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Financial Markets and the Macroeconomy: Theory, Evidence and Policy Prescriptions

Author
Dordal Carreras, Marc

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Financial Markets and the Macroeconomy: Theory, Evidence and Policy Prescriptions

by

Marc Dordal Carreras

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Committee in charge:

Professor Yuriy Gorodnichenko, Chair
Professor Andrés Rodríguez-Clare
Professor Christina Romer

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Abstract

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This dissertation investigates the role of financial markets as a driving force behind business cycle fluctuations and studies effective monetary policy responses that mitigate the negative economic impact of such fluctuations. The first chapter empirically investigates the consequences of the 2007 interbank market freeze and provides a new theoretical framework to study the economic benefits and risks arising from complex financial networks. I use highly detailed proprietary microdata from the German Bundesbank to provide evidence that exposure to the US financial market had a negative impact across several measures on domestic German monetary financial institutions (MFIs) and their clients. I develop a dynamic stochastic general equilibrium model of the macroeconomy with a banking sector that intermediates funds between depositors and firms. I show that interbank fund transactions improve the allocation of household savings across the economy, but also affect its volatility by determining how sensitive the aggregate supply of credit becomes to individual-bank shocks. I use the model to provide estimates of the welfare contribution of the interbank market to the German economy and the costs of bank disintermediation that followed the 2007 financial crisis. I study the welfare benefits of standard monetary policy and central bank lender-of-last-resort interventions, and I find that policies that actively target the credit spread arising from the banking sector are more effective.

The second chapter studies the historical (1868-1930 period) propagation of banking panics across the United States. I develop a partial equilibrium model of the interbank market consistent with the historical pyramidal reserve structure of deposits that was in place throughout the period. The model presents a simple tradeoff between an efficient allocation of bank funds and exposure to cross-border deposit fluctuations. I empirically estimate the dynamic spatial propagation of panics and I find that panics are accompanied by moderate

but temporary drops in variables capturing banking sector activity, together with a robust spatial propagation consistent with the model.

The third chapter investigates the welfare costs of the zero-lower bound (ZLB) on nominal interest rates and presents a theoretical New-Keynesian framework that incorporates the main empirical properties of ZLB spells. Employing a regime-switching (RS) risk-premium process to bring rates to the ZLB, I demonstrate how both frequency and duration of ZLB episodes can be jointly matched to realistic values. I find that duration exerts a strongly non-linear negative effect on welfare, which leads traditional models of the ZLB to seriously underestimate the costs of ZLB episodes. I conclude the chapter by discussing the optimal monetary policy inflation target and its relationship to the prevalence of ZLB episodes. I show that the optimum target lies at the point in which the marginal costs and benefits of trend inflation are equalized, and a calibration of the model to the U.S. economy generates optimal inflation mandates consistent with the 2% target commonly followed in most advanced economies.

Als meus pares.

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Chapter 1

Dissertation Introduction

This dissertation investigates the role of financial markets as a driving force behind business cycle fluctuations and studies effective monetary policy responses that mitigate the negative economic impact of such fluctuations. The first chapter empirically investigates the consequences of the 2007 interbank market freeze and provides a new theoretical framework to study the economic benefits and risks arising from complex financial networks. I use highly detailed proprietary microdata from the German Bundesbank that allows me to observe the network of bilateral transactions for the universe of domestic German monetary financial institutions (MFI) and its individual exposure to US financial institutions. I provide reduced form evidence that exposure to the US financial market had a negative impact across several measures on domestic German MFIs and their clients. I find that more exposed MFIs disproportionately reduce their borrowing volume on the interbank market, raise interest rates on their loans to non-financial clients and cut their aggregate lending activity. I also show that the results are not driven by any omitted characteristics of the German MFIs. I construct a bank-level measure of indirect exposure to the US market through domestic German interbank partners prior to the 2007 crisis. Under the plausible assumption that the relationship intensity with domestic banks is uncorrelated with their exposure to the US markets prior to crisis, I obtain causal estimates that are consistent with results exposed. In the second half of the chapter, I use techniques from the trade literature to develop a dynamic stochastic general equilibrium model of the macroeconomy with a banking sector that intermediates funds between depositors and firms and that in equilibrium nests within a standard New-Keynesian model of the economy. The banking sector contains a discrete number of heterogeneous banks that are differentiated by balance sheet size, depositor base and interbank participation. Banks in this model trade among themselves on an interbank market, giving rise to a complex network of overlapping financial claims. Interbank fund transactions improve the allocation of household savings across the economy, increasing output and societal welfare. They also affect the volatility of the economy by determining how sensitive the aggregate supply of credit becomes to individual-bank shocks. I calculate the welfare contribution of the interbank market to the German economy by structurally estimating the model and comparing it to the counterfactual autarky scenario without interbank market in which banks have to rise their funding exclusively through their depositor base. The results suggest welfare gains around 0.88% of consumption per-quarter, coming from a combination of efficiency gains in the allocation of funds across the bank network and decreased volatility through diversification of the bank's funding sources, with both channels roughly contributing 50% each to the estimated welfare gains. Assuming a structural break on bank lending relationships following the onset of the 2007 financial crisis, I estimate that the persistent shrinkage of the interbank market that ensued led to a welfare loss equivalent to 0.35% of consumption per-quarter. I conclude the chapter by studying the welfare benefits of lender-of-last-resort interventions. I find that access to the Central Bank's discount window increases welfare by up to 2.5%, with most of the gains arising from granting access to the large and well-connected banks at the top end of the bank size distribution.

The second chapter presents an empirical inquiry into the historical (1868-1930 period) propagation of banking panics across the United States. I begin the chapter by developing a partial equilibrium model of the interbank market consistent with the historical pyramidal reserve structure of deposits that was in place throughout the period. The model presents a simple tradeoff between an efficient allocation of bank funds and exposure to cross-border deposit fluctuations. Access to the interbank markets allows banks to tap into cheaper sources of funding and provide, on average, higher levels of credit to their home state. On the other hand, cross-state financial obligations expose banks activity to deposit fluctuations outside their states and exposes them to the effects of banking panics originated outside their regional borders. The second half of the chapter estimates the dynamic propagation of panics using Jordà local projection methods. I find that panics are accompanied by moderate and temporary drops in deposits and lending, increased liquidity, and a small negative impact on bank capital and number of banks, with the results being statistically significant up to two years from the onset of a panic. I also find that regional panics display a robust spatial propagation consistent with the model.

The third chapter investigates the welfare costs of the zero-lower bound (ZLB) on nominal interest rates and presents a theoretical New-Keynesian framework that incorporates the main empirical properties of ZLB spells. The chapter begins by presenting empirical evidence that ZLB episodes have been historically characterized by being infrequent but long-lived. As it is common in many traditional ZLB settings, I bring the nominal rate of interest to zero by introducing a reduced-form risk-premium shock that exerts downward pressure on the equilibrium nominal rate, and which captures shifts on the perception of risk by investors or the disturbances of the financial sector that I modeled in more detail in chapter 1. Traditional literature on the topic assumes that the risk-premium shock follows an auto-regressive (AR) process and calibrates its volatility and/or auto-regressive parameter to match the historical frequency at which the ZLB binds in the data. This approach omits the duration of ZLB spells, and I show that reasonable calibrations of the AR process generate frequent but short-lived ZLB episodes, which is contrary to what we observe in the data. Employing an alternative regime-switching (RS) representation of the risk-premium process, I demonstrate how both frequency and duration of ZLB episodes can be jointly matched to realistic values. Using standard non-linear solution methods, I provide estimates of the welfare losses associated to ZLB episodes under the alternative AR and RS shock representations. I find that duration exerts a strongly non-linear negative effect on welfare, which leads traditional AR models to seriously underestimate the costs of ZLB episodes. I conclude the chapter by discussing the optimal monetary policy inflation target and its relationship to the prevalence of ZLB episodes. Higher inflation targets increase the average value of the nominal interest rate, reducing the frequency and duration of ZLB spells. On the other hand, higher inflation generates additional welfare costs from price dispersion. I show that the optimum target lies at the point in which the marginal costs and benefits of trend inflation are equalized, and a calibration of the RS model to the U.S. economy generates optimal inflation mandates consistent with the 2% target commonly followed in most advanced economies.

Chapter 2

A Trade Model of the Banking Sector

This chapter is the product of joint work with Matthias Hoelzlein and Jens Orben. I thank them for allowing me to use our joint work as part of this dissertation. The opinions discussed in this chapter do not necessarily reflect the views of the authors' employing institutions.

2.1 Introduction

The onset of the 2007 crisis and subsequent turmoil made abundantly clear that the plumbing of financial markets can play a key role in the propagation of shocks to the real economy. Indeed, every banking panic or financial crisis renews interest in studying the workings of financial intermediation, but the interbank market — a core market for allocation of short-term funding for the U.S. and many other advanced economies — remains largely outside quantitative macroeconomic models. Furthermore, networks of financial intermediation are rarely incorporated in the analysis of business cycles.

In this paper, we develop a quantitative, tractable model of the interbank market that is nested in an otherwise standard New Keynesian framework. Building on recent methodological advances in international trade, we model trade in funds arising from liquidity mismatches across banks and our model can capture salient features in the data such as market concentration, network structure and varying degrees of participation in the interbank market. This framework allows us to study explicitly the role of the central bank as the lender-of-last-resort and thus study discount window policy (and similar tools) for macroeconomic dynamics and countercyclical monetary policy. We also use the framework to quantify gains from financial market integration, its interaction with monetary policy, and discuss the benefits of extending discount window access to a larger set of financial institutions.

In the first step of our analysis, we use unique data for German banks to document a series of stylized facts about the interbank market. We construct a database based on proprietary microdata from the Deutsche Bundesbank containing information on individual balance sheets and bilateral transactions for the universe of active MFIs in Germany. First, there are large differences in bank size, with high concentration at the top of the distribution. From the 1500+ active monetary financial institutions (MFIs) in Germany, the 4 (200) largest represent the 30% (80%) of all combined assets. This fact suggests that individual-level bank shocks might drive aggregate economic fluctuations as in the granularity hypothesis advanced by Gabaix (2011). Second, banks rely on the interbank market to cover structural funding deficits/surpluses, in addition to short-term liquidity mismatches. This distinction is important to understand the response of credit to the non-financial sector during periods of financial distress. Third, interbank connections per bank tend to be few but stable. Except for a small core of large banks with +100 connections, the median bank of our sample has no more than ten interbank partners. This limits the capacity of banks to substitute funding sources during an interbank credit freeze, while subjecting the market to the shocks of a small set of key intermediators.

In a second step, we provide causal, reduced form evidence on the effects of the US financial crisis on the German interbank market. Using detailed data on domestic interbank positions and direct foreign exposure of German banks to the US, we construct a bank-level measure of indirect exposure to the US through the domestic interbank partners' US asset holdings prior to the crisis. Following the Lehman collapse, we find that banks with high indirect exposure to US bank assets reduce interbank borrowing relative to banks with lower indirect exposure due to a reduction in domestic activity by their directly exposed lenders. More affected banks increase interest rates charged on non-financial loans (10 basis points per billion Euros of indirect exposure) and reduce lending to firms and consumers (2% drop per billion Euros of indirect exposure). These effects are statistically significant and economically important: the German interbank market persistently shrank following the Lehman collapse, and our measure of indirect exposure to the US is able to account for half of its size reduction.

From a theoretical perspective, however, the creation of a business cycle model that realistically captures these stylized facts while remaining analytically tractable is quite challenging. Among the few papers that study the effects of the interbank market on the macroeconomy, strong concessions are made in favor of tractability: Gertler et al. (2016) simplify the problem by assuming two types of banks, retail banks that obtain deposits from households, and wholesale banks that borrow from retail banks. Piazzesi et al. (2019) and De Fiore et al. (2018) build on search models that assume a continuum of atomistic banks differentiated only by the size of the liquidity shock that they receive each period. In this paper, we propose an alternative route by adapting the Eaton and Kortum (2002) model of international trade to the banking sector. Trade models are naturally well suited to the task, as they often feature a discrete number of heterogeneous agents and simple expressions for the volume of trade between them.

Our framework combines features of standard New Keynesian DSGE models with the market structure of current models of International Trade. Banks intermediate funds between households and firms, which use them to finance the capital investment necessary for production. Households receive utility from holding real deposit balances in banks, thanks either to a preference for liquidity or to the usefulness of deposits in the completion of consumption transactions. In addition, households might have stronger preferences for some banks over others, and these relative preferences might change over the time. This creates fluctuations in the funding costs of banks and provides an incentive to trade in the interbank market with banks that enjoy a cheaper access to deposits. However, banks face transaction costs with each other, and they will borrow from the interbank market only when the cost of obtaining funds through their own depositors exceeds a certain threshold. By calibrating these transaction costs to the data, our model can replicate the basic structure and strength of interbank connections, as well as the relative importance of each bank within the system.

We model transaction costs in the interbank market as volatile but remain ambivalent about the source of such costs shocks. Possible micro-foundations for such shocks are asymmetric information and varying capacity to assess the quality of the counter-party's collateral, which is consistent with the problems that arose during the 2007 crisis in assessing the value of mortgage backed securities and related assets. A sudden shock to transaction costs generates a credit freeze in the interbank market (or a subset of its participants), and banks are forced to rely on their own depositors (at higher funding costs), reduce their lending operations, or both. This gives rise to an efficiency-volatility trade-off in welfare that is at the core of our paper. Participation in the interbank market improves the allocation of funds among banks, therefore, lowers capital costs of firms that rely on bank credit. It also reduces the volatility of banks' funding costs by allowing them to diversify their funding sources. At the same time, however, banks become more exposed to interbank credit freezes, which increases the volatility of the economy.

Central to our analysis will be the notion of gains from trade (see Arkolakis et al. (2012), henceforth ACR), understood in our paper as the benefits of an interbank market where banks settle their funding mismatches. Our paper extends ACR gains from trade to stochastic environments, deriving an analytical approximation to welfare with which we explore the trade-off between efficiency and risk that accompanies processes of increased financial integration. We show that simple statistics like interbank trade shares and Herfindahl indexes of loan concentration serve as sufficient statistic for the consequences of financial market freezes, and more generally the complex relationships that arise in the interbank market. Banks do not take into account the effect of their decisions on aggregate financial volatility, which might lead them to over-rely on the interbank market. Processes of financial integration that concentrate the funding sources of banks and/or firms on a small subset of large banks might prove detrimental to social welfare. We discuss those situations in Section 2.8.

We study central banks' lender-of-last-resort policies, and show that the most effective discount window policies involve an active provision of credit to distressed banks. We also find that the discount window and open market operations are complementary tools and reduce the volatility of the economy through different channels: while the lender of last resort reduces the volatility of financial markets by setting a cap on the funding costs of banks, adjustments on the level of interest rates minimize the pass-through of financial market fluctuations to inflation and output gap.

Finally, we pair our model with detailed data on German MFIs provided by the Deutsche Bundebank. We link the structure of the model to banks' indirect exposure to the US crisis and the resulting plausibly exogenous interest rate shock to estimate the two key elasticities of the banking market. Equipped with these parameters, we calibrate the model to the German economy and estimate the welfare gains of financial market integration and the effects of monetary policy counterfactuals. Our findings suggest that the interbank market increases welfare on net through improved allocation of household deposits and by reducing financial volatility through diversification of the funding sources, which in practice surpasses the volatility costs of counter-party exposure. In our preferred calibration, the welfare gain of moving from the current level of financial integration to financial autarky (defined as a financial market with infinite transactions costs) amounts to 0.88% of quarterly consumption. The Great Recession and the European Bond crisis persistently reduced the participation of MFIs on the interbank markets and raised credit spreads. Our model provides a connection between the two events and estimates utility losses from the reduction in interbank market activity at around 0.35% of consumption per quarter.

The rest of the paper is organized as follows. Section 2.2 further relates our contributions to the existing literature. Section 2.3 presents the data employed in this paper. Section 2.4 shows some stylized facts about the German banking system. Section 2.5 provides reduced form evidence on the effects of the Great Recession on the German interbank market. Section 2.6 introduces the model. Section 2.7 empirically estimates the main parameters of the model and calibrates it to the data. Section 2.8 derives an analytical expression for welfare and estimates the gains from interbank trade to the German economy. Section 2.9 studies monetary policy in the context of our model. Section 2.10 concludes.

2.2 Related Literature

Building on earlier work by Bernanke and Gertler (1986), Kiyotaki and Moore (1997) and the financial accelerator of Bernanke et al. (1998), a prolific literature developed to explain how the financial system, in its role as intermediator between household savings and firms' investment, generates credit frictions that amplify business cycle fluctuations. The common story behind these papers argues that banks are subject to some sort of operative constraint (on leverage, liquidity, or others) that loosens (tightens) during booms (recessions), making their provision of credit to firms (or acquisition of funds from depositors) procyclical. While most of these papers focus on the bank-to-firms or the depositor-to-banks portion of the financial channel, others like Cingano et al. (2016) empirically show that an important fraction of the drop in bank lending observed during the Great Recession can be attributed to the freezing of interbank markets, which would be the bank-to-bank piece that lies in the center of financial markets. Our paper provides a theoretical framework that can be used to analyze this channel.

The literature on banking and financial networks, building on earlier theoretical work by Allen and Gale (2000) and more recently Acemoglu et al. (2015), describes the conditions under which interbank markets emerge and give rise to a trade-off between an efficient allocation of funds and a heightened risk of contagion (or default, volatility, etc.). In most of these papers though, the focus remains on the banking system itself. By linking the banking system to the rest of the economy, we are able to study welfare implications of financial market networks over the business cycle and under different monetary policy regimes.

Few papers in the International Trade literature have evaluated dynamic gains from trade. In recent work, Caselli et al. (2020) explore the gains and loses from international trade liberalization in volatile economies. On one hand, trade openness leads to specialization and industry concentration, increasing the volatility of national economies. On the other hand, it allows countries to diversify the sources of demand and supply of tradable goods, reducing the volatility of national income. In a quantitative evaluation of their model, the authors conclude that the later effect dominates, resulting in additional gains from trade integration. Allen and Atkin (2016) study a similar question focused on the agricultural sector of India, concluding that increased volatility led farmers to substitute towards crops with less risky yields as the economy became more open. The second order costs and benefits of integration will feature prominently in our model as well, and to our knowledge our paper is the first to study them in the context of financial intermediation.

Craig and Ma (2017) develop an alternative model of financial intermediation for the German banking system. In their model, large banks become core intermediators due to their comparative advantage in assuming fixed costs associated to interbank transactions. A periphery of smaller banks then clears their funding mismatches through them. Instead of trying to explain the reason why the interbank market developed its current structure, we take it as given and ask the question of how this structure contributes to the volatility of the economy, and how monetary policy can mitigate its negative effects on welfare.

Models of trade typically achieve aggregate economic expressions through the assumption of an infinite amount of varieties or geographic locations that differentiate individual products. In a recent paper, Farrokhi (2020) proposes an alternative formulation in which aggregation comes from productivity differentiation across time, instead of variety or space. This specification will help us adapt trade models to the context of financial markets.

2.3 Data

In this section, we describe how we combine several proprietary and confidential datasets provided by the Deutsche Bundesbank, the supervisory entity for the German financial market within the system of Eurozone central banks. The resulting database contains detailed, quarterly information on the balance sheets of monetary financial institutions[1] (MFI), borrowing and lending connections with other German MFIs and other information such as the type of banking group[2] and headquarter location for the universe of German MFIs over the years 2004-2016.
For the construction of this database, we start with the MFI Masterdata (MaMFI[3]) which includes meta-information about banks such as MFI type and headquarter location on a monthly basis and allows us to account for mergers and acquisitions in all other datasets. In order to avoid sudden discontinuities in the balance sheet size and its subcategories, we treat MFIs before and after a merger or acquisition as a single entity and add up the relevant categories for the MFIs participating in the M&A. Next, we add the Monthly Balance Sheet Statistics (BISTA[4]) that covers balance sheet positions at the end of each month for the entire universe of German domestic MFIs, disaggregated into several broad asset and liability categories. To be able to better account for each MFIs' business model we complement the broad loan categories in the BISTA with a more detailed breakdown of loans by sectors, borrower type and maturities from the Quarterly Borrowers' Statistics (VJKRE[5]). Finally, the Credit register of loans of 1 million Euro or more (Millionenkreditregister) provides MFI-level supervisory information on all loans that exceeded 1 million Euro (1.5 million Euro before 2014) within each quarter. The dataset contains loan amounts, on-/off-balance sheet exposure, write-offs, and, most importantly, it covers the vast majority of loans at all maturities

[1]Monetary and financial institutions are defined by the European Central Bank as "financial institutions whose business is to receive deposits and/or close substitutes for deposits from entities other than MFIs and, for their own account (at least in economic terms), to grant credits and/or make investments in securities", url-https://www.ecb.europa.eu/stats/financial_corporations/list_of_financial_institutions/html/index.en.html. However, for the remainder of the paper we will us the terms "MFI" and "bank" interchangeably.

[2]There are 9 different banking groups in the Deutsche Bundesbank statistical definition. The largest among them are credit banks, state banks, savings banks, mortage banks and cooperative banks.

[3]See Stahl (2018) for a description of the MaMFI dataset.

[4]See Beier et al. (2017) for a description of the BISTA dataset.

[5]See Beier et al. (2018) for a description of the VJKRE dataset.

between domestic MFIs[6]. We scale individual positions between two MFIs such that the total MFI lending in this dataset is consistent with domestic MFI loans in the BISTA data. The bilateral loan amounts allow us to analyze the interbank market in its full granularity, thereby, characterizing size and volatility of transaction costs at the level of individual relationships between MFIs.

We complement our main database with interest rates for a sample of around 200 to 240 MFIs representing 65% to 70% of total lending activities[7]. The Monthly Interest Rate Statistics (ZISTA[8]) reports average interest rates on loans and deposits vis-a-vis firms and households and their respective volumes. It offers a breakdown by different remaining maturities (outstanding loans) or initial time of rate fixation (new business). For most of our analysis we use the average interest rate on outstanding loans for each MFI which we calculate as the average across all maturities and borrower types (households, firms and others) weighted by their respective loan volumes. In the empirical section of the paper we account for the MFI heterogeneity in loan products by controlling for the detailed breakdown along borrower type and maturity.[9]

Lastly, we measure the exposure of each domestic MFI to the US financial market before the Great Recession using the assets and liabilities of German banks vis-à-vis US residents available in the External Position of MFIs (AUSTA[10]). This dataset contains the gross foreign positions of the 80 largest German banks and their foreign branches on a monthly basis, covering 90% of the value all foreign positions involving a German MFI. It allows a further breakdown of exposures vis-à-vis banks, enterprises, households and governments, by recipient country and the maturity of the respective investment.

We leverage our main database to provide evidence for the role of interbank markets in bank behavior during the Great Recession in section 2.5, estimate key model elasticities in section

[6]We find that a comparison of liabilities and assets with MFIs in the balance sheet data and aggregated loans in the credit registry line up very tightly. This suggest that the reporting threshold of 1 million Euro (1.5 million Euro before 2014) is a not serious concern for lending between MFIs.

[7]The sample is designed to be representative and, at the same time, capture a large share of the financial sector. The first stratification criterion is a combination of state and banking group in order to capture regional and institutional heterogeneity. Within each of the strata, the largest banks in terms of lending were selected. Throughout our analysis we tried to address this selection bias whenever possible.

[8]See Beier and Bade (2017) for a description of the ZISTA dataset.

[9]Another adjustment to our data comes from the 2010 German Accounting Modernization Law (see Bundesbank (2010) for a description of the Accounting Modernization Law.), that, among other changes for firms, caused a break in banks' balance sheet sizes. Generally, the most prominent change was the introduction of a fair value of the trading portfolio, partly adapting to the International Financial Reporting Standards (IFRS). While this change affects only larger banks with a trading book, it was left to their discretion at what point in the course of 2010 they applied the new rules in monthly balance sheet statistics (BISTA). We mitigate this circumstance by deducting the derivative exposures of the trading book from total assets.

[10]See Gomolka et al. (2019) for a description of the AUSTA dataset.

2.7.1, recover the model's fundamentals in section 2.7.2 and document stylized facts about the German financial market in next section.

2.4 Stylized Facts

The combined balance sheet assets of the German MFIs stood at 7.8tr euros (∼250% of GDP) in December 2016. Compared to investment funds (1.9tr euros in assets under custody), insurers (2.2tr euros total assets), and other financial services providers, MFIs stand as the central players in the German financial sector. Our dataset contains 1552 distinct MFIs. Figure 2.1 plots the cumulative share of total assets held by the n-largest banks as of December 2016. We can see that the four largest banks control ∼30% of the market, and the 200 largest banks control around 80%.

Turning to the interbank market, panel (a) of Figure 2.2 shows the share of combined liabilities that banks obtain through it. Approximately one-third of interbank borrowing is in overnight loans, another third in short term (≤1 year maturity) liabilities and the rest on medium and long-term maturities. The pre-cisis share of interbank liabilities on the balance sheet of banks stood stable ∼29% of total liabilities, with a 9% drop following 2007 that only seems to recover towards previous levels at the end of the sample. Panel (b) shows the share of assets, excluding MFI loans, that banks are able to fund with their own resources (deposits or capital), as opposed to interbank borrowing. Following 2007 banks gradually start relying more on their own funding sources, without any clear sign of reversion to previous levels.

Figure 2.3 shows the evolution of the interbank credit spread at different maturities. We observe two large spikes following the 2007 financial crisis and the european debt crisis, respectively. After 2014, the spread stabilizes, but it does never return to its previous level, especially evident at longer maturities. Our model will draw a link between the credit spread and the share of interbank liabilities consistent with the evolution of both variables.

We now look at the structure of interbank assets and liabilities of each bank. Figure 2.4a divides MFIs into bins according to their interbank shares and reports the number of entities within each bucket. Figure 2.4b reports the share of total MFI assets contained in each bin. We first notice that a large fraction of banks are simultaneously active as both borrowers and lenders, which would be consistent with the traditional hypothesis that the interbank market serves the purpose of covering temporary MFI liquidity shortfalls. But, as first noted by Craig and Ma (2017), many of these banks also take a net lender or borrower position in the interbank market, with net positions larger than 10% being common. Hence, the interbank market is a channel through which some banks cover their structural funding needs, while some others use it to allocate their structural surpluses. The distinction between the hypothesis that interbank markets primarily ease temporal liquidity mismatches and the hypothesis that interbank markets primarily support structural funding positions is relevant, as a market freeze like the one experienced after 2007 is intuitively more likely to have serious consequences when banks are unable to cover their structural positions and are forced to cut back on credit to the real (non-financial) sector. Data back the latter theory: Table 2.1 contains estimates of the Spearman rank correlation for the share of interbank assets and liabilities. The high correlation at horizons of up to two years indicates that bank's net positions are indeed very persistent across time. A market for structural funding is also consistent with the non-trivial amount of medium and long term borrowing that we observed in figure 2.2. Our model will be able to accommodate both a role for temporal liquidity and structural borrowing in the interbank market, and adequately capture the relative importance that each of these factors have.

A comparison between panels (a) and (b) of figure 2.4 also suggests that a subset of few large MFIs plays an important role in the allocation of funds (see bin on the 40% assets, 30% liabilities position). This becomes more clear by looking at the concentration of interbank commitments by bank and by bilateral connections displayed in Figure 2.5, in which we see that a disproportionate amount of interbank funds flow through a relatively small group of connections and banks. Figure 2.6 plots the average number of distinct borrowing connections by deciles. Deciles in panel (a) are based on the number of connections, while those in panel (b) are based on bank balance sheet size. The similarity between the two graphs indicates that central positions in the market are highly correlated with bank size. Large banks have access to a diversified pool of funding sources with close to hundred fifty unique interbank lenders, while smaller banks do not typically possess more than twenty connections. On the asset side (figure not shown) a similar pattern emerges, with large banks acting as lenders to the rest of the system. Concentration of funding sources is important because shocks to large lenders might drive the aggregate volatility of the financial sector, similar in spirit to how large firms drive economic fluctuations in Gabaix (2011). Our model will be able to adequately capture these concentration patterns.

2.5 Reduced Form Evidence

In this section, we use the detailed data on Germany's banking market summarized in more detail in section 2.3 to present causal reduced form evidence on the effect of the US financial crisis on the German banking market. We develop a difference-in-difference framework to show that banks which are more indirectly exposed to the US financial crisis beginning in 2007/2008 through their network of domestic lenders charge higher interest rates and provide less credit to the real economy. Moreover, exposed banks borrow less from their network of domestic MFIs in response to the crisis and rely more heavily on own funds to finance loans to the real economy. This empirical section serves two purposes: first, it provides intuition for the key mechanism in the model, namely, that a bank's access to the interbank market is an important driver of its lending decisions, interest rates and funding choices. Second, it introduces the exogenous variation we will leverage to estimate the key elasticities of our model in section 2.7.

2.5.1 A Measure of Banks' Indirect Exposure to the US Financial Crisis in 2008

Prior to the US financial crisis a subset of German banks were heavily invested in loans (broadly defined) to banks domiciled in the US. With the onset of the financial crisis in 2007/2008 German banks directly exposed to US bank assets, in particular, those derived from the US "subprime" mortgage market, experienced serious liquidity problems and had to significantly reduce their lending activity in both, the real economy and the interbank market. To highlight the role of the interbank market in the transmission of the US financial crisis to the German banking sector we focus our analysis on banks that borrow from those *directly exposed* banks in the domestic interbank market prior to 2007/2008. Specifically, we construct a measure of each bank's *indirect exposure* to the US financial market prior to the Great Recession according to

$$Exposure_{t0}^{US,n} = \sum_{i \neq n}^{N} \frac{M_{t0}^{in}}{\sum_{i' \neq n}^{N} M_{t0}^{i'n}} \mathcal{M}_{t0}^{US,i} \ .$$

The first component of our exposure measure is the value of assets (in billion Euros) bank n's lenders report with US MFIs in $t0$, $\mathcal{M}_{t0}^{US,i}$, reported in the External Position of MFIs (AUSTA) dataset.[11] We, then, weight each lender i's direct exposure by bank n's liabilities M^{in} with lender i out of total interbank liabilities in the initial period $t0$. More *indirectly* exposed banks either borrow a lot from *directly* exposed lenders or their lenders are heavily invested in the US banking sector. For the base period $t0$ we choose 2006Q1, 6 quarters before the first signs of the financial crisis in the summer of 2007 and 10 quarters prior to the collapse of Lehman Brothers in 2008Q3 which we ultimately set as our event date for the onset of the US financial crisis. In general, choosing the exact event date for the financial crisis proves to be somewhat ambigous. Hence, we opted to show our non-parametric event-study with 2008Q3 as the single event date but drop 2007Q3-2008Q2 as the event "period" from the sample for our parametric regressions and the estimation of the model elasticities in section 2.7. Our identification strategy builds on the idea that German banks who are *directly* exposed to the US financial crisis due to their asset position in US banks have to cut lending

[11] We have to restrict the lenders who report direct exposure to the US to the 80 banks present in the External Position of MFIs dataset. However, these 80 banks cover 90% of all foreign assets by the German financial sector.

in the German financial market once the crisis picks up pace in 2008.[12] With our exposure measure we capture that other German banks who rely on those directly exposed banks for their own funding on the German market are *indirectly* exposed through their funding sources and, hence, face higher funding costs during the period after the Lehman collapse compared to less indirectly exposed banks. We argue that more or less indirectly exposed German banks would have had similar changes in funding costs and, consequently, loan interest rate after 2007/2008 in the absence of the US financial crisis.

2.5.2 Non-parametric Difference-in-Difference Estimates

In our first specification we non-parametrically trace out the effect of indirect exposure on bank n's outcome variable y_t^n (e.g. final loan interest rate) over time using a standard event-study design,

$$\log y_t^n = \rho_n + \mu_t + \sum_{\tau=2006Q1}^{2011Q4} \delta_\tau \left(Exposure_{2006Q1}^{US,n} \times \mu_\tau \right) + \beta' X_t^n + u_t^n \,, \tag{2.1}$$

where ρ_n is a bank fixed effect and μ_t is a fixed effect for every quarter in our sample. We include a vector of time-varying controls X_t^n that contains the shares of different loan products in a bank n's aggregate loans (i.e. different types of borrowers or maturities) and bank n's direct exposure to US MFI assets. By controlling for the composition of a bank's loan portfolio we want to avoid picking up variation in the average loan rate coming from adjustments in the types of loan products a bank is selling. Accounting for a bank's direct exposure to US MFI assets helps avoid a spurious correlation with the banks indirect exposure.[13] We end the sample in 2011Q4 to avoid confounding the effect of the US financial crisis with the subsequent Euro-crisis. Due to the lack of interest rate data for the full dataset on the universe of German banks we need to restrict the estimation sample to the around 240 banks in the interest rate statistics (ZISTA) and we cluster robust standard errors at the bank group-quarter level.[14]

We are interested in the time path of the coefficients on the interaction of exposure and the quarter fixed effects, δ_τ since they capture the exogenous effect of indirect exposure on outcome variable y_t^n. We normalize the size of the effect relative to 2008Q2, the quarter

[12] Through the lens of the model in section 2.6 we can interpret the shock to a lender's balance sheet from outside the German market as a shock to T_t^n since it restricts the bank's ability to provide funding which is equivalent to saying the bank has less deposits.

[13] Since we don't observe US MFI assets for all but 80 banks we set the direct exposure measure to zero for these banks.

[14] Bank group refers the type of bank e.g. savings bank, credit bank, cooperative banks etc.

before the Lehman collapse, which we drop from the set of fixed effects. In figure 2.7 we plot these coefficient and 95% confidence intervals for four different bank variables over our sample period. In the upper left graph our point estimates imply that the interest rate on outstanding final loans to firms and consumers increased rapidly after Lehman by around 10 basis point per 1 billion Euros of indirect exposure and stays elevated for over 3 years. This result is consistent with our hypothesis that more indirectly exposed banks face higher funding costs due to their exposed lenders cutting lending in response to the US financial crisis and, therefore, charge higher interest rates. Our estimated effect on interest rates during the crisis are quite large: on average, a bank at the mean indirect exposure of 2.3 billion Euros contracts on a 25 basis points higher interest rate than a otherwise similar bank with zero exposure. Reassuringly, there is no significant difference in interest rate between more and less exposed banks before the crisis. To give further evidence for the interbank channel we plot the same coefficients for the log value of borrowing from other domestic banks in the upper right graph. At around 5 quarters into the recession banks reduce their liabilities with domestic banks by up to 10% per one billion Euro indirect exposure and liabilities stay at this level thereafter. Again, we find no significant pre-trends in interbank borrowing before the crisis. In the lower left graph we look at the effect of indirect exposure on log aggregate, outstanding loans to firms and consumers. We also find a significant drop of around 2% per one billion Euro after 5 quarters into the recession. However, there is a significant negative pre-trend in the 4 quarters prior to the Lehman collapse which we attribute to the fact that the financial crisis already unfolded in the second half of 2007 and more exposed banks anticipated lower loan demand. Lastly, in the lower right graph we depict the regression coefficients for the log share of final loans funded from own sources (i.e. deposits, equity etc.), henceforth, "own share". Since it is constructed as the difference between final loans minus domestic interbank borrowing over final loans for each bank it shows the combined effect of exposure on interbank borrowing and loans. If banks reduced final loans in the same proportion as interbank borrowing we should not see any effect of indirect exposure on own share. However, more exposed banks reduced interbank borrowing considerably more than final loans during the crisis which leads to a significant increase in the own share. Our regression estimates suggest that banks slowly increased their own share by up to 2% per 1 billion Euro. A bank with mean level of exposure of 2.3 billion Euros increases its reliance on own funding sources by up to 4.6%. To get a sense of the magnitude of this estimate we can compare it to the increase in aggregate own share during and after the Great Recession in figure 2.2. Between 2008 and 2011 the aggregate own share increased by roughly 6 percentage points or 9.4% (from a base of roughly 64%). Hence, around half of the aggregate increase in the own share can be attributed to the indirect exposure to the US financial crisis and subsequent reduction in interbank activity.

2.5.3 Parametric Difference-in-Difference Estimates

As an alternative, we also provide parametric estimates of the indirect exposure to the US financial crisis on the same set of bank outcomes. We will use this specification to structurally estimate two key model elasticities in section 2.7. Our specification is the same as in equation 2.1 but we estimate the average effects of indirect exposure over the 14 quarters following the Lehman collapse, we drop the 4 quarters preceding this date due to the somewhat unclear timing of the financial crisis and, lastly, we start the sample in 2006Q1, the quarter at which we construct the exposure variable. We estimate,

$$\log y_t^n = \rho_n + \tilde{\mu}_t + \delta Exposure_{2006Q1}^{US,n} \times Post_{2008Q3} + \beta' X_t^n + u_t^n \ , \tag{2.2}$$

where $Post_{2008Q3}$ is dummy for all quarters after the third quarter of 2008 and $\tilde{\mu}$ is the restricted set of quarter fixed effect.

Table **??** reports results from estimating 2.2 without controls for loan composition and direct exposure in columns 1-4 and with controls in columns 5-8. Both sets of coefficients are remarkably similar which further supports our claim that indirect exposure interacted with US financial crisis event represents exogenous variation from a German bank's perspective. The effect on loan interest rates is slightly smaller than the peak of our non-parametric estimate: a bank at mean indirect exposure charges an interest rate that is 14 basis points higher than a comparable bank with zero exposure. The effect on loan quantity exceeds the earlier result due to much higher pre-crisis level once the 4 quarters leading up to the Lehman collapse are dropped. The coefficient on the own share is smaller but still significant and quantitatively large. We find that around a third of the increase in the aggregate own share can be attributed to the indirect exposure of banks to the US financial crisis and the resulting reduction in borrowing from the interbank market.

In Table **??** we show coefficients from a pre-trends test. We now include 5 quarters before 2006Q1 (when we construct the exposure measure) and define the Post dummy for the 6 quarters until 2007Q2 (our pre-period in the main estimation). Using the same specification 2.2 we can test whether indirect exposure before the crisis in 2007/2008 compared to even earlier can predict trends in our outcome variables. There are no significant pre-trends in final loans and borrowing. However, we find a small but significant negative effect on loan interest rates in column 5. This implies that more exposed banks charged lower interest rates in the run-up to the crisis which then reversed to higher interest rates post-crisis. Hence, we argue that we underestimate the effect of the exposure to the US financial crisis. Moreover, we are comfortable to say that this an artifact of the relatively short pre-period, in particular, in the context of the clear pattern in figure 2.7. Similarly, we find a significant negative pre-trend for the own share of banks in columns 3 and 7. More exposed banks reduced their reliance on own funds prior to the crisis but then the pattern reverses a during the crisis which implies

that our estimated effect post-crisis might be a lower bound.
We have shown that access to interbank markets affects banks' decisions about interest rates and their funding choices. In particular, high exposure to the US financial crisis through banks' connections with other German banks led to an increase in final loan rates and a reduction in funding through the interbank market. In the next section, we develop a model that provides a theoretical link between these findings and allows us to assess the welfare effects of the decline in interbank market activity in response to the Great Recession.

2.6 Model

The model features discrete quarters indexed by t. Each quarter is divided in a $[0, 1]$ continuum in which agents take actions, like consumption or employment. A moment within the continuum will be indexed by τ, and the continuum can be interpreted as a smooth approximation to the days that comprise a quarter. The pair (t, τ) will serve us to identify the period and point in the continuum at which we are referring to.

2.6.1 Non-technical summary

We start by providing a non-technical overview of the agents in our model and how they interact with each other. There is a representative household and a continuum of firms as in the standard New-Keynesian model. Firms use labor and capital to produce a differentiated good, are subject to Calvo price stickiness, and finance their capital investment through bank loans. Firms pay a wage to the households in exchange for labor, and the household allocates its income between consumption and savings in the form of bank deposits. The banking sector is comprised of a discrete number of N banks, and a central bank that conducts monetary policy and lender-of-last-resort operations. Figure 2.8 depicts the different components of the financial channel. First, households allocate their savings among banks, with the exact distribution of deposits determined by the interest rate that each individual bank pays its depositors and by the household preferences for each bank. Banks must satisfy a constant loan demand from firms at a fixed interest rate agreed between them at the beginning of the quarter, which prevents them from passing any funding mismatch through quantities or prices to the firms.

As household preferences on where to allocate their deposits vary along the continuum τ, banks' capacity to attract deposits is affected. Figure 2.9 shows an hypothetical example of the evolution of deposits along the quarter. If the bank is unable to attract sufficient deposits at the current interest rate it is paying, it has two options: either raise the interest rate on deposits until it attracts sufficient funds to fulfill its loan commitments, or obtain the necessary funds on the interbank market from a bank experiencing a deposit surplus.

The latter is displayed in Figure 2.8 by the arrows linking the deposits and loans of different banks, representing the transfer of funds between them. Interbank loans are repaid, with interest, after one quarter. If interbank rates are lower than the necessary deposit rate to fulfill the firm-loan commitments, the latter option is preferable, and the funding costs of the borrowing bank will decrease. We also introduce a third alternative in the form of the lender-of-last-resort. If banks are unable to borrow at reasonable rates from their interbank counterparties or households, the central bank will offer them a loan at a penalty rate over the average interbank rate.

In equilibrium, an improvement in the allocative efficiency of the interbank market will result in lower interest rates charged to firms, and hence more investment and production. It will also result in less volatile interest rates, as banks can access alternative funding sources as financial markets integrate. On the other hand, as the interbank market grows bigger the banking system becomes more exposed to shocks of large banks and interbank credit freezes, increasing the volatility of the economy. Our model provides analytical expressions that we will use to study each one of these effects separately. The following sections lay out the formal microfoundations of the model and its solution.

2.6.2 Representative Household

Households obtain positive utility from consuming an aggregate good and supply their labor in exchange for a wage to the firms producing it. They have the option to save part of their income in a one-period risk-free bond or as deposits on any of the N banks that constitute the financial system of this model. Households derive positive utility from holding real deposit balances in banks, capturing a preference for liquidity or the usefulness of money in the completion of consumption transactions. The representative household maximizes the following objective function:

$$\max \quad E_t \sum_{j=0}^{\infty} \beta^j \left[\log\left(X_{t+j}\right) - \left(\frac{\eta}{\eta+1}\right) \int_0^1 N_{t+j,\tau}^{1+1/\eta} \, \mathrm{d}\tau \right] ,$$

where $N_{t,\tau} = \left[\int_0^1 N_{t,\tau}(\nu)^{1+1/\eta} \, \mathrm{d}\nu\right]^{\frac{\eta}{\eta+1}}$ is the aggregate labor index and $N(\nu)$ labor supplied to intermediate industry ν, η is the Frisch labor supply elasticity and variable X is a composite of consumption and bank deposit balances. In particular,

$$X_t = C_t + \sum_{n=1}^{N} \int_0^1 \left(1 - T_t^n \cdot z_{t,\tau}^n\right) \frac{D_{t,\tau}^n}{P_t} \, \mathrm{d}\tau ,$$

where $C_t = \int_0^1 C_{t,\tau}\, \mathrm{d}\tau$ is aggregate consumption in t, $D^n_{t,\tau}$ are one-period nominal deposits at bank n, P_t is the aggregate price index of the economy, $(1 - T^n_t)$ is the average utility of deposits at bank n and $z^n_{t,\tau}$ is an exogenous shock to those preferences.We model $z^n_{t,\tau}$ as a Weibull-distributed shock with mean one and shape parameter κ, and assume it to be i.i.d. across banks n, quarters t and time continuum τ. Parameter κ determines the volatility of these shocks. Movements of $z^n_{t,\tau}$, through a shift in preferences, generate a reallocation of deposits across banks that gives rise to the market for interbank loans, as we will show later.

Period (t, τ) budget constraint is

$$\begin{aligned} C_{t,\tau} + \frac{\sum_{n=1}^N D^n_{t,\tau}}{P_t} + \frac{B_{t,\tau}}{P_t} = & \frac{\sum_{n=1}^N (1+\varsigma_D) \cdot R^{D,n}_{t-1,\tau} D^n_{t-1,\tau}}{P_t} + \frac{R^B_{t-1} B_{t-1,\tau}}{P_t} \\ & + \int_0^1 \frac{W_t(\nu) N_{t,\tau}(\nu)}{P_t}\, \mathrm{d}\nu + \frac{\Upsilon_{t,\tau}}{P_t} \,, \end{aligned} \tag{2.3}$$

where $B_{t,\tau}$ and R^B_t are one-period government bonds and the rate paid on them, $R^{D,n}_{t,\tau}$ is the rate on household deposits paid by bank n, ς_D is a savings subsidy, $W_t(\nu)$ is the wage paid by industry ν and Υ_t are transfers from different sources, like government lump sum taxation and bank and firm profits.

Maximizing the representative household problem we obtain the following equilibrium condition for deposit rates:

$$R^{D,n}_{t,\tau} = (1+\varsigma_D)^{-1} \cdot R^B_t \cdot T^n_t \cdot z^n_{t,\tau} \,, \qquad \forall n \,. \tag{2.4}$$

A change in the return of bonds R^B_t has a proportional impact on the rate paid on deposits by all banks, while movements of $z^n_{t,\tau}$ will change the relative costs among them, leading to a reallocation of deposits across banks at each moment τ.

2.6.3 Firms

There is a continuum $\nu \in [0, 1]$ of intermediate goods, each produced by a monopolist with the following production function employing capital and labor

$$Y_{t,\tau}(\nu) = \left(\frac{K_t(\nu)}{\alpha}\right)^{\alpha} \left(\frac{exp(u^A_t) \cdot N_{t,\tau}(\nu)}{1-\alpha}\right)^{1-\alpha} \,, \tag{2.5}$$

where u^A_t is an aggregate technology process defined as $u^A_t = \rho_A\, u^A_{t-1} + \varepsilon^A_t$, $\varepsilon^A_t \sim N(0, \sigma^2_A)$. We impose the restriction that firms must employ a constant level of capital across the period, $K_{t,\tau}(\nu) = K_t(\nu)$, $\forall \tau$, so all production adjustments happen through the labor margin.

Aggregate capital is a CES composite of N distinct types of capital

$$K_t(\nu) = \left[\sum_{n=1}^{N} (a_t^n)^{1/\sigma} \, K_t^n(\nu)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \,, \tag{2.6}$$

Parameter $\sigma > 1$ is the elasticity of substitution between different types of capital and a_t^n is a time-varying demand shock for each type. Without loss of generality to the qualitative results that we will show later, we simplify the model by assuming full depreciation and instant build-up of capital from investment

$$K_{t,\tau}^n(\nu) = \frac{I_{t,\tau}^n(\nu)}{P_t} \,, \quad \forall \; n \,,$$

where $I_t^n(\nu)$ stands for investment in capital of type n. Firms finance their investment at each moment τ with credit obtained from banks. There are N distinct banks in the model and each one specializes in providing loans $L_{t,\tau}^n(\nu)$ for a different type of capital. Loans are repaid after one quarter at gross interest rate $R_t^{F,n}$, and firms are subject to the following investment constraints

$$I_{t,\tau}^n(\nu) \leq L_{t,\tau}^n(\nu) \,, \quad \forall \; n \,,$$

which hold with equality in equilibrium. We do not consider capital financed by the firm itself, but such distinction would not affect the qualitative results of our model.

A representative, perfectly competitive firm aggregates intermediate products into a final good according to

$$Y_t = \left[\int_0^1 Y_t(\nu)^{\left(\frac{\epsilon-1}{\epsilon}\right)} \, \mathrm{d}\nu \right]^{\left(\frac{\epsilon}{\epsilon-1}\right)} \,,$$

where $\epsilon > 1$ is the elasticity of substitution between varieties. Individual demand for intermediates is given by

$$Y_t(\nu) = \left(\frac{P_t(\nu)}{P_t} \right)^{-\epsilon} Y_t \,,$$

where $P(\nu)$ is the price of intermediate ν. The aggregate price index is

$$P_t = \left[\int_0^1 P_t(\nu)^{1-\epsilon} \, \mathrm{d}\nu \right]^{\frac{1}{1-\epsilon}} \,.$$

Intermediate producers have sticky prices *á la* Calvo (1983) and they reset their price at the beginning of the quarter with probability $1-\theta$. All firms reset to the same optimal price (in equilibrium) within a given period, which we denote by $P^{\star}$. This allows us to recursively express the previous expression as

$$P_t^{1-\epsilon} = (1-\theta)\cdot(P_t^{*})^{1-\epsilon} + \theta\cdot(P_{t-1})^{1-\epsilon} \ .$$

Intermediate firm ν maximizes the discounted stream of profits

$$\max \quad \sum_{j=0}^{\infty} E_t\left[Q_{t,t+j}\int_0^1 (1+\varsigma_F)\cdot P_{t+j}(\nu)Y_{t+j,\tau}(\nu) - W_{t+j}(\nu)N_{t+j,\tau}(\nu) - \sum_{n=1}^{N} R^{F,n}_{t+j-1}L^n_{t+j-1}(\nu)\,\mathrm{d}\tau\right] ,$$

where $Q_{t,t+j} = \beta^j\left(\frac{P_{t+j}}{P_t}\cdot\frac{X_{t+j}}{X_t}\right)^{-1}$ is the firm's stochastic discount factor between periods t and $t+j$ and ς_F is a production subsidy. Solving the firm's optimization problem and adding the loan demand across the continuum ν of firms, we obtain an expression of the aggregate demand for bank n loans as

$$L_t^n = a_t^n\left(\frac{R_t^{F,n}}{R_t^F}\right)^{-\sigma} L_t \ , \tag{2.7}$$

where $L_t = \left[\sum_{n=1}^{N}(a_t^n)^{1/\sigma}(L_t^n)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$ is an aggregate loan index and R_t^F is an aggregate interest rate index defined as

$$R_t^F = \left[\sum_{n=1}^{N} a_t^n\left(R_t^{F,n}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \ . \tag{2.8}$$

Note that from these expressions we can alternatively interpret σ as the elasticity of substitution between loans and a_t^n as a loan demand shock. Aggregate firm profits over the period are

$$\Upsilon_t^F = (1+\varsigma_F)\cdot P_tY_t - \int_0^1 W_t(\nu)N_t(\nu)\,\mathrm{d}\nu - \sum_{n=1}^{N} R^{F,n}_{t-1}L^n_{t-1} \ .$$

2.6.4 Banking Sector

Each bank performs three activities: they obtain deposits from the representative household, provide credit to firms and lend funds to each other in the interbank market. For expositional purposes we will assume that each bank is divided in two divisions, each one responsible for a different set of these tasks. The Loan Division provides credit to firms and secures the necessary funding through internal funds or interbank loans. The Deposit Division procures deposits from the representative household and distributes them to the Loan Divisions through the interbank market or internal transfer.

Loan Division

Loans granted by bank n to firms at each point τ of the continuum are subject to the following constraints:

$$L^n_{t,\tau} \leq M^n_{t,\tau} \;, \tag{2.9a}$$

$$M^n_{t,\tau} \geq 0 \;, \tag{2.9b}$$

where $M^n_{t,\tau}$ is the amount of internal and/or interbank funding available to the Loan Division at time τ. Equation (2.9a) captures the constraint that banks can only provide credit up to the amount that they have readily available to be lent, and holds with equality in equilibrium. Interbank loans taken in (t,τ) are repaid next quarter. Bank funds are perfect substitutes, and Loan Divisions obtain them from the bank that offers the lowest rate at each moment τ. Formally,

$$M^n_{t,\tau} = M^{in}_{t,\tau}, \quad i_{t,\tau}(n) = \arg_j \min \left\{ R^{I,jn}_{t,\tau} \right\} \;,$$

where $M^{in}_{t,\tau}$ are the interbank funds lent by Deposit Division i to Loan Division n and $R^{I,in}_{t,\tau}$ the gross rate on interbank loans that bank i charges to bank n. Next period profits of the Loan Division are

$$\int_0^1 (1+\varsigma_B) \cdot R^{F,n}_t L^n_{t,\tau} - R^{I,n}_{t,\tau} M^n_{t,\tau} \, \mathrm{d}\tau \;,$$

$$\text{where} \quad R^{I,n}_{t,\tau} = \min_i \left\{ R^{I,in}_{t,\tau} \right\} \;, \tag{2.10}$$

where ς_B is a subsidy to firm loans. Variable $R^{I,n}_{t,\tau}$ is the gross interest rate at which interbank loans (or own funds) at point τ are obtained. Banks know their individual firm loan demands given by equation (2.7) and act as monopolistic competitors, taking the aggregate index R^F as given. Banks and firms meet at the beginning of the quarter and agree on an interest rate that will prevail throughout the period[15]. This results in a constant firm loan demand along the continuum τ which banks have to finance while experiencing a varying capacity to attract funds due to the shocks to depositor preferences, forcing them to borrow from

[15] An alternative assumption with equivalent results would be that firm interest rates are sticky *within* the continuum, and can only be reset at the beginning of each quarter. Sørensen and Werner (2006) provide supporting empirical evidence for the stickiness of firm interest rates.

the interbank market or charge higher interest rates to firms. On the other hand, interbank rates are renegotiated each instant τ and reflect this shifting capacity to provide funds by the emitting bank. Solving the maximization problem we obtain the optimal interest rate on firm loans as a constant mark-up over the average cost of funds.

$$R_t^{F,n} = \left(\frac{\sigma \cdot (1+\varsigma_B)^{-1}}{\sigma - 1} \right) R_t^{I,n} \,, \qquad R_t^F = \left(\frac{\sigma \cdot (1+\varsigma_B)^{-1}}{\sigma - 1} \right) R_t^I \,, \tag{2.11}$$

where we defined $R_t^{I,n} \equiv \int_0^1 R_{t,\tau}^{I,n} \, \mathrm{d}\tau$ and $R_t^I = \left[\sum_{n=1}^N a_t^n \cdot \left(R_t^{I,n} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$.

Deposit Division

This Division obtains deposits from the representative household and converts them into internal funding or interbank loans to other banks. The amount of funds that n can provide is given by

$$M_{t,\tau}^{nn} + \sum_{i \neq n} d_t^{ni} \cdot M_{t,\tau}^{ni} = D_{t,\tau}^n \,, \tag{2.12}$$

subject to

$$M_{t,\tau}^{ni} \geq 0 \,, \quad D_{t,\tau}^n \geq 0 \,, \quad \forall n, i \,,$$

where $d^{ni} \geq 1$ are transaction costs associated to moving funds from lender bank n to borrower bank i and which should be interpreted as containing screening, enforcement, or other costs related to a transaction. Uncertainty about the value of mortgage-backed securities and related assets following the 2007 financial crises can be interpreted through the lens of the model as an increase in the costs of collateral screening, driving d^{ni} up. We implicitly normalized to one the transaction costs between Divisions of the same bank, $d^{nn} = 1$, $\forall n$.

Next period profits of Deposit Division n are

$$\int_0^1 \sum_{i=1}^N R_{t,\tau}^{I,ni} M_{t,\tau}^{L,ni} - R_{t,\tau}^{D,n} D_{t,\tau}^n \, \mathrm{d}\tau \,.$$

The markets for interbank loans and deposits are perfectly competitive and banks act as price takers. Solving the optimization problem and using equation (2.4), we obtain an expression for the interest rate charged by bank n to bank i at moment τ as a function of the bank fundamentals,

$$R^{I,ni}_{t,\tau} = (1+\varsigma_D)^{-1} \cdot R^B_t \cdot d^{ni}_t \cdot T^n_t \cdot z^n_{t,\tau} \,. \tag{2.13}$$

2.6.5 Central Bank

The central bank in our model can affect the risk-free rate of the economy through conventional open market operations as well as provide direct credit to banks in the system in its role as lender-of-last-resort. We describe both types of intervention in this Section.

Lending Facility

We allow the central bank to provide direct credit to banks through the discount window, which captures its functions as lender-of-last-resort and more broadly the various interventions in the financial markets witnessed during the Great Recession. We assign subindex zero to the central bank and model it as an additional bank within the system, but with some unique characteristics. The central bank does not obtain deposits from the representative household, and differentiates itself by its capacity to freely create money. This translates in the central bank being able to arbitrarily set the interest rate at which it is willing to lend. We model it as a penalty rate over the average rate at which each bank is able to borrow from the rest of its funding sources, formally

$$R^{I,0n}_{t,\tau} = penalty^n_{t,\tau} \cdot \Phi^n_t \cdot R^B_t \,, \tag{2.14}$$

where $\Phi^n_t \cdot R^B_t \equiv E_t \left[\min_{i \in \{1,\dots,N\}} \left\{ R^{I,in}_{t,\tau} \right\} \right]$ is bank n's average cost of funds from its non-central bank sources. We can interpret variable Φ^n_t as the credit spread between the interbank funding costs of bank n (excluding central bank credit) and the risk-free rate. We study different lending policies by assigning the following functional form to the penalty rate:

$$penalty^n_{t,\tau} = e^{\varpi_1} \cdot \underbrace{\left(\frac{\Phi^n_t}{\Phi^n} \right)^{-\varpi_2} \cdot z^0_{t,\tau}}_{\text{variable component}} \,,$$

where ϖ_1 is a parameter that controls the steady state size of the penalty, and ϖ_2 its response to deviations of the bank's funding costs from steady state. $z^0_{t,\tau}$ is a policy shock which we model for analytical convenience as being distributed Weibull with mean one and shape parameter κ. When $\varpi_1 \to +\infty$, the interest rate charged by the central bank becomes prohibitively expensive and the model collapses to what would be an equivalent version of it without lender of last resort intervention.

Any profits made by the central bank are returned to the representative household via lump-sum transfer,

$$\Upsilon_t^{CB} = \sum_{n=1}^{N} \int_0^1 R_{t-1,\tau}^{I,0n} M_{t-1,\tau}^{0n} \, d\tau \ .$$

Policy Rule

The central bank also determines the nominal risk-free rate R_t^B of the economy through conventional open market operations. We assume that it follows a Taylor rule of the form

$$R_t^B = R^B \cdot \left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\pi} \left(\frac{Y_t}{Y_t^n}\right)^{\gamma_y} \left(\frac{\widetilde{R}_t^I}{\widetilde{R}^I}\right)^{\gamma_I} \cdot exp\left(u_t^R\right) \ ,$$

where $\Pi_t \equiv P_t/P_{t-1}$ stands for gross inflation, Y_t^n is output under flexible prices, $\widetilde{R}^I \equiv R_t^I/R_t^B$ is the aggregate interbank credit spread (including lender-of-last-resort cost of credit), and u^R an exogenous monetary policy shock. We allow the central bank to respond to deviations of the credit spread $\widetilde{R}_t^I$ because we want to study whether there are additional welfare benefits of establishing such policy compared to the standard targeting of inflation and output gap.

2.6.6 Banking sector aggregation

Plugging equations (2.13) and (2.14) into (2.10), we obtain the distribution of the interbank credit spread $\widetilde{R}_t^{I,n} \equiv R_t^{I,n}/R_t^B$ paid by n,

$$\widetilde{R}_t^{I,n} = \Phi_t^n \cdot \left[1 + e^{-\kappa\varpi_1} \cdot \left(\frac{\Phi_t^n}{\Phi^n}\right)^{\kappa\varpi_2}\right]^{-1/\kappa} , \qquad (2.15)$$

where

$$\Phi_t^n = \left[\sum_{i=1}^{N} \left((1+\varsigma_D)^{-1} \cdot d_t^{in} \cdot T_t^i\right)^{-\kappa}\right]^{-1/\kappa} .$$

We used the property that the minimum of a group of Weibull random variables is also distributed Weibull[16]. To understand the determinants of the credit spread Φ^n (spread excluding Central Bank), note that the term $(1+\varsigma_D)^{-1} \cdot d_t^{in} \cdot T_t^i$ is equal to the average spread over the continuum between the interest rate $R_{t,\tau}^{ni}$ at which bank i is willing to lend funds to bank n and the risk-free rate R_t^B. Therefore, Φ^n is an average of the cost over the risk-free rate at

[16] Note that, $\widetilde{R}_{t,\tau}^{I,n} \sim W(\widetilde{R}_t^{I,n} \cdot \Gamma\left(1+1/\kappa\right)^{-1}, \kappa)$

which bank n can obtain funding from its interbank connections or its own depositors. If funds from a particular connection are relatively costly compared to the rest of bank n sources, the exponent $-\kappa$ ensures that this connection becomes less important in determining the final value of Φ^n. Intuitively, bank n will infrequently borrow from relatively expensive sources, so they will have a smaller effect on the determination of the average cost of bank n funds.

Transaction volumes and Deposits

Define λ_t^{0i} as the share of funding that bank i obtains from the central bank. We obtain an expression for it as

$$\lambda_t^{0i} = \left[1 + e^{\kappa\varpi_1} \cdot \left(\frac{\Phi_t^i}{\Phi^i}\right)^{-\kappa\varpi_2}\right]^{-1} .$$

Relevant to the solution of the model, we define the aggregate central bank trade share as

$$\lambda_t^0 = \sum_{n=1}^{N} s_t^i \cdot \lambda_t^{0i} , \qquad s_t^i = a_t^i \cdot \left(\frac{\widetilde{R}_t^{l,i}}{\widetilde{R}_t^l}\right)^{1-\sigma} ,$$

where s_t^i is the share of the firm loan market supplied by bank i. We define λ^{ni} as the share of funding that bank i would obtain from bank n if the lender of last resort wasn't present, for which we find an expression as

$$\lambda_t^{ni} = \left(\frac{(1+\varsigma_D)^{-1} \cdot d_t^{ni} T_t^n}{\Phi_t^i}\right)^{-\kappa} . \tag{2.16}$$

When the lender-of-last-resort is present, we just have to multiply the previous expression by $\left(1 - \lambda_t^{0i}\right)$ to obtain the appropriate trade share. Integrating equation (2.9) over the continuum we obtain the volume of funds transacted between any pair of banks as

$$M_t^{ni} = \left(1 - \lambda_t^{0i}\right) \cdot \lambda_t^{ni} \cdot L_t^i . \tag{2.17}$$

Intuitively, $(1+\varsigma_D)^{-1} \cdot d_t^{ni} \cdot T_t^n$ is the average cost above the risk-free rate R_t^B at which deposits received in bank n are *offered* to bank i over the period, while Φ^i is the average credit spread at which i *effectively* borrows over the period. The larger the ratio between the two, the less funds originated in n will reach i, both in absolute and relative terms. Banks suffer from a funding mismatch in which they have to supply a constant firm loan demand along the continuum while facing a varying capacity to attract deposits. Hence, even if funding from certain counter-parties is more expensive on average, there will be instances in the continuum in which borrowing from them is optimal due to the mismatch.

Note that we solved our model under the assumption that interbank funds supplied by distinct banks are perfect substitutes at any moment τ, but nonetheless we obtain a downward-sloping CES demand function for interbank funds when we aggregate demand over the continuum. This follows from the volatility of depositor preferences $\left\{z^j\right\}_{j=1}^N$ and its effect on the relative cost to banks of obtaining household deposits. Even banks with high transaction costs will eventually experience moments of high deposit influx within the continuum and temporarily become the least-cost suppliers in the interbank market. The elasticity of demand κ in the equation above is the same parameter that controls the variance of depositor preference shocks. Demand reacts more strongly to differences between $d_t^{ni} \cdot T_t^n$ and Φ^i when κ is high. This happens because a high κ implies a low variance of depositor preferences, so there are fewer instances in which funding costs are lowered enough by an excess influx of deposits to be able to compensate for the effect of high transaction costs.

Integrating equation (2.12) along the continuum and using (2.9) we obtain an expression for bank n and aggregate deposits

$$D_t^n = M_t^{nn} + \sum_{i \neq n} d_t^{ni} \cdot M_t^{ni} ,$$

$$D_t + \sum_{n=1}^{N} M_t^{0n} = \sum_{n=1}^{N} L_t^n + \sum_{n=1}^{N} \sum_{i=1}^{N} \left(d_t^{ni} - 1\right) \cdot M_t^{ni} ,$$

which tells us that in equilibrium aggregate deposits and central bank money must be equal to the total volume of loans plus interbank transaction costs.

Aggregate banking sector profits over period t are

$$\Upsilon_t^B = (1 + \varsigma_B) \cdot \sum_{n=1}^{N} R_{t-1}^{F,n} L_{t-1}^n - \sum_{n=1}^{N} \int_0^1 R_{t-1,\tau}^{D,n} D_{t-1,\tau}^n \, \mathrm{d}\tau - \sum_{n=1}^{N} \int_0^1 R_{t-1,\tau}^{I,0n} M_{t-1,\tau}^{0n} \, d\tau .$$

2.6.7 Government

The government in this model provides subsidies to firms, banks and depositors, which are funded through lump sum taxation of the representative household. The expression for government transfers is

$$\Upsilon_t^G = -\left[\varsigma_F \cdot P_t Y_t + \varsigma_B \cdot \sum_{n=1}^{N} R_{t-1}^{F,n} L_{t-1}^n + \varsigma_D \cdot \sum_{n=1}^{N} R_{t-1}^{D,n} D_{t-1}^n\right] .$$

Optimal subsidies

We assume that the government sets its subsidies to firms and banks in order to offset steady state real distortions from monopolistic competition, which corresponds to

$$\varsigma_F^* = \frac{1}{\epsilon - 1} \,, \quad \varsigma_B^* = \frac{1}{\sigma - 1} \,.$$

When the central bank provides credit to the interbank market in its role as lender-of-last-resort, it depresses the steady state rate paid on deposits and distorts the intratemporal substitution between consumption and labor, $-U_X/U_N$. We assume a subsidy to depositors that keeps this relationship stable and is given by

$$\varsigma_D^* = \frac{\lambda^0}{1 - \lambda^0} \,.$$

The elimination of real distortions via subsidies greatly simplifies the analytical welfare expressions that we will present in Section 2.8, and is a widely employed device in the business cycle literature for this reason. From an empirical standpoint, we can justify our assumption on deposit subsidies by the fact that there is no evidence on lender-of-last-resort interventions depressing the steady state interest rate paid on bank deposits. We leave the relaxation of these assumptions in the context of our model to future research.

2.6.8 Market clearing

Total profit transfers to the representative household become

$$\Upsilon_t \equiv \Upsilon_t^F + \Upsilon_t^B + \Upsilon_t^G + \Upsilon_t^{CB} = P_t Y_t - \int_0^1 W_t(\nu) N_t(\nu) \, \mathrm{d}\nu - (1 + \varsigma_D) \cdot \sum_{n=1}^{N} \int_0^1 R_{t-1,\tau}^{D,n} D_{t-1,\tau}^{n} \, \mathrm{d}\tau \,.$$

Aggregating the representative household budget constraint (2.3) over the τ continuum and making use of the previous expression, we obtain the following aggregate market clearing condition

$$C_t + \frac{D_t}{P_t} = Y_t \,.$$

2.6.9 Shock processes

We define the functional form of the shocks affecting the banking sector as:

CES firm weights

$$a_t^i = \frac{a^i \cdot exp\left(u_t^{a,i}\right)}{\sum_{j=1}^N a^j \cdot exp\left(u_t^{a,j}\right)} \ , \qquad u_t^{a,i} = \rho_I \cdot u_{t-1}^{a,i} + \varepsilon_t^{a,i} \ , \qquad \forall i \ ,$$

$$\sum_{i=1}^N a^i = 1 \ .$$

Depositor Preferences

$$T_t^i = T^i \cdot exp\left(u_t^{T,i}\right) \ , \qquad u_t^{T,i} = \rho_I \cdot u_{t-1}^{T,i} + \varepsilon_t^{T,i} \ , \qquad \forall i \ .$$

Transaction costs

$$d_t^{ni} = \begin{cases} \left(d^{ni}\right)^{\varrho} \cdot exp(u_t^{I,ni}) & \text{, if } i \neq n \ , \\ 1 & \text{, otherwise.} \end{cases} \ , \qquad u_t^{I,ni} = \rho_I \cdot u_{t-1}^{I,ni} + \varepsilon_t^{I,ni} \ , \qquad \forall n, i \ , \quad (2.18)$$

where $\varepsilon^{a,i}$, $\varepsilon^{T,i}$ and $\varepsilon^{I,ni}$ are mean-zero, exogenous (but potentially correlated) stochastic shocks. Parameter ϱ will allow us to modify the size of transaction costs once we look at banking system integration counterfactuals in Section 2.8.

2.6.10 Steady-state relationships

We now define the average own trade share of the banking system as

$$\lambda_t^{Own} = \left[\sum_{i=1}^N s_t^i \cdot \left(\lambda_t^{ii}\right)^{\frac{\sigma-1}{\kappa}}\right]^{\frac{\kappa}{\sigma-1}} \ .$$

After some manipulations we obtain the following expression for the steady state credit spread

$$\widetilde{R}^I = \left(1-\lambda^0\right)^{1+1/\kappa} \cdot \left(\lambda^{Own}\right)^{1/\kappa} \cdot \left[\sum_{i=1}^N a^i \cdot \left(T^i\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \ .$$

The first term reveals that the steady state credit spread is made smaller by higher central bank participation. Intuitively, central bank interventions as a lender-of-last-resort set an upper bound on the funding costs of banks, closing the gap between the interbank and risk-free rates. The second term shows that the own trade share is inversely related to the credit spread, as banks can access potentially cheaper funding sources when they participate on the interbank market. Otherwise, they have to absorb the liquidity shocks of their depositors along the continuum, which increases their funding costs. Finally, the last term captures the preference of households for holding their money on bank deposits versus riskless bonds. The stronger this preference is (lower T's), the lower the credit spread will be, as banks have to compensate depositors less for their money. There is suggestive evidence for this relationship in the data. Looking back at Figures 2.2 and 2.3 we see that the share of interbank liabilities on bank balance sheets converged to a persistently lower level following the 2007 financial crises, which would imply a higher value of the own trade share. Meanwhile, euribor credit spreads stabilized at a permanently higher level after the end of the euro crisis. That would suggest that the banking system intermediation capacity deteriorated and created an increase in bank funding costs.

2.6.11 Log-Linearized system

In this section we present the dynamic solution of the model under a first-order approximation. We use lower-case letters to denote the logarithm of a variable, while hats correspond to deviations from steady state. We discuss the key assumptions and equilibrium equations here and relegate the detailed derivations to Appendix A.1.

Banking sector variables

After some manipulations, we obtain the evolution of the interbank rate and central bank trade share as a function of the fundamental shocks

$$\hat{\hat{r}}^{I}_{t} = \rho_I \cdot \hat{\hat{r}}^{I}_{t-1} + (1-\varpi_2\lambda^0)(1-\lambda^0)\cdot\left[\varepsilon^T_t + \varepsilon^I_t\right] - \frac{\varepsilon^a_t}{\sigma-1} \ ,$$

$$\widehat{\log(\lambda^0_t)} = \rho_I \cdot \widehat{\log\left(\lambda^0_{t-1}\right)} + \kappa\varpi_2(1-\lambda^0)^2\cdot\left[\hat{\varepsilon}^T_t + \hat{\varepsilon}^I_t\right] \ ,$$

where ε^T_t, ε^I_t and ε^a_t are average combinations of the individual bank shocks to depositor preferences, transaction costs and firm loan demand, respectively. The structure of the banking system affects the size and volatility of these aggregate shocks, as the combination of individual shocks that comprises them depends on the bilateral bank trade shares λ^{ni} and the share of the firm loan market s^i controlled by each bank. Also note that lender of last resort policy parameters multiply some of these shocks, determining the strength with which they affect the variables.

Equilibrium conditions

The New-Keynesian Phillips Curve is

$$\hat{\pi}_t = \Omega \,\hat{\tilde{y}}_t + \beta E_t \left[\hat{\pi}_{t+1}\right] \ , \qquad \tilde{y}_t \equiv y_t - \bar{y}_t \ ,$$

where $\tilde{y}_t$ is the output gap and Ω is a constant defined in the Appendix as a combination of the Calvo price resetting parameter and several labor and production elasticities.

The Dynamic IS Equation is

$$\hat{\tilde{y}}_t = -\left[1 + \alpha\left(\frac{\eta}{\eta+1}\right)\right] \cdot \left[\hat{r}^B_t - E_t\left[\hat{\pi}_{t+1}\right] - \hat{\iota}^n_t\right] + E_t\left[\hat{\tilde{y}}_{t+1}\right] \ ,$$

where $\iota^n_t \equiv \left[(1-\rho_I)\left(\frac{\alpha}{1-\alpha}\right) \cdot \tilde{r}^I_t + (1-\rho_I)\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\eta}{\eta+1}\right)\left(\frac{\lambda^0}{1-\lambda^0}\right) \cdot \log\left(\lambda^0_t\right) - (1-\rho_A)\cdot u^A_t\right]$ stands for the natural interest rate under flexible prices. The term $\left[1+\alpha\left(\frac{\eta}{\eta+1}\right)\right] > 1$ captures the sensitivity of output gap to deviations of the real interest rate, $r^B_t - E_t\left[\pi_{t+1}\right]$, from its natural counterpart, ι^n_t. In standard New Keynesian models, the coefficient multiplying this deviation is equal to one. In our model, the additional sensitivity comes from the inclusion of the non-separable utility for deposits on the household's utility function[17]. Excluding this small difference, our model equilibrium equations and their interpretation remains essentially the same as in standard New Keynesian models. But in contrast to them, our model features real economic effects from central bank lender of last resort policies, and links the complex structure of the banking system to the volatility of financial shocks. We hope that our model provides the basic foundations to future research on the effects of lender-of-last-resort policies and financial structure within the New Keynesian framework. We develop some of these exercises in Sections 2.8 and 2.9.

2.7 Estimation and Calibration

In this section, we take the model to the detailed data on Germany's banking market introduced in section 2.3. First, we leverage the plausibly exogenous shock to loan interest rates from banks' exposure to the US financial crisis introduced in section 2.5 to causally estimate loan demand elasticity σ and interbank supply elasticity κ. The former captures the degree to which firms substitute between banks in loan demand. The latter governs the volatility of preference shocks for bank deposits and, consequently, the supply of funds into

[17]See Fisher (2015) for a discussion on how non-separable preferences for assets in the utility due to liquidity and/or safety motives modify the dynamic IS equation of the standard New Keynesian model.

the interbank market. In the second step, we recover the model-implied fundamentals of the banking sector (i.e. loan demand parameters a_t^n, deposit supply parameters T_t^n and bilateral transaction costs d_t^{ni}) for each bank and quarter using our data and estimated elasticities from the first step. We then use the resulting time series of each parameter to characterize their underlying stochastic processes.

2.7.1 Estimation of Banking Sector Elasticities σ and κ

For the estimation of demand elasticity σ we begin by taking the logs of both sides of bank n's loan demand in equation 2.7 and replacing aggregate, time-varying variables with a time fixed effect,

$$\log L_t^n = \mu_t - \sigma \log R_t^{F,n} + \log a_t^n \ , \tag{2.19}$$

where $\log a_t^n$ captures time-varying preferences for MFI n at time t. First, we assume $\log a_t^n = \rho_n + \beta' X_t^n + \epsilon_t^n$ where X_t^n are time-varying controls such as the composition of a bank's loan portfolio, ρ_n is a bank fixed effect and error term ϵ_t^n is a loan demand shock. By controlling for the shares of different loan products in a bank's aggregate loans (i.e. different borrowers or maturities) we want to avoid picking up variation in the average loan rate coming from adjustments to the loan portfolio that would be otherwise attributed to changes in preferences for bank n. Second, as is well known from the literature on demand estimation that causal identification of σ requires exogenous variation in interest rates by bank n at time t. We argue that our identification strategy in section 2.5 provides such exogenous variation in loan interest rates. More indirectly exposed banks contract on higher interest rates after the Lehman collapse compared to less indirectly exposed banks. Hence, we can interpret our difference-in-difference specification for loan interest rates as the first stage in estimating equation 2.19 with Two-Stage-Least-Squares and the specification with final loans to firms and consumers on the left-hand side as the corresponding reduced form. In practice, we use the interaction term of indirect exposure and the Post-dummy for quarters after the Lehman event as an instrument for $\log R_t^{F,n}$ in equation 2.19. For identification of σ, we find it reasonable to argue that our instrument is uncorrelated with loan demand shocks ϵ_t^n. Table **??** reports our estimates of $-\sigma$ without controls in column 1 and with controls in column 3. Our preferred estimate of σ is 37.8 in column 3 which is highly statistically significant and a first stage F-statistic of around 25. Our estimate of the elasticity of substitution of bank demand is quite high but falls into the range of other estimates from the literature.

We proceed analogously with our estimation of funding supply elasticity κ. In particular, this elasticity captures how banks reallocate between own deposits and accessing funds through the interbank market in order to finance final loans. Substituting equation 2.16 into 2.17 for the probability that bank n obtains funds from its own deposits the "own" share of bank n, i.e. the ratio of M_t^{nn} and loans L_t^n at time t can be expressed as

$$\frac{M_t^{nn}}{L_t^n} = (d_t^{nn})^{-\kappa} (T_t^n)^{-\kappa} \left(\frac{R_t^{F,n}}{R_t^B}\right)^{\kappa} \left(\frac{\sigma-1}{\sigma}\right)^{\kappa}.$$

Taking logs, assuming $d_t^{nn} = 1, \ \forall t$ and collecting all constant and aggregate, time-varying variables into a time fixed effects we arrive at

$$\log\left(\frac{M_t^{nn}}{L_t^n}\right) = \mu_t + \kappa \log R_t^{F,n} - \kappa \log T_t^n. \tag{2.20}$$

Equation 2.20 states that bank n resorts to funding through own funds when they receive a positive shock to depositors' preferences (negative T_t^n shock) relative to the total cost of funds including funding from the interbank market which is summarized by the interest rate on outstanding firm loans. To account for the level effect of depositor preferences for n and detailed, time-varying loan product shares X_t^n we assume $\log T_t^n = \rho_n + \beta' X_t^n + v_t^n$ where ρ_n is a bank fixed effect and we interpret v_t^n as a deposit supply shock. Equation 2.20 turns out to be exactly like equation 2.19 with the difference that we use the own share as the dependent variable. We make the analogous argument as above, namely, that indirect exposure to the US financial crisis after Lehman is uncorrelated with bank-level deposit shocks. Moreover, we control for banks' direct exposure to US financial crisis since it seems possible a bank's indirect exposure is correlated with deposits shocks through its relationship with direct exposure. Table **??** shows our IV estimates of κ with and without controls in columns 2 and 4 respectively. We find that the funding cost shock in the interbank leads to a significant increase in banks' reliance on own funding sources with an elasticity of around 21.3 with controls and 26.6 without controls. Hence, for our welfare analysis below we choose $\kappa = 21.3$ as our preferred estimate for the interbank supply elasticity.

2.7.2 Model Calibration

Having estimated the elasticities σ and κ in the previous subsection, we now turn to the parameters related to the financial shocks $\left\{a_t^n,\ T_t^n,\ d_t^{ni}\right\}_{\forall n,i}$, for which we specified a functional form in section 2.6.9. In a first step, we use observed bank-level data on $\left\{R_t^{F,n},\ L_t^n,\ \lambda_t^{ni}\right\}_{\forall n,i,t}$ together with the equilibrium relationships implied by the model to recover estimates of the financial shocks for each quarter. We back out estimates for $\{T_t^n\}_{\forall n}$ as

$$\hat{T}_t^n = (\lambda_t^{nn})^{-1/\kappa} \left(1-\lambda_t^{0n}\right)^{-1/\kappa} (1+\varsigma_D^*) \left(\frac{\sigma \cdot (1+\varsigma_B^*)^{-1}}{\sigma-1}\right)^{-1} R_t^{F,n},$$

which is an expression that we obtain after combining equation (2.16) for the own trade share together with equations (2.10) and (2.15). Using the same set of equations but employing the formula for the bilateral trade share between any $(n,\ i)$ pair of banks, we obtain our estimates for $\left\{d_t^{ni}\right\}_{\forall n,i}$ as

$$\hat{d}_t^{ni} = \left(\hat{T}_t^n\right)^{-1}\left(\lambda_t^{ni}\right)^{-1/\kappa}\left(1-\lambda_t^{0i}\right)^{-1/\kappa}\left(1+\varsigma_D^*\right)\left(\frac{\sigma\cdot\left(1+\varsigma_B^*\right)^{-1}}{\sigma-1}\right)^{-1} R_t^{F,i}\ .$$

Lastly, we recover the $\{a_t^n\}$ shocks by using the CES loan demand from equation (2.7).

In a second step, we use the time series of the recovered shocks to obtain estimates of their steady state values $\left\{a^n,\ T^n,\ d^{ni}\right\}_{\forall n,i}$, autoregressive coefficient ρ_l and variance-covariance matrix between shocks, for which we impose a specific structure later in section 2.8.2.

The only technical difficulty that we have to address relates to the fact that the ZISTA dataset containing information on interest rates only reports values for a sample of between 200 and 240 representative banks per quarter, as explained in section 2.3. The reported banks are selected through stratified sampling, assigning banks to between 15 and 17 groups using a criterion that combines state and banking categories in order to capture regional and institutional heterogeneity. Then, the largest banks within each strata are selected into the ZISTA sample. We construct predicted interest rates for the remaining banks in the main sample by computing a regression of interest rates on observable bank characteristics and detailed balance sheet composition, which we observe for all banks and quarters in the BISTA and VJKRE datasets. We test for sample selection bias and prediction performance by excluding the two smallest ZISTA banks of each strata ($\sim$ 35 banks per quarter). The R-squared of our prediction is 81.5%, and the average out-of-sample deviation of the predicted versus observed interest rates for the excluded sample is +4 basis points, which suggests that using this highly selected sample does not create sizable bias smaller banks outside the sample.

Finally, the remaining parameters (Frisch elasticity, Calvo stickiness,...) are commonly featured in most New-Keynesian models and we calibrate them to reasonable values within the literature's accepted range. Appendix A.4 provides a summary of the selected parameter values.

2.8 Welfare

2.8.1 Static Welfare

We introduced parameter ϱ in equation (2.18) as a modeling device to manipulate the level of transaction costs. We now define autarky (AU) as the situation in which $\varrho \to \infty$ and transaction costs approach infinity, resulting in an equilibrium where banks only have access to their own household deposits or credit from the central bank. Under autarky, the individual cost of funds becomes tied to their capacity to attract deposits from households, $\Phi_t^{i,AU} = T_t^i$, $\forall i$. Following the definition of static trade models, we define gains from interbank trade as the steady state change in welfare with respect to autarky, expressed as

$$\mathbb{J}^{ss} = \frac{U - U^{AU}}{U_X X} \ ,$$

where $U_X \equiv \mathrm{d}U/\mathrm{d}X$ is a normalization that allows us to express trade gains as a fraction of steady state consumption of X. In the appendix we show that after some manipulations, static trade gains can be expressed as

$$\mathbb{J}^{ss} = -\left(\frac{\alpha}{1-\alpha}\right)\frac{1}{\kappa} \cdot \log\left(\lambda^{Own}\right) \ , \tag{2.21}$$

which is a formula that highly resembles the Arkolakis et al. (2012) trade gains expression from the international trade literature. The term $-\log\left(\lambda^{Own}\right) \geq 0$ captures the degree of financial market integration. A lower λ^{Own} means higher participation on the interbank market and increased gains from trade due to improved allocation of funds across the banking sector. Elasticity κ measures the substitutability between alternative funding sources across the time continuum, as explained in section 2.6.6. When κ is low, substitution is less likely and gains from financial integration are larger. Parameter α measures the importance of capital in production, and capital investment is financed through the banking sector in our model. Hence, the term $\frac{\alpha}{1-\alpha}$ captures the importance of bank financing in the production of the final good which scales the gains for interbank trade. The denominator $1-\alpha$ follows from an input-output multiplier in which loan supply generates additional output which is partly allocated to new capital investment.

The main implication from this formula is that static gains from trade are monotonously increasing with the degree of financial market integration, which in the steady state efficiency in allocation of funds. The formula is also appealing as it gives us an *ex-ante* measure of welfare, in the sense that knowledge about the underlying structure of the model, or the counterfactual autarky scenario, is not necessary to provide an estimate of the gains from trade. The trade share λ^{Own} becomes a sufficient statistic which we can directly observe in the data.

One limitation of the static welfare formulation is that it excludes higher order terms related to the volatility of financial markets, which the literature generally considers as important. We look at them in the following section where we study welfare of the dynamic model.

2.8.2 Dynamic Welfare

The model is subject to the stochastic volatility of the financial sector shocks $\left\{T_t^n,\ d_t^{ni},\ a_t^n\right\}$, which propagate to the rest of the economy through the capital investment decisions of firms and generate business cycle fluctuations. We define the dynamic gains from trade as the unconditional expected change in utility between the current level of integration and autarky, formally

$$\mathbb{J} = E\left[\frac{U_t - U_t^{AU}}{U_X X}\right] . \tag{2.22}$$

Contrary to the steady state gains above there is no closed-form expression for dynamic gains. To shed some light on its main determinants, we conduct a second-order approximation around the static gains (2.21) and impose the following assumptions on the covariance structure of the model shocks.

Assumptions:

1. *CES firm weights:* $E\left[u_t^{a,i} \cdot u_t^{a,n}\right] = \begin{cases} \sigma_a^2 & \text{, if } n = i \text{ .} \\ \zeta_a \cdot \sigma_a^2 & \text{, otherwise.} \end{cases}$

2. *Depositor Preferences:* $E\left[u_t^{T,i} \cdot u_t^{T,i}\right] = \begin{cases} \sigma_T^2 & \text{, if } n = i \text{ .} \\ \zeta_T \cdot \sigma_T^2 & \text{, otherwise.} \end{cases}$

3. *Bilateral Transaction Costs:*

$$E\left[u_t^{I,ji} \cdot u_t^{I,kn}\right] = \begin{cases} 0 & \text{, if } j = i \text{ or } k = n \text{ .} \\ \sigma_I^2 & \text{, if } k = j \text{ , } n = i \text{ .} \\ \zeta_{I,B} \cdot \sigma_I^2 & \text{, if } k \neq j \text{ , } n = i \text{ .} \\ \zeta_{I,L} \cdot \sigma_I^2 & \text{, if } k = j \text{ , } n \neq i \text{ .} \\ \zeta_{I,X} \cdot \sigma_I^2 & \text{, otherwise.} \end{cases}$$

4. *Zero Cross-Correlation:* $E\left[u_t^{I,ji} \cdot u_t^{a,k}\right] = E\left[u_t^{I,ji} \cdot u_t^{T,k}\right] = E\left[u_t^{a,j} \cdot u_t^{T,k}\right] = 0 \text{ ,} \quad \forall j, i, k \text{ .}$

Assumptions 1 and 2 impose discipline on the structure of the firm-loan and depositor preferences shocks by assuming equal variance and covariance between pairs. Assumption 3 imposes similar restrictions for interbank transaction costs, with the addition that we allow for different covariances if the connections share the same borrower bank, $\zeta_{I,B}$, same lender bank, $\zeta_{I,L}$, or unrelated lender and borrower, ζ_X. The covariances involving trade with oneself are always zero as we normalized own transaction cost to one at all times, $d_t^{ii} = 1,\ \forall i$. The last assumption imposes zero correlation between the distinct shocks affecting the banking sector.

Under Assumptions 1-4, the second order components of an approximation to (2.22) depend on the change in business cycle volatility associated to the financial shocks $\left\{T_t^n,\ d_t^{ni},\ a_t^n\right\}$ (Proof: Appendix A.2). The expression for the dynamic gains from trade becomes

$$\mathbf{J} = \ \mathbf{J}^{ss} - \ \frac{1}{2}\left[\sigma_T^2 \cdot \mathfrak{J}^T + \sigma_a^2 \cdot \mathfrak{J}^a + \sigma_I^2 \cdot \mathfrak{J}^I\right] \ , \tag{2.23}$$

where $\mathfrak{J}^m,\ m \in \{T, a, I\}$ are multipliers capturing the second-order effects on welfare of the changes in the volatility of depositor preferences, firm-loan demand and transaction costs, respectively, that accompany the process of integration starting from the counterfactual scenario of financial autarky. A negative multiplier is possible and indicates that the volatility costs of its associated component decrease as the market integrates. The first term of the equation are the static gains from trade discussed in the previous section. The second term are the gains/costs from the volatility of depositor preferences. As the concentration of the funding sources is reduced, banks are more likely to find cheap sources of credit among their connections when they face a shock to their deposits. On the other hand, specialization in the collection of deposits might lead to higher volatility. As an example, the effect of depositor bank runs (which we can capture as a sudden increase of T_t^n in our model) on large interbank lenders will be felt more strongly by its borrowers the more open to each other they are. The third term is related to the concentration in the market for firm loans. If financial integration allows big banks to expand and capture a larger fraction of the firm-loan market, the volatility of the economy will increase due to concentration of shocks on fewer, larger entities. An intuition for this channel is provided by Huber (2018), that shows how credit shocks to *Commerzbank* (a large German bank) following the 2007 crisis were able to influence the aggregate German economy. If the opposite is true and small banks are able to grow due to their access to interbank credit, gains from trade improve. The last line is related to the economic volatility created by interbank transaction costs. Lehman Brothers is a good example of this channel: interbank markets allow banks to diversify their partners and decrease the volatility of interbank funding, but when a key player is unable/unwilling to lend, the banking system will be increasingly affected as a function of its participation in the market. We can look at this last channel more formally. By setting the covariances between distinct transaction costs to zero $\zeta_{I,B} = \zeta_{I,L} = \zeta_{I,X} = 0$, we obtain an intuitive expression for the multiplier as

$$\mathfrak{J}^{l} = \underbrace{\Theta^{\lambda} \cdot \sum_{n=1}^{N} \omega^{\lambda,n} \left[1 - (\lambda^{nn})^2\right]}_{\text{Exposure}} - \underbrace{\Theta^{H} \cdot \sum_{n=1}^{N} \omega^{H,n} \left[1 - H^{l,n}\right]}_{\text{Diversification}} ,$$

where $\left\{\Theta^{j,n}\right\}_{j\in\{H,\lambda\}}$ are positive constants, $\left\{\omega^{j,n}\right\}_{j\in\{H,\lambda\}}$ are weights such that $\sum_{n=1}^{N} \omega^{j,n} = 1$, and $H^{l,n} = \sum_{j=1}^{N} \left(\lambda^{jn}\right)^2$ is the Herfindahl concentration index of the funding sources of bank n (excluding the central bank). This expression has the following economic interpretation: Transactions costs with oneself are constant over time, as we normalized $d_t^{nn} = 1,\ \forall n,\ t$, but transactions costs with others are volatile. Hence, by increasing their participation on the interbank market, banks reduce the reliance on their own depositors (lower λ^{nn}) at the expense of more volatile funding costs. Unexpected changes in funding costs are eventually passed to firms and interact with their sticky price decisions, increasing the volatility of inflation and output gap. The *Exposure* term captures this effect in the equation above. On the other hand, when banks diversify their funding sources (lower $H^{l,n}$) they partially insure themselves from this costs by being able to rely on alternative connections, resulting on a positive contribution to welfare as captured by the *Diversification* term. Hence, whether the gains from diversification or the costs from interbank exposure will dominate as the financial system integrates will depend both on the particular structure of the banking system and on the integration path that is followed. Regulators have traditionally focused their attention on an array of leverage, capital and liquidity ratios (see Basel Accords for example) to reduce the risk of counter-party exposure. Our model would suggests adding to this list the concentration of the bank's funding sources as well.

2.8.3 Gains from financial integration in Germany

We can now look at the gains from financial integration in Germany using the estimate parameters and shocks from section 2.7. In order to study counterfactual scenarios with different levels of integration, we are going to proportionally increase/reduce the steady state bilateral transaction costs between banks by setting parameter ϱ in equation (2.18) to different values between zero (no costs) and infinity (autarky). A value of one corresponds to their present level. Figure 2.10 shows the gains from trade at different levels of integration, with a gain of 0.88% consumption per-quarter under the current regime and a theoretical maximum of 25% when trade costs are eliminated. Table **??** shows gains from trade under alternative calibrations of σ and κ elasticities, with values oscillating between a maximum of 2.05% and a minimum of 0.45%. Note that dynamic gains approximately double the size of static gains, which indicates that second-order reductions in economic volatility due to financial integration are important to understand welfare. Figure 2.11 plots the share of total gains from trade explained by each component of equation (2.23). Gains from interbank diversification and static gains are the two largest contributors with roughly the same importance. Depositor preferences have a slightly negative contribution, and loan demand shocks have close to zero. We observe that at moderate levels of integration the second-order term related to the volatility of the interbank market is very important in determining the gains from trade of the economy, while at higher levels of integration most of the gains from diversification have already been realized and the static efficiency component dominates.

As we mentioned in Section 2.4, Figure 2.2 shows a very persistent drop in the share of interbank liabilities held by German banks following the 2007 financial crisis. Under the assumption that the crisis generated a structural break on the banking sector parameters, we split the sample in two halves at 2007Q2 and 2008Q3 (we exclude the middle period to avoid the uncertain start of the crises, as we did in section 2.7.1) and compute the change in gains from trade between the two samples. We estimate that pre-crisis gains stood at 1.18% of consumption, while on the second second halve of the sample they drop to 0.83%, a 0.35% drop in steady state consumption per quarter.

2.9 Monetary policy

In this section we study the welfare effects of monetary policy. The central bank has two tools at its disposal: conventional manipulation of the economy risk-free rate, which we model as a standard Taylor rule, or direct lending on the interbank market. Even though both policies will prove useful, their impact on welfare will come through very different channels. In the former case, the central bank takes interbank volatility as given, and risk-free rates adjust to minimize the distortions on inflation and output gap that it creates. On the later, the central bank attempts to make interbank volatility smaller by setting an upper bound on the cost of funding.

2.9.1 Taylor rule

We define consumer welfare losses as a fraction of the steady state consumption of X, as it is common in the business cycles literature

$$\mathbb{L} = -\,E\left[\frac{U_t - U}{U_x X}\right] .$$

In the appendix we show that a second order approximation of this expression becomes

$$\mathbb{L} = E\left[\left(\frac{\alpha}{1-\alpha}\right)\cdot\underbrace{\left[\hat{\hat{r}}_t^I + \left(\frac{\eta}{\eta+1}\right)\left(\frac{\lambda^0}{1-\lambda^0}\right)\cdot\widehat{\log\left(\lambda_t^0\right)}\right]}_{\text{Terms independent of Taylor rule}} + \frac{1}{2\cdot\Xi}\left[\epsilon\cdot\hat{\pi}_t^2 + \Omega\cdot\hat{\hat{y}}_t^2\right]\right] + t.i.p. + h.o.t.\,.$$

The last two terms of this expression look familiar as they capture the welfare losses from inflation and output gap volatility present in standard New Keynesian models (see for example Galí (2015)). The first two terms are new to our model, and capture deviations of the interbank rate and central bank lending that affect utility directly and *independent from* their effect through inflation and output gap. Intuitively, they come from the effect of these variables on the level of production (and hence, consumption) itself. Standard monetary policy through risk-free rate adjustments is unable to affect those terms, but direct central bank lending will be.

Taking the Taylor rule response to inflation and output gap as given, we can analytically solve for the optimal reaction to movements in the interbank rate, γ_I^*, as

$$\gamma_I^* = \left(\frac{\alpha}{1-\alpha}\right)\cdot(1-\rho_I) > 0\,.$$

Within the simple framework of our model without adjustment lags and with perfect observation of the economic variables, optimal policy is able to completely eliminate the effect of interbank rate movements on inflation and output gap. Calibrating the model to the German economy and comparing to a situation with no response ($\gamma_I = 0$), welfare modestly improves by 0.0007 percentage points, suggesting that credit spread targeting isn't very effective at further reducing the costs of financial market volatility beyond what can be achieved with usual output gap and inflation targets. It is also worth noting that the optimal coefficient is positive, implying that the central bank should *raise* the risk-free rate in response to positive deviations of the interbank rate. The intuition for this result follows from the fact that an increase in intermediation costs is akin to a negative technological shock as it raises the cost of capital accumulation, reducing potential output and putting upward pressure on the output gap and inflation as firms adjusts to a lower level of production.

2.9.2 Lender-of-last-resort

Central banks, in their role as lenders-of-last-resort of the banking system, directly lend to entities experiencing liquidity shortfalls and set a cap on their funding costs at the interbank market. ECB's marginal lending facility (or the Fed's discount window) served that function historically. Following the 2007 crises new forms of intervention appeared, like the Long

Term Refinancing Operations (LTRO) of the ECB or the broadening of the categories of acceptable collateral to include Mortgage Backed Securities and commercial paper of good standing, which indirectly lowered the costs of accessing the discount window. We do not explicitly differentiate between these types of intervention in our model, and instead focus on the desirable characteristics that they should possess in order to maximize welfare. We start by looking at the steady state central bank share

$$\lambda^0 = \frac{1}{1 + e^{\kappa \varpi_1}} \ .$$

Hence, choosing the fix component of the penalty rate ϖ_1 is equivalent to setting the degree of intervention that the central bank desires. Parameter ϖ_2 controls its variance. We will be looking at the effect of different policies on total utility and gains from trade by comparing them to the equilibrium without central bank lending (equivalent to $\varpi_1 \to +\infty$).

We start by looking at the case in which $\varpi_2 = 0$ and the central bank imposes a fixed penalty over the average cost of funds, so $\widetilde{R}_t^{0n} = e^{\varpi_1} \cdot \Phi_t^n$. It is easy to see this implies a perfect positive correlation between the central bank rates and the average costs of funds. From a historical perspective, this is similar to the "Real Bills Doctrine" popular during the first half of the 20th century, which advocated for the Fed discount rate to track the average interest rate of the economy[18]. Looking at the change in total utility and gains from trade

$$\begin{aligned} E\left[U_t\left(\varpi_2 = 0\right) - U_t^{no-LoLR}\right] &= -\left(\frac{\alpha}{1-\alpha}\right) \cdot \left(1 + \frac{1}{\kappa}\right) \cdot \log\left(1 - \lambda^0\right) > 0 \ , \\ \mathbb{J}\left(\varpi_2 = 0\right) - \mathbb{J}^{no-LoLR} &= 0 \ . \end{aligned}$$

Central bank lending improves utility in this setup. Setting the central bank trade share λ^0 at 3.5%, its pre-crisis average, increases the utility of the German representative household by about 2.5%. However, there is no change in the gains from trade. Central bank lending only affects gains from trade by reducing the volatility of its stochastic components. But when $\varpi_2 = 0$, the central bank always provides the same fraction of funding $\lambda_t^0 = \lambda^0$, $\forall t$, regardless of the state of the interbank market. In this situation, changes in utility come exclusively from a steady state reduction in the level of interbank funding costs, not their volatility.

The general case in which $0 \leq \varpi_2 \leq 1$ has a more complicated analytical expression. We provide the solution for $\mathbf{J}$ in equation (2.23) and the formula for the utility level can be found

[18]See Richardson and Troost (2009) for an empirical assessment of the effects of such policy during the Great Depression.

in appendix A.3.2. Figure 2.12 shows the relationship between gains from trade and the policy choice parameters of the central bank, λ^0 and ϖ_2. The isoquants depicting different levels of trade gains are increasing in both parameters, and they also show us the complementary relationship between them: at high levels of central bank participation, a small increase in the countercyclical response of central bank rates is able to achieve high levels of welfare, while the reverse is true at low levels. For reasonable values of central bank participation λ^0 below 10%, gains from trade increase up to 20 basis points. This results suggest that most of the welfare gains from lender-of-last-resort intervention come from the *existence* of the discount window itself, with the intensity of the countercyclical response providing a moderate additional increase in welfare.

Figure 2.13 responds the question of how welfare gains from lender-of-last resort policies are increased by granting discount window access to a wider set of banks. We proceed as follows. First, we order the MFIs in our sample by the size of their balance sheet. Then, we compute households' utility under the assumption that none of the MFIs is able to borrow from the central bank, and progressively increase the number of banks with discount window access, from smallest to largest. As we can see in the figure, extending access to a larger subset of banks is always beneficial. Nonetheless, most of the gains come from granting access to the largest MFIs in the sample. Similar in nature to the common concept of "too big to fail", changes in the interbank borrowing conditions of the largest banks of the system are capable of generating aggregate economic fluctuations, and hence account for the majority of the welfare gains from the central bank's lender-of-last-resort policy. This result also suggests that there would be potentially large welfare benefits from expanding discount window access to financial entities that have traditionally been excluded from it, like investment funds and insurers.

Finally, we ask whether the two tools at the central bank's disposal (discount window and open market operations) complement each other in stabilizing financial volatility. Figure 2.14 shows different combinations between the three policy parameters under central banks' control (λ^0, ϖ_2 and γ_l) that keep gains from trade constant at a given level. As the Taylor rule response approaches its optimal level $\gamma_l^* = 0.14$, the same level of gains can be achieved with lower discount window lending and/or smaller countercyclical response of the penalty rate on central bank credit. The resulting policy prescription for central banks thus becomes a combined use of all the tools at their disposal.

2.10 Conclusions

In this paper we develop a new macroeconomic DSGE model of the interbank market capable of accommodating an heterogeneous banking system with complex relationships between its participants. Nonetheless, the model remains tractable and allows us to derive an analytical approximation to the gains from financial integration. We show that integration brings firs-order welfare gains from enhanced efficiency in the allocation of resources but entails second order trade-offs between gains from diversification and losses from increased counter-party risk exposure.

Using a collection of proprietary data for German banks, we document a high degree of heterogeneity in the banking sector. Banks present large size differences and concentration at the top, they use the interbank market to cover their structural funding deficits in addition to short-term liquidity mismatches, and individually rely on a small subset of interbank partners which limit their capacity to substitute funding sources in the event of an interbank credit freeze, while leaving the system exposed to the shocks of a few but large core intermediators.

We construct a measure of indirect exposure to the US through domestic interbank partners and provide reduced form evidence on the effects of the Great Recession on the German interbank market. Following the Lehman collapse, we find that interbank credit dried for the banks with higher indirect exposure, which led them to increase the interest rate charged on non-financial loans (10 basis points per billion Euros of indirect exposure) and reduce lending (2% drop per billion Euros of indirect exposure). These results are statistically significant and economically important: indirect exposure to the US is able to account for half of the reduction of the German interbank market that followed the 2007 crisis.

We then proceed to calibrate the model and evaluate welfare and policy counterfactuals for the German economy. We estimate gains from trade to be around 0.88% of consumption per-quarter, coming from a combination of efficiency gains in the allocation of funds across the bank network and decreased volatility through diversification of the bank's funding sources, which in practice outweigh the costs of increased exposure risk. We study two different central bank policies aimed at reducing the negative effects of financial volatility. Targeting steady state deviations of the interbank credit spread yields only modest increases in welfare beyond standard output gap and inflation targeting rules, while lender-of-last-resort intervention proves to be a more effective tool at reducing the costs of financial market fluctuations. Nonetheless, policy instruments complement each other, suggesting that a successful central bank policy should employ a combination of lender-of-last-resort and open market operations.

Finally, we believe that our model will prove useful in the study of several important topics, which we leave to future research. Among them, we consider the study of lender-of-last-resort policies at the zero lower bound and research on international processes of financial integration, like the ones that followed the creation of the European Union and the adoption of the euro, as some of the most interesting.

Figures

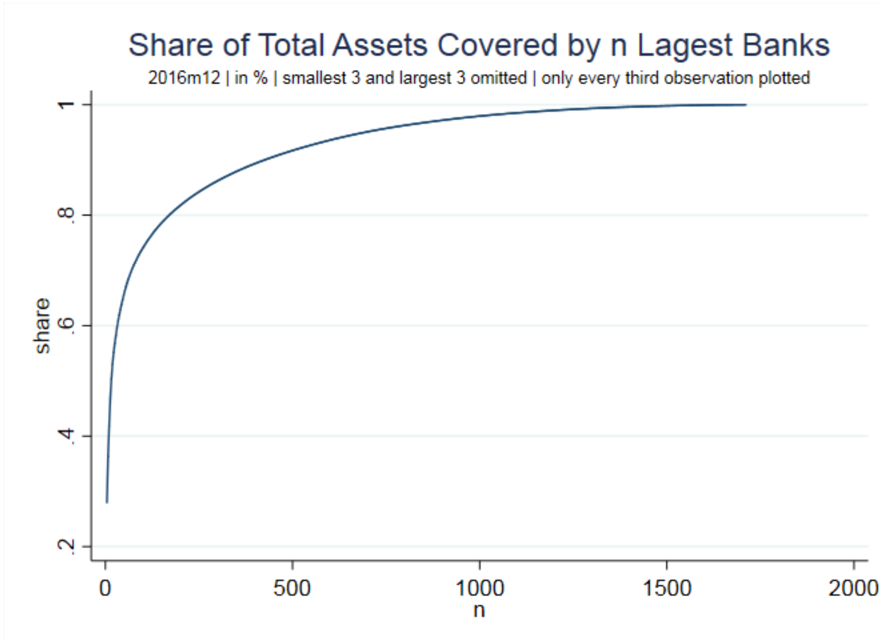
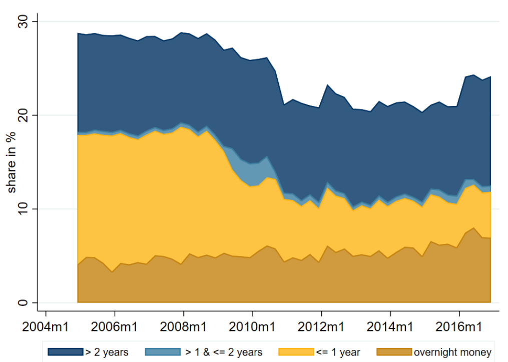
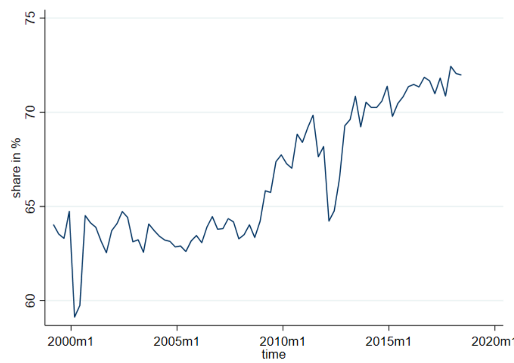


Figure 2.1: Cumulative share of total MFI assets by the n largests MFIs. Smallest and largest three MFI omitted, only every third observation plotted due to confidentiality requirements. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, BISTA, 2016m12, own calculations.



(a) Interbank Liabilities



(b) Interbank Trade openness

Figure 2.2: (a) Share of Interbank Liabilities in Total Liabilities by maturity. (b) Interbank Trade openness index, defined as $1 - \frac{\text{Interbank Liabilities}}{\text{Assets}-\text{Interbank Assets}}$. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, BISTA, 2004m12 - 2018m12, own calculations.

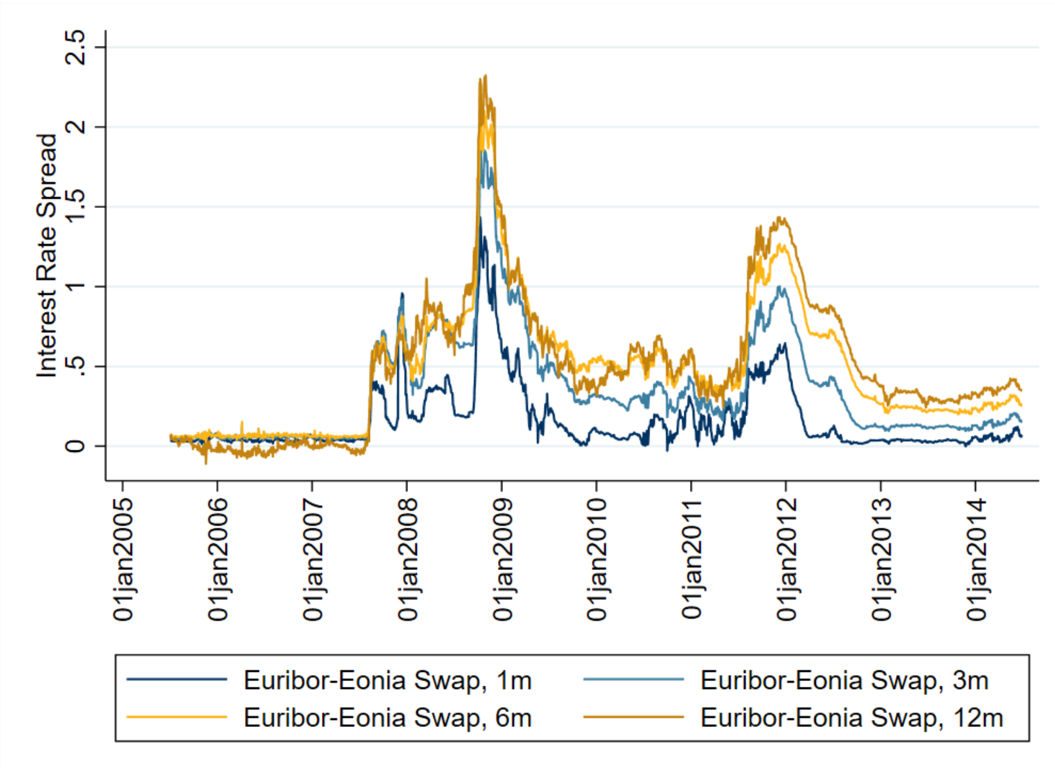
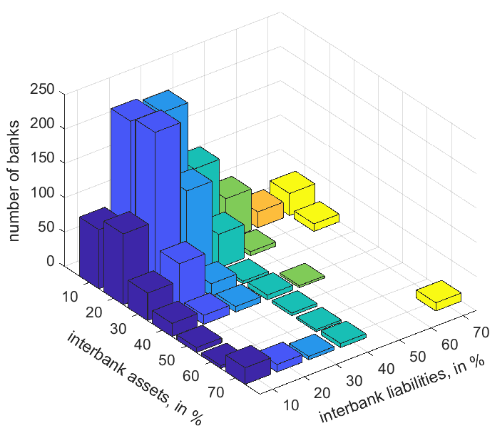
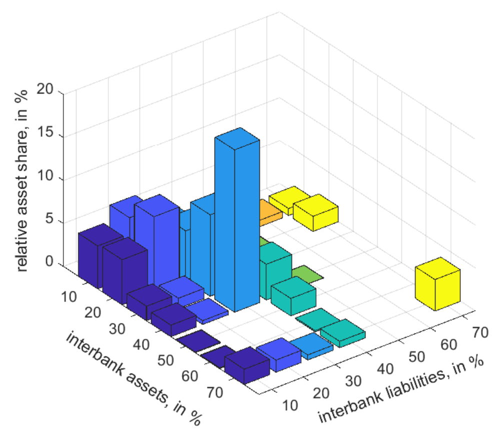


Figure 2.3: Interbank credit spread at different maturities, computed as the difference between the Euribor rate and the EONIA swap index. The Euribor is an average of the unsecured interbank rate at which Eurozone banks are willing to lend funds to each other. The EONIA Swap is a financial instrument commonly used to hedge against overnight moves of the unsecured interbank rate.

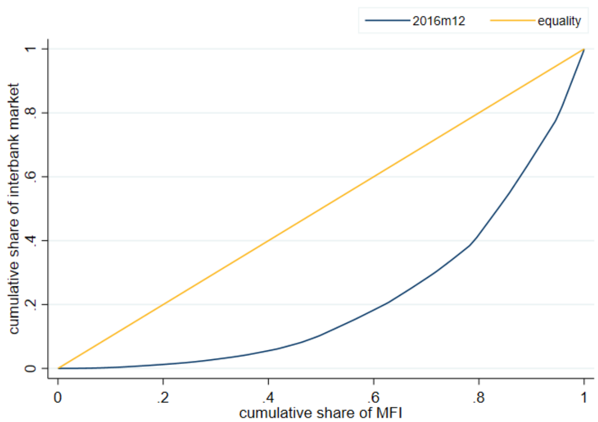


(a) Bank count

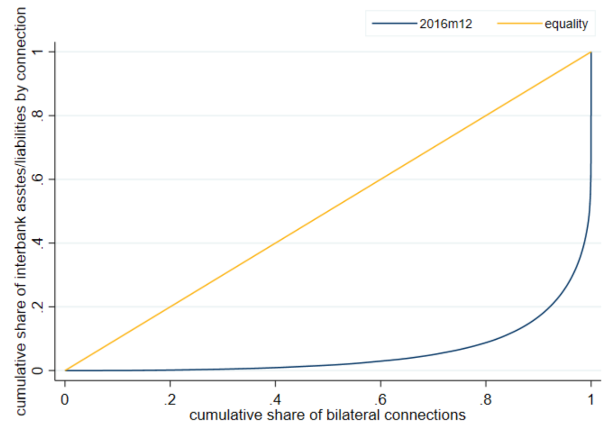


(b) Total MFI Asset share

Figure 2.4: Share of interbank assets and liabilities on the balance sheet, in percentages. The vertical axes in Figure 2.4a displays the number of banks within each bin. Figure 2.4b displays the share of total MFI assets that banks within the bin represent. Bins with less than three observations are not reported due to confidentiality requirements. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, BISTA, 2016m12, own calculations.

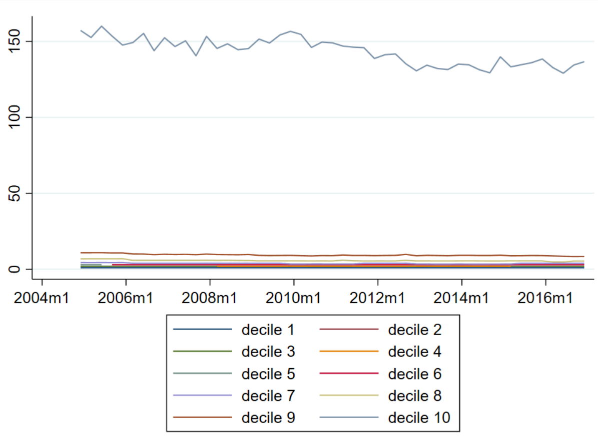


(a) By bank

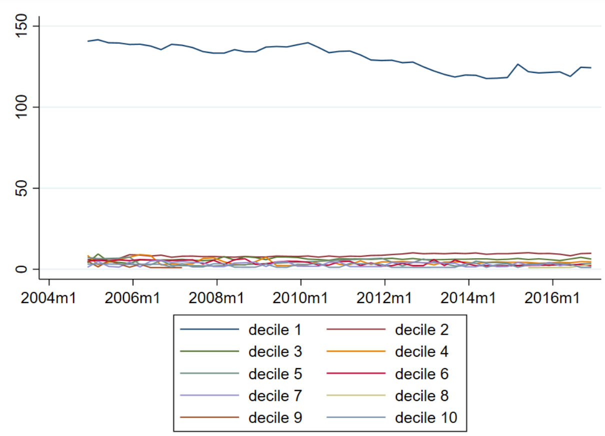


(b) By bilateral connection

Figure 2.5: Interbank concentration. Figure 2.5a shows the amount of interbank assets and liabilities held by the n-smallest banks. Figure 2.5b plots the cumulative share of interbank assets and liabilities that flow across individual bilateral connections among banks. 45 degree line indicates perfect equality in the distribution of interbank positions. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, BISTA and Credit Registry, 2016m12, own calculations.



(a) By number of connections



(b) By bank size

Figure 2.6: Average number of distinct interbank funding sources, by deciles. Figure 2.6a constructs deciles based on the number of distinct interbank funding sources. Figure 2.6b defines deciles with respect to total asset size of the MFIs. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, BISTA and Credit Registry, 2004m12 - 2018m12, own calculations.

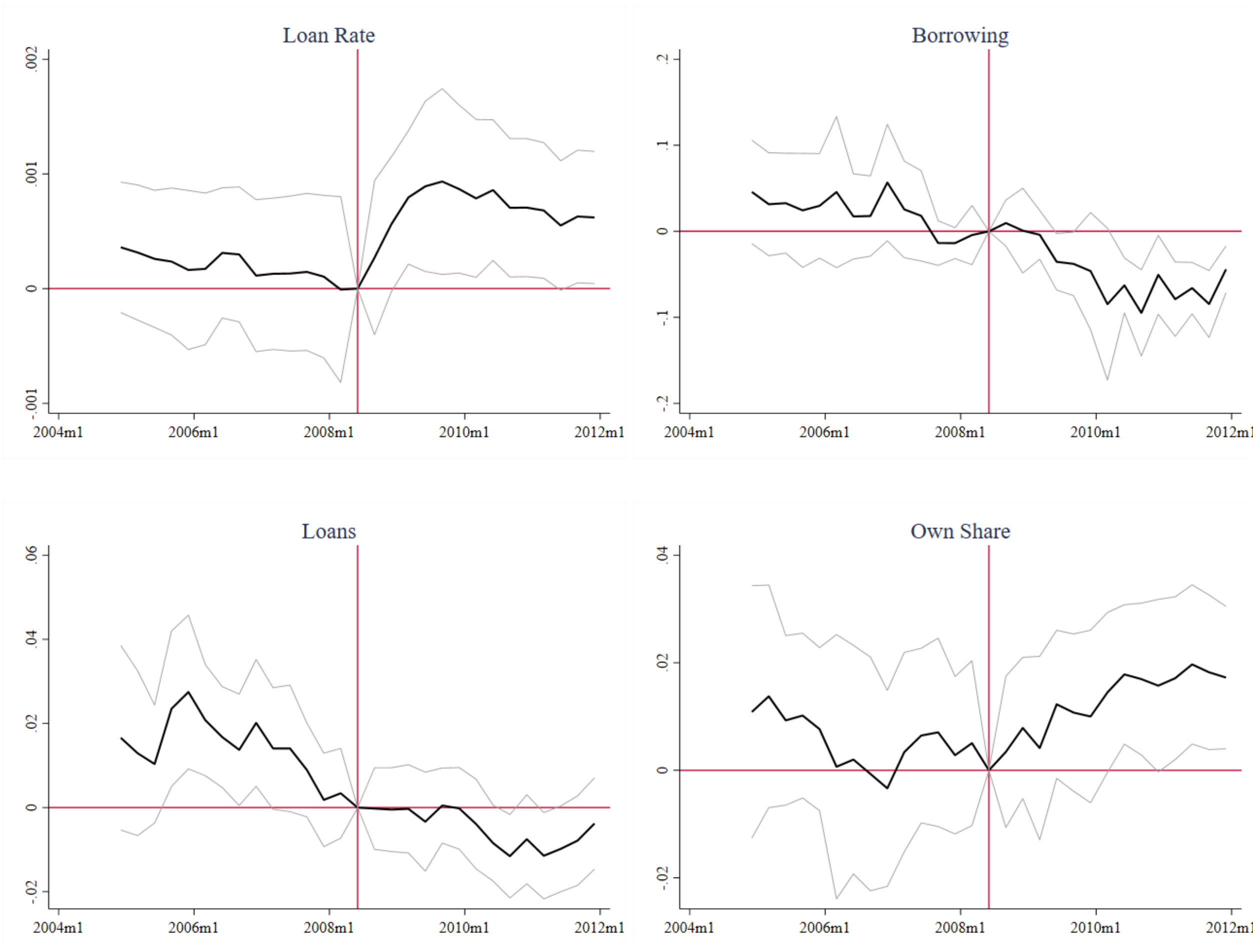


Figure 2.7: Event-study for indirect exposure to US financial crisis on interest rates of final loans (upper left), borrowing from other German MFIs (upper right), quantity of final loans (lower left) and share of funding from own sources (lower right). Each figure plots coefficients on $Exposure^{US}_{2006Q1} \times Quarter - FE$ and 95% confidence intervals. Red vertical lines marks 2008Q2, the event quarter just before the Lehman collapse. The regression includes quarter fixed effects, bank fixed effects, direct asset exposure to US and loan shares of non-mfi and households, each broken into maturity of less than 1 year, between 1 to 5 years and more than 5 years as well as separate shares for secured and unsecured mortgages. Robust standard errors are clustered at the bank group-quarter level. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, AUSTA, BISTA, VJKRE, ZISTA, 2004m1 - 2012m1, own calculations.

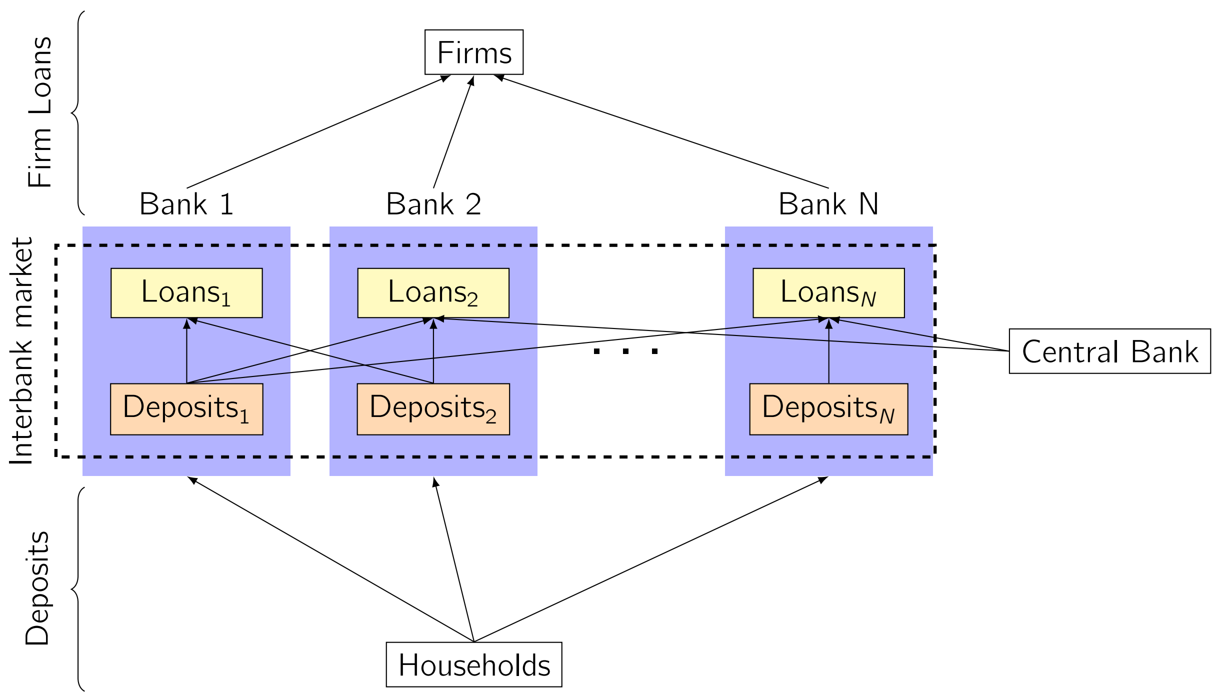


Figure 2.8: Components of the financial channel. Households distribute savings across banks in the form of one-period deposits. Banks lend these funds to firms, which use them to finance capital investment. Mismatches between available deposit funds and firm loan demand are settled in the interbank market. The central bank provides lender-of-last-resort credit at a penalty over the average interbank interest rate. Arrows indicate flow and direction of funds between agents.

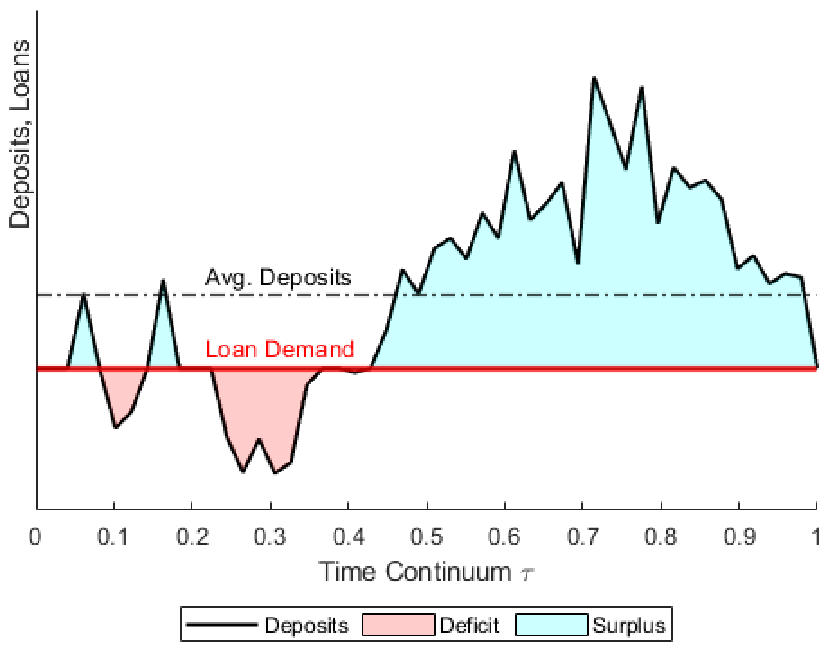


Figure 2.9: Evolution of bank deposits within a quarter (example). The figure displays the mismatch between loan demand and deposit availability for an individual bank. Vertical axis are the nominal dollar value of deposits and loans. Horizontal axis display the time continuum within the quarter. Red areas indicate a shortfall in deposits with respect to the loan commitments of the bank (red line). Blue areas indicate a deposit surplus. A bank is a net interbank lender (borrower) if the average amount of deposits attracted during the quarter is larger (smaller) than its individual loan demand.

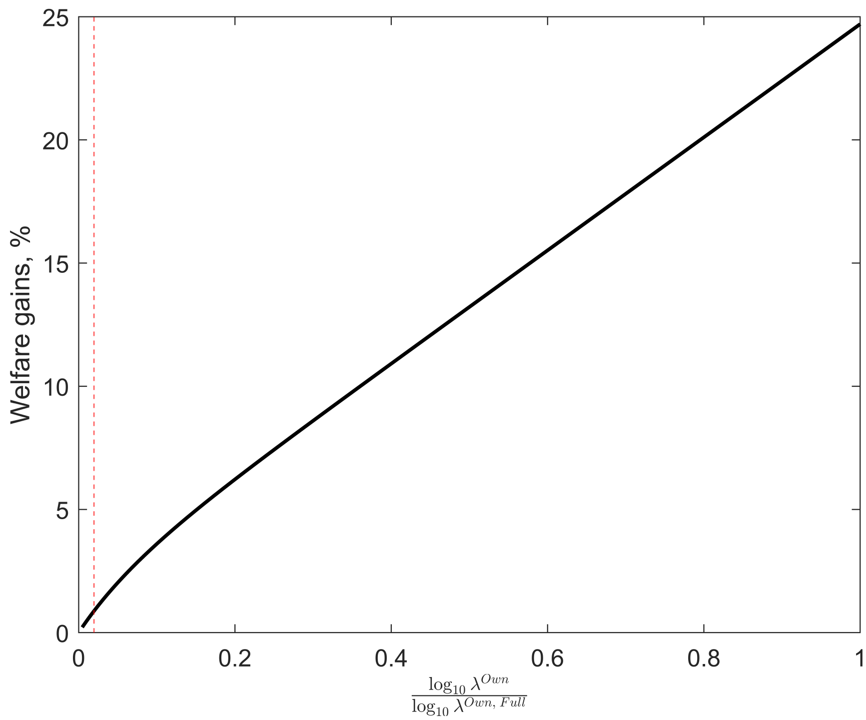


Figure 2.10: Gains from trade following proportional reduction/increase of bilateral transaction costs between banks, own calculations. Horizontal axis are normalized using the value of the own trade share at maximum level of financial integration. Dashed red line marks present level.

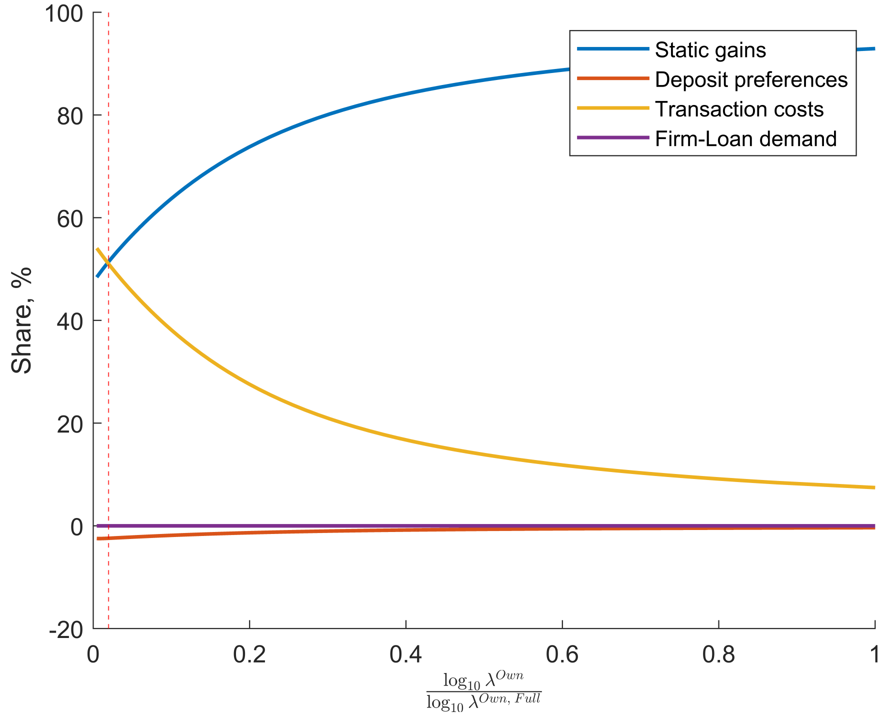


Figure 2.11: Share of total gains from financial integration explained by its main components, own calculations. Horizontal axis are normalized using the value of the own trade share at maximum level of financial integration. Dashed red line marks present level.

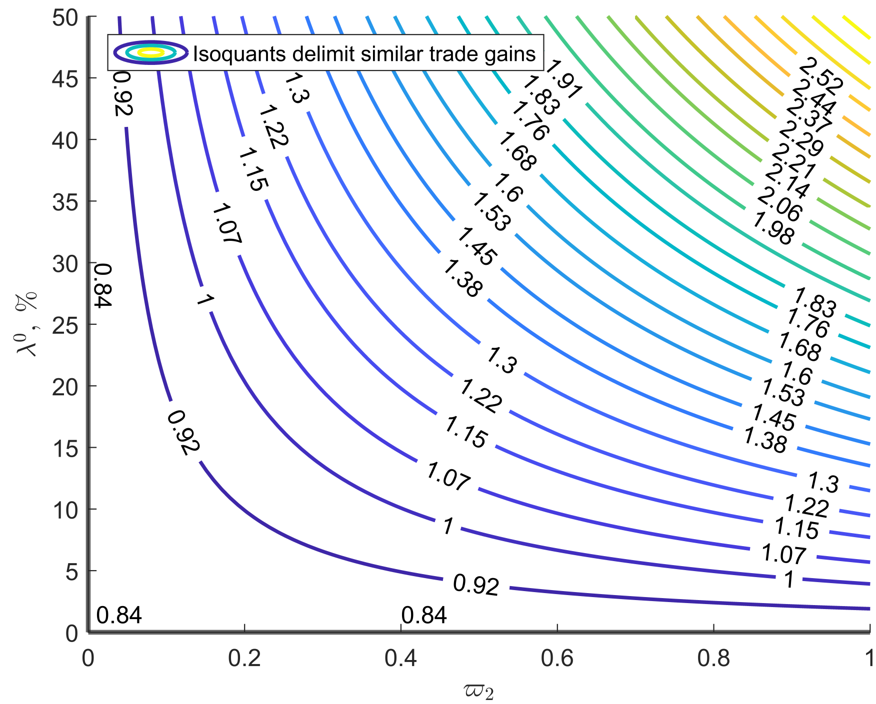


Figure 2.12: Gains from trade at different Central Bank lending calibrations, own calculations. Y-axis display the percentage participation of the Central Bank in the interbank market, x-axis the responsiveness of the penalty rate to deviations of funding costs from steady state. Isoquants display constant levels of gains from trade (in percentage) across the parameter space.

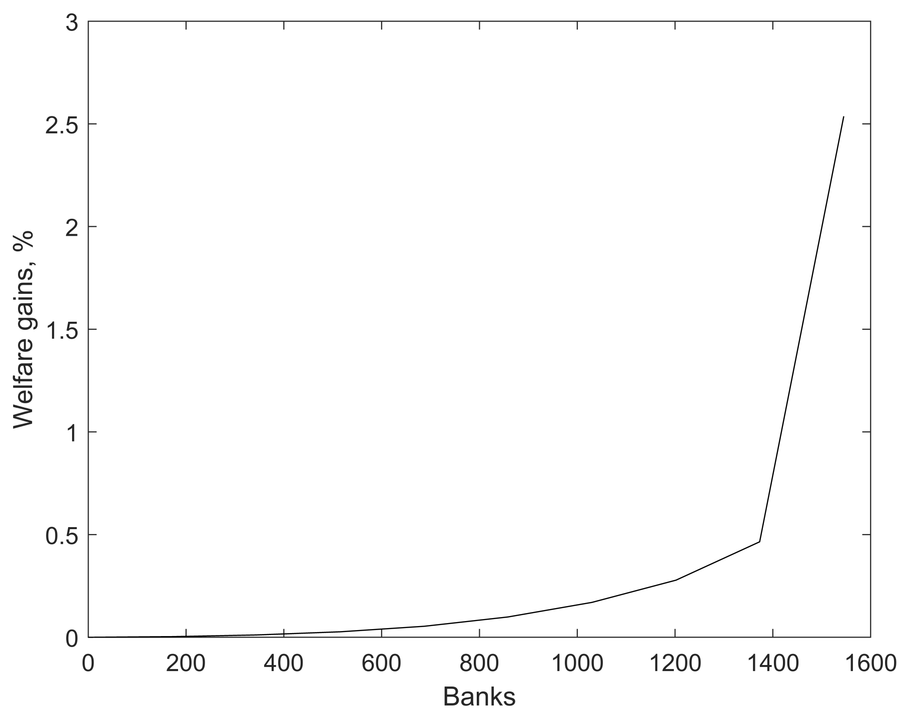
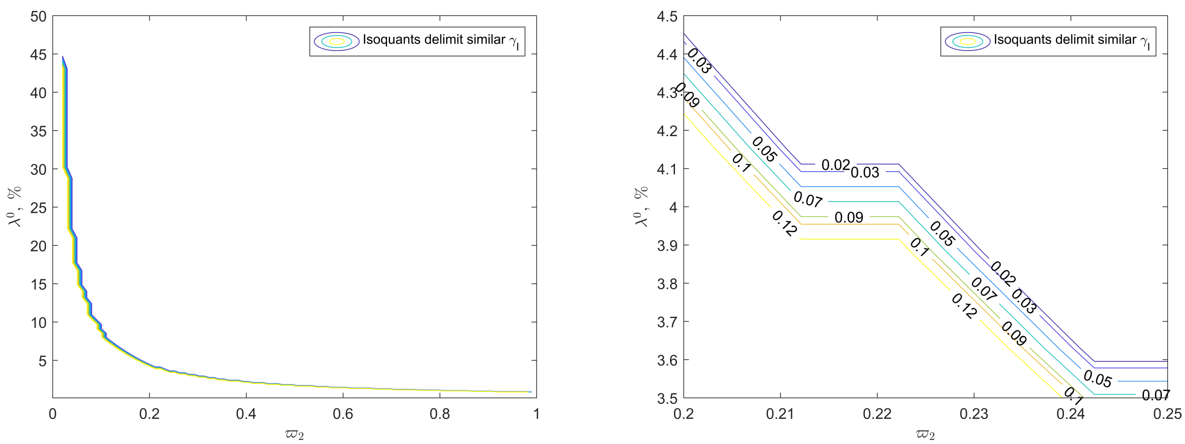


Figure 2.13: Welfare gains from expanding discount window access to a broader set of banks, own calculations. X-axis display the number of banks with access to the discount window in the counterfactual scenario. Banks are ordered by balance sheet size, with smaller banks given access first. Y-axis display welfare gains (in percentage) with respect to no-access counterfactual.



(a) (b)

Figure 2.14: Complementarity between Central Bank instruments, own calculations. Isoquants display the value of the Taylor rule response to interbank rate deviations γ_I that keep gains from trade constant, for a given parametrization of Central Bank lending to banks. Y-axis display the percentage participation of the Central Bank in the interbank market, x-axis the responsiveness of the penalty rate to deviations of funding costs from steady state. Figure 2.14b zooms into the isoquants of Figure 2.14a.

Tables

Months	1	6	12	24
Interbank asset share	0.954	0.882	0.839	0.765
Interbank liability share	0.977	0.947	0.923	0.877

Table 2.1: Spearman rank correlation tests. We construct the table by ranking the interbank market share of each bank and estimating the correlation with the ranking m-Months ahead. First row contains correlation of the interbank asset share, second row shows correlation of the liability share. All coefficients are statistically significant at <1% threshold. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, BISTA, 2004m12 - 2018m12, own calculations.

	(1) Loan Rate	(2) Loans	(3) Own Share	(4) Borrowing	(5) Loan Rate	(6) Loans	(7) Own Share	(8) Borrowing
$Expo_{t0} \times Post_t$	0.0005***	-0.0188***	0.0116**	-0.0671***	0.0006***	-0.0229***	0.0125***	-0.0789***
	(0.0001)	(0.0024)	(0.0047)	(0.0178)	(0.0001)	(0.0032)	(0.0046)	(0.0177)
Observations	3,612	3,612	3,578	3,609	3,612	3,612	3,578	3,609
R-squared	0.9262	0.9916	0.8758	0.9533	0.9299	0.9925	0.8805	0.9556
Controls	no	no	no	no	yes	yes	yes	yes
Mean of Expo	2.275	2.275	2.275	2.275	2.275	2.275	2.275	2.275

Table 2.2: Difference-in-difference results on indirect exposure to U.S. financial crisis. Regression compares outcomes between 2006Q1 to 2007Q2 and after Lehman collapse in 2008Q3 until 2011Q4 for more or less indirectly exposed banks to US financial crisis. Initial asset exposure to lenders in US market taken in 2006Q1. Controls include direct asset exposure to US and loan shares of non-MFI and household loans, each broken down into maturity of less than 1 year, between 1 and 5 years and more than 5 years as well as separate shares for secured and unsecured mortgages. All regressions include bank fixed effects and quarter fixed effects. Standard errors clustered at level of bank group-quarter. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, AUSTA, BISTA, VJKRE, ZISTA, 2004m12 - 2011m12, own calculations.
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

	(1) Loan Rate	(2) Loans	(3) Own Share	(4) Borrowing	(5) Loan Rate	(6) Loans	(7) Own Share	(8) Borrowing
$Expo_{t0} \times Post_{t0}$	-0.0000	0.0023	-0.0102***	0.0127	-0.0002*	-0.0035	-0.0101***	0.0030
	(0.0001)	(0.0054)	(0.0034)	(0.0083)	(0.0001)	(0.0034)	(0.0037)	(0.0085)
Observations	1,812	1,812	1,801	1,812	1,812	1,812	1,801	1,812
R-squared	0.9540	0.9950	0.9477	0.9877	0.9585	0.9958	0.9496	0.9884
Controls	no	no	no	no	yes	yes	yes	yes
Mean of Expo	2.275	2.275	2.275	2.275	2.275	2.275	2.275	2.275

Table 2.3: Pre-trends for results on indirect exposure to US financial crisis. Regression compares outcomes between 2004Q4 to 2005Q4 and the pre-period in main regression (2006Q1 until 2007Q2) for more or less indirectly exposed banks to US financial crisis. Initial asset exposure to lenders in US market taken in 2006Q1. Controls include direct asset exposure to US and loan shares of non-MFI and household loans, each broken down into maturity of less than 1 year, between 1 and 5 years and more than 5 years as well as separate shares for secured and unsecured mortgages. All regressions include bank fixed effects and quarter fixed effects. Standard errors clustered at level of bank group-quarter. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, AUSTA, BISTA, VJKRE, ZISTA, 2004m12 - 2007m6, own calculations.
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

	(1)	(2)	(3)	(4)
	$-\sigma$	κ	$-\sigma$	κ
$\log \tilde{R}_t^{F,i}$	-40.60***	26.56**	-37.78***	21.26***
	(9.10)	(10.64)	(5.99)	(7.45)
Observations	3,612	3,578	3,612	3,578
Controls	no	no	yes	yes
1st Stage F-Stat	15.14	12.97	24.61	22.14

Table 2.4: Two-Stage-Least-Squares estimates for σ and κ. Loan interest rate instrumented with $Exposure^{US}_{2006Q1} \times Post_{2008Q3}$. Data rages from 2006$Q$1 to 2007$Q$2 and after Lehman collapse in 2008Q3 until 2011Q4. Initial asset exposure to lenders in US market taken in 2006Q1. Controls include direct asset exposure to US and loan shares of non-MFI and household loans, each broken down into maturity of less than 1 year, between 1 and 5 years and more than 5 years as well as separate shares for secured and unsecured mortgages. All regressions include bank fixed effects and quarter fixed effects. Standard errors clustered at level of bank group-quarter. Source: Research Data and Service Centre (RDSC) of Deutsche Bundesbank, AUSTA, BISTA, VJKRE, ZISTA, 2004m12 - 2011m12, own calculations.
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Dynamic gains, in %

		σ		
		8	37.8	100
	10	2.05	1.58	1.22
κ	21.3	1.05	0.88	0.70
	40	0.61	0.53	0.45

Static gains, in %

		σ		
		8	37.8	100
	10	1.12	0.81	0.57
κ	21.3	0.57	0.45	0.35
	40	0.32	0.27	0.35

Table 2.5: Gains from trade under alternative calibrations. We construct the table by calibrating the model with different values for the elasticities of demand σ and supply κ. Left panel displays dynamic gains from trade, in percentages of steady state consumption. Right panel shows static gains from trade. Estimates under preferred calibration are colored in red.

2.11 Transitional section

In Chapter 2 I study the importance of the interbank market as a driver of business cycles fluctuations and provide empirical evidence on the welfare costs of the German interbank market freeze following the 2007 financial crisis. Chapter 2 also provides a theoretical model of the interbank market that I use to study counterfactual welfare scenarios like the costs of financial autarky and the effectiveness of several lender-of-last-resort policies. The model provides a general framework to study the linkages between financial institutions that can be broadly applied to other settings. In Chapter 3 I study the historical spatial propagation of banking panics across the United States. Methodologically, both chapters are linked by the development of closely related theoretical models of the interbank market. While chapter 2 relies on the combination of detailed proprietary micro-data and theoretical equilibrium relationships to empirically estimate the bank-level strength of bilateral connections existent on the German economy, chapter 3 adapts the model for its application to the study of aggregate state-level data and the study of the transmission of panics across different regions.

Chapter 3

Spatial Transmission of U.S. Banking Panics

3.1 Introduction

The United States features a prominent history of banking panics dating back to the eighteenth century, experiencing a minimum of fifteen panic waves in the period comprised between 1865 and 1930 (Jalil (2015)), many of them believed to have had a profound impact on the economy. In this chapter I investigate the domestic spatial spread of U.S. banking panics and quantify the distortions in banking sector activity that they generated.

The relative instability of the U.S. banking system can be traced to its specific institutional organization, in particular to the lack of a central bank (or equivalent) until 1913 and to the unit banking regulations that severely limited the capacity of banks to branch and diversify (Calomiris and Haber (2014)). Wicker (2006) studies the major panics occurred between 1873-1907 and provides a comprehensive narrative account of the events.

Kemmerer (1910) provides an early identification of these episodes through the reading of historical newspaper reports. Modern studies include DeLong and Summers (1986), Gorton (1988), Reinhart and Rogoff (2009) and Jalil (2015) which provide alternative classifications based on different criterions and varying regional detail. Jalil (2015), in addition, tackles the problem of quantification by providing econometric estimates of the impact of major, nationwide panics on industrial production and prices, with results suggesting large and persistent negative effects on production. My paper differs by focusing instead on the regional spatial transmission of localized panics. In section 3.4.2 I find that regional panics had a moderate impact on banking sector activity, with deposits and lending declining between 2% and 4% and the accumulation of additional liquidity reserves on its aftermath. I also find that the effects of these panics were largely transitory, with most of the variables studied returning to their pre-crisis trends within two years. More surprisingly, I find a lagged but robust response of the banking system outside the state borders in which the panics originated, which I attribute to inter-state financial linkages between banks that I explain below.

The United States at the time was characterized by the unit banking system (single office banks). Due to the lack of branching, individual banks created a network of inter-bank deposits in order to clear the interregional transactions of their clients as well as to pursue better investment opportunities. In addition, banks were allowed to hold their required reserves as interest earning deposits in banks located in reserve and central reserve cities (Chicago, New York, St. Louis). This generated a pyramidal structure of inter-bank deposits with central reserve cities (specially New York) on top. Reserve cities, in its turn, played an important role as liquidity providers at the regional and national levels. The literature on panics has long argued that distress in the upper layers of the pyramid, either through temporary suspensions of deposit convertibility or squeezes in interbank lending, was one of the main drivers of panic propagation and amplification[1]. Section 3.3 develops a model of the interbank market consistent with the overlapping inter-state financial relationships established by banks under the pyramidal reserve system. The model presents a simple trade-off between efficient fund allocation and market volatility. Participation on the interbank market allows banks to access cheaper funding sources and sustain, on average, higher levels of credit. On the other hand, it exposes them to deposit fluctuations outside their state borders, from which they would have otherwise been insulated by the unit banking system. This result provides the theoretical link between localized banking panics and their spatial spread across the United states. Section 3.4.1 provides the link between the equilibrium conditions of the model and the empirical specifications of this paper.

Section 3.3 introduces the theoretical model of the interbank market. Section 3.2 presents the data used in this study and its sources. Section 3.4 develops an estimation methodology consistent with the model and presents the main results. Section 3.5 concludes the paper.

3.2 Data

Data on state-level bank balance sheet aggregates comes from the Abstract of Reports contained within the Annual Report of the Office of the Comptroller of the Currency. I obtained the digitized series for the 1880-1910 period from Weber (2000), while the 1868-1879 and 1911-1930 periods have been digitized by myself. The data consists of self-reported balance sheets of all existing banks with a national charter, aggregated by the Comptroller at the reserve city and state level. See Table 3.5 for an overview of the contained categories. The District of Columbia is included in the sample and treated as an additional state. I

[1] Calomiris and Carlson (2017) test the importance of this channel through a detailed analysis of the correspondent network around the 1893 panic, finding that banks with a higher exposure to New York City were more likely to suspend activity or close.

exclude Alaska and Hawaii due to their distance to continental U.S.. Balance sheet data is reported at varying frequencies, with quarterly reporting being common. Years with higher frequency reporting are converted to quarterly observations by averaging the observations contained within the same quarter.

I use the banking panic series developed by Jalil (2015) because of its accurate geographical detail and panic dating. See Table 3.2 for a reproduction of Jalil's series for the sample period. He narrowly defines a panic as a "widespread run by private agents in financial markets. . . [in order to] convert deposits into currency", which makes for an homogeneous set of events across the sample. Jalil differentiates between Major and Minor panics. The later are regionally delimited and generally thought as less severe, while the former are characterized by rapidly engulfing most of the United States and accompanied by serious distress. Given the focus on spatial transmission of this paper, I will exclude the three Major panics contained in my sample, as they do not have clearly defined panic start regions.
Modern financial crises are the result of complex economic processes and it is difficult to consider them as exogenous to the business cycle. The situation is quite different for the banking panics that I am considering, most of them starting as bank runs created by events relatively uncorrelated to the state of the national and regional economies. Jalil (2015) provides an appendix with ample narrative support for this view, with common bank run triggers ranging from fraudulent behavior of individual bank directors to failed investments of individual banks, all of them paired with the lack of deposit insurance and the unit banking system, which made for small and undiversified banks. For this reason, I will be treating panics as exogenous throughout the paper. I conduct a more formal Granger causality test in section 3.4.1 that supports this view. If this assumption fails, then the reported estimates should be considered as an upper bound to the effect of panics.

3.3 Model

This section presents a simple partial equilibrium model that theoretically establishes the link between deposit fluctuations and spatial transmission of panics. The economy of this model consists of N regions, each one containing a representative bank. Banks raise deposits from their own region, and supply them in the form of loans to the domestic market or to banks in other regions through the interbank market. Time is discrete and quarters are indexed by t. Each quarter is comprised of a continuum [0, 1] of moments indexed by τ in which loan contracts are signed between distinct banks or with private borrowers. For expositional purposes I assume that each bank is divided in two divisions responsible for different tasks.

Loan Division

The loan division of bank n supplies credit to the regional economy, and faces an exogenous and constant loan demand throughout the quarter t given by

$$L^n_{t,\tau} \equiv L^n_t = \left(R^{F,n}_t\right)^{-\sigma} \cdot \varepsilon^n_t \ , \qquad \forall i \ , \tag{3.1}$$

where $R^{F,n}_t$ is the interest rate on private consumer loans and $\varepsilon^n t$ a regional demand shock. Loan supply of division n is subject to the following constraints

$$\begin{aligned} L^n_{t,\tau} &\geq 0 \ , \\ L^n_{t,\tau} &\leq M^n_{t,\tau} \ , \end{aligned}$$

where $M^n_{t,\tau}$ is the total funding raised from local depositors or through the interbank market. Both equations hold with equality in equilibrium. The division solves the following profit maximization problem

$$\max_{\{M^n_{t,\tau}\}} \int_0^1 R^{F,n}_t L^n_t - R^{I,n}_{t,\tau} M^n_{t,\tau} \, \mathrm{d}\tau \ ,$$

where ${R^I}^n_{t,\tau}$ represent the interest rate charged on funds obtained from the own deposit division (shadow value) or that of other banks through the interbank market. I assume that the interest rate $R^{F,n}_t$ is sticky and can only be reset at the beginning of each quarter. Assuming perfect competition on the credit supply market, the solution of the problem brings the following first order condition

$${R^F}^n_{t,\tau} = \int_0^1 R^{I,n}_{t,\tau} \, \mathrm{d}\tau \ . \tag{3.2}$$

Interbank funds $R^{I,n}_{t,\tau}$ are an homogeneous good (money), and loan division n borrows them from the cheapest source at each moment τ, formally

$$\begin{aligned} R^{I,n}_{t,\tau} &= \min_n \{R^{I,in}_{t,\tau}\} \ , \\ M^n_{t,\tau} &= M^{in}_{t,\tau}, \quad i = \arg\min_j \{R^{I,nj}_{t,\tau}\} \ . \end{aligned}$$

Deposit Division

The deposit division n receives deposits D^n_t from the inhabitants of region n at the beginning of the quarter and distributes them to its own loan division or to others through the interbank market. The allocation of deposits to this activities entails a production cost given by

$$\sum_{i=1}^{N}\int_0^1 T_t^{ni}\cdot z_{t,\tau}^{ni}\cdot M_{t,\tau}^{ni}\,\mathrm{d}\tau=(D_t^n)^{\alpha}\,,\qquad 0<\alpha<1\,, \tag{3.3}$$

where $M_{t,\tau}^{ni}$ are loans made to loan division i and D_t^n are local deposits. Parameter α captures the economies of scale of allocating and/or storing a large supply of deposits, and parameter $T_t^{ni}\geq 1$ the costs associated to trading with different regions. I normalize the cost of trading with the own deposit division to one for all banks, $T_t^{nn}=1$, $\forall n$. Variable $z_{t,\tau}^{ni}$ is an exogenous technology shock that follows a Weibull distribution with mean one and shape parameter κ that captures within-quarter varying difficulty to create loans (ex. unmodelled seasonal demand/supply peaks). Profit maximization problem of deposit division n is given by

$$\max_{\{M_{t,\tau}^{ni}\}}\sum_{i=1}^{N}\int_0^1 R_{t,\tau}^{I,ni}M_{t,\tau}^{ni}\,\mathrm{d}\tau-\bar{R}_t\cdot D_t^n\,,$$

where $\bar{R}_t$ is an exogenously given compensation to bank deposits[2]. The first order condition of the problem pins down the interbank rate charged on any bilateral transaction at each moment τ

$$R_{t,\tau}^{I,ni}=z_{t,\tau}^{ni}\left(\frac{\bar{R}_t}{\alpha}\right)D_{n,t}^{1-\alpha}\,. \tag{3.4}$$

Equilibrium

Equations (3.2) and (3.4) together with the properties of the Weibull distribution bring the following expression for the loan rate of region n

$$R_{i,t}^{F}(\tau)=\left(\frac{\bar{R}_t}{\alpha}\right)\cdot\left[\sum_{n=1}^{N}\left(T_t^{ni}\right)^{-\kappa}D_{n,t}^{-\kappa(1-\alpha)}\right]^{-1/\kappa}\,.$$

Combining the previous expression with loan demand (3.1) we obtain the connection between domestic credit in region n and deposit variation in other regions.

$$L_t^n=\left(\frac{\bar{R}_t}{\alpha}\right)^{-\sigma}\cdot\left[\sum_{i=1}^{N}\left(T_t^{in}\right)^{-\kappa}\left(D_t^i\right)^{-\kappa(1-\alpha)}\right]^{\frac{\sigma}{\kappa}}\cdot\varepsilon_{n,t}\,,\qquad\forall n\,. \tag{3.5}$$

[2]We can micro-found this assumption by linking the compensation of deposits to the prevailing risk-free rate on government bonds.

To better understand the implications of this model, I will assume that deposits on all regions are equal $D_t^n = \bar{D}$, $\forall n$, and transaction costs between different regions are infinite $T_t^{ni} \rightarrow \infty$, $i \neq n$. The previous equation becomes

$$L_t^n = \left(\frac{\bar{R}_t}{\alpha}\right)^{-\sigma} \cdot \bar{D}^{-\sigma(1-\alpha)} \cdot \varepsilon_{n,t} , \qquad \forall n ,$$

and banks are forced to entirely rely on their domestic depositors to fund their lending activity. The opposite case with no transaction costs $T_t^{ni} = 1$, $\forall i$ brings the following expression

$$L_t^n = N^{\frac{\sigma}{\kappa}} \cdot \left(\frac{\bar{R}_t}{\alpha}\right)^{-\sigma} \cdot \bar{D}^{-\sigma(1-\alpha)} \cdot \varepsilon_{n,t} , \qquad \forall n ,$$

where banks are able to supply $N^{\frac{\sigma}{\kappa}} > 1$ times more credit. The gains from a market with realistic transaction costs are likely to be lower, but still above those of an economy without a working interbank market. An interesting observation emerges from these exercises: interbank transactions improve the allocation of funding across the banking sector and allow to sustain higher (on average) levels of credit. But on the other hand (as shown by equation (3.5)), credit supply becomes linked to deposit fluctuations outside its regional borders, setting the theoretical foundations for the spatial spread of panics.

3.4 Empirical Estimation

3.4.1 Methodology

A log-linear approximation of equation (3.5) brings the following expression

$$\log\left(L_t^n\right) = \mu_{in} + \mu_t + \sum_{i=1}^{N} \gamma_{in} \cdot \log\left(D_t^i\right) + \varepsilon_t^n , \qquad \forall n , \tag{3.6}$$

where μ_{in} and μ_t are bilateral and seasonal fixed effects, respectively, γ_{in} captures the intensity of response of loans in region n to deposit movements in region i. In order to study the spatial propagation of panics, I assume the following functional specification for γ_{in}

$$\gamma_{in} = \lambda_1 + \lambda_2 \log\left(Distance_{in}\right) + \lambda_3 Neighbor_{in} + \lambda_4 Own_{in} ,$$

where $Distance_{in}$ is measured as the euclidean distance between state i and state n geographical centroids, $Neighbor_{in}$ is a binary variable equal to one if the states pair i and n are neighbors and Own_{in} is a binary variable equal to one if state i and n are the same. Assuming a linear relationship between deposits and panics, we can rewrite (3.6) as

$$\log\left(L_t^n\right) = \eta_n + \sum_{j=1}^{4} \theta_j F_t^{j,n} + \varepsilon_t^n \ , \tag{3.7}$$

where

$$F_t^{1,n} = \sum_{i=1}^{N} Panic_t^i \qquad F_t^{3,n} = \sum_{i=1}^{N} Neighbor_{in} \cdot Panic_t^i$$

$$F_t^{2,n} = \sum_{i=1}^{N} \log\left(Distance_{in}\right) \cdot Panic_t^i \qquad F_t^{4,n} = \sum_{i=1}^{N} Own_{in} \cdot Panic_t^i \ ,$$

which has the advantage of allowing us to summarize the spatial effects of banking panics with four factor variables $\left\{F^{j,n}\right\}_{j=1}^{4}$. Generalizing upon the simple model presented in section 3.3 and equation (3.7), I evaluate the spatial and dynamic propagation of panics by estimating the following set of Jordà Local Projections

$$y_{t+h}^n = \eta_{n,h}^y + s_{t,h}^y + \sum_{j=1}^{4} \theta_{j,h}^y F_t^{j,n} + \sum_{l=1}^{L} \beta_{l,h}^y X_{t-l}^n + \varepsilon_{t+h}^n \ , \qquad h = 1, \ldots, H \ , \tag{3.8}$$

where $\eta_{n,h}^y$ and $s_{t,h}^y$ are state and seasonal fixed effects and X_{t-l}^n is a set of control variables that includes four lags of variables F and y. As dependent variables y, I consider: number of active banks, average bank capital, loans, deposits and liquidity ratio, defined as the ratio of cash, species and short-term assets to total assets. Index h indicates the estimation horizon, with $h = 0$ being the quarter in which a panic originates.

The coefficients $\left\{\theta_{j,h}^y\right\}$ capture the causal spatial dynamic transmission of panics under the assumption that panics are uncorrelated with the error term ε_{t+h}^n. Jalil (2015) provides narrative evidence that backs this assumption, with individual episodes of fraud, foreign shocks or even weather acting as the trigger of panics. Note also, that even if this assumption is violated within the states where the panics originated, the estimates of spatial spread are still likely to remain causal as long as the regional economy of non-origin regions is uncorrelated with the causes of the panic. The plausibility of this hypothesis is reinforced by the unit banking system and restrictions on interstate branching throughout the sample period, leaving the interbank market as the most obvious source of spatial transmission. Table 3.3 contains the results of a Granger causality test in which I regress lags of deposits, loans and number of banks on the $Panic_t^i$ variable, formally

$$Panic_t^i = \mu_i + \mu_t + \sum_{l=1}^{4} \left[\beta_l^D \Delta \log\left(Deposits_{t-l}^i\right) + \beta_l^L \Delta \log\left(L_{t-l}^i\right) + \beta_l^B \Delta \log\left(Bank_{t-l}^i\right)\right] + \varepsilon_{i,t} \ .$$

The table reports results on the joint null hypothesis $H_0: \ \beta_l^D = \beta_l^L = \beta_l^B = 0 \ \forall l$. Columns 1 and 2 report the results only using Jalil (2015) nationwide major panics, while columns 3 and 4 report the test using regional panics. As we can see, the null is rejected for both specifications using major panics as the dependent variable, but we are not able to reject it when using the regional series. As major panics do not contribute much to the identification of spatial transmission due to their nationwide nature, I exclude them from the sample as explained in section 3.2.

3.4.2 Results

Figures (3.1)-(3.5) provide a graphical overview of the results. The figures are constructed as follows:

1. I estimate equation (3.8) for all horizons h, obtain $\{\hat{\theta}_{j,h}\}_{j=1}^4$ as explained in section 3.4.1.
2. Assume a panic in the state of New York, generate $\{F_{i,t}^j\}_{j=1}^4$ for all i.
3. Report $\sum_{j=1}^4 \hat{\theta}_{j,h}^y F_{i,t}^j$ as the predicted response at horizon h for state i and variable y.

P-values are constructed using Driscoll-Kraay standard errors in order to provide consistent estimates to spatial correlation, heteroskedasticy and auto-correlated error terms.

Figure (3.1) reports the evolution of deposits following a panic. We observe that the impact of the panic ranges from -3% in the simulated origin state of New York to around -1.5% on the remaining states, suggesting a rapid spatial propagation. The effects of the panic after one year suggests a lagged response by depositors outside the origin state, thought the result is not statistically significant. After two years, we see in figure three that deposits have returned to their pre-panic trend everywhere. Figure (3.2) depicts a similar pattern for bank lending, with an initial 4% drop in the origin state and mild but significant reductions of bank lending throughout the country. The second graph also displays the same lagged response of deposits, with lending falling by 3% across the country after one year. The last picture shows that lending has almost returned to its pre-crisis level everywhere except on the origin state and its neighbors after two years, though the results are no longer statistically significant. Figure 3.3 shows the evolution of the liquidity ratio, which increases on impact on the origin state and persistently raises above its pre-crisis level across the country thereafter.

The result is consistent with the stronger response of bank lending vis-à-vis deposits, which suggests that banks reallocate their portfolio towards safer assets like bonds following a panic. Figures (3.4) and (3.5) show the evolution of bank capital and number of banks, respectively. While there is no discernible effect on impact, bank capital diminishes by up to 1.5% after two years. Similarly, the number of banks decreases by 1-1.2% after two years, thought only the origin and neighboring states are affected.

The overall picture that emerges from these figures is consistent with the results found by the literature on financial crisis and Jalil (2015) price and output result for the same time period. Panics had a moderate impact on the banking sector across several dimensions and their effects largely vanished after two years. More surprisingly, I find that panics displayed a robust spread outside their initial state boundaries.

3.5 Conclusions

This paper quantifies the historical impact and geographical spread of banking panics on the U.S. banking system. I find that panics are accompanied by moderate and temporary drops in deposits and lending, increased liquidity, and a small negative impact on bank capital and number of banks, with the results being statistically significant up to two years from the onset of a panic. I also find that regional panics display a robust spatial propagation, which might be explained by the pyramidal reserve system prevalent during the historical period studied. Section 3.3 formalizes this intuition by providing a theoretical model of interbank trade in which banks from different regions establish bilateral loan contracts across time to smooth funding needs. The model shows that while these agreements are generally beneficial for the economy and allow for an expanded supply of credit, they also expose banks' activity to fluctuations originated outside their state borders, such as is the case with banking panics.

Figures

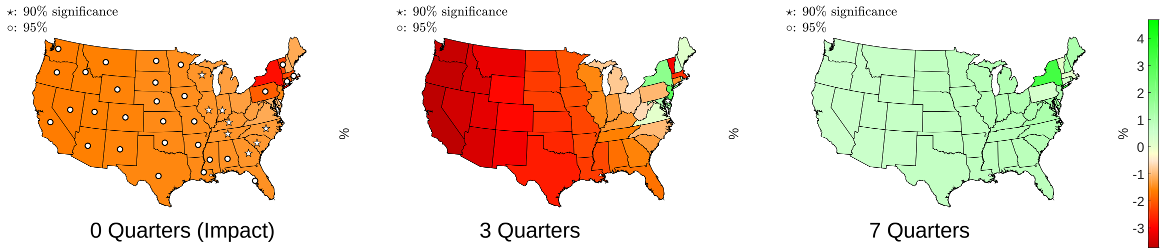


Figure 3.1: Impulse-response of bank deposits to a panic in New York. Right bar reports graph estimates color scale. P-values constructed using Driscoll-Kraay standard errors.
○ p<0.05, ⋆ p<0.1

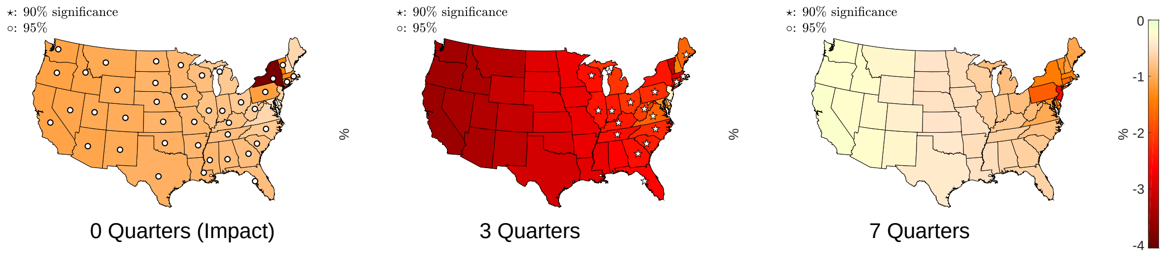


Figure 3.2: Impulse-response of bank loans to a panic in New York. Right bar reports graph estimates color scale. P-values constructed using Driscoll-Kraay standard errors.
○ p<0.05, ⋆ p<0.1

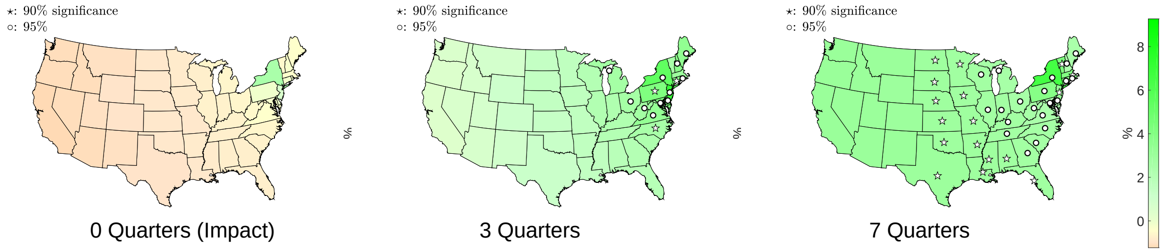


Figure 3.3: Impulse-response of liquidity ratio to a panic in New York. Right bar reports graph estimates color scale. P-values constructed using Driscoll-Kraay standard errors.
∘ $p<0.05$, ⋆ $p<0.1$

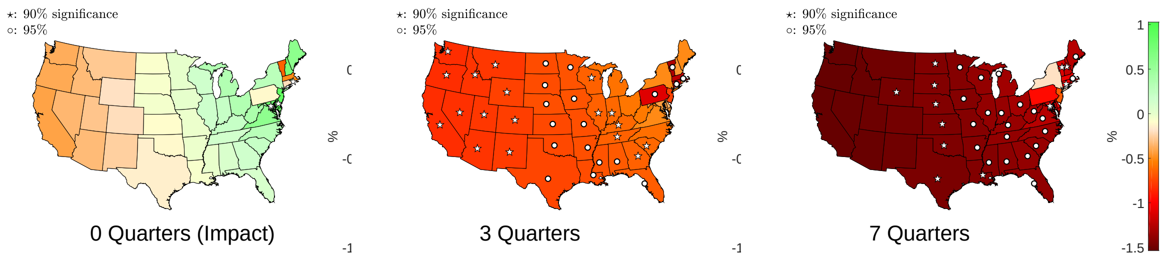


Figure 3.4: Impulse-response of bank capital to a panic in New York. Right bar reports graph estimates color scale. P-values constructed using Driscoll-Kraay standard errors.
∘ $p<0.05$, ⋆ $p<0.1$

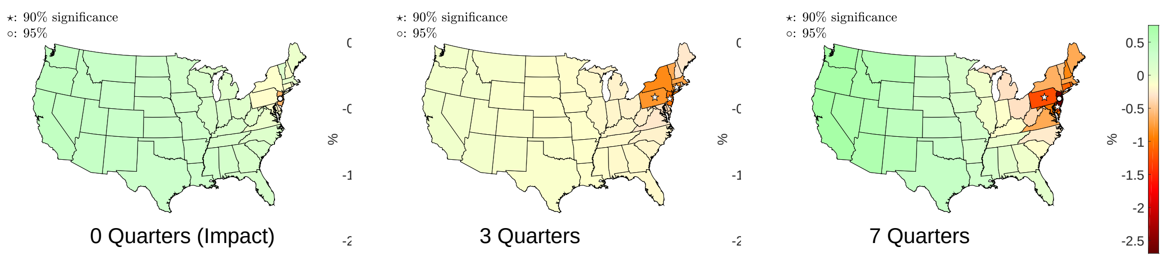


Figure 3.5: Impulse-response of number of banks to a panic in New York. Right bar reports graph estimates color scale. P-values constructed using Driscoll-Kraay standard errors.
∘ $p<0.05$, ⋆ $p<0.1$

Tables

Resources	Liabilities
Loans and discounts	Capital Stock
Overdrafts	Surplus fund
Bonds for circulation	Undivided profits
Bonds for deposits	National bank circulation
Other bonds for deposits	State bank circulation
U.S. Bonds on hand	Due to national banks
Premium on bonds	Due to state banks
Bonds, securities, etc	Due to trust companies, etc
Banking house, furniture, etc	Due to reserve agents
Real state, etc	Dividends unpaid
Current expenses	Individual deposits
Due from national banks	Certified checks
Due from state banks	U.S. deposits
Due from reserve agents	Deposits U.S. disbursing officers
Internal revenue stamps	Bonds borrowed
Cash items	Notes rediscounted
Clearing-house exchanges	Bills payable
Bills of other banks	Clearing-house certificates
Fractional currency	Other liabilities
Trade dollars	Specie
Legal-tender notes	
U.S. certificates of deposit	
Three per cent certificates	
5% fund with Treasury	
Clearing-house certificates	
Due from U.S. Treasury	
Total	Total

Table 3.1: Balance sheet original categories, Abstract of Reports. The Abstract of Reports, contained in the Annual Report of the Comptroller of the Currency, provides regional aggregates of the categories that I list in the following table. Categories reported tend to vary slightly across time, typically due to the subdivision of big categories into smaller ones on the latest reports. For example, the category “Loans and discounts” initially contains “Overdrafts”, which eventually becomes a category on its own.

States	Panic, start	Panic, end	Reporting date	Time to start (days)
All (Major)	18sep1873	30sep1873	26dec1873	99
NY, PA, NJ	13may1884	31may1884	20jun1884	38
NY	10nov1890	22nov1890	19dec1890	39
All (Major)	13may1893	19aug1893	12jul1893	60
IL, MN, WI	26dec1896	26dec1896	09mar1897	73
MA, NY	16dec1899	31dec1899	13feb1900	59
NY	27jun1901	06jul1901	15jul1901	18
PA, MD	18oct1903	24oct1903	17nov1903	30
All (Major)	12oct1907	30nov1907	03dec1907	52
NY	25jan1908	01feb1908	14feb1908	20
MA	12aug1920	02oct1920	08sep1920	27
ND	27nov1920	19feb1921	29dec1920	32
FL, GA	14jul1926	21aug1926	31dec1926	170
FL	08mar1927	26mar1927	23mar1927	15
FL	20jul1929	07sep1929	04oct1929	76
			Median	38.5

Table 3.2: Banking panics chronology (sample period). The series is extracted from Jalil (2015). The first column reports the states in which the panic initially originated. Panic start and end dates are obtained from Jalil (2015) classification appendix when possible or by reading the original listed sources. First column reports affected states. The fifth column reports days elapsed between the start of the crisis and the first Abstract of Reports from the Comptroller of the Currency observed.

	Specification			
	1	2	3	4
Joint F-test, p-value	***	***	H_0	H_0
R-squared	0.4	1.88	0.06	0.95
All panics	X	X		
Minor panics			X	X
Indiv. fix effects		X		X
Seasonal dummies		X		X

Table 3.3: Granger causality test. I regress panic episodes on lagged changes of deposits, loans and number of banks according to $Panic_t^i = \mu_i + \mu_t + \sum_{l=1}^{4} \left[\beta_l^D \Delta \log \left(Deposits_{t-l}^i\right) + \beta_l^L \Delta \log \left(L_{t-l}^i\right) + \beta_l^B \Delta \log \left(Bank_{t-l}^i\right)\right] + \varepsilon_{i,t}$. The table reports results for the null hypothesis H_0: $\beta_l^D = \beta_l^L = \beta_l^B = 0 \ \ \forall l$. Type of panics and controls included in each specification are indicated with a X.
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$, H0 $p \geq 0.1$

3.6 Transitional section

In chapters 2 and 3 I studied the consequences of financial distress on the interbank market, quantified the welfare costs of the 2007 financial crisis and estimated the spatial spread of historical banking panics. In chapter 2 I examined the benefits of several monetary policy and lender-of-last resort interventions, but I excluded the study of optimal policy at the zero lower bound (ZLB) on nominal interest rates. This particular case is of special interest to the study of efficient policy during financial crisis, as financial shocks can quickly bring nominal rates to the ZLB, rendering standard policy ineffective. Chapter 2 micro-founded the transmission of these financial shocks across the interbank network and showed they can be aggregated into a single credit production/risk-premium shock. Chapter 4 picks up this result and introduces an exogenous risk-premium shock to an otherwise standard New-Keynesian model, and studies in detail the modeling of ZLB episodes and the optimal trend inflation target of central banks.

Chapter 4

Infrequent but Long-Lived Zero-Bound Episodes and the Optimal Rate of Inflation

This chapter is the product of joint work with Olivier Coibion, Yuriy Gorodnichenko and Johannes Wieland. I thank them for allowing me to use our joint work as part of this dissertation. The opinions discussed in this chapter do not necessarily reflect the views of the authors' employing institutions.

4.1 Introduction

When the U.S. Federal Reserve finally raised its target for the Federal Funds Rate in December 2015, this likely marked the end of the zero-bound on short-term nominal interest rates for the United States after a staggering *seven years*. Japan's zero-bound period will most likely exceed this duration under Abenomics, while the Bank of England has similarly had near-zero interest rates since March of 2009. The Euro Central Bank is also not expected to raise interest rates for years. Combined with the previous experiences with the zero-bound on interest rates that occurred during the Great Depression and in Japan during the 1990s-2000s, this suggests that the two most prominent empirical features of zero-bound episodes are that they are *rare* but *long-lived*.

The zero-bound on interest rates raises a number of profound problems for monetary policymakers, one of which is the traditional question of what the optimal inflation rate should be. While it is well-understood that even stable inflation has costs (such as those arising from price dispersion), higher average inflation is also associated with higher nominal interest rates, which can benefit policymakers by giving them extra room to avoid running into the zero-bound. Quantifying the optimal rate of inflation then requires balancing the costs of inflation against its benefits, such as minimizing the frequency and severity of zero lower bound (ZLB) episodes.

But quantifying this potential benefit of higher inflation is difficult because the paucity of ZLB episodes makes their frequency and duration hard to gauge. For example, Schmitt-Grohé and Uribe (2010) calibrated their model prior to the start of the Great Recession and had no post-WWII zero-bound episodes in the U.S. to guide their choice over the frequency of hitting the zero-bound. This resulted in a calibration with very rare and short-lived ZLB episodes. Coibion et al. (2012) used the fact that the U.S. had spent 3 years at the ZLB at the time of their writing out of the post-WWII period to fix their frequency, yielding more frequent but still mostly short-lived episodes. In each case, these authors conclude that the optimal rate of inflation is unlikely to be much above 2% despite the zero-bound on interest rates. But given the actual durations of the most recent ZLB experienced by developed economies, each of these papers likely underestimated the average duration of ZLB episodes and therefore the potential benefits of higher levels of target inflation on the part of central banks.

In this paper, we revisit the topic of longer-lived ZLB episodes in two steps. First, following previous work, we generate longer-lived ZLB episodes by either increasing the persistence or the volatility of AR(1) risk-premium shocks which push the economy into the ZLB in our model. By doing so, we can generate a longer average duration of ZLB episodes consistent with the data. In our benchmark New Keynesian model, increasing the average duration of ZLB episodes (for a given steady-state level of inflation) through either more persistent or more volatile shocks can have large positive effects on the optimal inflation rate. For example, moving from an average duration of ZLB episodes of 5 quarters to just 6 quarters, holding the frequency of ZLB episodes fixed (by varying the persistence and volatility of shocks accordingly), can raise the optimal inflation rate from 1.3% to 2.2% in our baseline calibration, a very high sensitivity.

This sensitivity, however, reflects an unappealing characteristic common to all standard models of the ZLB in which normally distributed shocks drive the economy into the ZLB: the vast majority of ZLB episodes in the model are extremely short-lived. Significantly raising the average duration therefore requires large tail events, and these episodes have disproportionately large welfare costs. Policymakers become very willing to tolerate higher average inflation rates to avoid these episodes, leading to significantly higher optimal rates of inflation for even small changes in average durations of ZLB episodes. Hence, the high sensitivity of the optimal rate of inflation to the average duration of ZLB episodes in our benchmark model is a reflection of the counterfactual distribution of ZLB episodes, namely that they are too frequent and short-lived compared to the rare and long-lived episodes that we observe in the data.

Our second step is then to incorporate an alternative modeling strategy for the shocks that drive the economy into the ZLB which generates an empirically realistic distribution of ZLB episodes, namely that they tend to be rare but long-lived. We assume that each period, risk premia follow a regime-switching process. In each period when the economy is not at the ZLB, there is a fixed probability that the risk premium will rise sharply for a set number of periods. If the increase in the premium is large enough, this shock will give rise to a distribution with very long-lived ZLB durations, thereby more closely representing the empirical distribution of ZLB episodes. As a result, we can more carefully assess whether (or how much) raising the rate of inflation is optimal, in a welfare sense, to offset the presence of the zero bound on interest rates.

Unlike the AR(1) model, the regime switching approach does not display an excessive sensitivity of the optimal inflation rate to the average duration of ZLB episodes. Nonetheless, long-lived ZLB episodes generate large welfare costs in the model, which higher levels of steady state inflation can help avoid by reducing their frequency. We find that depending on our calibration of the average duration and the unconditional frequency of ZLB episodes, the optimal inflation rate can range from 1.5% to 4% . This uncertainty stems ultimately from the paucity of historical experience with ZLB episodes, which makes pinning down these parameters with any degree of confidence very difficult. A key conclusion of the paper is therefore that much humility is called for when making recommendations about the optimal rate of inflation since this fundamental data constraint is unlikely to be relaxed anytime soon.

Our paper builds on a broad literature on the optimal rate of inflation. This literature has covered a wide range of costs and benefits, with the zero bound on interests only recently coming to the forefront as a plausible source for positive optimal rates of inflation. In a survey of pre-Great Recession work, Schmitt-Grohé and Uribe (2010) highlighted that, although the quantitative conclusions about the optimal rate of inflation were potentially sensitive to the choice of the model used to assess the costs and benefits of inflation (or deflation) in the steady-state, one generally found that it was optimal to have a small amount of deflation. For example, using a standard model with demand for money, Schmitt-Grohé and Uribe (2010) estimated the optimal inflation rate at -0.6 percent per year with the Ramsey optimal policy. While this rate of deflation is considerably smaller than the rate of deflation originally suggested by Milton Friedman (approximately equal to the real interest rate), the optimality of deflation in steady state is inconsistent with the 1-3% /year inflation rates currently targeted by most central banks.

Even when one moves to cashless economies, it is difficult to push the optimal rate of inflation near the levels commonly targeted by modern central targets using traditional arguments for positive levels of inflation such as downward wage rigidity. Indeed, New Keynesian models generally suggest that the optimal rate of inflation should be close to zero because price dispersion generated by non-zero trend inflation is costly (see Benigno and Woodford

(2005)).[1] However, one may be able to raise the optimal rate of inflation by departing from the workhorse specifications to incorporate e.g. foreign demand for currency (Schmitt-Grohé and Uribe (2012)), firm-specific productivity growth (Weber (2012)), occasionally binding financial constraints (Abo-Zaid (2015)), or tax evasion (Schmitt-Grohé and Uribe (2010)). Pre-Great Recession work (e.g., Summers (1991)) discussed the zero lower bound (ZLB) on nominal interest rates as a potential reason for positive inflation but generally considered the ZLB as an improbable event. As a result, the models were calibrated to generate infrequent and short-lived ZLB episodes. For example, Schmitt-Grohé and Uribe (2010) indicate that to violate the zero bound "... the nominal interest rate ... must fall more than 4 standard deviations below its target level" thus making ZLB an extremely rare event.

Many others found similar results. For example, Reifschneider and Williams (2000) and Chung et al. (2012) document that the frequency of ZLB for three popular dynamic stochastic general equilibrium (DSGE) models estimated on the post-WWII, pre-2007 data is typically less than 5 percent.[2] Furthermore, ZLB episodes longer than 8 quarters can be observed less than 1 percent of the time. If one uses data from the Great Moderation period to assign parameters in DSGE models, ZLB episodes are even shorter and less frequent. Similarly, Adam and Billi (2007) find that, with optimal monetary policy, conditional on hitting ZLB, the likelihood of being at the ZLB for more than 4 quarters is a mere 1.8% . In Billi (2011) calibration, the ZLB binds 4 percent of the time and the average duration of ZLB is only 2 quarters. Using non-linear methods to solve and simulate calibrated DSGE models, Amano and Shukayev (2012) report that the probability of hitting the ZLB is 1.7% per quarter (i.e., a 4-quarter ZLB episode occurs once every 60 years). In other words, ZLBs in models used by researchers and policymakers were too short and too rare to matter.

As the welfare costs of short ZLB episodes tend to be small, the ZLB was found to have tiny effects on the estimated optimal rate of inflation. For example, the optimal rate of inflation in the Schmitt-Grohé and Uribe (2010) calibration increased modestly from -0.6 to -0.4 percent per year in light of the ZLB. In short, the consensus view before the Great Recession was that, although the ZLB was an interesting and curious possibility, one could treat it as remote and largely irrelevant.

With policy interest rates in major developed economies having spent years at the zero lower-bound during the Great Recession and its aftermath, there has of course been a shift in thinking about the frequency and nature of the ZLB. Examination of new cross-country evidence and long time series (i.e., series including the Great Depression) suggests that ZLB episodes are potentially costly (e.g., Williams (2009) estimated that four years at the ZLB

[1]Wolman (2011) and others show that even in the absence of money demand considerations, the optimal rate of inflation in New Keynesian models can be negative. For example, in Wolman's model and calibration the optimal deflation is 0.4% per year.

[2]Coibion et al. (2012) calibrate the frequency of ZLB in the basic New Keynesian model at 5 percent.

can cost as much as $ 1.8 trillion), more frequent (e.g., Chung et al. (2012) indicate that, based on pre-2010 data, one should double the probability of being at the ZLB in calibrated models), and more persistent. The latter point is particularly important as ZLB episodes in the U.S. and elsewhere are not characterized by a series of short intervals of constrained policy rates. Instead, the Great Depression and the Great Recession in the U.S. or the crash in Japan indicate that ZLB episodes can last for years if not decades.

Incorporating these changes in the way we model the ZLB can have dramatic effects on the optimal rate of inflation. Indeed, apart from ZLB episodes being modeled as more frequent and thus costlier, we know from Coibion et al. (2012) and others that the cost of ZLB is increasing steeply in its duration. That is, an 8-quarter ZLB is costlier than two 4-quarter ZLB episodes. Thus, the cost of ZLB in a new calibration can be considerably larger than in previous calibrations and can entail an optimal rate of inflation higher than the conventionally suggested 2 percent per year.

While the treatment of the ZLB is one important source of differences in the estimated optimal rate of inflation, there are other factors and modelling choices that can affect the optimal rate. For example, Coibion et al. (2012) show that how one models price stickiness can also influence results. Using the Calvo (1983) approach tends to yield a lower optimal inflation rate because Calvo-style pricing generates a larger increase in cross-sectional price dispersion for a given increase in trend inflation than e.g. Taylor (1979) pricing.[3] Intuitively, firms with Calvo pricing may be stuck at suboptimal prices for a long time while Taylor pricing guarantees that prices can be reset after a fixed number of periods which caps the size of departures from optimal levels of prices. Because cross-sectional price dispersion is the main cost of non-zero steady-state inflation in New Keynesian models, the choice of pricing assumptions can alter the point at which the cost of positive inflation balances the benefit of positive inflation (e.g., avoid ZLB). Consistent with this logic, Coibion et al. (2012) find the optimal rate of inflation to be 1.5% under Calvo pricing (when the probability of price adjustment is set at 0.55) and 1.8% under Taylor pricing (when the duration of contracts is set at 3 quarters).

In a similar spirit, menu-cost models limit the degree of cross-sectional price dispersion (since a firm can reset its price whenever it deviates too far from the optimal price) and thus could reduce the cost of non-zero steady-state inflation. As a result, it may be optimal in such models to target a higher rate of inflation which will reduce the probability of hitting the ZLB, but the exact magnitude depends on the details of menu cost models. While the optimal rate of inflation in the Dotsey et al. (1999) model is below 2 percent (see Coibion et al. (2012)), Blanco (2016) found that in the Golosov and Lucas Jr (2007) model welfare is maximized at approximately 5% /year inflation rate. Because the computational demands

[3]Using a medium-scale DSGE model, Ascari and Sbordone (2014) estimate that a consumption-equivalent welfare loss from of raising inflation from 2% to 4% can be as large as 7 percent.

become exceedingly high for long-lasting ZLB periods even in linearized models, we will focus on the Calvo approach to model price stickiness.

The paper is organized as follows. Section 4.2 presents the model and the two ways of modeling shocks that drive the economy into the zero bound. Section 4.3 then presents the main results of the paper, including comparing the distribution of ZLB episodes under the two assumptions about shock processes and their implications for optimal inflation. Section 4.4 concludes.

4.2 Model

In our quantitative analyses, we use the standard New Keynesian model similar to the framework in Coibion et al. (2012). To preserve space, we describe the main building blocks of the model and relegate derivations and various details to the Appendix.

4.2.1 Households

The representative consumer maximizes the present discounted value of the utility stream from consumption and leisure

$$\max E_t \sum_{j=0}^{\infty} \beta^j \left\{ \log \left(C_{t+j} - h \cdot GA_{t+j} \cdot C_{t+j-1} \right) - \frac{\eta}{\eta+1} \int_0^1 N_{t+j}(i)^{1+1/\eta} \, di \right\} \tag{4.1}$$

where C is consumption of the final good, $N(i)$ is labor supplied to individual industry i, GA is the gross growth rate of technology, η is the Frisch labor supply elasticity, h the internal habit parameter and β is the discount factor. The budget constraint in each period t is given by

$$\varrho_t : C_t + \frac{S_t}{P_t} + T_t \leq \int_0^1 \left(\frac{N_t(i) W_t(i)}{P_t} \right) di + \frac{S_{t-1} q_{t-1} R_{t-1}}{P_t} + \Gamma_t \tag{4.2}$$

where S is the stock of one-period bonds held by the consumer, R is the gross nominal interest rate, P is the price of the final good, $W(i)$ is the nominal wage earned from labor in industry i, T is real lump sum taxation (or transfers), Γ are real profits from ownership of firms, q is a risk premium shock, and ϱ is the shadow value of wealth (i.e., the Lagrange multiplier on constraint (4.2). As we discuss below, the risk premium shock plays a central role in generating binding ZLB.

4.2.2 Firms

For each intermediate good $i \in [0,1]$, a monopolist generates output using a production function linear in labor

$$Y_t(i) = A_t N_t(i) \tag{4.3}$$

where A denotes the level of technology, common across firms. The time series of technology is described by a random walk process: $A_t = exp\left(u_t^A\right)$, $u_t^A = \mu + u_{t-1}^A + \varepsilon_t^A$ with $\varepsilon_t^A \sim iid\ N\left(0, \sigma_A^2\right)$. Parameter μ sets the average growth rate of technology in the model.

A perfectly competitive sector combines intermediate goods into a final good using the Dixit-Stiglitz aggregator

$$Y_t = \left[\int_0^1 Y_t(i)^{(\theta-1)/\theta}\, di\right]^{\theta/(\theta-1)} \tag{4.4}$$

where Y is the final good and θ denotes the elasticity of substitution across intermediate goods, yielding the following demand curve for goods of intermediate sector i:

$$Y_t(i) = Y_t\left(P_t(i)/P_t\right)^{-\theta} \tag{4.5}$$

and the following expression for the aggregate price level

$$P_t = \left[\int_0^1 P_t(i)^{(1-\theta)}\, di\right]^{1/(1-\theta)}. \tag{4.6}$$

Each intermediate good producer has sticky prices, modeled as in Calvo (1983) where $1-\lambda$ is the probability that each firm will be able to reoptimize its price each period. Denoting the optimal reset price of firm i by B (all firms choose the same rest price), re-optimizing firms solve the following profit-maximization problem

$$\max_{B_t(i)} E_t \sum_{j=0}^{\infty} \lambda^j Q_{t,t+j}\left[Y_{t+j}(i)\, B_t(i) - W_{t+j}(i)\, N_{t+j}(i)\right] \tag{4.7}$$

where $Q_{t,\ t+j} = \beta^j E_t \left\{\frac{\varrho_{t+j}}{\varrho_t}\frac{P_t}{P_{t+j}}\right\}$ is the stochastic discount factor. The optimal reset price B_t is then given by

$$\frac{B_t}{P_t} = \frac{E_t \sum_{j=0}^{\infty} \lambda^j Q_{t,t+j} Y_{t+j}\left(\frac{P_{t+j}}{P_t}\right)^{\theta+1}\left(\frac{\theta}{\theta-1}\right)\left(MC_{t+j}(i)/P_{t+j}\right)}{E_t \sum_{j=0}^{\infty} \lambda^j Q_{t,t+j} Y_{t+j}\left(P_{t+j}/P_t\right)^{\theta}} \tag{4.8}$$

where $MC_t(i) = \frac{W_t(i)}{A_t}$ is the marginal cost of firm i .[4] Given these price-setting assumptions and price index in (4.6), the dynamics of the price level are governed by

$$P_t^{1-\theta} = (1-\lambda) B_t^{1-\theta} + \lambda P_{t-1}^{1-\theta} \tag{4.9}$$

Firms' aggregate real profits are

$$\begin{aligned}\Gamma_t &= \int_0^1 \Gamma_t(i)\, di = \frac{1}{P_t}\int_0^1 \left[P_t(i) Y_t(i) - N_t(i) W_t(i)\right] di \\ &= Y_t - \int_0^1 \left(\frac{N_t(i) W_t(i)}{P_t}\right) di\ .\end{aligned} \tag{4.10}$$

We define the aggregate labor input as

$$N_t = \left[\int_0^1 N_t(i)^{(\theta-1)/\theta}\, di\right]^{\theta/(\theta-1)} = \left[\int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)} = \frac{Y_t}{A_t}. \tag{4.11}$$

4.2.3 Government

We allow for government consumption of final goods (G) with the good market clearing condition $Y_t = C_t + G_t$. The government budget constraint is defined as

$$\frac{T_t + S_t}{P_t} = G_t + \frac{S_{t-1} q_{t-1} R_{t-1}}{P_t}, \tag{4.12}$$

where $G_t = \overline{G}_t \exp\left(u_t^G\right)$, $\overline{G}_t$ is the path of government spending such that the share of government spending in the economy is fixed when prices are flexible, and u_t^G is an exogenous, forcing variable: $u_t^G = \rho_G u_{t-1}^G + \varepsilon_t^G$ with $\varepsilon_t^G \sim iid\ N(0, \sigma_G^2)$.

The policy rule followed by the central bank is

$$R_t = max\{1, R_t^*\} \tag{4.13}$$

$$R_t^* = \overline{R}\left(\frac{R_{t-1}^*}{\overline{R}}\right)^{\rho_1}\left(\frac{R_{t-2}^*}{\overline{R}}\right)^{\rho_2}\left[\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\overline{Y}_t}\right)^{\phi_Y}\left(\frac{GY_t}{\overline{GY}}\right)^{\phi_{GY}}\right]^{(1-\rho_1-\rho_2)} exp\left(\varepsilon_t^R\right) \tag{4.14}$$

where R is the *realized* gross interest rate, R^* is the *desired* gross interest rate, GY is the gross growth rate of output, $\overline{\Pi}$ is the gross, steady-state level of inflation, $\overline{GY}$ is the steady state growth rate of output, $\overline{Y}_t$ is the flexible-price level of output, $\overline{R}$ is the steady state nominal interest rate, and ε^R is an i.i.d policy shock. Equation (4.13) is responsible for

[4] Labor employed by firm each period is obtained through the minimization of production costs.

introducing the zero lower bound to the model. We abstract from alternative monetary policy actions during ZLB episodes, such as quantitative easing. While these could potentially lower the costs of ZLB episodes, there is little evidence suggesting that these policies have had large economic effects.[5]

4.2.4 Risk premium shocks

As discussed in Amano and Shukayev (2012), the risk premium shock is the main "tool" that can generate a binding ZLB in standard New Keynesian models. To be clear, this shock should be interpreted broadly as capturing a variety of forces that bring interest rates to ultra-low levels. We consider two general approaches to model the dynamics of the shock.

The first approach is to describe the time series of the shock as an AR(1) process similar to what is usually assumed for other forcing variables in DSGE models (e.g., Coibion et al. (2012)):

$$q_t = \exp\left(u_t^q\right), \quad u_t^q = \rho_q u_{t-1}^q + \varepsilon_t^q \quad \text{with } \varepsilon_t^q \sim \; iid\; N\left(0, \sigma_q^2\right). \tag{4.15}$$

By adjusting ρ_q and σ_q^2 , one can regulate the frequency and duration of ZLB episodes. As we will show later, a major shortcoming of this approach to modeling the risk premium is that it cannot replicate the main qualitative empirical properties of ZLB episodes, namely that they are rare but long-lived. Instead, AR(1) shocks primarily deliver frequent and short-lived ZLB episodes.

As a result, we also consider a second approach which allows for two regimes of risk premia. For example, Christiano et al. (2011), Eggertsson and Woodford (2003), and Guerrieri and Lorenzoni (2009) assume that the ZLB is binding for a fixed number of periods or that, conditional on being at the ZLB, every period there is a random, i.i.d. draw determining exit from the ZLB; that is, with some probability the risk premium declines from a high level (ZLB is binding) to a low level (ZLB is not binding). This line of work typically assumes that after

[5] Coibion et al. (2012) (CGW henceforth) examine how the optimal rate of inflation varies if the central bank can implement an optimal stabilization policy with commitment. One can think of the commitment policy as introducing a very powerful form of forward guidance. CGW find that in this case the optimal rate of inflation shrinks to zero considerably. Intuitively, with a strong form of forward guidance delivered by fully credible commitment to keep low interest rates far into the future, the stabilization powers of monetary policy remain large unaffected by the ZLB. As a result, there is no need for a "cushion" created by positive trend inflation. CGW also show that if a Taylor rule includes an element of price level targeting, the central bank can nearly achieve the welfare one can obtain under the optimal policy with commitment because current below-target inflation is compensated with above-target inflation in the future. Since our objective is to consider scenarios that should push up the optimal rate of inflation (most importantly, increase in the duration of ZLB episodes), we do not cover the optimal policy with commitment as these move the optimal rate of inflation in the opposite direction.

exiting ZLB the economy does not return to it.

To permit recurrent ZLB episodes, we consider the following regime-switching process. The risk premium can take two values: zero and $\Delta > 0$. Each period when the risk premium is zero, there is a random, i.i.d. draw such that with probability p_{12} the risk premium switches from zero to Δ and stays at this elevated level for T_q periods. After T_q periods with low interest rates, the risk premium returns to zero. By varying Δ, p_{12}, T_q , we can obtain variation in the frequency and duration of ZLB. Note that $\Delta > 0$ does not guarantee that the interest rate will be literarily stuck at zero: other shocks (e.g., productivity) can lift the economy off the ZLB. However, by making Δ large enough, we can reduce the incidence of such lift-offs. We solve the model by adapting the solution algorithm in Coibion et al. (2012) to these deterministic regime-switching processes.[6]

While the difference in modeling the risk premium shock may seem subtle, these two approaches can generate different distributions for ZLB durations with important implications for calculating welfare losses arising from binding ZLB. As we demonstrate below, the AR(1) approach tends to yield frequent, short-lived ZLB episodes. Such a distribution of ZLB episodes appears to be inconsistent with the experience of the U.S. and other developed economies during the Great Recession or in other instances. In contrast, the regime-switching approach can produce long-lived ZLB episodes, similar to what we observe in the data.

4.2.5 Log-linearized system

Using lower-case letters with hats to denote variables log-linearized around the stochastic trend in technology, we can summarize the system of optimality conditions and budget constraints by the familiar equations.

Phillips curve:

$$\begin{aligned}
\left(1+\frac{\theta}{\eta}\right)\left(\frac{\lambda\overline{\Pi}^{(\theta-1)}}{1-\lambda\overline{\Pi}^{(\theta-1)}}\right)\hat{\pi}_t &= \sum_{j=0}^{\infty}\left[\gamma_2^j\left(1-\gamma_2\right)-\gamma_1^j\left(1-\gamma_1\right)\right]\left[\hat{y}_{t+j}+\hat{\varrho}_{t+j}\right] \\
&+\left(1-\gamma_2\right)\sum_{j=0}^{\infty}\gamma_2^j\left[\frac{1}{\eta}\hat{y}_{t+j}-\hat{\varrho}_{t+j}\right] \\
&+\sum_{j=0}^{\infty}\left[\gamma_2^{j+1}\theta\left(1+\frac{1}{\eta}\right)-\gamma_1^{j+1}\left(\theta-1\right)\right]E_t\left[\hat{\pi}_{t+j+1}\right]+\hat{u}_t^m, \qquad (4.16)
\end{aligned}$$

[6] We fix the duration so we only have to solve backward once from period By contrast, if the exit were stochastic we would have to solve backward from every possibly realization and weigh these paths by their probability.

where $\gamma_1 = \lambda\beta\overline{\Pi}^{(\theta-1)}$ and $\gamma_2 = \gamma_1\overline{\Pi}^{(1+\theta/\eta)}$ and $\hat{u}_t^m$ is an ad hoc cost-push shock such that $\hat{u}_t^m = \rho_m \hat{u}_{t-1}^m + \varepsilon_t^m$ and $\varepsilon_t^m \sim iidN\left(0, \sigma_m^2\right)$.

IS curve (consumption Euler equation):

$$\hat{\varrho}_t = E_t\left[\hat{\varrho}_{t+1} + \hat{r}_t - \hat{\pi}_{t+1} + \hat{u}_t^q\right] \tag{4.17}$$

where $\hat{\varrho}_t = \frac{h}{(1-h)(1-\beta h)}\hat{c}_{t-1} - \frac{1+\beta h^2}{(1-h)(1-\beta h)}\hat{c}_t + \frac{\beta h}{(1-h)(1-\beta h)}E_t\hat{c}_{t+1}$.

Taylor rule:

$$\begin{aligned} \hat{r}_t &= max\{\hat{r}_t^*, -\overline{r}\}, \\ \hat{r}_t^* &= \rho_1\hat{r}_{t-1}^* + \rho_2\hat{r}_{t-2}^* + (1-\rho_1-\rho_2)\left[\phi_\pi\hat{\pi}_t + \phi_y\hat{y}_t + \phi_{gy}\widehat{gy}_t\right] + \varepsilon_t^r, \end{aligned} \tag{4.18}$$

where $\widehat{gy}_t = \hat{y}_t - \hat{y}_{t-1} + \varepsilon_t^A$ is the log-linearized growth rate of output.

Market clearing:

$$\hat{y}_t = (1-s_G)\,\hat{c}_t - s_G\hat{g}_t, \tag{4.19}$$

where $s_G = \overline{G}_t/\overline{Y}_t$.

4.2.6 Welfare

Proposition 1 in Coibion et al. (2012) derives the second order approximation to expected per period utility in eq. (4.1) when steady state inflation is different from zero:

$$\Theta_0 + \Theta_1 var\left(\hat{y}_t\right) + \Theta_2 var\left(\hat{\pi}_t\right) + \Theta_3 var\left(\hat{c}_t\right) \tag{4.20}$$

where parameters $\Theta_k, k = \{0,1,2,3\}$ depend on the steady state inflation $\overline{\pi}$. As discussed in Coibion et al. (2012), this approximation has an intuitive interpretation and properties. The term Θ_0 captures the cost of cross-sectional price dispersion arising from positive trend inflation. For quantitatively relevant inflation rates, Θ_0 becomes more negative as steady-state inflation increases. Because of the functional assumption about the household's utility, $\Theta_1 < 0$ but Θ_1 does not depend directly on steady-state inflation. The coefficient on the variance of inflation $\Theta_2 < 0$, which is the main cost of business cycle in the standard New Keynesian model like ours, is decreasing in steady state inflation. Finally, the coefficient on the variance of consumption $\Theta_3 < 0$ captures the desire of habit-driven consumers to smooth consumption.

4.2.7 Calibration

We calibrate the model as in Coibion et al. (2012), see Table 4.1. This parametrization uses values standard in the literature. Parameter values governing the frequency and duration of ZLB (that is, ρ_q, σ_q^2 for the AR(1) model and Δ, p_{12}, T_q for the regime switching model) are harder to pin down because ZLB episodes are rare. Consequently, we will consider combinations of parameter values that yield a spectrum of durations and unconditional frequencies of ZLB episodes. As a baseline, we will focus on parameter values that generate an unconditional frequency of the ZLB equal to 10% , which corresponds to the U.S. post-WWII experience (seven years at the ZLB over seventy years), although we relax this assumption later on. In the case of the regime switching model, we have an extra free parameter. As a baseline, we choose to set $\Delta = 0.09$ 0 to ensure that the risk premium shock almost always yields a binding ZLB. For robustness, we will also consider two additional values of Δ . One is based on setting $\Delta = \overline{R} - 1 = \frac{1}{\beta}\overline{\Pi}(1+\mu) - 1$. That is, the size of the premium is equal to the steady-state level of the nominal rate, which in turn depends on the time preference parameter β , the steady state level of inflation $\overline{\Pi}$, and the growth rate of output (and technology) in the economy μ . Given the calibration of other parameters, we have $\Delta \approx 6\%$ per year in this case. Note that because R_t may be larger than $\overline{R}$, the risk premium $\Delta = \overline{R} - 1$ may be not large enough to push interest rates all the way to zero. Even when they do, the duration of the ZLB episode may be very short-lived if some other positive shocks hit the economy. The alternative calibration is to set a much higher value of $\Delta = 0.12$. This value will ensure that ZLB episodes are almost always long-lived.

4.3 Results

For each calibration, we simulate the model for 10,000 periods to calculate welfare and various statistics such as the frequency and duration of ZLB episodes. Because Coibion et al. (2012) provide an exhaustive description of mechanisms and results for the conventionally calibrated model, we focus our analysis on the effects of alternative calibrations of risk premium shocks that govern the properties of ZLB.

4.3.1 Parameters of Risk Premium Shock and the Properties of ZLB Episodes

We first consider how different parameter values in each representation of the risk premium shock process affect the properties of ZLB episodes. Panel (a) of Figure 4.1 illustrates how

ρ_q and σ_q^2 in the AR(1) models affect the unconditional frequency of the economy being at ZLB (that is, the fraction of periods when $R_t = 1$). By raising either ρ_q and σ_q^2 , we increase the unconditional frequency of the ZLB. This is intuitive: more persistent shocks (higher ρ_q) naturally tend to leave the economy depressed longer and more volatile shocks (higher σ_q^2) imply that large enough shocks that push the economy into the ZLB happen relatively more frequently. At the same time, there is a clear trade-off between ρ_q and σ_q^2 : one can sustain a given level of the unconditional frequency of ZLB episodes by reducing σ_q^2 (i.e., making the risk premium shocks less volatile) and increasing ρ_q (i.e., making the shocks more persistent) or vice versa. Hence, one can in principle achieve a target frequency of ZLB episodes through different combinations of σ_q^2 and ρ_q . However, changing the parameter values of σ_q^2 and ρ_q in such a way that the unconditional frequency of ZLB episodes is unchanged still changes the nature of ZLB episodes. When ρ_q is relatively high for a given unconditional frequency of ZLB episodes (and σ_q^2 is therefore relatively low), ZLB episodes will tend to be rare but longer-lived, as suggested by the historical experience. Panel (a) of Figure 4.2 demonstrates this result: as ρ_q rises and we move along an isoquant for a given frequency of ZLB episodes (so σ_q^2 falls by the necessary amount), the average duration of ZLB episodes also rises. This suggests that, within the context of AR(1) risk-premium shocks, we can model the notion of rare but long-lived ZLB episodes by raising ρ_q and lowering σ_q^2 , thereby changing the distribution of ZLB episodes from being frequent and short-lived to being rare and long-lived.

However, Panel (a) of Figure 4.2 also reveals that the average duration of ZLB episodes is fairly insensitive to changes in ρ_q when these are offset by corresponding changes in σ_q^2 that leave the ZLB frequency unchanged. It takes very large changes in ρ_q to raise the duration of ZLB by a quarter. For example, if we focus on the unconditional probability of 0.1, one has to increase ρ_q from 0.97 to 0.985 (that is, increase the half-life of the risk premium shock from ≈ 23 quarters to ≈ 46 quarters) to raise the average ZLB duration by just one quarter[7].

To further explore why the average ZLB duration is relatively unresponsive to changes in ρ_q , we examine the distribution of ZLB durations in the AR(1) model for different calibrations of σ_q^2 and ρ_q in Figure 4.3. In each case, we choose σ_q^2 and ρ_q such that the unconditional frequency of ZLB episodes is 0.10 but the average duration of ZLB episodes varies from a little over two quarters to almost seven quarters in duration. A striking feature common to all calibrations is that the distribution of ZLB episodes has a very heavy left tail: most ZLB episodes are just one- or two-quarters long while the share of ZLB episodes longer than 12 quarters is less than 20% . Similar results have been found in other studies (e.g., Chung et al. (2012)) using an AR(1) process for shocks akin to our risk premium shock. This characteristic of the ZLB distribution is largely invariant to the average duration. As ρ_q increases, there are relatively more very long-lived episodes. But higher values of ρ_q also require lower values of σ_q^2 , so the share of 1-quarter ZLB episodes falls only gradually. These two nearly off-setting

[7] The half life is given by $\log(0.5)/\log(\rho_q)$.

effects explain the pattern noted in Panel (a) of Figure 4.2 that even large increases in ρ_q have very modest effects on average ZLB durations.[8] In short, it is very difficult to generate an empirically realistic pattern of ZLB episodes using AR(1) shocks to the risk premium.

As a result, we also consider an alternative modeling strategy of regime switching risk-premia, as described in section 4.2.4. There are now three parameters of interest: p_{12} (the probability of a risk-premium increase when the economy is outside the ZLB), T_q (the duration of the high risk premium period), and Δ (the size of the risk premium shock). In Panel (b) of Figure 4.1, we illustrate that, for a fixed value of $\Delta = 9\%$, by changing p_{12} and T_q we can maintain a given unconditional frequency of ZLB, which is qualitatively similar to the AR(1) case. Increasing p_{12} means raising the probability the risk premium going up when the economy is outside the ZLB, which is similar to raising σ_q^2 in the AR(1) case. Increasing T_q makes the length of the risk premium shock longer, which is akin to increasing ρ_q in the AR(1) case. Hence, raising either parameter serves to increase the frequency of ZLB episodes and there is a tradeoff between the two parameters that can be utilized to maintain a fixed unconditional frequency of ZLB episodes, as in the AR(1) case. In this respect, the two ways of modeling risk premia appear similar.

However, the regime switching model is much more successful at allowing us to change average durations of ZLB episodes. Panel (b) of Figure 4.2 plots, again for a fixed value of $\Delta = 9\%$, how the average duration of ZLB episodes changes as one increases T_q (the length of risk premium shocks) while changing p_{12} by just enough to maintain a fixed unconditional frequency of ZLB episodes (as indicated by isoquants in the Figure). In contrast to the very flat slopes obtained with the AR(1) model, the regime switching model yields an approximately linear increase with a slope just above one in the average duration of ZLB episodes.

The reason for this difference lies in the distribution of ZLB episodes generated by the regime switching model. Figure 4.4 plots these distributions for four different values of Δ : 6% , 9% , 12% , and 18% . In each case, T_q is held fixed at 12 quarters while p_{12} is chosen to generate an unconditional frequency of ZLB episodes of 0.10. When the size of the risk premium shock is low ($\Delta = 6\%$), the distribution of ZLB episodes is very similar to the AR(1) case. Even though the risk-premium shocks are long-lived, they are not large enough to keep the economy in the ZLB for extended periods because other shocks tend to quickly push the economy out. As a result, ZLB episodes end up being frequent and short-lived, as in the AR(1) case. But as the size of the risk-premium shock goes up, the distribution of ZLB durations shifts away from short durations and toward longer-lived episodes. In part, this increase in the duration of ZLB episodes is generated by eliminating short breaks in periods with low interest rates. For example, a risk premium shock lasting 8 quarters can push the

[8] In principle, it is possible to push close to one and make ZLB episodes potentially very long. In this case, however, we start to face numerical issues. Once we have very long periods with the Taylor principle being violated, the model generates indeterminacy and thus can break down.

nominal interest rate towards zero but an expansionary demand can interrupt the spell of low interest rates. As a result, the simulated path may have three periods at the ZLB, then one period outside the ZLB, and then another four periods at the ZLB even though these eight periods are effectively the same episode. A sufficiently high Δ ensures that such interruptions are minimized which raises the average duration of ZLB episodes. In contrast, the AR(1) model does not allow for a straightforward treatment of such breaks. Once Δ is large enough, we see almost no short-lived ZLB episodes because the size of the risk premium shock is too large to be offset by other economic shocks and the duration of ZLB episodes is generally close to, albeit somewhat less than, the duration of the risk premium shock. Hence, this alternative modeling strategy is much more successful at replicating the empirical pattern of ZLB episodes being both rare and long-lived.

It's also worth noting that as Δ becomes large, the distribution of ZLB episodes becomes increasingly tight around T_q , a feature which may seem unrealistic given that ZLB episodes have been varied in duration across countries and time. This reflects our assumption that T_q is deterministic and constant. One could readily assume a stochastic process for T_q , which would generate much more variation in the distribution of durations of ZLB episodes. Unfortunately, because of the lack of historical data on ZLB episodes, it is not clear a priori how one might best characterize this distribution. As a result, and because our baseline calibration of $\Delta = 9\%$ already seems to yield a reasonable distribution of ZLB episodes, we prefer to treat T_q as a constant.

4.3.2 ZLB Duration, Welfare, and Optimal Inflation

We now consider how changes in the duration of ZLB episodes affect welfare. To do so, we first illustrate how welfare changes with different levels of steady state inflation under different calibrations of the risk premium process. Parameters for the risk premium are chosen to achieve different average durations of ZLB episodes but a fixed unconditional frequency of the ZLB equal to 0.1 when the steady state level of inflation in the model is equal to 3.5% (the historical average for the U.S.). We then simulate the model for each set of parameter values under different levels of steady state inflation to quantify changes in welfare.

The results for the AR(1) assumption for risk premia are plotted in Panel (a) of Figure 4.5 for average ZLB durations ranging from a little over two quarters to almost seven quarters. When the average ZLB duration is very low (about two quarters), welfare losses are very high at all levels of inflation. This is because achieving short durations of ZLB episodes for this fixed frequency requires very volatile risk premium shocks, and this volatility generates a very high level of welfare losses. These losses decline as average durations rise to around five quarters because the latter requires much less volatile shocks to the risk premium.

As ZLB durations get much higher, the welfare losses experienced at low levels of steady state inflation become extremely high, the welfare curves start to shift down, and the peaks of the curves start to move to the right. The first and second observations suggest that the cost of ZLB episodes increases in the duration of ZLB episodes. As a result, it is optimal to trade off some steady-state inflation for a reduced incidence of the ZLB.

To confirm this intuition, we plot the cost of the ZLB per quarter for the same combination of parameters in Panel (a) of Figure 4.6. As the duration of ZLB episodes increases, the welfare cost per period of ZLB rises. Furthermore, the increase in the cost is non-linear and rapid. If steady-state inflation is zero, then for the combination of ρ_q and σ_q^2 with the implied average duration of ZLB episodes equal to approximately 7 quarters, the *permanent* consumption-equivalent cost of a quarter at ZLB is a whopping 13% . This cost, however, rapidly declines with the average duration of ZLB episodes. For example, with the same unconditional frequency of binding ZLB but an average duration equal 4 quarters, the cost is around 1.3% . These costs also decline sharply with higher levels of trend inflation, since the latter reduce the duration of ZLB episodes. For example, the same calibration that yields a 13% cost of a quarter at the ZLB when steady-state inflation is zero yields a much smaller ZLB cost per quarter of just over 2% when steady-state inflation is 3% .

With AR(1) shocks, a small increase in the duration of ZLB from around 5.5 quarters to almost 7 quarters is associated with an increase in the optimal steady-state level of inflation from 1.5% per year to around 2.5% per year. From a policy point of view, this is a dramatic difference in the inflation rate coming from a relatively small change in the average duration of ZLB episodes. This sensitivity of the optimal rate of inflation reflects the fact that that while the average duration of the ZLB may be rising only little, engineering this change with AR(1) shocks requires generating some dramatically longer-lived ZLB episodes in the tail of the distribution of ZLB durations to make up for the fact that most episodes remain very short-lived, as illustrated in Figure 4.3. Because long-lasting episodes are extremely costly in the model, even a very rare occurrence of such episodes translates into a non-trivial unconditional cost of the ZLB. These episodes are extremely costly because the Taylor principle is not satisfied for a long time and thus a large volatility of output, inflation and consumption is possible. Because the cost of the ZLB is convex in ZLB duration, the welfare loss essentially explodes with these very long-lived ZLB episodes. As a result, raising the steady state inflation rate becomes worthwhile to offset these otherwise extremely rare and costly events.

The very high sensitivity of the optimal inflation rate to the average duration of the ZLB therefore appears to be an artefact of the empirically unrealistic distribution of ZLB episodes generated by AR(1) shocks, making it an unreliable guide to policy. We therefore turn to the predictions of the regime switching approach, which can generate more empirically realistic distributions of ZLB episodes. First, the shapes of the welfare curves in the regime-switching model (Panel (b) of Figure 4.5) are qualitatively similar to those of the AR(1) model. When average ZLB durations are relatively high, the welfare losses of low trend inflation are particularly large. This again reflects the disproportionately high cost of ZLB episodes when average durations are higher, as illustrated in Panel (b) of Figure 4.6. Second, the optimal inflation rate is rising with the average duration of ZLB episodes (once these are sufficiently high) as higher levels of inflation work to reduce the incidence of these episodes that induce such high welfare costs.

However, there are also some important differences between the results for the AR(1) and regime-switching models. One is that the curvature in Panel (b) is weaker than that in Panel (a), especially at higher durations of ZLB episodes. Another is that the peaks of the curves in Panel (b) are closer to zero than in Panel (a), such that welfare is generally higher in the regime-switching model than in the AR(1) model. The latter reflects the fact that ZLB episodes are less costly in the regime-switching model than in the AR(1) model even when we use high values of Δ . Panel (b) of Figure 4.6 confirms this conjecture: the costs of the ZLB per quarter of hit are more compressed and flatter in the regime switching model than in the AR(1) model. For example, at $\overline{\pi} = 0$, an average duration of ZLB episodes of 7 quarters yields a welfare cost of 13% in consumption equivalent per quarter of binding ZLB under AR(1) shocks but only around 3.5% with regime switching in the risk premium. This much smaller cost suggests that raising steady-state inflation levels might be less effective at combating ZLB in the regime-switching model than in the AR(1) model. Indeed, in Panel (b) of Figure 4.5, we see that raising the average duration of ZLB episodes by a full year raises the optimal inflation rate by less than a percentage point, a significantly reduced sensitivity relative to the AR(1) case.

Since the cost of the ZLB is lower in the regime-switching model than in the AR(1) model, the implied optimal steady-state rate of inflation rate is also lower in the regime-switching model. For example, when average durations of ZLB episodes are around 5-5.5 quarters, the optimal inflation rate is 1.4% with regime switching risk premia but approximately 1.7% with AR(1) shocks. When average durations are higher, the differences are even more pronounced: the optimal inflation rate with AR(1) shocks is nearly 3% when ZLB episodes have an average duration of 6.8 quarters whereas it is only 1.8% with regime switching in risk premia.

In short, these results highlight the pitfalls associated with relying on AR(1) shocks to study how economies hit the ZLB. Because this approach necessarily implies the existence of many very short-lived ZLB episodes, generating longer *average* durations requires hitting the economy with extremely long-lived and disproportionately costly episodes that drive welfare and policy results. In contrast, the regime switching approach can deliver a more realistic distribution of ZLB episodes and this distribution implies a smaller sensitivity of the optimal inflation rate to the average duration of ZLB episodes.

4.3.3 Optimal Inflation Rates for Different Durations and Frequencies of the Zero Bound

In Figure 4.5, we provided some results on optimal inflation rates for a few average durations of ZLB episodes and a single unconditional frequency of ZLB episodes. But as discussed earlier, the paucity of historical experience with this type of episode should make anyone wary of taking a strong stand on the precise values of these parameters. As a result, we now consider a much wider range of both frequencies and durations of ZLB episodes and characterize optimal inflation rates in each case.

Our results for AR(1) shocks are presented in Panel (a) of Figure 4.7 while analogous results for regime switching model are in Panel (b) of Figure 4.7. In each case, we plot optimal inflation rates (vertical axis) associated with different average durations of ZLB episodes (horizontal axis) and unconditional frequencies of the ZLB (captured by isoquants), where the latter two are measured at a 3.5% steady state inflation rate. The key result in the case of AR(1) shocks is, regardless of the specific frequency of the ZLB, the optimal inflation rate rises extremely rapidly with the average duration, as indicated by the slope of the isoquants. For example, going from an average duration of the ZLB of five quarters at an unconditional frequency of ZLB episodes equal to 7% to an average duration of eight quarters raises the optimal inflation rate from about 2% to almost 4.5% . But as discussed earlier, this excessive sensitivity reflects the unrealistic distribution of ZLB episodes generated by AR(1) shocks to risk premia.

A second unappealing feature of the AR(1) approach to modeling shocks is the fact that low frequency isoquants are to the left of higher frequency isoclines. This implies that for a given average duration of ZLB episodes, a higher frequency of the ZLB is associated with a *lower* optimal rate of inflation. The reason is that we cannot separately calibrate the volatility of the risk premium and the frequency and duration of ZLB episodes with only two parameters for the shock process, which is yet another undesirable property of AR(1) shocks.

Panel (b) of Figure 4.7 presents the analogous results from the regime switching approach to modeling shocks that push the economy into the zero bound on interest rates. The first difference to note is that, as expected, the slopes of the isoquants are now much flatter: optimal inflation rates rise less rapidly with average ZLB durations. This reflects the fact that one does not need to introduce extremely long-lived ZLB periods to change the average duration as is the case with AR(1) shocks. Nonetheless, high inflation rates can be sustained as optimal if one believes that average durations of ZLB episodes are sufficiently high or sufficiently frequent.

A second difference to note is that the regime switching approach now implies that, holding the average duration constant, higher frequencies of the ZLB would be associated with higher optimal rates of inflation. This result holds even at lower levels of Δ , as illustrated in Appendix Figure B.1. Hence, the alternative modeling strategy of regime switching shocks can fix this additional undesirable property of AR(1) shocks when it comes to characterizing the tradeoffs faced by policymakers.

More broadly, the results in Panel (b) of Figure 4.7 suggest that a wide range of optimal inflation rates can potentially be defended, depending on what one perceives to be the correct values for the average duration and frequency of ZLB episodes are. For example, relying only on the U.S. post-war experience of a single ZLB episode lasting seven years over a seventy year period points to an unconditional frequency of 10% and an average duration of twenty-eight quarters. This duration would justify an optimal inflation rate of approximately five percent, well above the Federal Reserve's current objective of two percent but in line with recommendations made by economists like Olivier Blanchard and Paul Krugman. Of course, other countries such as Canada experienced much shorter ZLB episodes during the Great Recession and other advanced economies such as Australia and New Zealand did not reach the ZLB at all. This suggests that the U.S. experience is likely not an average experience.

To get a better sense of the cross-country experience with the zero bound on interest rates since World War II, we summarize the experience of a range of advanced economies since 1950 in Table 4.2. Only Switzerland has experienced one-quarter long ZLB experiences, in 1972Q1 and again in 1972Q3. All other durations have been of at least one year. There are several episodes of approximately one year in length, although some of these are what one might consider interrupted sequences of longer underlying periods of economic weakness, such as the Euro-zone and Sweden from 2009Q3-2010Q3. For countries still at the ZLB, we assume that they will remain at the ZLB until 2015Q4 and measure durations and frequencies using this final date.

Figure 4.8 plots the distribution of the duration of ZLB episodes from Table 4.2 as well as a kernel estimate of the density. Despite the small number of observations, we can see that this distribution resembles, at least loosely, the distribution of ZLB episodes of the regime switching model with $\Delta = 9\%$. There is a small but non-zero share of very short ZLB episodes, but the vast majority of ZLB episodes are longer-lived, with significant dispersion in terms of actual durations of ZLB episodes. We interpret this as suggesting that our calibration of the model is indeed well suited to characterize the empirical distribution of ZLB episodes and also that the U.S. experience with the ZLB is in the upper ranges, at least in terms of durations. The mean duration across all episodes, for comparison, is fourteen quarters, less than half what the U.S. will have spent at the ZLB since the start of the Great Recession. If Japan is excluded from the sample, or if we include all of the Euro-zone countries as separate observations, the mean durations are lower still, at 11.5 and 12.3 quarters respectively. Hence, we interpret a reasonable estimate for average durations at the ZLB as somewhere between ten and fifteen quarters.

For each country in this sample, we also estimate the frequency of the ZLB as the share of quarters spent at the ZLB between 1950Q1 and 2015Q4. These frequencies range from a high of 23% for Japan to a low of 0% for Norway, Australia, and New Zealand, the three countries who experienced no ZLB periods at all over this time period. The U.S. frequency, at 11% , is again well above the average frequency across all countries in the sample, which is given by 7.5% . Excluding Japan lowers this frequency further to just under 6% , although it can go as high as 11% if we exclude those countries which have never experienced a ZLB episode from our estimate. Again, the U.S. is on the high end of estimates, and the cross-country experience suggests a plausible range of values for the frequency of ZLB estimates ranging anywhere from 6% to 11% .[9]

With these of ranges of values for the frequency and duration of ZLB episodes in mind, we can then reexamine optimal inflation rates in Panel (b) of Figure 4.7. If we consider the very low end of our estimates for both frequency and duration of ZLB episodes (a duration of around 10 quarters and a frequency of 6%), then our model points toward an optimal level of the annual inflation rate of 1.5% . The high end of our estimates for both frequency and duration of ZLB episodes (a duration of around 15 quarters and a frequency of 11%) points instead toward an optimal rate of inflation of 3% . Hence, these results suggest that reasonable bounds for the optimal rate of inflation are 1.5-3% .

The results vary little if we consider alternative calibrations of our regime switching model. In Appendix Figure B.1, we plot equivalent results as those in Figure 4.7 using $\Delta = 6\%$ and $\Delta = 12\%$. With the former, ZLB episodes are less costly, so optimal inflation rates are somewhat lower. But the equivalent range is only slightly changed, to 1.5-2.8% . With a higher value of Δ , ZLB episodes become more costly and optimal inflation rates rise. In this case, the range of optimal inflation rates becomes 1.6-4.0% . But as illustrated in Figure 4.4, neither calibration is as successful in replicating the historical distribution of ZLB durations as our baseline calibration.

4.4 Conclusion

Economies rarely hit the zero bound on interest rates, but when they do, these episodes tend to be long-lived. This simple empirical pattern is one that is not replicated by traditional models of optimal inflation that incorporate the zero bound on interest rates. We show that the most common approach to modeling shocks that drive the economy into the zero bound yields a distribution of ZLB episodes that is counterfactual: ZLB episodes are frequent and short-lived rather than rare and long-lived. And this counterfactual distribution is not

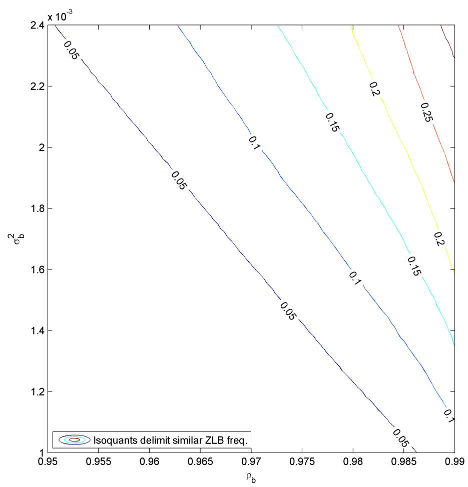
[9]Dropping the high-inflation period (e.g. 1968-1982), when reaching the ZLB was less likely given higher nominal interest rates, reduces the length of the sample and therefore raises measured frequencies somewhat. Monetary policy prior to this period appears to have been made similarly to that of post-Volcker era (Romer and Romer (2002)), so there is little reason to disregard the earlier time period.

innocuous. The fact that AR(1) shocks have to generate a large share of very short-lived ZLB episodes implies that longer average durations of ZLB episodes can only occur if there are also extremely long-lived episodes that generate disproportionately large welfare costs. The latter make policymakers very willing to raise inflation rates to avoid the ZLB, making the optimal inflation rate exceedingly sensitive to the average duration rate of ZLB episodes.

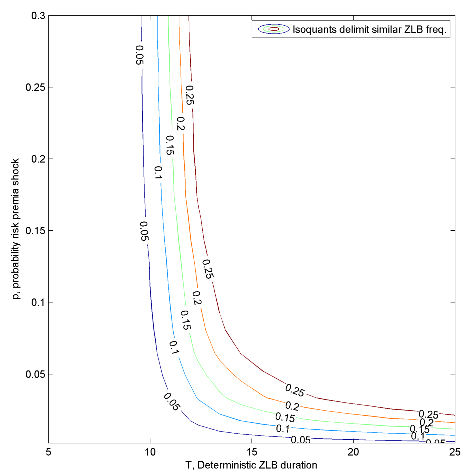
In contrast, we show that a regime switching approach to modeling the shocks that push the economy into the ZLB can generate an empirically realistic distribution of ZLB episodes and that this approach does not generate the same sensitivity of the optimal inflation rate to the average duration of ZLB episodes. The optimal rate of inflation is still sensitive to the ZLB since the latter is costly, and more frequent or long-lived episodes increase the incentives of policymakers to raise the target rate of inflation, but this incentive is much reduced relative to what is implied by the standard approach to modeling shocks.

The specific optimal rate of inflation implied by the model remains very sensitive to one's beliefs about the frequency and duration of ZLB episodes, values for which history provides only limited guidance. Using only the U.S. post-WWII experience, for example, our model would imply an optimal rate of inflation above 4% . From a broader cross-section of countries' historical experiences with the ZLB, one could just as readily conclude that the optimal rate of inflation is 2% , with a plausible range of values running from 1.5% to at least 3% . Given the uncertainty associated with measuring historical frequencies and durations associated with ZLB episodes, the range of plausible outcomes for optimal inflation rates implies that profound humility is called for by anyone advocating a specific inflation target.

Figures

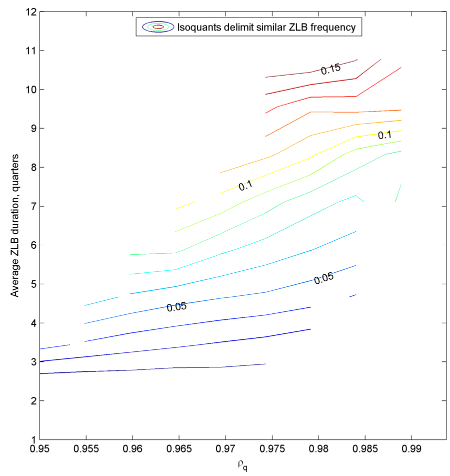


(a) AR(1) model

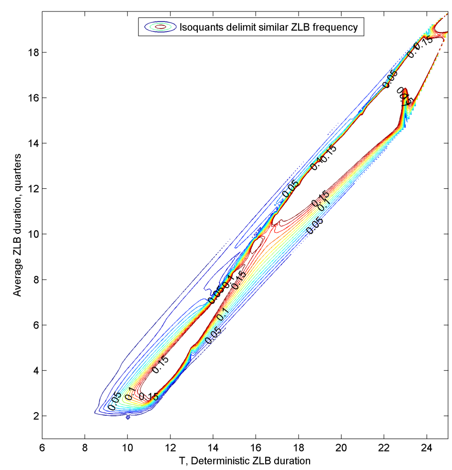


(b) Regime-switching model

Figure 4.1: Unconditional frequency of ZLB episodes. Each figure plots combinations of parameters that yield specific frequencies of ZLB episodes in simulated data, as indicated by isoquants. In the left panel, the two parameters are the persistence of the AR(1) process for the risk premium (x-axis) and its volatility (y-axis). In the right panel, the two parameters are the duration of the risk premium shock (x-axis) and the probability that a risk premium shock will occur in periods when the economy is not in the ZLB (y-axis). The size of the risk premium shock in Panel (b) is 9% . See section 4.3.1 for details.



(a) AR(1) model



(b) Regime-switching model

Figure 4.2: Duration of ZLB episodes. Each figure plots how average durations of ZLB episodes change as persistence of risk premium shocks varies holding constant the unconditional frequency of ZLB episodes. In panel (a), shocks are AR(1), so persistence depends on ρ_q , with σ_q^2 being changed in offsetting way to hold frequency of ZLB constant along isoquants. In panel (b), shocks follow regime-switching, so T_q determines the duration of the risk premium shock while p_{12} is changed in offsetting manner to hold frequency of ZLB constant along isoquants. The size of the risk premium shock in Panel (b) is 9% . See section 4.3.1 for details.

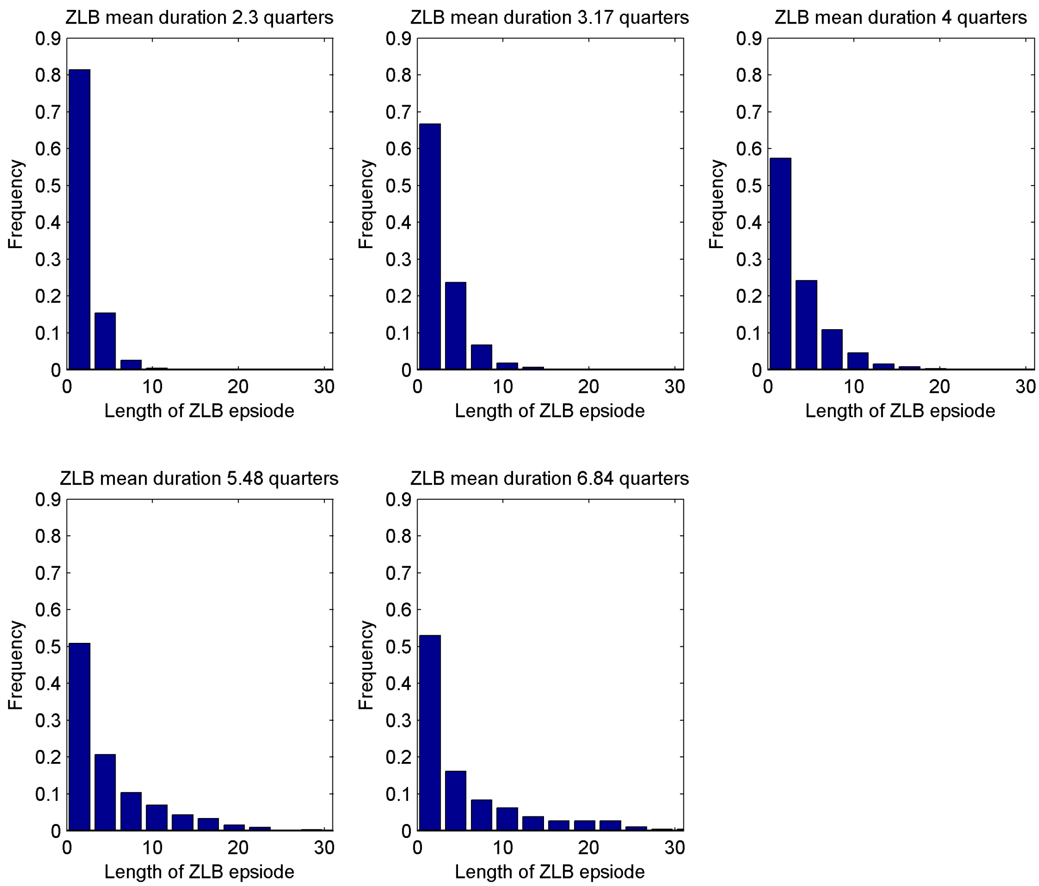


Figure 4.3: Duration of ZLB Episodes with AR(1) Shocks.The figure plots the distribution of durations of ZLB episodes from simulating the model with AR(1) shocks for different average durations of ZLB episodes but a constant unconditional frequency of ZLB episodes. This is done by changing the persistence and volatility of the shock process. See section 4.3.1 for details.

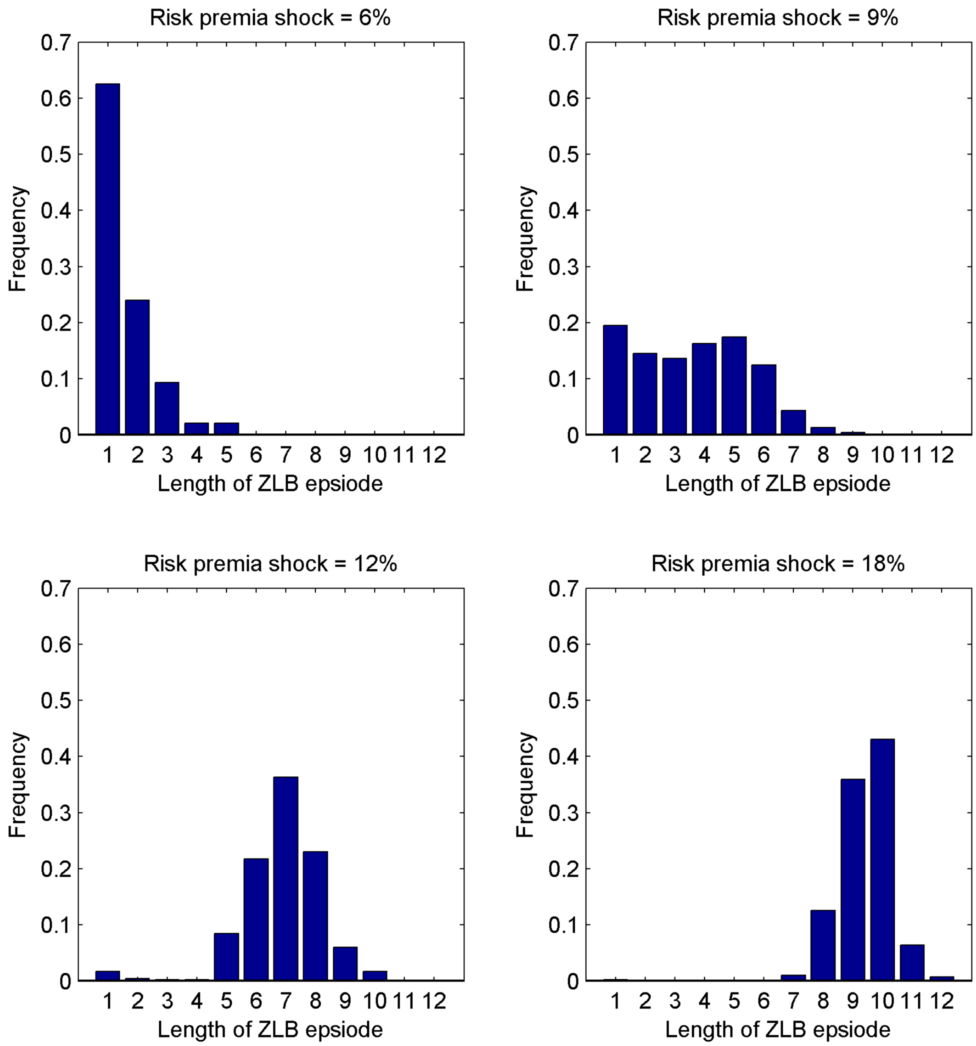
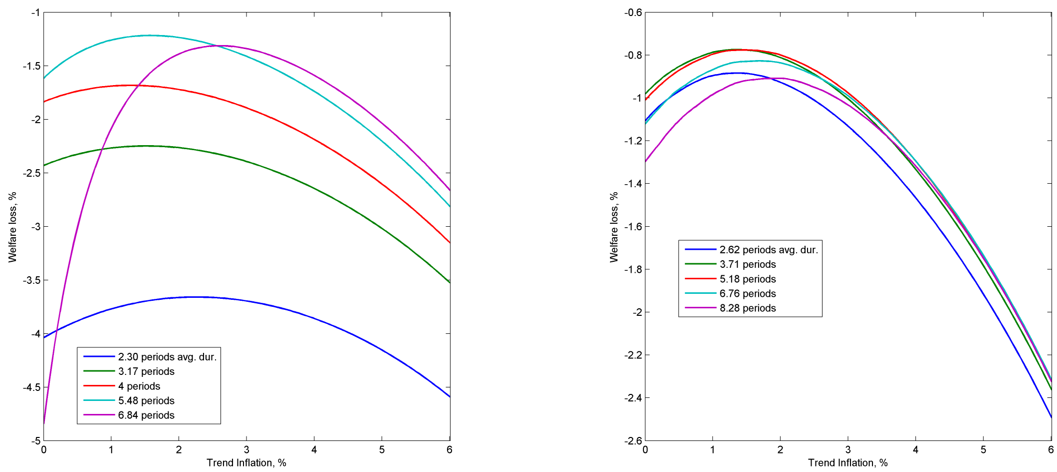


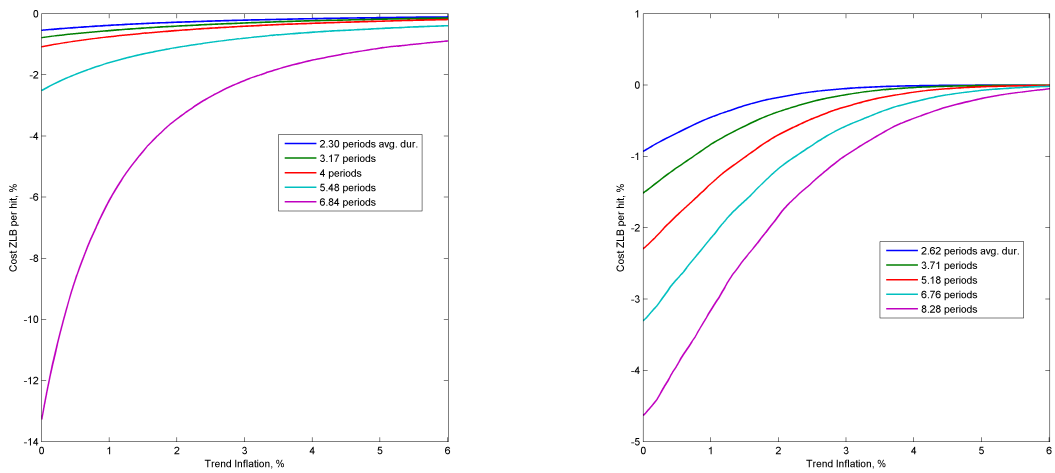
Figure 4.4: Duration of ZLB Episodes with Regime-Switching Shocks. The figure plots the distribution of durations of ZLB episodes from simulating the model with a regime switching risk premium shock for different average durations of ZLB episodes but a constant unconditional frequency of ZLB episodes. The duration of the risk premium shock (T_q) is held constant but the size of the risk premium shock (Δ) and the probability of a high risk premium occurring outside the ZLB (p_{12}) are allowed to vary to achieve the changing average duration of ZLB episodes with fixed unconditional frequency. See section 4.3.1 for details.



(a) AR(1) model

(b) Regime-switching model

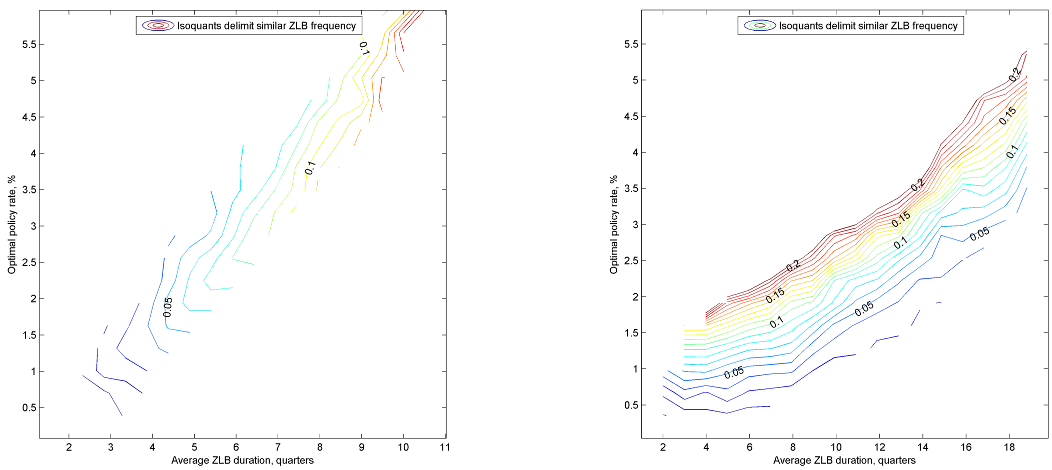
Figure 4.5: Welfare Losses at Fixed Frequency of ZLB. The figures plots welfare losses (y-axis) associated with different levels of annual trend inflation (x-axis) for different calibrations of the average duration of ZLB episodes (different colored lines) holding the unconditional frequency of ZLB episodes fixed at 0.10 for a trend inflation level of 3.5% . Panel (a) is done for AR(1) shocks, in which case ρ_q and σ_q^2 are varying to change the ZLB durations. Panel (b) is done for regime switching risk premia, with Δ =9% while T_q and p_{12} are varied to change the average durations of ZLB episodes. See section 4.3.2 for details.



(a) AR(1) model

(b) Regime-switching model

Figure 4.6: Cost of ZLB per Hit. The figures plots the cost of each ZLB period (y-axis) associated with different levels of annual trend inflation (x-axis) for different calibrations of the average duration of ZLB episodes (different colored lines) holding the unconditional frequency of ZLB episodes fixed at 0.10 for a trend inflation level of 3.5% . Panel (a) is done for AR(1) shocks, in which case ρ_q and σ_q^2 are varying to change the ZLB durations. Panel (b) is done for regime switching risk premia, with Δ =9% while T_q and p_{12} are varied to change the average durations of ZLB episodes. See section 4.3.2 for details.



(a) AR(1) model

(b) Regime-switching model

Figure 4.7: Optimal Inflation for Different Frequencies and Durations of ZLB Episodes. The figures plots the optimal annualized inflation rate (y-axis) associated with different levels of average ZLB durations (x-axis) and unconditional frequencies of the ZLB (indicated by isoquants). Panel (a) is done for AR(1) shocks, in which case ρ_q and σ_q^2 are varying to change the ZLB durations. Panel (b) is done for regime switching risk premia, with Δ =9% while T_q and p_{12} are varied to change the average durations of ZLB episodes. See section 4.3.2 for details.

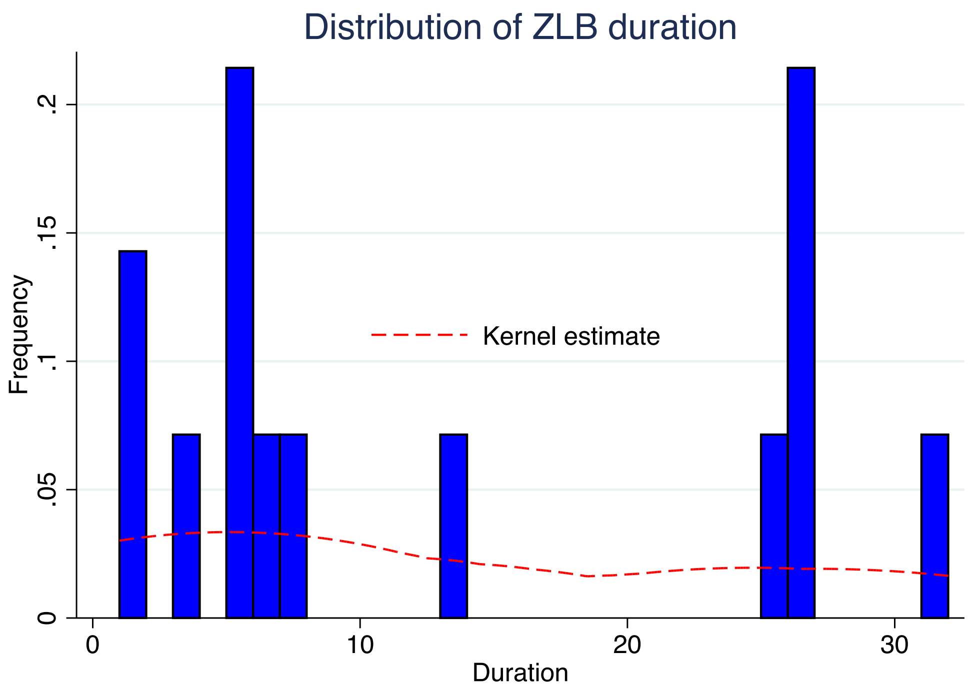


Figure 4.8: Distribution of Historical ZLB Durations. The figure plots the distribution of durations of historical ZLB episodes from Table 4.2. Durations are in quarters.

Tables

Parameters of Utility Function		*Steady-State Values*	
η: Frisch Labor Elasticity	1.00	μ: Growth Rate of RGDP/cap	1.5% p.a.
β: Discount factor	0.998	$\overline{c_y}$: Consumption Share of GDP	0.80
h: Habit in consumption	0.7	$\overline{g_y}$: Government Share of GDP	0.20
Pricing Parameters		*Shock Persistence*	
θ: Elasticity of substitution	7	ρ_g: Government Spending Shocks	0.97
λ: Degree of Price Stickiness	0.55	ρ_m: Cost-Push Shocks	0.90
ω: Price indexation	0.00		
Taylor Rule Parameters		*Shock Volatility*	
ϕ_π: Long run response to inflation	2.50	σ_g: Government Spending Shocks	0.0052
ϕ_{gy}: Long run response to output growth	1.50	σ_m: Cost-Push Shocks	0.0014
ϕ_x: Long run response to output gap	0.11	σ_a: Technology Shocks	0.0090
ρ_1: Interest smoothing	1.05	σ_r: Monetary Policy Shocks	0.0024
ρ_2: Interest smoothing	-0.13		

Table 4.1: Baseline Parameter Values. The table presents the baseline parameter values assigned to the model in section 4.2.1 and used for solving for the optimal inflation rate in section 4.2.2. "p.a." means per annum.

	Start of ZLB Episode	End of ZLB Episode	Duration (quarters)	Duration (years)	Unconditional Frequency of ZLB for Country
Australia	N/A	N/A	N/A	N/A	0.00
Canada	2009Q2	2010Q2	5	1.25	0.02
Germany/ECB	2009Q3	2010Q4	6	1.50	0.08
Germany/ECB	2012Q1	2015Q4∗	16	4	0.08
Japan	1998Q4	2006Q3	32	8	0.23
Japan	2008Q4	2015Q4∗	29	7.25	0.23
New Zealand	N/A	N/A	N/A	N/A	0.00
Norway	N/A	N/A	N/A	N/A	0.00
Sweden	2009Q3	2010Q3	5	1.25	0.04
Sweden	2014Q3	2015Q4∗	6	1.50	0.04
Switzerland	1972Q1	1972Q1	1	0.25	0.16
Switzerland	1972Q3	1972Q3	1	0.25	0.16
Switzerland	1978Q1	1979Q1	5	1.25	0.16
Switzerland	2003Q1	2004Q3	7	1.75	0.16
Switzerland	2008Q4	2015Q4∗	29	7.25	0.16
United States	2008Q4	2015Q4∗	29	7.25	0.11
United Kingdom	2009Q1	2015Q4∗	28	7	0.11
Average:			14.2	3.6	0.075
Average with all Euro countries:			12.3	3.1	0.085
Average w/o Japan:			11.5	2.9	0.058
Average w/o Norway, Australia and New Zealand:			14.2	3.6	0.108
Average without the 1968-1984 period			17.5	4.4	0.098

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Table 4.2: Post-War Experiences with the ZLB. The table presents a summary of ZLB episodes for advanced economies since World War II. We use Germany as representative of Euro-zone economies. For countries currently at the ZLB and expected to remain so through the end of 2015, we list the end of the episode as 2015Q4. Frequencies are measured using data starting in 1950Q1 going until 2015Q4. "Average with all Euro countries" incorporates eleven additional countries with the same ZLB experience as Germany.

Chapter 5

Dissertation Conclusion

This dissertation studies how complex financial networks interact with the business cycles and studies optimal monetary policy under different scenarios and policy tools. Chapter 2 provides the theoretical foundations for a new model of the interbank market. The complex network of bilateral financial claims that arises from participation on the interbank market boosts aggregate credit supply through an improved allocation of funds across the economy, but also affects the volatility of the business cycle by linking the network of banks to individual bank shocks. The structure of the resulting interbank network can dampen or amplify the volatility of financial shocks, with an interesting trade-off between efficiency and volatility arising in the later case. Chapter 2 provides an analytical approximation to the welfare gains and costs of financial integration, and calibrates the model to the German economy using proprietary micro-data on the universe of German monetary financial institutions. I find that the existence of the German interbank market improves the allocation of credit and reduces the volatility of economy, raising welfare by an equivalent of 0.88% consumption-quarter with respect to the counterfactual without market. I find that a monetary policy that responds to deviations of the credit spread is more effective at stabilizing the economy, and lender-of-last resort interventions improve welfare as well by limiting the fluctuations of the credit spread of distressed banks.

Chapter 3 applies the interbank model developed on the previous chapter to the study of the spatial propagation of historical U.S. banking panics. I theoretically show how the pyramidal structure of bank reserves present during the period considered (1868-1930) enables the propagation of panics across different regions, and I develop a theory-consistent empirical estimation of dynamic spatial propagation. I find that panics feature a robust spatial propagation across the United States, and a moderate but temporary negative effect on deposits, lending and related banking variables.

Chapter 4 studies the economic consequences of hitting the zero lower-bound (ZLB) on nominal interest rates and optimal monetary policy in that context. I show that the theoretical welfare costs of the ZLB are heavily influenced by the stochastic shock process that brings rates to the ZLB. I find that a regime-switching risk premia properly accounts for the infrequent but long ZLB spells that we observe in the data and substantially raises the costs of the ZLB vis-à-vis the autoregressive shocks commonly employed in the literature. Optimal policy by the central bank chooses the optimal trend inflation target that balances the marginal costs of price dispersion with the gains from more infrequent ZLB episodes. Calibration of the model to the U.S. economy implies an optimal inflation target around 3%, which is in the neighborhood of commonly accepted ranges for developed economies.

Chapter A

Appendices to Chapter 2

A.1 Detailed model derivation

Representative Household

The Representative Household maximizes the following objective function

$$\max \quad E_t \sum_{j=0}^{\infty} \beta^j \left[\log \left(X_{t+j} \right) - \left(\frac{\eta}{\eta+1} \right) \int_0^1 N_{t+j,\tau}^{1+1/\eta} \, \mathrm{d}\tau \right]$$

where $N = \left[\int_0^1 N(\nu)^{1+1/\eta} \, \mathrm{d}u \right]^{\frac{\eta}{\eta+1}}$ is the aggregate labor index and $N(\nu)$ labor supplied to intermediate industry u, η is the Frisch labor supply elasticity and X is a composite of consumption and bank deposit balances defined as

$$X_t = C_t + \sum_{n=1}^{N} \int_0^1 \left(1 - T_t^n \cdot z_{t,\tau}^n \right) \frac{D_{t,\tau}^n}{P_t} \, \mathrm{d}\tau \tag{A.1}$$

with $C = \int_0^1 C_\tau \, \mathrm{d}\tau$ representing aggregate consumption in t, D^n one-period deposits at bank n and z^n an utility shifter that affects the utility from deposits alongside the continuum. It is distributed Weibull

$$z_{t,\tau}^n \sim W \left(\frac{1}{\Gamma \left(1 + 1/\kappa \right)}, \kappa \right)$$

where $(1 - T^n)$ is the average utility of deposits at bank n, κ is the parameter controlling the variance of the shock and $\Gamma(\cdot)$ is the Gamma function. Shocks to z^n will generate a reallocation of deposits across banks that in turn will give rise to the market for interbank loans, as we will show later.

The period (t,τ) budget constraint is

$$\xi_{t,\tau}: \qquad C_{t,\tau} + \frac{\sum_{n=1}^{N} D^n_{t,\tau}}{P_t} + \frac{B_{t,\tau}}{P_t} = \frac{\sum_{n=1}^{N} (1+\varsigma_D)\cdot R^{D,n}_{t-1,\tau} D^n_{t-1,\tau}}{P_t} + \frac{R^B_{t-1} B_{t-1,\tau}}{P_t} + \int_0^1 \frac{W_t(\nu) N_{t,\tau}(\nu)}{P_t}\,\mathrm{d}\nu + \frac{\Upsilon_{t,\tau}}{P_t}, \tag{A.2}$$

where B and R^B are one-period government bonds and the rate paid on them, R^D_n is the rate paid on deposits by bank n, $W(\nu)$ is the wage paid by industry u, P is the aggregate price index, Υ are firm and bank profits lump-sum transferred to the agent, and ξ is the Lagrange multiplier associated to the constraint.

Maximizing the Representative Agent problem we obtain the following First Order

$$C_{t,\tau}: \qquad \xi_{t,\tau} = X_t^{-1} \tag{A.3}$$

$$N_{t,\tau}(\nu): \qquad N_t(\nu)^{1/\eta} = \xi_t \frac{W_t(\nu)}{P_t}, \qquad \forall u \tag{A.4}$$

$$B_t: \qquad 1/R^B_t = \beta E_t\left[\frac{\xi_{t+1}}{\xi_t \Pi_{t+1}}\right] \tag{A.5}$$

$$D^n_{t,\tau}: \qquad 1/R^{D,n}_{t,\tau} = (1+\varsigma_D)\cdot\beta E_t\left[\frac{\xi_{t+1}}{T^n_t z^n_{t,\tau}\xi_t\Pi_{t+1}}\right], \qquad \forall n \tag{A.6}$$

Combining equations (A.5) and (A.6) we obtain

$$R^{D,n}_{t,\tau} = (1+\varsigma_D)^{-1}\cdot T^n_t \cdot z^n_{t,\tau}\cdot R^B_t \tag{A.7}$$

Firms

There is a continuum $\nu \in [0,1]$ of intermediate goods, each respectively produced by a monopolist with the following production function employing capital and labor

$$Y_{t,\tau}(\nu) = \left(\frac{K_t(\nu)}{\alpha}\right)^{\alpha}\left(\frac{exp(u^A_t)\cdot N_{t,\tau}(\nu)}{1-\alpha}\right)^{1-\alpha}, \tag{A.8}$$

where u^A_t is an aggregate technology process defined as $u^A_t = \rho_A\, u^A_{t-1} + \varepsilon^A_t$, $\varepsilon^A_t \sim N(0,\sigma^2_A)$. We impose the restriction that firms must employ a constant level of capital across the period, $K_{t,\tau}(\nu) = K_t(\nu)$, $\forall\tau$, so all production adjustments happen through the labor margin.

Aggregate capital is a CES composite of N distinct types of capital

$$K_t(\nu) = \left[\sum_{n=1}^{N} (a_t^n)^{1/\sigma} \, K_t^n(\nu)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} , \qquad \text{(A.9)}$$

Parameter $\sigma > 1$ is the elasticity of substitution between different types of capital and a_t^n is a time-varying demand shock for each type. Without loss of generality to the qualitative results that we will show later, we simplify the model by assuming full depreciation and instant build-up of capital from investment

$$K_{t,\tau}^n(\nu) = \frac{I_{t,\tau}^n(\nu)}{P_t} \ , \quad \forall \, n \ ,$$

where $I_t^n(\nu)$ stands for investment in capital of type n. Firms finance their investment at each moment τ with credit obtained from banks. There are N distinct banks in the model and each one specializes in providing loans $L_{t,\tau}^n(\nu)$ for a different type of capital. Loans are repaid after one quarter at gross interest rate $R_t^{F,n}$, and firms are subject to the following investment constraints

$$I_{t,\tau}^n(\nu) \le L_{t,\tau}^n(\nu) \ , \quad \forall \, n \ ,$$

which hold with equality in equilibrium. We do not consider capital financed by the firm itself, but such distinction would not affect the qualitative results of our model.

A representative, perfectly competitive firm aggregates intermediate products into a final good according to

$$Y_t = \left[\int_0^1 Y_t(\nu)^{\left(\frac{\epsilon-1}{\epsilon}\right)} \, \mathrm{d}\nu \right]^{\left(\frac{\epsilon}{\epsilon-1}\right)} ,$$

where $\epsilon > 1$ is the elasticity of substitution between varieties. Individual demand for intermediates is given by

$$Y_t(\nu) = \left(\frac{P_t(\nu)}{P_t} \right)^{-\epsilon} Y_t \ ,$$

where $P(\nu)$ is the price of intermediate ν. The aggregate price index is

$$P_t = \left[\int_0^1 P_t(\nu)^{1-\epsilon} \, \mathrm{d}\nu \right]^{\frac{1}{1-\epsilon}} .$$

Intermediate producers have sticky prices *á la* Calvo (1983) and they reset their price at the beginning of the quarter with probability $1-\theta$. All firms reset to the same optimal price (in equilibrium) within a given period, which we denote by $P^{\star}$. This allows us to recursively express the previous expression as

$$P_t^{1-\epsilon} = (1-\theta)\cdot(P_t^{*})^{1-\epsilon} + \theta\cdot(P_{t-1})^{1-\epsilon} \ . \tag{A.10}$$

Intermediate firm ν maximizes the following discounted stream of present and future profits

$$\sum_{j=0}^{\infty} E_t\left[Q_{t,t+j}\int_0^1 (1+\varsigma_F)P_{t+j}(\nu)Y_{t+j,\tau}(\nu) - W_{t+j}(\nu)N_{t+j,\tau}(\nu) - \sum_{n=1}^{N} R_{t+j-1}^{F,n}L_{t+j-1}^{n}(\nu)\,\mathrm{d}\tau\right]$$

where $Q_{t,t+j} = \beta^j \dfrac{\xi_{t+j}}{\xi_t \Pi_{t+j}}$ is the firm's stochastic discount factor between periods t and $t+j$, and ς_F is a government production subsidy. Minimizing firm ν production costs with respect to labor and loans we obtain the following demand for inputs

$$N_t(\nu) = (1-\alpha)\cdot\frac{Y_t(\nu)}{A_t}\left(\frac{\widetilde{R}_t^F}{W_t(\nu)/P_tA_t}\right)^{\alpha} \tag{A.11}$$

$$\frac{L_t(\nu)}{P_tA_t} = \alpha\cdot\frac{Y_t(\nu)}{A_t}\left(\frac{\widetilde{R}_t^F}{W_t(\nu)/P_tA_t}\right)^{-(1-\alpha)} \tag{A.12}$$

$$L_t^n(\nu) = a_t^n\left(\frac{R_t^{F,n}}{R_t^F}\right)^{-\sigma} L_t(\nu) \tag{A.13}$$

where $\widetilde{R}_t^{F,n} = E_t\left[Q_{t,t+1}\right]\cdot R_t^{F,n}$ is the expected discounted gross rate on a loan from bank n at period t. The aggregate gross rate index R_t^F is defined as

$$R_t^F = \left[\sum_{n=1}^{N} a_t^n \left(R_t^{F,n}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{A.14}$$

Solving for the optimal resetting price at period t we obtain

$$\frac{P_t^*}{P_t} = \frac{E_t\left[\sum_{j=0}^{\infty}\theta^j Q_{t,t+j}\left(\frac{P_{t+j}}{P_t}\right)^{\epsilon+1} Y_{t+j}\left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon-1}\right)\left(\frac{MC_{t+j|t}(\nu)}{P_{t+j}}\right)\right]}{E_t\left[\sum_{j=0}^{\infty}\theta^j Q_{t,t+j}\left(\frac{P_{t+j}}{P_t}\right)^{\epsilon} Y_{t+j}\right]} \tag{A.15}$$

where subindex $t+j|t$ represents the value of the variable conditional on the firm having reset its price last time at period t, and $MC_{t+j|t}(\nu)/P_t$ is the real marginal cost of production, defined as

$$\frac{MC_{t+j|t}(\nu)}{P_t} = \left(\widetilde{R}^F_{t+j}\right)^{\alpha} \left(\frac{W_{t+j|t}(\nu)}{P_{t+j}A_{t+j}}\right)^{1-\alpha}$$

where $\widetilde{R}^F_t = E_t\left[Q_{t,t+1}\right] \cdot R^F_t$. Aggregate firm profits over the period are

$$\Upsilon^F_t = (1+\varsigma_F)P_tY_t - \int_0^1 W_t(\nu)N_t(\nu)\,\mathrm{d}\nu - \sum_{n=1}^{N} R^{F,n}_{t-1}L^n_{t-1}$$

Banks

Each bank performs three different activities: Obtain deposits from the Representative Household, provide credit to firms and lend funds to each other in the interbank market. For expositional purposes we will assume that each bank is divided in two Divisions, each one responsible for performing a different set of these activities. Loan Divisions provide credit to firms and secure the necessary funding through internal funds or interbank loans. Deposit Divisions procure deposits from the Representative Household and distribute them to the Loan Divisions through the interbank market or internal funds transfer.

Loan Division

Loans granted by bank n to firms at each point τ of the continuum are subject to the following constraints:

$$L^n_{t,\tau} \leq M^n_{t,\tau}\ , \tag{A.16a}$$

$$M^n_{t,\tau} \geq 0\ , \tag{A.16b}$$

where $M^n_{t,\tau}$ is the amount of internal and/or interbank funding available to the Loan Division at time τ. Equation (A.16a) captures the constraint that banks can only provide credit up to the amount that they have readily available to be lent, and holds with equality in equilibrium. Interbank loans taken in (t,τ) are repaid next quarter. Bank funds are perfect substitutes, and Loan Divisions obtain them from the bank that offers the lowest rate at each moment τ. Formally,

$$M^n_{t,\tau} = M^{in}_{t,\tau}, \quad i_{t,\tau}(n) = \arg_j \min\left\{R^{I,jn}_{t,\tau}\right\}\ ,$$

where M^{in} are the interbank funds lent by Deposit Division i to Loan Division n and $R^{I,in}_{t,\tau}$ is the interest rate on interbank loans that i charges to n. Expected profits of the Division are

$$\int_0^1 (1+\varsigma_B) R^{F,n}_t L^n_{t,\tau} - R^{I,n}_{t,\tau} M^n_{t,\tau} \, \mathrm{d}\tau$$

$$\text{where:} \qquad R^{I,n}_{t,\tau} = \min_i \left\{ R^{I,in}_{t,\tau} \right\} \tag{A.17}$$

where ς_B is a government subsidy to banks and variable $R^{I,n}_{t,\tau}$ is the rate at which interbank loans (or own funds) at point τ are obtained by bank n. Banks know their individual firm loan demands given by equation (A.13) and act as monopolistic competitors, taking the aggregate gross rate index R^F as given. We assume that rates on firm loans are sticky *within* the continuum, and they can only be reset at the beginning of each period. On the other hand, interbank rates are fully flexible and will reflect the capacity to provide funds of the emitting bank at each moment τ. Solving the maximization problem we obtain the optimal interest rate on firm loans as a constant mark-up over the average cost of funds.

$$R^{F,n}_t = \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma - 1} \right) R^{I,n}_t; \qquad R^F_t = \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma - 1} \right) R^I_t \tag{A.18}$$

where we defined $R^{I,n}_t \equiv \int_0^1 R^{I,n}_{t,\tau} \, \mathrm{d}\tau$ and $R^I_t = \left[\sum_{n=1}^N a^n_t \cdot \left(R^{I,n}_t \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$.

Deposit Division

Deposit Divisions obtain deposits from the Representative Household and convert them into internal funding or interbank loans to other banks. The amount of funds that n can provide is given by

$$\sum_{i=1}^N d^{ni}_t \cdot M^{ni}_{t,\tau} = D^n_{t,\tau} \tag{A.19}$$

subject to

$$M^{ni}_{t,\tau} \geq 0; \quad D^n_{t,\tau} \geq 0 \qquad\qquad \forall n, i$$

where $d^{ni} \geq 1$ are transaction costs associated to moving funds from bank n to i, which should be interpreted as containing screening, enforcement, or other costs related to a transaction. We normalize to one the transaction costs between Divisions of the same bank, $d^{nn} = 1$, $\forall n$.

The expected profits of Deposit Division n are

$$\int_0^1 \sum_{i=1}^{N} R_{t,\tau}^{I,ni} M_{t,\tau}^{L,ni} - R_{t,\tau}^{D,n} D_{t,\tau}^{n} \, \mathrm{d}\tau$$

The markets for interbank loans and deposits are perfectly competitive and banks act as price takers. Solving the optimization problem, the interest rate charged by bank n to i at moment τ is

$$\begin{aligned} R_{t,\tau}^{I,ni} &= d_t^{ni} \cdot R_{t,\tau}^{n} \\ &= (1+\varsigma_D)^{-1} \cdot d_t^{ni} \cdot T_t^n \cdot z_{t,\tau}^n \cdot R_t^B \end{aligned} \tag{A.20}$$

where we used equation (A.7) to obtain the second line expression

Central Bank

The central bank in our model can affect the risk-free rate of the economy through conventional open market operations as well as provide direct credit to banks in the system in its role as lender-of-last-resort. We describe both types of intervention in this Section.

Lending Facility

We allow the central bank to provide direct credit to banks through the discount window, which captures its functions as lender-of-last-resort and more broadly the various interventions in the financial markets witnessed during the Great Recession. We assign subindex zero to the central bank and model it as an additional bank within the system, but with some unique characteristics. The central bank does not obtain deposits from the representative household, and differentiates itself by its capacity to freely create money. This translates in the central bank being able to arbitrarily set the interest rate at which it is willing to lend. We model it as a penalty rate over the average rate at which each bank is able to borrow from the rest of its funding sources, formally

$$R_{t,\tau}^{I,0n} = penalty_{t,\tau}^{n} \cdot \Phi_t^n \cdot R_t^B \ , \tag{A.21}$$

where $\Phi_t^n \cdot R_t^B \equiv E_t \left[\min_{i \in \{1,\ldots,N\}} \left\{ R_{t,\tau}^{I,in} \right\} \right]$ is bank n's average cost of funds from its non-central bank sources. We can interpret variable Φ_t^n as the credit spread between the interbank funding costs of bank n (excluding central bank credit) and the risk-free rate. We study different lending policies by assigning the following functional form to the penalty rate:

$$penalty^n_{t,\tau} = e^{\varpi_1} \cdot \underbrace{\left(\frac{\Phi^n_t}{\Phi^n}\right)^{-\varpi_2} \cdot z^0_{t,\tau}}_{\text{variable component}} ,$$

where ϖ_1 is a parameter that controls the steady state size of the penalty, and ϖ_2 its response to deviations of the bank's funding costs from steady state. $z^0_{t,\tau}$ is a policy shock which we model for analytical convenience as being distributed Weibull with mean one and shape parameter κ. When $\varpi_1 \to +\infty$, the interest rate charged by the central bank becomes prohibitively expensive and the model collapses to what would be an equivalent version of it without lender of last resort intervention.

Any profits made by the central bank are returned to the representative household via lump-sum transfer,

$$\Upsilon^{CB}_t = \sum_{n=1}^{N} \int_0^1 R^{I,0n}_{t-1,\tau} M^{0n}_{t-1,\tau} \, d\tau \ .$$

Policy Rule

The central bank also determines the nominal risk-free rate R^B_t of the economy through conventional open market operations. We assume that it follows a Taylor rule of the form

$$R^B_t = R^B \cdot \left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\pi} \left(\frac{Y_t}{Y^n_t}\right)^{\gamma_y} \left(\frac{\widetilde{R}^I_t}{\widetilde{R}^I}\right)^{\gamma_I} \cdot exp\left(u^R_t\right) \ ,$$

where $\Pi_t \equiv P_t/P_{t-1}$ stands for gross inflation, Y^n_t is output under flexible prices, $\widetilde{R}^I \equiv R^I_t/R^B_t$ is the aggregate interbank credit spread (including lender-of-last-resort cost of credit), and u^R an exogenous monetary policy shock. We allow the central bank to respond to deviations of the credit spread $\widetilde{R}^I_t$ because we want to study whether there are additional welfare benefits of establishing such policy compared to the standard targeting of inflation and output gap.

Banking sector aggregation

Plugging equations (A.20) and (A.21) into (A.17), we obtain the distribution of the interbank rate paid by n

$$\widetilde{R}^{I,n}_{t,\tau} \sim W\left(\frac{\widetilde{R}^{I,n}_t}{\Gamma\left(1+1/\kappa\right)}, \kappa\right); \quad \widetilde{R}^{I,n}_t = \Phi^n_t \cdot \left[1 + e^{-\kappa\varpi_1} \cdot \left(\frac{\Phi^n_t}{\Phi^n}\right)^{\kappa\varpi_2}\right]^{-1/\kappa}$$

$$\Phi^n_t = \left[\sum_{i=1}^{N}\left((1+\varsigma_D)^{-1} \cdot d^{in}_t \cdot T^i_t\right)^{-\kappa}\right]^{-1/\kappa}$$

where we used the property that the minimum of a group of Weibull random variables is distributed Weibull as well.

Transaction volumes and Deposits

Define λ^{0i}_t as the share of funding that bank i obtains from the central bank. We obtain an expression for it as

$$\lambda^{0i}_t = \left[1 + e^{\kappa\varpi_1} \cdot \left(\frac{\Phi^n_t}{\Phi^n}\right)^{-\kappa\varpi_2}\right]^{-1}$$

We define $\lambda^{ni,CB}$ as the funding share that i obtains from bank $n \in \{1,\ldots,N\}$, and λ^{ni} as the same share if the central bank was not present. This distinction would be useful when studying the effect on welfare of central bank intervention later on. Formally, we can express them as

$$\lambda^{ni,CB}_t = \left(1-\lambda^{0i}_t\right) \cdot \lambda^{ni}_t\ ; \qquad \lambda^{ni}_t = \left(\frac{(1+\varsigma_D)^{-1} \cdot d^{ni}_t T^n_t}{\Phi^{I,i}_t}\right)^{-\kappa} \tag{A.22}$$

Integrating equation (A.16) over the continuum τ we obtain the volume of funds transacted between any pair of banks as

$$M^{ni}_t = \lambda^{ni,CB}_t \cdot L^i_t \tag{A.23}$$

An intuition for these results is as follows: $d^{ni}_t \cdot T^n_t$ is the average interest rate at which deposits originated in n are *offered* to i over the period, while Φ^i is the average rate at which i *effectively* borrows over the period. The larger the gap between the two, the less funds originated in n will reach i, both in absolute and relative terms. Also, note that we solved our model under the assumption that interbank funds supplied by distinct banks are perfect substitutes at any moment τ, but nonetheless we obtain a downward-sloping CES aggregate demand function over the period. This follows from the volatility of bank preferences $\{z^j\}_{j=1}^N$ and its effect on the relative cost to banks of obtaining Household deposits. Even banks with high transaction costs will eventually become the least cost suppliers in the interbank market at some point. The elasticity of demand κ in the equation above is the same parameter that controls the variance of shocks $\{z^j\}_{j=1}^N$. Demand reacts more strongly to differences between $d_t^{ni} \cdot T_t^n$ and Φ^i when κ is high. This happens because a high κ implies a low variance of deposit preferences, so there are fewer instances in which low deposit costs compensate for the effect of high transaction costs.

Integrating equation (A.19) along the continuum and using (A.16) we obtain an expression for bank n and aggregate deposits

$$D_t^n = \sum_{i=1}^N d_t^{ni} \cdot M_t^{ni}$$
$$D_t + \sum_{n=1}^N M_t^{0n} = \sum_{n=1}^N L_t^n + \sum_{n=1}^N \sum_{i=1}^N \left(d_t^{ni} - 1\right) \cdot M_t^{ni}$$

The last equation shows that in equilibrium aggregate deposits and central bank money are equal to the total amount of loans plus the interbank transaction costs.

Finally, aggregate banking sector profits over period t are

$$\Upsilon_t^B = \sum_{n=1}^N R_{t-1}^{F,n} L_{t-1}^n - \sum_{n=1}^N \int_0^1 R_{t-1,\tau}^{D,n} D_{t-1,\tau}^n \, \mathrm{d}\tau - \sum_{n=1}^N \int_0^1 R_{t-1,\tau}^{I,0n} M_{t-1,\tau}^{0n} \, d\tau$$

Government

The government in this model provides subsidies to firms, banks and depositors, which are funded through lump sum taxation of the representative household. The expression for government transfers are

$$\Upsilon_t^G = -\left[\varsigma_F \cdot P_t Y_t + \varsigma_B \cdot \sum_{n=1}^N R_{t-1}^{F,n} L_{t-1}^n + \varsigma_D \cdot \sum_{n=1}^N R_{t-1}^{D,n} D_{t-1}^n\right]$$

Market Clearing

Total transfers to the Representative Household become

$$\Upsilon_t \equiv \Upsilon_t^F + \Upsilon_t^B + \Upsilon_t^{CB} + \Upsilon_t^G = P_t Y_t - \int_0^1 W_t(\nu) N_t(\nu)\, \mathrm{d}\nu - (1+\varsigma_D) \cdot \sum_{n=1}^{N} \int_0^1 R_{t-1,\tau}^{D,n} D_{t-1,\tau}^n \, \mathrm{d}\tau$$

Aggregating the Representative Household budget constraint (A.2) over the τ continuum and making use of the previous expression, we obtain the following aggregate market clearing condition

$$C_t + \frac{D_t}{P_t} = Y_t \tag{A.24}$$

Aggregation

Using equations (A.3), (A.4), (A.11) and (A.18) we can express firm-specific marginal costs as a function of the aggregate variables

$$\frac{MC_{t+j|t}(\nu)}{P_{t+j}} = (1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma-1}\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\frac{X_{t+j}}{A_{t+j}}\right)^{\frac{\eta(1-\alpha)}{\eta+\alpha}} \left(\frac{Y_{t+j}}{A_{t+j}}\right)^{\frac{1-\alpha}{\eta+\alpha}} \left(\widetilde{R}_{t+j}^l\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\frac{P_t^*}{P_{t+j}}\right)^{-\left(\frac{\epsilon(1-\alpha)}{\eta+\alpha}\right)} \tag{A.25}$$

Similarly, we can integrate loan and labor demand across the continuum of firms and obtain the following expressions

$$\frac{L_t}{A_t P_t} = \alpha(1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma-1}\right)^{-\left(\frac{\eta(1-\alpha)}{\eta+\alpha}\right)} \left(\frac{X_t}{A_t}\right)^{\frac{\eta(1-\alpha)}{\eta+\alpha}} \left(\frac{Y_t}{A_t}\right)^{\frac{\eta+1}{\eta+\alpha}} \left(\widetilde{R}_t^l\right)^{-\left(\frac{\eta(1-\alpha)}{\eta+\alpha}\right)} \Delta_t \tag{A.26}$$

$$N_t = (1-\alpha)^{\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma-1}\right)^{\alpha\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{X_t}{A_t}\right)^{(1-\alpha)\left(\frac{\eta}{\eta+\alpha}\right)} \left(\frac{X_t}{Y_t}\right)^{-\left(\frac{\eta}{\eta+\alpha}\right)} \left(\widetilde{R}_t^l\right)^{\alpha\left(\frac{\eta}{\eta+\alpha}\right)} \Delta_t^{\frac{\eta}{\eta+1}} \tag{A.27}$$

where Δ_t is a measure of price-dispersion that can be recursively defined as

$$\Delta_t = (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta \Pi_t^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} \Delta_{t-1} \tag{A.28}$$

Plug (A.25) and the expressions for Q_{t+j} into the optimal resetting price equation (A.15)

$$\frac{P_t^*}{P_t} = \left(\frac{F_t}{H_t}\right)^{1/\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]} \tag{A.29}$$

where

$$F_t = (1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon-1}\right) \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma-1}\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\frac{X_t}{A_t}\right)^{-\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\frac{Y_t}{A_t}\right)^{\frac{\eta+1}{\eta+\alpha}} \left(\widetilde{R}_t^l\right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} \tag{A.30}$$

$$+ \theta\beta \cdot E_t \left[\Pi_{t+1}^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} F_{t+1}\right]$$

$$H_t = \left(\frac{X_t}{Y_t}\right)^{-1} + \theta\beta \cdot E_t \left[\Pi_{t+1}^{\epsilon-1} H_{t+1}\right] \tag{A.31}$$

Using (A.10) on the previous equations, we obtain the following equilibrium condition for price-resetting in our model

$$\frac{F_t}{H_t} = \left(\frac{1-\theta}{1-\theta\Pi_t^{\epsilon-1}}\right)^{\left(\frac{1}{\epsilon-1}\right)\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]} \tag{A.32}$$

$$\Delta_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\left(\frac{\epsilon}{\epsilon-1}\right)\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta\Pi_t^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} \cdot \Delta_{t-1} \tag{A.33}$$

Substitution of (A.24) into (A.1) allows us to express the composite good as

$$X_t = Y_t - \sum_{n=1}^{N} \int_0^1 T_t^n \cdot z_{t,\tau}^n \frac{D_{t,\tau}^n}{P_t}\, \mathrm{d}\tau$$

We can relate the deposits expression on the previous equation to the model aggregate variables as

$$\sum_{n=1}^{N} \int_0^1 T_t^n \cdot z_{t,\tau}^n \frac{D_{t,\tau}^n}{P_t}\, \mathrm{d}\tau = \frac{1}{(1+\varsigma_D)^{-1}} \sum_{n=1}^{N}\sum_{i=1}^{N} \int_0^1 \widetilde{R}_{t,\tau}^{l,ni} \frac{M_{t,\tau}^{ni}}{P_t}\, \mathrm{d}\tau = \frac{1-\lambda_t^0}{(1+\varsigma_D)^{-1}} \cdot \widetilde{R}_t^l \cdot \frac{L_t}{P_t}$$

where λ_t^0 is a weighted share of the funding obtained from the central bank lending facility, defined as

$$\lambda_t^0 = \sum_{i=1}^{N} s_t^i \cdot \lambda_t^{0i} \; ; \qquad s_t^i = a_t^i \cdot \left(\frac{\widetilde{R}_t^{l,i}}{\widetilde{R}_t^l} \right)^{1-\sigma}$$

with s_t^i being the share of the firm loan market supplied by bank i. Finally, using aggregate loan demand (A.26) and the previous expressions, we obtain the following equilibrium condition

$$\frac{X_t}{Y_t} = 1 - \alpha(1-\alpha)^{\frac{1-\alpha}{\eta+\alpha}} \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma-1} \right)^{-\left(\frac{\eta(1-\alpha)}{\eta+\alpha}\right)} \cdot \left(\frac{X_t}{Y_t} \right)^{-\left(\frac{1-\alpha}{\eta+\alpha}\right)} \left(\frac{X_t}{A_t} \right)^{(1-\alpha)\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\widetilde{R}_t^l \right)^{\alpha\left(\frac{\eta+1}{\eta+\alpha}\right)} \left(\frac{1-\lambda_t^0}{(1+\varsigma_D)^{-1}} \right) \Delta_t \tag{A.34}$$

Flexible Price Equilibrium

We define the flexible equilibrium of the economy as the one in which all firms can reset prices every quarter ($\theta = 0$). Combining equations (A.30)-(A.34) we obtain

$$\begin{aligned} X_t^n =& (1-\alpha)^{-\left(\frac{1}{\eta+1}\right)} \left[1 - \alpha \left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon-1} \right)^{-1} \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma-1} \right)^{-1} \left(\frac{1-\lambda_t^0}{(1+\varsigma_D)^{-1}} \right) \right]^{\frac{1}{\eta+1}} \cdot \\ & \cdot \left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon-1} \right)^{-\left(\frac{1}{1-\alpha}\right)\left(\frac{\eta+\alpha}{\eta+1}\right)} \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma-1} \right)^{-\left(\frac{\alpha}{1-\alpha}\right)} A_t \cdot \left(\widetilde{R}_t^l \right)^{-\left(\frac{\alpha}{1-\alpha}\right)} \\ \frac{X_t^n}{Y_t^n} =& 1 - \alpha \left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon-1} \right)^{-1} \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma-1} \right)^{-1} \left(\frac{1-\lambda_t^0}{(1+\varsigma_D)^{-1}} \right) \end{aligned}$$

where index n stands for a variable under flexible prices. Define composite good and output gap as

$$\widetilde{X}_t = \frac{X_t}{X_t^n}; \quad \widetilde{Y}_t = \frac{Y_t}{Y_t^n}$$

Then

$$\frac{\widetilde{X}_t}{\widetilde{Y}_t} = \left[1 - \alpha \left(\frac{(1+\varsigma_F)^{-1}\epsilon}{\epsilon-1} \right)^{-1} \left(\frac{(1+\varsigma_B)^{-1}\sigma}{\sigma-1} \right)^{-1} \left(\frac{1-\lambda_t^0}{(1+\varsigma_D)^{-1}} \right) \right]^{-1} \cdot \frac{X_t}{Y_t}$$

Equilibrium conditions, summary

Here we summarize the equilibrium conditions of the economy under the assumption of zero trend inflation ($\Pi = 1$) and optimal government subsidies to firms, banks and depositors ($\varsigma_F^* = \frac{1}{\epsilon-1}$; $\varsigma_B^* = \frac{1}{\sigma-1}$; $\varsigma_D^* = \frac{\lambda^0}{1-\lambda^0}$) to offset steady state real distortions from monopolistic mark-ups and central bank intervention.

$$F_t = \left[1 - \alpha \cdot \frac{1-\lambda_t^0}{1-\lambda^0}\right]^{-1} \left(\frac{\widetilde{X}_t}{\widetilde{Y}_t}\right)^{-\left(\frac{\eta+1}{\eta+\alpha}\right)} \widetilde{X}_t^{(1-\alpha)\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta\beta \cdot E_t \left[\Pi_{t+1}^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} F_{t+1}\right] \tag{A.35}$$

$$H_t = \left[1 - \alpha \cdot \frac{1-\lambda_t^0}{1-\lambda^0}\right]^{-1} \left(\frac{\widetilde{X}_t}{\widetilde{Y}_t}\right)^{-1} + \theta\beta \cdot E_t \left[\Pi_{t+1}^{\epsilon-1} H_{t+1}\right] \tag{A.36}$$

$$\frac{F_t}{H_t} = \left(\frac{1-\theta}{1-\theta\Pi_t^{\epsilon-1}}\right)^{\left(\frac{1}{\epsilon-1}\right)\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]} \tag{A.37}$$

$$\Delta_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\left(\frac{\epsilon}{\epsilon-1}\right)\left(\frac{\eta+1}{\eta+\alpha}\right)} + \theta\Pi_t^{\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} \cdot \Delta_{t-1} \tag{A.38}$$

$$\left[1 - \alpha \cdot \frac{1-\lambda_t^0}{1-\lambda^0}\right] \cdot \frac{\widetilde{X}_t}{\widetilde{Y}_t} = 1 - \alpha\left(\frac{1-\lambda_t^0}{1-\lambda^0}\right)\left(\frac{\widetilde{X}_t}{\widetilde{Y}_t}\right)^{-\left(\frac{1-\alpha}{\eta+\alpha}\right)} \widetilde{X}_t^{(1-\alpha)\left(\frac{\eta+1}{\eta+\alpha}\right)} \Delta_t \tag{A.39}$$

$$\frac{1}{R_t^B} = \beta E_t \left[\frac{\widetilde{X}_t}{\widetilde{X}_{t+1}\Pi_{t+1}} \cdot \frac{X_t^n}{X_{t+1}^n}\right] \tag{A.40}$$

$$R_t^B = R^B \, \Pi_t^{\gamma_\pi} \, \widetilde{Y}_t^{\gamma_Y} \left(\frac{\widetilde{R}_t^I}{\widetilde{R}^I}\right)^{\gamma_I} exp\left(u_t^R\right) \tag{A.41}$$

$$N_t = (1-\alpha)^{\frac{\eta}{\eta+1}} \left[1 - \alpha \cdot \frac{1-\lambda_t^0}{1-\lambda^0}\right]^{-\frac{\eta}{\eta+1}} \left(\frac{\widetilde{X}_t}{\widetilde{Y}_t}\right)^{-\left(\frac{\eta}{\eta+\alpha}\right)} \widetilde{X}_t^{(1-\alpha)\left(\frac{\eta}{\eta+\alpha}\right)} \Delta_t^{\frac{\eta}{\eta+1}} \tag{A.42}$$

$$X_t^n = \left[1 - \alpha \cdot \frac{1-\lambda_t^0}{1-\lambda^0}\right]^{\frac{\eta}{\eta+1}} (1-\alpha)^{-\frac{\eta}{\eta+1}} A_t \cdot \left(\widetilde{R}_t^I\right)^{-\left(\frac{\alpha}{1-\alpha}\right)} \tag{A.43}$$

$$\lambda_t^0 = 1 - \frac{e^{\kappa\varpi_1}}{1+e^{\kappa\varpi_1}} \cdot \sum_{i=1}^{N} s_t^i \left(\frac{\Phi_t^i}{\Phi^i}\right)^{-\kappa(1-\varpi_2)} \tag{A.44}$$

$$\widetilde{R}_t^I = \left[\sum_{i=1}^{N} a_t^i \cdot \left(\widetilde{R}_t^{I,i}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{A.45}$$

$$\widetilde{R}_t^{I,i} = \left[1 + e^{-\kappa \cdot \varpi_1}\right]^{-1/\kappa} \cdot \Phi^i \cdot \left(\frac{\Phi_t^i}{\Phi^i}\right)^{\varpi_2} \tag{A.46}$$

$$\Phi_t^i = \left[\sum_{n=1}^{N} \left(\left(1-\lambda^0\right) \cdot d_t^{ni} T_t^n\right)^{-\kappa}\right]^{-\frac{1}{\kappa}} \tag{A.47}$$

$$A_t = exp\left(u_t^A\right) \tag{A.48}$$

$$T_t^n = T^n \cdot exp\left(u_t^{T,n}\right) \tag{A.49}$$

$$d_t^{ni} = \left(d^{ni}\right)^{\varrho} \cdot exp\left(u_t^{I,ni}\right) \tag{A.50}$$

$$a_t^n = \frac{a^n \cdot exp\left(u_t^{a,n}\right)}{\sum_{j=1}^{N} a^j \cdot exp\left(u_t^{a,j}\right)} \tag{A.51}$$

$$u_t^A = \rho_A \cdot u_{t-1}^A + \varepsilon_t^A \tag{A.52}$$

$$u_t^R = \rho_R \cdot u_{t-1}^R + \varepsilon_t^R \tag{A.53}$$

$$u_t^{T,n} = \rho_I \cdot u_{t-1}^{T,n} + \varepsilon_t^{T,n} \tag{A.54}$$

$$u_t^{a,n} = \rho_I \cdot u_{t-1}^{a,n} + \varepsilon_t^{a,n} \tag{A.55}$$

$$u_t^{I,ni} = \rho_I \cdot u_t^{I,ni} + \varepsilon_t^{I,ni} \tag{A.56}$$

Steady state variables

These are the steady state values of the variables under the assumption of zero trend inflation and optimal government subsidies.

$$F = \frac{1}{(1-\alpha)(1-\theta\beta)}; \qquad X^n = \left(\widetilde{R}^I\right)^{-\left(\frac{\alpha}{1-\alpha}\right)}$$

$$H = \frac{1}{(1-\alpha)(1-\theta\beta)}; \qquad u^A = 0$$

$$\Delta = 1; \qquad u^R = 0$$

$$\widetilde{X} = 1; \qquad u^{T,n} = 0$$

$$\widetilde{Y} = 1; \qquad u^{a,n} = 0$$

$$N = 1; \qquad u^{I,ni} = 0$$

$$R^B = \beta^{-1}$$

$$\lambda^0 = [1 + e^{\kappa\varpi_1}]^{-1}$$

$$\Phi^i = \left[\sum_{n=1}^{N} \left(\left(1-\lambda^0\right) \cdot d^{ni} T^n\right)^{-\kappa}\right]^{-\frac{1}{\kappa}}$$

$$\widetilde{R}^{I,i} = \left(1-\lambda^0\right)^{-1/\kappa} \cdot \Phi^i$$

$$\widetilde{R}^I = \left(1-\lambda^0\right)^{-1/\kappa} \cdot \left[\sum_{i=1}^{N} a^i \cdot \left(\Phi^i\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

we can further rewrite the steady state aggregate interbank rate as

$$\widetilde{R}^I = \left(\lambda^{Own}\right)^{1/\kappa} \cdot \left(1-\lambda^0\right)^{1+1/\kappa} \cdot \left[\sum_{i=1}^{N} a^i \cdot \left(T^i\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{A.57}$$

where λ^{Own} is an index of the share of funds that banks obtain from their own depositors, defined as

$$\lambda^{Own} = \left[\sum_{i=1}^{N} s^i \cdot \left(\lambda^{ii}\right)^{\frac{\sigma-1}{\kappa}}\right]^{\frac{\kappa}{\sigma-1}}$$

We will make use of both expressions when analysing welfare.

First order log-linear equilibrium conditions

In these subsection we compute the first order log-linear approximation to the efficient steady state. We use hats to denote deviations from the steady state and minor-case versions of the model variables to refer to its logarithm. We start by approximating equations (A.45) and (A.51)

$$\hat{\tilde{r}}_t^I = \sum_{i=1}^{N} s^i \cdot \left[\hat{\tilde{r}}_t^{I,i} - \frac{\widehat{\log\left(a_t^i\right)}}{\sigma - 1} \right] \tag{A.58}$$

$$\hat{\tilde{r}}_t^{I,i} = \left(1 - \varpi_2 \lambda^0\right) \cdot \hat{\phi}_t^i \tag{A.59}$$

$$\hat{\phi}_t^i = \left(1 - \lambda^0\right) \cdot \sum_{n=1}^{N} \lambda^{ni} \cdot \left[\hat{u}_t^{T,n} + \hat{u}_t^{I,ni}\right] \tag{A.60}$$

Combining (A.54)-(A.60) we obtain the following expression

$$\hat{\tilde{r}}_t^I = \rho_I \cdot \hat{\tilde{r}}_{t-1}^I + (1 - \varpi_2 \lambda^0)(1 - \lambda^0) \cdot \left[\hat{\varepsilon}_t^T + \hat{\varepsilon}_t^I\right] - \frac{\hat{\varepsilon}_t^a}{\sigma - 1} \tag{A.61}$$

$$\text{where:} \quad \hat{\varepsilon}_t^a = \sum_{i=1}^{N} \left(s^i - a^i\right) \cdot \hat{\varepsilon}_t^{a,i}; \quad \hat{\varepsilon}_t^T = \sum_{i=1}^{N} s^i \sum_{n=1}^{N} \lambda^{ni} \cdot \hat{\varepsilon}_t^{T,n}; \quad \hat{\varepsilon}_t^I = \sum_{i=1}^{N} s^i \sum_{n=1}^{N} \lambda^{ni} \cdot \hat{\varepsilon}_t^{I,ni}$$

Similarly, we obtain a log-linear approximation of (A.44) as

$$\widehat{\log\left(\lambda_t^0\right)} = \rho_I \cdot \widehat{\log\left(\lambda_{t-1}^0\right)} + \kappa \varpi_2 (1 - \lambda^0)^2 \cdot \left[\hat{\varepsilon}_t^T + \hat{\varepsilon}_t^I\right]$$

The first order approximation of (A.39) is

$$\hat{\tilde{x}}_t - \hat{\tilde{y}}_t = -\alpha \left(\frac{\eta + 1}{\eta}\right) \cdot \hat{\tilde{x}}_t \tag{A.62}$$

The approximation of (A.35) - (A.37) is

$$\begin{aligned}\hat{f}_t = &(1-\theta\beta)\left[-\left(\frac{\eta+1}{\eta+\alpha}\right)(\hat{\hat{x}}_t-\hat{\hat{y}}_t)+(1-\alpha)\left(\frac{\eta+1}{\eta+\alpha}\right)\hat{\hat{x}}_t-\alpha\left(\frac{\lambda^0}{1-\lambda^0}\right)\cdot\log\left(\lambda^0_t\right)\right] \quad (A.63)\\ &+\theta\beta\left[\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)E_t\left[\hat{\pi}_{t+1}\right]+E_t\left[\hat{f}_{t+1}\right]\right]\end{aligned}$$

$$\hat{h}_t = -(1-\theta\beta)\cdot\left[(\hat{\hat{x}}_t-\hat{\hat{y}}_t)+\alpha\left(\frac{\lambda^0}{1-\lambda^0}\right)\cdot\log\left(\lambda^0_t\right)\right]+\theta\beta\left[(\epsilon-1)E_t\left[\hat{\pi}_{t+1}\right]+E_t\left[\hat{h}_{t+1}\right]\right] \tag{A.64}$$

$$\hat{f}_t-\hat{h}_t = \left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]\left(\frac{\theta}{1-\theta}\right)\hat{\pi}_t \tag{A.65}$$

Combining (A.62)-(A.65) we obtain the New-Keynesian Phillips Curve

$$\hat{\pi}_t = \Omega\cdot\hat{\hat{y}}_t+\beta\cdot E_t\left[\hat{\pi}_{t+1}\right] \tag{A.66}$$

where

$$\Omega = \left(\frac{\eta+1}{\eta}\right)\left[1+\alpha\left(\frac{\eta+1}{\eta}\right)\right]^{-1}\Xi; \qquad \Xi = (1-\alpha)\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]^{-1}\left(\frac{(1-\theta)(1-\theta\beta)}{\theta}\right)$$

The log-linear approximation of equations (A.40) and (A.41) is

$$\hat{r}^B_t = \gamma_\pi\cdot\hat{\pi}_t+\gamma_y\cdot\hat{\hat{y}}_t+\gamma_I\cdot\hat{\hat{r}}^I_t+\hat{u}^R_t \tag{A.67}$$

$$\begin{aligned}-\hat{r}^B_t = &\left[\hat{\hat{x}}_t-E_t\left[\hat{\hat{x}}_{t+1}\right]\right]-E_t\left[\hat{\pi}_{t+1}\right]+(1-\rho_A)\cdot\hat{u}^A_t \quad (A.68)\\ &-\left(\frac{\alpha}{1-\alpha}\right)\cdot\left[\hat{\hat{r}}^I_t-E_t\left[\hat{\hat{r}}^I_{t+1}\right]\right]\\ &-\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\eta}{\eta+1}\right)\left(\frac{\lambda^0}{1-\lambda^0}\right)\left[\widehat{\log\left(\lambda^0_t\right)}-E_t\left[\widehat{\log\left(\lambda^0_{t+1}\right)}\right]\right]\end{aligned}$$

Combining (A.61),(A.62), (A.67) and (A.68) we obtain the Dynamic IS Equation

$$\hat{\hat{y}}_t = -\left[1+\alpha\left(\frac{\eta}{\eta+1}\right)\right]\cdot\left[\hat{r}^B_t-E_t\left[\hat{\pi}_{t+1}\right]-\hat{\iota}^n_t\right]+E_t\left[\hat{\hat{y}}_{t+1}\right] \tag{A.69}$$

where $\iota^n_t \equiv \left[(1-\rho_I)\left(\frac{\alpha}{1-\alpha}\right)\cdot\tilde{r}^I_t+(1-\rho_I)\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\eta}{\eta+1}\right)\left(\frac{\lambda^0}{1-\lambda^0}\right)\cdot\log\left(\lambda^0_t\right)-(1-\rho_A)\cdot u^A_t\right]$ stands for the natural interest rate under flexible prices.

Log-linearizing (A.43)

$$\hat{x}^n_t = \hat{u}^A_t-\left(\frac{\alpha}{1-\alpha}\right)\cdot\hat{\tilde{r}}^I_t-\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\eta}{\eta+1}\right)\left(\frac{\lambda^0}{1-\lambda^0}\right)\cdot\widehat{\log\left(\lambda^0_t\right)} \tag{A.70}$$

Log-linearizing (A.42) and using (A.62)

$$\hat{n}_t = \hat{\hat{x}}_t - \left(\frac{\eta}{\eta+1}\right)\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\lambda_0}{1-\lambda_0}\right)\cdot \widehat{\log\left(\lambda_t^0\right)}$$

Model solution

Now we compute the solution to 1st order log-linear approximation of the model. Equations (A.66) and (A.69) constitute a system of stochastic differential equations, which we can summarize in matrix form as

$$A_0 \cdot \begin{bmatrix} \hat{\hat{y}}_t \\ \hat{\pi}_t \end{bmatrix} = B_0 \cdot E_t \begin{bmatrix} \hat{\hat{y}}_{t+1} \\ \hat{\pi}_{t+1} \end{bmatrix} + C_0 \cdot \begin{bmatrix} \hat{\hat{r}}_t^I \\ \widehat{\log\left(\lambda_t^0\right)} \\ \hat{u}_t^A \\ \hat{u}_t^R \end{bmatrix}$$

$$\begin{bmatrix} \hat{\hat{r}}_t^I \\ \widehat{\log\left(\lambda_t^0\right)} \\ \hat{u}_t^A \\ \hat{u}_t^R \end{bmatrix} = G_0 \cdot \begin{bmatrix} \hat{\hat{r}}_{t-1}^I \\ \widehat{\log\left(\lambda_{t-1}^0\right)} \\ \hat{u}_{t-1}^A \\ \hat{u}_{t-1}^R \end{bmatrix} + G_1 \cdot \begin{bmatrix} \varepsilon_t^T \\ \varepsilon_t^I \\ \varepsilon_t^a \\ \varepsilon_t^A \\ \varepsilon_t^R \end{bmatrix}$$

where

$$A_0 = \begin{bmatrix} -\Omega & 1 \\ \left[1+\alpha\left(\frac{\eta+1}{\eta}\right)\right]^{-1} + \gamma_y & \gamma_\pi \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 0 & \beta \\ \left[1+\alpha\left(\frac{\eta+1}{\eta}\right)\right]^{-1} & 1 \end{bmatrix}$$

$$C_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ (1-\rho_I)\left(\frac{\alpha}{1-\alpha}\right) - \gamma_I & (1-\rho_I)\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\eta}{\eta+1}\right)\left(\frac{\lambda^0}{1-\lambda^0}\right) & -(1-\rho_A) & -1 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} \left(1-\varpi_2\lambda^0\right)\left(1-\lambda^0\right) & \left(1-\varpi_2\lambda^0\right)\left(1-\lambda^0\right) & -\frac{1}{\sigma-1} & 0 & 0 \\ \kappa\varpi_2(1-\lambda^0)^2 & \kappa\varpi_2(1-\lambda^0)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$G_0 = diag\left(\rho_I, \rho_I, \rho_A, \rho_R\right)$$

Solving forward

$$\begin{bmatrix} \hat{\hat{y}}_t \\ \hat{\pi}_t \end{bmatrix} = \underbrace{\left[\sum_{s=0}^{\infty} \left(A_0^{-1} B_0\right)^s \cdot \left(A_0^{-1} C_0\right) \cdot G_0^s \right]}_{\equiv \Psi} \cdot \begin{bmatrix} \hat{\tilde{r}}_t^I \\ \widetilde{\log\left(\lambda_t^0\right)} \\ \hat{u}_t^A \\ \hat{u}_t^R \end{bmatrix} \equiv \Psi \cdot \begin{bmatrix} \hat{\tilde{r}}_t^I \\ \widetilde{\log\left(\lambda_t^0\right)} \\ \hat{u}_t^A \\ \hat{u}_t^R \end{bmatrix}$$

We reexpress the infinite sum in Ψ as:

$$\Psi - A_0^{-1} B_0 \cdot \Psi \cdot G_0 = A_0^{-1} C_0$$

If $\left(I - G^T \otimes \left(A_0^{-1} B_0\right)\right)$ is invertible, we can vectorize the previous equation and obtain a solution as

$$vec(\Psi) = \left(I - G_0^T \otimes \left(A_0^{-1} B_0\right)\right)^{-1} \cdot vec(A_0^{-1} C_0)$$

A.2 Welfare

We are going to approximate utility around the efficient steady state with zero trend inflation $\bar{\Pi} = 1$.

$$U_t = U + U_X X \left(\frac{X_t - X}{X}\right) + U_N N \left(\frac{N_t - N}{N}\right) + \frac{1}{2} U_{XX} X^2 \left(\frac{X_t - X}{X}\right)^2 + \frac{1}{2} U_{NN} N^2 \left(\frac{N_t - N}{N}\right)^2$$

We will use the following second order approximation

$$\frac{Z_t - Z}{Z} = \hat{z}_t + \frac{1}{2}\hat{z}_t^2$$

Then, the previous expression can be rewritten as

$$U_t - \bar{U} = U_X X \left[\hat{x}_t + \frac{1}{2}\left(1 + \frac{U_{XX} X}{U_X}\right)\hat{x}_t^2\right] + U_N N \left[\hat{n}_t + \frac{1}{2}\left(1 + \frac{U_{NN} N}{U_N}\right)\hat{n}_t^2\right]$$

In our model we have

$$U_X X = 1; \qquad U_N N = -1$$
$$\frac{U_{XX} X}{U_X} = -1; \qquad \frac{U_{NN} N}{U_N} = \frac{1}{\eta}$$

The utility approximation becomes

$$U_t - \bar{U} = \hat{x}_t - \left[\hat{n}_t + \frac{1}{2}\left(1 + \frac{1}{\eta}\right)\hat{n}_t^2\right] + \text{h.o.t.}$$

Using (A.83) and (A.84) into the previous expression

$$U_t - \bar{U} = \hat{x}_t^n - \left(\frac{1}{1-\alpha}\right)\left(\frac{\eta+\alpha}{\eta+1}\right)\cdot \widehat{\log\left(\Delta_t\right)} - \frac{1}{2}\left(\frac{\eta+1}{\eta}\right)\left[1 + \alpha\left(\frac{\eta+1}{\eta}\right)\right]^{-1}\hat{\tilde{y}}_t^2 + \text{h.o.t.}$$

Using (A.70) and (A.78) we obtain:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t U_t =& x^n - E_0\sum_{t=0}^{\infty}\beta^t\left[\left(\frac{\alpha}{1-\alpha}\right)\left[\hat{\tilde{r}}_t^l + \left(\frac{\eta}{\eta+1}\right)\left(\frac{\lambda^0}{1-\lambda^0}\right)\cdot\widehat{\log\left(\lambda_t^0\right)}\right] + \frac{1}{2\cdot\Xi}\left[\epsilon\cdot\hat{\pi}_t^2 + \Omega\cdot\hat{\tilde{y}}_t^2\right]\right]\\ &+ t.i.p. + h.o.t.\end{aligned}$$

Using the DSGE solution of Appendix A.1, we can express the expected per-period utility as

$$E\left[U_t\right] = -E\Bigg[\left(\frac{\alpha}{1-\alpha}\right)\left[\left(1+\frac{1}{\kappa}\right)\cdot\log\left(1-\lambda^0\right)+\frac{1}{\kappa}\cdot\log\left(\lambda^{Own}\right)\right] \tag{A.71}$$
$$+\left(\frac{\alpha}{1-\alpha}\right)\left[\hat{\tilde{r}}_t^I+\left(\frac{\eta}{\eta+1}\right)\left(\frac{\lambda^0}{1-\lambda^0}\right)\cdot\widehat{\log\left(\lambda_t^0\right)}\right]$$
$$+\frac{\Lambda_I}{2}\cdot\left(\hat{\tilde{r}}_t^I\right)^2+\frac{\Lambda_{CB}}{2}\cdot\widehat{\log\left(\lambda_t^0\right)}^2+\Lambda_{IxCB}\cdot\widehat{\log\left(\lambda_t^0\right)}\cdot\hat{\tilde{r}}_t^I\Bigg]+t.i.p.+h.o.t.$$

where

$$\Lambda_I = \Xi^{-1}\left[\epsilon\cdot\left(\Psi_{21}\right)^2+\Omega\cdot\left(\Psi_{11}\right)^2\right]$$
$$\Lambda_{CB} = \Xi^{-1}\left[\epsilon\cdot\left(\Psi_{22}\right)^2+\Omega\cdot\left(\Psi_{12}\right)^2\right]$$
$$\Lambda_{IxCB} = \Xi^{-1}\left[\epsilon\cdot\Psi_{21}\cdot\Psi_{22}+\Omega\cdot\Psi_{11}\cdot\Psi_{12}\right]$$

We define welfare losses around the steady state as

$$\mathbb{L} = -E\left[\frac{U_t-U}{U_xX}\right]$$

Using (A.70) and (A.78) we obtain:

$$\mathbb{L} = E\left[\left(\frac{\alpha}{1-\alpha}\right)\cdot\left[\hat{\tilde{r}}_t^I+\left(\frac{\eta}{\eta+1}\right)\left(\frac{\lambda^0}{1-\lambda^0}\right)\cdot\widehat{\log\left(\lambda_t^0\right)}\right]+\frac{1}{2\cdot\Xi}\left[\epsilon\cdot\hat{\pi}_t^2+\Omega\cdot\hat{\tilde{y}}_t^2\right]\right]+t.i.p.+h.o.t.$$

The usual definition of gains from trade compares the steady state change in welfare with respect to autarky, formally

$$\mathbb{J}^{st.} \equiv \frac{U-U^{AU}}{U_xX} = -\left(\frac{\alpha}{1-\alpha}\right)\frac{1}{\kappa}\cdot\log\left(\lambda^{Own}\right)$$

which is equal to the standard ACR formula, with gains from trade being a monotonously increasing function of financial market integration. The problem of this definition is that it excludes higher order terms related to the volatility of financial markets that the literature generally considers as important. Therefore, we define the stochastic version of the gains from trade as

$$\mathbb{J} = E\left[\frac{U_t-U_t^{AU}}{U_xX}\right]$$

Using (A.71) on the previous equation we obtain

$$
\begin{aligned}
\mathbb{J} = {} & \mathbb{J}^{ss} \\
& - E\left[\left(\frac{\alpha}{1-\alpha}\right)\cdot\left[\hat{\tilde{r}}_t^I - \hat{\tilde{r}}_t^{I,AU} + \left(\frac{\eta}{\eta+1}\right)\left(\frac{\lambda^0}{1-\lambda^0}\right)\cdot\left[\log\widehat{\left(\lambda_t^0\right)} - \log\widehat{\left(\lambda_t^{0,AU}\right)}\right]\right]\right] \\
& - E\left[\frac{\Lambda_I}{2}\cdot\left[\left(\hat{\tilde{r}}_t^I\right)^2 - \left(\hat{\tilde{r}}_t^{I,AU}\right)^2\right] + \frac{\Lambda_{CB}}{2}\cdot\left[\log\widehat{\left(\lambda_t^0\right)}^2 - \log\widehat{\left(\lambda_t^{0,AU}\right)}^2\right]\right] \\
& - E\left[\frac{\Lambda_{IxCB}}{2}\cdot\left[\hat{\tilde{r}}_t^I\cdot\widehat{\log\left(\lambda_t^0\right)} - \hat{\tilde{r}}_t^{I,AU}\cdot\log\widehat{\left(\lambda_t^{0,AU}\right)}\right]\right] \\
& + h.o.t.\,,
\end{aligned}
\tag{A.72}
$$

where constants Λ_I, Λ_{CB} and Λ_{IxCB} capture the sensitivity of the economy to credit spread shocks and central bank lending. Central bank policy parameters are components of these constants, which implies that the central bank is able to affect the sensitivity of the economy to financial shocks by modifying its intervention rules. The first line of the expression are the static ACR gains from trade, that in an stochastic environment are valid only as a first-order (instead of exact) approximation to trade gains. The second line is related to the difference between the average value of the variables and their steady state, as $\tilde{r}_t^I$ and $\log\left(\lambda_t^0\right)$ do not necessarily fluctuate symmetrically around their steady state values. The third and fourth lines are respectively related to the changes in volatility and cross-correlation between the interbank rate and central bank trade share.

In order to obtain an analytical expression for the welfare gains as a function of the structural parameters of the model, we have to solve the expectation operator on the last two terms of the previous equation. Under the additional assumptions of Appendix A.3.2, equation (A.72) can be expressed as a function of the underlying model parameters

$$
\begin{aligned}
\mathbf{J} = \mathbf{J}^{ss} & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(A.73)}\\
&+ \frac{(1-\zeta_T)\cdot\sigma_T^2}{2}\cdot\Bigg[\left[\Theta_1+\Theta_3\cdot H^F\right]\cdot\sum_{i=1}^{N}\omega^{H,i}\cdot\left[1-H^{I,i}\right]\\
&\qquad\qquad -\Theta_3\cdot\sum_{i=1}^{N}\sum_{n\neq i}^{N}\sum_{j=1}^{N}s^i s^n\lambda^{ji}\lambda^{jn}-\Theta_3\cdot\left[H^F-H^{F,AU}\right]\Bigg]\\
&+\frac{\sigma_I^2}{2}\cdot\left[\left[\Theta_1+\Theta_3\cdot H^F\right]\cdot\sum_{i=1}^{N}\omega^{H,i}\cdot\left[1-H^{I,i}\right]-\left[\Theta_3\cdot H^F-\Theta_2\right]\cdot\sum_{i=1}^{N}\omega^{\lambda,i}\cdot\left[1-\left(\lambda^{ii}\right)^2\right]\right]\\
&-\frac{\zeta_{I,B}\cdot\sigma_I^2}{2}\cdot\Bigg[\left[\Theta_1+\Theta_3\cdot H^F\right]\cdot\sum_{i=1}^{N}\omega^{H,i}\cdot\left[1-H^{I,i}\right]\\
&\qquad\qquad -\left[\Theta_3\cdot H^F-\Theta_2\right]\cdot\sum_{i=1}^{N}\omega^{\lambda,i}\cdot\left[1-\left(1-\lambda^{ii}\right)^2-\left(\lambda^{ii}\right)^2\right]\Bigg]\\
&-\frac{\zeta_{I,L}\cdot\sigma_I^2}{2}\cdot\Theta_3\cdot\sum_{i=1}^{N}\sum_{j\neq i}\sum_{n\neq\{j,i\}}s^i s^n\lambda^{ji}\lambda^{jn}\\
&-\frac{\zeta_{I,x}\cdot\sigma_I^2}{2}\cdot\Theta_3\cdot\sum_{i=1}^{N}\sum_{n\neq i}s^i s^n\left[\left(1-\lambda^{ii}\right)\cdot\left(1-\lambda^{nn}\right)-\sum_{j\neq i}\lambda^{ji}\lambda^{jn}\right]\\
&-\frac{(1-\zeta_a)}{2}\left(\frac{\sigma_a}{\sigma-1}\right)^2\cdot\left[\Theta_5\cdot\left[H^F-H^{F,AU}\right]-2\cdot\Lambda_I\sum_{i=1}^{N}a^i\cdot\left(s^i-s^{i,AU}\right)\right]\ ,
\end{aligned}
$$

where $\Theta_m,\ m=1,\ldots,5$ are positive constants potentially affected by the central bank monetary and lending policies, $\sum_{i=1}^{N}\omega^{m,i}=1,\ m=\{H,\lambda\}$ are weights

$$
\omega^{H,i}=\frac{\Theta_1\cdot s^i+\Theta_3\cdot\left(s^i\right)^2}{\Theta_1+\Theta_3\cdot H^F}\ ,\quad \omega^{\lambda,i}=\frac{\Theta_3\cdot\left(s^i\right)^2-\Theta_2\cdot s^i}{\Theta_3\cdot H^F-\Theta_2}\ ,
$$

and $H^{I,i}$ and H^F are Herfindahl concentration indices of the bank's funding sources and firm's loan market, respectively defined as

$$
H^{I,i}=\sum_{n=1}^{N}\left(\lambda^{ni}\right)^2\ ,\quad H^F=\sum_{n=1}^{N}\left(s^n\right)^2\ ,
$$

The first line are the standard ACR static gains from trade. The second line are the gains/loses from deposit diversification. As the concentration of the funding sources is reduced, banks are more likely to find cheap sources of credit among their connections when they face a shock to their deposits. On the other hand, specialization in the collection of deposits might lead to higher volatility. The third line is related to the volatility costs/benefits arising from interbank transaction costs. Transactions costs with oneself are constant over time, as we normalized $d_t^{ii} = 1, \ \forall i, \ \ t$, but transactions with others are volatile. Hence, by increasing their participation on the interbank market, banks reduce the reliance on their own depositors at the expense of more volatile funding costs. Unexpected changes in funding costs are eventually passed to firms and interact with their sticky price decisions, increasing the volatility of inflation and output gap. On the other hand, when banks diversify of their funding sources, as captured by the first term of line three, they partially insure themselves from this costs by being able to rely on alternative connections. Lines four to six are the effect of the correlation between transaction costs, with high positive correlations diminishing the diversification benefits just mentioned. The final line is related to the concentration in the market for firm loans. If financial integration disproportionately benefits big banks allowing them to capture a larger fraction of the market for firm loans, the volatility of the economy will increase due to concentration on few large entities. If the opposite applies and small banks are able to grow due to their access to interbank credit, gains from trade will rise.

A.3 Welfare approximation, technical derivations

A.3.1 Second Order Approximation

In this Appendix we compute the second-order log-linear approximations of the variables that we will use in computing our Welfare approximation of Appendix A.2.

Price Dispersion

We defined dispersion and aggregate price index as

$$\Delta_t = \int_0^1 \left(\frac{P_t(\nu)}{P_t}\right)^{-\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} \mathrm{d}\nu \tag{A.74}$$

$$P_t = \left[\int_0^1 P_t(\nu)^{1-\epsilon} \, \mathrm{d}\nu\right]^{\frac{1}{1-\epsilon}} \tag{A.75}$$

Rearranging (A.75):

$$1 = \int_0^1 \left(\frac{P_t(\nu)}{P_t}\right)^{1-\epsilon} \, \mathrm{d}\nu$$

A second order approximation of the RHS

$$\left(\frac{P_t(\nu)}{P_t}\right)^{1-\epsilon} = 1 + (1-\epsilon)\cdot \widehat{\log(P_t(\nu))} + \frac{(1-\epsilon)^2}{2}\widehat{\log(P_t(\nu))}^2$$

Then, plugging into the main expression and rearranging we get

$$E_\nu\left[\widehat{\log(P_t(\nu))}\right] = \frac{\epsilon-1}{2}\cdot Var_\nu\left(\log\left(P_t(\nu)\right)\right) \tag{A.76}$$

Integrating the RHS of (A.74)

$$\int_0^1 \left(\frac{P_t(\nu)}{P_t}\right)^{-\epsilon\left(\frac{\eta+1}{\eta+\alpha}\right)} \, \mathrm{d}\nu = 1 + \frac{\epsilon}{2}\left(\frac{\eta+1}{\eta+\alpha}\right)\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right] Var_\nu\left(\log\left(P_t(\nu)\right)\right)$$

Plugging back into (A.74) and taking logs

$$\widehat{\log(\Delta_t)} = \frac{\epsilon}{2\cdot\Theta} Var_\nu\left(\log\left(P_t(\nu)\right)\right) \tag{A.77}$$

where $\Theta = \left(\frac{\eta+1}{\eta+\alpha}\right)^{-1}\left[1+\epsilon\left(\frac{1-\alpha}{\eta+\alpha}\right)\right]^{-1}$. A second order approximation of (A.38) becomes

$$\widehat{\log(\Delta_t)} = \theta\cdot\widehat{\log(\Delta_{t-1})} + \left(\frac{\theta}{1-\theta}\right)\frac{\epsilon}{2\cdot\Theta}\cdot\hat{\pi}_t^2$$

We can express this equations recursively as

$$\widehat{\log(\Delta_{t+j})} = \theta^j\cdot\widehat{\log(\Delta_{t-1})} + \left(\frac{\theta}{1-\theta}\right)\frac{\epsilon}{2\cdot\Theta}\cdot\sum_{k=0}^{j}\theta^k\hat{\pi}_{t+j-k}^2$$

Now, we compute the following expression, which will be used to replace price dispersion in our second-order welfare approximation

$$\sum_{t=0}^{\infty}\beta^t\cdot E_0\left[\widehat{\log(\Delta_t)}\right] = \frac{\theta\epsilon}{2(1-\theta)(1-\theta\beta)\Theta}\sum_{t=0}^{\infty}\beta^t\cdot E_0\left[\hat{\pi}_t^2\right] + t.i.p. \tag{A.78}$$

Aggregate variables

Using implicit differentiation on equation (A.39), we obtain a second order approximation to $\hat{\hat{x}}_t - \hat{\hat{y}}_t$ as a function of $\hat{\hat{x}}_t$ and $\widehat{\log(\Delta_t)}$

$$\hat{\hat{x}}_t - \hat{\hat{y}}_t = -\alpha \left(\frac{\eta+1}{\eta}\right) \cdot \hat{\hat{x}}_t - \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\eta+\alpha}{\eta}\right) \cdot \widehat{\log(\Delta_t)} - \frac{\alpha}{2}\left(\frac{\eta+1}{\eta}\right)^2 \left(\frac{\eta+\alpha}{\eta}\right) \cdot \hat{\hat{x}}_t^2 \tag{A.79}$$

Log-linearizing equation (A.42) and using (A.79) we obtain

$$\hat{n}_t = \hat{\hat{x}}_t + \left(\frac{1}{1-\alpha}\right)\left(\frac{\eta+\alpha}{\eta+1}\right) \cdot \widehat{\log(\Delta_t)} + \frac{\alpha}{2}\left(\frac{\eta+1}{\eta}\right)^2 \cdot \hat{\hat{x}}_t^2 \tag{A.80}$$

From the previous expressions we get that

$$\hat{n}_t^2 = \hat{\hat{x}}_t^2 \tag{A.81}$$

From (A.79) if follows that

$$\hat{\hat{x}}_t^2 = \left[1 + \alpha\left(\frac{\eta+1}{\eta}\right)\right]^{-2} \hat{\hat{y}}_t^2 \tag{A.82}$$

Using (A.82) into (A.80) and (A.81) we obtain the following expressions

$$\hat{n}_t = \hat{\hat{x}}_t + \left(\frac{1}{1-\alpha}\right)\left(\frac{\eta+\alpha}{\eta+1}\right) \cdot \widehat{\log(\Delta_t)} + \frac{\alpha}{2}\left(\frac{\eta+1}{\eta}\right)^2 \left[1 + \alpha\left(\frac{\eta+1}{\eta}\right)\right]^{-2} \hat{\hat{y}}_t^2 \tag{A.83}$$

$$\hat{n}_t^2 = \left[1 + \alpha\left(\frac{\eta+1}{\eta}\right)\right]^{-2} \hat{\hat{y}}_t^2 \tag{A.84}$$

Banking variables

We start by computing the log-deviation of the individual interbank rate as

$$\hat{\hat{r}}_t^{I,i} = \left(1 - \varpi_2 \lambda^0\right) \cdot \hat{\phi}_t^i - \frac{1}{2} \cdot \kappa\left(\varpi_2\right)^2 \left(\lambda^0 - \left(\lambda^0\right)^2\right) \cdot \left(\hat{\phi}_t^i\right)^2$$

Aggregate $\widetilde{R}_t^I$ and Φ_t^i second order approximations are

$$\begin{aligned}
\hat{\tilde{r}}_t^I =& \sum_{i=1}^{N} s^i \cdot \left[\left(1 - \varpi_2 \lambda^0\right) \cdot \hat{\phi}_t^i - \frac{\widehat{\log\left(a_t^i\right)}}{\sigma - 1} \right] \\
& - \frac{1}{2} \left[\kappa \left(\varpi_2\right)^2 \left(\lambda^0 - \left(\lambda^0\right)^2\right) + (\sigma - 1)(1 - \varpi_2 \lambda^0) \right] \cdot \sum_{i=1}^{N} s^i \cdot \left(\hat{\phi}_t^i\right)^2 \\
& - \left(\frac{\sigma - 1}{2}\right) \sum_{i=1}^{N} s^i \cdot \left[\left(\frac{1}{\sigma - 1}\right)^2 \widehat{\log\left(a_t^i\right)}^2 - 2\left(1 - \varpi_2 \lambda^0\right) \cdot \hat{\phi}_t^i \cdot \frac{\widehat{\log\left(a_t^i\right)}}{\sigma - 1} \right] \\
& + \left(\frac{\sigma - 1}{2}\right) \sum_{i=1}^{N} \sum_{j=1}^{N} s^i s^j \cdot \left[\left(1 - \varpi_2 \lambda^0\right)^2 \cdot \hat{\phi}_t^i \cdot \hat{\phi}_t^j + \left(\frac{1}{\sigma - 1}\right)^2 \widehat{\log\left(a_t^i\right)} \cdot \widehat{\log\left(a_t^j\right)} \right. \\
& \qquad \left. - \left(1 - \varpi_2 \lambda^0\right) \cdot \phi_t^i \cdot \frac{\widehat{\log\left(a_t^j\right)}}{\sigma - 1} \right]
\end{aligned} \tag{A.85}$$

$$\begin{aligned}
\hat{\phi}_t^i =& \sum_{n=1}^{N} \lambda^{ni} \cdot \left[\hat{u}_t^{T,n} + \hat{u}_t^{I,ni} \right] \\
& + \frac{\kappa}{2} \sum_{n=1}^{N} \sum_{j \neq n} \lambda^{ni} \lambda^{ji} \cdot \left[\hat{u}_t^{T,n} \cdot \hat{u}_t^{T,j} + \hat{u}_t^{I,ni} \cdot \hat{u}_t^{I,ji} + \hat{u}_t^{T,n} \cdot \hat{u}_t^{I,ji} + \hat{u}_t^{T,j} \cdot \hat{u}_t^{I,ni} \right] \\
& - \frac{\kappa}{2} \sum_{n=1}^{N} \lambda^{ni} (1 - \lambda^{ni}) \cdot \left[\left(\hat{u}_t^{T,n}\right)^2 + \left(\hat{u}_t^{I,ni}\right)^2 + 2 \cdot \hat{u}_t^{T,n} \cdot \hat{u}_i^{I,ni} \right]
\end{aligned} \tag{A.86}$$

It follows from the previous expressions that

$$\left(\hat{\tilde{r}}_t^I\right)^2 = \sum_{i=1}^{N} \sum_{n=1}^{N} s^i s^n \cdot \left[\left(1 - \varpi_2 \lambda^0\right)^2 \cdot \hat{\phi}_t^i \cdot \hat{\phi}_t^n + \frac{\widehat{\log\left(a_t^i\right)} \cdot \widehat{\log\left(a_t^n\right)}}{(\sigma - 1)^2} - 2\left(1 - \varpi_2 \lambda^0\right) \cdot \hat{\phi}_t^n \cdot \frac{\widehat{\log\left(a_t^i\right)}}{\sigma - 1} \right] \tag{A.87}$$

$$\hat{\phi}_t^i \cdot \hat{\phi}_t^n = \sum_{j=1}^{N} \sum_{k=1}^{N} \lambda^{ji} \lambda^{kn} \cdot \left[\hat{u}_t^{T,j} \cdot \hat{u}_t^{T,k} + \hat{u}^{I,ji} \cdot \hat{u}^{I,kn} + \hat{u}_t^{T,j} \cdot \hat{u}^{I,kn} + \hat{u}_t^{T,k} \cdot \hat{u}^{I,ji} \right] \tag{A.88}$$

$$\left(\hat{\phi}_t^i\right)^2 = \sum_{j=1}^{N} \sum_{k=1}^{N} \lambda^{ji} \lambda^{ki} \cdot \left[\hat{u}_t^{T,j} \cdot \hat{u}_t^{T,k} + \hat{u}^{I,ji} \cdot \hat{u}^{I,ki} + \hat{u}_t^{T,j} \cdot \hat{u}^{I,ki} + \hat{u}_t^{T,k} \cdot \hat{u}^{I,ji} \right] \tag{A.89}$$

The second order approximations to λ_t^{0i}, λ_t^0 and s_t^i are

$$\widehat{\log\left(\lambda_t^{0i}\right)} = \kappa\varpi_2\left(1-\lambda^0\right)\cdot\hat{\phi}_t^i - \frac{1}{2}\cdot\kappa^2\left(\varpi_2\right)^2\cdot\lambda^0\left(1-\lambda^0\right)\cdot\left(\hat{\phi}_t^i\right)^2$$

$$\begin{aligned}\widehat{\log\left(\lambda_t^{0}\right)} =& \sum_{i=1}^{N} s^i\cdot\left[\widehat{\log\left(s_t^i\right)}+\widehat{\log\left(\lambda_t^{0i}\right)}\right] \\ &+\frac{1}{2}\cdot\sum_{i=1}^{N} s^i\cdot\left[\widehat{\log\left(s_t^i\right)}^2+\widehat{\log\left(\lambda_t^{0i}\right)}^2+\widehat{\log\left(s_t^i\right)}\cdot\widehat{\log\left(\lambda_t^{0i}\right)}\right] \\ &-\frac{1}{2}\cdot\sum_{i=1}^{N}\sum_{j=1}^{N} s^i s^j\cdot\left[\widehat{\log\left(\lambda_t^{0i}\right)}\cdot\widehat{\log\left(\lambda_t^{0j}\right)}+\widehat{\log\left(s_t^i\right)}\cdot\widehat{\log\left(s_t^j\right)}+\widehat{\log\left(\lambda_t^{0i}\right)}\cdot\widehat{\log\left(s_t^j\right)}\right]\end{aligned} \tag{A.90}$$

$$\begin{aligned}\widehat{\log\left(s_t^i\right)} &= (\sigma-1)\left[\hat{\tilde{r}}_t^l-\left[\hat{\tilde{r}}_t^{l,i}-\frac{\widehat{\log\left(a_t^i\right)}}{\sigma-1}\right]\right] \\ &= (\sigma-1)\left[\sum_{j=1}^{N} s^j\cdot\left[\hat{\tilde{r}}_t^{l,j}-\frac{\widehat{\log\left(a_t^j\right)}}{\sigma-1}\right]-\left[\hat{\tilde{r}}_t^{l,i}-\frac{\widehat{\log\left(a_t^i\right)}}{\sigma-1}\right]\right] \\ &\quad-\frac{(\sigma-1)^2}{2}\sum_{k=1}^{N} s^k\cdot\Bigg[\left(1-\varpi_2\lambda^0\right)^2\cdot\left(\hat{\phi}_t^k\right)^2+\left(\frac{1}{\sigma-1}\right)^2\cdot\widehat{\log\left(a_t^k\right)}^2 \\ &\qquad\qquad -2\left(1-\varpi_2\lambda^0\right)\cdot\hat{\phi}_t^k\cdot\frac{\widehat{\log\left(a_t^k\right)}}{\sigma-1}\Bigg] \\ &\quad+\frac{(\sigma-1)^2}{2}\sum_{k=1}^{N}\sum_{j=1}^{N} s^k s^j\cdot\Bigg[\left(1-\varpi_2\lambda^0\right)^2\cdot\hat{\phi}_t^k\cdot\hat{\phi}_t^j+\left(\frac{1}{\sigma-1}\right)^2\cdot\widehat{\log\left(a_t^k\right)}\cdot\widehat{\log\left(a_t^j\right)} \\ &\qquad\qquad -\left(1-\varpi_2\lambda^0\right)\cdot\hat{\phi}_t^k\cdot\frac{\widehat{\log\left(a_t^j\right)}}{\sigma-1}\Bigg]\end{aligned} \tag{A.91}$$

Using equation (A.91) we compute

$$\begin{aligned}
\sum_{i=1}^{N} s^i \cdot \widehat{\log\left(s_t^i\right)} = & -\frac{(\sigma-1)^2}{2}\sum_{k=1}^{N} s^k \cdot \Bigg[\left(1-\varpi_2\lambda^0\right)^2 \cdot \left(\hat{\phi}_t^k\right)^2 + \left(\frac{1}{\sigma-1}\right)^2 \cdot \widehat{\log\left(a_t^k\right)}^2 \\
& \qquad - 2\left(1-\varpi_2\lambda^0\right) \cdot \hat{\phi}_t^k \cdot \frac{\widehat{\log\left(a_t^k\right)}}{\sigma-1} \Bigg] \\
& + \frac{(\sigma-1)^2}{2}\sum_{k=1}^{N}\sum_{j=1}^{N} s^k s^j \cdot \Bigg[\left(1-\varpi_2\lambda^0\right)^2 \cdot \hat{\phi}_t^k \cdot \hat{\phi}_t^j + \left(\frac{1}{\sigma-1}\right)^2 \cdot \widehat{\log\left(a_t^k\right)} \cdot \widehat{\log\left(a_t^j\right)} \\
& \qquad - \left(1-\varpi_2\lambda^0\right) \cdot \hat{\phi}_t^k \cdot \frac{\widehat{\log\left(a_t^j\right)}}{\sigma-1} \Bigg] \\
\sum_{i=1}^{N} s^i \cdot \widehat{\log\left(s_t^i\right)}^2 = & \ (\sigma-1)^2\sum_{k=1}^{N} s^k \cdot \Bigg[\left(1-\varpi_2\lambda^0\right)^2 \cdot \left(\hat{\phi}_t^k\right)^2 + \left(\frac{1}{\sigma-1}\right)^2 \cdot \widehat{\log\left(a_t^k\right)}^2 \\
& \qquad - 2\left(1-\varpi_2\lambda^0\right) \cdot \hat{\phi}_t^k \cdot \frac{\widehat{\log\left(a_t^k\right)}}{\sigma-1} \Bigg] \\
& - (\sigma-1)^2\sum_{k=1}^{N}\sum_{j=1}^{N} s^k s^j \cdot \Bigg[\left(1-\varpi_2\lambda^0\right)^2 \cdot \hat{\phi}_t^k \cdot \hat{\phi}_t^j + \left(\frac{1}{\sigma-1}\right)^2 \cdot \widehat{\log\left(a_t^k\right)} \cdot \widehat{\log\left(a_t^j\right)} \\
& \qquad - \left(1-\varpi_2\lambda^0\right) \cdot \hat{\phi}_t^k \cdot \frac{\widehat{\log\left(a_t^j\right)}}{\sigma-1} \Bigg]
\end{aligned}$$

$$\begin{aligned}
\sum_{i=1}^{N}\sum_{n=1}^{N} s^i s^n \cdot \widehat{\log\left(s_t^i\right)} \cdot \widehat{\log\left(s_t^n\right)} =& 0 \\
\sum_{i=1}^{N}\sum_{n=1}^{N} s^i s^n \cdot \widehat{\log\left(s_t^i\right)} \cdot \widehat{\log\left(\lambda_t^{0n}\right)} =& 0
\end{aligned}$$

Plugging all the previous summations into (A.90)

$$
\begin{aligned}
E\left[\widehat{\log\left(\lambda_t^0\right)}\right] = {} & \kappa\varpi_2\left(1-\lambda^0\right)\cdot\sum_{i=1}^{N} s^i\cdot E\left[\hat{\phi}_t^i\right] \\
& - \frac{1}{2}\Bigg[(\sigma-1)\kappa\varpi_2\left(1-\lambda^0\right)\left(1-\varpi_2\lambda^0\right) \\
& \qquad - \kappa^2\left(\varpi_2\right)^2\left[\left(1-\lambda^0\right)^2-\lambda^0\left(1-\lambda^0\right)\right]\Bigg]\cdot\sum_{i=1}^{N} s^i\cdot E\left[\left(\hat{\phi}_t^i\right)^2\right] \\
& + \frac{1}{2}\left[(\sigma-1)\kappa\varpi_2\left(1-\lambda^0\right)\left(1-\varpi_2\lambda^0\right)-\kappa^2\left(\varpi_2\right)^2\left(1-\lambda^0\right)^2\right]\cdot\sum_{i=1}^{N}\sum_{n=1}^{N} s^i s^n\cdot E\left[\hat{\phi}_t^i\cdot\hat{\phi}_t^n\right]
\end{aligned}
\tag{A.92}
$$

From the previous expression and equation (A.85) we compute

$$
E\left[\widehat{\log\left(\lambda_t^0\right)}^2\right] = \kappa^2\left(\varpi_2\right)^2\left(1-\lambda^0\right)^2\cdot\sum_{i=1}^{N}\sum_{n=1}^{N} s^i s^n\cdot E\left[\hat{\phi}_t^i\cdot\hat{\phi}_t^n\right] \tag{A.93}
$$

$$
E\left[\widehat{\log\left(\lambda_t^0\right)}\cdot\hat{r}_t^I\right] = \kappa\varpi_2\left(1-\lambda^0\right)\left(1-\varpi_2\lambda^0\right)\cdot\sum_{i=1}^{N}\sum_{n=1}^{N} s^i s^n\cdot E\left[\hat{\phi}_t^i\cdot\hat{\phi}_t^n\right] \tag{A.94}
$$

A.3.2 Welfare derivations

Additional Assumptions

To shed some light on the causes behind the evolution of welfare along the interbank market integration path, we impose additional assumptions on our model in order to obtain a tractable expression for the welfare equation in (**??**) as a function of the underlying model parameters.

1. *CES firm weights:* $E\left[u_t^{a,i} \cdot u_t^{a,n}\right] = \begin{cases} \sigma_a^2 & \text{if } n = i \\ \zeta_a \cdot \sigma_a^2 & \text{otherwise} \end{cases}$

2. *Depositor Preferences:* $E\left[u_t^{T,i} \cdot u_t^{T,i}\right] = \begin{cases} \sigma_T^2 & \text{if } n = i \\ \zeta_T \cdot \sigma_T^2 & \text{otherwise} \end{cases}$

3. *Bilateral Transaction Costs:*

$$E\left[u_t^{I,ji} \cdot u_t^{I,kn}\right] = \begin{cases} 0 & \text{if } j = i \text{ or } k = n \\ \sigma_I^2 & \text{if } k = j,\ n = i \\ \zeta_{I,B} \cdot \sigma_I^2 & \text{if } k \neq j,\ n = i \\ \zeta_{I,L} \cdot \sigma_I^2 & \text{if } k = j,\ n \neq i \\ \zeta_{I,X} \cdot \sigma_I^2 & \text{otherwise} \end{cases}$$

4. *Zero Cross-Correlation:* $E\left[u_t^{I,ji} \cdot u_t^{a,k}\right] = E\left[u_t^{I,ji} \cdot u_t^{T,k}\right] = E\left[u_t^{a,j} \cdot u_t^{T,k}\right] = 0\ ,\ \ \forall j, i, k.$

Welfare Expectations

A second order approximation of equation (A.51) is

$$\widehat{\log\left(a_t^n\right)} = \hat{u}_t^{a,n} - \sum_{i=1}^{N} a^i \hat{u}_t^{a,i} - \frac{1}{2}\sum_{i=1}^{N} a^i\left(1 - a^i\right)\cdot\left(\hat{u}_t^{a,i}\right)^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{n\neq i} a^i a^n \cdot \hat{u}_t^{a,i} \cdot \hat{u}_t^{a,n} \quad \text{(A.95)}$$

Using (A.95), we can compute the following second order approximations

$$\widehat{\log\left(a_t^i\right)}^2 = \left(\hat{u}_t^{a,i}\right)^2 - 2\cdot\sum_{k=1}^{N} a^k \cdot \hat{u}_t^{a,k}\hat{u}_t^{a,i} + \sum_{j=1}^{N}\sum_{k=1}^{N} a^j a^k \cdot \hat{u}_t^{a,j}\hat{u}_t^{a,k} \quad \text{(A.96)}$$

$$\sum_{i=1}^{N}\sum_{n=1}^{N} s^i s^n \cdot \widehat{\log\left(a_t^i\right)} \cdot \widehat{\log\left(a_t^n\right)} = \sum_{i=1}^{N}\sum_{n=1}^{N} s^i s^n \cdot \hat{u}_t^{a,i} \cdot \hat{u}_t^{a,n} - 2 \cdot \sum_{i=1}^{N}\sum_{k=1}^{N} s^i a^k \cdot \hat{u}_t^{a,i} \cdot \hat{u}_t^{a,k} + \sum_{i=1}^{N}\sum_{n=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} s^i s^n a^j a^k \cdot \hat{u}_t^{a,j} \cdot \hat{u}_t^{a,k} \tag{A.97}$$

Using all the previous assumptions, we obtain

$$E\left[\widehat{\log\left(a_t^n\right)}\right] = -\frac{\sigma_a^2}{2}\left(1-\zeta_a\right) \cdot \left[1 - H^a\right] \tag{A.98}$$

where we defined $H^a = \sum_{n=1}^{N}\left(a^n\right)^2$.

We can also compute

$$\sum_{i=1}^{N}\sum_{n=1}^{N} s^i s^n \cdot E\left[\widehat{\log\left(a_t^i\right)} \cdot \widehat{\log\left(a_t^n\right)}\right] = \sum_{i=1}^{N}\sum_{n=1}^{N} s^i s^n \cdot E\left[\hat{u}_t^{a,i} \cdot \hat{u}_t^{a,n}\right] - 2 \cdot \sum_{i=1}^{N}\sum_{k=1}^{N} s^i a^k \cdot E\left[\hat{u}_t^{a,i} \cdot \hat{u}_t^{a,k}\right] + \sum_{i=1}^{N}\sum_{n=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} s^i s^n a^j a^k \cdot E\left[\hat{u}_t^{a,j} \cdot \hat{u}_t^{a,k}\right]$$

Solving the expectations of each individual component

$$\sum_{i=1}^{N}\sum_{n=1}^{N} s^i s^n \cdot E\left[\hat{u}_t^{a,i} \cdot \hat{u}_t^{a,n}\right] = \sigma_a^2\left[\zeta_a + \left(1-\zeta_a\right) \cdot H^F\right]$$

$$\sum_{i=1}^{N}\sum_{k=1}^{N} s^i a^k \cdot E\left[\hat{u}_t^{a,i} \cdot \hat{u}_t^{a,k}\right] = \zeta_a \cdot \sigma_a^2 + \sigma_a^2\left(1-\zeta_a\right)\sum_{i=1}^{N} s^i a^i$$

$$\sum_{i=1}^{N}\sum_{n=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} s^i s^n a^j a^k \cdot E\left[\hat{u}_t^{a,j} \cdot \hat{u}_t^{a,k}\right] = \zeta_a\sigma_a^2 + \sigma_a^2(1-\zeta_a) \cdot H^a$$

where we defined $H^F = \sum_{m=1}^{N}\left(s^m\right)^2$ as the Herfindahl concentration index of the firm loan market.

Plugging back on the original expression and simplifying

$$\sum_{i=1}^{N}\sum_{n=1}^{N} s^i s^n \cdot E\left[\widehat{\log\left(a_t^i\right)} \cdot \widehat{\log\left(a_t^n\right)}\right] = \sigma_a^2 \cdot (1-\zeta_a) \cdot \left[H^F + H^a - 2\sum_{i=1}^{N} a^i s^i\right] \tag{A.99}$$

We now compute the expectations of the following expression

$$E\left[\widehat{\log\left(a_t^i\right)}^2\right] = E\left[\left(\hat{u}_t^{a,i}\right)^2\right] - 2 \cdot \sum_{k=1}^{N} a^k \cdot E\left[\hat{u}_t^{a,k}\hat{u}_t^{a,i}\right] + \sum_{j=1}^{N}\sum_{k=1}^{N} a^j a^k \cdot E\left[\hat{u}_t^{a,j}\hat{u}_t^{a,k}\right]$$

Solving the expectations of the individual components

$$\sum_{j=1}^{N}\sum_{k=1}^{N} a^j a^k \cdot E\left[\hat{u}_t^{a,j}\hat{u}_t^{a,k}\right] = \zeta_a\sigma_a^2 + \sigma_a^2(1-\zeta_a) \cdot H^a$$

$$\sum_{k=1}^{N} a^k \cdot E\left[\hat{u}_t^{a,k}\hat{u}_t^{a,i}\right] = \zeta_a\sigma_a^2 + \sigma_a^2(1-\zeta_a) \cdot a^i$$

Plugging back on the original expression and simplifying

$$E\left[\widehat{\log\left(a_t^i\right)}^2\right] = \sigma_a^2 \cdot (1-\zeta_a) \cdot \left[1 + H^a - 2 \cdot a^i\right]$$

We now compute

$$\sum_{i=1}^{N} s^i \cdot E\left[\widehat{\log\left(a_t^i\right)}^2\right] = \sigma_a^2 \cdot (1-\zeta_a) \cdot \left[1 + H^a - 2 \cdot \sum_{i=1}^{N} s^i \cdot a^i\right] \tag{A.100}$$

Using all the previous information, we now compute

$$\sum_{i=1}^{N} s^i \cdot E\left[\widehat{\log\left(a_t^i\right)}^2\right] - \sum_{i=1}^{N}\sum_{j=1}^{N} s^i s^j \cdot E\left[\widehat{\log\left(a_t^i\right)} \cdot \widehat{\log\left(a_t^j\right)}\right] = \sigma_a^2 \cdot (1-\zeta_a) \cdot \left[1 - H^F\right] \tag{A.101}$$

Computing expectations of equation (A.86) we obtain

$$\begin{aligned} E\left[\hat{\phi}_t^i\right] =& \frac{\kappa}{2}\sum_{n=1}^{N}\sum_{j\neq n}\lambda^{ni}\lambda^{ji}\cdot\left[\zeta_T\cdot\sigma_T^2+\zeta_{I,B}\cdot\sigma_I^2\right]-\frac{\kappa}{2}\sum_{n=1}^{N}\lambda^{ni}(1-\lambda^{ni})\cdot\left[\sigma_T^2+\sigma_I^2\right] \\ =& -\frac{\kappa}{2}\cdot\left(1-H^{I,i}\right)\cdot\left[(1-\zeta_T)\cdot\sigma_T^2+(1-\zeta_{I,B})\cdot\sigma_I^2\right] \end{aligned}$$

Using the previous equation, we compute

$$\sum_{i=1}^{N}s^i\cdot E\left[\hat{\phi}_t^i\right]=-\frac{\kappa}{2}\cdot\left(1-\sum_{i=1}^{N}s^i\cdot H^{I,i}\right)\cdot\left[(1-\zeta_T)\cdot\sigma_T^2+(1-\zeta_{I,B})\cdot\sigma_I^2\right] \tag{A.102}$$

Taking expectations of the following components of equation (A.89) we obtain

$$\sum_{j=1}^{N}\sum_{k=1}^{N}\lambda^{ji}\lambda^{ki}\cdot E\left[\hat{u}_t^{T,j}\cdot\hat{u}_t^{T,k}\right]=\sigma_T^2\cdot\sum_{j=1}^{N}\left(\lambda^{ji}\right)^2+\zeta_T\cdot\sigma_T^2\cdot\sum_{j=1}^{N}\sum_{k\neq j}\lambda^{ji}\lambda^{ki} \tag{A.103}$$
$$=\sigma_T^2\cdot\left[\zeta_T+(1-\zeta_T)\cdot H^{I,i}\right]$$

$$\sum_{j=1}^{N}\sum_{k=1}^{N}\lambda^{ji}\lambda^{ki}\cdot E\left[\hat{u}^{I,ji}\cdot\hat{u}^{I,ki}\right]=\sigma_I^2\cdot\left[H^{I,i}-\left(\lambda_t^{ii}\right)^2\right]+\zeta_{I,B}\cdot\sigma_I^2\cdot\sum_{j\neq i}\sum_{k\neq\{j,i\}}\lambda^{ji}\lambda^{ki} \tag{A.104}$$
$$=\sigma_I^2\cdot\left[H^{I,i}-\left(\lambda_t^{ii}\right)^2\right]+\zeta_{I,B}\cdot\sigma_I^2\cdot\left[\left(1-\lambda^{ii}\right)^2-\left[H^{I,i}-\left(\lambda^{ii}\right)^2\right]\right]$$

Using (A.103) and (A.104), the expectation of (A.89) becomes

$$\begin{aligned} E\left[\left(\hat{\phi}_t^i\right)^2\right]=&\ \sigma_T^2\cdot\left[\zeta_T+(1-\zeta_T)\cdot H^{I,i}\right] \\ &+\sigma_I^2\cdot\left[H^{I,i}-\left(\lambda_t^{ii}\right)^2\right] \\ &-\zeta_{I,B}\cdot\sigma_I^2\cdot\left[H^{I,i}-\left[\left(1-\lambda^{ii}\right)^2+\left(\lambda^{ii}\right)^2\right]\right] \end{aligned}$$

Using the previous expression, we compute

$$\begin{aligned} \sum_{i=1}^{N}s^i\cdot E\left[\left(\hat{\phi}_t^i\right)^2\right]=&\ (1-\zeta_T)\cdot\sigma_T^2\cdot\left[\frac{\zeta_T}{1-\zeta_T}+\sum_{i=1}^{N}s^i\cdot H^{I,i}\right] \\ &+\sigma_I^2\cdot\sum_{i=1}^{N}s^i\cdot\left[H^{I,i}-\left(\lambda_t^{ii}\right)^2\right] \\ &-\zeta_{I,B}\cdot\sigma_I^2\cdot\sum_{i=1}^{N}s^i\cdot\left[H^{I,i}-\left[\left(1-\lambda^{ii}\right)^2+\left(\lambda^{ii}\right)^2\right]\right] \end{aligned} \tag{A.105}$$

Using equation (A.88) we compute

$$\sum_{i=1}^{N}\sum_{n=1}^{N} s^i s^n \cdot E\left[\hat{\phi}_t^i \cdot \hat{\phi}_t^n\right] =$$

$$\sum_{i=1}^{N}\sum_{j=1}^{N} \left(s^i\right)^2 \left(\lambda^{ji}\right)^2 \cdot \left(\hat{u}_t^{T,j}\right)^2 \tag{A.106a}$$

$$+\sum_{i=1}^{N}\sum_{n\neq i}^{N}\sum_{j=1}^{N} s^i s^n \lambda^{ji}\lambda^{jn} \cdot \left(\hat{u}_t^{T,j}\right)^2 \tag{A.106b}$$

$$+\sum_{i=1}^{N}\sum_{n=1}^{N}\sum_{j=1}^{N}\sum_{k\neq j} s^i s^n \lambda^{ji}\lambda^{kn} \cdot \hat{u}_t^{T,j} \cdot \hat{u}_t^{T,k} \tag{A.106c}$$

$$+\sum_{i=1}^{N}\sum_{j\neq i} \left(s^i\right)^2 \left(\lambda^{ji}\right)^2 \cdot \left(\hat{u}_t^{I,ji}\right)^2 \tag{A.106d}$$

$$+\sum_{i=1}^{N}\sum_{j\neq i}\sum_{n\neq\{j,i\}} s^i s^n \lambda^{ji}\lambda^{jn} \cdot \hat{u}_t^{I,ji} \cdot \hat{u}_t^{I,jn} \tag{A.106e}$$

$$+\sum_{i=1}^{N}\sum_{j\neq i}\sum_{k\neq\{j,i\}} \left(s^i\right)^2 \lambda^{ji}\lambda^{ki} \cdot \hat{u}_t^{I,ji} \cdot \hat{u}_t^{I,ki} \tag{A.106f}$$

$$+\sum_{i=1}^{N}\sum_{n\neq i}\sum_{j\neq i}\sum_{k\neq\{j,n\}} s^i s^n \lambda^{ji}\lambda^{kn} \cdot \hat{u}_t^{I,ji} \cdot \hat{u}_t^{I,kn} \tag{A.106g}$$

Now we are going to take expectations of each of the components of (A.106). Equation (A.106a) becomes

$$\sum_{i=1}^{N}\sum_{j=1}^{N} \left(s^i\right)^2 \left(\lambda^{ji}\right)^2 \cdot E\left[\left(\hat{u}_t^{T,j}\right)^2\right] = \sigma_T^2 \cdot \sum_{i=1}^{N} \left(s^i\right)^2 \cdot H^{I,i}$$

Equation (A.106b) becomes

$$\sum_{i=1}^{N}\sum_{n\neq i}^{N}\sum_{j=1}^{N} s^i s^n \lambda^{ji}\lambda^{jn} \cdot E\left[\left(\hat{u}_t^{T,j}\right)^2\right] = \sigma_T^2 \cdot \left[\sum_{i=1}^{N}\sum_{n\neq i}^{N}\sum_{j=1}^{N} s^i s^n \lambda^{ji}\lambda^{jn}\right]$$

Equation (A.106c) becomes

$$\sum_{i=1}^{N}\sum_{n=1}^{N}\sum_{j=1}^{N}\sum_{k\neq j} s^i s^n \lambda^{ji}\lambda^{kn}\cdot E\left[\hat{u}_t^{T,j}\cdot\hat{u}_t^{T,k}\right]$$
$$=\zeta_T\cdot\sigma_T^2\cdot\left[\sum_{i=1}^{N}\sum_{n=1}^{N}\sum_{j=1}^{N}\sum_{k\neq j} s^i s^n \lambda^{ji}\lambda^{kn}\right]$$
$$=\zeta_T\cdot\sigma_T^2\cdot\left[\sum_{i=1}^{N}\sum_{n=1}^{N}\sum_{j=1}^{N} s^i s^n \lambda^{ji}\left(1-\lambda^{jn}\right)\right]$$
$$=\zeta_T\cdot\sigma_T^2\cdot\left[\sum_{i=1}^{N}\sum_{n=1}^{N} s^i s^n \left(1-\sum_{j=1}^{N}\lambda^{ji}\lambda^{jn}\right)\right]$$
$$=\zeta_T\cdot\sigma_T^2\cdot\left[\sum_{i=1}^{N}\left(s^i\right)^2\left(1-\sum_{j=1}^{N}\left(\lambda^{ji}\right)^2\right)+\sum_{i=1}^{N}\sum_{n\neq i}^{N} s^i s^n \left(1-\sum_{j=1}^{N}\lambda^{ji}\lambda^{jn}\right)\right]$$
$$=\zeta_T\cdot\sigma_T^2\cdot\left[\sum_{i=1}^{N}\left(s^i\right)^2\left(1-H^{I,i}\right)+\sum_{i=1}^{N}\sum_{n\neq i}^{N} s^i s^n \left(1-\sum_{j=1}^{N}\lambda^{ji}\lambda^{jn}\right)\right]$$
$$=\zeta_T\cdot\sigma_T^2\cdot\left[1-\sum_{i=1}^{N}\left(s^i\right)^2\cdot H^{I,i}-\sum_{i=1}^{N}\sum_{n\neq i}^{N}\sum_{j=1}^{N} s^i s^n \lambda^{ji}\lambda^{jn}\right]$$

Equation (A.106d) becomes

$$\sum_{i=1}^{N}\sum_{j\neq i}\left(s^i\right)^2\left(\lambda^{ji}\right)^2\cdot E\left[\left(\hat{u}_t^{I,ji}\right)^2\right]=\sigma_I^2\cdot\sum_{i=1}^{N}\left(s^i\right)^2\left[H^{I,i}-\left(\lambda^{ii}\right)^2\right]$$

where we defined $H^{I,i}=\sum_{j=1}^{N}\left(\lambda^{ji}\right)^2$ as the Herfindahl index of concentration of bank i funding sources.

Equation (A.106e) becomes

$$\sum_{i=1}^{N}\sum_{j\neq i}\sum_{n\neq\{j,i\}} s^i s^n \lambda^{ji}\lambda^{jn}\cdot E\left[\hat{u}_t^{I,ji}\cdot\hat{u}_t^{I,jn}\right]=\zeta_{I,L}\cdot\sigma_I^2\left[\sum_{i=1}^{N}\sum_{j\neq i}\sum_{n\neq\{j,i\}} s^i s^n \lambda^{ji}\lambda^{jn}\right]$$

Equation (A.106f) becomes

$$
\begin{aligned}
&\sum_{i=1}^{N}\sum_{j\neq i}\sum_{k\neq\{j,i\}}\left(s^{i}\right)^{2}\lambda^{ji}\lambda^{ki}\cdot E\left[\hat{u}_{t}^{I,ji}\cdot\hat{u}_{t}^{I,ki}\right]\\
&\qquad=\zeta_{I,B}\cdot\sigma_{I}^{2}\cdot\sum_{i=1}^{N}\sum_{j\neq i}\sum_{k\neq\{j,i\}}\left(s^{i}\right)^{2}\lambda^{ji}\lambda^{ki}\\
&\qquad=\zeta_{I,B}\cdot\sigma_{I}^{2}\cdot\sum_{i=1}^{N}\sum_{j\neq i}\left(s^{i}\right)^{2}\left[\lambda^{ji}\left(1-\lambda^{ii}\right)-\left(\lambda^{ji}\right)^{2}\right]\\
&\qquad=-\zeta_{I,B}\cdot\sigma_{I}^{2}\sum_{i=1}^{N}\left(s^{i}\right)^{2}\left[H^{I,i}-\left[\left(1-\lambda^{ii}\right)^{2}+\left(\lambda^{ii}\right)^{2}\right]\right]
\end{aligned}
$$

Equation (A.106g) becomes

$$
\begin{aligned}
&\sum_{i=1}^{N}\sum_{n\neq i}\sum_{j\neq i}\sum_{k\neq\{j,n\}}s^{i}s^{n}\lambda^{ji}\lambda^{kn}\cdot E\left[\hat{u}_{t}^{I,ji}\cdot\hat{u}_{t}^{I,kn}\right]\\
&\qquad=\zeta_{I,x}\cdot\sigma_{I}^{2}\cdot\sum_{i=1}^{N}\sum_{n\neq i}\sum_{j\neq i}\sum_{k\neq\{j,n\}}s^{i}s^{n}\lambda^{ji}\lambda^{kn}\\
&\qquad=\zeta_{I,x}\cdot\sigma_{I}^{2}\cdot\sum_{i=1}^{N}\sum_{n\neq i}\sum_{j\neq i}s^{i}s^{n}\lambda^{ji}\left(1-\lambda^{nn}-\lambda^{jn}\right)\\
&\qquad=\zeta_{I,x}\cdot\sigma_{I}^{2}\cdot\sum_{i=1}^{N}\sum_{n\neq i}s^{i}s^{n}\left[\left(1-\lambda^{ii}\right)\cdot\left(1-\lambda^{nn}\right)-\sum_{j\neq i}\lambda^{ji}\lambda^{jn}\right]
\end{aligned}
$$

Therefore we can express (A.106) as

$$\begin{aligned}
&\sum_{i=1}^{N}\sum_{n=1}^{N} s^i s^n \cdot E\left[\hat{\phi}_t^i \cdot \hat{\phi}_t^n\right] \qquad \text{(A.107)}\\
&\quad = (1-\zeta_T)\cdot\sigma_T^2\cdot\left[\frac{\zeta_T}{1-\zeta_T} + \sum_{i=1}^{N}\left(s^i\right)^2\cdot H^{I,i} + \sum_{i=1}^{N}\sum_{n\neq i}^{N}\sum_{j=1}^{N} s^i s^n \lambda^{ji}\lambda^{jn}\right]\\
&\qquad + \sigma_I^2\cdot\sum_{i=1}^{N}\left(s^i\right)^2\cdot\left[H^{I,i} - \left(\lambda_t^{ii}\right)^2\right]\\
&\qquad - \zeta_{I,B}\cdot\sigma_I^2\cdot\sum_{i=1}^{N}\left(s^i\right)^2\cdot\left[H^{I,i} - \left[\left(1-\lambda^{ii}\right)^2 + \left(\lambda^{ii}\right)^2\right]\right]\\
&\qquad + \ \zeta_{I,L}\cdot\sigma_I^2\cdot\sum_{i=1}^{N}\sum_{j\neq i}\sum_{n\neq\{j,i\}} s^i s^n \lambda^{ji}\lambda^{jn}\\
&\qquad + \ \zeta_{I,x}\cdot\sigma_I^2\cdot\sum_{i=1}^{N}\sum_{n\neq i} s^i s^n\left[\left(1-\lambda^{ii}\right)\cdot\left(1-\lambda^{nn}\right) - \sum_{j\neq i}\lambda^{ji}\lambda^{jn}\right]
\end{aligned}$$

Using (A.101)-(A.102), (A.105) and (A.107) to compute the expectation of (A.85) we obtain

$$
\begin{aligned}
&E\left[\hat{\hat{r}}_t^I\right] = \qquad\text{(A.108)}\\
&\frac{(1-\zeta_T)\cdot\sigma_T^2}{2}\cdot\sum_{i=1}^{N}\Bigg[\left[(\kappa-\sigma+1)(1-\varpi_2\lambda_0)-\kappa\left(\varpi_2\right)^2\left(\lambda^0-\left(\lambda^0\right)^2\right)\right]\cdot s^i\\
&\qquad\qquad\qquad\qquad+(\sigma-1)(1-\varpi\lambda_0)^2\cdot\left(s^i\right)^2\Bigg]\cdot\left[H^{I,i}-1\right]\\
&+\frac{(1-\zeta_T)\cdot\sigma_T^2}{2}\cdot(\sigma-1)\left(1-\varpi_2\lambda^0\right)^2\cdot\left[\sum_{i=1}^{N}\sum_{n\neq i}\sum_{j=1}^{N}s^is^n\lambda^{ji}\lambda^{jn}+H^F\right]\\
&-\frac{(1-\zeta_T)\cdot\sigma_T^2}{2}\cdot\left[\kappa\left(\varpi_2\right)^2\left(\lambda^0-\left(\lambda^0\right)^2\right)+(\sigma-1)\lambda^0(1-\varpi_2\lambda^0)\right]\cdot\frac{1}{1-\zeta_T}\\
&+\frac{\sigma_I^2}{2}\cdot\sum_{i=1}^{N}\left[\left[(\kappa-\sigma+1)(1-\varpi_2\lambda_0)-\kappa\left(\varpi_2\right)^2\left(\lambda^0-\left(\lambda^0\right)^2\right)\right]\cdot s^i+(\sigma-1)(1-\varpi\lambda_0)^2\cdot\left(s^i\right)^2\right]\cdot\left[H^{I,i}-1\right]\\
&+\frac{\sigma_I^2}{2}\cdot\sum_{i=1}^{N}\left[\left[\kappa\left(\varpi_2\right)^2\left(\lambda^0-\left(\lambda^0\right)^2\right)\right]\cdot s^i-(\sigma-1)\left(1-\varpi_2\lambda^0\right)^2\cdot\left(s^i\right)^2\right]\cdot\left[\left(\lambda^{ii}\right)^2-1\right]\\
&-\frac{\zeta_{I,B}\cdot\sigma_I^2}{2}\cdot\sum_{i=1}^{N}\Bigg[\left[(\kappa-\sigma+1)(1-\varpi_2\lambda_0)-\kappa\left(\varpi_2\right)^2\left(\lambda^0-\left(\lambda^0\right)^2\right)\right]\cdot s^i\\
&\qquad\qquad\qquad\qquad+(\sigma-1)(1-\varpi\lambda_0)^2\cdot\left(s^i\right)^2\Bigg]\cdot\left[H^{I,i}-1\right]\\
&-\frac{\zeta_{I,B}\cdot\sigma_I^2}{2}\cdot\sum_{i=1}^{N}\left[\left[\kappa\left(\varpi_2\right)^2\left(\lambda^0-\left(\lambda^0\right)^2\right)\right]\cdot s^i-(\sigma-1)\left(1-\varpi_2\lambda^0\right)^2\cdot\left(s^i\right)^2\right]\cdot\left[\left(1-\lambda^{ii}\right)^2+\left(\lambda^{ii}\right)^2-1\right]\\
&+\frac{\zeta_{I,L}\cdot\sigma_I^2}{2}\cdot(\sigma-1)\left(1-\varpi_2\lambda^0\right)^2\cdot\sum_{i=1}^{N}\sum_{n\neq i}s^is^n\lambda^{ji}\lambda^{jn}\\
&+\frac{\zeta_{I,X}\cdot\sigma_I^2}{2}\cdot(\sigma-1)\left(1-\varpi_2\lambda^0\right)^2\cdot\sum_{i=1}^{N}\sum_{n\neq i}s^is^n\left[\left(1-\lambda^{ii}\right)\left(1-\lambda^{nn}\right)-\sum_{j\neq i}\lambda^{ji}\lambda^{jn}\right]\\
&-\left(\frac{\sigma-1}{2}\right)\left(\frac{\sigma_a}{\sigma-1}\right)^2(1-\zeta_a)\cdot\left[1-H^F\right]
\end{aligned}
$$

Using (A.99), (A.102), (A.105) and (A.107) to compute the expected value of (A.87) we obtain

$$E\left[\left(\hat{\tilde{r}}_t^I\right)^2\right] = (1-\zeta_T)\,\sigma_T^2\cdot\left(1-\varpi_2\lambda^0\right)^2\cdot\left[\frac{\zeta_T}{1-\zeta_T}+\sum_{i=1}^{N}\left(s^i\right)^2\cdot H^{I,i}+\sum_{i=1}^{N}\sum_{n\neq i}^{N}\sum_{j=1}^{N}s^i s^n\lambda^{ji}\lambda^{jn}\right] \tag{A.109}$$

$$+\,\sigma_I^2\cdot\left(1-\varpi_2\lambda^0\right)^2\cdot\sum_{i=1}^{N}\left(s^i\right)^2\cdot\left[H^{I,i}-\left(\lambda^{ii}\right)^2\right]$$

$$-\,\zeta_{I,B}\cdot\sigma_I^2\cdot\left(1-\varpi_2\lambda^0\right)^2\cdot\sum_{i=1}^{N}\left(s^i\right)^2\cdot\left[H^{I,i}-\left[\left(1-\lambda^{ii}\right)^2+\left(\lambda^{ii}\right)^2\right]\right]$$

$$+\,\zeta_{I,L}\cdot\sigma_I^2\cdot\left(1-\varpi_2\lambda^0\right)^2\cdot\sum_{i=1}^{N}\sum_{j\neq i}\sum_{n\neq\{j,i\}}s^i s^n\lambda^{ji}\lambda^{jn}$$

$$+\,\zeta_{I,x}\cdot\sigma_I^2\cdot\left(1-\varpi_2\lambda^0\right)^2\cdot\sum_{i=1}^{N}\sum_{n\neq i}s^i s^n\left[\left(1-\lambda^{ii}\right)\cdot\left(1-\lambda^{nn}\right)-\sum_{j\neq i}\lambda^{ji}\lambda^{jn}\right]$$

$$+\left(\frac{\sigma_a}{1-\sigma}\right)^2\cdot\left[\zeta_a+(1-\zeta_a)\cdot H^F\right]$$

Using (A.108) and (A.109) we compute

$$
\begin{aligned}
&-E\left[\left(\frac{\alpha}{1-\alpha}\right)\cdot\hat{\tilde{r}}^{I}_{t}+\frac{\Lambda_I}{2}\cdot\left(\hat{\tilde{r}}^{I}_{t}\right)^2\right]= \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(A.110)}\\
&\frac{(1-\zeta_T)\cdot\sigma_T^2}{2}\cdot\left[\sum_{i=1}^{N}\left[\chi_1\cdot s^i+\chi_3\cdot\left(s^i\right)^2\right]\cdot\left[1-H^{I,i}\right]-\chi_3\cdot\sum_{i=1}^{N}\sum_{n\neq i}^{N}\sum_{j=1}^{N}s^is^n\lambda^{ji}\lambda^{jn}-\chi_3\cdot H^F\right]\\
&-\frac{\sigma_T^2}{2}\cdot\chi_4\\
&+\frac{\sigma_I^2}{2}\cdot\left[\sum_{i=1}^{N}\left[\chi_1\cdot s^i+\chi_3\cdot\left(s^i\right)^2\right]\cdot\left[1-H^{I,i}\right]-\sum_{i=1}^{N}\left[\chi_3\cdot\left(s^i\right)^2-\chi_2\cdot s^i\right]\cdot\left[1-\left(\lambda^{ii}\right)^2\right]\right]\\
&-\frac{\zeta_{I,B}\cdot\sigma_I^2}{2}\cdot\left[\sum_{i=1}^{N}\left[\chi_1\cdot s^i+\chi_3\cdot\left(s^i\right)^2\right]\cdot\left[1-H^{I,i}\right]\right.\\
&\qquad\qquad\qquad\left.-\sum_{i=1}^{N}\left[\chi_3\cdot\left(s^i\right)^2-\chi_2\cdot s^i\right]\cdot\left[1-\left(1-\lambda^{ii}\right)^2-\left(\lambda^{ii}\right)^2\right]\right]\\
&-\frac{\zeta_{I,L}\cdot\sigma_I^2}{2}\cdot\chi_3\cdot\sum_{i=1}^{N}\sum_{j\neq i}\sum_{n\neq\{j,i\}}s^is^n\lambda^{ji}\lambda^{jn}\\
&-\frac{\zeta_{I,x}\cdot\sigma_I^2}{2}\cdot\chi_3\cdot\sum_{i=1}^{N}\sum_{n\neq i}s^is^n\left[\left(1-\lambda^{ii}\right)\cdot\left(1-\lambda^{nn}\right)-\sum_{j\neq i}\lambda^{ji}\lambda^{jn}\right]\\
&+\frac{(1-\zeta_a)}{2}\left(\frac{\sigma_a}{1-\sigma}\right)^2\cdot\left[\chi_5\cdot\left[1-H^F\right]+2\Lambda_I\sum_{i=1}^{N}a^i\cdot s^i-\chi_6\right]
\end{aligned}
$$

where we defined

$$
\begin{aligned}
&\chi_1=\left(\frac{\alpha}{1-\alpha}\right)\cdot\left[(\kappa-\sigma+1)\left(1-\varpi_2\lambda^0\right)-\kappa\left(\varpi_2\right)^2\left(\lambda^0-\left(\lambda^0\right)^2\right)\right]; && \chi_4=\Lambda_I\left(1-\varpi_2\lambda^0\right)^2\cdot\zeta_T-\chi_2\\
&\chi_2=\left(\frac{\alpha}{1-\alpha}\right)\cdot\left[\kappa\left(\varpi_2\right)^2\left(\lambda^0-\left(\lambda^0\right)^2\right)+(\sigma-1)\left(1-\varpi_2\lambda^0\right)\right]; && \chi_5=\left(\frac{\alpha}{1-\alpha}\right)(\sigma-1)+\Lambda_I\\
&\chi_3=\left(\frac{\alpha}{1-\alpha}\right)\left(1-\varpi_2\lambda^0\right)^2\left[\sigma-1+\cdot\Lambda_I\right]; && \chi_6=\Lambda_I\cdot\left[1+H^a\right]
\end{aligned}
$$

Substituting (A.102), (A.105) and (A.107) on (A.92) and rearranging

$$
\begin{aligned}
&\left(\frac{\lambda^0}{1-\lambda^0}\right)\cdot E\left[\widehat{\log\left(\lambda^0_t\right)}\right] = \qquad \text{(A.111)}\\
&\frac{(1-\zeta_T)\cdot\sigma_T^2}{2}\cdot\sum_{i=1}^{N}\Bigg[\lambda^0\left[\kappa^2\varpi_2+\kappa^2\left(\varpi_2\right)^2\left(1-2\lambda^0\right)-(\sigma-1)\kappa\varpi_2\left(1-\varpi_2\lambda^0\right)\right]\cdot s^i\\
&\qquad +\lambda^0\left[(\sigma-1)\kappa\varpi_2(1-\varpi_2\lambda^0)-\kappa^2\left(\varpi_2\right)^2\left(1-\lambda^0\right)\right]\cdot\left(s^i\right)^2\Bigg]\cdot\left[H^{I,i}-1\right]\\
&+\frac{(1-\zeta_T)\cdot\sigma_T^2}{2}\cdot\lambda^0\left[(\sigma-1)\kappa\varpi_2(1-\varpi_2\lambda^0)-\kappa^2\left(\varpi_2\right)^2\left(1-\lambda^0\right)\right]\cdot\sum_{i=1}^{N}\sum_{n\neq i}\sum_{j=1}^{N}s^is^n\lambda^{ji}\lambda^{jn}\\
&+\frac{(1-\zeta_T)\cdot\sigma_T^2}{2}\cdot\lambda^0\left[(\sigma-1)\kappa\varpi_2(1-\varpi_2\lambda^0)-\kappa^2\left(\varpi_2\right)^2\left(1-\lambda^0\right)\right]\cdot H^F\\
&+\frac{\sigma_T^2}{2}\cdot\Bigg[\lambda^0\left[(\sigma-1)\kappa\varpi_2(1-\varpi_2\lambda^0)-\kappa^2\left(\varpi_2\right)^2\left(1-\lambda^0\right)\right]\zeta_T\\
&\qquad -\lambda^0\left[(\sigma-1)\kappa\varpi_2(1-\varpi_2\lambda^0)-\kappa^2\left(\varpi_2\right)^2\left(1-2\lambda^0\right)\right]\Bigg]\\
&+\frac{\sigma_I^2}{2}\cdot\sum_{i=1}^{N}\Bigg[\lambda^0\left[\kappa^2\varpi_2+\kappa^2\left(\varpi_2\right)^2\left(1-2\lambda^0\right)-(\sigma-1)\kappa\varpi_2\left(1-\varpi_2\lambda^0\right)\right]\cdot s^i\\
&\qquad +\lambda^0\left[(\sigma-1)\kappa\varpi_2(1-\varpi_2\lambda^0)-\kappa^2\left(\varpi_2\right)^2\left(1-\lambda^0\right)\right]\cdot\left(s^i\right)^2\Bigg]\cdot\left[H^{I,i}-1\right]\\
&-\frac{\sigma_I^2}{2}\cdot\sum_{i=1}^{N}\Bigg[\lambda^0\left[(\sigma-1)\kappa\varpi_2(1-\varpi_2\lambda^0)-\kappa^2\left(\varpi_2\right)^2\left(1-\lambda^0\right)\right]\cdot\left(s^i\right)^2\\
&\qquad -\lambda^0\left[(\sigma-1)\kappa\varpi_2(1-\varpi_2\lambda^0)-\kappa^2\left(\varpi_2\right)^2\left(1-2\lambda^0\right)\right]\cdot s^i\Bigg]\cdot\left[\left(\lambda^{ii}\right)^2-1\right]\\
&-\frac{\zeta_{I,B}\cdot\sigma_I^2}{2}\cdot\sum_{i=1}^{N}\Bigg[\lambda^0\left[\kappa^2\varpi_2+\kappa^2\left(\varpi_2\right)^2\left(1-2\lambda^0\right)-(\sigma-1)\kappa\varpi_2\left(1-\varpi_2\lambda^0\right)\right]\cdot s^i\\
&\qquad +\lambda^0\left[(\sigma-1)\kappa\varpi_2(1-\varpi_2\lambda^0)-\kappa^2\left(\varpi_2\right)^2\left(1-\lambda^0\right)\right]\cdot\left(s^i\right)^2\Bigg]\cdot\left[H^{I,i}-1\right]\\
&+\frac{\zeta_{I,B}\cdot\sigma_I^2}{2}\cdot\sum_{i=1}^{N}\Bigg[\lambda^0\left[(\sigma-1)\kappa\varpi_2(1-\varpi_2\lambda^0)-\kappa^2\left(\varpi_2\right)^2\left(1-\lambda^0\right)\right]\cdot\left(s^i\right)^2\\
&\qquad -\lambda^0\left[(\sigma-1)\kappa\varpi_2(1-\varpi_2\lambda^0)-\kappa^2\left(\varpi_2\right)^2\left(1-2\lambda^0\right)\right]\cdot s^i\Bigg]\cdot\left[\left(1-\lambda^{ii}\right)^2+\left(\lambda^{ii}\right)^2-1\right]
\end{aligned}
$$

$$-\frac{\zeta_{I,L}\sigma_I^2}{2}\cdot\lambda^0\left[(\sigma-1)\kappa\varpi_2(1-\varpi_2\lambda^0)-\kappa^2\left(\varpi_2\right)^2\left(1-\lambda^0\right)\right]\cdot\sum_{i=1}^{N}\sum_{j\neq i}\sum_{n\neq\{j,i\}}s^i s^n\lambda^{ji}\lambda^{jn}$$

$$-\frac{\zeta_{I,X}\cdot\sigma_I^2}{2}\cdot\lambda^0\Bigg[(\sigma-1)\kappa\varpi_2(1-\varpi_2\lambda^0)$$

$$-\kappa^2\left(\varpi_2\right)^2\left(1-\lambda^0\right)\Bigg]\cdot\sum_{i=1}^{N}\sum_{n\neq i}s^i s^n\left[\left(1-\lambda^{ii}\right)\cdot\left(1-\lambda^{nn}\right)-\sum_{j\neq i}\lambda^{ji}\lambda^{jn}\right]$$

Substituting (A.102), (A.105) and (A.107) on the expectation of (A.93) and rearranging

$$E\left[\widehat{\log\left(\lambda_t^0\right)}^2\right]= \tag{A.112}$$

$$\left(1-\zeta_T\right)\cdot\sigma_T^2\cdot\kappa^2\left(\varpi_2\right)^2\left(1-\lambda^0\right)^2\cdot\left[\frac{\zeta_T}{1-\zeta_T}+\sum_{i=1}^{N}\left(s^i\right)^2\cdot H^{I,i}+\sum_{i=1}^{N}\sum_{n\neq i}\sum_{j=1}^{N}s^i s^n\lambda^{ji}\lambda^{jn}\right]$$

$$+\sigma_I^2\cdot\kappa^2\left(\varpi_2\right)^2\left(1-\lambda^0\right)^2\cdot\sum_{i=1}^{N}\left(s^i\right)^2\cdot\left[H^{I,i}-\left(\lambda^{ii}\right)^2\right]$$

$$-\zeta_{I,B}\cdot\sigma_I^2\cdot\kappa^2\left(\varpi_2\right)^2\left(1-\lambda^0\right)^2\cdot\sum_{i=1}^{N}\left(s^i\right)^2\cdot\left[H^{I,i}-\left(1-\lambda^{ii}\right)^2-\left(\lambda^{ii}\right)^2\right]$$

$$+\zeta_{I,L}\cdot\sigma_I^2\cdot\kappa^2\left(\varpi_2\right)^2\left(1-\lambda^0\right)^2\cdot\sum_{i=1}^{N}\sum_{j\neq i}\sum_{n\neq\{j,i\}}s^i s^n\lambda^{ji}\lambda^{jn}$$

$$+\zeta_{I,X}\cdot\sigma_I^2\cdot\kappa^2\left(\varpi_2\right)^2\left(1-\lambda^0\right)^2\cdot\sum_{i=1}^{N}\sum_{n\neq i}s^i s^n\left[\left(1-\lambda^{ii}\right)\cdot\left(1-\lambda^{nn}\right)-\sum_{j\neq i}\lambda^{ji}\lambda^{jn}\right]$$

Substituting (A.102), (A.105) and (A.107) on the expectation of (A.94) and rearranging

$$E\left[\widehat{\log\left(\lambda_t^0\right)}\cdot\hat{\tilde{r}}_t^I\right]= \tag{A.113}$$

$$(1-\zeta_T)\cdot\sigma_T^2\cdot\kappa\varpi_2\left(1-\lambda^0\right)\left(1-\varpi_2\lambda^0\right)\cdot\left[\frac{\zeta_T}{1-\zeta_T}+\sum_{i=1}^{N}\left(s^i\right)^2\cdot H^{I,i}+\sum_{i=1}^{N}\sum_{n\neq i}\sum_{j=1}^{N}s^is^n\lambda^{ji}\lambda^{jn}\right]$$

$$+\sigma_I^2\cdot\kappa\varpi_2\left(1-\lambda^0\right)\left(1-\varpi_2\lambda^0\right)\cdot\sum_{i=1}^{N}\left(s^i\right)^2\cdot\left[H^{I,i}-\left(\lambda^{ii}\right)^2\right]$$

$$-\zeta_{I,B}\cdot\sigma_I^2\cdot\kappa\varpi_2\left(1-\lambda^0\right)\left(1-\varpi_2\lambda^0\right)\cdot\sum_{i=1}^{N}\left(s^i\right)^2\cdot\left[H^{I,i}-\left(1-\lambda^{ii}\right)^2-\left(\lambda^{ii}\right)^2\right]$$

$$+\zeta_{I,L}\cdot\sigma_I^2\cdot\kappa\varpi_2\left(1-\lambda^0\right)\left(1-\varpi_2\lambda^0\right)\cdot\sum_{i=1}^{N}\sum_{j\neq i}\sum_{n\neq\{j,i\}}s^is^n\lambda^{ji}\lambda^{jn}$$

$$+\zeta_{I,X}\cdot\sigma_I^2\cdot\kappa\varpi_2\left(1-\lambda^0\right)\left(1-\varpi_2\lambda^0\right)\cdot\sum_{i=1}^{N}\sum_{n\neq i}s^is^n\left[\left(1-\lambda^{ii}\right)\cdot\left(1-\lambda^{nn}\right)-\sum_{j\neq i}\lambda^{ji}\lambda^{jn}\right]$$

Using (A.111), (A.112) and (A.113) we compute

$$-E\left[\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\eta}{\eta+1}\right)\left(\frac{\lambda^0}{1-\lambda^0}\right)\cdot\widehat{\log\left(\lambda_t^0\right)}+\frac{\Lambda_{CB}}{2}\cdot\widehat{\log\left(\lambda_t^0\right)}^2+\Lambda^{I\times CB}\cdot\widehat{\log\left(\lambda_t^0\right)}\cdot\hat{\tilde{r}}_t^I\cdot\right]= \tag{A.114}$$

$$\begin{aligned}
&\frac{(1-\zeta_T)\cdot\sigma_T^2}{2}\cdot\left[\sum_{i=1}^N\left[\upsilon_1\cdot s^i+\upsilon_3\cdot\left(s^i\right)^2\right]\cdot\left[1-H^{I,i}\right]-\upsilon_3\cdot\sum_{i=1}^N\sum_{n\neq i}^N\sum_{j=1}^N s^is^n\lambda^{ji}\lambda^{jn}-\upsilon_3\cdot H^F\right]\\
&-\frac{\sigma_T^2}{2}\cdot\upsilon_4\\
&+\frac{\sigma_I^2}{2}\cdot\left[\sum_{i=1}^N\left[\upsilon_1\cdot s^i+\upsilon_3\cdot\left(s^i\right)^2\right]\cdot\left[1-H^{I,i}\right]-\sum_{i=1}^N\left[\upsilon_3\cdot\left(s^i\right)^2-\upsilon_2\cdot s^i\right]\cdot\left[1-\left(\lambda^{ii}\right)^2\right]\right]\\
&-\frac{\zeta_{I,B}\cdot\sigma_I^2}{2}\cdot\left[\sum_{i=1}^N\left[\upsilon_1\cdot s^i+\upsilon_3\cdot\left(s^i\right)^2\right]\cdot\left[1-H^{I,i}\right]\right.\\
&\qquad\qquad\left.-\sum_{i=1}^N\left[\upsilon_3\cdot\left(s^i\right)^2-\upsilon_2\cdot s^i\right]\cdot\left[1-\left(1-\lambda^{ii}\right)^2-\left(\lambda^{ii}\right)^2\right]\right]\\
&-\frac{\zeta_{I,L}\cdot\sigma_I^2}{2}\cdot\upsilon_3\cdot\sum_{i=1}^N\sum_{j\neq i}\sum_{n\neq\{j,i\}}s^is^n\lambda^{ji}\lambda^{jn}\\
&-\frac{\zeta_{I,x}\cdot\sigma_I^2}{2}\cdot\upsilon_3\cdot\sum_{i=1}^N\sum_{n\neq i}s^is^n\left[\left(1-\lambda^{ii}\right)\cdot\left(1-\lambda^{nn}\right)-\sum_{j\neq i}\lambda^{ji}\lambda^{jn}\right]
\end{aligned}$$

where we defined

$$\begin{aligned}
\upsilon_0&=\kappa^2(\varpi_2)^2\left(1-\lambda^0\right)^2\cdot\Lambda_{CB}+2\kappa\varpi_2\left(1-\lambda^0\right)\left(1-\varpi_2\lambda^0\right)\cdot\Lambda_{I\times CB}\\
\upsilon_1&=\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\eta}{\eta+1}\right)\lambda^0\left[\kappa^2\varpi_2+\kappa^2\left(\varpi_2\right)^2\left[1-2\cdot\lambda^0\right]-(\sigma-1)\kappa\varpi_2\left(1-\varpi_2\lambda^0\right)\right]\\
\upsilon_2&=\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\eta}{\eta+1}\right)\lambda^0\left[(\sigma-1)\kappa\varpi_2\left(1-\varpi_2\lambda^0\right)-\kappa^2\left(\varpi_2\right)^2\left[1-2\lambda^o\right]\right]\\
\upsilon_3&=\upsilon_0+\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\eta}{\eta+1}\right)\lambda^0\left[(\sigma-1)\kappa\varpi_2\left(1-\varpi_2\lambda^0\right)-\kappa^2\left(\varpi_2\right)^2\left(1-\lambda^o\right)\right]\\
\upsilon_4&=\zeta_T\cdot\upsilon_3-\upsilon_2\\
\upsilon_5&=0\\
\upsilon_6&=0
\end{aligned}$$

Combining (A.110) and (A.114) we obtain

$$\mathbf{L} = \frac{(1-\zeta_T)\cdot\sigma_T^2}{2}\cdot\left[\left[\Theta_1+\Theta_3\cdot H^F\right]\cdot\sum_{i=1}^{N}\omega^{H,i}\cdot\left[1-H^{I,i}\right]-\Theta_3\cdot\sum_{i=1}^{N}\sum_{n\neq i}^{N}\sum_{j=1}^{N}s^i s^n\lambda^{ji}\lambda^{jn}-\Theta_3\cdot H^F\right]-\frac{\sigma_T^2}{2}\cdot\Theta_4 \tag{A.115}$$

$$+\frac{\sigma_I^2}{2}\cdot\left[\left[\Theta_1+\Theta_3\cdot H^F\right]\cdot\sum_{i=1}^{N}\omega^{H,i}\cdot\left[1-H^{I,i}\right]-\left[\Theta_3\cdot H^F-\Theta_2\right]\cdot\sum_{i=1}^{N}\omega^{\lambda,i}\cdot\left[1-\left(\lambda^{ii}\right)^2\right]\right]$$

$$-\frac{\zeta_{I,B}\cdot\sigma_I^2}{2}\cdot\left[\left[\Theta_1+\Theta_3\cdot H^F\right]\cdot\sum_{i=1}^{N}\omega^{H,i}\cdot\left[1-H^{I,i}\right]-\left[\Theta_3\cdot H^F-\Theta_2\right]\cdot\sum_{i=1}^{N}\omega^{\lambda,i}\cdot\left[1-\left(1-\lambda^{ii}\right)^2-\left(\lambda^{ii}\right)^2\right]\right]$$

$$-\frac{\zeta_{I,L}\cdot\sigma_I^2}{2}\cdot\Theta_3\cdot\sum_{i=1}^{N}\sum_{j\neq i}\sum_{n\neq\{j,i\}}s^i s^n\lambda^{ji}\lambda^{jn}$$

$$-\frac{\zeta_{I,x}\cdot\sigma_I^2}{2}\cdot\Theta_3\cdot\sum_{i=1}^{N}\sum_{n\neq i}s^i s^n\left[\left(1-\lambda^{ii}\right)\cdot\left(1-\lambda^{nn}\right)-\sum_{j\neq i}\lambda^{ji}\lambda^{jn}\right]$$

$$+\frac{(1-\zeta_a)}{2}\left(\frac{\sigma_a}{\sigma-1}\right)^2\cdot\left[\Theta_5\cdot\left[1-H^F\right]+2\cdot\Lambda_I\sum_{i=1}^{N}a^i\cdot s^i-\Theta_6\right]$$

$$\mathbf{J} = -\frac{1}{\kappa}\left(\frac{\alpha}{1-\alpha}\right)\cdot\log\left(\lambda^{Own}\right) \tag{A.116}$$

$$+\frac{(1-\zeta_T)\cdot\sigma_T^2}{2}\cdot\left[\left[\Theta_1+\Theta_3\cdot H^F\right]\cdot\sum_{i=1}^{N}\omega^{H,i}\cdot\left[1-H^{I,i}\right]\right.$$
$$\left.-\Theta_3\cdot\sum_{i=1}^{N}\sum_{n\neq i}^{N}\sum_{j=1}^{N}s^i s^n\lambda^{ji}\lambda^{jn}-\Theta_3\cdot\left[H^F-H^{F,AU}\right]\right]$$

$$+\frac{\sigma_I^2}{2}\cdot\left[\left[\Theta_1+\Theta_3\cdot H^F\right]\cdot\sum_{i=1}^{N}\omega^{H,i}\cdot\left[1-H^{I,i}\right]-\left[\Theta_3\cdot H^F-\Theta_2\right]\cdot\sum_{i=1}^{N}\omega^{\lambda,i}\cdot\left[1-\left(\lambda^{ii}\right)^2\right]\right]$$

$$-\frac{\zeta_{I,B}\cdot\sigma_I^2}{2}\cdot\left[\left[\Theta_1+\Theta_3\cdot H^F\right]\cdot\sum_{i=1}^{N}\omega^{H,i}\cdot\left[1-H^{I,i}\right]-\left[\Theta_3\cdot H^F-\Theta_2\right]\cdot\sum_{i=1}^{N}\omega^{\lambda,i}\cdot\left[1-\left(1-\lambda^{ii}\right)^2-\left(\lambda^{ii}\right)^2\right]\right]$$

$$-\frac{\zeta_{I,L}\cdot\sigma_I^2}{2}\cdot\Theta_3\cdot\sum_{i=1}^{N}\sum_{j\neq i}\sum_{n\neq\{j,i\}}s^i s^n\lambda^{ji}\lambda^{jn}$$

$$-\frac{\zeta_{I,x}\cdot\sigma_I^2}{2}\cdot\Theta_3\cdot\sum_{i=1}^{N}\sum_{n\neq i}s^i s^n\left[\left(1-\lambda^{ii}\right)\cdot\left(1-\lambda^{nn}\right)-\sum_{j\neq i}\lambda^{ji}\lambda^{jn}\right]$$

$$-\frac{(1-\zeta_a)}{2}\left(\frac{\sigma_a}{\sigma-1}\right)^2\cdot\left[\Theta_5\cdot\left[H^F-H^{F,AU}\right]-2\cdot\Lambda_I\sum_{i=1}^{N}a^i\cdot\left(s^i-s^{i,AU}\right)\right]$$

where we defined

$$
\begin{aligned}
\Theta_l &= \chi_l + \upsilon_l,\ \ l = 1, \ldots, 6 \\
\omega^{H,i} &= \frac{\Theta_1 \cdot s^i + \Theta_3 \cdot \left(s^i\right)^2}{\Theta_1 + \Theta_3 \cdot H^F} \\
\omega^{\lambda,i} &= \frac{\Theta_3 \cdot \left(s^i\right)^2 - \Theta_2 \cdot s^i}{\Theta_3 \cdot H^F - \Theta_2}
\end{aligned}
$$

A.4 Parameter calibration

Parameter	Value	Description	Source
η	1	Frisch labor supply elasticity	Standard
β	0.99	Discount factor	Standard
ϵ	7	Elasticity of substitution intermediate output	Standard
θ	0.55	Calvo price stickiness	Standard
α	0.4	Capital share in production	Aggregate data
σ	37.8	Firm's loan elasticity of demand	Estimation, Sec. 2.7
κ	21.3	Interbank loan elasticity of demand	Estimation, Sec. 2.7
$\bar{\Pi}$	0	Target inflation rate	Standard
γ_π	2.5	Taylor rule inflation response	Standard
γ_y	1.5	Taylor rule output gap response	Standard
γ_I	0	Taylor rule interbank rate response	Baseline assumption
ϖ_1	0.47	Fixed penalty rate	Match 3.5% pre-crisis central bank trade share
ϖ_2	0.25	Variable penalty rate responsiveness	Educated guess
ρ_I	0.79	Persistence interbank shocks	Estimation, Sec. 2.7
ζ_T	0.31	Covariance depositor preferences shock	Estimation, Sec. 2.7
$\zeta_{I,B}$	0.054	Covariance interbank transactions shock, same borrower	Estimation, Sec. 2.7
$\zeta_{I,L}$	1	Covariance interbank transactions shock, same lender	Estimation, Sec. 2.7
$\zeta_{I,X}$	0.034	Covariance interbank transactions shock, different lender and borrower	Estimation, Sec. 2.7
$\sigma_a \cdot (1 - \zeta_a)$	0.0013	Standard deviation firm-loan demand shock and covariance, joint	Estimation, Sec. 2.7
σ_T	0.016	Standard deviation depositor preferences shock	Estimation, Sec. 2.7
σ_I	0.049	Standard deviation interbank transactions shock	Estimation, Sec. 2.7

Table A.1: The table presents the baseline parameter values used to calibrate the model in Section 2.7. Last column indicates reference source for parameter calibration.

Chapter B

Appendices to Chapter 4

B.1 Model

In this appendix, we present a more detailed version of the model from section 4.2 in the paper.

Household
The representative consumer maximizes the present discounted value of the utility stream from consumption and leisure

$$\max E_t \sum_{j=0}^{\infty} \beta^j \left\{ \log \left(C_{t+j} - hGA_{t+j} C_{t+j-1} \right) - \frac{\eta}{\eta+1} \int_0^1 N_{t+j}\left(i\right)^{1+1/\eta} di \right\} \tag{B.1}$$

where C is consumption of the final good, $N(i)$ is labor supplied to individual industry i, GA is the gross growth rate of technology, η is the Frisch labor supply elasticity, h the internal habit parameter and β is the discount factor.[1] The budget constraint each period t is given by

$$\varrho_t : C_t + \frac{S_t}{P_t} + T_t \leq \int_0^1 \left(\frac{N_t\left(i\right) W_t\left(i\right)}{P_t} \right) di + \frac{S_{t-1} q_{t-1} R_{t-1}}{P_t} + \Gamma_t \tag{B.2}$$

where S is the stock of one-period bonds held by the consumer, R is the gross nominal interest rate, P is the price of the final good, $W(i)$ is the nominal wage earned from labor in industry i, T is real lump sum taxation (or transfers), Γ are real profits from ownership of

[1] We use internal habits rather than external habits because they more closely match the (lack of) persistence in consumption growth in the data. The gross growth rate of technology enters the habit term to simplify derivations.

firms, q is a risk premium shock, and is the shadow value of wealth.[2]

The risk premium shock q is defined as follows:

$$q_t = \exp\left(u_t^q\right)$$
$$u_t^q = \rho_q u_{t-1}^q + \varepsilon_{t-1}^q$$

with ε_{t-1}^q iid normally distributed.

The first order conditions from this utility-maximization problem are then:

$$\varrho_t = \left(C_t - hGA_tC_{t-1}\right)^{-1} - \beta h E_t G A_{t+1}\left(C_{t+1} - hGA_{t+1}C_t\right)^{-1}, \tag{B.3}$$
$$N_t(i)^{1/\eta} = \varrho_t W_t(i)/P_t, \tag{B.4}$$
$$\varrho_t/P_t = \beta E_t\left[\varrho_{t+1} q_t R_t / P_{t+1}\right]. \tag{B.5}$$

Final Goods

Production of the final good is done by a perfectly competitive sector which combines a continuum of intermediate goods into a final good per the following aggregator

$$Y_t = \left[\int_0^1 Y_t(i)^{(\theta-1)/\theta}\, di\right]^{\theta/(\theta-1)} \tag{B.6}$$

where Y is the final good and $Y(i)$ is intermediate good i, while θ denotes the elasticity of substitution across intermediate goods, yielding the following demand curve for goods of intermediate sector i

$$Y_t(i) = Y_t\left(P_t(i)/P_t\right)^{-\theta} \tag{B.7}$$

and the following expression for the aggregate price level

$$P_t = \left[\int_0^1 P_t(i)^{(1-\theta)}\, di\right]^{1/(1-\theta)}. \tag{B.8}$$

[2]As discussed in Smets and Wouters (2007), a positive shock to q, which is the wedge between the interest rate controlled by the central bank and the return on assets held by the households, increases the required return on assets and reduces current consumption. The shock q has similar effects as net-worth shocks in models with financial accelerators. Such financial shocks have arguably played a major role in causing the zero lower bound to bind in practice. Amano and Shukayev (2012) also document that shocks like q are essential for generating a binding zero lower bound in the New Keynesian model.

Government
We allow for government consumption of final goods (G). Government budget constraint is defined as

$$\frac{T_t + S_t}{P_t} = G_t + \frac{S_{t-1} q_{t-1} R_{t-1}}{P_t} + \nu \int_0^1 \left(\frac{N_t(i) W_t(i)}{P_t} \right) di \tag{B.9}$$

$$G_t = \exp\left(u_t^G\right) \overline{G}_t \tag{B.10}$$

$$u_t^G = \rho_G u_{t-1}^G + \varepsilon_t^G$$

with ε_{t-1}^G iid normally distributed. $\overline{G}_t$ is the path of government spending such that the share of government spending in the economy is fixed when prices are flexible. Substituting household's budget constraint (B.2) into the government budget constraint (B.9)

$$C_t + G_t = \Gamma_t + (1-\nu) \int_0^1 \left(N_t(i) W_t(i) / P_t \right) di \tag{B.11}$$

Market Clearing
Firms' aggregate real profits are

$$\Gamma_t = \int_0^1 \Gamma_t(i)\, di = \frac{1}{P_t} \int_0^1 P_t(i) Y_t(i) - (1-\nu) N_t(i) W_t(i)\, di$$

$$= Y_t - (1-\nu) \int_0^1 \left(N_t(i) W_t(i) / P_t \right) di \tag{B.12}$$

Plug (B.12) in (B.11), this gives us the goods market clearing condition for the economy

$$Y_t = C_t + G_t. \tag{B.13}$$

Intermediate Goods
The production of each intermediate good is done by a monopolist facing a production function linear in labor

$$Y_t(i) = A_t N_t(i) \tag{B.14}$$

$$A_t = exp\left(u_t^A\right)$$

$$u_t^A = \mu + u_{t-1}^A + \varepsilon_{t-1}^A$$

with ε_{t-1}^A iid normally distributed, and A denotes the level of technology, common across firms. Each intermediate good producer has sticky prices, modeled as in Calvo (1983) where

$1-\lambda$ is the probability that each firm will be able to reoptimize its price each period. We allow for indexation of prices to steady-state inflation by firms who do not reoptimize their prices each period, with ω_s and ω_d respectively representing the degree of static and dynamic indexation (0 for no indexation to 1 for full indexation). Denoting the optimal reset price of firm i by $B(i)$, re-optimizing firms solve the following profit-maximization problem

$$\max_{B_t(i)} E_t \sum_{j=0}^{\infty} \lambda^j Q_{t,t+j} \left[Y_{t+j}(i) B_t(i) \overline{\Pi}^{j\omega_s} \left(\prod_{k=1}^{j} \Pi_{t+j-k} \right)^{\omega_d} - (1-\nu) W_{t+j}(i) N_{t+j}(i) \right] \quad \text{(B.15)}$$

where $Q_{t,\ t+j} = \beta^j E_t \left\{ \frac{\varrho_{t+j}}{\varrho_t} \frac{P_t}{P_{t+j}} \right\}$ is the stochastic discount factor, $\overline{\Pi}$ is the gross steady-state level of inflation and Π is gross level of inflation. The optimal relative reset price is then given by

$$\frac{B_t(i)}{P_t} = \frac{E_t \sum_{j=0}^{\infty} \lambda^j Q_{t,t+j} Y_{t+j} \left(\frac{P_{t+j}}{P_t} \right)^{\theta+1} \overline{\Pi}^{-j\omega_s\theta} \left(\prod_{k=1}^{j} \Pi_{t+j-k} \right)^{-\omega_d\theta} \left(\frac{\theta}{\theta-1} \right) \left(MC_{t+j}(i) / P_{t+j} \right)}{E_t \sum_{j=0}^{\infty} \lambda^j Q_{t,t+j} Y_{t+j} \left(P_{t+j}/P_t \right)^{\theta} \overline{\Pi}^{-j\omega_s(\theta-1)} \left(\prod_{k=1}^{j} \Pi_{t+j-k} \right)^{-\omega_d(\theta-1)}} \quad \text{(B.16)}$$

Labor employed by firms each period is obtained through the minimization of production costs

$$\min_{N_t(i)} Costs_t(Y_t(i)) = (1-\nu) W_t(i) N_t(i) \quad \text{s.t. } Y_t(i) = A_t N_t(i) \quad \text{(B.17)}$$

The FOC of problem (B.17) brings

$$MC_t(i) = \frac{(1-\nu) W_t(i)}{A_t} \quad \text{(B.18)}$$

Firm-specific marginal costs can be related to aggregate variables using

$$\frac{MC_{t+j}(i)}{P_{t+j}} = (1-\nu) \left(\frac{\varrho_{t+j}^{-1}}{A_{t+j}} \right) \left(\frac{Y_{t+j}}{A_{t+j}} \right)^{1/\eta} \left(\frac{B_t(i)}{P_t} \right)^{-\theta/\eta} \left(\frac{P_{t+j}}{\overline{\Pi}^{j\omega_s} \left(\prod_{k=1}^{j} \Pi_{t+j-k} \right)^{\omega_d} P_t} \right)^{\theta/\eta} \quad \text{(B.19)}$$

Note that in equilibrium all firms reoptimize to the same price, so $\frac{B_t}{P_t} = \frac{B_t(i)}{P_t}$. Plugging

(B.20) and the expression for $Q_{t,\ t+j}$ in (B.16) and rearranging

$$\left(\frac{B_t}{P_t}\right)^{\left(1+\frac{\theta}{\eta}\right)} = \frac{E_t \sum_{j=0}^{\infty} \left(\frac{(1-\nu)\theta}{\theta-1}\right) \left(\frac{\beta\lambda}{\overline{\Pi}^{\omega_s\theta\left(1+\frac{1}{\eta}\right)}}\right)^j \left(\prod_{k=1}^{j} \Pi_{t+j-k}\right)^{-\omega_d\theta\left(1+\frac{1}{\eta}\right)} \left(\frac{Y_{t+j}}{A_{t+j}}\right)^{\left(1+\frac{1}{\eta}\right)} \left(\frac{P_{t+j}}{P_t}\right)^{\theta\left(1+\frac{1}{\eta}\right)}}{E_t \sum_{j=0}^{\infty} \left(\frac{\beta\lambda}{\overline{\Pi}^{\omega_s(\theta-1)}}\right)^j \left(\prod_{k=1}^{j} \Pi_{t+j-k}\right)^{-\omega_d(\theta-1)} \left(A_{t+j}\varrho_{t+j}\right) \left(\frac{Y_{t+j}}{A_{t+j}}\right) \left(\frac{P_{t+j}}{P_t}\right)^{(\theta-1)}} \tag{B.20}$$

This equation can be log-linearized around the stochastic trend in technology as

$$\begin{aligned} \left(1+\frac{\theta}{\eta}\right)\hat{b}_t &= \sum_{j=0}^{\infty}\left[\gamma_2^j\left(1-\gamma_2\right)-\gamma_1^j\left(1-\gamma_1\right)\right]\left[\hat{y}_{t+j}+\hat{\varrho}_{t+j}\right]+\left(1-\gamma_2\right)\sum_{j=0}^{\infty}\gamma_2^j\left[\frac{1}{\eta}\hat{y}_{t+j}-\hat{\varrho}_{t+j}\right] \\ &+\sum_{j=0}^{\infty}\left[\gamma_2^{j+1}\theta\left(1+\frac{1}{\eta}\right)-\gamma_1^{j+1}\left(\theta-1\right)\right]E_t\left[\hat{\pi}_{t+j+1}\right]-\omega_d\sum_{j=0}^{\infty}\left[\gamma_2^{j+1}\theta\left(1+\frac{1}{\eta}\right)-\gamma_1^{j+1}\left(\theta-1\right)\right]\hat{\pi}_{t+j} \\ &+\hat{u}_t^m \end{aligned} \tag{B.21}$$

Define F_t as the numerator of (B.20). It can be recursively expressed as

$$F_t = \left(\left(\frac{(1-\nu)\,\theta}{\theta-1}\right)\left(\frac{Y_t}{A_t}\right)^{\left(1+\frac{1}{\eta}\right)} + \left(\frac{\beta\lambda}{\overline{\Pi}^{\omega_s\theta\left(1+\frac{1}{\eta}\right)}\Pi_t^{\omega_d\theta\left(1+\frac{1}{\eta}\right)}}\right) E_t\left[\Pi_{t+1}^{\theta\left(1+\frac{1}{\eta}\right)} F_{t+1}\right]\right)\exp\left(u_t^m\right) \tag{B.22}$$

$$u_t^m = \rho_m u_{t-1}^m + \varepsilon_{t-1}^m$$

with ε_{t-1}^m iid normally distributed, where u_t^m is an ad-hoc cost push shock. Define H_t as the denominator of (B.20). It can be recursively expressed as

$$H_t = \left(A_t\varrho_t\right)\left(\frac{Y_t}{A_t}\right) + \left(\frac{\beta\lambda}{\overline{\Pi}^{\omega_s(\theta-1)}\Pi_t^{\omega_d(\theta-1)}}\right) E_t\left[\Pi_{t+1}^{(\theta-1)} H_{t+1}\right] \tag{B.23}$$

Therefore, (B.20) can be expressed as

$$\frac{B_t}{P_t} = \left(\frac{F_t}{H_t}\right)^{\left(\frac{1}{1+\frac{\theta}{\eta}}\right)} \tag{B.24}$$

Given these price-setting assumptions, the dynamics of the price level are governed by

$$P_t^{1-\theta} = \left(1-\lambda\right)B_t^{1-\theta} + \lambda P_{t-1}^{1-\theta}\overline{\Pi}^{\omega_s(1-\theta)}\Pi_{t-1}^{\omega_d(1-\theta)}. \tag{B.25}$$

Dividing by $P_t^{1-\theta}$

$$1 = (1-\lambda)\left(\frac{B_t}{P_t}\right)^{1-\theta} + \lambda\left(\frac{\overline{\Pi}^{\omega_s}\Pi_{t-1}^{\omega_d}}{\Pi_t}\right)^{1-\theta} \tag{B.26}$$

Plugging (B.24) on (B.26) and rearranging

$$\frac{F_t}{H_t} = \left(\frac{1-\lambda\left(\frac{\overline{\Pi}^{\omega_s}\Pi_{t-1}^{\omega_d}}{\Pi_t}\right)^{1-\theta}}{1-\lambda}\right)^{\frac{1+\frac{\theta}{\eta}}{1-\theta}} \tag{B.27}$$

We define the aggregate labor input as

$$N_t = \left[\int_0^1 N_t\left(i\right)^{(\theta-1)/\theta}\, di\right]^{\theta/(\theta-1)} \tag{B.28}$$

Plugging (B.15) on (B.30)

$$N_t = \left[\int_0^1 \left(\frac{Y_t\left(i\right)}{A_t}\right)^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)} \tag{B.29}$$

Plugging (B.7) on (B.31)

$$N_t = \frac{Y_t}{A_t}\left[\int_0^1 \left(\frac{P_t\left(i\right)}{P_t}\right)^{1-\theta} di\right]^{-\left(\frac{\theta}{1-\theta}\right)} = \frac{Y_t}{A_t} \tag{B.30}$$

Monetary Policy

Finally, the policy rule followed by the monetary authority is

$$R_t = max\{1, R_t^*\} \tag{B.31}$$

$$R_t^* = \overline{R}\left(\frac{R_{t-1}^*}{\overline{R}}\right)^{\rho_1}\left(\frac{R_{t-2}^*}{\overline{R}}\right)^{\rho_2}\left[\left(\frac{\Pi_t}{\overline{\overline{\Pi}}}\right)^{\phi_\pi}\left(\frac{Y_t}{\overline{\overline{Y_t}}}\right)^{\phi_Y}\left(\frac{GY_t}{\overline{\overline{GY}}}\right)^{\phi_{GY}}\left(\frac{P_t}{\overline{\overline{P_t}}}\right)^{\phi_P}\right]^{(1-\rho_1-\rho_2)} exp\left(\varepsilon_t^R\right) \tag{B.32}$$

where R is *realized* gross interest rate, R^* is *desired* gross interest rate, GY is the gross growth rate of output and ε^R is an i.i.d policy shock. Note that equation (B.31) is responsible for introducing the zero lower bound to the model.

Equilibrium conditions

$$F_t = \left(\left(\frac{(1-\nu)\,\theta}{\theta-1}\right)\left(\frac{Y_t}{A_t}\right)^{\left(1+\frac{1}{\eta}\right)} + \left(\frac{\beta\lambda}{\overline{\Pi}^{\omega_s\theta\left(1+\frac{1}{\eta}\right)}\Pi_t^{\omega_d\theta\left(1+\frac{1}{\eta}\right)}}\right) E_t\left[\Pi_{t+1}^{\theta\left(1+\frac{1}{\eta}\right)} F_{t+1}\right]\right)\exp\left(u_t^m\right)$$

$$H_t = \left(A_t\varrho_t\right)\left(\frac{Y_t}{A_t}\right) + \left(\frac{\beta\lambda}{\overline{\Pi}^{\omega_s(\theta-1)}\Pi_t^{\omega_d(\theta-1)}}\right) E_t\left[\Pi_{t+1}^{(\theta-1)} H_{t+1}\right]$$

$$\frac{F_t}{H_t} = \left(\frac{1-\lambda\left(\frac{\overline{\Pi}^{\omega_s}\Pi_{t-1}^{\omega_d}}{\Pi_t}\right)^{1-\theta}}{1-\lambda}\right)^{\frac{1+\frac{\theta}{\eta}}{1-\theta}}$$

$$\varrho_t = \beta E_t\left[\frac{\varrho_{t+1} q_t R_t}{\Pi_{t+1}}\right]$$

$$\varrho_t = \left(C_t - hGA_tC_{t-1}\right)^{-1} - \beta h E_t G A_{t+1}\left(C_{t+1} - hGA_{t+1}C_t\right)^{-1}$$

$$Y_t = C_t + \exp\left(u_t^G\right)\overline{G}_t$$

$$R_t = max\{1, R_t^*\}$$

$$R_t^* = \overline{R}\left(\frac{R_{t-1}^*}{\overline{R}}\right)^{\rho_1}\left(\frac{R_{t-2}^*}{\overline{R}}\right)^{\rho_2}\left[\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\overline{Y_t}}\right)^{\phi_Y}\left(\frac{GY_t}{\overline{GY}}\right)^{\phi_{GY}}\left(\frac{P_t}{\overline{P_t}}\right)^{\phi_P}\right]^{(1-\rho_1-\rho_2)} exp\left(\varepsilon_t^R\right)$$

$$GY_t = \frac{Y_t}{Y_{t-1}}$$

$$GC_t = \frac{C_t}{C_{t-1}}$$

$$\overline{P}_t = \overline{\Pi}^t$$

$$A_t = exp\left(u_t^A\right)$$

$$q_t = exp\left(u_t^q\right)$$

$$u_t^A = \mu + u_{t-1}^A + \varepsilon_t^A$$

$$u_t^q = \rho_q u_{t-1}^q + \varepsilon_t^q$$

$$u_t^G = \rho_G u_{t-1}^G + \varepsilon_t^G$$

$$u_t^m = \rho_m u_{t-1}^m + \varepsilon_t^m$$

Equilibrium conditions, stationary variables

$$F_t = \left(\left(\frac{(1-\nu)\,\theta}{\theta-1}\right)\widetilde{Y}_t^{\left(1+\frac{1}{\eta}\right)} + \left(\frac{\beta\lambda}{\overline{\Pi}^{\omega_s\theta\left(1+\frac{1}{\eta}\right)}\Pi_t^{\omega_d\theta\left(1+\frac{1}{\eta}\right)}}\right)E_t\left[\Pi_{t+1}^{\theta\left(1+\frac{1}{\eta}\right)}F_{t+1}\right]\right)\exp\left(u_t^m\right)$$

$$H_t = \widetilde{\varrho}_t\widetilde{Y}_t + \left(\frac{\beta\lambda}{\overline{\Pi}^{\omega_s(\theta-1)}\Pi_t^{\omega_d(\theta-1)}}\right)E_t\left[\Pi_{t+1}^{(\theta-1)}H_{t+1}\right]$$

$$\frac{F_t}{H_t} = \left(\frac{1 - \lambda \left(\frac{\overline{\Pi}^{\omega_s} \Pi_{t-1}^{\omega_d}}{\Pi_t} \right)^{1-\theta}}{1 - \lambda} \right)^{\frac{1+\frac{\theta}{\eta}}{1-\theta}}$$

$$\widetilde{\varrho}_t = \left(\widetilde{C}_t - h\widetilde{C}_{t-1} \right)^{-1} - \beta h E_t \left(\widetilde{C}_{t+1} - h\widetilde{C}_t \right)^{-1}$$

$$\widetilde{\varrho}_t = \beta E_t \left[\frac{\widetilde{\varrho}_{t+1} q_t R_t}{GA_{t+1} \Pi_{t+1}} \right]$$

$$\widetilde{Y}_t = \widetilde{C}_t + \exp\left(u_t^G\right) \overline{\widetilde{G}}$$

$$GA_t = \exp\left(\mu + \varepsilon_t^A\right)$$

$$GY_t = \frac{\widetilde{Y}_t}{\widetilde{Y}_{t-1}} GA_t$$

$$GC_t = \frac{\widetilde{C}_t}{\widetilde{C}_{t-1}} GA_t$$

$$\widetilde{P}_t = \widetilde{P}_{t-1} \frac{\Pi_t}{\overline{\Pi}}$$

$$R_t^* = \overline{R} \left(\frac{R_{t-1}^*}{\overline{R}} \right)^{\rho_1} \left(\frac{R_{t-2}^*}{\overline{R}} \right)^{\rho_2} \left[\left(\frac{\Pi_t}{\overline{\Pi}} \right)^{\phi_\pi} \left(\frac{\widetilde{Y}_t}{\overline{\widetilde{Y}}} \right)^{\phi_Y} \left(\frac{GY_t}{\overline{GY}} \right)^{\phi_{GY}} \widetilde{P}_t^{\phi_P} \right]^{(1-\rho_1-\rho_2)} \quad exp\left(\varepsilon_t^R\right)$$

$$R_t = max\{1, R_t^*\}$$

$$q_t = exp\left(u_t^q\right)$$

$$u_t^q = \rho_q u_{t-1}^q + \varepsilon_t^q$$

$$u_t^G = \rho_G u_{t-1}^G + \varepsilon_t^G$$

$$u_t^m = \rho_m u_{t-1}^m + \varepsilon_t^m$$

where $\widetilde{\varrho}_t = A_t \varrho_t$, $\widetilde{C}_t = \frac{C_t}{A_t}$ and $\widetilde{Y}_t = \frac{Y_t}{A_t}$

Steady state values

$$\overline{\widetilde{Y}} = \left[\left(\frac{(\theta-1)(1-\beta h)\left(1-\beta\lambda\overline{\Pi}^{(1-\omega_s-\omega_d)\theta\left(1+\frac{1}{\eta}\right)}\right)}{(1-\nu)\,\theta\,(1-h)\left(1-\overline{G}\right)\left(1-\beta\lambda\overline{\Pi}^{(1-\omega_s-\omega_d)(\theta-1)}\right)} \right) \left(\frac{1-\lambda\overline{\Pi}^{(1-\omega_s-\omega_d)(\theta-1)}}{1-\lambda} \right)^{\frac{1+\frac{\theta}{\eta}}{1-\theta}} \right]^{\frac{1}{1+\frac{1}{\eta}}}$$

$$\overline{F} = \left(\frac{(1-\nu)\,\theta}{(\theta-1)\left(1-\beta\lambda\overline{\Pi}^{(1-\omega_s-\omega_d)\theta\left(1+\frac{1}{\eta}\right)}\right)} \right) \overline{\widetilde{Y}}^{\left(1+\frac{1}{\eta}\right)}$$

$$\overline{H} = \frac{1-\beta h}{(1-h)\left(1-\overline{G}\right)\left(1-\beta\lambda\overline{\Pi}^{(1-\omega_s-\omega_d)(\theta-1)}\right)}$$

$$\overline{\widetilde{C}} = \left(1-\overline{G}\right)\overline{\widetilde{Y}}$$

$$\overline{\widetilde{\varrho}} = \left(\frac{1-\beta h}{(1-h)\left(1-\overline{G}\right)} \right) \overline{\widetilde{Y}}^{-1}$$

$$\overline{R} = \frac{\overline{\Pi G Y}}{\beta}$$

$$\overline{R}^S = \overline{R}$$

$$\overline{GY} = \exp(\mu)$$

$$\overline{\tilde{P}} = 1$$

$$\overline{u^q} = 0$$

$$\overline{u^G} = 0$$

$$\overline{u^m} = 0$$

Log-linearized equilibrium conditions around stochastic trend in technology

$$\begin{aligned}\hat{f}_t &= \left(1 - \beta\lambda\overline{\Pi}^{(1-\omega_s-\omega_d)\theta\left(1+\frac{1}{\eta}\right)}\right)\left(1+\frac{1}{\eta}\right)\hat{y}_t \\ &+ \left(\beta\lambda\overline{\Pi}^{(1-\omega_s-\omega_d)\theta\left(1+\frac{1}{\eta}\right)}\right)\left[\theta\left(1+\frac{1}{\eta}\right)\hat{\pi}_{t+1} - \omega_d\theta\left(1+\frac{1}{\eta}\right)\hat{\pi}_t + \hat{f}_{t+1}\right] + \hat{u}_t^m\end{aligned}$$

$$\begin{aligned}\hat{h}_t &= \left(1 - \beta\lambda\overline{\Pi}^{(1-\omega_s-\omega_d)(\theta-1)}\right)\left[\hat{\varrho}_t + \hat{y}_t\right] \\ &+ \left(\beta\lambda\overline{\Pi}^{(1-\omega_s-\omega_d)(\theta-1)}\right)\left[(\theta-1)\hat{\pi}_{t+1} - \omega_d(\theta-1)\hat{\pi}_t + \hat{h}_{t+1}\right]\end{aligned}$$

$$\hat{f}_t - \hat{h}_t = \left(1+\frac{\theta}{\eta}\right)\left(\frac{\lambda\overline{\Pi}^{(1-\omega_s-\omega_d)(\theta-1)}}{1-\lambda\overline{\Pi}^{(1-\omega_s-\omega_d)(\theta-1)}}\right)\left(\hat{\pi}_t - \omega_d\hat{\pi}_{t-1}\right)$$

$$\hat{\varrho}_t = \frac{h}{(1-h)(1-\beta h)}\hat{c}_{t-1} - \frac{1+\beta h^2}{(1-h)(1-\beta h)}\hat{c}_t + \frac{\beta h}{(1-h)(1-\beta h)}E_t\hat{c}_{t+1}$$

$$\hat{\varrho}_t = E_t\left[\hat{\varrho}_{t+1} + \hat{r}_t - \hat{\pi}_{t+1} + \hat{u}_t^q\right]$$

$$\hat{y}_t = (1 - s_g)\,\hat{c}_t - s_g \hat{u}_t^G$$

$$\hat{r}_t = max\{\hat{r}_t^*, -\bar{r}\}$$

$$\hat{r}_t^* = \rho_1 \hat{r}_{t-1}^* + \rho_2 \hat{r}_{t-2}^* + (1 - \rho_1 - \rho_2)\left[\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \phi_{gy} \widehat{gy}_t + \phi_p \hat{p}_t\right] + \varepsilon_t^r$$

$$\widehat{gy}_t = \hat{y}_t - \hat{y}_{t-1} + \varepsilon_t^A$$

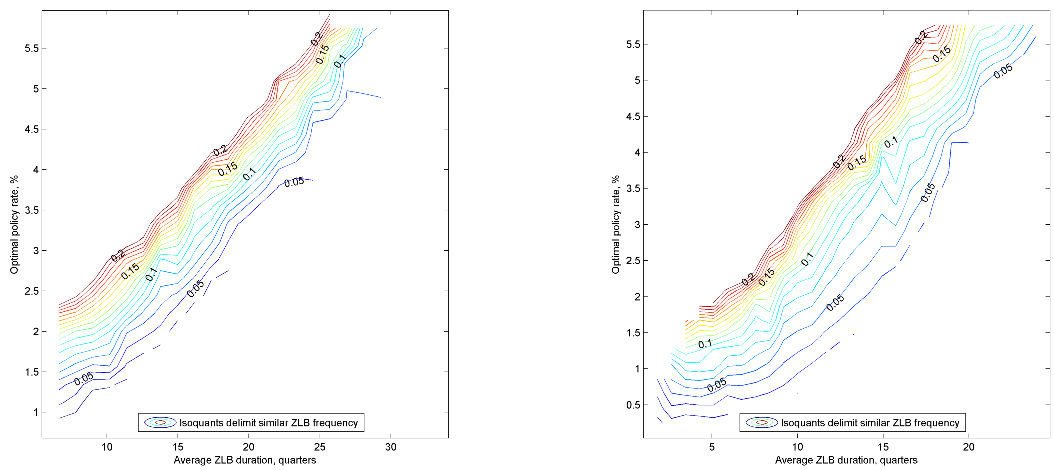
$$\hat{p}_t = \hat{p}_{t-1} + \hat{\pi}_t$$

$$\hat{u}_t^q = \rho_q \hat{u}_{t-1}^q + \varepsilon_t^q$$

$$\hat{u}_t^G = \rho_G \hat{u}_{t-1}^G + \varepsilon_t^G$$

$$\hat{u}_t^m = \rho_m \hat{u}_{t-1}^m + \varepsilon_t^m$$

B.2 Appendix Figures



(a) $\Delta = 6\%$ (b) $\Delta = 12\%$

Figure B.1: Optimal Inflation with Different Sizes of Shocks to Risk Premium. The figures plot the optimal annualized inflation rate (y-axis) associated with different levels of average ZLB durations (x-axis) and unconditional frequencies of the ZLB (indicated by isoquants). Panel (a) is done for regime switching risk premia, with $\Delta = 6\%$ while T_q and p_{12} are varied to change the average durations of ZLB episodes. Panel (b) is done with $\Delta = 12\%$. See section 4.3.2 for details.

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