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MULTIPERIPHERAL MODEL SUGGESTION OF A DAMPED OSCILLATORY COMPONENT IN HIGH ENERGY TOTAL CROSS SECTIONS*

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ABSTRACT

We use a simplified multiperipheral model to illustrate the possibility of complex Regge poles that contribute damped oscillations to the total cross section. The model suggests a relation between the period of the oscillations and the rate of growth of average multiplicity with energy.

Multiperipheral models are well known to predict Regge behavior in total cross sections at high energy. We here concern ourselves with complex Regge poles that may occur in the spectrum of such models, corresponding to oscillatory components of the total cross section [1]. We show how the wavelength of the lowest harmonic can be roughly estimated from the average multiplicity of particle production, together with the mean number of particles emitted from a single "link" of the multiperipheral chain. We also give an estimate for the rate of damping of the lowest harmonic as the energy increases. Possible relevance of complex Regge poles to the recent Serpukhov total cross-section measurements is discussed.

For a preliminary orientation recall that a pair of complex conjugate Regge poles gives a cross section contribution of the form

$$r s^{\alpha} + r^{*} s^{\alpha^{*}} = 2|r| s^{\mathrm{Re}\,\alpha} \cos[\mathrm{Im}\,\alpha \,\ln s + \arg r] ,$$

where r and r^{*} are the pole residues and s is the square of the c.m. energy. One full period of the oscillation thus corresponds to

$$\Delta (\ln s) = \frac{2\pi}{\mathrm{Im}\,\alpha} . \tag{1}$$

Now, a general characteristic of multiperipheral models is that as the energy increases the phase space in the small-momentum-transfer region opens up at a rate that permits one additional link in the multiperipheral chain for each addition of a certain increment in $\ln s$. This increment of $\ln s$, denoted by w, is determined by the ratio of the mass emitted at each link and the momentum transfer between links [2]. One ought not be surprised if the total cross section generated by the multiperipheral mechanism contains an oscillatory component corresponding to the interval $\Delta(\ell n\ s) = w$. Referring to Formula (1), we might expect to have a pair of complex Regge poles with

$$\mathrm{Im}\ \alpha \approx \pm \frac{2\pi}{w} . \tag{2}$$

We have verified the appearance of such complex poles in a simple and transparent multiperipheral model. The feature making the model extremely tractable is a kernel that is factorizable in its dependence on momentum transfers and subenergies. Physically interesting kernels do not factor, but the eigenvalue spectrum of nonfactorizable kernels, when evaluated in the first Fredholm approximation (the trace approximation), is the same as that of a factorizable kernel. We have checked in the ABFST model [2] that the trace approximation is not misleading for the leading eigenvalue.

With factorizability the multiperipheral equation can be reduced to an equation in one variable, the energy. The equation has the form

$$F(s) = F_0(s) + \int_1^s \frac{ds'}{s'} K\left(\frac{s}{s'}\right) F(s') , \tag{3}$$

where the total cross section is related to $F(s)$ by

$$\sigma_{tot}(s) = \frac{1}{s} F(s) .$$

It will be understood that the crossed-channel quantum numbers are those of the vacuum, so the relevant total cross section is an appropriate average over internal quantum numbers. For Eqn.(3), a Mellin transform constitutes the analogue of projection onto a representation of the

Lorentz group, the technique by which the full multiperipheral equation is diagonalized [3]. Thus, if

$$f(\lambda) = \int_1^\infty \frac{dx}{x} x^{-\lambda} F(x) ,$$

with an analogous definition of $f_0(\lambda)$ and $k(\lambda)$, we find

$$\begin{aligned} f(\lambda) &= f_0(\lambda) + k(\lambda) f(\lambda) \\ &= \frac{f_0(\lambda)}{1 - k(\lambda)} . \end{aligned}$$

The inverse transformation is

$$F(s) = \frac{1}{2\pi i} \int_c d\lambda \, f(\lambda) \, s^\lambda ,$$

with the contour c running vertically in the complex λ plane from $L - i\infty$ to $L + i\infty$, where L stands to the right of all singularities of $f(\lambda)$. Thus a pole at $\lambda = \alpha_i$ with residue r_i contributes a term $r_i \, s^{\alpha_i}$ to the asymptotic expansion of $F(s)$.

The particular model to be investigated here is characterized by a kernel

$$K\left(\frac{s}{s'} \right) = \gamma \left(\frac{s}{s'} \right)^\beta \theta\left(\ell n \frac{s}{s'} - w\right) , \qquad (4)$$

the crucial feature being the "threshold" at $\ell n \, s/s' = w$ which we have discussed above in our introduction. The factor $(s/s')^\beta$ is suggested by the ABFST model [2], which in the trace approximation can be approximately matched to the model here if we choose $\beta \approx 0$ [4]. The real positive parameter γ is determined by the requirement that the leading pole of $f(\lambda)$, i.e., the leading zero of $1 - k(\lambda)$, shall occur at $\lambda = 1$, corresponding to a constant high energy limit for the cross section. From Eqn. (4) we find

$$k(\lambda) = \frac{\gamma\, e^{-w(\lambda-\beta)}}{\lambda - \beta},$$

so

$$\gamma = (1 - \beta)e^{w(1-\beta)} . \qquad (5)$$

The complete spectrum of "Regge" poles is specified by the transcendental equation, $k(\alpha_j) = 1$, or

$$\alpha_j = \beta + (1 - \beta)e^{w(1-\alpha_j)} ,$$

from which it follows that the only real root is $\alpha_0 = 1$. An infinite sequence of complex roots, in complex conjugate pairs, extends in the array illustrated by Fig. 1, which corresponds to the case $\beta = 0$ and $w = 1.5$. If the sequence α_j with $\operatorname{Im} \alpha_j > 0$ is ordered according to increasing magnitude of the imaginary part we see that the real part of α_j decreases monotonically with increasing j.

A good approximation to the imaginary part of α is given by

$$\operatorname{Im} \alpha_1 \approx \frac{3\pi}{2w} ,$$

corresponding to an asymptotic period $(4/3)w$, so the pair of poles α_1 and α_1^* qualitatively possess the expected property (2).

Values of j greater than 1 correspond to higher harmonics and their contributions damp out with increasing energy faster than does the $j = 1$ contribution. The rate of damping of $j = 1$ is determined by $\operatorname{Re} \alpha_1$, which in general is less than 1. In the example of Fig. 1, with $\beta = 0$ and $w = 1.5$, it turns out that $\text{Re}\,\alpha_1 = 0.23$; so the damping of this oscillation in the cross section goes $\propto s^{-0.77}$.

To complete the model we assign to our inhomogeneous term in Eqn. (3) the form

$$F_0(s) = s_0 \gamma_0 \, \delta(s - s_0) ,$$

corresponding to the assumption that the mass emitted at the end of the chain is s_0. It follows that

$$f_0(\lambda) = \gamma_0 \, s_0^{-\lambda}$$

and

$$f(\lambda) = \frac{\gamma_0 \, s_0^{-\lambda}}{1 - k(\lambda)} ,$$

from which the pole residues are easily computed to be

$$r_j = \gamma_0 \, s_0^{-\alpha_j} \, \frac{\alpha_j - \beta}{1 + (\alpha_j - \beta) w} .$$

With the choice of parameters, $\beta = 0$ and $w = 1.5$, the two leading residues turn out to be

$$r_0 = \gamma_0 \, s_0^{-\alpha_0} \times 0.40 ,$$

$$r_1 = \gamma_0 \, s_0^{-\alpha_1} \times 0.29 \, e^{-1.20i} ,$$

corresponding to the asymptotic approximations to the total cross section shown in Fig. 2. Since the model has only one real pole at $\lambda = 1$ (i.e., no P' trajectory), the oscillating component is added to a constant instead of a decreasing smooth part. Also shown for comparison is the total cross section corresponding to the exact solution to Eqn. (3), which can be obtained by iteration to be [5]

$$F(s) = \gamma_0\, s_0\, \delta(s - s_0) + \gamma_0 \gamma \left(\frac{s}{s_0}\right)^{\beta} \sum_{n=0}^{\infty} \frac{\gamma^n}{n!} \left[\ell n \frac{s}{s_0} - (n+1)w\right]^n$$

$$\times\ \theta \left[\ell n \frac{s}{s_0} - (n+1)w\right]. \qquad (6)$$

We note that our simple model probably overestimates the amplitude of the oscillations because of the step function in our kernel. A more realistic kernel would be smoother.

Having argued the plausibility of complex Regge poles and the associated oscillatory component of high-energy total cross sections, we now suggest that the observed rate of multiplicity increase with increasing energy can be used to estimate the period w. Multiperipheral models predict that the average overall multiplicity behaves roughly as

$$\overline{N} \approx K\, \ell n\, s + \text{constant},$$

and if observed multiplicities are fitted to such a formula one finds $K \approx 2$. With weak coupling the constant K is proportional to coupling strength, but for strong coupling K is evidently bounded by $\overline{n}/w$, where $\overline{n}$ is the average number of particles emitted at each link. Models such as that discussed above are close to this upper limit, so it is possible to estimate w as

$$w \lesssim \frac{\overline{n}}{K}. \qquad (7)$$

A value for $\bar{n}$ in the neighborhood of 2 would lead to the estimate $w \lesssim 1$, qualitatively consistent with our example. Conversely, if w can be determined from an experimentally observed oscillation, one may use (7) to estimate $\bar{n}$.

We close with the observation that the apparent upturn in $\pi^{\pm}p$ and $k^{-}p$ total cross sections observed near 40 GeV lab energy in recent Serpukhov experiments [6] might result from a small (≈5%) oscillation associated with a complex Regge pole belonging to the vacuum quantum numbers. If this is the first upward oscillation it would imply $w \approx 1.5$ in our model, and we would predict that the following maximum occurs at a lab energy about 3 times as large as that at the first minimum. From Fig. 2 it is apparent that our simple model predicts a larger amplitude for the first oscillation than needed to explain the Serpukhov data. It would be easy to reduce this amplitude by choosing a smoother kernel; estimating the amplitude is much more difficult than estimating the period.

Note that if the Serpukhov phenomenon is associated with a complex Regge pole, the asymptotic cross section limits may lie quite close to the values attained at the highest currently accessible energies, in contrast to the situation when a strong Regge cut is invoked [7].

FOOTNOTES AND REFERENCES

* Supported in part by the U.S. Atomic Energy Commission.

1. Complex poles are also well known to occur in potential models at energies below the physical threshold, as discussed with explicit examples by N. F. Bali, S. Y. Chu, R. W. Haymaker, and C-I Tan, in Phys. Rev. 161, 1450 (1967). It is normal for a secondary trajectory, which eventually passes through physical J values corresponding to bound states, to be complex over a finite interval of real negative kinetic energy.

2. In the model of L. Bertocchi, S. Fubini, and M. Tonin, Nuovo Cimento 25, 626 (1962), further developed by D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento 26, 6 (1962), one finds, with a form factor to constrain the range of momentum transfer, that

$$\cosh w \approx 1 + \frac{m^2}{2\Delta^2} ,$$

if m is the average mass emitted at each link and Δ is the upper limit of momentum transfer.

3. G. F. Chew and C. DeTar, Phys. Rev. 180, 1577 (1969).

4. The result $\beta \approx 0$ will be shown in a separate paper by T. Rogers and the authors of this paper to follow in the ABFST model (Ref. 2) from the very small pion mass together with the zero pion spin. Multi-Regge models also suggest $\beta \approx 0$, but on the basis of a leading octet trajectory intercept near 0.5.

5. Corresponding to Eqn. (6), we may identify the following formula for σ_n^{ba}, the partial cross section to produce n + 1 links of the multiperipheral chain in a collision between particles b and a:

$$\sigma_n^{ba} = \gamma_a \gamma_b \left(\frac{s}{s_0}\right)^{\beta-1} \frac{\gamma^n}{n!} \left[\ln \frac{s}{s_0} - (n+1)w\right]^n .$$

This formula is an improvement over that given by Formula (5.3) in the paper by G. F. Chew and A. Pignotti, Phys. Rev. 176, 2112 (1968), which in effect sets w = 0. If the improved formula is used for phenomenological data fitting, it should be remembered that because of the constraint (5) only two of the three "internal" parameters γ, β and w are independent.

6. J. V. Allaby et al., Phys. Letters, to be published, 1969.

7. The hypothesis of a strong cut has been explored numerically by V. Barger and R. Phillips, submitted to Phys. Rev. Letters, 1969.

FIGURE CAPTIONS

Fig. 1. The Regge poles in the complex λ plane predicted by the model with $\beta = 0$ and $w = 1.5$.

Fig. 2. Total cross section from the model. The dashed lines are the individual contributions to the total cross section [5], the solid line is the exact cross section, Eqn. (6), and the dot-dash is the three-pole (one real and one complex-conjugate pair) approximation.

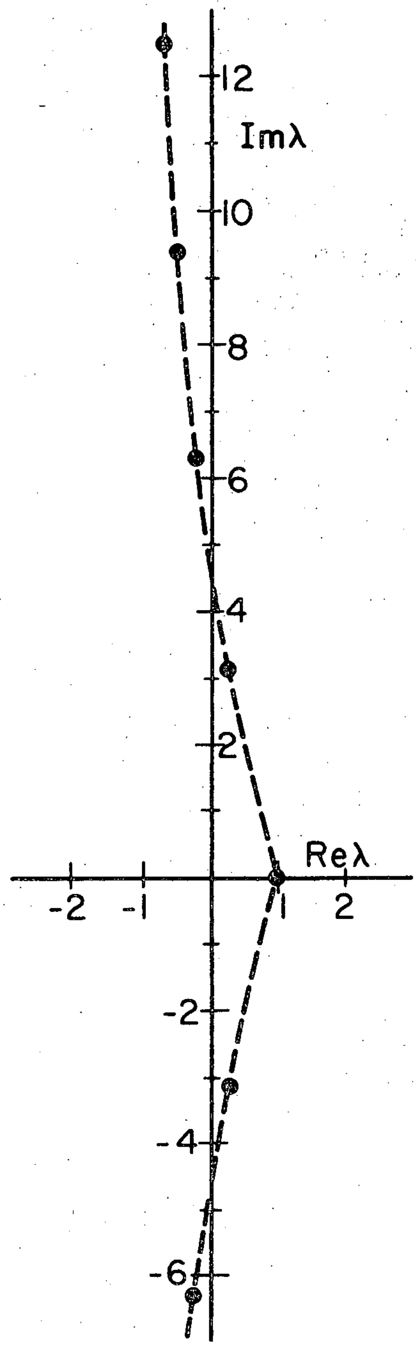

Fig. 1

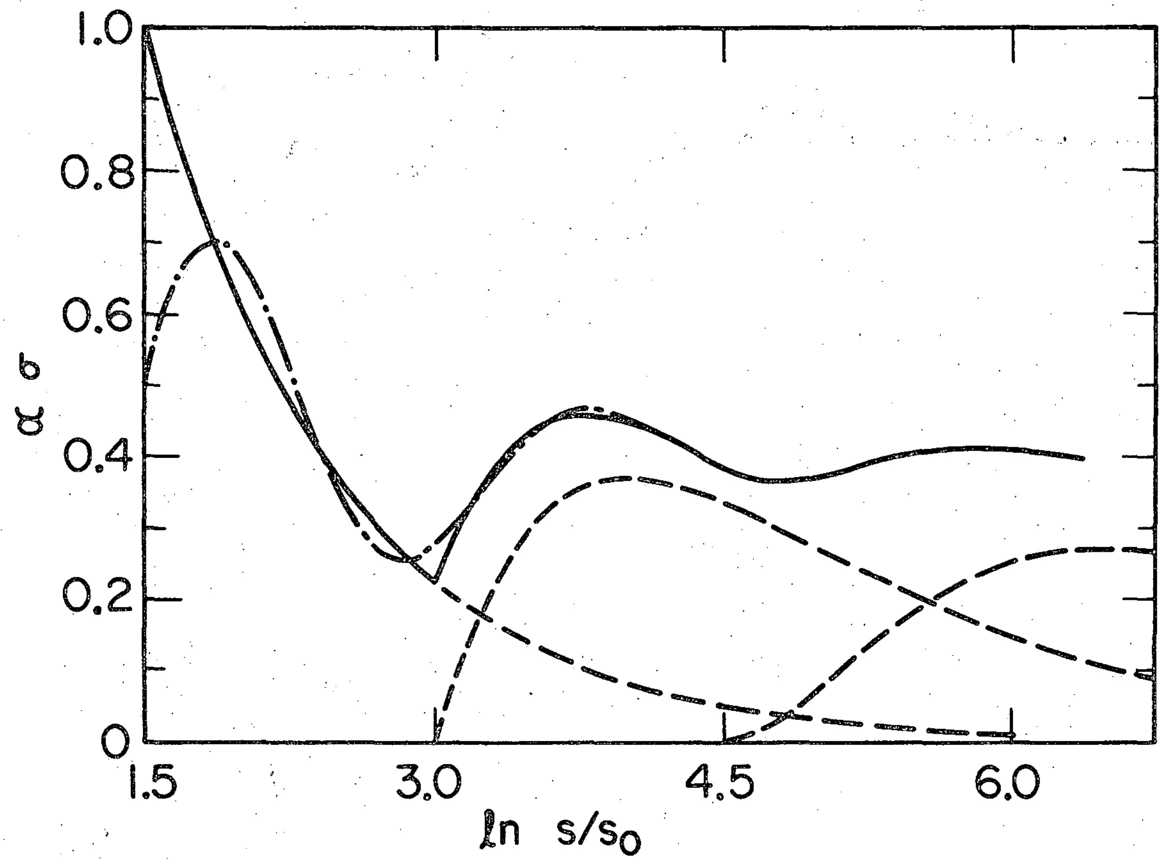

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Fig. 2

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