

Economic Growth with Constraints on Tax Revenues and Public Debt:
Implications for Fiscal Policy and Cross-Country Differences*

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Abstract

This paper evaluates optimal public investment and fiscal policy for countries characterized by limited tax and debt capacities. We study a non stochastic CRS endogenous growth model where public expenditure is an input in the production process, in countries where distortions and limited enforceability result in limited fiscal capacities, as captured by a maximal effective tax rate. We show how persistent differences in growth rates across countries could stem from differential public finance constraints, and differentiate between the case where the public expenditure finances the *flow* of recurring spending (such as law enforcement), versus the *stock* of tangible public infrastructure. Although the *flow* of public expenditure raises productivity, the government should not borrow to finance it as the resulting increase in public debt would lower welfare and the growth rate. With outstanding public debt, the optimal fiscal policy should keep the debt-to-GDP ratio constant in the economy with or without a binding constraint on tax revenues as a share of GDP – current non-durable public goods should be financed only from current revenue. With investment in the *stock* of public infrastructure, public sector borrowing to finance the accumulation of public capital goods may allow the economy to reach a long-run optimal growth path faster. With a binding tax capacity constraint, if the ratio of the initial public/private sector stock of capital is smaller than the sustainable balanced growth ratio, the optimal policy for the government is to purchase public capital, financed by debt, to immediately attain the sustainable ratio of public capital to private capital. The sustainable steady-state ratio is endogenous to the initial public-to-private capital ratio, the tax capacity and any exogenous debt limit (say, due to sovereign risk). With capital stock adjustment costs, these statements apply to a transition of finite duration rather than an instantaneous stock jump. With either a binding exogenous debt limit or solvency constrained borrowing, a more patient country will have a higher steady-state growth rate but a lower steady-state public-to-private capital ratio.

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1. Introduction

The purpose of this paper is to study the impact of limited tax and debt capacity on growth. This issue is pertinent for developing countries, where structural factors imply that capacity constraints are tighter than in the OECD countries. These are also the countries where limited infrastructure imposes bottlenecks inhibiting growth, and where fiscal adjustment falls disproportionately on public investment.¹ The goal of our paper is to identify the factors accounting for the growth and welfare effects associated with fiscal adjustment, and to provide testable implications of limited capacities on optimal fiscal and debt policies

It is widely acknowledged that fiscal policies can promote or deter economic growth. Current *flow* public expenditures can promote private investment in capital and improved techniques of production, for example, by maintaining the rule of law, enforcing contracts and regulating financial markets. Public investment in either the *stock* of physical infrastructure or human capital can increase the productivity of both capital and labor. The tax system can distort resource allocation reducing both growth and welfare. The economic, institutional and political environment influences the effectiveness of public spending for promoting growth and the capacity of the government to raise tax revenue or borrow to finance public goods while minimizing the cost to economic growth. The national environment imposes constraints on fiscal policy that can be reflected in cross-country differences in rates of economic growth.

The essence of Ricardian equivalence is that “optimal public debt” is only a meaningful concept when the fiscal capacity of the government is constrained. The implications of fiscal constraints for optimal public borrowing are particularly important for developing countries where distortions associated with tax collection are greater than in advanced industrialized countries. This paper contributes to the limited literature on growth and the public debt by studying how fiscal capacity limitations would impact the optimal path of growth and the public debt. Specifically, we consider how constraints on the capacity of the government to tax or debt

¹ See Servén and Burnside (2005) for a discussion of the international experience with public investment in the context of fiscal adjustment, and for analytical review of the links between fiscal targets, public investment and growth. Our paper addresses related issues for economies where, unlike their specification, public infrastructure is an essential input, constrained due to limited tax and debt capacities.

finance public expenditures affects economic growth when externalities generated by public goods raise the productivity of private investment and employment. The effectiveness of tax policies for raising revenue varies widely across developing countries. The divergence of statutory tax rates from effective tax rates is much higher for most developing countries than for advanced industrialized countries as is the variation in statutory rates across activities within economies.² The strength of institutions is an important determinant of the degree of tax compliance and of the adverse effect of the tax system on output, income and growth.³ The analysis in this paper first highlights how constraints on effective tax rates motivated by tax distortions and enforcement costs affect optimal fiscal policies in an endogenously growing economy.

The emphasis on maximal effective rates of taxation is a simple way to capture the role of institutional characteristics that increase the distortions due to taxation and limit the capacity of fiscal authorities to generate tax revenues, both conventional and inflationary, for financing public goods expenditures and public debt interest payments. We model the interaction between public sector borrowing and public revenue constraints, studying the role of both public and external debt, using a simple non-stochastic, endogenous growth model with constant returns to scale. We show that the impact of debt limits differs between the case where public spending provides a *flow* of services that enhance productivity (for example, law enforcement) and the case in which the government invests in a productive *stock* of public capital (for example, transportation infrastructure). The model follows the production structure of endogenous growth models with public goods externalities proposed by Barro (1990) and Barro and Sala-i-Martin (1992). The effectiveness of public goods spending is not analyzed because it is already incorporated into the parameters of the aggregate production function. The impact of institutions on the effectiveness of public expenditure for raising welfare and economic growth is reflected in the marginal productivity of public goods in the aggregate production function as discussed in Barro (1990). These assumptions are useful in providing a well-defined role for public expenditure, yet they fall short of defining the optimal path of public debt, and the implications

² See Gordon and Li (2005) and Tanzi and Zee (2000) and the references therein.

³ See Slemrod (1995), Aizenman and Jinjara (2005) and Cukierman, Edwards and Tabellini (1992) and the references therein.

of debt overhang [i.e., under these assumptions, past debt does not constrain present investment and public expenditure]. This shortfall is the outcome of the assumed efficiency of the tax system, ignoring considerations associated with limited tax and debt capacities. While the Barro (1990) assumptions may fit OECD countries well, they fall short of characterizing the challenges facing developing countries. The contribution of our paper is explicitly adding limited debt and tax capacities and studying the impact of these constraints on the optimal pattern of debt and fiscal policy.

The relevance of limited tax capacity has been vividly illustrated by Baunsgaard and Keen (2005), who found that for middle-income countries, revenue recovery following trade liberalization has been about 50 cents for each dollar of lost trade tax revenue, and the revenue recovery has been weak in low-income countries. Nor is there much evidence that the mere presence of a value-added tax has made it easier to cope with the revenue effects of trade liberalization. If countries are not successful in filling the “tax void” caused by trade and financial liberalization, their debt carrying capacity may go down at the same time that debt dynamics worsen (because primary deficits are going up with the decline in taxes). If this is compensated for by cutting public capital expenditure, then future growth could also be hurt. In the same vein, recent research on debt intolerance and debt crises associated with deteriorating debt dynamics highlighted the heterogeneity of external debt capacities in developing countries.⁴ These results are consistent with the finding reported later in Tables 1-2 of this paper, estimating the impact of public debt on private and public investment, while controlling for other factors. There is a sizable adverse effect of public debt overhang on both types of investments.⁵

The contribution of our paper is in mapping the implications of heterogeneous tax and debt capacities on fiscal policy, growth and (constrained) optimal debt dynamics. The impact of a binding constraint on tax revenues as a share of gross domestic product on the growth rate of a country is analyzed in a non-stochastic endogenous growth model with externalities generated by public goods. The analysis is simplified by using a model in which production displays constant returns to non-human factors of production so that the optimal fiscal policy supports equilibrium with a constant growth rate. The maximal effective tax rate is binding in that it reduces welfare

⁴ See Reinhart, Rogoff and Savastano (2003).

⁵ The regressions also suggest that private (public) investment is significantly higher (lower) in countries with greater capital account openness. Both effects are of similar magnitude, but opposite direction, suggesting that the net effect of greater financial openness on aggregate investment/GDP is small.

from its level in the unconstrained optimum. A tighter tax constraint reflects weaker fiscal institutions and lowers the equilibrium growth rate. This implies that otherwise identical countries have lower growth rates if they have larger tax distortions evidenced, for example, by larger discrepancies between statutory and effective tax rates.

The usefulness of public debt in the presence of constraints on maximal tax revenues as a proportion of the economy is also considered in the endogenous growth model with public goods. The benefits of public sector borrowing differ according to whether the public goods that raise private factor productivities are current, such as *flow* spending on current law enforcement, or the accumulated of the stock of capital goods over time, such as infrastructure. In the *flow* case, access to borrowing does not have a social benefit associated with the tax revenue constraint. Borrowing may be welfare improving but for reasons other than the tax constraint such as tax smoothing in the presence of stochastic productivity shocks. With public investment in the *stock* of infrastructure that complements private inputs, access to financial markets allows the government to pursue more efficient fiscal policies. In these circumstances, the tax constraint imposes a trade-off between higher public investment in transition and lower permanent economic growth that does not appear with unconstrained lump-sum taxes: as higher debt is associated with lower long-run growth rates, higher investment in the transition implies faster convergence to a lower long-run growth rate.

Developing country governments face constraints on borrowing on both international and domestic capital markets due to sovereign risk. The debt limits imposed by creditors facing sovereign immunity are tighter than the traditional solvency constraint that relies on a debtor's commitment to repay up to the point of exhausting all its resources. We also consider how an exogenous debt limit interacts with a constraint on the average effective tax rate for a government to restrain optimal fiscal policy. The analysis shows that the effects of debt limits on economic growth depend on the nature of the externality associated with public goods spending. If public spending is restricted to current *flows* of services, then higher levels of public debt are associated with lower permanent growth rates. If public spending finances investment in the *stock* of public infrastructure, then relaxation of a debt limit can raise welfare by increasing growth rates in transition to the steady state. This conclusion is tempered by the observation that higher debt is associated with lower long-run growth rates which are reached in

finite time in the model economy. A country with a lower debt-to-GDP ratio will have a higher balanced growth rate than a country with identical characteristics and higher debt.

Section 2 outlines the association between the statutory tax rate and net tax revenue. Sections 4 and 5 map the association between tax constraints and growth in an endogenous growth model where current public goods or infrastructure are essential inputs. Section 6 identifies the association of debt constraints and growth. Sections 6 and 7 conclude by reviewing the implications of fiscal constraints for growth, and future empirical work. The derivation of key results is provided in the Appendix.

2. Tax enforcement, distortions and effective tax rates

The incentive effects of taxes are both well-known and controversial. However, these are reflected in varying rates of successful tax collection with the type of tax, definition of the tax base and means of payment. Tax evasion tends to be higher with lower enforcement expenditures and weaker penalties for non-compliance. A constrained optimal fiscal weighs the costs of increasing tax compliance against the benefits of collecting more revenue. The choice of tax bases and rates should also reflect the impact of these on overall revenues net of collection and enforcement costs. The strength of fiscal institutions will be evidenced directly by the ratio of actual tax receipts to potential (statutory) tax revenues for any given tax imposed and indirectly by the effective aggregate tax rate measured by the ratio of revenues in the economy.

Similarly, actual tax instruments affect production, consumption, savings and investment decisions at the margin so that increasing tax rates lower output, growth and welfare. Raising tax rates can reduce the tax base lowering total revenues. Less effective means of taxation can lead to wider variation in tax rates across different activities exacerbating the distortions created by taxation. More generally, it should be anticipated that weaker fiscal institutions lead to lower tax collections in proportion to output or to lower output to generate a given effective tax rate.

The implications of the incentive effects of taxation for both tax payment and economic efficiency used here is that total tax revenue displays decreasing returns in tax rates. Further, the relationship between tax revenues and output depends on the strength of fiscal institutions in particular and quality of governance in general for a particular economy.

We model the relationship between tax rates and tax revenues using a simple function for illustration. With distortionary taxes or imperfect tax compliance, the tax base denoted decreases

with the tax rate and is denoted $z(\tau^s)$ where y is the aggregate tax base and τ^s measures statutory tax rates.⁶ Total tax revenues are given by $\tau^s z(\tau^s)$ and are maximized for the rate satisfying

$$\frac{d}{d\tau^s}(\tau^s z(\tau^s)) = \tau^s z'(\tau^s) + z(\tau^s) = 0.$$

The interior solution to this condition generates a maximal statutory tax rate and total tax revenue. We use this result to calculate a maximal effective tax rate. Although this approach applies to both tax distortions and costly tax enforcement, we restrict it in our analytical model to the case of declining tax compliance with increasing tax rates. This allows us to use a simple model in which aggregate output is independent of the tax rate and tax revenues are a proportion of GDP given by the effective tax rate. The effective tax rate has a maximum which is country specific and sensitive to fiscal institutions. The analysis, however, is fully consistent with interpreting the tax base, z , as aggregate GDP and the reduction in tax revenues with marginal tax rates as due to distortions that reduce the output of the economy for given factor supplies.

3. Public goods, tax constraints and growth

The impact of public expenditures on economic growth is formalized in a simple endogenous growth model following Barro (1990) and Barro and Sala-i-Martin (1992). Non-rival public goods increase the productivity of capital and labor employed by competitive firms. The production function of a single enterprise is given by

$$y_j = Ak_j^\alpha \ell_j^{1-\alpha} G^{1-\alpha}$$

where G represents aggregate public goods supply, and k_j and ℓ_j represent a typical firm j 's input of capital and labor, respectively. The parameter, A , is a constant. Each firm takes public goods

⁶ Endogenous tax capacity can be modeled along the line of Cukierman, Edwards and Tabellini (1992), which explains the obstacles to tax reforms in polarized countries, characterized by political instability. They focused on the case where fiscal revenue can be raised by taxes associated with collection costs [income taxes], and implicit taxes where the collection cost is zero [inflation tax]. They assumed implementation lags – the present policy maker determines the efficiency of the tax system next period. This implies that the choice of the tax system efficiency may be strategic – the current policy maker may choose an inefficient future tax system in order to constrain the fiscal revenue available to future policy makers. This prevents future policy makers from spending in ways that are viewed as inferior from the vantage point of the present policy maker. Their model can be extended to deal other fiscal issues pertinent for developing countries, like endogenous tax evasion, optimal enforcement of the “hard to collect” taxes, etc. [see Aizenman and Jinjarak (2005)].

supply as exogenous so that the production function displays constant returns to scale with respect to private inputs. The aggregate production is given by

$$Y = AK^\alpha L^{1-\alpha} G^{1-\alpha}$$

where uppercase denotes economy-wide aggregates. Aggregate production displays increasing returns to scale, but labor supply growth is exogenous. L is assumed constant and normalized to one for simplicity. The model is written in per capita terms from here. The inclusion of constant returns to private inputs means that there are no profits since payments to capital and labor exhaust the value of output. The benefits of public goods provision accrue to owners of capital and labor.

In the aggregate, public expenditures can finance either non-storable public goods or investment in public infrastructure. Examples of growth-enhancing recurrent public spending include expenditures to maintain the rule of law, enforce private contracts and property rights, as well as education, health and other social welfare spending. In the production function, G represents the services of both types of public goods. However, recurrent public spending and public infrastructure investment are treated separately in the analysis of tax distortions and borrowing constraints. We begin by considering only non-storable public goods.

The objective function for finding optimal policies with fiscal policy constraints is given by the utility for a representative household given by

$$U_t = \int_t^{\infty} \frac{c_s^{1-\sigma}}{1-\sigma} e^{-\rho(s-t)} ds$$

where c_s denotes consumption. A constant elasticity of substitution utility function is assumed to simplify the algebra by allowing constant growth rates in equilibrium. Without restrictions on the government's access to lump-sum financing of public expenditures, the optimal growth path is a balanced growth path, as shown by Barro (1990). The aggregate resource identity written in per capita terms,

$$(1) \quad \dot{k} = Ak_t^\alpha g_t^{1-\alpha} - g_t - c_t,$$

and the restriction that the capital stock cannot be negative complete the constraints for finding the optimum. In the unconstrained optimum, the growth rate is constant and the marginal productivity of public goods spending equals the marginal social opportunity cost of public goods, one. The growth rate is given by

$$\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{g}}{g} = \frac{r - \rho}{\sigma} \equiv \gamma,$$

where

$$r = \alpha A k^{\alpha-1} g^{1-\alpha} \quad \text{and} \quad g = (1-\alpha)^{\frac{1}{\alpha}} A^{\frac{1}{\alpha}} k.$$

The optimum growth rate is

$$\gamma = \frac{1}{\sigma} \left(\frac{\alpha}{1-\alpha} (1-\alpha)^{\frac{1}{\alpha}} A^{\frac{1}{\alpha}} - \rho \right),$$

and consumption is a constant share of wealth given by

$$c = A^{1/\alpha} (1-\alpha)^{\frac{1}{\alpha}} \left(\frac{\alpha}{1-\alpha} \right) k.$$

The balanced growth path is attained immediately in the unconstrained optimal fiscal policy. In a decentralized economy with lump-sum taxes available to the government, public debt is irrelevant. In our case, the government's capacity to tax is limited so that the optimum may not be attainable and borrowing might play a role in the transition to a balanced growth path.

Adding an upper bound on effective tax rates for the case of recurrent public goods spending introduces the possibility that the government borrows. The flow budget identity for the public sector is given by

$$(2) \quad \dot{d} = r_t d_t - \tau_t A k_t^{\alpha} g_t^{1-\alpha} + g_t,$$

where outstanding public debt per capita is denoted d_t and the effective tax rate expressed as a proportion of GDP is denoted τ_t .

We first consider the case in which the government runs a continuously balanced budget and has no outstanding debt. The upper bound constraint on the effective tax rate is binding if the maximal share of public expenditures in GDP, which is $\bar{\tau}$, is less than the efficient ratio of public spending in GDP,

$$\frac{g}{y} = (1-\alpha)^{\frac{1}{\alpha}} A^{\frac{1}{\alpha}} \frac{k}{y} = (1-\alpha).$$

Imposing the constraint that the public sector primary balance is non-negative,

$$\bar{\tau} A k^{\alpha} g^{1-\alpha} - g \geq 0,$$

when $\bar{\tau} < 1 - \alpha$, the constrained optimal growth path is a balanced growth path with a ratio of public goods spending given by

$$g = (\bar{\tau}A)^{\frac{1}{\alpha}} k,$$

and a growth rate equal to

$$\gamma = \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{g}}{g} = \frac{1}{\sigma} \left(\alpha A^{\frac{1}{\alpha}} \bar{\tau}^{\frac{1-\alpha}{\alpha}} - \rho \right)$$

which is below the unconstrained optimal growth rate. Equilibrium output and consumption are given by

$$y = A^{\frac{1}{\alpha}} \bar{\tau}^{\frac{1-\alpha}{\alpha}} k \quad \text{and} \quad c = A^{\frac{1}{\alpha}} \bar{\tau}^{\frac{1}{\alpha}} \left(\frac{1}{\bar{\tau}} - 1 \right) k .^7$$

Relaxation of the constraint on effective tax rates raises the share of public expenditure in GDP and the growth rate of GDP. It also lowers the capital-to-output ratio while increasing the marginal productivity of capital. The ratio of consumption to output and welfare rise with $\bar{\tau}$.

With outstanding public debt, a balanced growth rate path will keep the debt-to-output ratio constant. The public sector budget identity is satisfied in a balanced growth equilibrium if

$$\frac{\dot{d}}{d} = \gamma = r - \bar{\tau} \frac{Ak^{\alpha} g^{1-\alpha}}{d} + \frac{g}{d}$$

which solves for the share of public goods spending in output as

$$(3) \quad \frac{g}{y} = (\gamma - r) \frac{d}{y} + \bar{\tau},$$

where r is the equilibrium interest rate, $r = \alpha Ak^{\alpha-1} g^{1-\alpha}$. In a competitive equilibrium with no restrictions on asset substitution by households, the interest rate on public debt is equal to the rate of return to capital. The ratio of public expenditure to output declines with an increase in the debt-to-GDP ratio.⁸ An increase in public debt as a share of GDP unambiguously lowers the return to capital and the balanced rate of growth.

Consider whether the government should optimally borrow to finance recurrent public spending with an upper bound constraint on tax revenues as a share of GDP. By borrowing, it

⁷ The derivations of the equilibrium solutions discussed in the text are explained in the Appendix.

⁸ Since the marginal productivity of capital and the growth depend on g/k , this can be verified by differentiating equation (3) with respect to the debt-to-GDP ratio, d/y . Dynamic efficiency implies that the interest rate exceeds the growth rate for all values of σ .

could provide a higher level of spending on public goods complementing capital accumulation early on. This would come at the cost of lowering the share of public expenditures in GDP later reducing the future growth rate of the economy. When the upper bound on the share of tax revenues in GDP does not bind, Ricardian equivalence holds. With the upper bound, raising current public goods spending by borrowing lowers future public expenditure from an already suboptimal level. The answer is intuitive for this model economy because tax revenues grow at the same rate as output and consumption in a balanced growth path. If the government borrows today and repays tomorrow, public goods spending grows at a different rate than capital and consumption violating the first-order condition for utility maximization between today and tomorrow.

The optimal borrowing strategy of the government keeps the share of public spending in GDP constant over time. This implies that the optimal policy given initial outstanding debt maintains a constant debt-to-output ratio. Either paying down the debt as a share of GDP or borrowing to raise temporarily public goods spending reduces welfare. The government has no incentive to borrow to finance recurrent public spending in the deterministically growing economy. Debt lowers both welfare and the equilibrium rate of growth. Current non-durable public goods should be financed from current revenues in the deterministic economy. Adding transitory productivity shocks to the economy would introduce conventional tax-smoothing motives for cyclical public sector deficits and surpluses (following Barro (1979)), but not trend borrowing.

The optimal growth path in the presence of a binding constraint on tax revenues when the government can issue debt is found by maximizing household welfare subject to the resource identity (1) and the public sector budget identity (2) imposing the conventional solvency constraint.⁹ The equilibrium growth rate is given by

$$\gamma = \frac{1}{\sigma} \left(\alpha A^{\frac{1}{\alpha}} \omega^{\frac{1-\alpha}{\alpha}} - \rho \right)$$

⁹ The Appendix contrasts two possible cases: ours, in which firms do not internalize the increase in public goods spending created by private investment through higher tax revenues, and the case in which the private return to capital internalizes this effect. The second case suggests that the constrained optimal fiscal policy could include a capital income subsidy, but the Appendix shows how the optimal subsidy is zero when it must be financed from constrained tax revenues.

where ω is the solution to equation (3) for the ratio of public spending to output, g/y . For zero inherited debt, this is the maximal tax rate, $\bar{\tau}$.

In summary, when the flow of public expenditure raises productivity, the government should not borrow to finance public spending in this economy. Existing public debt always lowers welfare and the growth rate. With outstanding ex ante public debt, the optimal fiscal policy should keep the debt-to-GDP ratio constant in the constant returns to scale economy with or without a binding constraint on tax revenues as a share of GDP.

4. Public infrastructure, tax constraints and growth

Government expenditures on public sector capital can also raise productivity and welfare. With investment in public infrastructure, public sector borrowing to finance the accumulation of public capital goods may allow the economy to reach a long-run optimal growth path faster. This turns out to be true in the endogenous growth model.

As above, we begin with the unconstrained economy by substituting public infrastructure for exhaustive public spending and characterize the optimum with lump-sum taxes constrained only by available resources. The resource identity (the equation of motion for private capital) becomes

$$(4) \quad \dot{k} = Ak_t^\alpha g_t^{1-\alpha} - x_t - c_t,$$

and the growth equation for public capital is

$$(5) \quad \dot{g} = x_t,$$

where public investment is denoted x_t and depreciation of either type of capital is left out for simplicity.

The optimum is a balanced growth path in which the ratio of public capital to private capital satisfies

$$\frac{g}{k} = \frac{1-\alpha}{\alpha}.$$

The growth rate of output, consumption and public and private capital in the optimum is

$$\gamma = \frac{1}{\sigma} \left(A \alpha^\alpha (1-\alpha)^{1-\alpha} - \rho \right),$$

and public investment as a share of GDP is

$$\frac{x}{y} = \gamma \frac{g}{y} = \gamma A^{-1} \left(\frac{1-\alpha}{\alpha} \right)^\alpha.$$

The optimum requires an immediate shift from any initial ratio of public to private capital, \bar{g}_0 / \bar{k}_0 , to the ratio $g/k = (1-\alpha)/\alpha$. This can be achieved without public debt through a lump-sum tax equal to

$$T_0 = (1-\alpha)\bar{k}_0 - \alpha\bar{g}_0.$$

This amounts to a capital levy, but the optimum can also be supported by issuing public debt to pay for the initial purchase of private capital for public use (due to Ricardian equivalence). The required debt issue (per capita) is $d_0 = T_0$. If the initial ratio of public capital to private capital exceeds the optimal ratio, then the government sells capital acquiring public credit in the amount $-T_0$. If lump-sum taxes are levied in the amount

$$(6) \quad T_t = (r-\gamma)\frac{d_0}{y_0}y_t + x_t = (r-\gamma)\frac{d_0}{y_0}A\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}k_t + \gamma\left(\frac{1-\alpha}{\alpha}\right)k_t,$$

then the optimal growth equilibrium is achieved with a constant public debt-to-GDP ratio, d_0/y_0 . In this model, investment is reversible. If private investment is irreversible, then the optimum includes a transition path that converges to the balanced growth path in finite time. We assume reversible investment to simplify the solution for the optimal debt-to-output ratio without the extra algebra of transition dynamics.

Imposing the upper bound on tax revenues as share of output we also rule out capital levies. The constraint on tax revenues binds if the government cannot raise the revenues given in equation (6),

$$T_t > \bar{\tau}y_t.$$

In this case, solving for the constrained optimum leads to a trade off between issuing debt to increase the initial ratio of public infrastructure to private capital and using tax revenue to pay for continued public infrastructure investment if the initial public to private capital stock ratio is less than the long-run ratio. Because tax revenues are proportionate to output, output displays constant returns to capital and utility displays constant elasticity of substitution, the solution is again a balanced growth path. The equilibrium debt-to-GDP ratio will be constant so that public debt grows at the same constant rate as output, public capital and private capital. This is verified by maximizing the representative household's utility with respect to consumption and public and

private investment given equations (3), (4) and (5) plus the flow budget identity for the public sector given by

$$(7) \quad \dot{d} = r_t d_t - \bar{\tau} A k_t^\alpha g_t^{1-\alpha} + x_t$$

and the conventional solvency constraint.

The constrained optimal growth path is found by first solving for the constant ratio of public sector capital to output from the public sector budget constraint for a balanced growth path. This is the analog of equation (6) in the presence of an upper bound on tax revenues,

$$\bar{\tau} y_t = (r - \gamma) \frac{d_0}{y_0} y_t + x_t = (r - \gamma) \frac{d_0}{y_0} y_t + \gamma g_t.$$

This is rewritten as

$$\bar{\tau} = (r - \gamma) \left(\frac{g_t}{y_t} - \frac{\bar{g}_0}{y_0} \right) + \gamma \frac{g_t}{y_t},$$

because the ratio of public infrastructure to output is constant so that debt-to-GDP ratio satisfies

$$\frac{d_0}{y_0} = \frac{g_t}{y_t} - \frac{\bar{g}_0}{y_0}.$$

The solution for the constant ratio $\omega = g_t / y_t$ is determined by

$$(8) \quad \bar{\tau} = (r - \gamma) \left(\omega - \frac{\bar{g}_0}{y_0} \right) + \gamma \omega$$

where $r = \alpha A^\alpha \omega^{\frac{1-\alpha}{\alpha}}$ and $\gamma = \frac{1}{\sigma} \left(\alpha A^\alpha \omega^{\frac{1-\alpha}{\alpha}} - \rho \right)$. Equilibrium output, consumption and public

investment share in output are given in terms of the solution ω to equation (8) by

$$y = A^\alpha \omega^{\frac{1-\alpha}{\alpha}} k, \quad c = A^\alpha \omega^{\frac{1-\alpha}{\alpha}} \left(\frac{1}{\omega} - 1 \right) k \quad \text{and} \quad \frac{x}{y} = \gamma \omega.$$

When the tax revenue constraint binds, the solution for the public infrastructure capital to output from equation (8) satisfies

$$\omega < A^{-1} \left(\frac{1-\alpha}{\alpha} \right)^\alpha.$$

In this case, the initial ratio of public sector capital to output can be either less or greater than the steady-state ratio, ω . It is easy to see that when the initial ratio of public sector capital to private capital is smaller than the sustainable balanced growth ratio that the government borrows to raise

the share of public infrastructure in the aggregate capital stock. If the initial ratio of public sector capital to GDP exceeds the solution to equation (8), then the constrained optimal policy requires the government to sell public capital in exchange for public credit in the amount

$$-\frac{d_0}{y_0} = \frac{\bar{g}_0}{y_0} - \frac{g_t}{y_t}.$$

The government optimally sells capital for bonds if the initial ratio of public infrastructure to output exceeds the solution ω and sells bonds for capital if the initial ratio is less than the solution ω . The government sells public capital in this case because the constraint on tax revenues as a share of output means that the initial ratio of public capital to output cannot be sustained in the long run. The revenue constraint does not allow sufficient public investment to keep the ratio of public infrastructure to output equal to its initial value. If the government did not sell public capital and invest the proceeds in debt reduction or credit accumulation when the infrastructure to output ratio cannot be sustained, then the ratio of public capital to private capital would decline over time as the economy converged to a balanced growth path. The economy could only converge to a lower ratio of public infrastructure to output than the solution ω . The long run growth rate is higher in the constrained optimum because the interest on the financial savings of the government pays for additional public investment in infrastructure.

In summary, with a binding constraint on proportional tax rates the optimal policy for the government is to purchase or sell public capital to immediately attain the sustainable ratio of public capital to private capital. A government purchase of capital should be financed by an issue of debt, while the proceeds of a government sale of capital should be used to increase public credit. The debt-to-output ratio should remain constant thereafter.

5. Debt constraints and growth

The endogenous growth model with productivity-enhancing public expenditures reveals some basic results for the role of public debt in optimal fiscal policy with constraints on the government's capacity to raise tax revenues. When recurrent, exhaustive public goods spending raises the productivity of capital and labor, public borrowing is not part of an optimal policy. In the presence of a binding constraint on tax revenues as a share of GDP, any pre-existing government debt reduces the equilibrium growth rate of the economy and the welfare of the

representative household. The optimal policy is to keep the debt-to-GDP ratio constant if the government inherits public debt.

In the case that public sector infrastructure increases the productivity of labor and private capital, public spending on infrastructure investment may be debt financed in an optimal fiscal policy. In the endogenous growth economy without capital stock adjustment costs, the government should borrow once to raise the initial ratio of public sector capital to private sector capital if a higher steady-state ratio can be sustained in the presence of the upper bound on the effective tax rate for the economy. As pointed out above, the optimal policy can also be an initial privatization of public capital increasing public credit to reach the long-run sustainable ratio of public infrastructure capital to private capital. With capital stock adjustment costs, these statements apply to a transition of finite duration rather than an instantaneous stock shift. These statements apply to external national debt just as well as to the debt of the public sector.

Consider an exogenous constraint on the outstanding debt as a share of GDP. With only current public goods spending (the first model), relaxing this constraint is of no benefit. Borrowing to finance public goods spending is not efficient. With productive public sector infrastructure, a constraint on indebtedness can bind so that the government cannot issue sufficient debt to finance the constrained optimal public infrastructure to output ratio defined by equation (8).

In terms of equation (8), a limit on the debt-to-output ratio, $\varphi \geq \frac{d_t}{y_t}$, is binding if $\varphi > \omega - \frac{\bar{g}_0}{y_0}$, where ω is the constrained optimal public infrastructure to output ratio satisfying equation (8). In the presence of a binding borrowing constraint, the optimal policy requires the government to issue debt up to the limit immediately. The debt-to-GDP ratio remains at the limit so that the public sector flow budget identity, equation (7), implies that the public sector investment rate is given by the condition,

$$(9) \quad \bar{\tau} = (r_t - \gamma_t)\varphi + \frac{x_t}{y_t},$$

where the interest rate, growth rate and infrastructure investment rate are not necessarily constant. At date zero, the government increases the public capital to output ratio by the amount

$$\frac{g_0}{y_0} = \varphi - \frac{\bar{g}_0}{y_0}$$

where \bar{g}_0 is the ex ante stock of public sector capital and g_0 is the constrained optimal level of public sector capital beginning at date 0. Since equation (8) cannot be satisfied at date 0, the share of public sector investment in GDP, x_0 / y_0 , is higher than in the absence of a binding debt limit by equation (9).

The economy must pass through a transition path with lower initial public infrastructure and higher public investment shares in output in the presence of the exogenous debt limit. Along the transition path, the constrained optimal policy obeys the equations of motion for private and public capital given in (2) and (3), the fiscal constraint given by equation (9) and the Euler condition for consumption,

$$\frac{\dot{c}}{c} = \frac{1}{\sigma} \left(\alpha A \left(\frac{g_t}{k_t} \right)^{1-\alpha} - \rho \right).^{10}$$

The path converges to the balanced growth path that satisfies the condition

$$\bar{r} = (r - \gamma)\varphi + \gamma \frac{g}{y}$$

where $r = \alpha A^{\frac{1}{\alpha}} \left(\frac{g}{y} \right)^{\frac{1-\alpha}{\alpha}}$ and $\gamma = \frac{1}{\sigma} \left(\alpha A^{\frac{1}{\alpha}} \left(\frac{g}{y} \right)^{\frac{1-\alpha}{\alpha}} - \rho \right)$. The steady-state ratio of public capital to

output, g/y , is higher when the debt limit is binding for the given bound on the share of tax revenues. As in other constant returns to scale in accumulable factors growth models, the transition path converges to the balanced growth path in finite time.

The welfare cost of the exogenous debt limit arises from the intertemporal distortion to the public infrastructure-to-output ratio. The debt constraint lowers this ratio initially which lowers welfare even though the long-run growth rate is higher under the constraint. Relaxing the debt limit during the transition phase necessarily raises welfare because it allows the current public capital-to-output ratio and the steady-state public capital-to-output ratio to move towards each other reducing the intertemporal distortion. However, once the economy converges to balanced growth relaxing the constraint can have no effect on welfare or growth because the

¹⁰ In the transition path, the growth rate of consumption is not equal to the growth rate of output denoted by γ_t .

public capital-to-output ratio is at a sustainable long-run level. That is, the gains to borrowing more to increase the public to private capital ratio are past, gone with the transition path.

The assumption that an exogenous debt limit exists can be motivated by indirect sanctions that support lending and repayment in the presence of sovereign risk. In many applications, sovereign default is assumed to result in a proportional exogenous reduction in national income. Lending is then constrained by the exogenous present value of such penalties for default. In the model of endogenous growth, this notion is simply expressed by the given debt limit, ϕy . The analysis shows that such limitations on borrowing due to sovereign risk can reduce welfare if both of two conditions hold. These are that there is public investment in infrastructure that enhances private factor productivity and the debt limit is binding given the current infrastructure-to-GDP ratio and the constraint on tax revenues in GDP.

Some theoretical models of sovereign debt derive the incentives to repay from the gains from borrowing itself (for example, Eaton and Gersovitz (1981) and Kletzer and Wright (2000)). If stochastic productivity shocks are added to the constant returns to scale production function, then borrowing to smooth aggregate consumption can be efficient even if only the flow of public expenditures raises productivity. If the Kletzer and Wright (2000) model of credible sanctions were adopted, borrowing to finance either recurrent public expenditures or investment in public sector infrastructure would be limited by the ongoing gains from smoothing against stochastic productivity shocks. In that case, the debt limit will be endogenous in a constrained efficient equilibrium. Relaxation of the constraint would require increasing opportunities for the sovereign to commit to repay.

In the deterministic model of public goods and growth, access to financial or commodity market trade does not provide an incentive to repay either foreign debt or domestic public debt. The costs of default need to be unrelated to the gains from borrowing implying the assumed exogenous debt limit. This means that increasing the costs of default can bring about an increase in welfare with productivity externalities generated by investment in public sector infrastructure. But, in the case that only current expenditure raises productivity additional borrowing strictly reduces welfare. Relaxing a constraint on borrowing is undesirable in that case.

6. Implications of fiscal constraints for growth

Two constraints on fiscal policy appear in the endogenous growth model with public goods. The upper bound on tax revenues as a share of domestic production or national income varies widely across countries. Improvement in fiscal institutions can raise effective tax rates by increasing revenue collection or reducing the distortions to resource allocation due to taxation. In the model a more efficient tax system is reflected by a higher bound on the effective tax rate in output. Raising the upper bound on the effective tax rate increases the growth rate of the economy permanently along with representative household welfare. Countries with stronger fiscal institutions should have higher shares of public spending in output, higher ratios of public sector infrastructure to output and higher growth rates of per capita GDP than those with the same social discount rates and access to technologies but less efficient fiscal institutions. The model explains how measures of the inefficiency of tax systems proposed by Aizenman and Jinjark (2005) and others should help explain persistent differences in growth rates across countries.

The other fiscal constraint introduced in this model is the exogenous debt limit. An important conclusion of the model, however, is that the relative significance of recurrent public spending to public infrastructure investment for enhancing factor productivity determines the impact of debt limits on the rate of economic growth. The welfare benefits of public sector or international borrowing depend on the relative importance of public sector capital goods for the impact of public goods spending on productivity growth. Higher indebtedness always lowers the growth rate if recurrent public expenditures enhance productivity growth by reducing public goods spending in the amount of the debt service required.

Even if only public sector infrastructure spending raises private factor productivity, the model economy does not simply imply that relaxing a bound on the debt-to-GDP ratio raises economic growth. If the economy is not initially in a balance growth path, then relaxing a binding debt limit will raise welfare and the growth rate of the economy. However, if the economy is already in balanced growth equilibrium, then its growth rate is negatively related to the debt-to-GDP ratio. In balanced growth, a country with a higher debt-to-GDP level will have a lower growth rate than a less indebted but otherwise identical country without regard to the relative importance of durable and non-durable public goods for raising aggregate productivity. Relaxing a debt limit due, for example, to sovereign risk can improve welfare for an

infrastructure poor country in transition by substituting more valuable current productivity growth for long-run productivity growth.

7. Conclusion and future empirical work

The above analysis maps the implications of fiscal constraints.

If only current flow of public spending influencing factor productivity, we find that:

- A country with a lower maximal effective tax rate (smaller $\bar{\tau}$) will have a lower equilibrium growth rate (ceteris p.) (given the bound is binding).
- A country with higher outstanding public debt will have a lower equilibrium growth rate. Debt reduction leads to a higher growth rate. Borrowing is not welfare improving in the deterministic model (in a stochastic model, it can be for conventional tax-smoothing reasons).
- As in existing models without our addition, a country with a lower rate of discount will have a higher growth rate. Also, the public-to-private capital ratio will only depend on the tax constraint and the existing debt-to-GDP ratio.

If public sector capital is an essential input for aggregate GDP, we find that

- Borrowing is welfare improving if the initial public-to-private capital ratio is not equal to the sustainable steady-state ratio. The sustainable steady-state ratio is endogenous to the initial public-to-private capital ratio, the maximal effective tax rate ($\bar{\tau}$) and any exogenous debt limit (say, due to sovereign risk).
- In the case that an exogenous debt limit does not bind (conventional solvency applies, however), a lower initial public-to-private capital ratio or a lower effective tax rate (either c.p.) leads to a lower equilibrium public-to-private capital ratio and a lower equilibrium growth rate.
- If an exogenous (tighter) debt limit binds, then the long-run growth rate is higher than if it did not. A tighter debt limit implies a higher steady-state growth rate, but a longer transition with a lower growth rate. That means that if the economy is not in the steady state, relaxation of the debt limit will increase the public-to-private capital ratio and growth rate immediately at the expense of steady-state growth.

- With either a binding exogenous debt limit or solvency constrained borrowing, a more patient country will have a higher steady-state growth rate but a lower steady-state public-to-private capital ratio.

In future work we plan to evaluate empirically these implications.

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Appendix

A. Productive current public goods expenditure

Begin by considering the problem of maximizing household welfare with a continuously balanced public sector budget and revenue constraint. The government maximizes

$$(A1) \quad U_t = \int_t^{\infty} \frac{c_s^{1-\sigma}}{1-\sigma} e^{-\rho(s-t)} ds$$

with respect to c_s and g_s subject to the resource identity,

$$(A2) \quad \dot{k} = Ak_t^\alpha g_t^{1-\alpha} - g_t - c_t$$

and the public sector revenue constraint,

$$(A3) \quad \bar{\tau}Ak^\alpha g^{1-\alpha} - g \geq 0,$$

along with non-negativity constraints for c_s , k_s , and g_s . The necessary conditions for an optimum include

$$(A4) \quad \sigma \frac{\dot{c}}{c} = \left(1 - \bar{\tau} \frac{\lambda}{q}\right) \alpha Ak_t^{\alpha-1} g_t^{1-\alpha} - \rho$$

$$(A5) \quad \frac{\lambda}{q} = \frac{(1-\alpha)Ak^\alpha g^{-\alpha} - 1}{(1-\alpha)\bar{\tau}Ak^\alpha g^{-\alpha} - 1}$$

$$(A6) \quad \bar{\tau}Ak^\alpha g^{1-\alpha} = g$$

$$(A7) \quad \frac{\dot{k}}{k_t} = A \left(\frac{g_t}{k_t}\right)^{1-\alpha} - \frac{g_t}{k_t} - \frac{c_t}{k_t}$$

and the transversality condition

$$(A8) \quad \lim_{t \rightarrow \infty} q_t k_t e^{-\rho t} = 0,$$

where λ is the multiplier for the budget constraint and q is the costate variable for k . The solution path is a balanced growth path with a ratio of public goods spending to capital,

$$(A9) \quad \frac{g}{k} = \left(\bar{\tau}A\right)^{\frac{1}{\alpha}},$$

and growth rate,

$$(A10) \quad \frac{\dot{k}}{k} = \frac{\dot{c}}{c} = \frac{1}{\sigma} \left(\left(\frac{1-\bar{\tau}}{\bar{\tau}} \right) (\bar{\tau}A)^{\frac{1}{\alpha}} - \rho \right).$$

This growth path can be sustained as a competitive equilibrium if the household rate of interest equals

$$(A11) \quad r = \left(\frac{1 - \bar{\tau}}{\bar{\tau}} \right) (\bar{\tau}A)^{\frac{1}{\alpha}}$$

where

$$(A12) \quad \frac{\dot{k}}{k} = \frac{\dot{c}}{c} = \frac{1}{\sigma} (r - \rho).$$

This interest rate is the post-tax marginal productivity of capital when the endogeneity of government expenditures to income, $g_t = \bar{\tau}y_t$, is taken into account. This interest rate and growth rate are the same expressions as for the unconstrained optimum but with the share of public goods spending in output set equal to $\bar{\tau}$ instead of $(1 - \alpha)$.

Implementing this equilibrium when firms and households take public goods spending as exogenous requires a proportionate capital income subsidy because the private marginal productivity of capital is

$$(A13) \quad r = \alpha A k^{\alpha-1} g^{1-\alpha} = \frac{\alpha}{\bar{\tau}} (\bar{\tau}A)^{\frac{1}{\alpha}} < \frac{1 - \bar{\tau}}{\bar{\tau}} (\bar{\tau}A)^{\frac{1}{\alpha}}.$$

The inequality results from the positive externality generated by public goods. The optimization exercise above imposes a constraint on public goods spending but does not take account of the fiscal cost of paying a capital income subsidy. The subsidy, s , is added to the balanced budget constraint as

$$(A14) \quad \bar{\tau}A k^{\alpha} g^{1-\alpha} = g + sk.$$

Adding the cost of the capital income subsidy to the public sector budget leads to solutions for the growth rate and interest rate given by

$$(A15) \quad \frac{\dot{k}}{k} = \frac{\dot{c}}{c} = \frac{1}{\sigma} \left(\left(\frac{1 - \bar{\tau}}{\omega - \bar{\tau}(1 - \alpha)} \right) \alpha (\omega A)^{\frac{1}{\alpha}} - \rho \right)$$

$$(A16) \quad r = \left(\frac{1 - \bar{\tau}}{\omega - \bar{\tau}(1 - \alpha)} \right) \alpha (\omega A)^{\frac{1}{\alpha}} = \frac{\alpha}{\omega} (\omega A)^{\frac{1}{\alpha}} + s$$

where ω is the share of public goods spending in income (that is, $g = \omega y = \bar{\tau}y - sk$).

Household welfare can now be calculated directly as a function of ω after integrating equation (A15) for consumption. Differentiating household welfare with respect to ω is

straightforward and shows that household welfare increases in ω up to the limit $\omega = \bar{\tau}$. The reason is simple: the marginal productivity of public goods spending exceeds the marginal productivity of capital with a binding upper bound on public expenditure. Therefore, the optimal fiscal policy given a binding constraint on the average tax rate ($\bar{\tau} < 1 - \alpha$) involves no capital income subsidy so that the equilibrium growth rate is given by

$$(A17) \quad \gamma = \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{g}}{g} = \frac{1}{\sigma} \left(\alpha A^{\frac{1}{\alpha}} \bar{\tau}^{\frac{1-\alpha}{\alpha}} - \rho \right).$$

Substitution using the resource identity yields consumption as reported in the text,

$$(A18) \quad c = A^{\frac{1}{\alpha}} \bar{\tau}^{\frac{1}{\alpha}} \left(\frac{1}{\bar{\tau}} - 1 \right) k.$$

The optimal level of public borrowing can be found by maximizing utility again after replacing the balanced public sector budget constraint with the flow budget identity for the government,

$$(A19) \quad \dot{d} = r_t d_t - \tau_t A k_t^\alpha g_t^{1-\alpha} + g_t.$$

The first-order conditions for a constrained optimal fiscal policy given that firms and households take public goods spending as exogenous now include

$$(A20) \quad \sigma \frac{\dot{c}}{c} = r_t - \rho$$

$$(A21) \quad \frac{\dot{k}}{k_t} = A \left(\frac{g_t}{k_t} \right)^{1-\alpha} - \frac{g_t}{k_t} - \frac{c_t}{k_t}$$

and

$$(A22) \quad \frac{\dot{d}}{d} = r_t - \frac{\bar{\tau} A k_t^\alpha g_t^{1-\alpha}}{d_t} + \frac{g_t}{d_t}$$

where

$$r_t = \alpha A k_t^{\alpha-1} g_t^{1-\alpha}.$$

The equilibrium path is a balanced growth path with growth rate γ ,

$$\gamma = \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{g}}{g} = \frac{\dot{d}}{d} = \frac{1}{\sigma} (r - \rho),$$

where the ratio g/k is determined by the initial public debt to output ratio, d/y , as

$$(A23) \quad \frac{\dot{d}}{d} = \gamma = \alpha A \left(\frac{g}{k} \right)^{1-\alpha} + \left(A^{-1} \left(\frac{g}{k} \right)^{\alpha} - \bar{\tau} \right) \frac{y}{d}.$$

Again denoting $\omega = g / y$, the equilibrium growth rate is

$$(A24) \quad \gamma = \frac{1}{\sigma} \left(\frac{\alpha}{\omega} (\omega A)^{\frac{1}{\alpha}} - \rho \right)$$

and the share of public goods spending in output is determined by

$$(A25) \quad r - \frac{1}{\sigma} (r - \rho) = (\bar{\tau} - \omega) \frac{y}{d}$$

for $r = \frac{\alpha}{\omega} (\omega A)^{\frac{1}{\alpha}}$.

B. Productive public infrastructure

With productive public capital, the constrained optimum for the government is found by maximizing household utility given by (A1) with respect to consumption, c_s , and public investment, x_s , subject to the resource identity,

$$(A26) \quad \dot{k} = A k_t^{\alpha} g_t^{1-\alpha} - x_t - c_t$$

the accumulation equation for public infrastructure,

$$(A27) \quad \dot{g} = x_t,$$

and the public sector budget identity,

$$(A28) \quad \dot{d} = r_t d_t - \tau_t A k_t^{\alpha} g_t^{1-\alpha} + x_t$$

along with non-negativity constraints along with non-negativity constraints for c_s , x_s , k_s and g_s .

As before, maximizing utility with the upper bound on the tax rate leads to a solution in which the household rate of interest takes account of the endogenous public investment as for the flow public goods case. The necessary conditions for such an optimum include

$$(A29) \quad -\frac{\dot{q}_1}{q_1} = \sigma \frac{\dot{c}}{c} = \left(1 - \bar{\tau} \frac{q_3}{q_1} \right) \alpha A k_t^{\alpha-1} g_t^{1-\alpha} - \rho$$

$$(A30) \quad -\frac{\dot{q}_2}{q_2} = \frac{q_1}{q_2} \left(1 - \bar{\tau} \frac{q_3}{q_1} \right) (1 - \alpha) A k_t^{\alpha} g_t^{-\alpha} - \rho$$

$$(A31) \quad -\frac{\dot{q}_3}{q_3} = r_t - \rho$$

$$(A32) \quad q_1 = q_2 + q_3$$

where q_1 , q_2 and q_3 are the costate variables associated with k_s , g_s and d_s , respectively. The equilibrium that satisfies the three standard transversality conditions and the initial conditions is a balanced growth path in which the rate of interest exceeds the private marginal productivity of capital. Equations (A29) through (A32) can be used to eliminate the costate variables by setting the growth rates of the three costate variables equal and solving for the balanced growth path. As before, when the effect of increased private investment on public revenues and public investment is not internalized by firms, a capital income subsidy is required to support this balanced growth path. In this case, it is no longer a constrained optimum because the cost of the subsidy reduces public investment and the marginal productivity of public capital exceeds the marginal productivity of private capital. This can be verified in the same way as for the model with nondurable public goods.

The constrained optimal fiscal policy with the public goods externality supports the balanced growth path determined by setting the interest rate for households equal to the marginal productivity of private capital. This is given by the conditions

$$(A33) \quad \gamma = \sigma \frac{\dot{c}}{c} = \alpha A \left(\frac{g_t}{k_t} \right)^{1-\alpha} - \rho$$

$$(A34) \quad \gamma = \frac{\dot{k}}{k} = A \left(\frac{g_t}{k_t} \right)^{1-\alpha} - \frac{x_t}{k_t} - \frac{c_t}{k_t}$$

$$(A35) \quad \gamma = \frac{\dot{g}}{g} = \frac{x_t}{g_t}$$

and

$$(A36) \quad \gamma = \frac{\dot{d}}{d} = \alpha A \left(\frac{g_t}{k_t} \right)^{1-\alpha} + \left(A^{-1} \left(\frac{g_t}{k_t} \right)^\alpha - \bar{\tau} \right) \frac{y}{d}.$$

The debt-to-output ratio is determined by the initial conditions to satisfy the transversality conditions as

$$(A37) \quad \frac{d}{y} = \frac{g_0 - \bar{g}_0}{y_0} = \frac{g_t - \bar{g}_0}{y_t} = \frac{\bar{k}_0 - k_t}{y_0} = \frac{k_t}{y_t}$$

where $y_t = A k_t^\alpha g_t^{1-\alpha}$ can be used to solve out for the balanced growth path public to private capital ratio, the equilibrium interest rate and the growth rate.

Table 1: System GMM Estimators

	Dependant Variable: Gross Public Investment as Share of GDP ¹¹			
	(1)	(2)	(3)	(4)
L.Pvt Fixed Invst Rate	-0.019 (0.018)	-0.029 (0.019)	-0.031 (0.019)	-0.019 (0.018)
L2.Pub. Invst rate	0.185 (0.091)**	0.164 (0.094)*	0.181 (0.093)*	0.184 (0.091)**
L. Pub. Invst rate	0.544 (0.101)***	0.523 (0.105)***	0.510 (0.104)***	0.538 (0.102)***
L.Debt-GDP ratio	-0.005 (0.002)**	-0.003 (0.002)	-0.004 (0.002)*	-0.005 (0.002)**
L.M2/GDP	0.018 (0.006)***	0.019 (0.006)***	0.019 (0.006)***	0.019 (0.006)***
L.Tax Rev/GDP	-0.006 (0.015)	-0.017 (0.015)	-0.013 (0.015)	-0.005 (0.015)
L.K A/c Openness	-0.120 (0.060)**	-0.126 (0.061)**	-0.558 (0.214)***	-0.228 (0.223)
L.Corruption	-0.060 (0.099)			-0.064 (0.100)
L.Law & Order		0.203 (0.109)*	0.186 (0.107)*	
L.K Open*Corrup			0.161 (0.077)**	
L.K Open*Law				0.032 (0.063)
Constant	1.725 (0.497)***	1.324 (0.471)***	1.346 (0.464)***	1.763 (0.501)***
Observations	344	344	344	344
Number of Ctrys	47	47	47	47
Sargan Test χ^2	73.78	69.34	69.18	72.73
Prob. > χ^2	0.52	0.66	0.67	0.55
AR(1) Test	-5.17	-5.01	-4.96	-5.10
Pr > z	0.00	0.00	0.00	0.00
AR(2) Test	0.85	0.87	0.66	0.83
Pr > z	0.40	0.38	0.51	0.41

Standard errors in parentheses

* significant at 10%; ** significant at 5%; *** significant at 1%

Dataset: The data used covers the period 1990-2004

Variable	Description	Source
Dependent Variables:		
Private Fixed Investment Rate	Constructed as Private Gross Fixed Capital Formation in current LCU/ GDP in current LCU (as percentage)	IMF World Economic Outlook
Public Investment Rate	Constructed as Public Gross Capital Formation in current LCU/ GDP in current LCU (as percentage)	IMF World Economic Outlook
Explanatory Variables:		
Debt-GDP Ratio	Ratio of Central Government Debt to GDP (as percentage)	Jaimovich and Panizza (2006)
M2/GDP	Ratio of broad money to GDP	World Bank, WDI
Tax Revenue/GDP		World Bank, WDI
Capital Account Openness	Chinn-Ito financial openness variable. Takes on higher values the more open the country is to cross-border capital transactions. Measures regulatory restrictions, not de-facto openness.	Chinn, Menzie and Hiro Ito (2006)
Law and Order Corruption	Defined so that a higher value means lower risk.	PRS Group, International Country Risk Guide

¹¹ The difference GMM estimator estimates a dynamic panel data model by first differencing the regression equation and then using the lags of explanatory and lagged dependent variables as instruments. GMM is used on stacked observations to deal with serial correlation of first differenced errors. The system GMM estimator estimates in addition, the levels equation in which the instruments are the lagged differences of the relevant explanatory variables, and is useful when the dependent variable is highly persistent. See Arellano and Bond (1991), Arellano and Bover (1995) and Blundell and Bond (1997)

Table 2: System GMM Estimators

	Dependant Variable: Gross Private Fixed Investment as Share of GDP			
	(1)	(2)	(3)	(4)
Lagged Pub Invst Rate	0.052 (0.068)	0.041 (0.066)	0.048 (0.066)	0.058 (0.068)
L.Pvt Fixed Invst Rate	0.852 (0.048)***	0.851 (0.049)***	0.846 (0.049)***	0.856 (0.047)***
L.Debt-GDP ratio	-0.000 (0.003)	-0.000 (0.003)	0.002 (0.003)	-0.000 (0.003)
L.M2/GDP	-0.001 (0.007)	-0.000 (0.007)	-0.001 (0.007)	-0.002 (0.007)
L.Tax Rev/GDP	0.051 (0.023)**	0.052 (0.023)**	0.045 (0.023)*	0.051 (0.024)**
L.K A/c Openess	0.148 (0.088)*	0.148 (0.088)*	0.747 (0.310)**	0.205 (0.309)
L.Law & Order	0.005 (0.132)			-0.000 (0.131)
L.Corruption		-0.016 (0.147)	-0.000 (0.146)	
L.K Open*Law			-0.180 (0.089)**	
L.K Open*Corrup				-0.021 (0.109)
Constant	1.176 (0.743)	1.281 (0.762)*	1.300 (0.751)*	1.122 (0.739)
Observations	395	395	395	395
Number of coden	47	47	47	47
Sargan Test χ^2	214.05	214.27	213.36	213.57
Prob. > χ^2	0.22	0.22	0.23	0.23
AR(1) Test	-6.63	-6.64	-6.74	-6.65
Pr > z	0.00	0.00	0.00	0.00
AR(2) Test	-1.28	-1.27	-1.22	-1.28
Pr > z	0.20	0.20	0.22	0.20

Standard errors in parentheses

* significant at 10%; ** significant at 5%; *** significant at 1%