias, the concept of a drift criterion, may not be the whole story. Rather, it seems that the drift rate bias may be also based on what the participant knows about the environment. Usually, such environmental information is assumed to either adjust the prior probability of the different responses, or modify the lens through which stimuli are evaluated. Our data suggest that environmental information may also be accumulated as evidence, at least when the stimulus information is lacking.

Furthermore, some studies have examined how information can be weighted differently in their integration in their response based on their reliability. There has been previous research which show that individuals are able to integrate information from multiple sources, weighing them based on their reliability. (Ernst & Banks, 2002; Fetsch, Pouget, DeAngelis, & Angelaki, 2012; Ohshiro, Angelaki, & DeAngelis, 2011). This has been supported using a modified version of the diffusion model in order to account for the time course of the process (Turner, Gao, Koenig, Palfy, & McClelland, 2017). This further supports the possibility that individuals may be integrating both stimulus information and environmental information when accumulating evidence. When the stimulus is uninformative, individuals may give greater weight to the environmental information which results in the diffusion model showing stimulus bias in the parameter estimates.

When discussing these findings in the context of a diffusion model, it is important to keep in mind that the current set of analyses is a redescription of the observed data through the diffusion model and may not represent the 'true' underlying model. Some alternative models that may be able to account for the results of the current experiment are the leaky, competing accumulator (LCA) model proposed by Usher and McClelland (2001), and Kvam's (2019) theory of bias based on split attention and racing diffusion processes. Usher and McClelland's (2001) LCA model suggests that the observed stimulus bias may be caused by a lateral inhibition between accumulators for the two alternative choices. On the other hand, Kvam (2019) puts forward a model based on a continuous orientation judgement paradigm which suggests that stimulus information and predecision information (such as base rates) compete with each other as separate accumulators and cues can also moderate attention given to a stimulus. Although these models are outside of the scope of this paper, further research in this area should consider these models.

The results of the current experiment highlight that limited information can induce stimulus bias in blocks with uneven stimulus frequencies. Potential avenues for future research include investigating whether this stimulus bias can be induced by other manipulations such as stimulus difficulty or stimulus ambiguity, as well as using other modelling approaches, which may provide alternative explanations for the observed stimulus bias effects. This may help to shed light on the underlying mechanism through which information is processed and how it can result in stimulus bias. Investigating the source of these effects have important implications for understanding how individuals make decisions with different levels of information and may give some deeper insight into the roles of different types of information in decision making.

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