Lawrence Berkeley National Laboratory

Recent Work

Title

SCALING, DEEP INELASTIC ELECTRON-NUCLEON SCATTERING, AND THE MULTIPERIPHERAL MODEL

Permalink

<https://escholarship.org/uc/item/94f220dm>

Authors

Shei, Sun-Sheng
Tow, Don M.

Publication Date

1970-12-01

SCALING, DEEP INELASTIC ELECTRON-NUCLEON SCATTERING,
AND THE MULTIPERIPHERAL MODEL

Sun-Sheng Shei and Don M. Tow

December 9, 1970

AEC Contract No. W-7405-eng-48

TWO-WEEK LOAN COPY

*This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 5545*

34
LAWRENCE RADIATION LABORATORY
UNIVERSITY of CALIFORNIA BERKELEY

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

SCALING, DEEP INELASTIC ELECTRON-NUCLEON SCATTERING,
AND THE MULTIPERIPHERAL MODEL*

Sun-Sheng Shei and Don M. Tow

Lawrence Radiation Laboratory
University of California
Berkeley, California 94720

December 9, 1970

ABSTRACT

The ABFST multiperipheral model is modified to study deep inelastic electron-nucleon scattering. In particular, the scaling property of the structure functions is derived. The multiperipheral derivation is compared with derivations that use other models.

A great deal of attention has recently been devoted to the study of inelastic electron-nucleon scattering from both experimental and theoretical points of view.¹ If only the final electron is observed in unpolarized inelastic e-N scattering, then all information is contained in $W_{\mu\nu}$ or the two structure functions W_1 and W_2 .²

Many theoretical models have been proposed for deep inelastic e-N scattering. They include parton model,³ canonical field theory model,² diffraction model,⁴ vector meson dominance model,⁵ and Veneziano-like model.⁶ Particular emphasis is on the scaling property of W_1 and νW_2 , as first suggested by Bjorken,⁷ in the Bjorken limit of $-q^2 \rightarrow \infty$

with $\frac{2m\nu}{-q^2}$ fixed. Here q is the virtual-photon momentum, ν is its energy in the laboratory system, and m is the nucleon mass.

The purpose of this letter is to present another model for deep inelastic e - N scattering. Our model is a simple modification of the ABFST multiperipheral model (MPM),⁸ which is known to explain many qualitative and quantitative features of high energy hadronic interactions.⁸⁻¹⁰ We assume that the high energy virtual photon-nucleon scattering amplitude is dominated by t -channel pion poles, with the exception of the last link that couples directly to the photon; for this last link we assume a t -channel nucleon pole. In other words, except for the last link our model is described by the ABFST MPM. Our model is shown in Fig. 1.

First, we present our results and interpretation and then describe the essential steps in our derivation. A more detailed description will appear in a forthcoming paper.¹¹

In our model we find that a cutoff (or an extra damping factor) for the momentum transfer of the exchanged nucleon is needed. This cutoff is consistent with the experimental fact that for high-energy scattering the transverse momentum of secondaries is limited. When this cutoff is introduced, we find in the limit of

$$\nu \rightarrow \infty, \quad -q^2 \rightarrow \infty, \quad \text{and} \quad \omega \equiv \frac{2m\nu}{-q^2} \text{ fixed but large;} \quad (1)$$

$$W_1 = c\omega^\alpha, \quad \nu W_2 = 2mc\omega^{\alpha-1}, \quad (2)$$

which implies

$$\frac{W_1}{\nu W_2} = \frac{\omega}{2m}, \quad (3)$$

$$\frac{\sigma_S}{\sigma_T} = \frac{-q^2}{\nu^2} \rightarrow 0, \quad (4)$$

where σ_S and σ_T are the "longitudinal" and "transverse" photoabsorption cross sections. Result (4) corresponds to spin- $\frac{1}{2}$ partons³ and to the Callan-Gross result¹² for a spin- $\frac{1}{2}$ quark current. Similarly, in the limit (1) we have

$$W_1^{(p)} = W_1^{(n)}, \quad \nu W_2^{(p)} = \nu W_2^{(n)}, \quad (5)$$

where superscripts p and n refer to proton and neutron, respectively. Result (5) is the same as that of the parton model³ for the case of the nucleon consisting of three quarks and a quark-antiquark "sea."

The multiplicity of pions is given by $\langle n_\pi \rangle \sim \ln \omega$. Our results are consistent with present experimental data^{1,13} and are very close to those of DLY.² Later we will compare the two models.

In our derivation we do not have to assume current conservation. But it can be shown that in the limit of ν , $-q^2 \rightarrow \infty$, our model is consistent with current conservation. In other words, in the limit of ν , $-q^2 \rightarrow \infty$, our model describes correctly the gauge invariant aspect of deep inelastic $e-N$ scattering.

We now present a brief derivation of our results. The absorptive part of the forward Compton amplitude is given by the sum of diagrams of Fig. 1; this sum, together with the kinematics, is represented in Fig. 2. We first neglect the photon link on the left. Averaging and summing over the spins of the nucleons on the right, we can consider the external nucleon to be spinless. From ABFST,⁸ the

absorptive part of this forward N-N amplitude satisfies the equation

$$-A(p', p) + r \cdot [p B_1(p', p) + p' B_2(p', p)] = \frac{1}{8\pi^4} \iint ds_0 d^4 p'' \frac{[-A^R(s_0) + r \cdot (p'' B_1^R(s_0) + p' B_2^R(s_0))] \delta[(p' - p'')^2 - s_0] A_{\pi N}(p'', p)}{(p''^2 - \mu^2)^2}, \quad (6)$$

where μ is the pion mass, the superscript R refers to low energy π -N scattering. Taking the trace, we get two equations:

$$A(p', p) = \frac{1}{8\pi^4} \iint ds_0 d^4 p'' \frac{A^R(s_0) \delta[(p' - p'')^2 - s_0] A_{\pi N}(p'', p)}{(p''^2 - \mu^2)^2}, \quad (7a)$$

$$r \cdot [p B_1(p', p) + p' B_2(p', p)] = \frac{r}{8\pi^4} \cdot \iint ds_0 d^4 p'' \frac{[p'' B_1^R(s_0) + p' B_2^R(s_0)] \delta[(p' - p'')^2 - s_0] A_{\pi N}(p'', p)}{(p''^2 - \mu^2)^2}. \quad (7b)$$

After attaching the photon link, we get for $W_{\mu\nu}$ the expression

$$W_{\mu\nu}(q, p) = \frac{f^2}{16m\pi^5} \int d^4 p' \frac{\delta[(p' - q)^2 - m^2]}{(p'^2 - m^2)^2}$$

$$\times \text{Trace} \{ [-A(p', p) + p B_1(p', p) + p' B_2(p', p)] (\not{p}' + m) \gamma_\mu (\not{p}' - q + m) \gamma_\nu (\not{p}' + m) \}, \quad (8)$$

where f is the γNN vertex function, which has been assumed to be a constant. We discuss only the B_1 term; similar arguments apply to A and B_2 terms. Designating the trace in (8) by $T_{\mu\nu}^{B_1}(q, p', p)$ and defining $W_{\mu\nu}^{B_1}(s, u)$ and $B_1(s', u')$ by

$$W_{\mu\nu}^{B_1}(q, p) = \iint ds du W_{\mu\nu}^{B_1}(s, u) \delta[(p + q)^2 - s] \delta(u + q^2), \quad (9a)$$

$$B_1(p', p) = \iint ds' du' B_1(s', u') \delta[(p + p')^2 - s'] \delta(u' + p'^2), \quad (9b)$$

we get

$$W_{\mu\nu}^{B_1}(s,u) = \frac{f^2}{16m_\pi^5} \iint ds' du' \frac{B_1(s',u')}{(u' + m^2)^2} Q_{\mu\nu}^{B_1}(s,u; s',u'; m^2, \mu^2), \quad (10)$$

where

$$\begin{aligned} Q_{\mu\nu}^{B_1}(s,u; s',u'; m^2, \mu^2) \\ = \int d^4p' T_{\mu\nu}^{B_1}(q,p',p) \delta[(p' - q)^2 - m^2] \delta[(p + p')^2 - s'] \delta(u' + p'^2). \end{aligned} \quad (11)$$

We can write $Q_{\mu\nu}^{B_1}$ as the sum $(c_1 g_{\mu\nu} + c_2 q_\mu q_\nu + c_3 p_\mu p_\nu + c_4 p_\mu q_\nu + c_5 q_\mu p_\nu)$. In the limit of large s and u , one can show that current is conserved. We therefore can write $Q_{\mu\nu}^{B_1}$ as

$$Q_{\mu\nu}^{B_1} = -\eta_1^{B_1} \left(g_{\mu\nu} + \frac{q_\mu q_\nu}{u} \right) + \frac{\eta_2^{B_1}}{m^2} \left[p_\mu + \frac{(p \cdot q) q_\mu}{u} \right] \left[p_\nu + \frac{(p \cdot q) q_\nu}{u} \right]. \quad (12)$$

The structure functions are given by

$$W_i^{B_1}(s,u) = \frac{f^2}{16m_\pi^5} \iint ds' du' \frac{B_1(s',u')}{(u' + m^2)^2} \eta_i^{B_1}(s,u; s',u'; m^2, \mu^2). \quad (13)$$

After a long but straightforward calculation, one can show from (11)

and (12) that to leading order in the limit of $s \rightarrow \infty$, $u \rightarrow \infty$, and $\frac{s}{u}$

fixed but large, one has

$$\frac{s^2}{4m^2 u} \eta_2^{B_1} \approx \eta_1^{B_1} \approx 2(us' - u's) Q_{ABFST}^{B_1}(s,u; s',u'; m^2), \quad (14)$$

where $Q_{ABFST}^{B_1}$ is the ABFST boundary function.⁸

To evaluate (13) in the Bjorken limit, we need to know the behavior of $B_1(s',u')$ for large s' and u' . Using (7b) and defining $A_{\pi N}(s'',u'')$ as in (9), we can show that for large s' and u' , with s'/u' fixed but large, we have

$$B_1(s', u') \approx - \frac{u'}{s'} \left\{ \frac{1}{8\pi^4} \int ds_0 B_1^R(s_0) \iint ds'' du'' \frac{A_{\pi N}(s'', u'')}{(u'' + \mu^2)^2} \right. \\ \left. \times Q_{ABFST}(s', u'; s'', u''; s_0) \right\} . \quad (15)$$

From Regge pole dominance in the ABFST MPM, we know that^{8,14}

$$A_{\pi N}(s'', u'') \underset{s'' \text{ large}}{\sim} s''^\alpha \phi_\alpha^{\pi N}(u''), \quad (16a)$$

where

$$\phi_\alpha^{\pi N}(u'') \underset{u'' \text{ large}}{\sim} \frac{1}{(u'')^{\alpha+1}} . \quad (16b)$$

From (15) and (16) we can show that

$$B_1(s', u') \underset{s' \text{ large}}{\sim} s'^{\alpha-1} \phi_\alpha^{B_1}(u') , \quad (17a)$$

where

$$\phi_\alpha^{B_1}(u') \underset{u' \text{ large}}{\sim} \frac{1}{(u')^\alpha} . \quad (17b)$$

Introducing a cutoff (or an extra damping factor) for u' , we obtain in the limit of $s \rightarrow \infty$, $u \rightarrow \infty$, and $\frac{s}{u}$ fixed but large,

$$W_1^{B_1}(s, u) \approx c' \left(\frac{s}{u}\right)^\alpha, \quad W_2^{B_1}(s, u) \approx \frac{4m^2 c'}{u} \left(\frac{s}{u}\right)^{\alpha-2}, \quad (18)$$

where c' is a proportionality constant. Similar results are obtained from A and B_2 terms, except that no cutoff is needed for the A term. Remembering that $u = -q^2$ and $s \approx 2m\nu$ for large ω , we obtained the results quoted in Eqs. (1)-(4). Equation (5) follows from the fact that the vacuum Regge pole couples identically to $p\bar{p}$ and $n\bar{n}$ channels. If the vacuum Regge pole has intercept one, $\nu W_2(\omega)$ should approach a constant.

We conclude this letter with the following remarks.

1. Our multiperipheral derivation is concerned with unitarity diagrams, and not Feynman diagrams, so it is not necessary to consider

renormalizations. In DLY's field theory derivation,² only the leading term is kept in each order; they had to assume the sum of these leading terms is the dominant contribution. For this reason, they cannot determine the constant in Eq. (2). On the other hand, the multiperipheral integral approach keeps all terms; only at the end does one approximate by keeping the leading Regge poles. In this respect, the multiperipheral derivation is more rigorous.¹⁵ Furthermore, in principle we can determine the eigenfunction of the inhomogeneous integral equation and therefore the constant in (2).

2. Our model is consistent with the ABFST multiperipheral assumption of meson-exchange dominance. Note that our model has only one exchanged nucleon, as compared with DLY's, in which all exchanged particles are nucleons. We have shown that deep inelastic e-N scattering can be adequately described by a minimal modification of the ABFST MPM.

3. We have also calculated the diagram with no nucleon exchange, i.e., the photon couples to two pions. In this case we get the same functional dependence for W_2 and for W_1 an extra factor of $\ln(-q^2)/(-q^2)$, i.e., $W_1 \rightarrow 0$ in the limit (1).¹⁶ Therefore, at least for W_1 , in the limit of large $(-q^2)$ the diagram with γ coupling to spin- $\frac{1}{2}$ particle dominates over diagram with γ coupling to spin-0 particle.

4. Our model predicts that an energetic $N\bar{N}$ pair is produced in deep inelastic e-N scattering. If this is not observed experimentally, it means that the meson-exchange ladder is not yet negligible in the present kinematic region. Since our derivation of scaling (as in almost all other models) is an asymptotic derivation, it still leaves unanswered

why does scaling set in so early.

5. In our model, the contribution of any Regge trajectory scales, in contrast to Harari's diffraction model,⁴ in which contributions of "ordinary" trajectories do not scale.

6. The cutoff or extra damping factor can be justified because the nucleon propagator does not sufficiently damp out large transverse momenta (corresponding to large momentum transfers). This cutoff also allows all other momentum transfers to be small, consistent with the notion of peripheralism. Note that no cutoff is needed for momentum transfers corresponding to exchanged pions.

7. We have assumed a constant for the off-shell γNN vertex function in the asymptotic limit we are interested in. This is not an unreasonable assumption, because we can generalize the plausibility argument of Matsuda and Suzuki¹⁷ to include spin as in our case. What is essential in this generalization is that electromagnetism is minimally coupled to spin- $\frac{1}{2}$ particles and there is no derivative coupling among hadrons.

8. Our derivation may actually be more general than the ABFST MPM, since all that is needed in our derivation is Regge behavior as in (16a) and a particular asymptotic off-shell dependence, with the latter depending on the particular extra damping factor introduced. It is not necessary to know how this asymptotic behavior is derived.

9. The method of this paper can of course be used to study ν -N and $\bar{\nu}$ -N scatterings. By crossing, it can also be used to study $e^- + e^+ \rightarrow \bar{p} + \text{anything}$.

We thank Professors Geoffrey F. Chew and Mahiko Suzuki for valuable comments and for reading the manuscript.

FOOTNOTES AND REFERENCES

- * This work was supported in part by the U.S. Atomic Energy Commission.
- 1. For a review, see articles by F. Gilman and R. E. Taylor, in International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, 1969, edited by D. W. Braben (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970).
- 2. For their definitions, see S. D. Drell, D. J. Levy, and T.-M. Yan, Phys. Rev. Letters 22, 744 (1969); Phys. Rev. 187, 2159 (1969); Phys. Rev. D1, 1035 (1970). In this paper their model is designated as DLY.
- 3. J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).
- 4. H. D. I. Abarbanel, M. L. Goldberger, and S. B. Treiman, Phys. Rev. Letters 22, 500 (1969); H. Harari, Phys. Rev. Letters 22, 1078 (1969); *ibid.*, 24, 286 (1970).
- 5. J. J. Sakurai, Phys. Rev. Letters 22, 981 (1969); C. F. Cho and J. J. Sakurai, Phys. Letters 31B, 22 (1970); W. K. Tung, Phys. Rev. Letters 23, 1531 (1969).
- 6. P. V. Landshoff and J. C. Polkinghorne, Nucl. Phys. B19, 432 (1970).
- 7. J. D. Bjorken, Phys. Rev. 179, 1547 (1969).
- 8. L. Bertocchi, S. Fubini, and M. Tonin, Nuovo Cimento 25, 626 (1962); D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento 26, 896 (1962). This model is called the ABFST multiperipheral model.
- 9. D. M. Tow, Phys. Rev. D2, 154 (1970).
- 10. G. F. Chew, T. Rogers, and D. R. Snider, Phys. Rev. D2, 765 (1970).
- 11. S.-S. Shei and D. M. Tow, to be published.
- 12. C. G. Callan and D. J. Gross, Phys. Rev. Letters 22, 156 (1969).

13. E. D. Bloom et al., Recent Results in Inelastic Electron Scattering, SLAC preprint, Sept. 1970.
14. C. Ceolin, F. Duimio, R. Stroppolini, and S. Fubini, Nuovo Cimento 26, 247 (1962).
15. Our derivation is similar to that of Abarbanel, Goldberger, and Treiman (Ref. 4), who used a Bethe-Salpeter integral equation approach to study a ϕ^3 model with a spinless photon.
16. G. Altarelli and H. R. Rubinstein, Phys. Rev. 187, 2111 (1969), also obtained this result in a ϕ^3 model.
17. S. Matsuda and M. Suzuki, Phys. Rev. D1, 1778 (1970).

FIGURE CAPTIONS

- Fig. 1. Diagram representing our model; solid, dotted, and wavy lines are nucleons, pions, and photons, respectively.
- Fig. 2. Diagram representing unitarity sum and defining kinematic variables; solid, dotted, and wavy lines are nucleons, pions, and photons, respectively.

Fig. 1.

XBL 7012 - 4212

Fig. 2.

XBL 7012-4211

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

- A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or*
- B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.*

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720