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PION MULTIPLICITY DISTRIBUTIONS IN
HEAVY ION COLLISIONS*

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Abstract

We calculate the π^- multiplicity distribution for
Ar + Pb₃O₄ at 1.8 GeV/nuc. Good agreement with data is found.
Effects of multipion correlations are investigated.

Recently, the negative pion multiplicity distribution $P(n)$ has been measured¹ for a variety of heavy ion targets and projectiles at beam energies ~ 2 GeV/nucleon. ($P(n)$ is the probability of producing n negative pions in a given heavy ion collision). In order to assess the implications of the data and to help in planning future experiments, we present here the results of model calculations of $P(n)$. Our study is motivated by the following questions: (1) can a simple model account for the measured $P(n)$, (2) how sensitive is $P(n)$ to the details of heavy ion dynamics, (3) what information does $P(n)$ contain that is not available from single pion inclusive cross sections², and (4) how are multipion correlations reflected in $P(n)$.

We show below that a thermodynamic "fireball" model^{3,4} gives $P(n)$ in good agreement with data.¹ Then through a dynamical calculation of $P(n,t)$ based on a master equation, we show that there exists a large class of models that lead to the same result. In particular, we prove that for a fixed impact parameter b , $P(n,t;b)$ is a Poisson distribution independent of the dynamical model as long as multipion correlations can be neglected. The average multiplicity, $\langle n(b) \rangle$, is model dependent. However, in the absence of correlations, $P(n)$ contains less information than the single pion inclusive cross section. Finally, we show that correlations due to Bose statistics and coherent pion production mechanisms lead to convoluted multiple Poisson distributions.

To compute $P(n)$ in the fireball model,³ the pions are assumed⁴ to come to chemical as well as thermal equilibrium during heavy ion collisions. We will relax the assumption of equilibrium later, but for now we consider an isospin saturated ($N=Z$) nuclear system of volume V and temperature T with pion states $|i\rangle$ having corresponding frequencies ω_i . Treating pions as an ideal boson gas, the probability of finding n negative pions in the system is

$$P(n) = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \dots \left\{ \prod_{i=1}^{\infty} x_i^{n_i} (1-x_i) \right\} \delta(n - \sum_{j=1}^{\infty} n_j) \quad (1)$$

with $x_i = \exp(-\omega_i/T)$, in units $\hbar = c = k_B = 1$, and δ is a Kronecker delta constraint. The $P(n)$ are most easily computed via the generating function

$$F(\lambda) = \sum_{n=0}^{\infty} \lambda^n P(n) \quad (2a)$$

$$= \exp \left\{ \sum_i \ln \left[\frac{1-x_i}{1-\lambda x_i} \right] \right\} \quad (2b)$$

$$= \exp \left\{ \sum_{k=1}^{\infty} N(k) (\lambda^k - 1) \right\} \quad (2c)$$

-3-

where

$$N(k) = \frac{1}{k} \left(\frac{V m_\pi^3}{2\pi^2} \right) \left(\frac{T}{k m_\pi} \right) K_2 \left(\frac{k m_\pi}{T} \right), \quad (3)$$

follows from Eq. (2b) with K_2 being the modified Bessel function.

Expanding the exponential in Eq. (2c) and using the multinomial theorem, $P(n)$ is found to be the following finite sum of terms, each having a finite number of factors

$$P(n) = \exp \left[- \sum_{k=1}^{\infty} N(k) \right] \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \dots \left\{ \prod_{k=1}^{\infty} \frac{N(k)^{n_k}}{n_k!} \right\} \delta(n - \sum_{j=1}^{\infty} j n_j) \quad (4)$$

In fact, we have shown that any discrete distribution on the non-negative integers having $P(0) > 0$ — a category including every possible multiplicity distribution of collision-created particles — can be expressed as a convoluted multiple Poisson distribution, as in Eq. (4), with the $N(k)$ uniquely determined by the $P(n)$, $n=0,1,\dots,k$. In the fireball model, $N(k)$ is given by Eq. (3) and decreases rapidly with k . In fact, for $T < m_\pi$, $N(2)/N(1) < 0.04 \ll 1$, and a good approximation to Eq. (4) is obtained by setting $N(k) = 0$ for $k \geq 2$. Therefore, for $T < m_\pi$, $P(n)$ appears to reduce to a simple Poisson⁵:

$$P(n) \approx e^{-\langle n \rangle} \frac{\langle n \rangle^n}{n!}, \quad (5)$$

where $\langle n \rangle = dF/d\lambda \big|_{\lambda=1} = \sum_{k=1}^{\infty} k N(k) \approx N(1)$ is the average number of negative pions. More precisely, Eq. (4) shows that corrections to Eq. (5) arise to orders $N(2)/N(1)$ and $N(2)/(N(1))^2$. The latter correction depends on the volume V and becomes important only if V is small compared to the pion thermal quantum volume $\propto (m_\pi T)^{-3/2}$.

To understand the conditions leading to Eq. (5), a physical interpretation of $N(k)$ is needed. We only quote here the results of a calculation

that shows that $kN(k)$ is the average number of states $|i\rangle$ with occupation number $n_i \geq k$. We refer to a group of k pions in the same state $|i\rangle$ as statistically correlated due to Bose statistics. The condition $N(1) \gg N(2)$ then means that only a very small fraction of the pions are statistically correlated. Thus, $P(n)$ is a simple Poisson when such correlations can be neglected.

In heavy ion collisions temperatures are typically³ $T \lesssim 100$ MeV, and therefore Eq. (5) should apply if equilibrium is established. To compare to experiment, $P(n)$ must be averaged over impact parameters.

$$P(n) = \int_0^{b_{\max}} \frac{2b db}{b_{\max}^2} e^{-\langle n(b) \rangle} \frac{\langle n(b) \rangle^n}{n!} \quad (6)$$

In the fireball model,^{3,4} sharp sphere geometry and relativistic kinematics give the total number of interacting nucleons $N_{\text{nuc}}(b)$ and the excitation energy per nucleon $E^*(b)$ as a function of b . The relation between $T(b)$ and $E^*(b)$ then follows⁴ from the assumption of equilibrium between nucleons and pions and depends on the volume $V(b)$. In this model, $V(b)$ is taken to be the volume of the expanding fireball at the point where the nucleon density reaches a critical "freeze out" density ρ_c . Typically, $\rho_c \approx (1/4 \text{ to } 1/2) \rho_0$, $\rho_0 = 0.17 \text{ fm}^{-3}$, and corresponds to a density below which no further pion production or absorption is assumed to take place. The key assumption is that experimentally only the properties of the fireball at this freeze out density are observed. Thus $V(b) = N_{\text{nuc}}(b)/\rho_c$. For $\rho_c = \rho_0/3$, $T(b)$ was found to be related to $E^*(b)$ approximately as⁴

$$T(b) \approx T_0 (1 - \exp(-2E^*(b)/3T_0)) \quad (7)$$

where $T_0 \approx 92$ MeV. We take $\rho_c = \rho_0/3$ from Ref. (4), where ρ_c was adjusted to fit the low energy pion inclusive cross section. Since $d^2\sigma_\pi/d\Omega dE$ is proportional to the average pion multiplicity, adjusting ρ_c to obtain the correct normalization of $d^2\sigma_\pi$ insures that the $\langle n(b) \rangle$ entering Eq. (6) are also of the correct magnitude.

The comparison of $P(n)$ from Eq. (6) to experiment¹ is given in Fig. 1(a) for the reaction: Ar incident on Pb_3O_4 at 1.8 GeV/nucleon. For this mixed target $P(n)$ is appropriately averaged over Pb and O targets. Curve 1 is obtained by integrating Eq. (6) over all impact parameters up to $R_p + R_T$ = sum of projectile and target radii. Curve 1 should be compared to untrigged data. However, in Ref. (1) data were selected in an "inelastic" trigger mode, which, according to their measured total charge multiplicity distribution, corresponds to ~ 5 or more charged tracks per event. This trigger can be taken into account by an impact parameter cutoff, $b_{\max} < R_p + R_T$. In curve 2 we took $b_{\max} \approx R_p + R_T - 1$ fm, which placed a lower bound on the number of interacting nucleons in the fireball to be $N_{\text{nuc}}(b_{\max}) \approx 8$. As seen in Fig. 1(a), curve 2 fits all features of the data well including the bump at $n = 1$ and the rapid fall-off for $n > 10$. The contribution of the Pb and O targets to curve 2 are indicated by dashed lines.

To see in detail why the bump at $n = 1$ is due to a trigger bias, we plot in Fig. 1(b), $P(n;b)$ as a function of impact parameter for Ar + Pb. Larger b correspond to smaller interaction volumes $V(b)$ and hence smaller $\langle n(b) \rangle$. Thus, $P(n;b)$ is shifted to lower n as b increases. In Eq. (6), $b \approx b_{\max}$ is weighted the most, hence reducing $b_{\max}$ shifts the whole distribution $P(n)$ to higher n as seen in Fig. 1(a).

Having found good agreement with data using the fireball model, we turn now to the question of whether the assumption of equilibrium is necessary. To this end we formulate a dynamical theory of multiplicities. We only outline the main results here. Details will be given elsewhere. Our approach is motivated by the results of Ref. (6). In Ref. (6), the number of binary collisions in a heavy ion scattering was shown to be Poisson distributed by solving a master rate equation. We extend here the method to time dependent pion production and absorption rates to study the general conditions under which $P(n,t)$ is a simple Poisson.

The dynamical model is specified by a set of average rates, $\gamma_k^\pm(t)$, for producing and absorbing correlated groups of k pions. For example, $\gamma_1^\pm(t)$ could specify the average rates for $NN \rightleftharpoons NN\pi$, while $\gamma_k^\pm(t)$ for $k \geq 2$ could describe multi-pion resonance production and absorption (such a resonance would have to include $k \pi^-$'s in its possible decay products to be included in $\gamma_k^\pm(t)$ for the data studied here). We expect that all rates must be strongly time dependent since the nucleon space and momentum distribution $f(\tilde{x}, \tilde{p}, t)$ varies rapidly during heavy ion collisions.

The probability of producing a correlated group of k pions between times t and $t + dt$ is then $\gamma_k^+(t) dt$. Also, the probability of coherently absorbing a group of k pions between t and $t + dt$, when there were n pions at t , is $C(n,k) \gamma_k^-(t) dt$, where $C(n,k) = n!/((n-k)!k!)$. The master equation for $P(n,t)$ may then be obtained by expressing $P(n,t + dt)$ in terms of $P(n \pm k, t)$ and the above probabilities

$$\begin{aligned} \frac{dP(n,t)}{dt} = & \sum_{k=1}^{\infty} \gamma_k^+(t) \left[P(n-k,t) - P(n,t) \right] \\ & + \sum_{k=1}^{\infty} \gamma_k^-(t) \left[C(n+k,k) P(n+k,t) - C(n,k) P(n,t) \right], \end{aligned} \quad (8)$$

where $P(n,t) \equiv 0$ for $n < 0$ is understood. Note that in this formulation, we ignore statistical correlations due to Bose statistics and consider only dynamic correlations due to coherent production and absorption mechanisms.

The time dependent generating function, Eq. (2a), then satisfies

$$\frac{\partial F}{\partial t}(\lambda, t) = \sum_{k=1}^{\infty} (\lambda^k - 1) \left[\gamma_k^+(t) F(\lambda, t) - \gamma_k^-(t) \frac{1}{k!} \frac{\partial^k F}{\partial \lambda^k} \right], \quad (9)$$

The boundary conditions $F(1, t) = 1$ and $F(\lambda, 0) = 1$ stem from the normalization of $P(n, t)$ and the initial condition $P(n, 0) = \delta_{n0}$.

The general solution of Eqs. (8,9) will be discussed elsewhere.

Here, we note the result that if multipion absorption can be neglected, i.e., $\gamma_k^- = 0$ for $k \geq 2$, then the solution of Eq. (9) is given by Eq. (2c) with $N(k)$ replaced by $N(k, t)$. These $N(k, t)$ satisfy the system of rate equations:

$$\begin{aligned} \frac{dN(k, t)}{dt} = & \gamma_k^+(t) + \gamma_1^-(t) \left\{ (k+1)N(k+1, t) \right. \\ & \left. - k N(k, t) \right\} \end{aligned} \quad (10)$$

Thus, $P(n, t)$ is given by Eq. (4) with $N(k) \rightarrow N(k, t)$. Note that $N(k, t)$ converges to a unique equilibrium solution as $t \rightarrow \infty$. However, $N(k, \infty)$ for $k > 1$ arise here from dynamic correlations rather than from the statistical correlations leading to Eq. (3).

Our main result can be stated as follows: when correlations between pions can be neglected, i.e., $N(1, t) \gg N(k \geq 2, t)$ at the

observation time $t = t_{\text{obs}}$ (as, for example, when $\gamma_k^{\pm}(t)=0$ for $k \geq 2$), then $P(n, t_{\text{obs}})$ is a simple Poisson with mean $N(1, t_{\text{obs}})$ (at a fixed impact parameter). This means that a very large class of dynamical models exists that give identical results to the fireball model. In particular, equilibrium need not be reached! The only requirements on those models is that $N(1, t_{\text{obs}}) \gg N(k \geq 2, t_{\text{obs}})$ and that the average pion multiplicity $\langle n \rangle \approx N(1, t_{\text{obs}})$ is correctly given.

We can now comment on the multiplicity calculation of Ref. (7) represented by the points ∇ in Fig.1(a). That calculation is based on an independent nucleon-nucleon collision model with pion absorption neglected. In terms of Eq. (9), γ_1^+ is taken as a constant, while $\gamma_1^- = 0$. Consequently, this model predicts too many high multiplicity events. While the condition $N(1, t_{\text{obs}}) \gg N(k \geq 2, t_{\text{obs}})$ is satisfied in that model, $\langle n \rangle$ is overestimated for central collisions due to the neglect of pion absorption.

To conclude, we suggest that future experiments on pion multiplicities should be conducted in a "highly inelastic" trigger mode in order to measure $P(n, b \approx 0)$ directly. Any deviations from a simple Poisson form would then be a truly significant signal of unusual coherent pion processes in heavy ion collisions.

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FIGURE CAPTION

Fig. 1 (a) π^- multiplicity distribution for Ar + Pb_3O_4 at 1.8 GeV/nuc:

Φ data, Ref. (1); ∇ - Ref. (7);

(1) - Eq. (6), no b cutoff, $\rho_c = 0.16 \text{ m}_\pi^3$;

(2) - Eq. (6) with $b_{\text{max}}(\text{Pb}) = 9.6 \text{ fm}$, $b_{\text{max}}(\text{O}) = 5.0 \text{ fm}$;

dashed curves give contributions to (2) from Pb and O targets.

(b) π^- multiplicity distribution as a function of impact parameter b for Ar + Pb at 1.8 GeV/nuc. (XBL-778-1599 Fig. 1(a) and 1(b)).

XBL 778-1599

Fig. 1

0 0 1 1 8 9 2 6 4

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