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The Pomeranchuk theorems[1] have played an important role in recent discussions of the high-energy asymptotic behavior of scattering amplitudes. The assumed constancy of high-energy cross sections has been used by Chew and Frautschi[2,3] in their statement of the principle of maximum strength for strong interactions. Recent experiments have shown, however, that the $\pi^{\pm}$-p, K^--p, and $\bar{p}$-p total cross sections are still falling at 20 Bev, and that they are doing so at different rates. On the other hand, the K^+-p and p-p total cross sections appear to have attained constant values. This has often led to the statement that the conditions for the validity of the Pomeranchuk theorems have not yet been attained in any of the elementary particle scattering processes, at energies accessible to the present-day accelerators. Discussions on this question have been clouded by the complete absence of a theoretical framework prescribing how high the energy has to be for the validity of these theorems, what is the rate at which the different cross sections are to approach the constant limit postulated by Pomeranchuk, or how the differences of particle-antiparticle cross sections are to approach zero. Therefore, it has not been possible so far, even empirically, to get a clear and reliable idea regarding the trends of these cross sections at high energies.[4]

It is the purpose of this letter to point out that one does have
such a framework now, following from the proposal of Chew and
Frautschi that the asymptotic behavior of the elastic scattering
amplitude for any process is dominated by the Regge poles[6] in the
crossed channels. We can already recognize why certain cross sections
are falling more rapidly than others at high energies while certain
others are more or less constant. With more extensive and accurate
experimental data on the cross sections at high energies on the one
hand, and with a better theoretical understanding of the trajectories
and residues of the Regge poles corresponding to the pionic resonances
in the low-energy region on the other, it should be possible in the
near future to understand how the different cross sections are
approaching the Pomeranchuk limit.

The relation between the asymptotic behaviors of the cross
sections and the Regge poles in the crossed channels will be seen
quite clearly if we consider the example of $\pi^{\pm}$-p cross sections in
some detail. These cross sections may be expressed as

$$\frac{\sigma(\pi^+ p) + \sigma(\pi^- p)}{2} = \frac{1}{q_L} \ \text{Im} \ T^+(\theta=0)$$

and

$$\frac{\sigma(\pi^- p) - \sigma(\pi^+ p)}{2} = \frac{1}{q_L} \ \text{Im} \ T^-(\theta=0) \ ,$$

where q_L is the laboratory-system momentum of the pion and $T^{\pm}$ are
combinations of $I = 1/2$ and $I = 3/2$ amplitudes given by
$$T^+ = \frac{1}{3} (T^{1/2} + 2T^{3/2}), \quad T^- = \frac{1}{3} (T^{1/2} - T^{3/2}) \ .$$

The sum and difference of the π^+-p and π^--p cross sections are therefore governed by the Regge poles that are present in the $\pi+\pi \longrightarrow N+\bar{N}$ channel of the T^+ and T^- amplitudes respectively. Now T^+ and T^- for the $\pi+\pi \longrightarrow N+\bar{N}$ channel have $I = 0$, $G = +1$ and $I = 1$, $G = +1$ respectively, and so the only known Regge poles that could be present in them are the vacuum pole and ABC pole[7] in the case of T^+, and the ρ-meson pole in the case of T^-. The vacuum pole trajectory, of course, passes through 1 at $s = 0$ in order to ensure constancy of high-energy cross sections; if the trajectories of the ABC pole and ρ-meson pole pass through $\alpha_{ABC}(0)$ and $\alpha_\rho(0)$ respectively at $s = 0$, then we immediately see[8]

$$\sigma(\pi^- p) - \sigma(\pi^+ p) \propto E^{-(1-\alpha_\rho(0))}$$

and

$$\sigma(\pi^- p) + \sigma(\pi^+ p) \propto a + b E^{-(1-\alpha_{ABC}(0))} \quad,$$

where E is the energy of the pion in the laboratory system. The ABC pole shows itself as a virtual state near the threshold of π-π scattering, i.e. at $s \approx 4$, and so has a value close to zero at $s \approx 4$. Therefore, it is expected to have a value ≈ 0 at $s = 0$. If we put $\alpha_{ABC}(0) = 0$, we get

$$\sigma(\pi^+ p) + \sigma(\pi^- p) = a + b/E \quad.$$

It is interesting to note that such a fit was actually used by S. J. Lindenbaum et al.[5] for their data purely on empirical grounds. One now sees a possible reason why it works so well. It is worth while

to emphasize here the close relationship between the zero-energy $I = 0$ scattering of the π-π system (ABC pole) and the sum of high-energy $\pi^{\pm}$-p cross sections. The coefficient b of the $E^{-(1-\alpha_{ABC}(0))}$ term should eventually be related to the parameters of the $I = 0$ π-π scattering at low energies.

The ρ-meson pole passes through 1 at $s = 29$, and has a value < 1 at $s = 0$, and is thus responsible for the decrease with energy of the difference of $\pi^{\pm}$-p cross sections. The CERN data[9] from 4.5 Bev/c to 10 Bev/c can be fitted by

$$\sigma(\pi^- p) - \sigma(\pi^+ p) \varpropto E^{-1/2} ,$$

indicating $\alpha_\rho(0) \approx 0.5$. The Brookhaven data at higher energies[5] also seem to be consistent with such a fit. But in view of the small value of the difference of these cross sections, and the consequent large error in the experimental values, one has to wait for more data before making a more definitive fit.

One important point which already emerges from the above discussion is that in attempting empirical fits to the high-energy cross sections, one must use certain definite linear combinations of the cross sections. Thus it is meaningless to say that $\sigma(pp)$ or $\sigma(K^+ p)$ alone satisfies the conditions for the validity of Pomeranchuk theorem. In the table I below we have shown the Regge poles governing various sums and differences of cross sections, and the expected asymptotic behaviors arising therefrom.[10,11] The Regge poles in question are those corresponding to vacuum, $\eta, \rho, \omega, \pi, ABC$ --i.e., six

Table I.

Regge poles contributing to various cross sections.

Cross sections	Contributing Regge poles	Expected high-energy behavior
$\sigma(\pi^- p) + \sigma(\pi^+ p)$	vacuum, ABC	$a + bE^{-(1-\alpha_{ABC}(0))}$
$\sigma(\pi^- p) - \sigma(\pi^+ p)$	ρ	$cE^{-(1-\alpha_\rho(0))}$
$\sigma(K^- p) - \sigma(K^+ p)$	η,ω,ρ	$dE^{-(1-\alpha_\eta(0))} + eE^{-(1-\alpha_\omega(0))} - gE^{-(1-\alpha_\rho(0))}$
$\sigma(K^- p) + \sigma(K^+ p)$	vacuum, ABC	$f + hE^{-(1-\alpha_{ABC}(0))}$
$\sigma(\bar{p}p) - \sigma(pp)$	η,ω,ρ	$\ell E^{-(1-\alpha_\eta(0))} + mE^{-(1-\alpha_\omega(0))} - qE^{-(1-\alpha_\rho(0))}$
$\sigma(\bar{p}p) + \sigma(pp)$	vacuum, π, ABC	$p + nE^{-(1-\alpha_\pi(0))} + rE^{-(1-\alpha_{ABC}(0))}$
$\sigma(pp) - \sigma(np)$	ρ,π	$qE^{-(1-\alpha_\rho(0))} + nE^{-(1-\alpha_\pi(0))}$
$\sigma(pp) + \sigma(np)$	vacuum, ABC, η,ω	$p + rE^{-(1-\alpha_{ABC}(0))} - \ell E^{-(1-\alpha_\eta(0))} - mE^{-(1-\alpha_\omega(0))}$
$\sigma(K^+ p) + \sigma(K^+ n)$	vacuum, ABC, η,ω	$f + hE^{-(1-\alpha_{ABC}(0))} - dE^{-(1-\alpha_\eta(0))} - eE^{-(1-\alpha_\omega(0))}$
$\sigma(K^+ p) - \sigma(K^+ n)$	ρ	$gE^{-(1-\alpha_\rho(0))}$

in all. It is therefore not surprising that the high-energy cross
sections for different channels approach the Pomeranchuk limit quite
differently.

The coefficients a,b,···occurring in the above table are
related to the residues of the various Regge poles in the corresponding
channels, and it should be possible to obtain information on them from
an analysis of data on the electromagnetic structure of the nucleon,
and low-energy π-π, π-N, N-N, K-N and π-K scattering. Some of
these relationships are being considered in detail. In the meantime,
it appears on empirical grounds that the signs of all our coefficients
as written in the table are positive. For any given two-particle
channel, the vacuum pole alone contributes to the forward coherent
amplitude at the highest energies, giving rise to a constant total
cross section and the Pomeranchuk theorems. At lower energies, more
and more Regge poles show up and produce "deviations" from the
Pomeranchuk theorems.[12] We have already seen in the case of
$[\sigma(\pi^-p) + \sigma(\pi^+p)]$ that even though $\alpha_{vac}(0) - \alpha_{ABC}(0) \approx 1$, the
ABC pole makes a substantial contribution in the energy region of
approx 20 Bev/c, making this cross section fall by about 15% between
5 Bev/c and 20 Bev/c. Since all the Regge poles we are considering
have trajectories lying between the vacuum pole and the ABC pole, we
may expect that whenever more than one of them are capable of
contributing, their contributions will be comparable in this energy
range. Now in the case of $\sigma(pp)$ and $\sigma(K^+p)$, all or almost all of
them are contributing, some with positive sign and some with negative.
The observed near constancy of these cross sections therefore seems

to be a rather complicated effect arising from the more or less complete cancellation of the contributions of all except the vacuum pole.

It should be noticed that some of the coefficients occur more than once in the table. For example, the coefficients of the ρ-pole term are equal but opposite in sign in $[\sigma(\bar{p}p) - \sigma(pp)]$ and $[\sigma(pp) - \sigma(np)]$; the coefficients of η and ω poles are equal but opposite in sign in $[\sigma(\bar{p}p) - \sigma(\bar{p}p)]$ and $[\sigma(pp) + \sigma(np)]$; and so on. This means, for example, that whereas $\sigma(\bar{p}p)$ approaches the Pomeranchuk limit from above, $\sigma(np)$ should do so from below. The scanty information on $\sigma(np)$ is consistent with this expectation. At 5 Bev,[13] $\sigma(np) = 33.6 \pm 1.6$ mb, and thus is about 9 mb less than $\sigma(pp)$, and this difference is comparable with $\sigma(\bar{p}p) - \sigma(pp)$ at the same energy,[14] as expected. It is to be hoped that enough data will become available in the near future to test these predictions in detail.

Recent data[14,15] on $K^{\pm}$-p, p-p and $\bar{p}$-p scattering seem to bear out the above expectations, though since differences of cross sections are to be fitted, and these have large errors, a detailed quantitative comparison will have to be postponed. $[\sigma(K^+p) + \sigma(K^-p)]$ seems consistent with $\alpha_{\rho}(0) \approx 0.5$, and $[\sigma(K^-p) - \sigma(K^+p)]$ can be fitted with a single Regge pole with an effective $\alpha(0) \approx 0.4$, which we may for the time being attribute to the η . Actually ρ, η and ω are all involved in the latter, and with more accurate data it should be possible to separate their effects. For the present, it is very

encouraging that the value of $\alpha_\rho(0)$ from $[\sigma(K^+p) + \sigma(K^-p)]$ is consistent with that from $[\sigma(\pi^-p) - \sigma(\pi^+p)]$ and that $\alpha_\eta(0)$ has a value consistent with our ideas regarding the slopes of the Regge trajectories.[3] The data on $\sigma(\bar{p}p)$ and $\sigma(pp)$ also fit into the above pattern. An analysis of the combined data to get $\alpha(0)$ for all the poles is in progress.

To conclude, we would like to emphasize that even though there may be no structure in the cross sections, there is considerable information hidden in the gentle slopes with which the total cross sections are approaching the Pomeranchuk limit.

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REFERENCE AND FOOTNOTES

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2. G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 7, 394 (1961).

3. G. F. Chew and S. C. Frautschi, Phys. Rev. Letters (to be published).

4. For example, in a recent paper, S. J. Lindenbaum et al. (reference 5)
   have given an empirical fit for their data on $\pi^{\pm}$-p cross sections
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   finite limit ($\approx$ 2 mb) as energy becomes infinite. Such a behavior
   would, of course, be inconsistent with the Pomeranchuk theorem.

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7. The ABC pole refers to the I = 0 two-pion anomaly observed by
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8. Note that $\cos \theta_t \big|_{t=0} \sim E$ in the π-N channel.

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10. Note that only $B = 0$, $S = 0$ trajectories contribute. In preparing this table we have assumed that η and ω have $I = 0$, $G = -1$; and that ρ has $I = 1$, $G = +1$. The Regge poles which contribute can be determined from considerations of G parity and iso-spin. In the case of $\sigma(\bar{p}p) \pm \sigma(pp)$ they follow directly from the concept of J-parity introduced by Chew and Frautschi (reference 5). I am thankful to Dr. Evan J. Squires for several interesting discussions on this point.

11. We have retained only the leading term in the expansion of $P_\alpha(z)$ for large z. The more exact expressions will be considered in another communication in connection with the angular distributions in the diffraction peak.

12. In our table we have included only the known Regge poles whose trajectories lie high enough for them to show up as particles. These appear to be sufficient for a description of the cross sections in the region of several Bev. At low energies more Regge poles may be expected to contribute as well.

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