Title
ENERGY DEPENDENCE OF THE LOW-ENERGY K- -PROTON AND K+ -PROTON CROSS SECTIONS

Authors
Ferrari, Fabio
Frye, Graham
Pusterla, Modesto.

Publication Date
1960-10-13

UCRL-9434

UNIVERSITY OF CALIFORNIA

Ernest O. Lawrence Radiation Laboratory

BERKELEY, CALIFORNIA

UCRL-9434

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

ENERGY DEPENDENCE OF THE LOW-ENERGY K^--PROTON AND K^+-PROTON CROSS SECTIONS

Fabio Ferrari, Graham Frye, and Modesto Pusterla

October 13, 1960

ENERGY DEPENDENCE OF THE LOW-ENERGY K^--PROTON AND K^+-PROTON CROSS SECTIONS

Fabio Ferrari, Graham Frye, and Modesto Pusterla

Lawrence Radiation Laboratory
University of California
Berkeley, California

October 13, 1960

ABSTRACT

The K^--proton and K^+-proton S-wave scattering is analyzed by using a relativistic effective range formula derived by studying the analytic properties of partial wave scattering amplitudes. The influence of the pion-pion interaction on the elastic and reaction cross sections is discussed.

ENERGY DEPENDENCE OF THE LOW-ENERGY K^--PROTON AND K^+-PROTON CROSS SECTION*

Fabio Ferrari†, Graham Frye§, and Modesto Pusterla**

Lawrence Radiation Laboratory
University of California
Berkeley, California

October 13, 1960

I. INTRODUCTION

An interesting analysis of the low energy K^- elastic and reaction processes on protons, has been worked out by using an effective range expansion for S-state interactions. Jackson, Ravenhall and Wyld have shown that in order to describe these processes, only two complex phase shifts are needed.[1] In particular, extensive calculations carried out by Dalitz and Tuan on the basis of zero range formulas, and considering explicitly the $\bar{K}^0$-K^- mass difference, have succeeded in reproducing all the gross features of the elastic and reaction cross sections.[2]

Unfortunately, although these phenomenological approaches give a reasonable fit to the experimental data, they are deficient from a theoretical point of view inasmuch as there are no clear relations between the phenomenological parameters and the dynamics of the interaction. On the other hand, more fundamental theoretical attempts based on conventional field theory

---

* This work was supported by the U. S. Atomic Energy Commission, a grant from the National Academy of Science (F.F.) and the U. S. Air Force, and monitored by the Air Force Office of Scientific Research of the Air Research Development Command.

† Now at Istituto di Fisica, Universita' di Padova, Padova, Italy.

§ Now at the Physics Department, University of Washington, Seattle, Washington.

** Now at Istituto di Fisica Teorica, Universita' di Napoli, Napoli, Italy.

have so far been meaningless because of the difficulties of handling strong interactions. The static model, although tractable, is inadequate for the K-nucleon problem.

The aim of this paper is to discuss the K-nucleon interaction in terms of relativistic effective range formulas, derived by studying the analytic properties of partial wave scattering amplitudes. This new way of looking at strong interaction theory, originated by the works of Chew-Mandelstam and Chew-Low,[3] marked an important step forward with respect to previous theoretical attempts. The fundamental principle of the theory is that the "interactions" are associated with singularities of the scattering amplitudes in the unphysical region. The location of these singularities is determined by the masses of the physical systems involved, whereas their strengths are related to physical cross sections and restricted by the unitarity condition.[4] It follows that the "long range forces", due to the exchange of one or two particles, are associated with "near by" singularities. At the present stage, it is possible to calculate the "long range forces", while the "short range forces", due to the exchange of three or more particles, have still to be treated phenomenologically.

Returning to the K-nucleon interaction, we would like to emphasize the fact that, as a consequence of the small mass of the pion with respect to the K-meson, the singularity associated with the exchange of a two-pion system is very close to the K-nucleon physical threshold and, more important, may be considered as the only "near-by" singularity.[5] From this point of view, the "long range part" of the K-nucleon interaction should, in principle, give clearer information on the pion-pion interaction than that obtained by studying the nucleon-nucleon and pion-nucleon problems. It is unfortunate that, due to the complexity of the K-nucleon force, other strongly interacting particles tend to confuse the situation with major contributions to the S-wave phase shifts. Nevertheless, the presence of a strong pion-pion interaction should affect in a detectable way the energy dependence of the low energy K^--proton elastic and reaction cross sections.

In a previous note,[6] we have analyzed the elastic and charge exchange K^--proton cross sections on the basis of a model in which there is a strong pion-pion interaction in the $I = 1$, $J = 1$ state. It has been found that a sharp resonance, such as suggested by Frazer and Fulco,[7] is consistent with the present experimental data. Here we will discuss the influence of the pion-pion interaction on hyperon production.

Finally, the relation between the "long range force" of the K-nucleon and $\bar{K}$-nucleon systems is pointed out,[8] which follows from charge conjugation symmetry. There is no simple relation for the short range forces that involve exchange of particles other than pions.

II. RELATIVISTIC EFFECTIVE RANGE FORMULAS FOR THE $\overline{K}$-NUCLEON INTERACTION

Let us consider the $\overline{K}$-nucleon interaction under the assumption of charge independence and neglecting mass corrections. The partial-wave scattering amplitudes for the processes

$$\overline{K} + N \to \overline{K} + N$$

$$\overline{K} + N \to Y + \pi$$

$$Y + \pi \to Y + \pi$$

can be written in terms of two matrices $T^{(I)}$ (I=0,1 are the isotopic spin):

$$T^{(I)} = \begin{vmatrix} T^{(I)}_{\overline{K}N,\overline{K}N} & T^{(I)}_{\overline{K}N,Y\pi} \\ T^{(I)}_{Y\pi,\overline{K}N} & T^{(I)}_{Y\pi,Y\pi} \end{vmatrix} \qquad \text{(II.1)}$$

For $I = 0$, $T^{0}_{Y\pi,Y\pi}$ and $T^{0}_{Y\pi,\overline{K}N}$ are 1-by-1 matrices, whereas for $I = 1$, $T^{1}_{Y\pi,Y\pi}$ and $T^{1}_{\overline{K}N,YT}$ are 2-by-2 and 2-by-1 submatrices referring to $\Sigma\pi$ and $\Lambda\pi$ channels. As pointed out in FNP and FFP, it is convenient to express the analytic behavior of the $T^{(I)}$ matrix in terms of a matrix $G^{(I)}$ defined by:

$$T^{(I)} = \frac{1}{\sqrt{s}} \left[\frac{q^{2\ell+1}}{E + M}\right]^{1/2} G^{(I)} \left[\frac{q^{2\ell+1}}{E + M}\right]^{1/2} \qquad \text{(II.2)}$$

where $G^{(I)}$ is of the form

$$G^{(I)} = \frac{1}{16\pi} \Big\{ (E + M) q^{-\ell} [A_{\ell}^{(I)} + (\sqrt{s} - \overline{M}) B_{\ell}^{(I)}] q^{-\ell} (E + M)$$

$$- q^{-\ell+1} [A_{\ell\pm1}^{(I)} - (\sqrt{s} + \overline{M}) B_{\ell\pm1}^{(I)}] q^{-\ell+1} \Big\} . \qquad \text{(II.3)}$$

A_ℓ and B_ℓ are the partial-wave projections of the amplitudes A and B which satisfy the Mandelstam representation; q, E, and M are diagonal matrices with components equal to the center-of-mass momentum, baryon energy and baryon mass, respectively; $\overline{M}_{ij} = \frac{1}{2}(M_i + M_j)$ and $\sqrt{s}$ is the total energy. G has been defined so as to contain, in the cut $\sqrt{s}$-plane, only singularities connected with the Mandelstam representation. (In what follows we will suppress, for simplicity, the isotopic spin index.)

From Eq. (II.2), it follows that the unitarity condition requires:

$$\text{Im}\, G^{-1}(s) = -\frac{1}{\sqrt{s}} \frac{q^{2\ell+1}}{E + M} \Theta \tag{II.4}$$

where Θ is a diagonal matrix of step functions

$$\Theta_i = \begin{cases} 1 & W_i < \sqrt{s} \\ 0 & W_i > \sqrt{s} \ . \end{cases} \tag{II.5}$$

W_i being the threshold energy of the ith channel.

The location of the singularities of G is discussed in Ref. (9). The main point is that all the dynamic cuts, except that arising from the $K\overline{K}$-$\pi\pi$ interaction, lie below the threshold for the $\Lambda\pi$ channel, ($s_{\Lambda\pi} \sim 80 m_\pi^2$). Therefore, restricting our discussion to energies close to the $\overline{K}$-nucleon physical threshold ($p_{lab} \lesssim$ 250 Mev/c, i.e., $104\, m_\pi^2 \leq s \leq 113\, m_\pi^2$), the single baryon poles, the cuts associated with the absorptive part of the processes $\pi + \pi \rightarrow Y + \overline{Y}$, the crossed K-nucleon and π-hyperon cuts, should affect weakly the energy dependence of the scattering amplitudes. Moreover, only some of the discontinuities across these cuts may be considered as known functions (for example, the single baryon terms containing the baryon masses, the K-nucleon and the π-hyperon coupling constants). Thus, it is plausible to describe the effect of these singularities by phenomenological constants or by "far away" poles.[10] The introduction of phenomenological parameters makes it immaterial, for S-wave processes, to work in the cut $\sqrt{s}$-plane; for simplicity, we shall use the cut s-plane later on (see Appendix A).

The dynamic singularity due to the $K\bar{K}$-$\pi\pi$ interaction instead extends to $s = [\sqrt{M_N^2 - m_\pi^2} + \sqrt{m_K^2 - m_\pi^2}]^2$ and should produce the strongest energy dependence in the physical amplitudes. The discontinuity across this cut, i.e. $[\mathrm{Im}\, G(s)]_{\pi\pi}$, depends on the matrix elements for the reactions $\pi + \pi \to K + \bar{K}$ and $\pi + \pi \to N + \bar{N}$.[8] Extensive calculations are required to determine the influence the long range force has on the physical processes. To simplify the problem, we will represent the distributed spectrum $[\mathrm{Im}\, G(s)]_{\pi\pi}$ by the function $R\delta(s - a_1)$. This approximation is quite accurate for a narrow pion-pion resonance of the type introduced by Frazer and Fulco in the $I = 1$, $J = 1$ state. In particular, if the two pion system interacts prevalently in this state, the long range contributions to the $I = 0$ and $I = 1$ $\bar{K}$-nucleon states are in the ratio ~ -3.[8]

Let us now go on to calculate the scattering amplitudes. We define[11]

$$G^{-1}(s) = D(s)\, N^{-1}(s) \tag{II.6}$$

where the matrix $N(s)$ has only the left hand cuts and the matrix $D(s)$ has only the unitarity cuts starting from the physical thresholds s_0 for the different $\bar{K}$-nucleon and π-hyperon processes. Applying Cauchy's theorem in the complex s-plane, we have for a given set of poles on the dynamical cuts:

$$N(s) = -\frac{1}{\pi} \Sigma_i \frac{R_i D(a_i)}{s - a_i}$$

(II.7)

$$D(s) = 1 - \frac{s - \omega}{\pi^2} \int_{s_0}^{\infty} ds' \quad \Sigma_i \frac{\text{Im } G^{-1}(s') R_i D(a_i)}{(s' - s)(s' - a_i)(s' - \omega)}$$

where $D(s)$ is normalized by setting $D(\omega) = 1$. R_i are matrices in channel space. In particular, R_1 refers to the $\pi\pi$ interaction and has only the (1,1) element $R_{KK,\pi\pi}$ different from zero.

A two-pole approximation to the left hand cuts leads to the following expressions for the elastic and reaction amplitudes:

(II.8)

$$A^{(I)}_{\overline{K}N,\overline{K}N}(s) = \frac{1}{(E + M)\sqrt{s}} G^{(I)}_{\overline{K}N,\overline{K}N}(s) = \frac{1}{(E + M)\sqrt{s}} \left[\ell(s) + \frac{1}{f_I(s) + z_I} - i \frac{q_{N\overline{K}}}{(E + M)\sqrt{s}} \right]^{-1}$$

$$A^{(I)}_{\overline{K}N,\Sigma\pi}(s) = \left(\frac{q_{\Sigma\pi} M}{q_{N\overline{K}} M_\Sigma} \right)^{1/2} \frac{z_I \eta_I}{z_I + f_I(s)} A^{(I)}_{\overline{K}N,\overline{K}N}(s) , \qquad (II.9)$$

$$A^{(1)}_{\overline{K}N,\Lambda\pi}(s) = \left(\frac{q_{\Lambda\pi} M}{q_{N\overline{K}} M_\Lambda} \right)^{1/2} \frac{z_1 \nu_1}{z_1 + f_1(s)} A^{(1)}_{\overline{K}N,\overline{K}N}(s) \qquad (II.10)$$

where

$$\ell(s) = \frac{(s - a_1)(s - a_2)}{\pi} P \int_{s_0}^{\infty} ds' \frac{\text{Im } G^{-1}(s')}{(s' - s)(s' - a_1)(s' - a_2)}$$

$$f_I(s) = -\frac{R^{(I)}_{KK,\pi\pi}}{\pi(s - a_1)(1 + \beta R^{(I)}_{KK,\pi\pi})}$$

$$\beta = \frac{a_1 - a_2}{\pi^2} \int_{s_0}^{\infty} ds' \frac{\operatorname{Im} G^{-1}(s')}{(s' - a_1)^2 (s' - a_2)} \quad .$$

z_0, z_1, η_0, η_1 and ν_1 depend on the parameters $(R_{ij})_2$ and a_2 of the second pole and on the center-of-mass momenta $q_{\Sigma\pi}$ and $q_{\Lambda\pi}$ for the pion-hyperon systems; they may be regarded as complex constants if we neglect the energy dependence of $q_{\Sigma\pi}$ and $q_{\Lambda\pi}$ with respect to q_{NK} (Appendix B).

Finally, by considering the unitarity condition we have the following relations between the constants z_0, z_1, η_0, η_1, and ν_1 :

$$q_{\Sigma\pi} \mid \eta_0 \mid^2 = - \operatorname{Im} (1/z_0) \tag{II.11}$$

$$q_{\Lambda\pi} \mid \nu_0 \mid^2 + q_{\Sigma\pi} \mid \eta_0 \mid^2 = - \operatorname{Im} (1/z_1) \quad . \tag{II.12}$$

It is easy to see that the elastic scattering amplitudes, Eq. (II.8), reduce to the two complex scattering lengths of JRW if the $K\bar{K}\pi\pi$ interaction is disregarded, i.e. if $f_I(s) \sim 0$. Note that the energy dependence of $\ell(s)$ is negligible in the approximation $q_{Y\pi} \sim$ constant.

The generalization of Eqs. (II.8), (II.9) and (II.10) for any finite number of poles on the left hand cuts is straightforward.

III. EFFECT OF $\overline{K}^0$ K^- MASS DIFFERENCE ON THE SCATTERING AMPLITUDES

The kinematic effects which result from the mass difference between K^0 and K^- mesons have been discussed in detail by Jackson and Wyld.[12] Owing to the mass difference the isotopic spin is not strictly conserved in reactions involving both $\overline{K}^0$ and K^- mesons. It is then necessary to transform the matrix $G^{-1}(s)$, defined by Eq. (II.6), from the isotopic spin representation to a charge representation. In this new representation, we have to deal with a 5-by-5 matrix in which the rows and columns refer to the elastic and charge exchange processes $K^-p \rightarrow K^-p$, $\overline{K}^0n \rightarrow \overline{K}^0n$ and to the reactions $K^-p \rightarrow (\Sigma\pi)_I$ in the two isotopic spin state $I = 0,1$ and $K^-p \rightarrow \Lambda\pi$, respectively. If we indicate with g^0_{ij} and g^1_{ij} the elements of the matrix $\sqrt{q}\ T^{-1}\sqrt{q}$ for the two isotopic spin states 0,1 we have explicitly:

$$M = \begin{vmatrix} \frac{1}{2}(g^0_{11}+g^1_{11}) - iq_{\overline{K}N} & \frac{1}{2}(g^1_{11} - g^0_{11}) & \frac{1}{\sqrt{2}}\, g^0_{12} & \frac{1}{\sqrt{2}}\, g^1_{12} & \frac{1}{\sqrt{2}}\, g^1_{13} \\ \frac{1}{2}(g^1_{11} - g^0_{11}) & \frac{1}{2}(g^0_{11}+g^1_{11}) - iq'_{\overline{K}N} & -\frac{1}{\sqrt{2}}\, g^0_{12} & \frac{1}{\sqrt{2}}\, g^1_{12} & \frac{1}{\sqrt{2}}\, g^1_{13} \\ \frac{1}{\sqrt{2}}\, g^0_{12} & -\frac{1}{\sqrt{2}}\, g^0_{12} & g^0_{22} - iq_{\Sigma\pi} & 0 & 0 \\ \frac{1}{\sqrt{2}}\, g^1_{12} & \frac{1}{\sqrt{2}}\, g^1_{12} & 0 & g^1_{22} - iq_{\Sigma\pi} & g^1_{23} \\ \frac{1}{\sqrt{2}}\, g^1_{13} & \frac{1}{\sqrt{2}}\, g^1_{13} & 0 & g^1_{23} & g^1_{33} - iq_{\Lambda\pi} \end{vmatrix} \qquad \text{(III.1)}$$

where q' is the momentum in the $\overline{K}^0$ n channel. By inverting the matrix M and recalling Eq. (II.2), the elastic reaction and scattering amplitudes are given by:

$$A_{K^-p,K^-p} = \frac{1}{2}\,\frac{A^1_{\bar{K}N,\bar{K}N}(1 - i\, q'_{\bar{K}N}\, A^0_{\bar{K}N,\bar{K}N}) + A^0_{\bar{K}N,\bar{K}N}(1 - i\, q'_{\bar{K}N}\, A^1_{\bar{K}N,\bar{K}N})}{\Delta} \qquad \text{(III.2)}$$

$$A_{K^-p,K^0n} = \frac{1}{2}\,\frac{A^1_{\bar{K}N,\bar{K}N} - A^0_{\bar{K}N,\bar{K}N}}{\Delta} \qquad \text{(III.3)}$$

$$A_{K^-p,(\Sigma\pi)_0} = -\frac{1}{\sqrt{2}}\,\eta_0\, A^0_{\bar{K}N,\bar{K}N}\,\frac{1 - i\, q'_{\bar{K}N}\, A^1_{\bar{K}N,\bar{K}N}}{\Delta} \qquad \text{(III.4)}$$

$$A_{K^-p,(\Sigma\pi)_1} = -\frac{1}{\sqrt{2}}\,\eta_1\, A^1_{\bar{K}N,\bar{K}N}\,\frac{1 - i\, q'_{\bar{K}N}\, A^0_{\bar{K}N,\bar{K}N}}{\Delta} \qquad \text{(III.5)}$$

$$A_{K^-p,\Lambda\pi} = -\frac{1}{\sqrt{2}}\,\nu_1\, A^1_{\bar{K}N,\bar{K}N}\,\frac{1 - i\, q'_{\bar{K}N}\, A^0_{\bar{K}N,\bar{K}N}}{\Delta} \qquad \text{(III.6)}$$

where

$$\Delta = \frac{1}{A^0_{\bar{K}N,\bar{K}N}\; A^1_{\bar{K}N,\bar{K}N}}\Big[1 - \frac{i}{2}\,(q_{\bar{K}N} + q'_{\bar{K}N})(A^0_{\bar{K}N,\bar{K}N} + A^1_{\bar{K}N,KN})$$

$$- q_{\bar{K}N}\, q'_{\bar{K}N}\; A^0_{\bar{K}N,\bar{K}N}\; A^1_{\bar{K}N,\bar{K}N}\Big] .$$

The two elastic amplitudes $A^0_{\bar{K}N,\bar{K}N}$ and $A^1_{\bar{K}N,\bar{K}N}$, as well as the parameters η_0, η_1 and ν_1 are defined in Sec. II.

Sufficiently above the threshold for the charge exchange scattering these formulas reduce to those given by Eqs. (II.8), (II.9) and (II.10).

IV. TOTAL ELASTIC AND REACTION CROSS SECTIONS

The available experimental data on the K^--proton elastic and reaction cross sections are too crude in order to get any reliable information on the pion-pion forces.[13] However, we think it important to recognize that a strong pion-pion interaction, like that discussed by Frazer and Fulco in their analysis of the nucleon electromagnetic structure, is consistent with both the K^--proton and K^+-proton data (see also Sec. V).

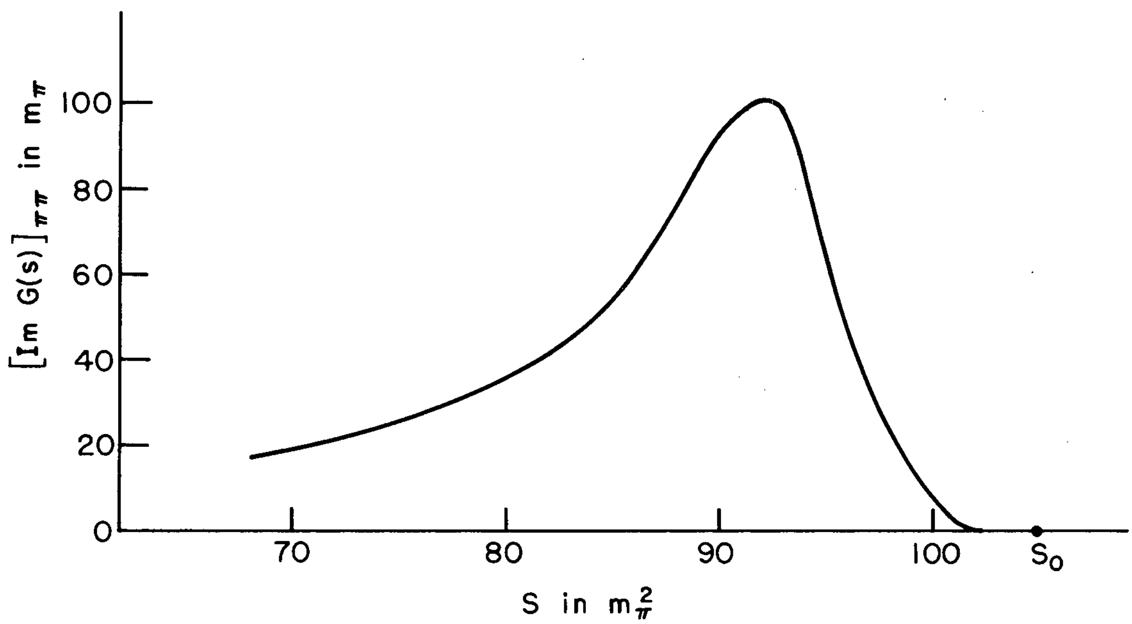
As an example of the effects the pion-pion forces may have on the $\overline{K}$-nucleon S-wave scattering, we have calculated the elastic, charge exchange and hyperon production cross sections according to Eqs. (II.11-12) and (III.1-3). Assuming a pion-pion interaction in the $I = 1$, $J = 1$ state, with a resonant energy $\omega_R \sim 3.6\, m_\pi$ and $|F_\pi(\omega_R)|^2 \sim 16$, the spectral function $[\mathrm{Im}\, G(s)]_{\pi\pi}$ can be calculated in terms of the parameter f_K describing the charge structure of the K-meson[8] (Fig. 1). The corresponding function $R^{(I)}_{KK,\pi\pi}\delta(s - a_1)$ is given by:

$$R^{(0)}_{KK,\pi\pi} \simeq -2.8\, f_K\, M^4 \text{ fermi}$$

$$a_1 \simeq 93\, m_\pi^2 .$$

The elastic and charge exchange cross sections are given in Fig. 2; the $\Sigma^+\pi^-$ and $\Sigma^-\pi^+$ production cross sections are given in Fig. 3. The dotted curves refer to the Dalitz and Tuan solution (a+).[2]

Inspection of Figs. 2 and 3 exhibits some interesting features related to the long range contributions to the S-wave $\overline{K}$-nucleon interaction. In particular, it is worthwhile to note:



MU-21795

Fig. 1. Spectral function $\left.\mathrm{ImG}(s)\right]_{\pi\pi}$ assuming the pion form factor of Frazer and Fulco with $\omega_R \approx 3.6m_\pi$ and $|F_\pi(\omega_R)|^2 \sim 16$. S_0 is the $\bar{K}$-nucleon physical threshold.

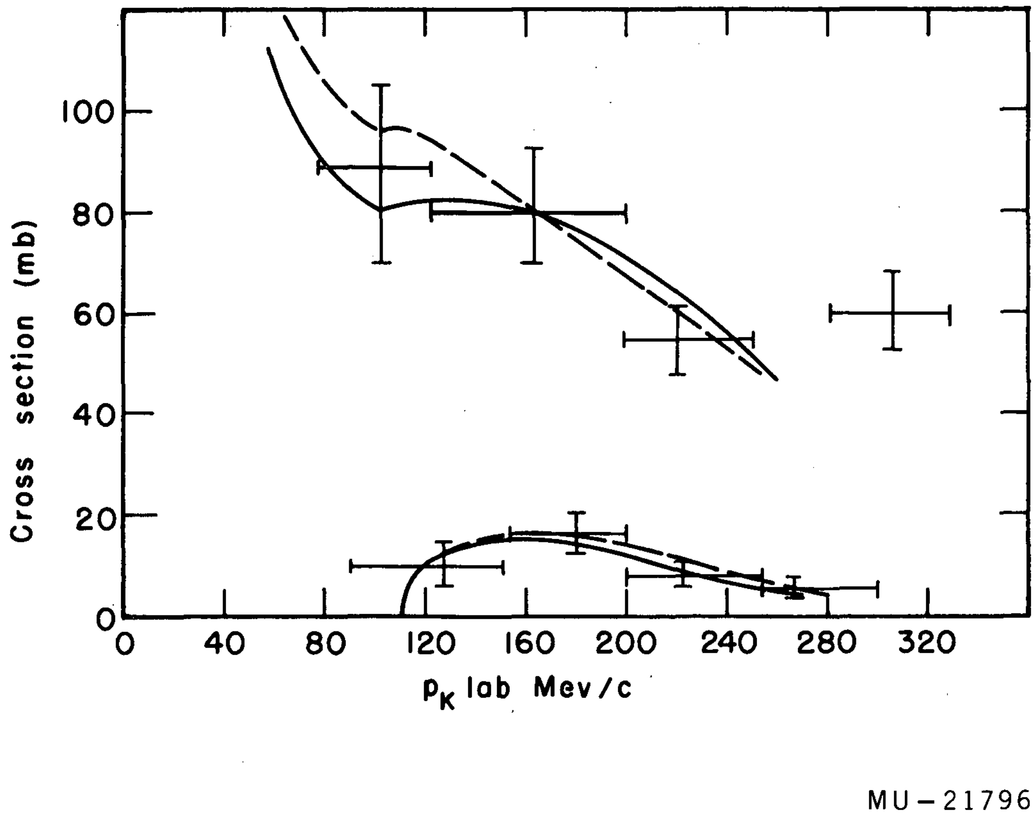


Fig. 2. Cross section for elastic and charge exchange K^--proton scattering. The cross sections are normalized at $p_{K, Lab}$ = 172 Mev/c. The parameters used are $z_0 = [-0.41+i\ 1.17]$; $z_1 = [2.01+i\ 0.20]$ M_N^2 fermi; $a_2 = 55\ m_\pi^2$.

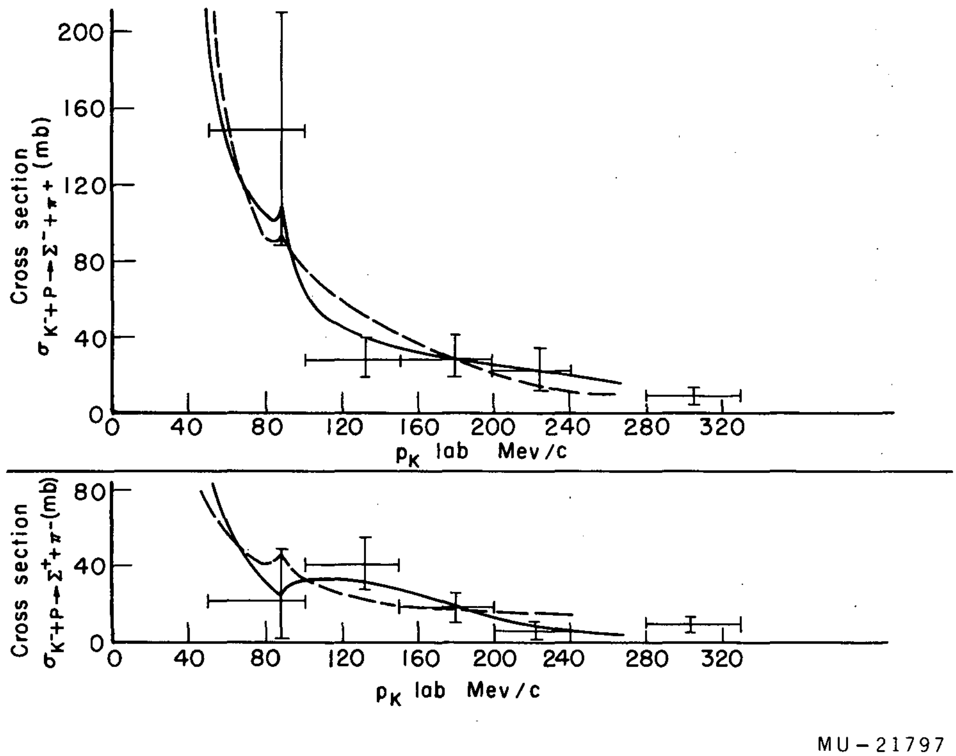


Fig. 3. Cross section for $\Sigma^-\pi^+$ and $\Sigma^+\pi^-$ production. The cross sections are normalized at $p_{K,\ Lab} = 172$ Mev/c. The phase difference between the I = 0 and I = 1 $\Sigma\pi$ production amplitudes is -112°.

(a) The levelling off of the low energy ($p_L \sim 150$ Mev/c) K^--proton elastic scattering cross section;

(b) The different form of the cusps for the $\Sigma^+\pi^-$ and $\Sigma^-\pi^+$ production cross sections.

These results are extremely sensitive to the ratio between the long range contributions in the two $I = 0$ and $I = 1$ $\bar{K}$-nucleon isotopic spin states which, if the two pion pair interacts in the $I = 1$, $J = 1$ state only, is -3. Since the curves plotted in the figures are obtained with the ratio $R^{(0)}_{KK,\pi\pi}/R^{(1)}_{KK,\pi\pi} \sim -3.8$, we have a feeble indication in favor of a strong pion-pion interaction in the $I = 1$, $J = 1$ state.

If the last conclusion is qualitatively correct, the picture that seems to be emerging is that for the $\bar{K}$-nucleon system the long range force is repulsive in the isotopic spin $I = 0$ state and attractive for $I = 1$, whereas the net K^--proton short range interaction is attractive. The same experimental fit requires $f_K \sim 1.5$.

V. RELATIVISTIC EFFECTIVE RANGE FORMULA FOR THE K^+-PROTON INTERACTION

In this section we shall discuss briefly the K^+-proton interaction. Two considerations determine the nature of our results: (a) The dynamical singularity closest to the physical threshold is again due to the $K\bar{K},\pi\pi$ interaction; (b) The long range forces, arising from the exchange of a pion pair, give opposite contributions to the $\bar{K}$-nucleon and K-nucleon systems.[8]

Using the same technique as outlined in Sec. II, a two-pole approximation to the left hand cuts leads to the following expression for the elastic $\langle K^+p \mid K^+p \rangle$ amplitude:

$$A_{K^+p,K^+p} = \frac{1}{(E+M)\sqrt{s}} \left[\ell(s) + \frac{1}{f_{1+}(s) + z_+} - i \frac{q_{NK}}{\sqrt{s}\,(E+M)} \right]^{-1} \tag{V.1}$$

Here

$$f_{1+}(s) = \frac{R^{(1+)}_{K\bar{K},\pi\pi}}{(s-a_1)(1+\beta R^{(1+)}_{K\bar{K},\pi\pi})}$$

and z_+ is a real constant depending on the parameters R_2 and a_2 of the second pole. As the K^+-proton is a pure $I = 1$ state the distributed spectrum $[\mathrm{Im}\, G(s)]_{\pi\pi}$ will be described by the function $R^{(1+)}_{K\bar{K},\pi\pi}\, \delta(s-a_1)$ where

$$R^{(1+)}_{K\bar{K},\pi\pi} \sim -0.72\, f_K\, M^4 \text{ fermi}$$

$$a_1 \simeq 93\, m_\pi^2 .$$

Very few experimental data are yet available for the low energy K^+-proton scattering. The cross section increases slightly with increasing K^+ momentum and is of the order of 15 mb at 250-300 Mev/c. Using Eq. (V.1) and normalizing the cross section at 250 Mev/c, we get a smoothly varying monotonic cross section ranging between 10 mb for $p_{lab} = 0$ and 15 mb for $p_{lab} = 250$ Mev/c. It is gratifying that the long range forces here do not produce sharp variations in the cross section, in contrast to the K^--proton results described above.

APPENDIX A

It is known that, in order to avoid kinematic singularities, the boson-baryon partial wave amplitudes should be given as functions of $W = \sqrt{s}$.

We define the scattering amplitude as:

$$f_0 = \frac{1}{W}(E + M)^{1/2} G (E + M)^{1/2} ,$$

where

$$G = \frac{1}{16\pi}\left\{ [A_0 + (W - \overline{M})B_0] + \frac{E - M}{q}[-A_1 + (W + \overline{M})B_1]\frac{E - M}{q}\right\}$$

shows the proper threshold behavior at $W = W_0$ and $W = -W_0$.

Following the method outlined in Sec. II, we have:

$$N(W) = \frac{1}{E + M}\left[\frac{\eta}{W + \sqrt{a_1}} - \frac{\eta}{W - \sqrt{a_1}} + \frac{\nu}{W + \sqrt{a_2}} - \frac{\nu}{W - \sqrt{a_2}}\right]$$

$$D(W) = 1 + \frac{W - \sqrt{a_2}}{\pi}\int_{W_0}^{\infty} dW' \frac{(-q')(E' + M)\, N(W')}{W'(W' - \sqrt{a_2})(W' - W)}$$

$$+ \frac{W - \sqrt{a_2}}{\pi}\int_{-\infty}^{-W_0} dW' \frac{q'(E' + M)\, N(W')}{W'(W' - \sqrt{a_2})(W' - W)} .$$

It is then easy to see that Eq. (II.7) are obtained assuming:

$$2\sqrt{a_2}\ \nu = \frac{R_2}{2M}$$

$$2\sqrt{a_1}\ \eta = -\frac{R_1}{2M} .$$

APPENDIX B

In this Appendix we give the relations between the z_0, z_1, η_0, η_1 and ν_1 and the parameters of the second pole.

Isotopic Spin I = 0

$$\frac{1}{z_0} = (n_{11})_0 - \frac{(n_{12}^2)_0}{\Delta_0}$$

$$\eta_0 = - \frac{(n_{12})_0}{\Delta_0}$$

where

$$\Delta_0 = (n_{22})_0 - i\, q_{\Sigma\pi} \;.$$

Isotopic Spin I = 1

$$\frac{1}{z_1} = (n_{11})_1 - (n_{12}^2)_1 \frac{(n_{33})_1 - i\, q_{\Lambda\pi}}{\Delta_1} - (n_{13}^2)_1 \frac{(n_{22})_1 - i\, q_{\Sigma\pi}}{\Delta_1}$$

$$+ 2(n_{12})_1 (n_{13})_1 \frac{(n_{23})_1}{\Delta_1}$$

$$\eta_1 = -(n_{12})_1 \frac{[(n_{33})_1 - i\, q_{\Lambda\pi}]}{\Delta_1} + (n_{13})_1 \frac{(n_{23})_1}{\Delta_1}$$

$$\nu_1 = - (n_{13})_1 \frac{[(n_{22})_1 - i\, q_{\Sigma\pi}]}{\Delta_1} + (n_{12})_1 \frac{(n_{23})_1}{\Delta_1}$$

where

$$\Delta_1 = (n_{22})_1 (n_{33})_1 - (n_{23}^2)_1 - q_{\Sigma\pi}\, q_{\Lambda\pi} - i[(n_{33})_1\, q_{\Sigma\pi} + (n_{22})_1\, q_{\Lambda\pi}] \;.$$

The $(n_{ij})_I$ are related with the parameters of the second pole by:

$$(n_{ij})_I = -\frac{1}{\pi} \frac{R_{ij}^{(I)}}{s - a_2} .$$

ACKNOWLEDGMENTS

The authors would like to thank Professor Geoffrey F. Chew for invaluable advice and for many illuminating discussions, and Dr. David Judd for the hospitality at the Lawrence Radiation Laboratory

REFERENCES

1. J. D. Jackson, D. G. Ravenhall, and H. W. Wyld, Nuovo cimento 9, 893 (1958).
2. R. H. Dalitz and S. F. Tuan, Ann. Phys. 3, 307 (1960).
3. G. F. Chew and F. Low, Phys. Rev. 101, 1570 (1956); and G. F. Chew and S. Mandelstam, Phys. Rev. 119, 467 (1960).
4. G. F. Chew, Double Dispersion Relations and Unitarity as the Basis for a Dynamical Theory of Strong Interactions, UCRL-9289.
5. Very little is known about a possible three-pion resonant state. We will avoid here any detailed analysis of such a system.
6. F. Ferrari, G. Frye and M. Pusterla, Phys. Rev. Letters 4, 614 (1960).
7. W. R. Frazer and J. Fulco, Phys. Rev. 117, 1609 (1960).
8. F. Ferrari, G. Frye and M. Pusterla, Long Range Interaction in $\bar{K}$-Nucleon and K-Nucleon Elastic Amplitudes, UCRL-9421, October 5, 1960.
9. F. Ferrari, M. Nauenberg and M. Pusterla, UCRL-8985.
10. The importance of the single nucleon term for the low energy behavior of pion-nucleon scattering has been emphasized by Chew and Low. For analogy, it may be interesting to fix the position of the "far away" poles by considering the single baryon terms and regard only the residues as phenomenological parameters.
11. Technique introduced by Chew and Mandelstam for the pion-pion problem. Extension of the method to the many-channel case requires some caution in order to continue analytically the scattering amplitudes below physical thresholds, as usually there are anomalous thresholds and superpositions of left and right cuts (see also J. Bjorken, Phys. Rev. Letters 4, 473 (1960)).
12. J. D. Jackson and H. W. Wyld, Nuovo cimento 13, 85 (1959).
13. Kiev Report, 1959; Data of Rosenfeld, Solmitz, Tripp, and Ross, reported by L. W. Alvarez at the 9th Annual Conference on High Energy Nuclear Physics.