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Journal

Physics of Fluids B, 2(3)

ISSN

0899-8221

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Publication Date

1990

DOI

10.1063/1.859342

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The transport of ions in a turbulent plasma

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(Received 9 August 1989; accepted 9 November 1989)

Direct, experimental results show cross-field majority species ion transport which is linearly proportional to electrostatic fluctuation levels. Laser-induced fluorescence was used to tag ions within a plasma. The ion diffusive and convective motion could be observed. In a quiet plasma the ion cross-field diffusion agreed with classical predictions. A controlled level of electrostatic turbulence was then introduced into the plasma. The resulting increase in cross-field diffusion was consistent with $D_{\perp} \approx 4(cT_e/eB)(\delta n_i/n_0)$.

I. INTRODUCTION

Statistical physics plays an important underlying role in the understanding of plasma physics. Laboratory and space plasma transport processes occur over a wide range of parameters, from those dominated by classical collisional effects to plasmas filled with super high intensity turbulence. By looking in virtually any journal where plasma research is published, one may see that tremendous experimental and theoretical efforts have been directed at understanding irreversible processes. There remains a need to make a specific experimental connection between transport processes and the responsible microturbulent or classical mechanisms. A commonly used foundation for attempting theoretically to understand irreversible processes in plasmas is the use of the dressed test particle concept.¹⁻⁴ In this paper, an experiment is described wherein majority species test ions were created within a turbulent plasma and their subsequent transport was measured. Previous measurements by McWilliams and Okubo⁵ in a quiet plasma show test ion transport consistent with classical predictions.

The motivation for this paper is stated thusly: the desire is to understand the role that microscopic processes play in plasma transport. One might wish to begin by studying transport under the simplest macroscopic geometries and parameters possible and introduce turbulence independently. After understanding the transport dependence on both the macroscopic and microscopic variables, one might then proceed to much more complex experiments, where transport depended on magnetic shear, curvature, spatial location, device-specific instabilities, nonlinear saturation mechanisms, recycling, etc. The question that arises here is are there transport properties that depend on microscopic turbulence but are independent of boundary conditions?

Experimentally, extensive work has been done to measure transport of particles across magnetic fields. Most experimenters have been unable to create test particles and hence have measured bulk plasma motion rather than the movement of test particles. Commonly in these experiments the observer has control over macroscopic parameters and

observes primarily macroscopic changes in the plasma. However, it is widely suspected that varying the macroscopic parameters leads to changes in microscopic activity in the plasma. It is also conjectured that these microscopic activities then result in changes in macroscopic observables. For example, experimenters may control the macroscopic variables of magnetic field strength, plasma current, plasma density, and plasma temperature. It is common that these variables may be inextricably connected. These macroscopic properties may result in microscopic events such as plasma waves, instabilities, and associated saturation levels, as well as spatial and temporal dependence of these microscopic phenomena. These microscopic phenomena may in turn be responsible for macroscopic particle and energy confinement times in plasmas, because of their role in producing transport. Often in these circumstances only global properties, such as confinement times, can be inferred and transport coefficients are described by fitting power laws to the data, a technique of considerable value. A concise assessment of such experiments has been given by Surko and Slusher.⁶ One may refer to the review papers of Hinton and Hazeltine⁷ and Liewer⁸ to gain access to the body of knowledge obtained in these cases. Experiments such as the heavy-ion beam probe work of Hallock, Wootton, and Hickok⁹ and the far-infrared laser measurements of Kim *et al.*¹⁰ are recent examples of techniques used to measure local plasma parameters that relate to cross-field transport in these complicated experiments. In the present paper we discuss an experiment where some measurable control over microscopic plasma turbulence was available, without requiring substantial changes in the basic macroscopic plasma parameters.

The technique of laser-induced fluorescence (LIF), described in detail in Refs. 11–15, can be very valuable in plasma transport experiments. Using LIF, and particularly the extensions of it to optical tagging (first demonstrated by Stern, Hill, and Rynn¹⁶) and optical tomography (by McWilliams and Koslover^{17,18}), has allowed direct measurements of test particle transport and forms the basis of the experimental technique used for the present paper. Preliminary results showing anomalous cross-field transport in the presence of sizable wave fluctuations were reported by McWilliams *et al.*¹⁹

In this paper we describe an experiment that quantified test ion transport in a homogeneous plasma where homogeneous turbulence was introduced and where the amplitude

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of the turbulence was under the control of the experimenter. The test ions created in the plasma belonged to the majority plasma ion species (no beam injections), and, except for the distinguishing label, were statistically indistinguishable from the background ions. Experimentally, a small population of the existing plasma ions were tagged (without introducing perturbations to the plasma by the tagging process) and the motion of these ions was followed after tagging. Direct, nonperturbing, high resolution measurements of the test ion cross-field transport were made as a function of turbulence fluctuation level. As a foundation for comparison of these results, measurements were made of the transport of test ions in a quiet plasma and were reported by McWilliams and Okubo.⁵ The results of the McWilliams–Okubo paper showed that test ions in a quiet plasma with ion density fluctuations of $\langle (\delta n_i/n_0)^2 \rangle^{1/2} \approx 2 \times 10^{-3}$ were transported across the magnetic field in agreement with classical transport predictions of the modified Langevin equation, Rostoker,¹ and Klimontovich.²⁰ The references cited in the McWilliams–Okubo paper, the experimental techniques, and the method of data analysis are all relevant to the work on turbulence reported here and it is suggested that a copy of Ref. 5 be at hand while perusing the present paper.

II. WAVE THEORY

To simplify interpretation, it was desired in the experiment to produce turbulence that was spatially homogeneous and isotropic perpendicular to the confining magnetic field. A second requirement was that the turbulence amplitude should be controlled directly by the experimenter so that other plasma parameters could be held constant. The method devised to achieve these goals was to launch high-frequency electron plasma pump waves from an external antenna, these waves affecting the ions only when parametric decay produced low-frequency ion wave turbulence in the plasma. External control of the pump wave amplitude thus controlled the turbulence amplitude.

The turbulence was found to be fairly broadband, spatially homogeneous, and isotropic perpendicular to the confining magnetic field. The ion waves propagate away from their point of origin and appear to scatter substantially, perhaps off the plasma edge, before being damped. This is evidenced by taking interferograms, which show a loss of phase correlations in about one third of a wavelength across the magnetic field. Another factor that might contribute to phase decorrelation is the spectral width of the ion waves, which was seen to be about 36% [at full width at half-maximum (FWHM)] of the fundamental wave frequency.

The electron plasma pump wave (EPW) frequency was chosen to be well above the ion cyclotron and ion plasma frequencies so that the wave would propagate via electron response, without substantial ion interaction. At power levels below the parametric decay threshold, neither ion heating, diffusion increases, nor ion turbulence increases were observed. For the regime $\omega_{ci}, \omega_{pi} \ll \omega_0$, $\omega_2 \ll \omega_{pe} \ll \omega_{ce}$, where ω_0 is the pump wave frequency and ω_2 is the lower sideband frequency, the nearly electrostatic electron plasma wave dispersion relation simplifies (for $\omega = \omega_0$ or ω_2) to ω

$\approx \omega_{pe} k_{\parallel} / k_{\perp}$, where $k_{\parallel}$ ($k_{\perp}$) is the wavenumber parallel (perpendicular) to the confining magnetic field.²¹

By increasing the pump wave power, the parametric decay instability may produce electrostatic ion cyclotron waves (EICW). These waves, $\omega \equiv \omega_1$, follow the dispersion relation

$$\omega = \omega_{ci} + (T_e/T_i) \omega_{ci} \Gamma_1(k_{\perp}^2 \rho_i^2),$$

for $v_e \gg \omega/k_{\parallel} \gg v_i$, where v_e (v_i) is the electron (ion) thermal speed, $(2T_e/m_e)^{1/2}$, and $\Gamma_1(x) = e^{-x} I_1(x)$. The theory of parametric decay²² has been applied to electron plasma waves by several authors^{23–26} for a dipole pump approximation, $k_0 = 0$, and some finite pump effects.²⁶ From Wong, Wilson, and Porkolab²⁷ we estimate the parametric coupling coefficient to be dominated by the $E \times B$ drift. The decay threshold is also modified by convection.²⁸

III. DIFFUSION THEORY

A common view of diffusive processes in plasmas begins with the assumption of the indistinguishability of particles belonging to the same species. With this view, like particle collisions should not lead to substantial transport across magnetic fields. However, there is also a need to consider distinguishable particles. Questions of particle transport, energy confinement, impurity transport in plasmas, etc., ultimately require knowledge of the motion of individual particles through the background plasma via random walk or nonrandom processes. Much can be learned about transport in plasmas when the time-dependent motion of individual particles can be followed without perturbing the plasma by the interrogating experimental technique.

The classical diffusion of test ions in a nonturbulent plasma is described by two-body Coulomb collisions modified by collective plasma effects which shield the range of interaction. When two indistinguishable ions collide, the individual before-and-after collision orbits cannot be connected unambiguously and little measurable diffusion occurs. This leads to the requirement for interspecies collisions to cause large measurable particle transport across magnetic fields. When distinguishable ions collide, the individual before-and-after collision orbits can be connected unambiguously. This gives the ability to study the migration of ions within plasmas. This technique of following the orbit of individual ions of the majority species plasma was demonstrated by Stern, Hill, and Rynn¹⁴ and was used shortly thereafter by McWilliams *et al.*,¹⁹ who reported preliminary measurements of ion cross-field transport. For distinguishable ions in a quiet plasma the leading diffusive term can be ion–ion collisions. In such a case the classical ion diffusion coefficient is predicted^{1,20} to be

$$D_{\perp, \text{class}} = \frac{1}{2} \rho_i^2 \nu_{ii}, \quad (1)$$

for a test ion in a quiet Maxwellian plasma with $T_i \approx T_e$, where ρ_i is the thermal ion Larmor radius and ν_{ii} is the ion–ion collision frequency. Experiments by McWilliams and Okubo⁵ showed agreement with Eq. (1). For the experiments described in the present paper, the base level of diffusion is measured to be essentially classical, given by Eq. (1). Turbulent effects then lead to an increase in diffusion above

the classical level. Hence we may say that the reference level of diffusion in the present experiment is reasonably well understood and described by theory and experiment in Ref. 5. The reference level of diffusion is not described by the term "anomalous" and comparisons between theory and experiment (both here and in Ref. 5) are not made by normalizing the data or the theory to arbitrary values or best fits.

In general, the turbulent diffusion of ions is a very complex problem. The basic elements of diffusion theories put forward by Rostoker,¹ Dupree,²⁹ and Taylor and McNamara³⁰ are noted here. These theories should be taken as sample theoretical predictions that possibly do not include some physical elements required for a precise description of the transport of ions in a turbulent plasma.

Dupree²⁹ presented a heuristic model for the diffusion coefficient, due to drift waves, as

$$D_1 = (1/\sqrt{2})(c\bar{E}_1/k_1 B), \quad (2)$$

which shows that strong electric fields may lead to a linear dependence of D_1 on the electric field.

Using the test-particle approach Rostoker¹ shows D_1 should be calculated according to

$$D_1 = \frac{4c^2}{B^2} \int_0^\infty d\tau \langle \mathbf{E}_1(\mathbf{x}, t) \cdot \mathbf{E}_1(\mathbf{x} + \mathbf{r}(\tau), t + \tau) \rangle. \quad (3)$$

Taylor and McNamara³⁰ used an approach similar to Rostoker's, using the particles' velocity correlation function, based on $\mathbf{v} = c\mathbf{E} \times \mathbf{B}/B^2$. Their result is

$$D_1 = \sqrt{2}(c\langle\phi^2\rangle^{1/2}/B), \quad (4)$$

where ϕ is the magnitude of the potential fluctuation. In the case of our experiment, the fluctuations are due to electrostatic ion cyclotron waves and the $\mathbf{E} \times \mathbf{B}$ drift is modified to

$$v_1 = [\omega\omega_{ci}/(\omega^2 - \omega_{ci}^2)]c\mathbf{E}_1/B. \quad (5)$$

For the experiment, the central frequency of the turbulence is about $1.32\omega_{ci}$. Equation (4) would be modified to our experiment to be approximately

$$D_1 \approx 4.47(c\langle\phi^2\rangle^{1/2}/B). \quad (6)$$

If the linearized Boltzmann relation is satisfied for these waves, then

$$D_{1,\text{turb}} \approx 4(cT_e/eB)(\delta n_i/n_0)\text{rms}. \quad (7)$$

That the linearized Boltzmann relation is satisfied for our experiment was shown by Lang and Boehmer,³¹ who also showed how the density fluctuations depend on potential when the linearized relation is violated.

Many present day experiments for which D_1 is inferred from experiment compare results to theories involving trapped particle effects, i.e., the bouncing of particles due to magnetic mirror effects. When these trapped particle effects are thought to be significant, the largest vector component of the magnetic field may not play a significant role in the D_1 theories, which then leads to large cross-field transport coefficient predictions compared to the equations presented above, for which no component of $\mathbf{B}$ is ignored. In the Irvine experiment, the magnetic field ripple is about 3% peak to peak and the experiments were performed with plasma populations that had drifted through a full ripple before entering

the experimental region. It is estimated that the fraction of the flux, measured by the laser in the experiment, which is due to particles that become trapped, is less than 2×10^{-3} of the total laser detected signal. Hence no trapped particle effects are invoked for the theory of the equations above and none are invoked in the interpretation of the experiment. The experiment was designed to have the simplest magnetic field geometry possible so that initial understanding of turbulent transport might be achieved without resorting to complex interpretation. In the future, if trapped particle effects are to be investigated, the magnetic field geometry should be changed accordingly.

IV. LIF TAGGING AND OPTICAL TOMOGRAPHY

A. Laser-induced fluorescence (LIF) and ion tagging

Experimentally, there is a strong motivation for performing direct, noninferential measurements of transport in plasmas. The power of the LIF diagnostic is that it can measure directly the motion, both diffusive and convective, of majority species ions in a plasma. The technique used to measure ion diffusion is described in detail in Ref. 5 and is outlined only briefly here.

Essentially, LIF may be used to tag optically a small fraction of the majority species ions within a small volume of the plasma (see Fig. 1). Laser interrogation of the surrounding plasma at later times following the tagging yields a direct measurement of the time history of the location of the tagged ions. One may observe the diffusive spreading of the ions from the initial tagging volume as well as observing motion such as convection. All of these tagging and detecting measurements do so without perturbing the plasma parameters. Only the atomic states of the ions are changed, but not the degree of ionization, ion temperature, ion density, etc. Hence LIF may be used to create test ions in a plasma.

FIG. 1. General plasma configuration and diagnostic geometry. The cylindrical plasma formed at the hot plate drifts down the magnetic field to the endplate, where it is lost. The T and D beams are the tagging and detection laser beams, respectively. Bottom of figure: horizontal profile of tagged particles at two different axial locations.

B. Optical tomography

The measurement of diffusion and its relation to plasma parameters requires a careful characterization of the plasma ions. The technique of optical tagging of plasmas was demonstrated first by Stern, Hill, and Rynn¹⁴ and shown to be usable for diffusion measurements by McWilliams *et al.*¹⁹ The LIF diagnostic also has been shown to give precise characterization of the two- and three-dimensional ion phase space distributions by the invention of optical tomography by McWilliams and Koslover^{17,18} so that the full $f_i(x, v, t)$ may be measured without perturbing the plasma. That is to say, high space and velocity space resolution measurements of the ion distribution function may be made by using optical tomography. For example, the initial plasma used in these experiments can be shown to be a drifting Maxwellian distribution (see Fig. 2), information that needs to be known for the analysis of the present experiments. In the presence of the parametric decay induced electrostatic ion cyclotron waves, the ion temperature perpendicular to the confining magnetic field increased while there were no substantial changes in the parallel distributions (see Fig. 3), again, such information being required for proper knowledge of ion Larmor orbits when determining diffusive motion, as opposed to reversible processes.

V. EXPERIMENTAL RESULTS

The experiments were performed in the University of California, Irvine Q Machine,³² which was run single ended (see Fig. 1). The nearly completely ionized barium plasma was formed via contact ionization on a rhenium-plated tungsten hot plate. The resulting 5 cm diam plasma flows down the axial magnetic field, $B_0 = B_0 \hat{z}$ with $B_0 < 7$ kG, typically at a speed of $5 \times 10^4 < v_z < 1 \times 10^5$ cm/sec, and is buried at a cold floating plate 1.4 m downstream. Initial electron and ion temperatures are approximately 0.2 eV. Electron temperature was inferred from a Langmuir probe while ion velocity distributions were obtained using laser-induced fluorescence techniques. Plasma densities were $5 \times 10^9 < n_e < 1.7 \times 10^{10}$ cm⁻³, the density being inferred from Langmuir probes and through the angle of propagation of a lower hybrid test wave ($\theta \approx \omega/\omega_{pe}$ with respect to B_0).

FIG. 2. Ion distribution function of undisturbed, drifting nearly Maxwellian plasma. Shown are contours of constant f_i in 10% increments from 100% to 20%.

FIG. 3. Ion distribution function in the presence of low frequency parametric decay ion waves induced from the electron plasma pump waves, $B = 6$ kG.

A. Turbulence characterization

Low-frequency ion fluctuations were produced throughout the plasma by the use of parametric decay-induced electrostatic ion cyclotron waves. Electron plasma waves were launched from a plane array antenna consisting of eight dielectric-shielded parallel metallic elements, similar to coaxial loop antennas used in the diffusion experiments reported in Ref. 19. These electron plasma waves were launched with a 30 MHz pump frequency with the antenna phased to excite a principal parallel wavelength of 12 cm. Interferograms of the pump wave verified that the propagation characteristics of the waves were as expected from cold plasma theory.

There is no noticeable change in the ion distribution function until the pump wave power is raised above a threshold power, typical of parametric decay processes. At the threshold power level, where changes in the ion distribution are first observed, the presence of decay waves is also noted. The pump wave decays into a daughter lower sideband electron plasma wave and a low-frequency electrostatic ion cyclotron wave. The growth rate of the decay process is not very sensitive to the ion wave frequency and hence a broad spectrum of low-frequency decay waves is observed. The fundamental elements of the decay process are displayed in Fig. 4. Figure 4(a) shows the peak frequency of each observed low-frequency ion wave and the difference of the pump and lower sideband frequency, as a function of confining magnetic field. Frequency conservation is observed to be satisfied as well as the magnetic field dependence of the low-frequency dispersion relation. Figure 4(b) shows the ion wave amplitude and observed perpendicular ion temperature as a function of pump wave power. Above threshold power levels, the ion temperature and the low-frequency ion wave amplitude increase with increasing pump power. These results are not surprising, because the pump wave, whose energy resides in perpendicular field energy and parallel electron kinetic energy, does not interact significantly with the ions for two reasons: the pump wave phase velocity is well above the ion thermal velocity and the energy transfer time from collisionally heated electrons to ions is longer than an ion transit time in the plasma. The amplitude of the ion fluctuations thus may be controlled by adjusting the pump wave power level.

FIG. 4. Identification of parametric decay waves. (a) Demonstration of frequency matching of parametric decay produced waves: triangles, ion waves; circles, difference of pump and sideband electron plasma waves; solid lines, f_{ci} . (b) The plots of ion wave potential amplitude and ion temperature versus rf pump power show the instability threshold and ion heating only above the threshold. Circles, ion wave amplitude; triangles, perpendicular ion temperature, $B = 6$ kG, -20 dB = 0.5 W.

Figure 5 shows a typical frequency spectrum of the low-frequency ion waves. By examining Fig. 5, one sees that the ion waves follow $\omega_i \approx l(1 + \delta_i)\omega_{ci}$ with $\delta_{l+1} < \delta_l$ [$l \leq 4$ shown in Fig. 4(a) although l up to 9 was observed commonly]. Hence this decay does not resemble a cascade pump depletion. As seen in the figure, the spectral width of each peak of the ion waves is substantial; typically the spectral width of the fundamental was $(\delta\omega_1/\omega_1) = 0.36$ at full width at half-maximum.

The low-frequency decay waves are formed within the resonance cone of the pump wave, yet were observed to be fairly uniform in amplitude across the plasma. The calculated spectrum of the ion cyclotron damping length covered the range of 1–4 cm across the magnetic field, and as the plasma edge was about 2.5 cm from the plasma center, there probably was significant scattering and randomization of the ion wave phase throughout the plasma. Additionally, the broad frequency spectrum of the low frequency ion waves leads to a breadth in the expected k spectrum for the waves. Interferograms of the ion waves yielded phase information for about one third of the expected wavelengths, possibly due either to

FIG. 5. Low-frequency ion wave spectrum during parametric decay. Note that $B = 6$ kG, $f_{ci} \sim 66$ kHz.

the scattering or to the spectral width of the waves. The range of expected $k_{\perp}$ values for the ion waves was about $4\text{--}8$ cm^{-1} . Using this range of $k_{\perp}$ values and the measured width in frequency one may estimate the autocorrelation time $\tau_{ac} \sim [k\delta(\omega/k)]^{-1}$ for these waves to be about $\tau_{ac} = 1/(3f_{ci})$.

B. Turbulent diffusion

It was desired to produce an experiment where the dependence of the cross-field diffusion coefficient $D_{\perp}$ on electrostatic turbulence could be determined. There are many models of turbulent diffusion and these models predict many different dependencies on the ion fluctuation level. While most plasma physicists agree that fluctuations should contribute to transport, there have been no direct experimental confirmations of the functional dependence of $D_{\perp}$ on fluctuations, only indirect, inferential conclusions noting "anomalous" levels of transport at the same time as fluctuations were reported. McWilliams *et al.*¹⁹ reported the first direct observations that majority ion diffusion depended on fluctuation level, but did not determine the functional dependence. It was the purpose of the presently reported experiment to determine the functional dependence of cross-field diffusion of majority species ions on the plasma fluctuation levels. This experiment was performed by creating a controllable level of plasma fluctuations, as described above, and measuring the diffusion coefficient by direct observation of the diffusive spreading of a tagged population of majority species ions.

A plot of the diffusion coefficient versus the rf pump power (see Fig. 6) shows a power threshold, below which the observed diffusion agreed with classical collisional ion processes as reported in Ref. 5. Above this threshold power, which was coincident with the threshold for the parametric decay of EPW into EICW, diffusion driven by turbulent, low-frequency rf waves became dominant over classical collisional diffusion.

Plasma rotation driven by fluctuation-induced changes in the plasma potential must be considered since it would lead to an overestimate of the diffusion coefficient, due to apparent spreading in x -direction test ion profiles as a result of the tagging laser beam geometry. Because the laser tags a vertical column, the horizontal spreading of the tagged profile will be enhanced by $(\cos \theta)^{-1}$, where θ is the angle of plasma rotation that occurs between ion tagging and detec-

FIG. 6. Cross-field ion diffusion coefficient versus rf pump power, showing enhancement of $D_{\perp}$ above classical levels only when low-frequency ion turbulence from parametric decay is present, -20 dB = 1.25 W.

tion. For any width enhancement of $(\cos \theta)^{-1}$, total integrated flux along a horizontal detection scan will increase by the same factor. From rf levels below parametric threshold to the maximum applied power level, the largest observed increase in integrated horizontal detection flux was 5.5%. This indicates a maximum plasma rotation of 0.23 radians between the vertical tagging signal and the horizontal detection signal. This would lead to a maximum overestimate of $D_{\perp}$ by 12%. There was no noticeable trend in plasma rotation with rf power.

The cross-field diffusion coefficient versus ion fluctuation level is plotted in Fig. 7. Several conclusions may be drawn from this figure. First, at ion fluctuation levels of $(\delta n/n)_{\text{rms}} \geq 10^{-2}$, turbulent diffusion dominates classical collisional cross-field diffusion in these experiments. There is no radical departure of the diffusion coefficient in the juncture between classical and turbulent regimes of diffusion; rather the data indicate a smooth transition between the two regimes. Second, the functional dependence of $D_{\perp}$ on $(\delta n/n)_{\text{rms}}$ can be determined. The diffusion coefficient is seen to increase monotonically with increasing low-frequency ion fluctuation level. Third, the diffusion coefficient is nearly two orders of magnitude larger than classical values by the time the ion fluctuation level is only $(\delta n/n)_{\text{rms}} = 4 \times 10^{-2}$. It should be noted that many laboratory plasmas and natural plasmas have fluctuation levels that exceed a few percent, and hence classical diffusion would rarely be the applicable diffusion model; rather, a turbulent diffusion model would be required and direct measurements of fluctuation levels would be needed to determine correctly the expected diffusion.

The dependency of cross-field diffusion on ion fluctuation level can be determined by analysis of the data of Fig. 7. Trial of a least-squares quadratic fit to the data indicates that

FIG. 7. Cross-field ion diffusion versus low-frequency ion density fluctuation level. Dashed line, theory equation (8); solid line, linear least-squares fit to data.

any term quadratic in $(\delta n/n)_{\text{rms}}$ would contribute less than 5% of the linear term over the entire range of the experiment. Thus it may be concluded that models of turbulent diffusion which would predict a quadratic dependence of $D_{\perp}$ on $(\delta n/n)_{\text{rms}}$ for this experiment are incorrect. A power law fit of the form $D_{\perp} = a(\delta n/n)_{\text{rms}}^y$ yields a power of $y = 1.13$, with $r = 0.9653$. This reinforces the conclusion that a quadratic fit is inappropriate. The power law fit intuitively might be felt to hold an exponent slightly over unity since the classical diffusion zero fluctuation level offset is not considered in the fit [such a power law fit forces the diffusion coefficient to be zero at $(\delta n/n)_{\text{rms}} = 0$].

Next, consider a least-squares linear fit to the data. The data are fit by

$$D_{\perp, \text{exp}} = D_{\perp, \text{class}} + 4.39 \times 10^4 [(\delta n/n)_{\text{rms}} - 1.37 \times 10^{-3}] \text{ cm}^2/\text{sec}, \quad (8)$$

with σ (slope) = 1.48% of the slope value of 4.39×10^4 cm²/sec. This is to be compared to the theoretical value predicted earlier, that is, combining Eqs. (1) and (7), of

$$D_{\perp, \text{thy}} \approx D_{\perp, \text{class}} + 4(cT_e/eB)(\delta n_i/n_i)_{\text{rms}}, \quad (9)$$

which yields a prediction for the experiment of

$$D_{\perp, \text{thy}} \approx D_{\perp, \text{class}} + 5 \times 10^4 (\delta n_i/n_i)_{\text{rms}} \text{ cm}^2/\text{sec}. \quad (10)$$

The experimental least-squares linear fit to the data is plotted in Fig. 7 as the solid line, while the theoretical prediction of Eq. (8) is plotted as the dashed line.

Two conclusions may be drawn from this analysis. First, the functional dependence of cross-field diffusion is described well by a linear dependence of the diffusion coefficient on ion fluctuation level in this experiment. Second, the turbulent diffusion model described by Eq. (7), taken in combination with the previously observed classical diffusion coefficient, agrees well with the experimental results, both in functional dependence on fluctuation level and in absolute magnitude. The difference between theory and experiment is within 15%.

VI. CONCLUSIONS

Experiments reported here show a directly measured cross-field majority species ion transport dependence on electrostatic fluctuation level. For ion density fluctuation levels below 2×10^{-3} the observed diffusion coefficient is in agreement with the directly measured classical diffusion experiments of McWilliams and Okubo.⁵ For higher ion fluctuation levels, the increased diffusion coefficient was linearly proportional to the fluctuation level. The experimental comparison with simple turbulent and classical theories is quantitative and does not require the use of "anomalous" coefficients.

ACKNOWLEDGMENTS

Illuminating discussions were held with Dr. D. N. Hill, Dr. J. Krommes, Dr. N. Rostoker, Dr. N. Rynn, Dr. R. Stern, Dr. W. Thompson, and Dr. R. Walkup. Mr. V. Laul and S. Roe provided valuable technical assistance.

This work was supported by the National Science Foundation, Grants No. PHY-8606081 and No. ATM87-16631.

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