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ANALYSIS OF THE WINDOW DISSIPATION FORMULA ON THE BASIS OF LINEAR RESPONSE THEORY

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Analysis of the window dissipation formula on the basis of linear response theory *

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Abstract:

Linear response theory is employed to analyse the rate of energy dissipation in a binary one-body potential well whose two parts are connected by a small "window" and are in slow relative motion. It is shown that suitable randomization assumptions lead to the "completed wall-and-window formula", including the contribution from the change in the mass asymmetry. The developed general formal framework provides a well-founded basis for systematic calculation of corrections in cases that are less than ideal, such as are encountered in quasi-fission reactions.

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1 Introduction

As was first recognized by Hill and Wheeler, [1] the long nucleonic mean free path has profound consequences for the character of large-scale nuclear dynamics. The first comprehensive study of this "new dynamics", often referred to as *one-body nuclear dynamics* (since the motion of the nucleons is governed by the changing one-body mean field), was carried out about ten years ago within the framework of classical kinetic theory.[2] It led to two remarkably simple formulas for the rate of energy dissipation: the *wall formula* pertaining to a slowly deforming mononucleus, and the *window formula* pertaining to a dinucleus whose two parts are in slow relative motion. The wall and window dissipation formulas have been employed extensively, with a considerable degree of success, to low-energy nuclear dynamical processes as occuring in fusion, fission, and damped reactions. It is our aim, in this paper, to clarify the conditions for the validity of the standard window formula and to develop a formal framework for systematic improvement in cases where the conditions are less than ideal.

Such developments are particularly relevant for the study of quasi-fission reactions, which were discovered only relatively recently and have provided new testing ground for theories of nuclear dynamics (see, for example, ref. [3]). These reactions are believed to proceed through shapes which are somewhat intermediate between mononuclei and dinuclei: they are rather compact and yet they possess a well-defined (and slowly evolving) mass asymmetry. As a consequence of this more complicated geometry, quasi-fission reactions are harder to treat theoretically and calculations have, so far, employed simple *ad hoc* interpolations between the wall and window formulas.[4] This situation is far from satisfactory. In order to make progress in our understanding of these processes, it is necessary to develop the one-body dissipation theory to encompass also such transitional shapes. It is also towards this goal that the present study is oriented.

2 Characterization of the problem

The validity of the simple wall formula has been studied in a variety of more refined formal frameworks. Most relevant for the present study is the work by Koonin and Randrup based on linear response theory.[5] In that work it was shown that the one-body energy dissipation rate can be expressed as

$$\dot{Q} = \lim_{t\to-\infty} \langle \dot{H}_1 [\int_t^0 dt' \hat{U}_0(t') - \hat{U}_0(t) t] \, \dot{H}_1 \frac{\partial f_0}{\partial H_0} \rangle \ . \tag{1}$$

The instantaneous one-body field is described by the Hamiltonian H_0 and $\hat{U}_0$ is the associated evolution operator. The slow distortion of the nucleus is described by the time-dependent perturbation $H_1(t)$. The above expression can be interpreted as follows. At $t = 0$ the nucleons have the phase-space distribution $f_0(H_0)$, which is assumed to depend only on the energy $H_0(\mathbf{r}, \mathbf{p})$. (Note that the factor $\partial f_0/\partial H_0$ in

(1) ensures that only nucleons near the Fermi surface contribute.) Each phase-space point $(\mathbf{r},\mathbf{p})$ is traced back in time from $t=0$ to $t\to-\infty$; the brackets indicate the corresponding phase-space integral over $(\mathbf{r},\mathbf{p})$. The first term follows the trajectory $(\mathbf{R},\mathbf{P})$ of an individual nucleon as it bounces around in the *unperturbed* field H_0, receiving impulses $\dot{H}_1(\mathbf{R},\mathbf{P})$ along the way due to the perturbation H_1. The second term is a correction which is instrumental in ensuring convergence when regularities are present in the shape or its rate of distortion. The expression (1) is identical to equation (4.19a) of ref. [5], except for an inversion of the time direction.

If the perturbation consists of inducing local movements of the surface elements in a leptodermous cavity, the impulses $\dot{H}_1$ are received as the nucleon is reflected from the wall. The dissipation rate then has the form of a double surface integral,

$$\dot{Q}=\int d^2a\int d^2b\; u(a)\gamma(a,b)u(b)\,, \tag{2}$$

where $u(a)$ and $u(b)$ are the (normal) velocities of the surface at the points a and b. The dissipation kernel $\gamma(a,b)$ is non-local, *i.e.* depends on points far apart on the surface. However, if the nuclear shape and its rate of distortion are sufficiently irregular, only the last reflection at $t\approx 0$ contributes to the time integral in (1) and, consequently, only the local part of γ contributes, $\gamma(a,b)\approx\rho\bar{v}\,\delta^2(a-b)$, and the standard wall formula emerges, $\dot{Q}_{\rm wall}=\rho\bar{v}\int d^2a\,u(a)^2$. In ref. [5] the important role of regularities was illustrated for the especially simple cases of slab geometry and nearly spherical shapes. The regularities conspire to induce correlations between impulses received at subsequent wall reflections, in such a manner as to diminish or, for the smallest multipolarities, completely cancel the local contribution stemming from the first reflection. (In particular, if the nucleus is subjected to an overall uniform translation or rotation, the cancellation is complete and there is no associated energy dissipation.)

The work reported in ref. [5] can be seen as providing a "proof" of the simple wall formula, by establishing a formal tool for studying the conditions of its validity and incorporating corrections arising from regularities. The present paper reports an analogous application of (1) to a dinuclear geometry, in which the system consists of two distinct parts, in relative motion, joined by a small "window." This will provide a similar "proof" of the window formula and clarify the conditions for its validity, particularly the role played by the character of the nucleonic motions. Moreover, the work brings us in a good position to confront the transitional shapes characteristic of quasi-fission reactions, since we have now a general treatment which gives the proper description in the mononuclear and dinuclear extremes.

The type of system considered is illustrated in fig. 1. The dinuclear potential well has two distinct parts, A and B, joined by a small planar window whose normal direction is chosen as the z-axis. The two parts are subjected to small uniform translations, $\mathbf{U}_A$ and $\mathbf{U}_B$, but are otherwise not changing in time. This yields the simplest situation for which window friction should arise. Any intrinsic distortions of A or B are expected to contribute separate dissipation terms of the mononuclear type discussed above (and approximately given by the wall formula).

In the usual derivation of window friction [2] the nucleons in part A are assumed to have a velocity distribution which is shifted by the amount $\mathbf{U} = \mathbf{U}_A - \mathbf{U}_B$ relative to those in part B. More precisely, these velocity distributions are assumed for those particles which are about to cross the window. This corresponds closely to how the distributions would actually develop, provided the two parts are irregular and the window is small. In the linear-response treatment there is only *one* velocity distribution, namely the one associated with the unperturbed potential H_0, which in the present case is the dinuclear potential displayed in fig. 1 *without* any relative motion. The effect of the relative dinuclear motion is manifested in the impulses impacted to the particles when they interact with the nuclear boundary, as explained below eq. (1). The analogue of having displaced velocity distributions is then that the unperturbed trajectories be suitably random.

As will become clear from the developments later on, the standard window formula can be derived under suitable idealizations regarding the character of the nucleonic motions in the binary container. These are summarized below. To specify position near the window, it is convenient to employ cylindrical coordinates, so that the window plane corresponds to $z = 0$ and a location on the window is specified by the transverse position $\boldsymbol{\rho}$.

I. *Leptodermous window.* It is assumed that the opening between the two parts of the system is leptodermous, *i.e.* the environment felt by a particle at the window is as in the bulk parts of the system, $H(\mathbf{r} = (\boldsymbol{\rho}, z = 0), \mathbf{p}) = p^2/2\mu + V_0$, except possibly for a relatively thin region near the window boundary. Thus the window is assumed to be "fully open", without any potential barrier between the two bulk parts. This requirement is stronger than the usual leptodermous condition which only refers to the two main parts of the system.

II. *Ergodicity.* The particles within a phase-space element $\mathbf{dRdP}$ originally located at the window will in the course of time eventually cover the corresponding energy shell uniformly. (Note that this requirement does not imply that such relaxation is achieved between two successive window crossings.)

III. *Randomization of direction.* When an average is performed over the element's initial position on the window, $\boldsymbol{\rho}$ (which is equivalent to considering a thin phase-space slice covering the window uniformly), then the particles appear to be randomized when they return to cross the window. This requirement implies that particles that have crossed the window at some random position have, at any subsequent crossings, a momentum direction which is randomly distributed over the unit hemisphere.

IV. *Randomization of residence time.* When an average is performed over the element's initial position on the window, as in III above, then the time between two successive window crossings, the particle's *residence time*, is a random variable distributed according to a standard exponential-decay law. (Contrary to this, for any localized phase-space element **dRdP**, as considered in II above, such behavior is expected only if the window is small compared with the overall surface area of the binary container.)

3 Derivation of the window formula

The time-dependent perturbation describing the relative dinuclear motion of the system shown in fig. 1 is

$$\dot{H}_1 = -\mathbf{u}(\mathbf{r}) \cdot \frac{\partial H_0}{\partial \mathbf{r}} = \mathbf{u}(\mathbf{r}) \cdot \dot{\mathbf{p}} \ , \tag{3}$$

where the local surface velocity is given by

$$\mathbf{u}(\mathbf{r}) = \begin{cases} \mathbf{U}_A & \text{for } \mathbf{r} \in A \\ \mathbf{U}_B & \text{for } \mathbf{r} \in B \end{cases} \ . \tag{4}$$

In actual applications of eq. (1), it is convenient [5] to invert the time integration, and the first term of eq. (1) then reads

$$\begin{aligned} &\langle -\dot{H}_1 \int_0^t dt' \hat{U}_0(t') \dot{H}_1 \frac{\partial f_0}{\partial H_0} \rangle \qquad (5) \\ &= -\int \mathbf{dr} \int \frac{\mathbf{dp}}{(2\pi)^3} [\int_0^t dt' \ \dot{H}_1(\mathbf{R}(\mathbf{r},\mathbf{p};t'), (\mathbf{P}(\mathbf{r},\mathbf{p};t'); t')] \ \dot{H}_1(\mathbf{r},\mathbf{p}; t=0) \frac{\partial f_0}{\partial H_0} \ . \end{aligned}$$

Here $(\mathbf{R}, \mathbf{P})$ denotes the phase space coordinates at time t' for the trajectory originating at the point $(\mathbf{r}, \mathbf{p})$, at the time $t = 0$. With the expression (3) for $\dot{H}_1$, the time integrand is discontinuous each time the trajectory starting in $(\mathbf{r}, \mathbf{p})$ crosses the window. As illustrated in fig. 2, the values of $(\mathbf{R}, \mathbf{P}; t)$ at the time of window crossings are denoted by $(\boldsymbol{\rho}_i, \mathbf{P}_i; t_i)$, $i = 1, 2, \ldots$. For a trajectory starting in $A, \mathbf{r} \in A$, the time integral in (5) then becomes

$$\begin{aligned} & \int_0^t dt' \ \mathbf{u}(\mathbf{R}(\mathbf{r},\mathbf{p};t')) \cdot \dot{\mathbf{P}}(\mathbf{r},\mathbf{p};t') \\ = \ & \mathbf{U}_A \cdot (\mathbf{P}_1 - \mathbf{p}) + \mathbf{U}_B \cdot (\mathbf{P}_2 - \mathbf{P}_1) + \mathbf{U}_A \cdot (\mathbf{P}_3 - \mathbf{P}_2) + \ldots \qquad (6) \\ = \ & -\mathbf{U}_A \cdot \mathbf{p} + (\mathbf{U}_A - \mathbf{U}_B) \cdot \sum_{n=1}^{N} (-1)^{n+1} \mathbf{P}_n \ + \mathbf{u}(\mathbf{R}(\mathbf{r},\mathbf{p};t)) \cdot \mathbf{P}(\mathbf{r},\mathbf{p};t) \ . \end{aligned}$$

A similar result holds for $\mathbf{r} \in B$. The number of window crossings, N, depends on the initial phase-phase position $(\mathbf{r},\mathbf{p})$ and the elapsed time t. For $t \to \infty$, the term $\mathbf{u}(\mathbf{R}(\mathbf{r},\mathbf{p};t)) \cdot \mathbf{P}$ will cancel when integrated over a small part of phase space, since values of $\mathbf{P}$ and - $\mathbf{P}$ will be equally probable due to the assumption II of ergodic motion. Also, the term $-\mathbf{U}_A \cdot \mathbf{p}$ will cancel when integrated over $\mathbf{p}$, since H_0 is even in $\mathbf{p}$ and $\dot{H}_1$ depends only on $\mathbf{r}$.

3.1 Substitution of integration variables

A key step in the derivation is the substitution of each phase space element $\mathbf{drdp}$ by one around the first window crossing $\mathbf{dR}_1\mathbf{dP}_1$. Let t'' denote the time it takes a particle to propagate from $(\mathbf{r},\mathbf{p})$ to $(\mathbf{R}_1, \mathbf{P}_1)$. The time dt'' spent within the phase-space element at $(\mathbf{R}_1, \mathbf{P}_1)$ is determined by its extension dz along the z-axis through the relation $\mu dz = -\mathbf{P}_1 \cdot \hat{\mathbf{z}} dt''$, where μ is the nucleon mass. (The z-axis points from B to A, as shown on fig. 1.) Employing t'' as an integration variable, we obtain

$$\mathbf{drdp} = -\mathbf{d}\boldsymbol{\rho}_1 \frac{\mathbf{P}_1 \cdot \hat{\mathbf{z}}}{\mu} dt'' \mathbf{dP}_1 \ , \tag{7}$$

where $\mathbf{d}\boldsymbol{\rho}_1$ is the two-dimensional interval on the window. The integral over $\boldsymbol{\rho}_1$ yields the window area σ_{window}. For each $(\mathbf{R}_1, \mathbf{P}_1)$, the upper limit t_0 of the t'' integration is found by following the trajectory backwards from $(\mathbf{R}_1, \mathbf{P}_1)$, until the previous window crossing is reached. It is natural to call that window crossing number zero and denote the corresponding momentum by $\mathbf{P}_0$, as shown in the example in fig. 2. When the integral over t'' is performed, each window phase space element substitutes a part of phase space forming a tube around the trajectory propagating backwards in time from $(\mathbf{R}_1, \mathbf{P}_1)$. For ergodic motion, all trajectories will eventually cross the window (see idealization II), so the entire phase space is covered (uniformly) by such tubes. After carrying out the substitution (7), the phase space integral with restriction to $\mathbf{r} \in A$ reads

$$\begin{aligned}
&\langle -\dot{H}_1 \int_0^t dt' \hat{U}_0(t') \dot{H}_1 \frac{\partial f_0}{\partial H_0} \rangle_{\mathbf{r}\in A} \\
&= -(\mathbf{U}_A - \mathbf{U}_B) \cdot \left(\int \mathbf{dr} \int \frac{\mathbf{dp}}{(2\pi)^3} \sum_{n=1}^{N} (-1)^{n+1} \mathbf{P}_n \frac{\partial f_0}{\partial H_0} \dot{\mathbf{p}} \right) \cdot \mathbf{U}_A \\
&= (\mathbf{U}_A - \mathbf{U}_B) \cdot \left(\int \mathbf{d}\boldsymbol{\rho}_1 \int_- \frac{\mathbf{dP}_1}{(2\pi)^3} \frac{\partial f_0}{\partial H_0} \frac{\mathbf{P}_1 \cdot \hat{\mathbf{z}}}{\mu} \int_0^{t_0} dt'' \, [\sum_{n=1}^{N} (-1)^{n+1} \mathbf{P}_n \, \dot{\mathbf{p}}(t'')] \right) \cdot \mathbf{U}_A \ ,
\end{aligned} \tag{8}$$

where the subscript on the momentum integral indicates that it extends over the negative hemisphere only, $\mathbf{P}_1 \cdot \hat{\mathbf{z}} < 0$.

Until now, only the requirement of ergodic motion has been invoked, and the expression (8) is not less complex than the starting point (5). The virtues of the changes made are the simple properties of the sum over momenta $(-1)^{n+1}\mathbf{P}_n$, when expressed in the new variables $\boldsymbol{\rho}_1, \mathbf{P}_1, t''$. The first term is just $\mathbf{P}_1$ itself, and requirements III and IV imply that the higher terms in the sum depend only on $t - t''$, after the integration over the window location $\boldsymbol{\rho}_1$ has been performed.

3.2 Contribution from the first window crossing

First the term for $n = 1$ of expression (8) is evaluated. For this term, the integral over t'' is straightforward, since the dependence on t'' is restricted to $\dot{\mathbf{p}}(t'')$. Inserting the values $\mathbf{p}(t''=0) = -\mathbf{P}_1$ and $\mathbf{p}(t''=t_0) = -\mathbf{P}_0$, one obtains

$$\begin{aligned}
&\langle -\dot{H}_1 \int_0^t dt' \hat{U}_0(t') \dot{H}_1 \frac{\partial f_0}{\partial H_0} \rangle_{\mathbf{r}\in A, n=1} \\
&= (\mathbf{U}_A - \mathbf{U}_B) \cdot \sigma_{\text{window}} \left(\int_0^\infty dP \frac{P^2}{(2\pi)^3} \frac{df_0(\frac{P^2}{2\mu})}{dP} \frac{\partial P}{\partial H_0} \frac{P}{\mu} P^2 \right) \\
&\qquad 2\pi\, \overline{\hat{\mathbf{P}}_1 (\hat{\mathbf{P}}_1 \cdot \hat{\mathbf{z}})(-\hat{\mathbf{P}}_0 + \hat{\mathbf{P}}_1)} \cdot \mathbf{U}_A \,.
\end{aligned} \tag{9}$$

Here the bar denotes the average over momentum directions $\hat{\mathbf{P}}_1$, under the condition $\hat{\mathbf{P}}_1 \cdot \hat{\mathbf{z}} < 0$. With the integral over the window position $\boldsymbol{\rho}_1$ carried out, $\mathbf{P}_0$ is uncorrelated with $\mathbf{P}_1$, see idealization III. If one orders the values of $\boldsymbol{\rho}_1$, $\mathbf{P}_1$ according to intervals of t_0, the average of $\hat{\mathbf{P}}_0$ within such an interval is given by the flux-weighted average, $\frac{2}{3}\hat{\mathbf{z}}$. Inserting this, the directional averages are readily calculated

$$\begin{aligned}
\overline{\hat{\mathbf{P}}_1 (\hat{\mathbf{P}}_1 \cdot \hat{\mathbf{z}}) \hat{\mathbf{P}}_1} &= -\frac{1}{8} (\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}} + 2\hat{\mathbf{z}}\hat{\mathbf{z}}) \,, \\
\overline{\hat{\mathbf{P}}_1 (\hat{\mathbf{P}}_1 \cdot \hat{\mathbf{z}}) \hat{\mathbf{P}}_0} &= \frac{2}{9} \hat{\mathbf{z}}\hat{\mathbf{z}} \,.
\end{aligned} \tag{10}$$

Here, the signs follow from the condition $\hat{\mathbf{P}}_1 \cdot \hat{\mathbf{z}} < 0$. The integral over the momentum magnitude P gives the result

$$\begin{aligned}
\int_0^\infty dP \frac{P^2}{(2\pi)^3} \frac{df_0(\frac{P^2}{2\mu})}{dP} \frac{\partial P}{\partial H_0} \frac{P}{\mu} P^2 &= -4 \int_0^\infty dP \frac{P^3}{(2\pi)^3} f_0(\frac{P^2}{2\mu}) \\
&= -\frac{1}{\pi} \int \frac{\mathbf{dP}}{(2\pi)^3} P f_0(\frac{P^2}{2\mu}) = -\frac{\rho \bar{v}}{\pi} \,.
\end{aligned} \tag{11}$$

Here $\rho\bar{v}$ is the product of the nuclear mass density and the mean nucleon speed in the nuclear interior, the standard one-body dissipation strength coefficient. Inserting these results into expression (9), and combining with the corresponding expression for $\mathbf{r} \in B$, the contribution from the first window crossing can be written

$$\begin{aligned}
&\langle -\dot{H}_1 \int_0^t dt' \hat{U}_0(t') \dot{H}_1 \frac{\partial f_0}{\partial H_0} \rangle_{n=1} \\
&= (\mathbf{U}_A - \mathbf{U}_B) \cdot \frac{1}{4} \rho \bar{v} \sigma_{\text{window}} (\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}} + 2\hat{\mathbf{z}}\hat{\mathbf{z}}) \cdot (\mathbf{U}_A - \mathbf{U}_B) \\
&+ (\mathbf{U}_A - \mathbf{U}_B) \cdot \frac{4}{9} \rho \bar{v} \sigma_{\text{window}} \hat{\mathbf{z}}\hat{\mathbf{z}} \cdot (\mathbf{U}_A - \mathbf{U}_B) \,.
\end{aligned} \tag{12}$$

Here, the first term is the standard window formula.[2] The second term will to a large extent be cancelled by terms stemming from the subsequent window crossings, as discussed below.

3.3 Contribution from subsequent window crossings

For $\mathbf{r} \in A$, the z-component of each term in the sum $(-1)^{n+1}\mathbf{P}_n$ is negative, since the alternating sign compensates for the alternating direction of passage of the window.

For a leptodermous window (see idealization I), the magnitude of the momentum $P_n = |\mathbf{P}_n|$ is the same for all window crossings, $P_0 = P_1 = \ldots \equiv P$. For given window location $\boldsymbol{\rho}_1$ and momentum direction $\hat{\mathbf{P}}_1$), the sum depends on the time elapsed since the first window crossing, $t - t''$. For sufficiently large times the ergodic limit can be invoked and the average time between successive window crossings, the residence times t_A and t_B, can readily be calculated as the inverse of the mean currents through the window. For example,

$$t_A^{-1} = \frac{\sigma_{\text{window}} 2\pi \int_0^{\frac{\pi}{2}} \frac{P}{\mu} \cos\theta \mathrm{d}(-\cos\theta)}{4\pi\Omega_A} = \sigma_{\text{window}} \frac{P}{4\mu} \frac{1}{\Omega_A} , \tag{13}$$

where Ω_A is the volume of part A. Therefore, for large times, the rate of change of the sum in eq. (8) is given simply by

$$\frac{d}{d(t-t'')} \left(\sum_{n=2}^{N} (-)^{n+1} \mathbf{P}_n \right) = -\frac{2}{3} \hat{\mathbf{z}} P \frac{2}{t_A + t_B} . \tag{14}$$

The solution to this differential equation is conveniently written as

$$\sum_{n=2}^{N} (-)^{n+1} \mathbf{P}_n = -\frac{2}{3} \hat{\mathbf{z}} P \frac{2}{t_A + t_B} [t - t'' - \Delta t_A(\boldsymbol{\rho}_1, \mathbf{P}_1)] , \tag{15}$$

where the integration constant is the time shift Δt_A and depends on $\boldsymbol{\rho}_1, \mathbf{P}_1$. For large times, when the above result holds, the time integral in (8) can be carried out by partial integration,

$$\begin{aligned} & \int_0^{t_0} dt'' \left(\sum_{n=2}^{N} (-)^{n+1} \mathbf{P}_n \dot{\mathbf{p}}(t'') \right) \qquad (16) \\ = \; & -\frac{2}{3} \hat{\mathbf{z}} P \left[\frac{2[t - \Delta t_A(\boldsymbol{\rho}_1, \mathbf{P}_1)]}{t_A + t_B} (-\mathbf{P}_0 + \mathbf{P}_1) + \frac{2t_0}{t_A + t_B} \mathbf{P}_0 + \frac{1}{t_A + t_B} \int_0^{t_0} \mathbf{p}(t'') dt'' \right] . \end{aligned}$$

The remaining integral of $\mathbf{p}(t'')$ will cancel when integrated over $\boldsymbol{\rho}_1, \mathbf{P}_1$, since it can be changed back into a phase space integral over $\mathbf{r}, \mathbf{p}$, which is odd in the momentum. In performing the integral over $\boldsymbol{\rho}_1, \mathbf{P}_1$, one needs the orientation averages

$$\overline{\hat{\mathbf{z}}(\hat{\mathbf{P}}_1 \cdot \hat{\mathbf{z}})\hat{\mathbf{P}}_1} = -\overline{\hat{\mathbf{z}}(\hat{\mathbf{P}}_1 \cdot \hat{\mathbf{z}})\hat{\mathbf{P}}_0} = \frac{1}{3} \hat{\mathbf{z}}\hat{\mathbf{z}} . \tag{17}$$

When these are inserted, the total contribution to (8) from all the crossings subsequent to the first one can be expressed as

$$\begin{aligned} & \langle -\dot{H}_1 \int_0^t dt' \hat{U}_0(t') \dot{H}_1 \frac{\partial f_0}{\partial H_0} \rangle_{\mathbf{r} \in A, n>1} \\ = \; & (\mathbf{U}_A - \mathbf{U}_B) \cdot \frac{8}{9} \rho \bar{v} \sigma_{\text{window}} \frac{2t}{t_A + t_B} \hat{\mathbf{z}}\hat{\mathbf{z}} \cdot \mathbf{U}_A \qquad (18) \\ - \; & (\mathbf{U}_A - \mathbf{U}_B) \cdot \frac{8}{9} \rho \bar{v} \sigma_{\text{window}} \frac{t_A + 2\overline{\Delta t}_A}{t_A + t_B} \hat{\mathbf{z}}\hat{\mathbf{z}} \cdot \mathbf{U}_A , \end{aligned}$$

where $\Delta\bar{t}_A$ is the average time shift, and the average value t_A of t_0 has been inserted.

The second term of (19) will partly cancel the second part of the contribution (12) from the first crossing. The first term of (19) diverges for large t, and should be cancelled by the second term of the expression (1), which corrects for the inclusion of non-dissipative contributions to the phase space integral (2) when the imposed surface velocities change the total volume.

3.4 Correction for volume change

For ergodic motion, and for a leptodermous potential, the second term of equation (1) is different from zero when the motion of the potential boundaries changes the volume of the system.[5] Indeed, by explicit evaluation one obtains

$$\langle \dot{H}_1 \hat{U}_0(t) \dot{H}_1 \frac{\partial f_0}{\partial H_0} t \rangle = \frac{10}{9} E \left(\frac{\dot{\Omega}}{\Omega} \right)^2 t \ , \tag{19}$$

where $\dot{\Omega}$ is the rate of change of the volume $\Omega = \Omega_A + \Omega_B$, and E is the total energy content of all particles. For the motion shown on fig. 1, the rate of change of the volume is equal to

$$\dot{\Omega} = \sigma_{\text{window}} (\mathbf{U}_A - \mathbf{U}_B) \cdot \hat{\mathbf{z}} \ , \tag{20}$$

and this can be inserted into the expression (19):

$$\frac{10}{9} E \left(\frac{\dot{\Omega}}{\Omega} \right)^2 t = -\frac{8}{9} \rho \bar{v} \ \frac{P_F}{2\mu} \ \sigma^2_{\text{window}} \frac{[(\mathbf{U}_A - \mathbf{U}_B) \cdot \hat{\mathbf{z}}]^2}{\Omega} t \ , \tag{21}$$

where P_F is the Fermi momentum. This verifies explicitly the cancellation of the term proportional to t in the phase space integral (19), when combined with the equivalent term from $\mathbf{r} \in B$, and the average residence times (see (13)) are inserted.

3.5 The time shifts

We now turn to the determination of the average time shifts $\Delta\bar{t}_A$ and $\Delta\bar{t}_B$. For that we consider the average number of crossings, $\mathcal{N}_A(t - t'')$, that particles first entering A at the time t'' have experienced by the time t (not counting their first crossing at time t''). An analogous definition holds for $\mathcal{N}_B(t - t'')$. These quantities are simply related to the sum appearing in eq. (8):

$$\sum_{n=2}^{N} (-1)^{n+1} \mathbf{P}_n = -\frac{2}{3} \hat{\mathbf{z}} P \, \mathcal{N}_A(t - t'') \ \text{ for } \mathbf{r} \in A \ , \tag{22}$$

and equivalently for $\mathbf{r} \in B$. It is now elementary to show that the two functions $\mathcal{N}_A(t)$ and $\mathcal{N}_B(t)$ obey the coupled integral equations

$$\begin{aligned} \mathcal{N}_A(t) &= \int_0^t dt_2 w_B(t_2) \left[1 + \mathcal{N}_B(t - t_2) \right] \ , \\ \mathcal{N}_B(t) &= \int_0^t dt_2 w_A(t_2) \left[1 + \mathcal{N}_A(t - t_2) \right] \ . \end{aligned} \tag{23}$$

where we have employed the residence-time distribution functions $w_A(t)$ and $w_B(t)$. The quantity $w_A(t)$ gives the probability that a particle after entering part A (through some random point on the window) will leave again after a specified length of time, t, and analogously for $w_B(t)$. It follows form the idealization IV that these distributions are given by

$$w_A(t) = \frac{1}{\bar{t}_A} e^{t/t_A} \; , \; w_B(t) = \frac{1}{\bar{t}_B} e^{t/t_B} \; . \tag{24}$$

Therefore, the integral equations (23) can readily be solved,

$$\mathcal{N}_A(t) = \frac{2t}{t_A + t_B} + \frac{t_A(t_A - t_B)}{(t_A + t_B)^2} \left[1 - e^{-(\frac{1}{t_A} - \frac{1}{t_B})t} \right] \; , \tag{25}$$

and equivalently for $\mathcal{N}_B$. By comparison with the large-time solution (15), and recalling the relationship (22), we then find

$$\Delta\bar{t}_A = \frac{t_A}{2} \frac{t_B - t_A}{t_A + t_B} \; , \; \Delta\bar{t}_B = -\frac{t_B}{2} \frac{t_B - t_A}{t_A + t_B} \; . \tag{26}$$

It should be noted that there is, of course, an inevitable time delay before a particle that has just entered A can leave A again, since it must first cross the bulk of A twice, at the least. This time delay is of the order of twice the transit time and falls in the category of microscopic time scales. As such it is assumed to be negligible in comparison with the above calculated time shifts (26) which arise from the nonuniformity in crossing times caused by the uneven size of the two containers and are of macroscopic size.

4 Discussion

The analyses in the preceding section allow us to write down the final result for the energy dissipation rate,

$$\begin{aligned} \dot{Q} &= (\mathbf{U}_A - \mathbf{U}_B) \cdot \frac{1}{4} \rho \bar{v} \sigma_{\text{window}} (\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}} + 2\hat{\mathbf{z}}\hat{\mathbf{z}}) \cdot (\mathbf{U}_A - \mathbf{U}_B) \\ &+ (\mathbf{U}_A - \mathbf{U}_B) \cdot \frac{4}{9} \rho \bar{v} \sigma_{\text{window}} \left(\frac{\Omega_A - \Omega_B}{\Omega_A + \Omega_B} \right)^2 \hat{\mathbf{z}}\hat{\mathbf{z}} \cdot (\mathbf{U}_A - \mathbf{U}_B) \; . \end{aligned} \tag{27}$$

Here, the first term is the standard window formula for the energy dissipation rate.[2] This term arises from the orientation average of $\hat{\mathbf{P}}_1(\hat{\mathbf{P}}_1 \cdot \hat{\mathbf{z}})\hat{\mathbf{P}}_1$, and is thus associated with the first window crossing only. Since the particles do not receive impulses at the window, the window dissipation stems from correlated impulses in trajectories that are first reflected a number of times in one part of the dinucleus, then cross the window, and are subsequently reflected a number of times in the other part. Preliminary estimates for nearly spherical nuclei indicate that a good convergence of the window formula is obtained after rather few reflections in both parts. Thus, the time required for obtaining the window formula (in the original time intergral formulation (1)) is expected to be of the order of a few times the microscopic transit time.

The second term of $\dot{Q}$ in the expression (27) arises because the average amounts of time $\bar{t_A}$ and $\bar{t_B}$ spent within the two parts of the system are not equal when the partition is asymmetric. This term can be associated with the rate of change in the relative mass asymmetry parameter

$$a \equiv \frac{\Omega_A - \Omega_B}{\Omega_A + \Omega_B} \tag{28}$$

implied by the motion depicted on fig. 1. Assuming that the change in volume is equally distributed on the two parts, *i.e.* $d(\Omega_A - \Omega_B)/dt = 0$, the second term of expression (27) can be rewritten in terms of the time derivative of a as

$$\dot{Q}_{\text{asym}} = \frac{4}{9}\frac{\rho\bar{v}}{\sigma_{\text{window}}}(\dot{a}\Omega)^2 \ . \tag{29}$$

In the case of the motion considered here, the change in the mass asymmetry arises from the radial motion. The more realistic and general case of volume conserving motion of the nuclear surface, and with arbitrary changes of the mass asymmetry, has been considered in [6], giving as the result the so-called *completed wall and window formula.* It is noteworthy that the dissipative resistance against changes in the mass asymmetry evaluated in [6] has exactly the form (29) when expressed in terms of the variable a. Thus we conclude that the present derivation of the expression (27) for the dissipation provides a proof based upon linear response theory of the completed wall and window formula for the special kind of motion shown in fig. 1.

The long mean free path dynamics of nuclei provides a unique dissipation mechanism, depending significantly on the symmetries of the motion. We have here elucidated the formal conditions under which the simple (completed) window is expected to be valid. In many situations of practical interest these conditions are only partially met, and in quasi-fission systems they are violated to a large degree. Indeed, the transition between the wall and window dissipation has posed a key problem in macroscopic nuclear dynamics for the last ten years and so far only *ad hoc* interpolations between the two dissipation formulas have been employed. The derivation of the wall formula in ref. [5], together with the present derivation of the window formula, establish linear response theory as a reliable starting point for investigating this question and the formalism developed here may provide a useful basis for systematic refinement of the treatment.

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Figure captions

Figure 1. The dinuclear cavity. In the dinucleus, the individual nucleons move in a leptodermous potential that has two distinct parts, A and B. The two parts are joined by a small planar "window" whose normal is chosen as the z-axis. The two dinuclear parts are endowed with the uniform translational velocities $\mathbf{U}_A$ and $\mathbf{U}_B$.

Figure 2. The window crossings. A nucleon initially located at the position $\mathbf{r}$and having the momentum $\mathbf{p}$has the momentum $\mathbf{P}_1$ when it first crosses the window, $\mathbf{P}_2$ at its next crossing, and so forth. Backwards propagation of the path yields the momentum at the most recent window crossing, $\mathbf{P}_0$. The contributing particles originate near the nuclear surface since only there is the effect of the imposed translation felt.

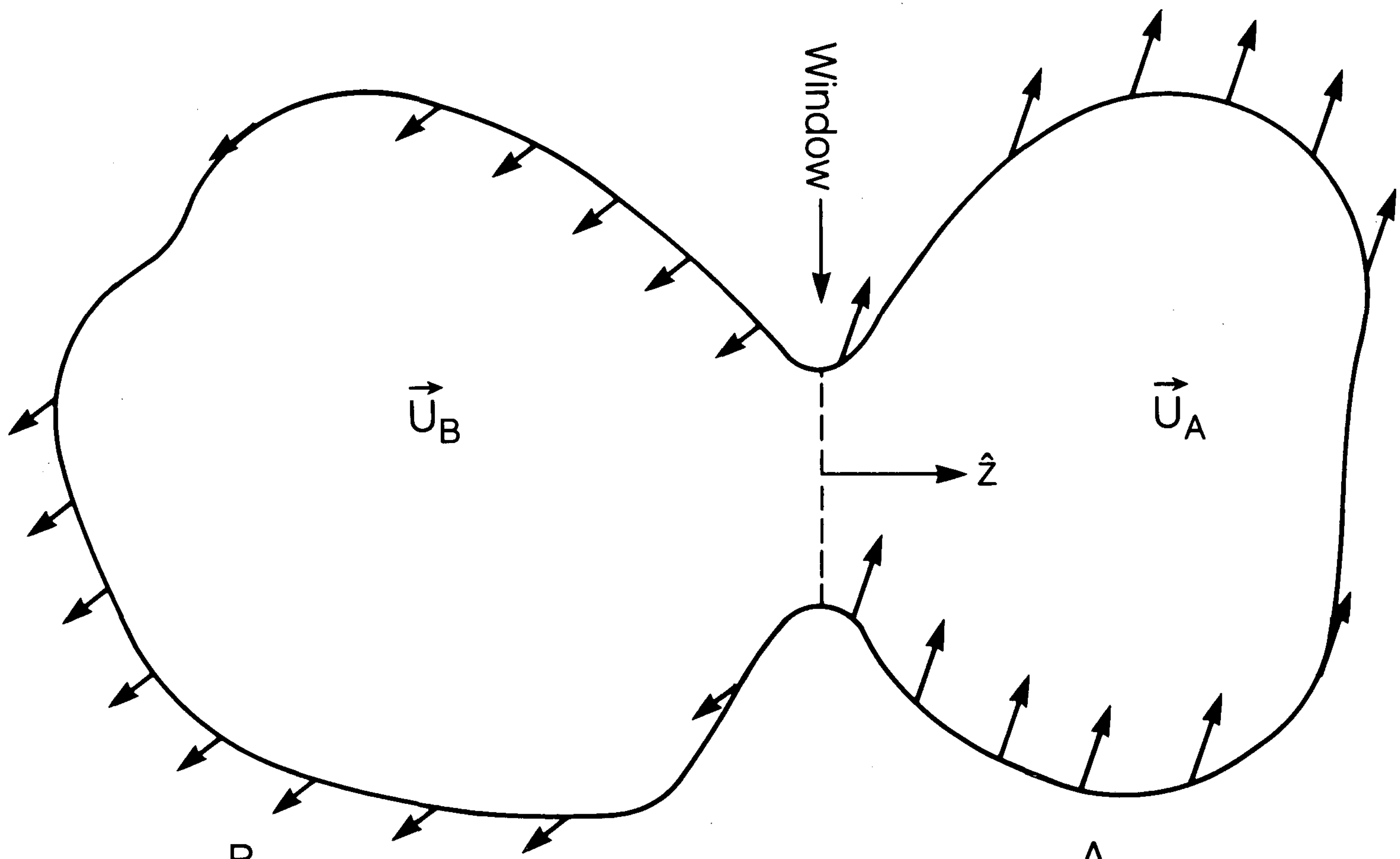

Figure 1.

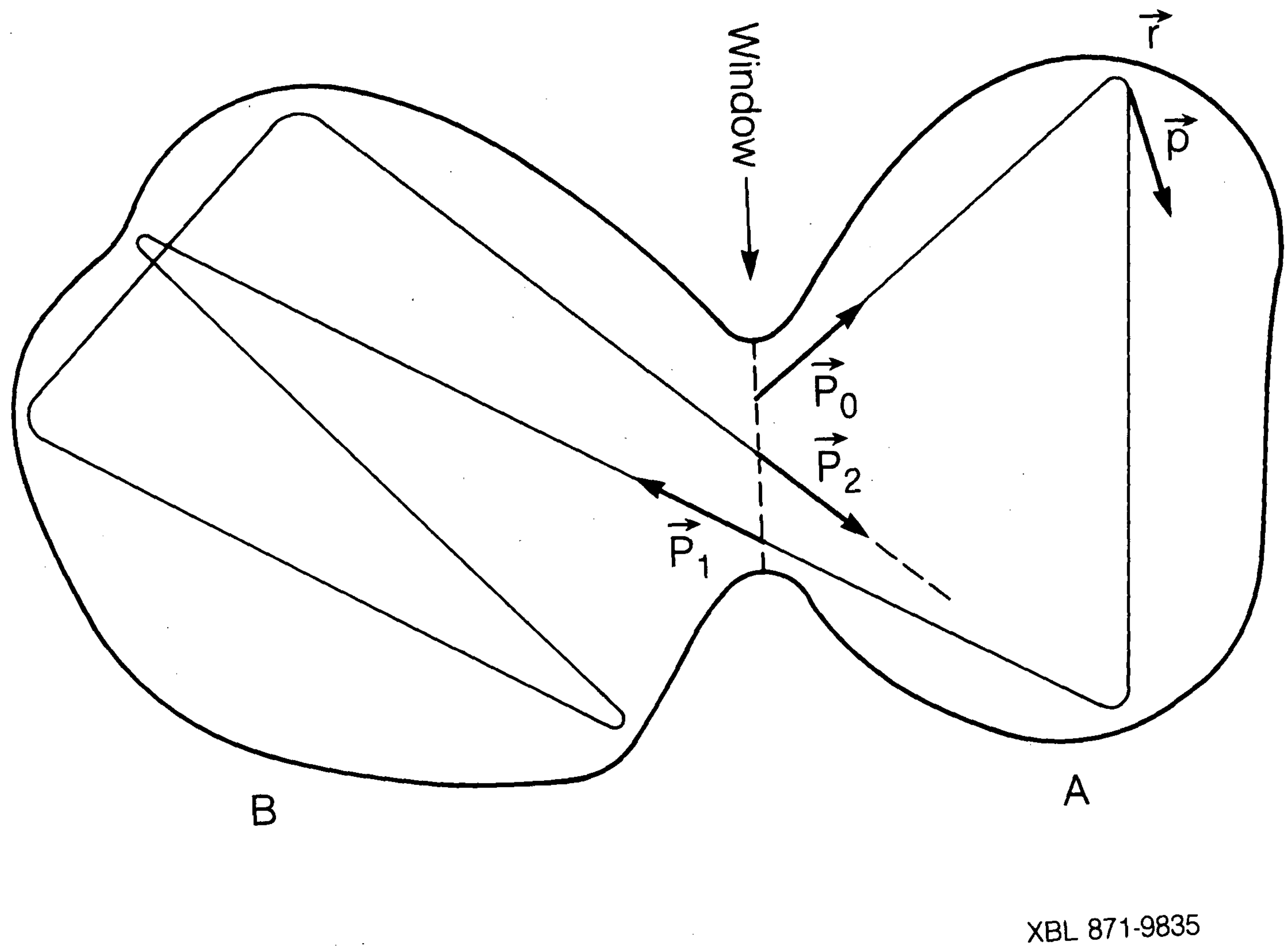

Figure 2.