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# Quantum Ratcheted Photophysics in Energy Transport

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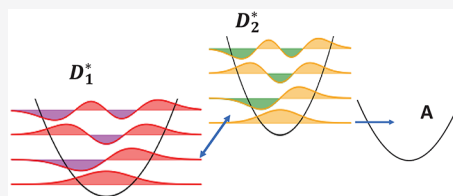
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**ABSTRACT:** In this paper, we explore the scope of vibrations as quantum ratchets that serve as nonthermal routes to achieving population transport in systems where excitation transport between molecules is otherwise energetically unfavorable. In addition to their role as channels of transport, we investigate the effect of resonance of the vibrations, which are described by Huang–Rhys mixing, with excitonic energy gaps, which leads to strongly mixed vibronic excitons. Finally, we explore the interplay of resonance and Huang–Rhys mixing with electronic coupling between the molecules, in the presence of a dissipative bath, in optimizing transport in such systems. We find that while resonance is desirable, a moderate electronic coupling has a stronger positive effect in contrast to a large electronic coupling, which results in delocalized excitations across molecules and hampers unidirectional transport. We also report a special resonance regime that is able to circumvent the transport problems arising from large electronic couplings.



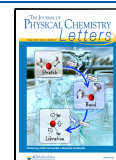
To exploit coherences to aid the architecture of synthetic light-harvesting machinery is an area of rapidly proliferating interest. Coherences in molecular systems occur due to large electronic couplings between molecules, resulting in delocalized excitations/excitons, which offer a route to manipulating molecular systems away from bath-induced dissipation and Boltzmann statistics. Theoretical calculations by Ishizaki and Fleming suggest that quantum coherence could be beneficial in overcoming local energy traps, thus mediating efficient EET (electronic energy transport) in the Fenna–Matthews–Olson photosynthetic complex.<sup>1</sup> A key benefit, thus, can be realized in the potential to control molecular dynamics and direct energy/charge transport by harvesting coherences. Conjugated molecular aggregates, for instance, are promising candidates for such control, due to the extended  $\pi$ -stacking prevalent in such systems due to the highly ordered microscopic assemblies. A major disadvantage, however, is posed by the fragility of molecular coherences, since they are extremely susceptible to destruction by their surroundings/bath. As a consequence, disorder and imperfections in the molecular assemblies will diminish the exciton length and limit coherence. A plethora of theoretical studies report that vibrationally enhanced transfer achieves an optimal transfer efficiency in such scenarios.<sup>2,3</sup> Also, vibrations, if they are resonant with an excitonic energy gap and are strongly coupled to the electronic transition, have the capacity to provide a means to circumvent this difficulty. This is because the excitation can switch back and forth between the exciton and the vibration that is less susceptible to decoherence, resulting in longer-lived coherences.<sup>4–7</sup> For instance, calculations by Plenio and co-workers suggest that nonequilibrium processes due to spectrally sharp vibrational modes can effectively compete with dissipative pathways induced by the thermal bath.<sup>8</sup> In another work reported by Cao and co-workers, it is

proposed that underdamped vibrations resonant with energy gaps in the system can drive excitation transport, in addition to retention of coherences for longer time scales.<sup>9</sup> The work by Bennett et al. suggests that incoherent vibronic transport results in more efficient excitation transfer, compared to coherent transport, if the energetic disorder is greater than the coupling between the donor and the vibrationally excited states on the acceptor.<sup>10</sup> A recent work from our group suggests that underdamped vibrations, resonant with excitonic energy gaps in the LHCII complex, can drive population transport across large energy gaps that are thermally inaccessible.<sup>11</sup> Therefore, vibrations and coherences are important tools that can be exploited for the design of efficient energy-harvesting and charge transport devices. Key advances that utilize such design principles are actively studied by Therien and co-workers, where they report the synthesis of “supermolecules”, which are composed of highly conjugated and strongly coupled PZn<sub>n</sub> porphyrin arrays.<sup>12–21</sup> The studies suggest that these molecules enjoy resonances between vibrations and excitons, as well as strong vibronic coupling between vibrations and excitonic transitions. This is expected to lead to vibronic mixing, which could pave the way to robust coherences. The studies also report transition dipole moments for the S<sub>0</sub> → S<sub>1</sub> transition that increase progressively as the number of PZn<sub>n</sub> units increase. A key manifestation of this is large electronic couplings across the porphyrin assemblies, which again ensures large exciton coherence lengths. In this work, we explore the

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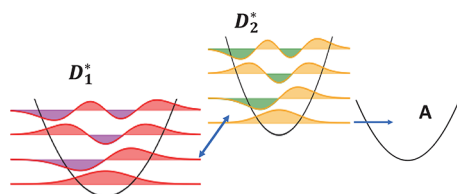
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interplay between resonance and electronic coupling and Huang–Rhys mixing, in an effort to formulate design principles that would ensure efficient energy harvesting in synthetic devices. We specifically investigate the role of vibrations as nonthermal routes to ratcheting populations between molecules.

In this work, our focus is on the scope of quantum ratcheted photophysics in model systems where energy transport is otherwise energetically unfavorable. This is demonstrated in Figure 1a, where the objective is to achieve a fast and efficient



**Figure 1.** Schematic depicting ratcheted energy transport. Direct transfer of excitation from  $D_1^*$  to A is unfavorable, both energetically and owing to a large spatial separation between them. To address this, we use an intermediate molecule  $D_2$ , which is placed between  $D_1$  and A. A vibration is used as a quantum ratchet to achieve nonthermal transport of population from  $D_1^*$  to  $D_2^*$ , followed by a unidirectional decay to A, which acts as an energy sink. The relaxation of the vibrational mode on  $D_1^*$ , due to its interaction with the bath, serves as a competing pathway.

transport of excitation from the electronically excited donor molecule  $D_1^*$  to an acceptor molecule A. A favorable electronic coupling between  $D_1^*$  and A would potentially lead to coherent transport but the spatial separation between them is large, resulting in small electrostatic couplings. The other possibility is incoherent Förster transport, which again is unfavorable due a large spatial separation. Also, the energetic constraint  $E_{D_1} < E_A$  necessitates an uphill transport from  $D_1^*$  to A. This pathway is thermally inaccessible if the energy gap  $\Delta E_{A,D_1} > k_B T$ , where  $T$  is the temperature of the thermal bath. To circumvent the transport problems, we use a second intermediate donor molecule  $D_2$ , which acts as a bridge between  $D_1$  and A, given that we have an efficient, nonthermal route to transporting excitation from  $D_1^*$  to  $D_2^*$ . Here,  $D_2^*$  is the electronically excited  $S_1$  state on  $D_2$ . The excitation, then, undergoes a downhill transport from  $D_2^*$  to A. We propose to accomplish this by exploiting a vibrational mode that ratchets population from  $D_1^*$  to  $D_2^*$  through the mechanism discussed below. Denoting the electronic excitons formed from a two-level electronic Hamiltonian defined by the  $S_1$  states of  $D_1^*$  and  $D_2^*$  as  $m$  and  $n$ , the transport between  $m$  and  $n$  can be effectively mediated by a vibrational mode that is resonant/quasi-resonant with the excitonic energy gap  $\Delta \epsilon_{mn}$ . In the simplest model, the presence of the vibrational mode is incorporated through its Huang–Rhys mixing. We consider A to be an energy sink, thus the energy transport from  $D_2^*$  to A is unidirectional and irreversible. Therefore, for a minimal description of the system Hamiltonian that adequately describes quantum ratcheting, we will consider only the electronic/vibronic states of  $D_1^*$  and  $D_2^*$ , while the excitation transfer from  $D_2^*$  to A is easily evaluated by a simple kinetic equation. We investigate the interplay between the Huang–Rhys mixing, the resonance/quasi-resonance of the vibrational mode with the excitonic energy gap and coherence, arising from the electronic coupling between  $D_1^*$  and  $D_2^*$ , in dictating

the transport between these molecules, in the presence of a dissipative bath. It is important to note here that the relaxation of the vibrationally excited state on  $D_1^*$ , due to its interaction with the bath, serves as a competing pathway to transport from  $D_1^*$  to  $D_2^*$ . In our calculations, we incorporate the effects of such relaxation.

To model the ratcheting dynamics, we define a system Hamiltonian that explicitly includes the vibrational mode, along with the electronically excited states of  $D_1$  and  $D_2$ . For efficient yet adequate calculations, we define our local basis as follows:  $\{|D_1 v_{D_1,0}^e v_{D_2,0}^g\rangle, |D_1 v_{D_1,1}^e v_{D_2,0}^g\rangle, |D_2 v_{D_1,0}^e v_{D_2,0}^g\rangle\}$ , where the first term in the ket indicates the species that is electronically excited.  $v_{D,l}^k$  contains the vibrational information on the donor species  $D$  ( $D \in \{D_1, D_2\}$ ), given by the vibrational quantum number  $l$  in the  $k$  electronic state, where  $k \in \{g, e\}$ , implying the ground and excited electronic states, respectively. The prime symbol “'” on the vibrational state  $l$  on  $e$  indicates that this vibrational wave function is structurally different from its counterpart on  $g$  due to a different charge distribution on  $e$ . It is worth emphasizing that it is always possible to add more vibrational information to the Hamiltonian, by incorporating a larger vibrational basis. In our studies, however, the focus is the ratcheting dynamics stemming from the excitation of the vibrational mode on  $D_1^*$  that places  $D_1$  at a higher energy compared to  $D_2^*$  and the subsequent excitation transfer due to resonance. Therefore, including up to the first vibrationally excited state on  $D_1^*$ , while considering the vibrational ground state on  $D_2^*$ , is adequate to infer more about the ratcheting photophysics.

The vibronic Hamiltonian of the system, with respect to the local basis described above, can be written as follows:

$$H_{\text{sys}} = \begin{pmatrix} E_{D_1} & 0 & JS_{00}^{D_1 D_2} \\ 0 & E_{D_1} + \Omega & JS_{01}^{D_1 D_2} \\ JS_{00}^{D_1 D_2} & JS_{01}^{D_1 D_2} & E_{D_2} \end{pmatrix} \quad (1)$$

Here,  $E_{D_1}$  and  $E_{D_2}$  are the energies of the ground vibrational states in the electronically excited  $e$  states for  $D_1$  and  $D_2$ , respectively. The vibrational mode has a frequency  $\Omega$ ,  $J$  is the electronic coupling between  $D_1$  and  $D_2$ , and  $S_{\nu\nu'}^D$  measures the overlap between the nuclear wave functions describing the vibrational states  $\nu$  (residing on the ground electronic state) and  $\nu'$  (residing on the electronically excited state) respectively, on  $D$  ( $D \in \{D_1, D_2\}$ ) and is expressed in terms of the Huang–Rhys factors. While  $H_{\text{sys}}$  describes the electronic and vibrational excitations, we use the following total Hamiltonian  $H$  that incorporates the phonons, also referred to as the bath/environment, which are modeled as harmonic oscillators in  $H_{\text{ph}}$ , and the system–bath interaction described by  $H_{\text{sys-ph}}$ .

$$H = H_{\text{sys}} + H_{\text{sys-ph}} + H_{\text{ph}} \quad (2)$$

where

$$H_{\text{ph}} = \sum_{j,k} \left( \frac{p_{jk}^2}{2m_{jk}} + \frac{1}{2} m_{jk} q_{jk}^2 \omega_{jk}^2 \right) \quad (3)$$

and  $H_{\text{sys-ph}} = \sum_j Q_j |j\rangle \langle j|$ , where

$$Q_j = \sum_k m_{jk} \nu_{jk} q_{jk} \quad (4)$$

Here,  $m_{jk}$ ,  $q_{jk}$ ,  $p_{jk}$  and  $\omega_{jk}$  describe the mass, position, momentum, and angular frequency, respectively, of the  $k$ th bath oscillator pertaining to the  $j$ th system state. The system–bath interaction, in eq 4, is diagonal with respect to the local basis and is considered to be adequately described by a linear dependence of the position coordinate  $q_{jk}$ .  $\nu_{jk}$  describes the coupling strength of the  $k$ th oscillator with the  $j$ th site.

For simulating dynamics, we will employ a reduced density matrix description of the system and use a near-analytic approximate approach.<sup>22–24</sup> This approach uses a unitary mapping that transforms to a stationary adiabatic basis, formed by diagonalizing  $H_{\text{sys}} + H_{\text{sys-ph}}$  and frozen at the equilibrium geometry. The technical definition for stationary adiabatic states, therefore, differs from that of excitons, which are eigenstates formed by diagonalizing only the system Hamiltonian and do not reflect any contribution from the bath. It is important to note here that though they differ in their definitions, the energies of the excitons formed by diagonalizing the system Hamiltonian are the same as those of the stationary adiabatic states.<sup>11</sup>

The chief benefit of the unitary mapping arises from a decoupling between decoherence, described by fluctuations in the energies of the dynamically evolving adiabatic states due to shifting of the bath position coordinates, and phonon-mediated population relaxation, described by a derivative operator that drives a change in electronic state, in conjunction with bath momenta. This is followed by a nonperturbative treatment of decoherence that captures the effects of the dynamically fluctuating energies of the stationary adiabatic states (see eqs 6 and 7), while the population relaxation is described perturbatively. The method has been described in rigorous detail in previous work; therefore, we will only discuss the final expression for the reduced density operator we use to describe the system.

While  $|\alpha\rangle\langle\alpha|$  describes the initial density operator at  $t = 0$ , where  $\alpha$  denotes a local state, we intend to determine the extent of overlap of the excitation at time  $t$  with the local basis coherence  $|\gamma\rangle\langle\beta|$ . In the expression below,  $m$ ,  $n$ ,  $m'$ , and  $n'$  denote the stationary adiabatic states,  $\epsilon_n$  denoting the energy of the stationary adiabatic state  $n$ . The bath is described by the spectral density function  $S(\omega)$ , defined below, at temperature  $T$ .

$$\begin{aligned} \rho_{\beta\gamma}(t) \approx & \sum_{m,n,m'} \langle\beta|m\rangle\langle n|\alpha\rangle\langle\alpha|n\rangle\langle m'|\gamma\rangle p_{mm',nn'}(t) \\ & + \sum_{m,n,n',m'} \langle\beta|m\rangle\langle n|\alpha\rangle\langle\alpha|n'\rangle\langle m'|\gamma\rangle p_{mm',n'n'}(t) \\ & \times e^{-i\epsilon_{nn'}t} e^{-\phi_{n,n'}(t)} \end{aligned} \quad (5)$$

Here,  $\epsilon_{nn'} = \epsilon_n^0 - \epsilon_{n'}^0$ ,  $\phi_{n,n'}(t) = \text{Re}[\phi_{n,n'}(t)] + i \text{Im}[\phi_{n,n'}(t)]$ , where  $\phi_{n,n'}(t)$  describes the effects of decoherence, with the real and imaginary components given as

$$\begin{aligned} \text{Re}[\phi_{n,n'}(t)] = & \int_0^\infty d\omega S(\omega) \frac{1 - \cos(\omega t)}{\omega^2} \coth\left(\frac{\omega}{2k_B T}\right) \\ & \times \sum_j \left[ \left( \frac{\partial \epsilon_n}{\partial Q_j} - \frac{\partial \epsilon_{n'}}{\partial Q_j} \right)_{Q_j=0} \right]^2 \end{aligned} \quad (6)$$

and

$$\begin{aligned} \text{Im}[\phi_{n,n'}(t)] = & \int_0^\infty d\omega S(\omega) \frac{\sin(\omega t) - \omega t}{\omega^2} \\ & \times \sum_j \left[ \left( \frac{\partial \epsilon_n}{\partial Q_j} \right)_{Q_j=0}^2 - \left( \frac{\partial \epsilon_{n'}}{\partial Q_j} \right)_{Q_j=0}^2 \right] \end{aligned} \quad (7)$$

Here,  $S(\omega)$  is the spectral density of the environment, defined as

$$S(\omega) = \sum_k \frac{m_{jk} \nu_{jk}^2}{2\omega_{jk}} \delta(\omega - \omega_{jk}) \quad (8)$$

While  $e^{-\phi_{n,n'}(t)}$  is evaluated nonperturbatively and describes the effects of decoherence, the term  $p_{mm',nn'}(t)$  addresses the effects of relaxation arising from a perturbative treatment of the nonadiabatic derivative operator, in conjunction with bath momenta, and is evaluated using a Markovian master equation, which is solved using matrix algebra. The relaxation rates, arising from the perturbative treatment of relaxation, for a pair of stationary adiabatic states  $m$  and  $m'$  are given as

$$\Gamma_{mm'} = 2\pi \frac{S(\omega_{mm'})}{e^{\omega_{mm'}/k_B T} - 1} \sum_j (\langle m'|l_j\rangle\langle j|m\rangle)^2 \quad (9)$$

and

$$\Gamma_{m'm} = e^{\omega_{mm'}/k_B T} \Gamma_{mm'} \quad (10)$$

In the above expression,  $\omega_{mm'} > 0$ , where  $\omega_{mm'} = \epsilon_m^0 - \epsilon_{m'}^0$ .

We will now apply the above formalism to investigate quantum ratcheted energy transport in various model systems. In our discussion forward, we will loosely refer to stationary adiabatic states as adiabatic states, for convenience. However, it is important to keep in mind the subtle distinction between excitons and adiabatic states.

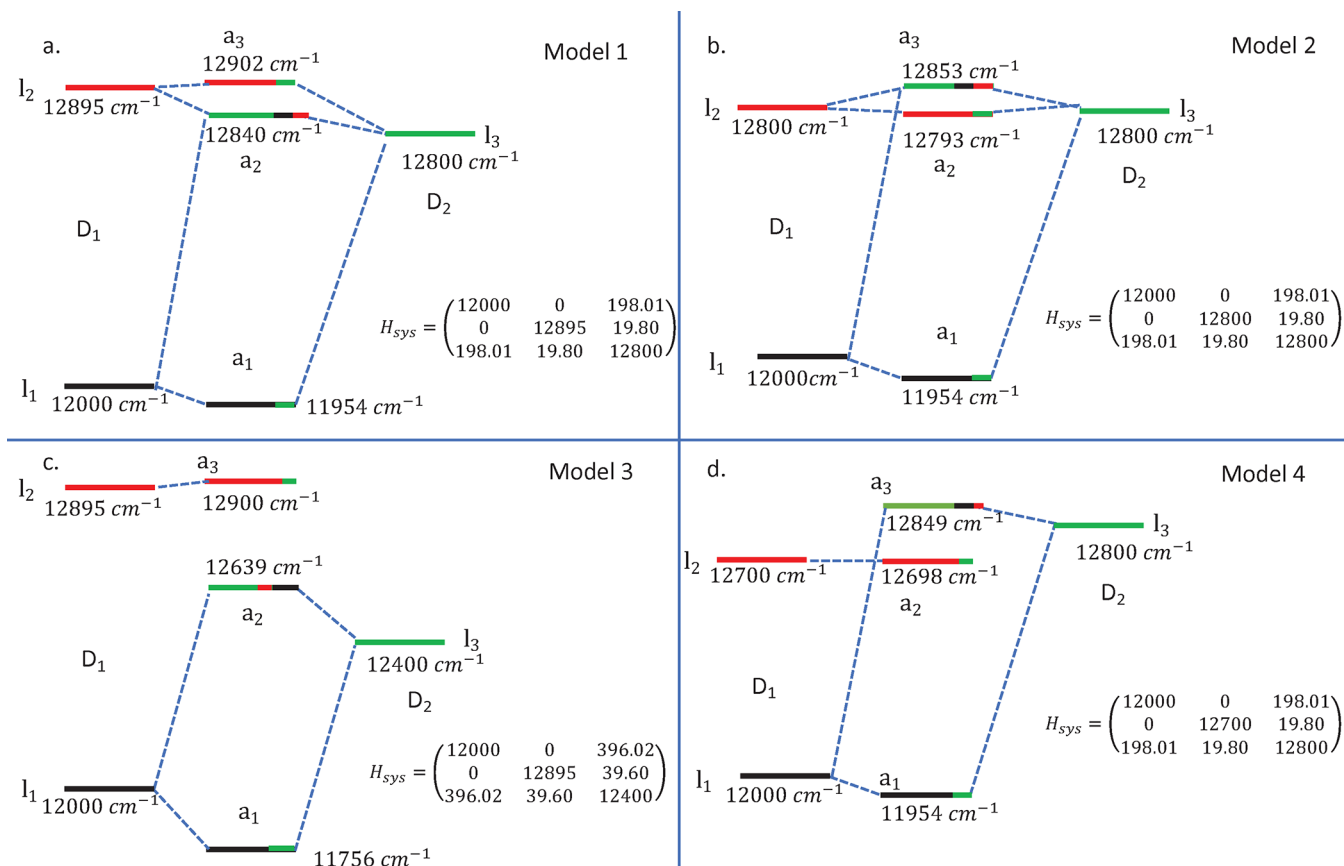
Table 1 enumerates the system parameters for the various models we consider. In Figures 2 and 4, we demonstrate the

**Table 1. System Parameters for the Various Models We Consider<sup>a</sup>**

	model 1	model 2	model 3	model 4	model 5	model 6
$\Delta E_{D_2,D_1}$ ( $\text{cm}^{-1}$ )	800	800	400	800	400	400
$J$ ( $\text{cm}^{-1}$ )	200	200	400	200	400	400
$\Omega$ ( $\text{cm}^{-1}$ )	895	800	895	700	646	646
$\sigma$	0.01	0.01	0.01	0.01	0.01	0.1

<sup>a</sup> $\Delta E_{D_2,D_1}$  denotes the difference between the bare electronic excitation energies on  $D_2$  and  $D_1$ , respectively.  $J$  denotes the electronic coupling between  $D_1$  and  $D_2$ , and  $\Omega$  is the excitation frequency of the vibrational mode.  $\sigma$  denotes the Huang–Rhys factor for the vibrational mode.

energy level diagrams for each of these models, where  $l_1$ ,  $l_2$ , and  $l_3$  label the local states.  $l_1$  and  $l_2$  are localized on  $D_1$  and label the states  $|D_1\nu_{D_1,0}^e\nu_{D_2,0}^g\rangle$  and  $|D_1\nu_{D_1,1}^e\nu_{D_2,0}^g\rangle$ , respectively.  $l_3$  is localized on  $D_2$  and labels the state  $|D_2\nu_{D_1,0}^e\nu_{D_2,0}^g\rangle$ .  $a_1$ ,  $a_2$ , and  $a_3$  label the stationary adiabatic states. We list the eigenvector contributions for the adiabatic states with respect to the local states, for all the six models, in the Supporting Information. The quantity we wish to investigate here is the population transport to  $l_3$ , while  $l_2$  is considered the initial seat of



**Figure 2.** Energy level diagrams for models 1, 2, 3, and 4.  $l_1$  and  $l_2$  are localized on  $D_1$  and label the states  $|D_1\nu_{D_1,0}^e\nu_{D_2,0}^g\rangle$  and  $|D_1\nu_{D_1,1}^e\nu_{D_2,0}^g\rangle$ , respectively.  $l_3$  is localized on  $D_2$  and labels the state  $|D_2\nu_{D_1,0}^e\nu_{D_2,0}^g\rangle$ .  $a_1$ ,  $a_2$ , and  $a_3$  label the stationary adiabatic states, and the color coding of the adiabatic states indicates the presence of the respective local states.

excitation. This is achieved by directly exciting  $l_2$  from the ground electronic state, by using a UV–vis radiation field of suitable frequency. A second way of placing the initial population at  $l_2$  is by first exciting  $l_1$  using a UV–vis pulse, followed by an IR pulse that creates the vibrational excitation, leading to the population being transported from  $l_1$  to  $l_2$ . Therefore, for ratcheted photophysics, we consider the temporal evolution of population, considering  $l_2$  to be the initial state.

For model 1, the vibronic system Hamiltonian and the energy level diagram are depicted in Figure 2a. If we consider the electronic-only Hamiltonian formed by the basis states  $|D_1\rangle$  and  $|D_2\rangle$ ,

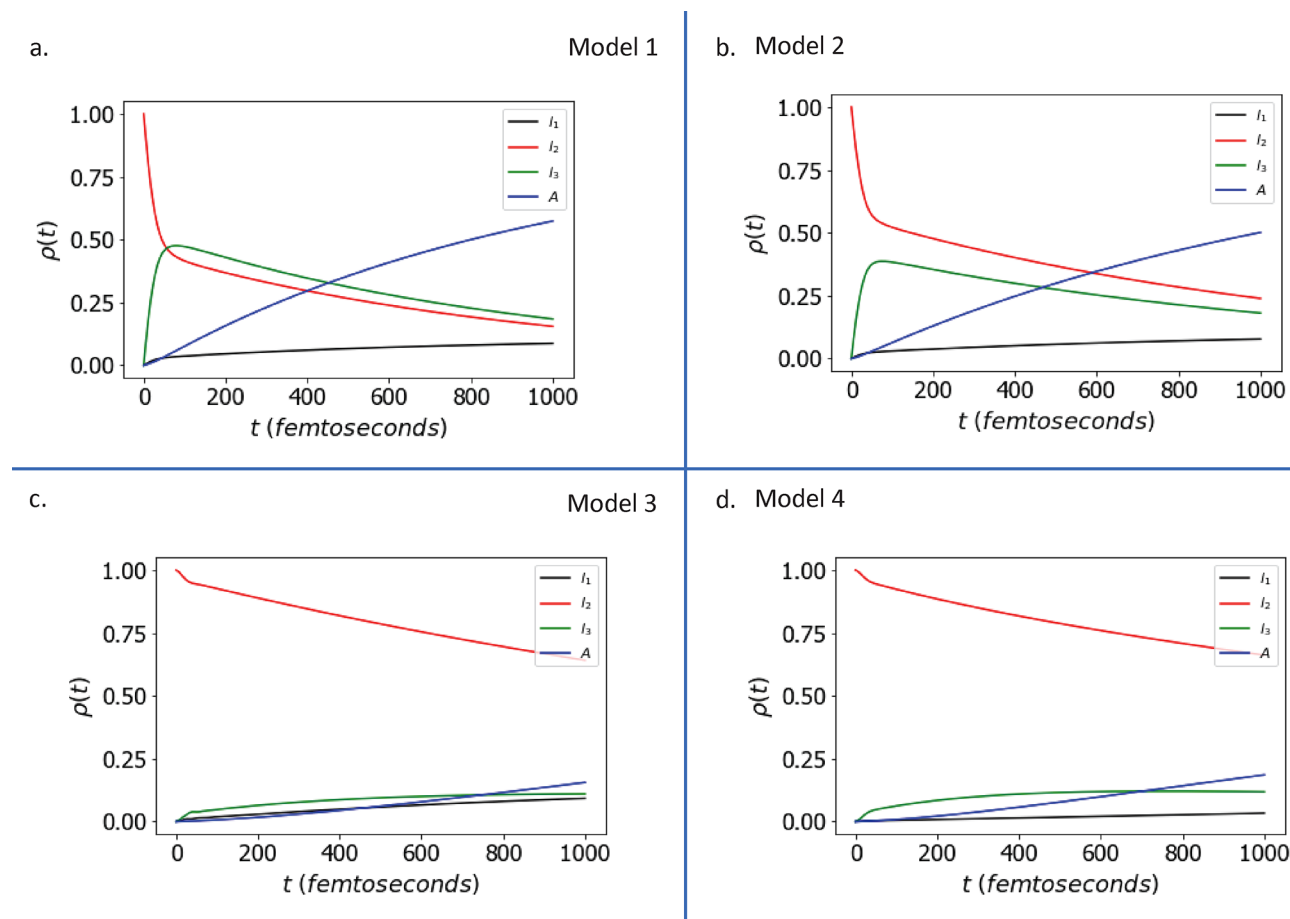
$$H_{\text{electronic}} = \begin{pmatrix} E_{D_1} & J \\ J & E_{D_2} \end{pmatrix} \quad (11)$$

the energy gap between the electronic excitons is given as  $\Delta\epsilon_{\text{electronic}} = 895 \text{ cm}^{-1}$ , which is used as the frequency of the vibrational mode. In other words, the vibrational mode is resonant with the energy gap between the electronic excitons. We incorporate vibrational relaxation in our calculations, since the relaxation of the vibrational mode on  $l_2$ , due to its interaction with the bath, acts as a competing pathway to  $l_2 \rightarrow l_3$  transport. The electronic and vibrational environments are both modeled by the Drude spectral density, where the reorganization energies are given as  $\lambda_{\text{elec}} = 100 \text{ cm}^{-1}$  and  $\lambda_{\text{vib}} = 10 \text{ cm}^{-1}$ .  $\lambda_{\text{vib}}$ , thus, corresponds to a vibrational reorganization

time given as  $\tau_{\text{vib}} \approx 3.3 \text{ ps}$ . The bath is considered to reside at  $T = 300 \text{ K}$ , and the phonon relaxation frequency for both electronic and vibrational excitations is  $\omega_c = 30 \text{ cm}^{-1}$ . We use these bath parameters for all the models. To evaluate excitation transport to the energy sink A, we choose  $\Gamma_{l_3 \rightarrow A} = 1.8 \times 10^{12} \text{ s}^{-1}$ , where  $\Gamma_{l_3 \rightarrow A}$  is the rate of decay of excitation from the local state  $l_3$  to A.

A closer look at the energy level diagram for model 1 tells us that the lowest energy adiabatic state  $a_1$  is mostly localized on the local state  $l_1$ , with a small contribution from  $l_3$ , and the highest energy adiabatic state  $a_3$  is localized on the local state  $l_2$  with a small contribution from  $l_3$ . In contrast, the adiabatic state  $a_2$  is more mixed, with it being localized mostly on  $l_3$ , but with modest contributions from both  $l_1$  and  $l_2$  (see Table S1). Since the quantity of interest is the temporal evolution of population at  $l_3$ , it is easy to see that the adiabatic states  $a_2$  and  $a_3$  will play prominent roles in dictating the dynamics, since they are mostly localized on  $l_3$  and  $l_2$ , respectively. Therefore, the faster the  $a_3 \rightarrow a_2$  pathway is, the faster the  $l_2 \rightarrow l_3$  pathway will be, since  $a_3 \rightarrow a_2$  is downhill transport, which has a greater relaxation rate than uphill transport. The uphill transport, nevertheless, has a sizable rate because the energy gap between  $a_3$  and  $a_2$  is thermally accessible at  $T = 300 \text{ K}$ . The relaxation rates between pairs of adiabatic states, computed using eqs 9 and 10, are listed in Table 2 for the various models. Apart from the energetics, the overlap between the local and the adiabatic states, as seen in eqs 9 and 10, will also dictate the rate of energy transport. A larger overlap for a pair of adiabatic states





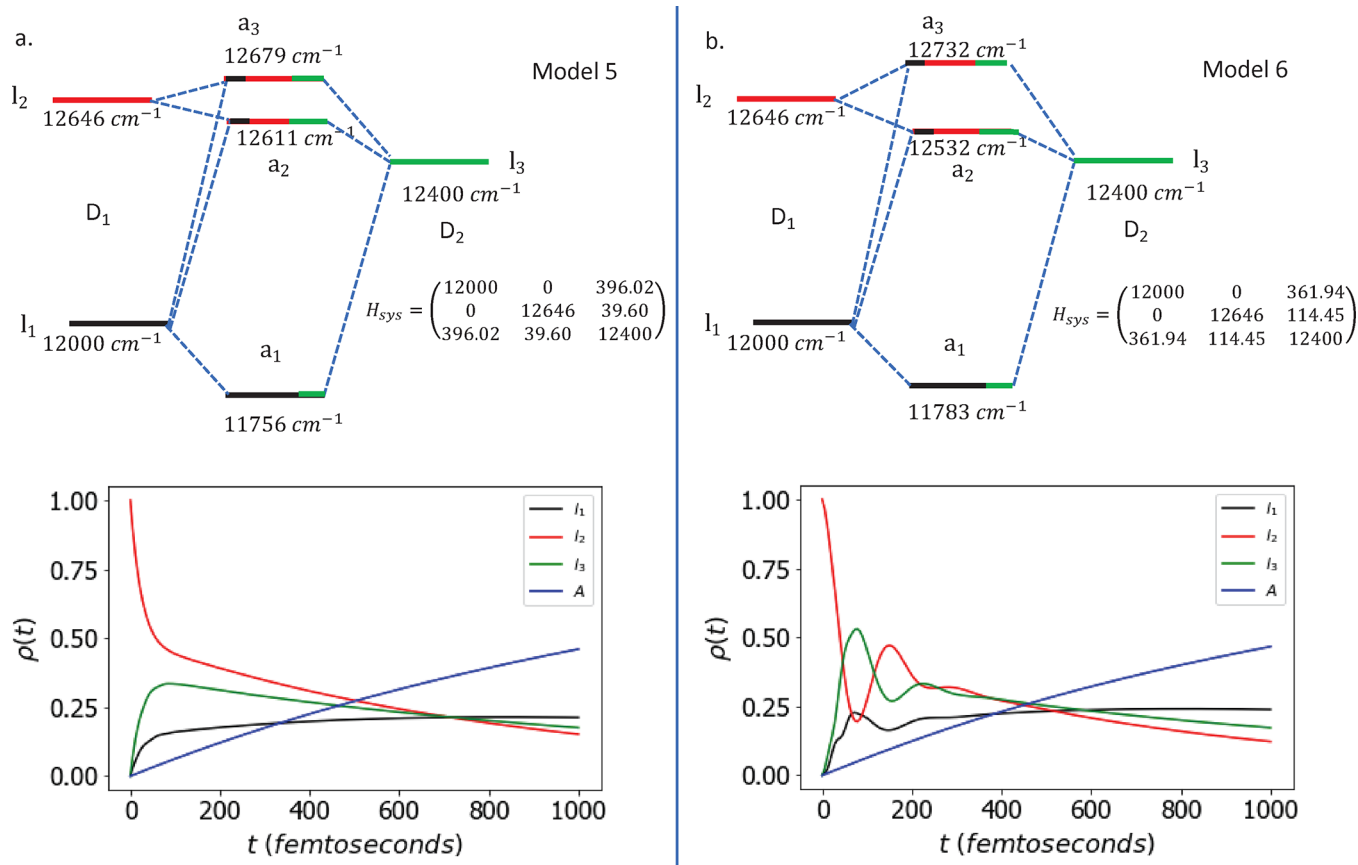
**Figure 3.** Population evolution at the local states  $l_1$ ,  $l_2$ , and  $l_3$  for models 1, 2, 3, and 4. The rate of decay of excitation from  $l_3$  to A is  $\Gamma_{l_3 \rightarrow A} = 1.8 \times 10^{12} \text{ s}^{-1}$ , where A acts as an energy sink. The blue curve depicts the growth of population at A.

with a given local state will contribute to an increased rate between the adiabatic states. Therefore, a more mixed adiabatic state that enjoys strong overlap with multiple local states is expected to boost transport. Strong electronic coupling between local states will also lead to more mixed states. Finally, the Huang–Rhys factor also determines mixing to an extent, since a larger Huang–Rhys factor implies a more nonlocal vibration that contributes to mixing through the off-diagonal overlap terms in eq 1. However, we should also note that a larger off-diagonal term will also lead to greater splittings between adiabatic states that will have an adverse effect on rates. Also, a greater mixing implies that the local states are strongly mixed to form the adiabatic states. Therefore, when we probe the population evolution at the local states, unidirectional transport between the local states is reduced since the adiabatic states are delocalized across the local states, thus enabling back-transport between the local states. Therefore, for maximum yield, the system will need to achieve the limit of optimum mixing, since this will determine both the energy splittings, and the overlap of adiabatic states with local states. Figure 3a depicts the local populations evolving with time for model 1, and we find that the population at  $l_3$  increases quite rapidly and overshoots the population at  $l_2$  before 100 fs.

We now compare the previous model with model 2, where  $l_2$  and  $l_3$  are degenerate, and the vibrational mode has a vibrational frequency  $800 \text{ cm}^{-1}$ , which is quasi-resonant with the excitonic energy gap  $\Delta\epsilon_{\text{electronic}} = 895 \text{ cm}^{-1}$ . The immediate

effects can be observed in the energy level diagram for model 2 in Figure 2b. We see that the mixing in the adiabatic states  $a_3$  and  $a_2$  is reversed compared with model 1.  $a_3$  is now the more mixed adiabatic state and is localized mostly on  $l_3$  with small contributions from both  $l_1$  and  $l_2$  (see Table S2).  $a_2$  is localized on  $l_2$ , with a small contribution from  $l_3$ .  $a_1$  is exactly the same for both models. While the energy gap between  $a_3$  and  $a_2$  is almost the same as in model 1, the rates are somewhat different (see Table 2) due to the fact that the higher energy  $a_3$  state is now the more mixed state compared to the mostly local lower energy  $a_2$ . A second important consequence is the pathway  $l_2 \rightarrow l_3$ , which we are interested in for the ratcheted photophysics and which is now facilitated through the uphill transport pathway  $a_2 \rightarrow a_3$  and, hence, has a lower transport rate compared to the downhill route. However, for a bath temperature of 300 K, it is a thermally accessible pathway. A comparison of the populations at  $l_3$  in Figure 3 for models 1 and 2 reinforces the energetic constraints of model 2 and depicts a lower thermal population at  $l_3$  for model 2.

In model 3, we reduce the electronic energy gap between the local sites  $D_1$  and  $D_2$  ( $\Delta E = 400 \text{ cm}^{-1}$ ) and increase the electronic coupling  $J$  in eq 11, such that we have the same energy gap between the electronic excitons  $\Delta\epsilon_{\text{electronic}} = 895 \text{ cm}^{-1}$ , as in models 1 and 2. This is used as the frequency of the vibrational mode in the vibronic Hamiltonian for model 3 in Figure 2c, and therefore, the vibrational mode is resonant with the energy gap between the electronic excitons. A direct consequence of increasing the value of  $J$  is larger off-diagonal



**Figure 4.** Energy level diagrams and population evolution at the local states  $l_1$ ,  $l_2$ , and  $l_3$  and acceptor A, for models 5 and 6. The oscillations in the local populations in model 6 have a frequency of  $200 \text{ cm}^{-1}$ , arising from the energy gap between the stationary adiabatic states  $a_3$  and  $a_2$ .

**Table 2. Relaxation Rates between the Stationary Adiabatic States, Computed Using Eqs 9 and 10<sup>a</sup>**

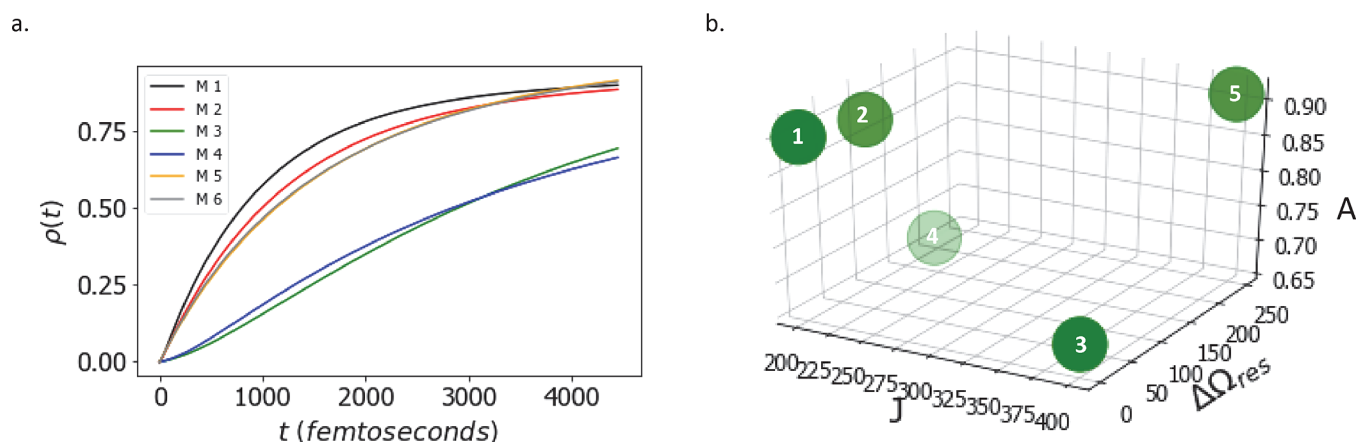
	model 1	model 2	model 3	model 4	model 5	model 6
$\Gamma_{a_3 \rightarrow a_1}$	0.02	0.22	0.01	0.25	0.47	0.37
$\Gamma_{a_3 \rightarrow a_2}$	23.87	26.40	0.39	0.97	46.91	8.07
$\Gamma_{a_2 \rightarrow a_1}$	0.23	0.03	1.04	0	0.58	0.78
$\Gamma_{a_2 \rightarrow a_3}$	17.71	19.77	0.11	0.47	33.93	3.09
$\Gamma_{a_1 \rightarrow a_2}$	0	0	0.01	0	0.01	0.02
$\Gamma_{a_1 \rightarrow a_3}$	0	0	0	0	0.01	0

<sup>a</sup>For example,  $\Gamma_{a_3 \rightarrow a_1}$  measures the downhill relaxation rate from the adiabatic state  $a_3$  to the adiabatic state  $a_1$ . All the values in the table are to be multiplied by a factor of  $10^{12} \text{ s}^{-1}$ .

couplings in the vibronic Hamiltonian. The coupling between the local states  $l_1$  and  $l_3$  is much larger compared to that between  $l_2$  and  $l_3$ . Consequently, the adiabatic states  $a_1$  and  $a_2$  have sizable overlap with the local states  $l_1$  and  $l_3$ , with  $a_2$  having a small contribution from  $l_2$ .  $a_3$  is predominantly localized on  $l_2$ , with a very small contribution from  $l_3$  due to a small, nonzero coupling between  $l_2$  and  $l_3$  (see Table S3). Despite the creation of more mixed states arising from a larger electronic coupling, the mixing here does not boost the transport between  $l_2$  and  $l_3$ , because (a) the mixing occurs mostly between  $l_1$  and  $l_3$ , leading to formation of  $a_1$  and  $a_2$ , and possibly a somewhat boosted transport between  $a_2$  and  $a_1$  due to the sizable overlap between the local and adiabatic states that affect the rates (eqs 9 and 10) ( $a_3$  that is localized on  $l_2$  is

mostly unaffected by this mixing) and (b) compared to models 1 and 2, the energy gap between  $a_3$  and  $a_2$  is larger, since we reduce the energy gap between  $l_1$  and  $l_3$  in this model at the cost of an increased value of  $J$  and a resonant vibrational mode, resulting in lower transport rates. This study shows that while a moderate amount of mixing might promote boosted transport as in model 1, a large value of an electronic coupling, while considering a resonant vibrational mode for a given  $\Delta E_{D_2, D_1}$ , could actually result in slower rates between  $l_2$  and  $l_3$ , as in model 3. Also, as discussed earlier, a large off-diagonal coupling implies reduced unidirectional transport to  $l_3$ . A comparison of the rates for models 1, 2, and 3 in Table 2 shows that rates for model 3 are significantly lower than those for models 1 and 2.

In models 1 and 3, we have considered vibrational modes resonant with the energy gap between electronic excitons. In model 2, the vibrational mode is quasi-resonant; however,  $l_2$  and  $l_3$  are degenerate, resulting in closely spaced adiabatic states and, therefore, sizable transport from  $l_2$  to  $l_3$ . In model 4, we consider a nonresonant vibrational mode with a vibrational frequency of  $700 \text{ cm}^{-1}$  while  $\Delta \epsilon_{\text{electronic}} = 895 \text{ cm}^{-1}$ . We find that the adiabatic states  $a_1$ ,  $a_2$ , and  $a_3$  are predominantly localized on  $l_1$ ,  $l_2$ , and  $l_3$ , respectively, and  $a_1$  and  $a_2$  have very small contributions from  $l_3$  and  $l_1$ , respectively (see Table S4). As a result of the negligible mixing, the term in the relaxation rates for a pair of adiabatic states in the rate equations, which arises from the overlap between the local states and the given pair of adiabatic states, is negligible and results in very small transport rates (see relaxation rates for model 4 in Table 2). Therefore, resonance/quasi-resonance is desirable since this



**Figure 5.** (a) Comparison of the acceptor populations for the six models discussed in our paper and designated as M1–M6, given that the rate of decay of excitation from  $l_3$  to A is  $\Gamma_{l_3 \rightarrow A} = 1.8 \times 10^{12} \text{ s}^{-1}$ . The growth of population at A is almost identical for models 5 and 6, which differ from each other with respect to the Huang–Rhys factor (0.01 for model 5 and 0.1 for model 6), despite the markedly different local state populations, as seen in Figure 4, and relaxation rates, listed in Table 2. Model 1 is seen to result in the fastest transport to A for subpicosecond time scales, followed by model 2. At longer times, models 5 and 6 take over. (b) Acceptor population at 4 ps, depicted by the z-axis label A, plotted with respect to the electronic coupling parameters  $J$  and  $\Delta\Omega$ , which denotes the shift of the vibrational frequency  $\Omega$  from the electronic-only stationary adiabatic energy gap between the  $S_1$  states of  $D_1$  and  $D_2$ , for models 1–5. Model 6 is not included explicitly since the acceptor population in model 6 has an almost identical evolution as in model 5, which differ with respect to the Huang–Rhys factor. The Huang–Rhys parameter  $\sigma$  is not included as a coordinate since the acceptor populations are seen to be almost identical for models that differ only with respect to  $\sigma$ .

enables the formation of mixed adiabatic states that leads to boosted transport.

Models 5 and 6 differ from each other with respect to the Huang–Rhys factor (0.01 for model 5 and 0.1 for model 6, implying a more nonlocal vibrational mode). While a larger Huang–Rhys factor implies the involvement of multiple vibrational states in the dynamics, a larger vibrational basis is a subject for future study. For these models, we explore a special case where the vibrational mode frequency of  $646 \text{ cm}^{-1}$  is resonant with the energy gap between the higher adiabatic state and the lower local state in the electronic-only Hamiltonian in eq 11. A significant consequence of this can be seen in the energy level diagrams for models 5 and 6 in Figure 4, where the adiabatic states are more strongly mixed when compared with the previous models. While the lowest adiabatic state  $a_1$ , for both the models, is given by a superposition of  $l_1$  and  $l_3$ , with a stronger contribution from the lower energy local state  $l_1$ , the higher energy adiabatic states  $a_2$  and  $a_3$  are strongly mixed adiabatic states, featuring sizable contributions from all the local states  $l_1$ ,  $l_2$ , and  $l_3$  (see Tables S5 and S6). While the off-diagonal couplings between  $l_1$  and  $l_3$  for these two models are not too different, the off-diagonal coupling between  $l_2$  and  $l_3$  in model 6 is nearly 3 times that of the coupling in model 5, resulting in a larger splitting ( $\approx 200 \text{ cm}^{-1}$ ) between the adiabatic states  $a_3$  and  $a_2$  in model 6, compared to an energy splitting of  $68 \text{ cm}^{-1}$  between  $a_3$  and  $a_2$ , in model 5. The larger off-diagonal coupling between  $l_2$  and  $l_3$  in model 6 is a consequence of the larger Huang–Rhys factor for model 6. From the energy level diagram, it is easy to see that the transport from  $l_2$  to  $l_3$  will mostly be mediated via the adiabatic states  $a_3$  and  $a_2$ . Therefore, the larger energy splitting between  $a_3$  and  $a_2$  gives smaller relaxation rates for model 6, as seen from the rates in Table 2 for models 5 and 6. From the population evolution plots in Figure 4, we can see that the population at  $l_1$  grows faster for models 5 and 6, compared to the previous models. This can be attributed to the fact that the local state  $l_1$  has presence in the adiabatic states  $a_3$  and  $a_2$  that feature a strong contribution

from the initial seat of excitation at  $l_2$ , which contributes to the  $l_2 \rightarrow l_1$  transport. The  $l_2 \rightarrow l_1$  transport is faster in model 6 compared to model 5 since the energy gap between  $a_2$  and  $a_1$ , where  $a_1$  features a strong contribution from  $l_1$ , is lesser in model 6, compared to model 5. The local state populations for model 6 exhibit oscillations at short times (up to  $\approx 200$  fs) that gradually decohere due to interaction with the bath. The oscillations have a frequency of  $200 \text{ cm}^{-1}$ , arising from the energy gap between the adiabatic states  $a_3$  and  $a_2$ . The local state population at  $l_3$  is seen to evolve similarly for models 5 and 6 at longer times, unlike the populations at  $l_2$  and  $l_1$  that evolve differently for the two models.

Figure 5a compares the temporal evolution of the acceptor population for all the six models. To reiterate,  $l_2$ , which labels  $|D_1 v_{D_1,1}^e v_{D_2,0}^e\rangle$ , is the initial seat of excitation localized on  $D_1$ , while  $l_3$ , which labels  $|D_2 v_{D_1,0}^e v_{D_2,0}^e\rangle$ , is localized on  $D_2$ . In our calculations, we investigate the transport from  $l_2$  to  $l_3$ , followed by a unidirectional decay to A, which serves as an energy sink. The rate of decay for the process is given as  $\Gamma_{l_3 \rightarrow A} = 1.8 \times 10^{12} \text{ s}^{-1}$ . Transforming to the stationary adiabatic basis, the rate of decay from a stationary adiabatic state to A can be expressed in terms of the respective eigenvector components with respect to  $l_3$ . Model 1 is seen to result in the fastest transport at subpicosecond time scales but at longer times, models 5 and 6 take over. Given the vibrational reorganization time of  $\tau_{\text{vib}} = 3.3$  ps, the excitation transport in model 1 outcompetes the vibrational relaxation the best, compared to the other models. It is surprising to note that the acceptor populations for models 5 and 6 grow almost together, despite markedly different relaxation rates for the two models. On the other hand, the eigenvector components that describe the overlap of the adiabatic states with the local states for both these models are almost identical. This is possibly the reason for similar acceptor populations for these two models, since the decay of population from an adiabatic state to the acceptor is described by only the eigenvector components with respect to  $l_3$ . It is to be noted that the rate  $\Gamma_{l_3 \rightarrow A}$  is a key parameter that dictates the



unidirectional transport. We have compared the acceptor populations for the above six models at  $\Gamma_{I_3 \rightarrow A} = 9 \times 10^{12} \text{ s}^{-1}$  and  $\Gamma_{I_3 \rightarrow A} = 0.45 \times 10^{12} \text{ s}^{-1}$ , in Figures S1 and S2. A larger value of  $\Gamma_{I_3 \rightarrow A}$  is seen to result in a faster growth at A for all models. For  $\Gamma_{I_3 \rightarrow A} = 9 \times 10^{12} \text{ s}^{-1}$ , the transfer of population to A is extremely rapid for models 1, 2, 5, and 6, compared to  $\Gamma_{I_3 \rightarrow A} = 0.45 \times 10^{12} \text{ s}^{-1}$ . Figure 5b depicts the acceptor population at 4 ps for all the models, plotted with respect to the electronic coupling  $J$  and  $\Delta\Omega$ , which denotes the shift of the vibrational frequency  $\Omega$  from the electronic-only stationary adiabatic energy gap between the  $S_1$  states of  $D_1$  and  $D_2$  in eq 11. The Huang–Rhys parameter  $\sigma$  is not included as a coordinate since the acceptor populations are seen to grow almost together for models that differ with respect to  $\sigma$ . It is noted that resonance/quasi-resonance, along with moderate  $J$  is desirable for enhanced transport to A. However, models 5 and 6, despite having  $\Delta\Omega = 249 \text{ cm}^{-1}$ , lead to fastest transport to A at longer times. The reason for this is that they explore a special resonance regime where the vibrational frequency is resonant with the energy gap between the higher adiabatic state and the lower local state in the electronic-only Hamiltonian in eq 11, which leads to optimally mixed adiabatic states and a boosted transport.

In the discussion above, we demonstrate that the system needs to achieve an optimum balance between electronic coupling, Huang–Rhys mixing, and resonance of the vibrational mode, to optimize  $I_2 \rightarrow I_3$  transport. Among all the models that we discuss, model 1 is best suited for a fast transport to  $I_3$  at subpicosecond time scales. While it enjoys a resonant vibrational mode, the electronic coupling between  $D_1$  and  $D_2$  is about half of the electronic coupling considered in model 3. An optimum electronic coupling is desirable for formation of mixed adiabatic states that can boost rates, since the rates feature an overlap between the local and adiabatic states. However, a very large electronic coupling in the presence of a resonant mode could have an adverse effect on transport as it diminishes unidirectional transport since the adiabatic states are now delocalized across the local states. On the other hand, if the vibrational mode is resonant/quasi-resonant with the energy gap between the higher adiabatic state and the lower local state in the electronic-only Hamiltonian, as seen in models 5 and 6, it explores a special regime where we have strongly mixed adiabatic states due to this specific form of resonance. Such models lead to enhanced transport. If the vibration is described by a large Huang–Rhys factor, as in model 6, coherent oscillations could be observed at shorter time scales that eventually decohere. A faster  $I_2 \rightarrow I_3$  transport leads to a quicker growth of the acceptor population. Therefore, models 1, 2, 5, and 6 are best suited for an efficient transport to A. It is important to note that though different values of Huang–Rhys parameters, for given values of  $J$  and  $\Delta\Omega$ , result in markedly different local state evolutions, they result in similar growth at A, as seen for models 5 and 6. While the calculations discussed in this work focus on energy transport, we can expect similar principles to apply to ratcheted electron transport. This is a subject of future research.

## ■ ASSOCIATED CONTENT

### SI Supporting Information

The Supporting Information is available free of charge at <https://pubs.acs.org/doi/10.1021/acs.jpcllett.0c02351>.

Tables S1–S6, contributions of the local states,  $I_1$ ,  $I_2$ , and  $I_3$ , to the adiabatic states  $a_1$ ,  $a_2$ , and  $a_3$  for the six models discussed in the main manuscript; Figures S1 and S2, comparison of the acceptor populations for the six models discussed in the main manuscript for different values of  $\Gamma_{I_3 \rightarrow A}$  [(a)  $\Gamma_1 = 9 \times 10^{12} \text{ s}^{-1}$ , (b)  $\Gamma_2 = 1.8 \times 10^{12} \text{ s}^{-1}$ , and (c)  $\Gamma_3 = 0.45 \times 10^{12} \text{ s}^{-1}$ ] (PDF)

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### Notes

The authors declare no competing financial interest.

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