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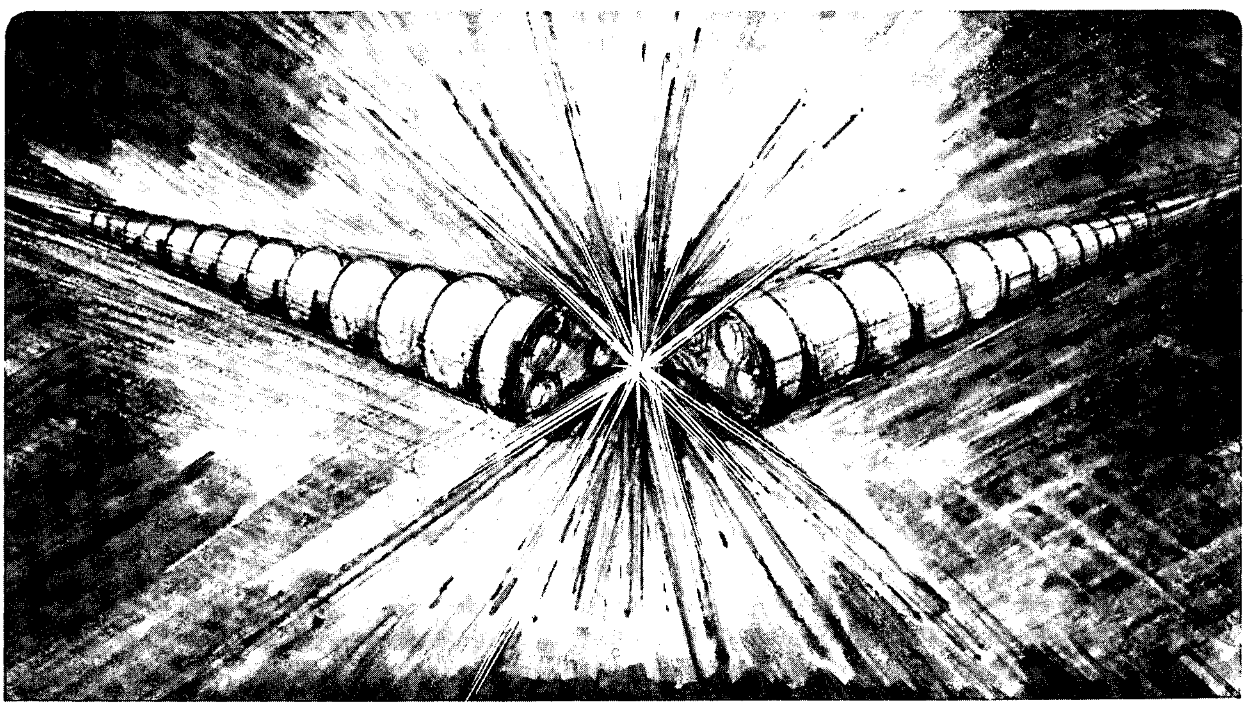

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**Abstract**

A quantum electrodynamic approach to the the temporal dependence of the first order correlation is presented using photon propagator techniques applied to the density operator. A diagrammatic interpretation and derivation of the spontaneous limited linewidth associated with stimulated emission is given and the Townes-Schawlow line narrowing formula derived.

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## Introduction

Correlation functions of the electro-magnetic field are of fundamental importance since they specify the coherence. At the most basic level these measure a specific temporal sequence of density matrix elements which can arise from the particle and photonic states. At any time the density matrix element can be either diagonal or off-diagonal in the particle or radiation field portion, providing several possibilities. The appropriate choice will be seen to be determined by the temporal sequence of both generation and detection verticies associated with the measurement, that is the quantum electro-dynamic interaction sequence involved in both the generation and detection associated with the correlation measurement.

The detailed relationship between the photon statistics and field correlations can thus be made precise, and we believe, is unique. We demonstrate this by a q.e.d. calculation of the Townes-Schawlow line narrowing formula for stimulated emission [1-8]. Although many calculations have been carried out from both the underlying photon statistics and the fields, the relationships between these diverse approaches are both obscure and most if not all of the fully quantum approaches use "Langevin Noise Sources", which are indirectly related to the requirements of q.e.d. This is not necessary when both the detection and generation are simultaneously considered.

The recent generation of non-Poissonian photon distributions, particularly squeezed states, have re-emphasized the underlying q.e.d. nature of the field correlations of various orders and has raised questions as to how they might be best calculated and experimentally measured [9]. In addition the generation of partially coherent radiation, particularly at short wavelengths, for which the photon density per mode is rather low and the generation is predominantly through spontaneous emission necessitates a conceptually more precise approach.

In this letter the fundamental relationships among field expectations, photonic distributions, and correlation functions are discussed in terms of a double Feynman diagramatic approach to the simultaneous temporal evolution of the field generation and detection processes. The "Langevin noise terms" are inherently obtained through a precise fully quantum electrodynamic derivation, as are the limits upon coherence implied by them. Since both detection and generation must be included it is useful to consider these aspects from the perspective of a specific experimental arrangement. For simplicity we consider linear stimulated and spontaneous emission in combination with a Michelson interferometer-square law detection arrangement as schematically shown in Fig 1. The temporal sequence of the relevant field generation and detection processes is depicted quantum electrodynamically in terms of the density operator in Figs. 1(b) and 1(c). The dashed lines refer to the detector density matrix temporal evolution, the detection itself assumed to be linear absorption with the signal proportional to the upper state transition rate. Absorption through negative energy photon exchange [10,11] has been chosen. This is because the contribution to the excited state transition rate with analytic signal $\mathbf{E}^-$ preceding $\mathbf{E}^+$ is desired. Terms with $\mathbf{E}^+$ preceding $\mathbf{E}^-$ can be handled similarly. The inclusion of the photon propagator for density operator calculations is discussed in [11]. Other relevant aspects of density matrix diagrams are discussed in references [13-19].

The density matrix equations directly give the detector upper state transition rate $Tr[H,\rho]/i\hbar = -Tr[\rho\overline{p}\cdot\mathbf{E}^+(t)/i\hbar]$ evaluated at time t. Here $\rho(t)$ is the direct product state of the detector medium and radiation field density operators. Evaluating the detector density matrix evolution of Fig. 1, using standard diagramatic techniques or, as originally, by Glauber [20] gives for this rate, the detection response function convolved with

$$G(t,t_1) = Tr(\mathbf{E}^+(t)\mathbf{E}^-(t_1)R)\rho_{ee} \tag{1}$$

where $R$ is the density matrix element describing the radiation field at time t, $t-t_1$ is the temporal delay implemented by the interferometer, and $\rho_{ee}$ is the initial upper state density matrix element of the emitting medium. The traced expression is the radiation field correlation function.

The loss in correlation over a macroscopic time delay $t-t_1 = \tau$ implemented by the interferometer is due to accumulated photocounts produced by the spontaneous emission events, as measured by the number of excited states produced in the detector. In terms of QED, calculating the total number of such events in time $t-t_1$ is a high-order process since each excited state excitation corresponds to an internal photon line (Feynman propagators). However, the Markovian nature of both the spontaneous and stimulated emission events, [21] allows one to obtain the loss in correlation over the macroscopic time delay from the fractional loss in correlation due to a single such event over a differential time $\Delta\tau$; that is, the

$$G(\tau) = \lim_{N\to\infty} G(0)(1+\frac{\Delta G}{G(0)})^N = G(0)e^{(\frac{\Delta G}{\Delta t}/G(0))N\,\Delta t} \tag{2}$$

where $N = (t-t_1)/\Delta t$. Thus the problem reduces to a calculation of $\alpha = \frac{\Delta G}{\Delta t}$, the rate of change of $G$ for a single such event.

The differential correlation change,$\alpha$, itself is a characteristic solely of the quantized radiation field generated by the emitting medium. To calculate it, however the relative time ordering of the $\mathbf{E}^-$ and the $\mathbf{E}^+$ operators, with respect to the interaction vertices of the emission (as established experimentally by the interferometer and detector) must be considered. This time-ordering establishes the relevant density operator temporal sequence, as shown in Fig. 1 for a particular case.

The temporal evolution depicted by Fig. 1 begins prior to any interactions, and thus the density matrix is diagonal, describing the probability of the emitting medium initially in the excited state and the detector in its ground state. This is $|1\rangle|e\rangle|n+1\rangle \mathbf{R}_{n+1,n+1}\rho_{11}\rho_{ee}\langle n+1|\langle e|\langle 1|$ where $\mathbf{R}$ is the probability of the $|n+1\rangle$ photon state being occupied, $\rho_{ee}$ that of the excited atomic state $|e\rangle$, and $\rho_{11}$ that of the detector ground state $|1\rangle$. The detector evolution has been shifted in time by the respective delay of the interferometer branches and thus is shown in the reference frame of the emitting medium. For the particular process diagramed, at $t''$ a polarization is induced in the emitting medium and the diagonal density matrix element is thereby correlated with the off-diagonal element $|1\rangle|g\rangle |n+2\rangle \mathbf{R}_{n+2,n+1}\rho_{11}\rho_{ge}\langle n+1|\langle e|\langle 1|$. This off-diagonal component induces a response in the detector at $t_1$. We note that since the interaction is expressed in the time frame of the medium, a medium polarization must be present for this to have a significant probability. The emitting medium subsequently completes a self-energy interaction through a photon absorption operation at $t'$. This results in a time rate of change of the off-diagonal photon density matrix element and the upper state atomic population. After this interaction occurs, the second field interaction with the detector takes place, resulting in an increase of the upper state detector population, and consequently, a photo-count. For simplicity it will be assumed that the presence of a pumping mechanism for the emitting medium maintains a constant upper level population for the emission medium, which is taken into account by taking the limit as the decay constant goes to zero.

Of interest is the calculation of the expectation just after the $\mathbf{E}^+$ vertex of Fig.(1) since this gives the contribution of the correlation to the transition rate of the upper detector state. To do this precisely the $\mathbf{R}$ density matrix under the trace of Eq. (1) must be calculated. The governing differential equations can be explicitly obtained from the Q.E.D. diagrams. This implies a general underlying photonic interpretation of double Feynman diagrams, which is treated in [11]. Here we wish to consider a perturbational treatment of the first order correlation directly to show that it inherently contains the limits traditionally imposed by stochastic noise terms and hence the resultant linewidth limitations given by the Townes-Schawlow formula, as expressed through the correlation function.

For the perturbation numerous time-ordered diagramatic contributions to the correlation, in addition to that explicitly shown in Fig. 1, need to be included. These various possible time-ordered evolutionary paths, are obtained by noting that the atomic system can undergo eight fundamental processes. These arise from having the two interaction vertices on either the bra or ket and for one on each two possible time orderings to give four. These are doubled by having a positive or negative photon energy line for each case (that is, absorption or emission). Each of these provides a possible contribution to the correlation and hence the detector interaction. The resultant eight basic diagrams with the specific time ordering of Fig. 1 are shown in Fig. 2. In addition to these, the time ordering of the photon interaction vertices of the medium with respect to the detector vertices must be taken into account. This provides a crucial aspect in the present analysis.

Each of the diagrams has a fourth order dependence upon the field operators. This, however, can be reduced by judiciously combining pairs of self-energy and exchange diagrams with the commutation relations, For the correlation, the interference results in terms having the same time-ordered operator dependence as the correlation itself (induced correlation terms). However, additional operator independent terms also originate from the spontaneous emission diagrams. These "stochastic" terms depend on the present approach and provide the expected lack of correlation of the field measured at time t with respect to that measured a time $\Delta t$ earlier. These are identified as the source of the Langevin noise terms.

The particular pairs which combine to give induced and/or stochastic correlation source terms could be obtained by physical arguments, however the number of processes makes this cumbersome. This is systematized by considering time-ordered field operator arrangements obtained by appropriately pairing the diagrams according to the type of photon process involved. This pairing is shown in Fig. 2(a) and (b) for instance, giving the two emission processes due to a positive energy photon self-energy and exchange interaction, respectively. If the algebraic form of the two terms of each pair is the same, then the terms can easily be combined as a common factor multiplied by the field operator sum of the two. This is the simplest situation which can arise and will for convenience be assumed. The combination of the terms of Fig. 2(a) and (b) is then proportional to

$$a \mid aa^+a^+ - a^+a \mid aa^+ = a \mid aa^+a^+ - a \mid aa^+a^+ = 0 \qquad (3)$$

Thus (a) and (b) do not contribute to a detected photo-count. The pair (c) and (d) of Fig.(2), an absorptive pair, also gives a zero annihilation and creation operator combination for the time ordering explicitly displayed:

$$a^+aa \mid a^+ - aa \mid a^+a^+ = a^+a^+aa \mid - a^+a^+aa \mid = 0 \qquad (4)$$

The emission pair (e) and (f) for the explicitly displayed time ordering on the other hand gives:

$$aa^+a \mid a^+ - a^+a \mid aa^+ = (a^+aa^+a - aa^+a^+a) \mid \; = - a^+a \mid \quad , \tag{5}$$

which leads to a term depending upon the correlation field pair itself and consequently an induced correlation contribution. If for these diagrams the vertex at time $t''$ precedes the vertex at $t_1$ (shown inset in (e) and (f)), then the combination of operators becomes:

$$\begin{aligned} aaa^+ \mid a^+ - aa^+ \mid aa^+ &= a^+aaa^+ \mid - aa^+aa^+ \mid \\ &= - aa^+ \mid \\ &= (-a^+a - 1) \mid \end{aligned} \tag{6}$$

resulting in the field pair correlation term, with a sign opposite to that of the absorption, plus a term independent of either the field pair or the electric field amplitude. This time-ordered emission pair can thus be identified with at least a portion of the stochastic driving term for a treatment of laser linewidth using the Langevin equation [ 4-8]. One observes that the emissive terms (a) and (b) gives a zero combined contribution for this particular time ordering, since interchanging $t''$ and $t_1$ does nothing, thus,

$$a \mid aa^+a^+ - a^+a \mid aa^+ = aa^+a^+a \mid - aa^+a^+a \mid = 0 \tag{7}$$

for both time orderings in which $t''$ is less than or greater than $t_1$.

Considering the absorptive pair (g) and (h), one obtains

$$\begin{aligned} a \mid a^+aa^+ - aa \mid a^+a^+ &= +a^+aa^+a \mid - a^+a^+aa \mid \\ &= -a^+(aa^+ - a^+a)a \mid \\ &= +a^+a \mid \end{aligned} \tag{8}$$

independent of the relative time ordering of $t_1$, $t'$, and $t''$. Thus as would be anticipated this combination contributes to the field pair but not to the stochastic noise term.

One concludes that the absorptive pair (g) and (h) and the emissive pair (e) and (f) of Fig. 2 are all that survive for $t'$ between $t_1$ and $t$, with the inset time-ordering of (e) and (f) in particular being responsible for the stochastic driving terms. For $t' < t_1$ these same two pairs provide induced terms but no stochastic terms. The stochastic contribution in this case arises from the emissive pair a and b, which also gives an induced response. Finally for $t' < t_1$, the absorptive pair c and d results in an induced term.

All these surviving induced correlation terms and stochastic terms inhibiting the correlation are respectively redrawn in Figs. 3 and 4. For the first four terms of Fig. 3, $t'$ and $t''$ vary from $-\infty$ to $t$, whereas for the latter four, $t' \leq t_1$ and $t''$ varies from $-\infty$ to $t'$.

The stochastic terms, shown in Fig. 4, are non-zero only for the specific time orderings indicated.

These diagrams emphasize two obvious but important conclusions. Firstly, the absorptive diagrams contribute only to the evolution of the induced correlation as expected. Moreover, the emissive terms are of opposite sign and provide additional stochastic terms responsible for the fundamental linewidth limitation of the Fourier transform of the correlation function. Secondly, only by considering the correlation diagrams could one obtain the stochastic contribution, since it depends upon the presence of the field interaction vertex at time $t_1$.

A detailed treatment of the correlation dependence upon $\Delta t$ requires a consideration of all of the time ordered diagrams of Figs. 3 and 4. However for the atomic population pumped to maintain a constant threshold value diagrams with noninterleaved detection and lasing medium vertices (such as Figs. 3(e-h) and 4(c-d) are unimportant since pumping compensates relaxation to give an effective zero population decay rate after the first two vertices. Consequently, the emission process is de-coupled from the detection, which consequently senses the medium as effectively quiescent during the measurement. This is also true for Figs. 3 (a-d) for the times $t', t'' < t_1''$. The remaining diagramatic contributions of Fig. 3(a-d) contribute a small perturbation of the induced correlation, as will be seen.

The processes represented by Figs. 4(a) and (b) give effects whose characteristics are generally inferred indirectly by zero point energy arguments [4-8] and are in particular responsible for linewidth limitations as given by the Townes-Schawlow narrowing formula [1,7]. This pair of diagrams thus provides a precise quantum-electrodynamic basis and generalization of these stochastic noise sources used as driving terms for Langevin type equations in various treatments of quantum fluctuation phenomena in the presence of stimulated emission.

Figure 4(b) is reproduced in Fig. 5. A dressed atom energy level diagram is also included to indicate the decay coefficients which determine the detailed algebraic form of this diagramatic contribution. $\Gamma_{eg}$ corresponds to the usual $T_2$ polarization interruption time. $\Gamma'_{ee}$, $\Gamma'_{gg}$, and $\Gamma'_{eg}$ refer to decay processes which have long effective decay times; $\Gamma'_{ee}$ and $\Gamma'_{gg}$ because states $g$ and $e$ are assumed pumped to maintain a constant population, and $\Gamma'_{eg}$ because this refers to a non-energy conserving decay process. These long decay times help assure the equality of the algebraic forms of Figs. 4(a) and 4(b) pair, thereby allowing these two terms to be simply expressed as a common algebraic expression multiplying the field operator combination obtained above, which in this case is 1. Any inequality of the algebraic factors can be included as an additional perturbation of the induced correlation. In the present analysis it will be assumed that $(t - t_1) \ll \Gamma'^{-1}_{ee}, \Gamma'^{-1}_{gg}, \Gamma'^{-1}_{eg}$ so that this situation does not arise. By further assuming that the time delay for the correlation measurement is composed of successive independent differential intervals each of duration dt, this can always be guaranteed.

The differential contribution to the detector transition rate, by the terms arising from Figs. (4a) and (4b) is then determined by the field correlation

$$dG(t,t_1) = \left[\frac{\hbar}{2\varepsilon}\right]^2 \left[\frac{e}{m\hbar}\right]^2 \int \frac{d^4q}{(2\pi)^4} \int_{t'=t_1}^{t=t_1+dt} dt'd\vec{r}'$$
$$\int_{t''=-\infty}^{t_1} dt''d\vec{r}'' \langle K^+(t,t')pK^+(t',t_1)K^+(t_1,t'')$$
$$pK^+(t'',t_0)\,\frac{\rho_{ee}}{q^2}\,K^-(t_0,t'')K^-(t'',t_1)K^-(t_1,t')K^-(t',t)\rangle p_0 \tag{9}$$

as written directly from the diagrams neglecting the detector response function. In this expression $p_0$ arises from the field factors at $t_1$ and $t$ as does one of the $(\frac{\hbar}{2\varepsilon})$ factors. The Trace over the field density matrix is 1 for the lowest order of perturbation. The combinations of propagators, for statistically independent time intervals between interactions and exponential decays are given by:

$$\langle K^+(t,t')K^-(t',t)\rangle = e^{-\Gamma_{ee}(t-t')} \tag{10a}$$

$$\langle K^+(t',t_1)K^-(t_1,t')\rangle = e^{-i(E_g-E_e)/\hbar\,(t'-t_1')-\Gamma_{eg}'(t'-t_1')} \qquad (10b)$$

$$\langle K^+(t_1,t'')K^-(t'',t_1)\rangle = e^{-i(E_g-E_e)/\hbar\,(t-t'')-\Gamma_{eg}(t_1-t'')} \qquad (10c)$$

Substituting these into the expression for the spontaneously developed correlation, integrating with respect to $t'$ from $t_1$ to $t = t_1 + dt$ and $t''$ from $-\infty$ to $t_1$, one obtains

$$dG(t-t_1) = 2\left[\frac{e}{m\hbar}\right]^2 \left[\frac{\hbar}{2\varepsilon}\right]^2 \int \frac{d^4q}{(2\pi)^4}\frac{\rho_{ee}}{q^2} \qquad (11)$$

$$\frac{[e^{(-\Gamma'_{eg}-i(E_g-E_e)/\hbar-ip_0)dt}-e^{-\Gamma'_{gg}dt}]}{(\Gamma'_{gg}-\Gamma'_{eg}-i(E_g-E_e)/\hbar-ip_0)(i(E_g-E_e)/\hbar+ip_0+\Gamma_{eg})}\, e^{ip_0dt}\,|p_{ge}|^2 p_0$$

where $|p| = |p_{ge}|$ is the matrix element (momentum) $\langle g|p|e\rangle$. Assuming a single radiation mode at frequency $p_0 = \omega_0$ the integral reduces to a single term. Assuming $dt \ll \frac{1}{\Gamma'_{eg}}$, $\frac{1}{\Gamma'_{gg}}$, and $(p_0-(E_e-E_g)/\hbar)^{-1}$, one obtains

$$dG(t-t_1) = \left[\frac{e}{m\hbar}\right]^2 \left[\frac{\hbar}{2\varepsilon_0}\right]^2 dt\, \frac{e^{i_0dt}}{(i(E_g-E_e)/\hbar+i\omega_0+\Gamma_{eg})}\,|p|^2 \qquad (12)$$

This expression provides real and imaginary contributions to the correlation expression, representing both a shift in the correlation oscillation frequency as well as a real decay of correlation with time. It is observed that $dG$ can be interpreted as the linear portion of the interference of two temporally oscillating terms; one at the emitting frequency, and the other at the atomic line center frequency. A real part which limits the linewidth is guaranteed since $\Gamma_{eg}$ must be at least as large as the inverse of the spontaneous emission time.

If the unperturbed contribution (Fig. 1(b)) is included, the correlation to first order in the differential time (assuming an instantaneous detector response of unit quantum efficiency) is:

$$\langle \mathbf{E}(t-dt)\mathbf{E}^{+}(t)\rangle = (\langle \mathbf{E}(0)\mathbf{E}^{*}(0)\rangle + dG)e^{i\omega_0 dt}$$
$$= \langle \mathbf{E}(0)\mathbf{E}^{*}(0)\rangle[1 + dG/\langle \mathbf{E}(0)\mathbf{E}^{*}(0)\rangle]e^{i\omega_0 dt} \tag{13}$$

Using the statistical independence of the differential intervals [21], the total fractional loss of correlation is then the product of that for each interval. One thus has as the number of differential intervals goes to infinity.

$$\langle \mathbf{E}(t_1)\mathbf{E}^{+}(t)\rangle = \lim_{n\to\infty} \langle \mathbf{E}(0)\mathbf{E}^{*}(0)\rangle\, [1 + G_s/(\langle \mathbf{E}(0)\mathbf{E}(0)\rangle]^n \; e^{i\omega_0(t-t_1)}$$
$$= \langle \mathbf{E}(0)\mathbf{E}^{*}(0)\rangle\, e^{G_s/\langle \mathbf{E}(0)\mathbf{E}(0)\rangle + i\omega_0(t-t_1)} \tag{14}$$

where $G_s$ is equal to $dG$ with $dt$ replaced by $t-t_1$. Substituting the expression for $G_s$ one obtains

$$\langle E(t_1)E^{+}(t)\rangle = G(t-t_1) = \langle E(0)E^{*}(0)\rangle \exp \tag{15}$$
$$\left[-\frac{\rho_{ee}}{\langle |E(0)|^2\rangle}\left(\frac{e}{m\hbar}\right)^2\left(\frac{\hbar}{2\varepsilon_0}\right)^2\frac{|p|^2}{\omega_0^2}\left[\frac{\Gamma_{eg}-i(\omega_0+(E_g-E_e)/\hbar)}{(\omega_0+(E_g-E_e)/\hbar)^2+\Gamma_{eg}^2}\right](t-t_1) + i\,\omega_0(t-t_1)\right]$$

the usual result including the slight correlation oscillation frequency shift from the lasing frequency.

The decaying temporal amplitude of the correlation deduced above gives the Fourier transform Lorentzian linewidth, $\Delta\nu$, equal to:

$$2\pi\Delta\nu = 2\times\frac{1}{W}\left[\frac{e\,|p|}{m}\right]^2\frac{1}{2\,\varepsilon_0}\times\frac{1}{2\omega_0}\;\frac{\Gamma_{eg}}{(\omega_0+(E_g-E_e)/\hbar)^2+\Gamma_{eg}^2}\;\rho_{ee} \tag{16a}$$

$$= \hbar\,\omega_0\,\frac{A\,\rho_{ee}}{2W} \tag{16b}$$

where $W$ is the energy density $2\varepsilon_0\langle |\mathbf{E}(0)|^2\rangle$. Using $A = h\,\nu B$ where $B$ is the stimulated emission coefficient, A is the Einstein coefficient for spontaneous emission into a single mode, and $h\,\nu_0(N_e-N_g)B = 2\pi\Delta\nu_{1/2}\,\frac{1}{c}$ from the threshold equation where $\Delta\nu_{1/2}$ is the cavity linewidth. This can be written as the usual linewidth formula:

$$\Delta\nu = \frac{\pi h \nu_0 (\Delta r_{1/2})^2 \rho_{ee}}{P(\rho_{ee} - \rho_{gg})} \tag{17}$$

Here P is the power emitted from the cavity, and $\nu_0$ the resonant frequency ($\omega_0/2\pi$).

This derivation establishes the basic approach of the present diagramatic treatment of the line-narrowing formula. If the saturation terms given by higher order perturbation diagrams are taken into account, an explicit expression for P will be obtained from the field expectation equation, and a fully quantum mechanical oscillator model demonstrating the amplitude stability should follow.

One observes that for this approach the induced terms, which are proportional to $a\, a^+$ would be a small correction to the diagram of Fig.(1b) since $E^+E$ is proportional to $aa^+$ as well. Thus $< E^+(0)E(0) >$ could be renormalized to reflect this.

At least two other approaches have been used to arrive at essentially the same result; the Langevin equation driven by a stochastic noise source for an oscillator ([5], for instance) and a photonic approach using the photon rate equations in the high photon number limit (see [22]).

It can be shown that the photonic density matrix equations, and hence this approach to the linewidth, are implicit in the diagramatic representation of the field expectations determined by the emission process. The off-diagonal components of the radiation field portion can be extracted, and hence the linewidth inferred. A complete treatment however requires the correlation diagrams of Figs.(3) and (4) to give the full particle radiation field density matrix element combinations. The Langevin approach provides a phenomenological model for this combination.

It is interesting that a detuning enhancement factor ($1+\alpha^2$) has not resulted directly from the present derivation [4,8,24-27]. In its simplest form $\alpha = \Delta\omega/\Gamma_{eg}$, first derived by Lax [8], whose quantized approach included a steady state solution of the Heisenburg equation for the field operator. Henry [4] provided a simple model based upon amplitude-phase coupling through the refractive index. Vahala and Yariv derived the same result by considering $\chi^{(3)}$ coupling [26]. All of these used phenomenological noise sources. In terms of the present formulation this amplitude-phase coupling should result from higher order self-energy terms which cause fluctuations in the transition frequency $(E_e - E_g)/h$ as well as lifetime broadening as modeled by $\Gamma_{eg}$. A relatively small fluctuation is expected for the diagrams contributing to the threshold equation because of the dominance of the stimulated emission (large photon number) associated with a single mode. A particularly simple approximation is to assume that the fluctuations are much larger than the linewidth for a given $p_0$ and that all values of $E_e - E_g$ are equally probable. This could be true for the large effects observed for semiconductor lasers. An integral over $E_e - E_g$ results multiplying the correlation diagram and hence $\Delta\nu$ by $1 + (\Delta\omega/\Gamma_{eg})^2$. A simple extension should give $\Delta n_r / \Delta n_i$ for $\alpha$

In conclusion we have considered a quantum electrodynamic approach for the calculation of stimulated emission linewidth as limited by spontaneous emission processes. This approach emphasizes the importance of simultaneously considering the generation and detection processes [28]. Generally the present approach is particularly useful for situations in which phase coherence is a consideration, which arises, for instance, when states having significant number population are present. The present results also raise a fundamental question with regard to self-energy diagrams. In terms of the density operator, these are seen to be on an equal footing with exchange diagrams and Ref.(11) shows that both of these are equally important in determining the expectation value of the electric field associated with the photonic distribution.

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[28] see Ref.(23) page 1 for an interesting comment relating to this.

## Figure Captions

Fig. 1. Classical Michelson interferometer measurement of field correlations (a) schematic of the experimental configuration, (b) density matrix diagram for the unperturbed contribution to a correlation measurement; showing the initial state $|1\rangle\ |n+1\rangle$, where $|1\rangle$ is the ground state of the detector, $|e\rangle$ the excited medium state and $|n+1\rangle$ the photon number state of the radiation field. (c) One possible density matrix diagram contributing to the first order spontaneous and stimulated contributions to the correlation measurement. $|g\rangle$ is the ground state of the medium and $|2\rangle$ the excited state of the detector.

Fig. 2. The diagrams for the eight possible first order processes which can contribute electric field "induced" and "spontaneous" correlation terms in a Michelson interferometer measurement. The insets in (e) and (f) show the specific time ordered diagrams providing stochastic source terms.

Fig. 3. Specific time-order pairs of terms of Fig. 2 which contribute only induced correlation terms. For (a) through (d) $t'' \le t'$ and $t' \le t$ for the time constraints. For (e) through (h) $t'' \le t'$ and $t' \le t_1$. As explained in the text these terms are perturbations with respect the zeroth order processes of Fig.(1b).

Fig. 4. The specific time-ordered diagrams of Fig. 2 contributing the quantum noise sources and associated with the noise sources in the Langevin equation for the field correlation calculation. For (a) and (b) $t \ge t' \ge t_1$ and $t'' \le t_1$. For (c) and (d) $t', t'' \le t_1$ and $t'' \le t'$.

Fig. 5. Decay coefficients which enter into the correlation coefficient calculation. (a) Fig. 3(d) with the decay processes indicated. (b) Dressed atom portrayal of the decay processes. Appropriate Green's functions for calculating emission rates.

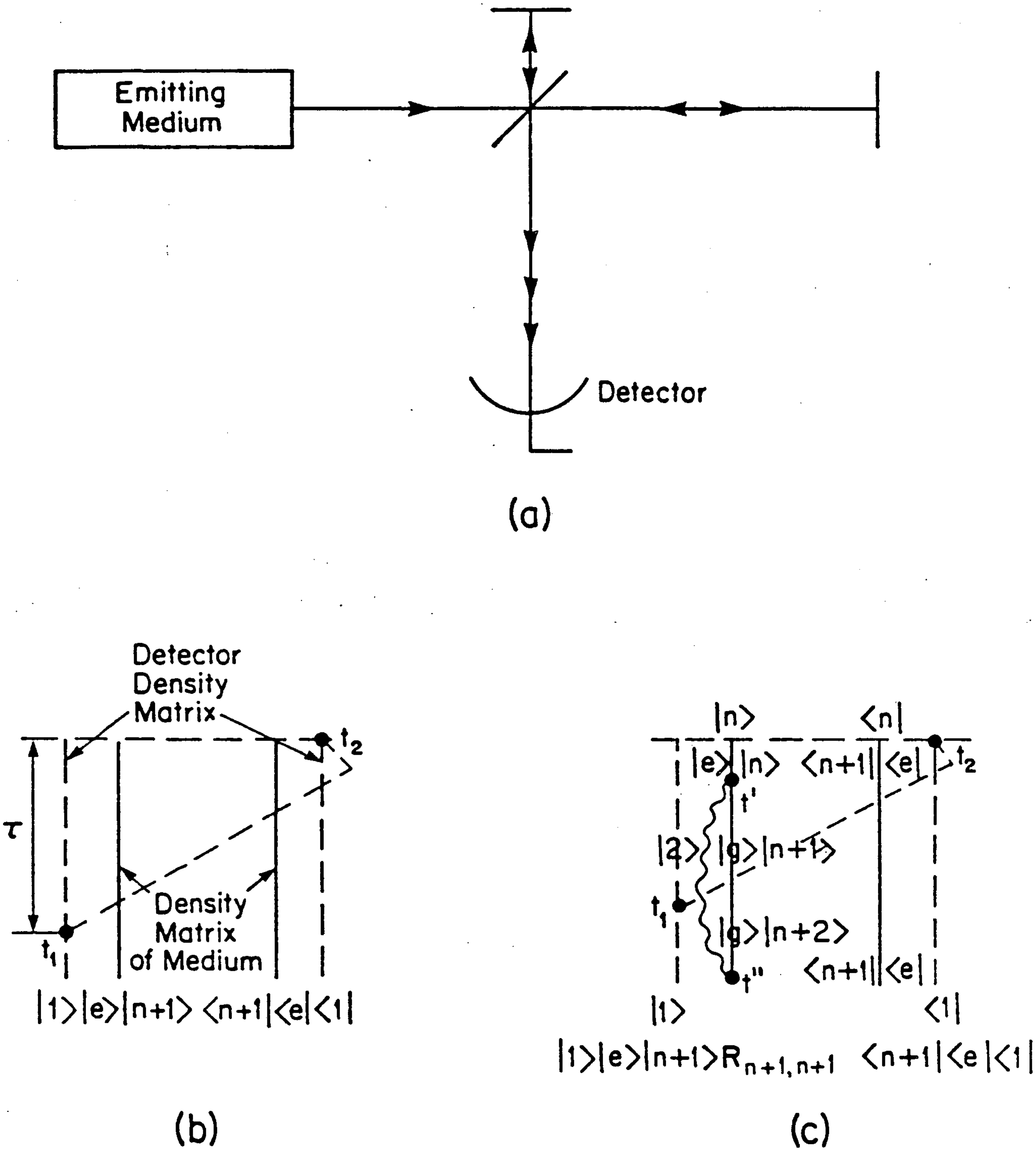
Emitting Medium
Detector
(a)
Detector Density Matrix
τ
Density Matrix of Medium
$t_1$
$t_2$
|1>|e>|n+1> <n+1|<e|<1|
(b)
|n>
<n|
|e>|n> <n+1||<e|
t'
|2>
|g>|n+1>
|g>|n+2>
t''
<n+1||<e|
|1>
<1|
|1>|e>|n+1>$R_{n+1,n+1}$ <n+1|<e|<1|
(c)

Figure 1

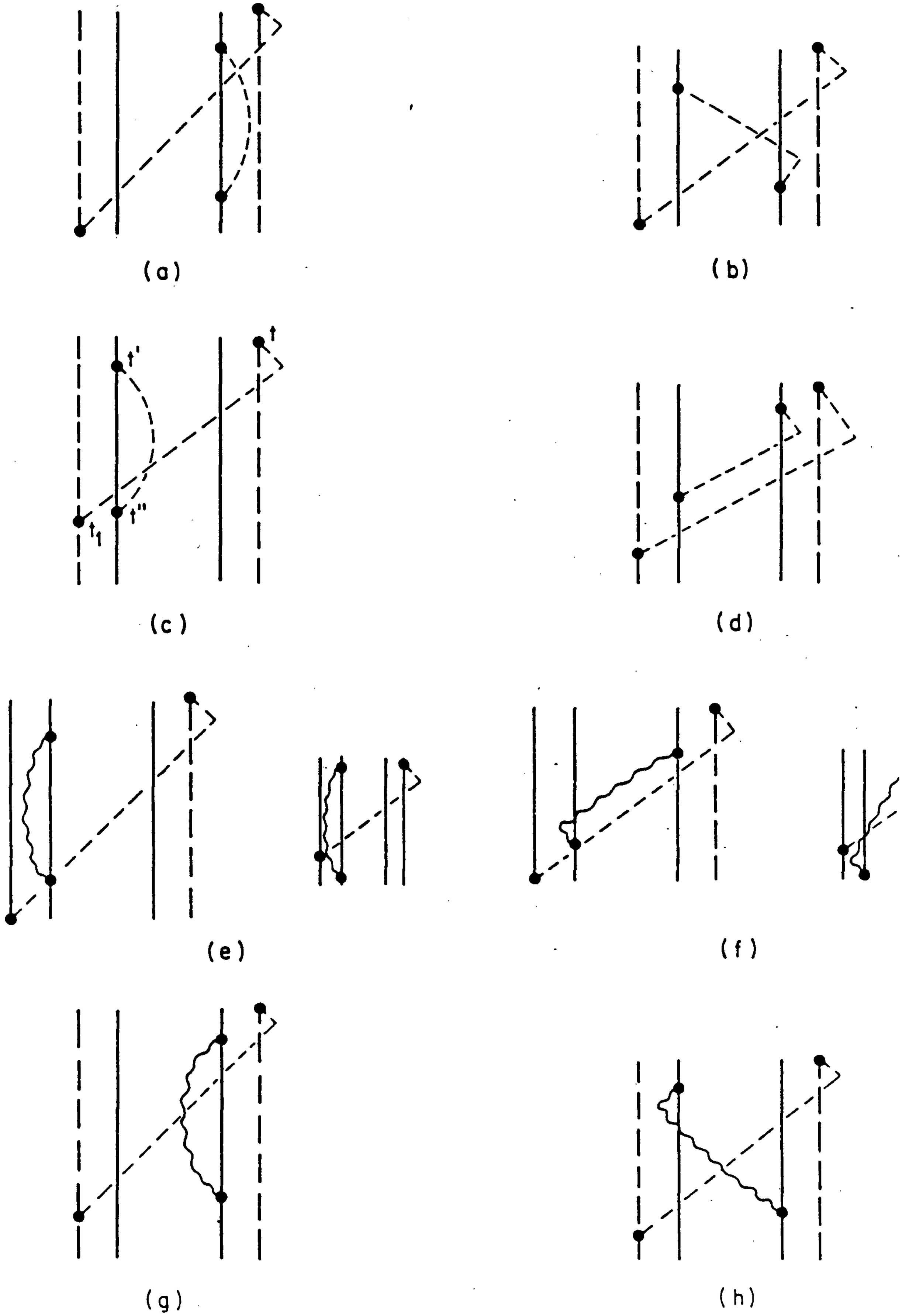


Figure 2

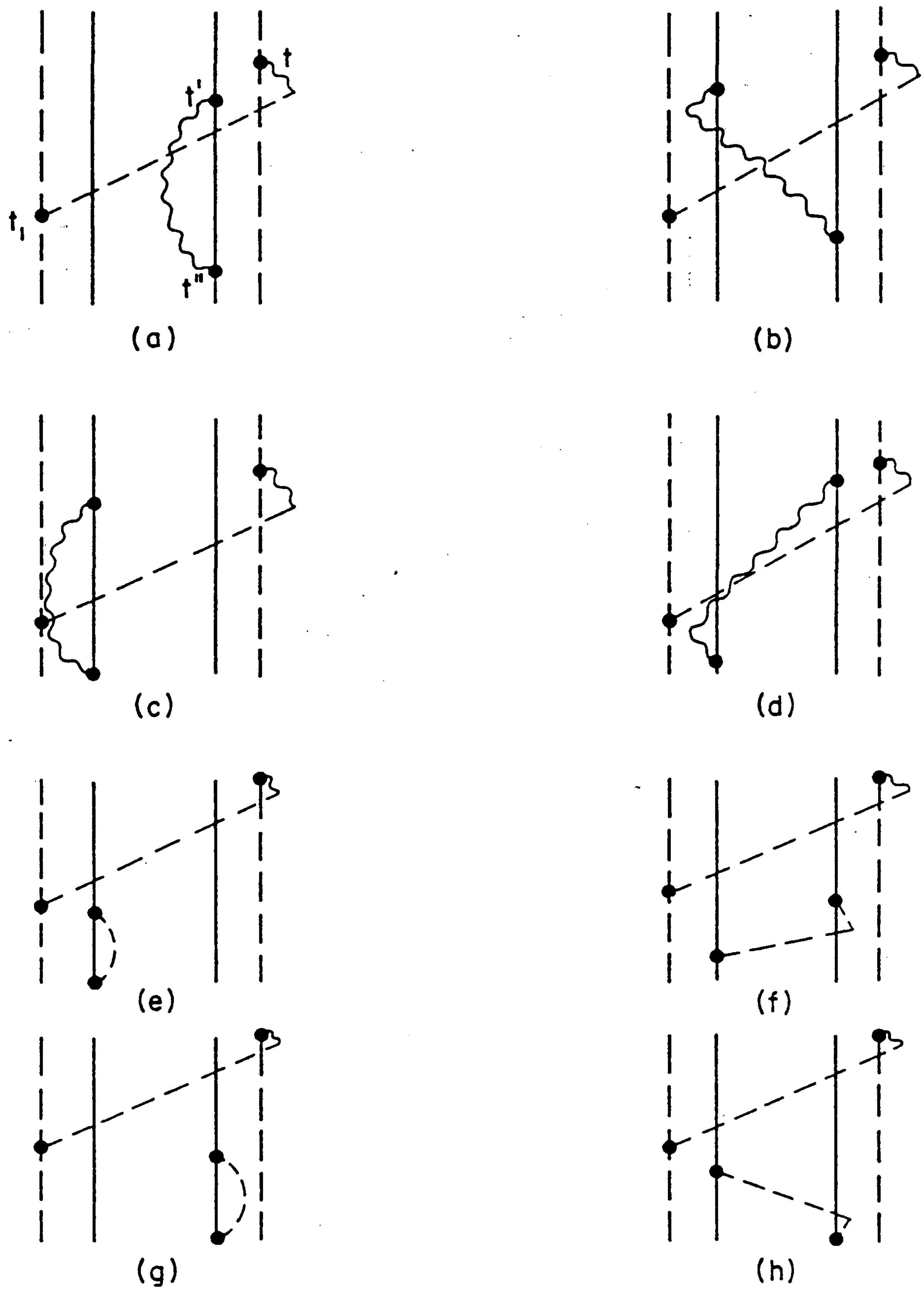


Figure 3

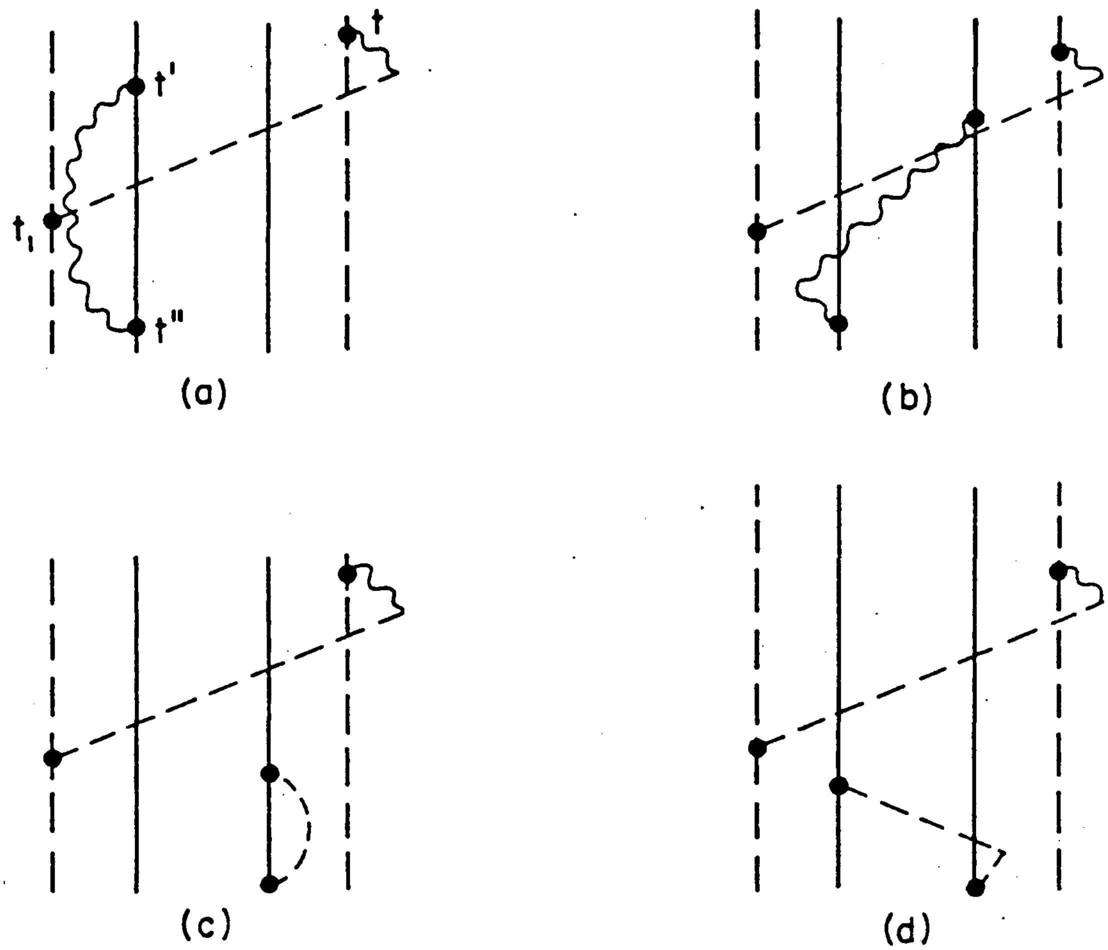


Figure 4

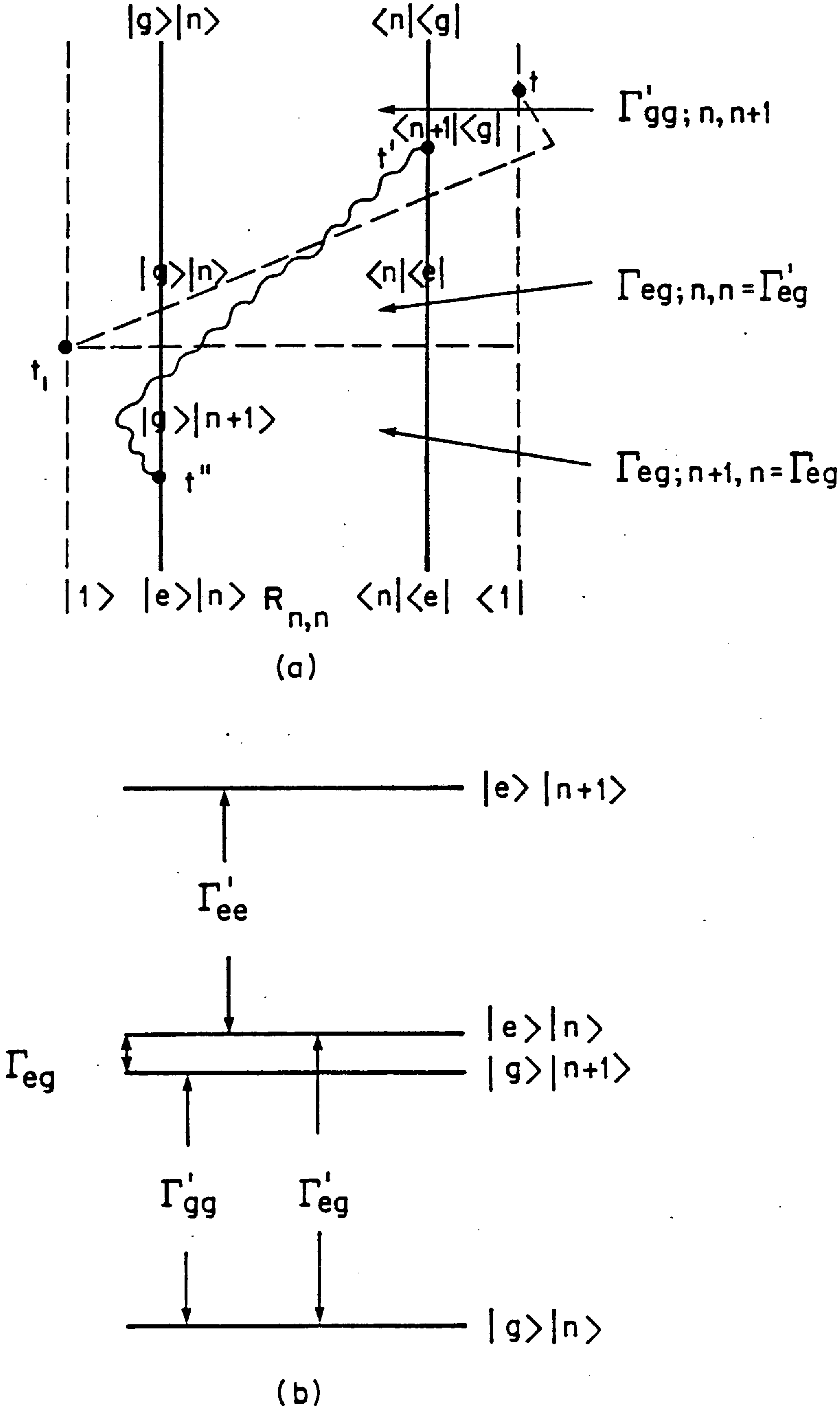
|g>|n>
<n|<g|
t
Γ'gg; n, n+1
<n+1|<g|
t'
|g>|n>
<n|<e|
Γeg; n,n = Γ'eg
t1
|g>|n+1>
Γeg; n+1, n = Γeg
t"
|1>
|e>|n>
R n,n
<n|<e|
<1|
(a)
|e> |n+1>
Γ'ee
|e>|n>
Γeg
|g>|n+1>
Γ'gg
Γ'eg
|g>|n>
(b)

Figure 5

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