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ABSTRACT

Geometry correction factors for proton-recoil track samples taken from nuclear emulsions have been derived. The factors apply to samples of tracks that have been selected at random from the emulsion volume, and to tracks created under conditions of s-wave scattering with fast neutrons. The derived factors as a function of track length ℓ, emulsion thickness T, and angle of scattering θ are

$2T/(2T-\ell)$ (when $\ell \leq T$; and $2\ell/T$ (when $\ell \geq T$) for isotropic exposure; $T/(T - \ell \cos \theta)$ for face-normal exposure; and $\pi T/(\pi T - 2a)$ when $a \leq T$, and

$$\frac{\pi T}{2(T \arcsin \frac{T}{a} + \sqrt{a^2 - T^2} - a}$$

(when $a \geq T$) for edge-normal exposure, if $a = \ell \sin \theta$.

I. INTRODUCTION

The development, in recent years, of microscopes for the semiautomatic scanning of tracks has given a new impetus to the use of nuclear track emulsions.[1] In the past, track measurement and analysis required a great deal of time, but now rapid scanning of emulsions and data analysis by electronic computer have partly overcome this difficulty. However, the efficient use of the new microscopes in fast-neutron analysis requires a new scanning technique: tracks are selected at random throughout the emulsion volume.[2] In earlier work scanning was usually restricted to tracks that lay within a right rectangular pyramid parallel to the beam axis of the incident neutrons.[3,4] Correction factors for such restricted samples have been derived by Richards.[5] It is necessary to correct a track sample by a factor that is based on the dimensions of the emulsion and of the scanned volume, and on the exposure direction. Otherwise the sample will be biased against the longer tracks that more frequently end outside the emulsion and therefore are not measured. Here correction factors are derived that apply to samples of proton recoil tracks selected at random throughout the emulsion volume. Isotropic, face-normal, and edge-normal exposures are considered.

II. THE PROBLEM

To find: Geometry correction factors $1/P(\ell)$ for samples of proton-recoil tracks, where $P(\ell)$ is the probability that a track of length ℓ that originates in the emulsion will also end in the emulsion.

Given:

(a) Samples constituted of tracks that both begin and end in the emulsion and that have been selected at random throughout the emulsion volume,

(b) An emulsion of infinite lateral (x, y) extent and a thickness T of a few hundred micrometers,

(c) Rectilinear tracks,

(d) Lengths and angles that apply to the emulsion at exposure (before processing),

(e) s-wave scattering.

The condition of infinite lateral extent is in general acceptable. For example, if a 600-μ emulsion is cut to a 1-in. square, the ratio of edge area to total surface area is about 0.05. Thus the edge factor will usually amount to no more than a 1 to 5% change in the infinite- extent correction factor.

It has been found empirically that the assumption of rectilinear tracks leads to a systematic underestimate of the escape probability that becomes important for long tracks.[3] This effect amounts to 20% for 1000-μ tracks in pyramidal samples. No similar measurements are available for random samples, but the error is about the same.

The condition of original dimensions is not restrictive, because the measured lengths and angles in processed emulsion may easily be corrected to re-create the original geometry. The condition of s-wave scattering is also not restrictive. When the s-wave phase shift determines the cross section for scattering of hydrogen nuclei by neutrons, the angular distribution in the center-of-mass system is isotropic.

Measurements in nuclear emulsions[6] at 8.8 and 13 MeV and by counter telescopes[7,8] at 12 to 21 MeV have shown no departure from isotropic scattering.

Case I. Isotropic Exposure

Consider the situation when the proton-recoil track length ℓ exceeds the emulsion thickness T, as in Fig. 1. The track begins at an arbitrary depth z and ends at the surface of the emulsion. Note that there is cylindrical symmetry about a line normal to the emulsion surfaces through the track origin. Isotropic exposure implies that there is no preferred direction for tracks of any length. Therefore, the relation

$$dN/d\Omega = N_0/4\pi = \text{constant} \tag{1}$$

applies separately for tracks of each length.

$$\text{Now}\quad dP = dN/N_0 = d\Omega/4\pi \ , \tag{2}$$

where P is the probability that a track will lie within the solid angle Ω.

This implies

$$P = (\Omega_2 - \Omega_1)/4\pi \ . \tag{3}$$

From Fig. 1, $\Omega_2 = 2\pi(1 - \cos\theta_2)$, $\Omega_1 = 2\pi(1 - \cos\theta_1)$, $\cos\theta_1 = z/\ell$, and $\cos\theta_2 = -(T-z)/\ell$.

Therefore
$$P(\ell) = \frac{1}{2}\left(1 + \frac{T-z}{\ell}\right) - \frac{1}{2}\left(1 - \frac{z}{\ell}\right) \tag{4}$$

$$= T/2\ell \quad (\text{for } \ell \geqslant T) \ . \tag{5}$$

In order to obtain $P(\ell)$ for $\ell < T$, the emulsion thickness is divided into two pieces, ℓ and $T-\ell$. For the fraction ℓ/T the probability is $\frac{1}{2}$ as given by Eq. (5), where ℓ = "T." For the fraction $(T-\ell)/T$ the probability is unity. Therefore

$$P(\ell) = 1/2\,(\ell/T) + (T-\ell)/T = 1 - \ell/2T \quad (\text{for } \ell \leqslant T). \tag{6}$$

The correction factor for isotropic exposure is independent of the neutron energy spectrum.

Case II. Face-Normal Exposure

Consider the situation when neutrons of a single energy E_n enter the emulsion on an axis that is normal to the surfaces of a piece of nuclear emulsion. Consider a proton track of length ℓ that originates at depth z in the emulsion, as in Fig. 2. Note that there is cylindrical symmetry about the beam axis, and that a track of proton length ℓ arising by collision with a neutron of energy E_n can occur only at the fixed angle θ. Here only those tracks of given length ℓ that originate at depth less than z will also end in the emulsion.

Therefore

$$P = z/T . \tag{7}$$

However,

$$z = T - \ell \cos\theta = T - \ell \ (E_p/E_n)^{\frac{1}{2}} = T - \ell \ (\ell/\ell_m)^{\frac{1}{2}n} , \tag{8}$$

where E_p is the energy of the proton recoil of length ℓ given by $\ell = 13.9\, E_p^n$ if ℓ is in units of μ and E_p in units of MeV. The parameter n varies from 1.38 when $\ell = 4\mu$ to 1.64 when $\ell = 2000\,\mu$.[9] The value of ℓ_m, the maximum track length, is obtained when $E_p = E_n$.

Now, $P(\ell, \ell_m) = 1 - (\ell_m/T)\,(\ell/\ell_m)^{(2n+1)/2n}$, (9)

or, in more useful form,

$$P\,(\ell, E_n) = 1 - (\ell/T)\,(E_p/E_n)^{\frac{1}{2}} . \tag{10}$$

Equation (10) is valid for a given E_n, when E_p corresponds to the length ℓ, and when $\ell \cos\theta \leqslant T$. Notice, with the help of Eq. (8) and Fig. 2, that $P = 0$ in all cases where $\ell \cos\theta$ exceeds T.

Case III. Edge-Normal Exposure

Consider a parallel beam of single-energy neutrons that enters a piece of nuclear emulsion normal to an edge that defines the x, z plane. Further consider the geometry when a neutron collides at Q with a proton that travels for a distance ℓ at angle θ from the path-axis of the incident neutron, as in Figs. 3 and 4. There is equal chance that the end of the track will lie at any point on the circumference of a circle of radius $\ell \sin\theta$ about V. Because there is symmetry in the +x and -x direction, and because there is a "back and forth" symmetry as Q moves between $z = 0$ and $z = T$, or from $z = T$ to $z = 0$, only the quadrant R S U V is used.

Situation when $\ell \sin\theta \leq T$.

If Q lies at a depth exactly equal to $\ell \sin\theta$, then all possible tracks (of the given ℓ, θ) end within the emulsion along R S U. However, if Q lies at a depth less than $\ell \sin\theta$, such as $\ell \sin\theta \sin\phi$, then only a fraction of the possible tracks--those ending on arc S U -- will end within the emulsion. In fact the probability that a track (of given ℓ, θ) will lie on arc S U is given by

$$P(z) = (2/\pi) \arcsin (z/\ell \sin\theta), \text{ for } z \leq \ell \sin\theta. \qquad (11)$$

The average probability that a track will lie along arc S U as z varies from 0 to $\ell \sin\theta$ is given by

$$P_{av} = \int_0^{\ell \sin\theta} \frac{dz}{\ell \sin\theta} \cdot P(z) = 1 - \frac{2}{\pi} . \qquad (12)$$

This average probability applies to the fraction $(\ell \sin\theta)/T$ of the emulsion. The probability for the fraction $1 - (\ell \sin\theta)/T$ is unity. Therefore

$$P(\ell, \theta) = (1 - (\ell \sin\theta)/T) + (1 - 2/\pi) (\ell \sin\theta)/T = 1 - (2/\pi) (\ell \sin\theta)/T. \qquad (13)$$

In more useful form,

$$P(\ell, E_n) = 1 - (2/\pi)\,(\ell/T)\,(1 - E_p/E_n)^{\frac{1}{2}} \;, \quad \text{for} \quad \ell \sin\theta \leqslant T, \tag{14}$$

is the probability that a proton track of energy E_p (and length ℓ), arising by collision with a neutron of energy E_n at any depth in the emulsion, will also end inside the emulsion.

When $\ell \sin\theta \geqslant T$, $P(z) = (2/\pi) \arcsin (z/\ell \sin\theta)$ (15)

as in Fig. 5, and

$$P_{av}(\ell, \theta) = (2/\pi) \int_0^T (dz/T)\, P(z); \tag{16}$$

$$P_{av}(\ell, \theta) = (2/\pi T)\,[T \arcsin(T/\alpha) + \sqrt{\alpha^2 - T^2} - \alpha] \;. \tag{17}$$

in which $\alpha = \ell \sin \theta$.

III. DISCUSSION

It should be emphasized that the derived factors of Cases II and III depend upon a knowledge of the angle of scattering as well as upon the length of the track. In this they differ from the pyramidal-sample correction factors that apply to all track lengths within some maximum acceptance angle. Also, because the track length is not independent of the angle of scattering, it is incorrect to obtain an "average" correction factor from Eqs. (10), (13), and (16) by integration over θ.

In practice, tracks must be sorted into θ intervals, and (within the appropriate θ interval) into length intervals, before suitable correction factors can be applied. Some examples of geometry correction factors for various exposures, scattering angles, and lengths are presented in Fig. 6.

FOOTNOTES AND REFERENCES

[*] Work done under the auspices of the U.S. Atomic Energy Commission.

[†] Now at Department of Physics, University of Arizona, Tucson.

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FIGURE CAPTIONS

1. Cross section of a piece of nuclear emulsion of thickness T. Geometry for isotropic exposure.

Fig. 2. Cross section of a piece of nuclear emulsion of thickness T. Geometry for face-normal exposure.

Fig. 3. Geometry of edge-normal exposure. QV is path axis of incident neutron.

Fig. 4. Cross section of a piece of nuclear emulsion of thickness T cut normal to path axis of incident neutron. Geometry for edge-normal exposure when $l \sin \theta \leq T$.

Fig. 5. Cross section of a piece of nuclear emulsion of thickness T cut normal to path axis of incident neutron. Geometry for edge-normal exposure when $l \sin \theta \geq T$.

Fig. 6. Geometry correction factors for proton-recoil tracks in nuclear emulsion of thickness 600 μ. $E_p = E_n$.

I: maximum for face-normal exposure,
II: minimum for face-normal exposure, $(E_n = 20 \text{ MeV})$,
III: isotropic exposure,
IV: maximum for edge-normal exposure, $(E_p = 0.75\, E_n)$
V: edge-normal exposure for $\theta = 15°$. $(E_p = 0.93\, E_n)$

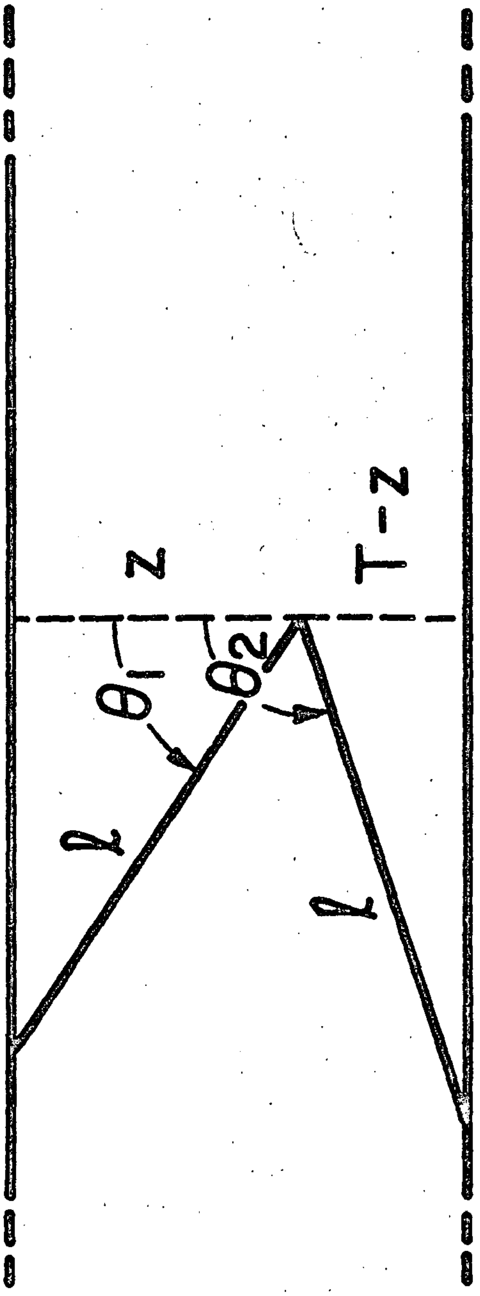

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Fig. 1

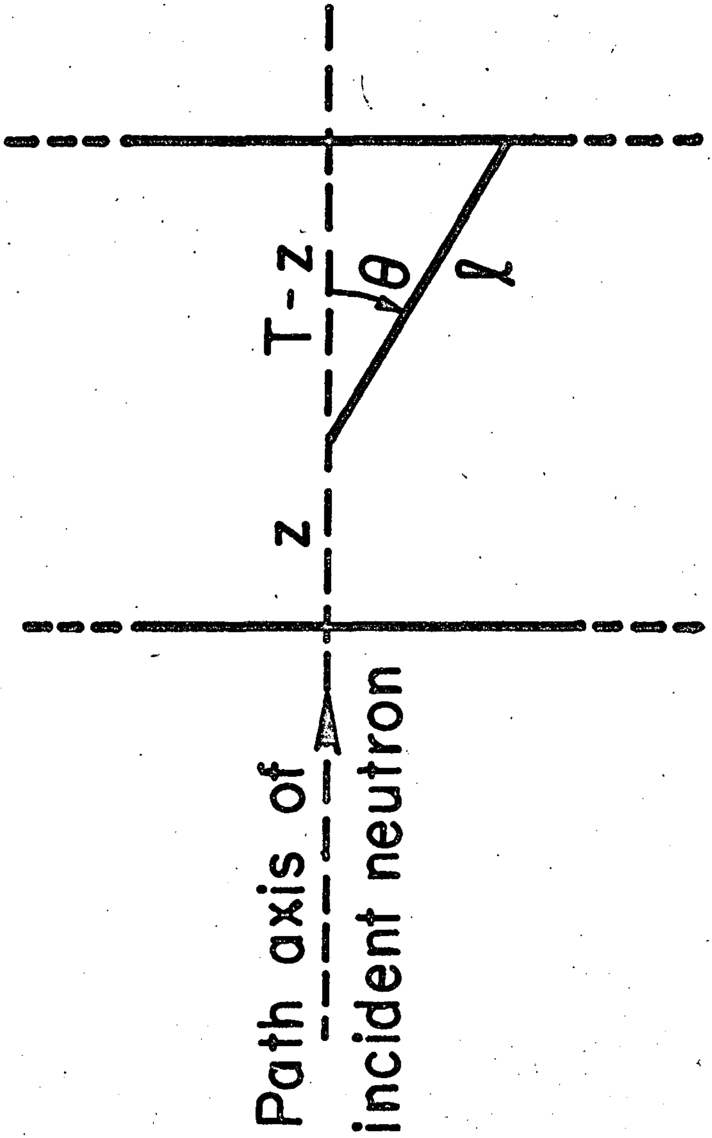

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Fig. 2

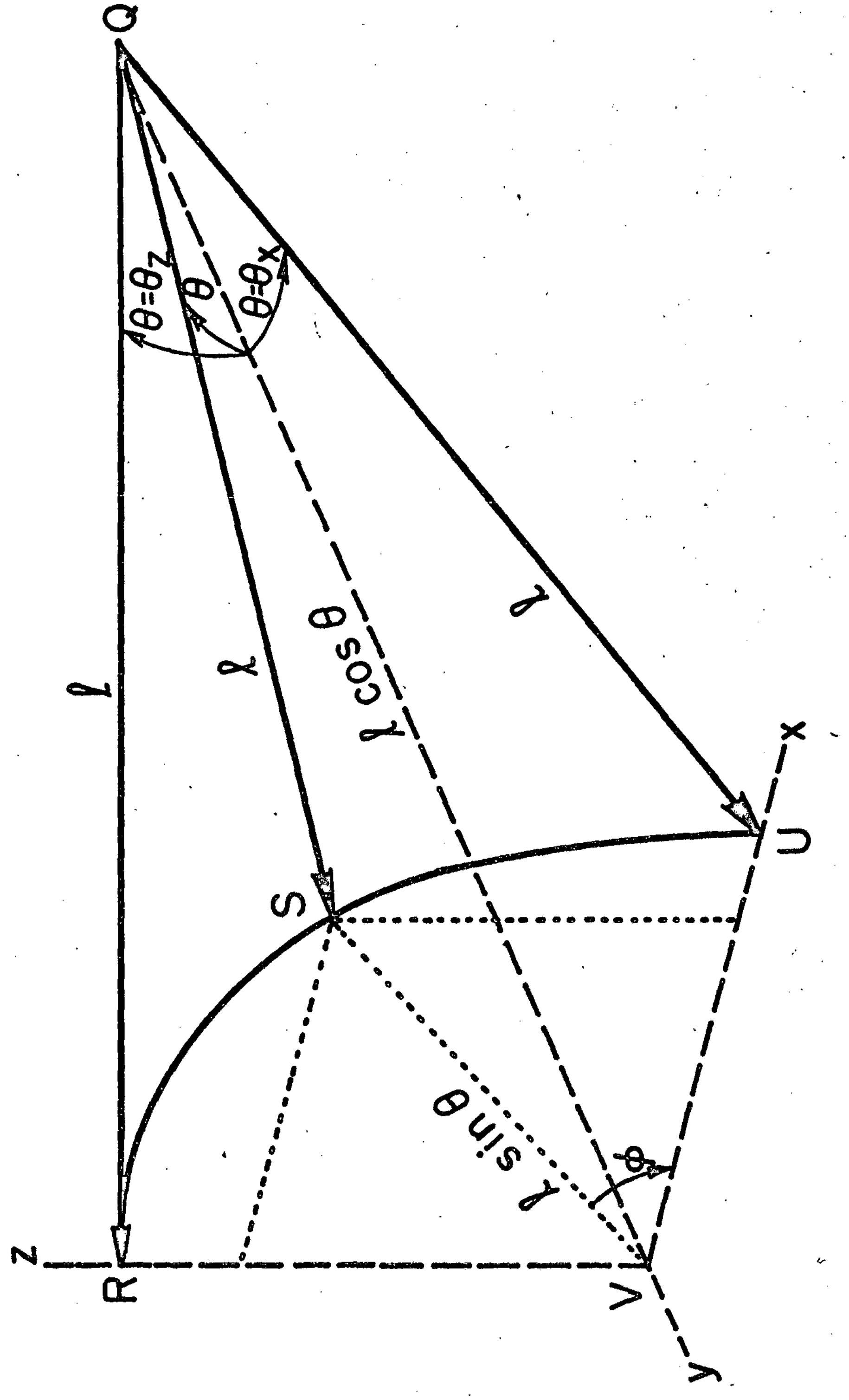

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Fig. 3

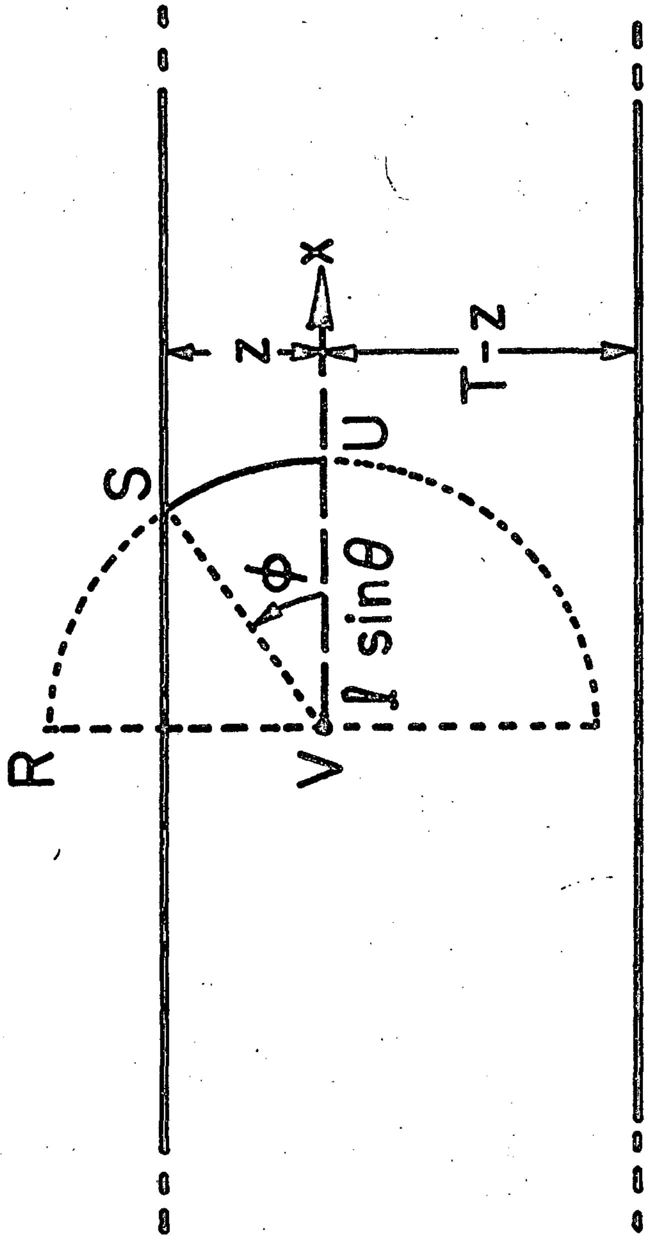

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Fig. 4

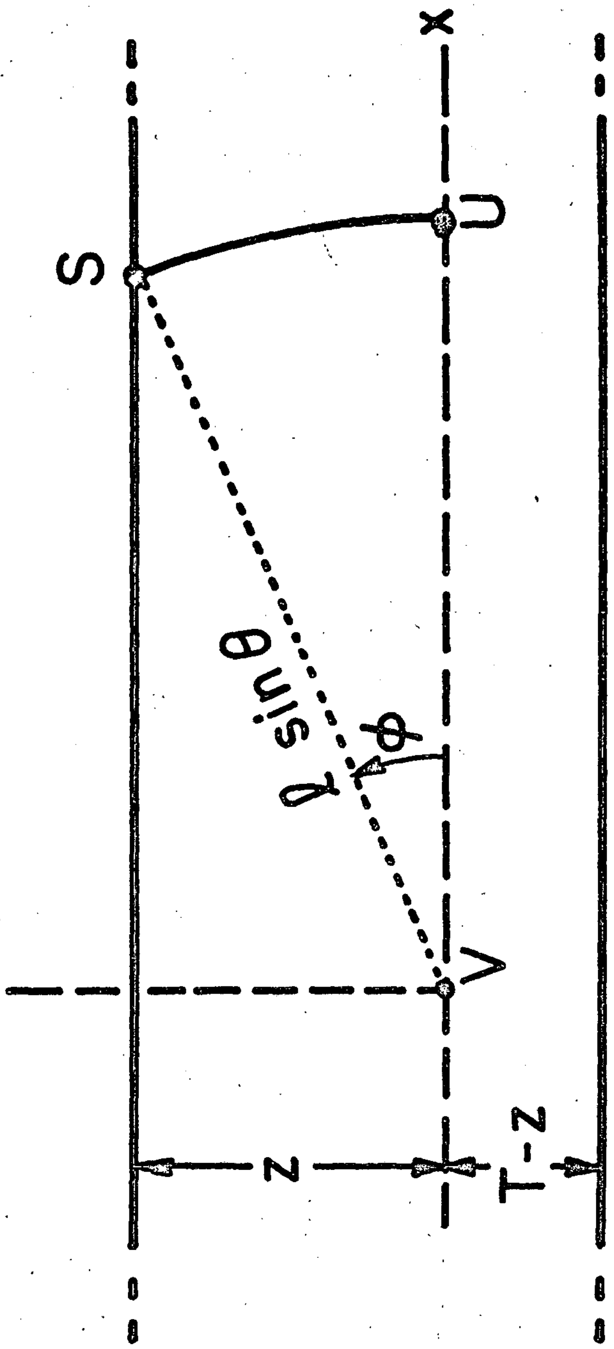

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Fig. 5

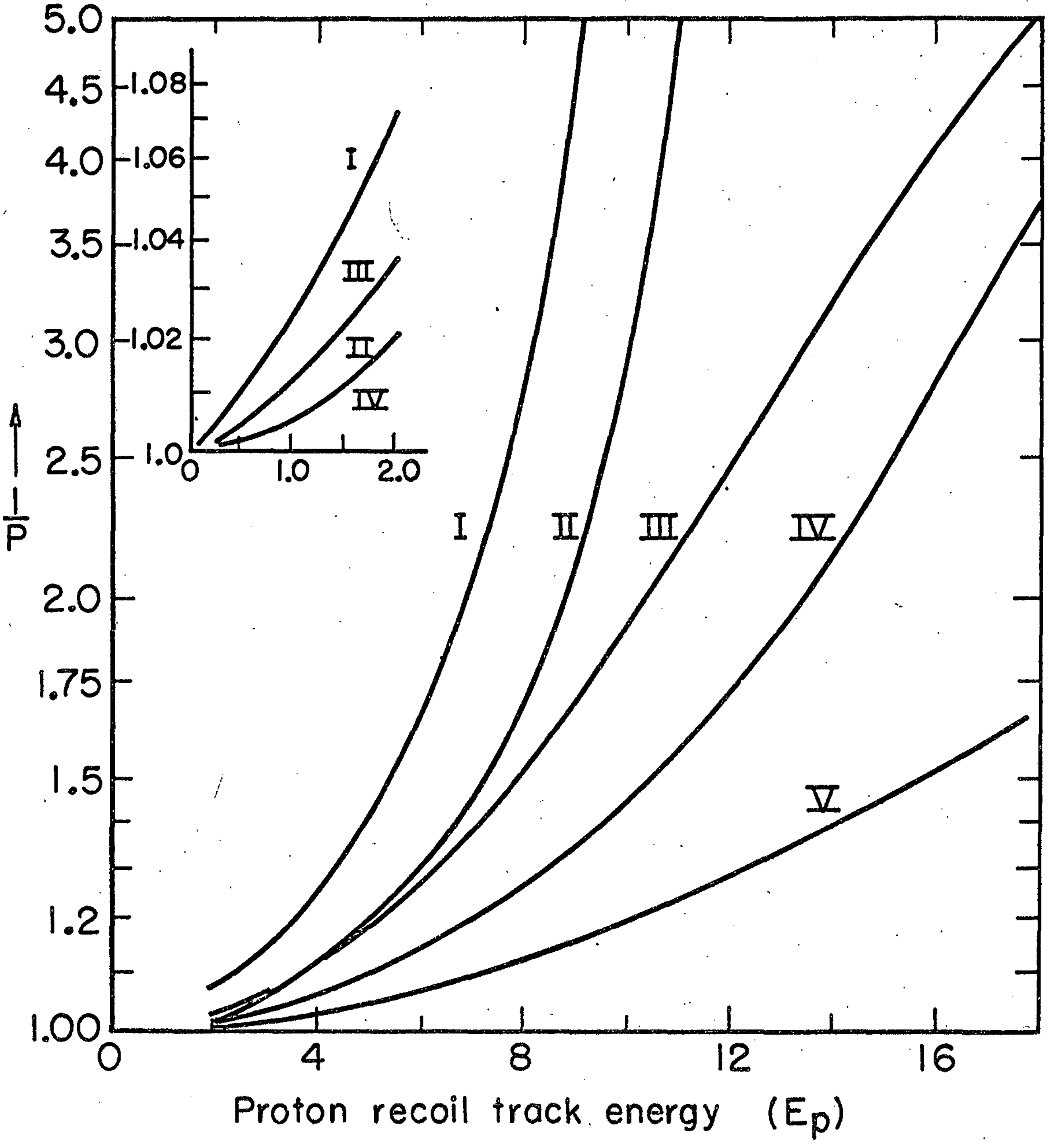

Fig. 6

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