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Journal
Physical Review D, 3(12)

ISSN
2470-0010

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Publication Date
1971-06-15

DOI
10.1103/physrevd.3.3173

PHYSICAL REVIEW D VOLUME 3, NUMBER 12 15 JUNE 1971

Time-Longitudinal Dilations and Scaling Behavior of High-Energy Collisions*

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(Received 10 February 1971)

The assumption that a dilation transformation in time and *one* space direction is implementable for matrix elements of certain operators between states with constituent particles moving mainly along that direction with large momenta, leads to scaling of inclusive processes in the high-energy limit. Without further restriction, certain exclusive processes likewise survive this limit. A nonzero limit for some of these processes, namely, those involving a transfer of quantum numbers, is unacceptable. This difficulty is overcome by postulating that conserved quantities may be broken up into right- and left-moving ones, and each is separately conserved.

I. INTRODUCTION

It has been recently noted that rates for many high-energy processes have certain scaling properties; they are functions of dimensionless ratios of variables describing these processes. It was conjectured by Bjorken[1] and experimentally confirmed[2] that the structure functions of inelastic electron scattering depend only on the ratio of the energy transfer and momentum transfer squared. This relation holds true only in the region where both variables are large. Likewise, for inclusive processes, Feynman[3] suggested that the spectrum of a definite particle emerging from a high-energy two-body collision depends only on the ratio of longitudinal momentum to center-of-mass energy, $q_\parallel/\sqrt{s}$, and on the transverse momentum $q_\perp$.

Such dependences on ratios of dynamical quantities are suggestive of the invariance of the forces causing these processes under scale transformations,[4]

$$x \to \lambda x, \quad p \to p/\lambda. \tag{1}$$

Invariance under such transformations implies the nonexistence of a scale of length and thus requires that all physical quantities depend only on dimensionless ratios. It is clear that the transformations generated by Eq. (1) are, at best, approximate symmetries valid at high energies. In this paper we shall be interested in processes initiated by two particles with large total center-of-mass energy. The experimental observation that transverse momenta[5] are bounded indicates that the complete four-dimensional dilation invariance is too strong. With this in mind, we shall explore invariance under time-longitudinal dilations (TLD),

$$\begin{aligned}(x_0, x_\parallel) &\to (\lambda x_0, \lambda x_\parallel), \quad x_\perp \to x_\perp, \\ (p_0, p_\parallel) &\to (p_0/\lambda, p_\parallel/\lambda), \quad p_\perp \to p_\perp.\end{aligned} \tag{2}$$

Again we expect this to be valid under very limited circumstances. *We shall assume that the transformations implied by Eq. (2) are valid for matrix elements of local operators, or products of local operators, taken between states of large total mass and with all constituent particles having large longitudinal and small transverse momenta.* We shall refer to such states as longitudinal states (LS). We note that in the center-of-mass system, the initial state of a high-energy two-body collision is a LS.

In the subsequent sections we shall examine the consequences of the assumption that the above transformation may be implemented. Among these are Feynman's conjecture[3] on inclusive processes, as well as the survival of certain exclusive processes at high energies, namely, those dominated by the exchange of a Pomeranchuk trajectory. Without further assumptions, we find that exclusive processes with an exchange of quantum numbers likewise survive at high energies. As this is unacceptable, there must be a scheme which ensures that the matrix elements for these reactions do vanish. We propose a scheme where any conserved operator may be decomposed into a part acting only on left- or on right-moving constituents. The assumption that each part, separately, is a symmetry eliminates the nonzero limit of the unwanted processes.

Before closing this section, it may be worthwhile to present a different way of understanding the transformations of Eq. (2) and Eq. (3). Rather than considering them to be a symmetry of some truncated Hamiltonian, they may be considered a symmetry of the equations describing some approximate model, as for instance, the multiperipheral model. Scaling properties of exclusive reactions have been obtained from these models.[6]

II. ASSUMED SYMMETRY

Let

$$|\cdots p_\parallel^{(i)}, p_\perp^{(i)}, \chi^{(i)} \cdots ; n\rangle$$

and

$$|\cdots q_{\parallel}^{(i)}, q_{\perp}^{(i)}, \eta^{(i)} \cdots ; m\rangle$$

be n- and m-particle LS, respectively. χ and η denote helicities, masses, and any other quantum numbers necessary to specify the state. We assume that the transformation of Eq. (2) may be implemented through a unitary transformation $U(\lambda)$. Normalization of the states leads to

$$U(\lambda)|\cdots p_{\parallel}^{(i)}, p_{\perp}^{(i)}, \chi^{(i)} \cdots ; n\rangle = \lambda^{-n/2}|\cdots p_{\parallel}^{(i)}/\lambda, p_{\perp}^{(i)}, \chi^{(i)} \cdots ; n\rangle . \quad (3)$$

Likewise, let $j_1(x_1)\cdots j_n(x_r)$ be local operators satisfying

$$U(\lambda) j_i(t^{(i)}, x_{\parallel}^{(i)}, x_{\perp}^{(i)}) U^\dagger(\lambda) = \lambda^{-d_i} j_i(\lambda t^{(i)}, \lambda x_{\parallel}^{(i)}, x_{\perp}^{(i)}) . \quad (4)$$

The assumed symmetry takes the following form as a condition on matrix elements of products of local operators between LS:

$$\langle\cdots q_{\parallel}^{(i)}, q_{\perp}^{(i)}, \eta^{(i)}\cdots ; m| j_1(x^{(1)})\cdots j_r(x^{(r)})|\cdots p_{\parallel}^{(i)}, p_{\perp}^{(i)}, \chi^{(i)}\cdots ; n\rangle$$
$$= \lambda^{-(m-n)/2}\prod_{i=1}^{n}\lambda^{-d_i}\langle\cdots q_{\parallel}^{(i)}/\lambda, q_{\perp}^{(i)}, \eta^{(i)}\cdots ; m| j_1(\lambda t^{(1)}, \lambda x_{\parallel}^{(1)}, x_{\perp}^{(1)})\cdots j_r(\lambda t^{(r)}, \lambda x_{\parallel}^{(r)}, x_{\perp}^{(r)})|\cdots p_{\parallel}^{(i)}/\lambda, p_{\perp}^{(i)}, \chi^{(i)}\cdots ; n\rangle . \quad (5)$$

The operators of interest will be the source currents for various particles. In Sec. III we obtain the dimensions d_i for these currents for particles of various spins.

III. DIMENSIONS OF SOURCE CURRENTS

A. Scalar Particles

If $\phi(x)$ is some local operator normalized through the condition

$$(2\pi)^3\sqrt{2p_0}\langle 0|\phi(0)|\vec{p}\rangle = 1 , \quad (6)$$

where $|\vec{p}\rangle$ denotes a state of the particle of interest, the source current $j(x)$ is defined by

$$(\partial^2 + m^2)\phi(x) = j(x) . \quad (7)$$

We wish to determine the dimensionality of $j(x)$ under TLD.

Let $|\alpha\rangle$, $|\beta\rangle$ be any multiparticle longitudinal states and let $|\vec{p}, \alpha\rangle$ be the state formed by adding the particle of interest to the state $|\alpha\rangle$. Let $|\vec{p}, \alpha\rangle$ likewise be a LS. Using the standard reduction technique, we obtain

$$\langle\beta, \text{out}|\vec{p}, \alpha, \text{in}\rangle = i\int\frac{e^{-ip\cdot x}d^4x}{(2\pi)^{3/2}\sqrt{2p_0}}\langle\beta; \text{out}|j^\dagger(x)|\alpha; \text{in}\rangle . \quad (8)$$

Performing a TLD on both sides we obtain in a straightforward manner

$$U(\lambda) j(x) U^\dagger(\lambda) = \lambda^2 j(\lambda t, \lambda x_{\parallel}, x_{\perp}) . \quad (9)$$

Thus we find that the d of Eq. (4), for scalar (or pseudoscalar) fields is $d = -2$.

B. Spinor Fields

Paralleling the previous discussion we consider a local spinor operator $\psi(x)$, normalized to

$$(2\pi)^{3/2}\sqrt{2p_0}\langle 0|\psi(0)|\vec{p}, \lambda\rangle = u(\vec{p}, \lambda) , \quad (10)$$

with $u(\vec{p}, \lambda)$ a Dirac spinor for a particle of momentum $\vec{p}$ and helicity λ. The source current for the particle under consideration is obtained from

$$(i\partial\!\!\!/ - m)\psi(x) = \chi(x) . \quad (11)$$

We again use the reduction formulas on states formed analogously to those of the previous section:

$$\langle\beta; \text{out}|\vec{p}\lambda, \alpha, \text{in}\rangle = i\int\frac{e^{-ip\cdot x}}{(2\pi)^{3/2}\sqrt{2p_0}}\langle\beta, \text{out}|\bar{\chi}(x) u(\vec{p}, \lambda)|\alpha, \text{in}\rangle . \quad (12)$$

In this limit, we may consider the spinors to be of the form $\sqrt{p_{\parallel}}(1 \pm \sigma_{\parallel})/2$ for helicities $\pm\frac{1}{2}$, respectively. Performing a TLD on both sides of Eq.(12), we obtain

$$(1 \pm \sigma_{\parallel}) U(\lambda)\chi(x) U^\dagger(\lambda) = \lambda^{3/2}(1 \pm \sigma_{\parallel})\chi(\lambda t, \lambda x_{\parallel}, x_{\perp}) . \quad (13)$$

Repeating this argument for the reduction of an antiparticle state, we get the unrestricted relation

$$U(\lambda)\chi(x) U^\dagger(\lambda) = \lambda^{3/2}\chi(\lambda t, \lambda x_{\parallel}, x_{\perp}) . \quad (14)$$

The dimension of the source currents for spinor particles is $-\frac{3}{2}$.

C. Vector Particles

Considering separately the reduction methods for transversely and longitudinally polarized vector particles, we find the following transformation properties for the source currents of vector particles:

$$\begin{aligned} U(\lambda) J_{0,\parallel}(x) U^\dagger(\lambda) &= \lambda J_{0,\parallel}(\lambda t, \lambda x_{\parallel}, x_{\perp}) , \\ U(\lambda) J_{\perp}(x) U^\dagger(\lambda) &= \lambda^2 J_{\perp}(\lambda t, \lambda x_{\parallel}, x_{\perp}) . \end{aligned} \quad (15)$$

It is tempting to assume that the 16 $SU(3)\otimes SU(3)$ currents have the transformation properties of Eq. (15). It is gratifying to note that the dimension of $J_0(x)$ is consistent with the current algebra of $SU(3)\otimes SU(3)$,

$$[J_0^\alpha(t, x_\parallel, x_\perp), J_0^\beta(t, y_\parallel, y_\perp)] = if^{\alpha\beta\gamma}\delta(x_\parallel - y_\parallel)\delta(x_\perp - y_\perp)J^\gamma(t, x_\parallel, x_\perp)\,. \tag{16}$$

IV. INCLUSIVE PROCESSES

A. Single-Particle Spectrum

We consider the high-energy process

$$a+b\to c+X \tag{17}$$

and are interested in the momentum spectrum of particle c when X is not observed. Let $j(x)$, $\chi(x)$, or $J_\mu(x)$ be the source currents for particle c in the cases where c is a scalar, spin-$\frac{1}{2}$, or spin-1 particle. This inclusive cross section in the center-of-mass system is

$$d\sigma = \frac{1}{8\sqrt{s}\,|p|}\times\begin{cases} W^{(0)}(p_\parallel, p_\perp, s)d^3p/p_0 \\ W^{(1/2)}_{\alpha\beta}(p_\parallel, p_\perp, s)\,u_\beta(p,\lambda)\,\bar{u}_\alpha(p,\lambda)d^3p/p_0 \\ W^{(1)}_{\mu\nu}(p_\parallel, p_\perp, s)\epsilon_\nu(p,\lambda)\epsilon^*_\mu(p,\lambda)d^3p/p_0 \end{cases} \tag{18}$$

with s being the center-of-mass energy, p and λ denoting the momentum and helicity of particle c, and the structure functions W being defined by

$$\begin{aligned} W^{(0)}(p_\parallel, p_\perp, s) &= (2\pi)^8(4p_{a0}p_{b0}) \times\int\langle a, b|j(x)j^\dagger(0)|a, b\rangle e^{-ip\cdot x}d^4x\,, \\ W^{(1/2)}_{\alpha\beta}(p_\parallel, p_\perp, s) &= (2\pi)^8(4p_{a0}p_{b0}) \times\int\langle a, b|\chi_\alpha(x)\,\bar{\chi}_\beta(0)|a, b\rangle e^{-ip\cdot x}d^4x\,, \\ W^{(1)}_{\mu\nu}(p_\parallel, p_\perp, s) &= (2\pi)^8(4p_{a0}p_{b0}) \times\int\langle a, b|J_\mu(x)J_\nu^\dagger(0)|a, b\rangle e^{-ip\cdot x}d^4x\,. \end{aligned} \tag{19}$$

We now concentrate on high incident energies and assume that $|a, b\rangle$ is a LS. With the dimensions obtained in the previous section the following scaling properties hold:

$$\begin{aligned} W^{(0)}(p_\parallel, p_\perp, s) &= \lambda^2 W^{(0)}(p_\parallel/\lambda, p_\perp, s/\lambda^2)\,, \\ W^{(1/2)}_{\alpha\beta}(p_\parallel, p_\perp, s) &= \lambda^1 W^{(1/2)}_{\alpha\beta}(p_\parallel/\lambda, p_\perp, s/\lambda^2)\,, \\ W^{(1)}_{\mu\nu}(p_\parallel, p_\perp, s) &= \lambda^{[a(\mu)+a(\nu)]}W^{(1)}_{\mu\nu}(p_\parallel/\lambda, p_\perp, s/\lambda^2)\,, \end{aligned} \tag{20}$$

with $a(\mu)=0$ for $\mu=0$ and parallel, and $a(\mu)=1$ for μ transverse. Implementing these scaling laws, we note that the above structure functions may be written as

$$\begin{aligned} W^{(0)}(p_\parallel, p_\perp, s) &= sF^{(0)}(p_\parallel/\sqrt{s}, p_\perp)\,, \\ W^{(1/2)}_{\alpha\beta}(p_\parallel, p_\perp, s) &= \sqrt{s}, F^{(1/2)}_{\alpha\beta}(p_\parallel/\sqrt{s}, p_\perp)\,, \\ W^{(1)}_{\mu\nu}(p_\parallel, p_\perp, s) &= s^{[a(\mu)+a(\nu)]/2}F^{(1)}_{\alpha\beta}(p_\parallel/\sqrt{s}, p_\perp)\,. \end{aligned} \tag{21}$$

Noting the scaling properties of the Dirac spinors u and polarization vectors ϵ_μ, we obtain Feynman's conjecture for the behavior of the inclusive one-particle spectrum (3):

$$d\sigma \sim F(p_\parallel/\sqrt{s}, p_\perp)\frac{d^3p}{p_0}\,. \tag{22}$$

B. Two-Particle Spectrum

A more complicated inclusive reaction is one in which two particles are detected:

$$a+b\to c+d+X\,. \tag{23}$$

In this reaction, c and d are detected and all other particles are summed over. The cross section in this case is proportional to

$$\langle a, b|T^*(j_c(x)\cdot j_d(y))T^*(j_c(z)\cdot j_d(0))|a, b\rangle\,, \tag{24}$$

where $T^*(j_c(x)j_c(y))$ is related to the ordinary time-ordered product by

$$T^*(j_c(x)j_d(y)) = T(j_c(x)j_d(y)) - \delta(x-y)\phi(x) - \partial_i\delta(x-y)\phi_i(x) - \cdots\,, \tag{25}$$

where $\phi(x)$, $\phi_i(x)\cdots$ are local operators, and the subtractions ensure that T^* is Lorentz-invariant.[7] Utilization of the reduction techniques permits us to establish the dimensions of these extra operators. Concentrating for definiteness on the case where c and d are spinless, we find

$$U(\lambda)T^*(j_c(x)j_d(y))U^\dagger(\lambda) = \lambda^4T^*(j_c(\lambda x_0, \lambda x_\parallel, x_\perp)j_d(\lambda y_0, \lambda x_\parallel, \lambda_\perp))\,. \tag{26}$$

From the above, we obtain the scaling property for the cross section:

$$d\sigma = \frac{d^3p_c}{p_{c0}}\,\frac{d^3p_d}{p_{d0}}F\left(\frac{p_{c\parallel}}{\sqrt{s}}, \frac{p_{d\parallel}}{\sqrt{s}}, p_{c\perp}, p_{d\perp}\right). \tag{27}$$

This may be generalized to multiparticle inclusive spectra.

V. EXCLUSIVE PROCESSES

Pursuing the ideas of time-longitudinal dilations, we may obtain scaling properties for exclusive reactions which are not so welcome as those for inclusive reactions. There are two typical exclusive reactions for which we may obtain such results:

$$a + b \to f\,, \tag{28a}$$

$$a + b \to c + f\,. \tag{28b}$$

In these reactions, $|a, b\rangle$ is a LS, $|f\rangle$ is a definite multiparticle LS, and c is a one-particle state. Let us first consider reaction (28a). The transition matrix is defined through

$$\langle f\,|\,a, b\rangle = -i\,\delta^4(p_f - p_a - p_b)T(p_a, p_b\,; p_1 \cdots p_n)\,. \tag{29}$$

TLD implies

$$T(p_a, p_b\,; p_1 \cdots p_n) = \lambda^{-(n-2)/2}\, T\left(\frac{p_{a\parallel}}{\lambda}, p_{a\perp} \cdots \frac{p_{n\parallel}}{\lambda}, p_{n\perp}\right). \tag{30}$$

This in turn implies that

$$|\,T(p_a, p_b\,; p_1 \cdots p_n\,|^2 = s\frac{G(p_{a\parallel}/\sqrt{s}, p_{a\perp}; \cdots p_{n\parallel}/\sqrt{s}, p_{n\perp})}{(\sqrt{s})^n}\,, \tag{31}$$

yielding for the cross section

$$d\sigma \sim sG\prod_{i=1}^{n}\frac{d^3p_i}{\sqrt{s}}\,\delta^4(p_a + p_b - p_f)\,. \tag{32}$$

For $|f\rangle$ a two-particle state, this would imply $d\sigma/dt = f(t)$ for all two-body final states. Clearly, this is an unwanted result. Similar scaling behavior is obtained for reaction (28b) by studying the transition matrix $\langle f\,|\,j_c(x)\,|\,a, b\rangle$.

In order to avoid this unpleasantness, we must find a mechanism which will ensure the vanishing of these exclusive spectra, i.e., make sure that G of Eq. (32) is zero. Section VI is devoted to a possible mechanism.

VI. LEFT- AND RIGHT-MOVING CONSTANTS OF THE MOTION

Following Feynman,[3] we shall assume that at very high energies, operators with a positive longitudinal component of momentum are separated from those with negative component. If Q is a conserved quantity, we decompose it into three parts,

$$Q = Q_R + Q_L + \text{rest}\,, \tag{33}$$

where $Q_{R(L)}$ acts on right- (left-) moving constituents and the "rest" term vanishes when acting on LS. We further assume that in the high-energy limit, both Q_R and Q_L are conserved.

This conservation does not exclude any of the inclusive processes but does exclude the exclusive ones where any quantum numbers are exchanged between the left- and right-moving constituents of the collision. This leaves us with exclusive reactions where no quantum numbers make a catastrophic shift. At zero momentum transfer, the survival of these reactions is consistent with dominance of the exchange of a Pomeranchuk trajectory. The question arises whether such processes should still survive at finite momentum transfers. If diffraction is dominated by a flat Pomeranchuk trajectory, then a nonzero limit of exclusive processes with no exchange of conserved quantum numbers would be consistent with our present ideas. The opposite case, namely, requiring all processes with finite momentum transfers to vanish, may be accomplished by requiring that the longitudinal momentum $P_\parallel$ be decomposable into $P_{\parallel R}$ and $P_{\parallel L}$ and that each be separately conserved. Thus only forward reaction would survive the high-energy limit.

ACKNOWLEDGMENTS

Part of this work was completed at Aspen Center for Physics, whose hospitality is acknowledged. I am grateful for discussions with Dr. D. Silverman and Dr. K. Wilson.

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*Partially supported by the National Science Foundation.

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