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Diffraction Effects in Neutron Attenuation Measurements

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DIFFRACTION EFFECTS IN NEUTRON ATTENUATION MEASUREMENTS

Edwin M. McMillan and Duane C. Sewell

November 20, 1947

Berkeley, California

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DIFFRACTION EFFECTS IN NEUTRON ATTENUATION MEASUREMENTS

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University of California, Berkeley

November 20, 1947

ABSTRACT

All errors due to diffraction effects in a neutron attenuation experiment are computed. Also a special experiment to measure the forward intensity of diffracted neutrons from lead and copper is described, and the results given. These agree with the theoretical values.

DIFFRACTION EFFECTS IN NEUTRON ATTENUATION MEASUREMENTS

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I. Introduction

In interpreting fast neutron scattering experiments, it has generally been assumed that the diffraction of the neutron waves by the nucleus accounts for a part of the total cross section equal to the projected collision area of the nucleus. (See, for example, refs. (1) and (2) and earlier work referred to in these papers.) This is

(1) R. Sherr, Phys. Rev. 68 240 (1945)

(2) E. Amaldi, D. Bocciarilli, B. H. Cacciapuoti, and G. C. Trabacchi, Nuovo Cimento 3 203 (1946)

a very reasonable assumption, particularly in cases where the neutron wave length is short compared to the nuclear diameter, since the diffracted intensity is mostly in the forward direction and the situation approximates that of diffraction by a disk-like obstacle. Comparison with the well-known equivalent optical problem shows that the total diffracted flux is indeed equal to the flux intercepted by the obstacle, and that its angular distribution is given by:

$$\sigma_d(\theta) = \left[\frac{RJ_1(kR\sin\theta)}{\sin\theta}\right]^2 \quad , \qquad (1)$$

where $\sigma_d(\theta)$ is the cross section per unit solid angle for diffraction at the angle θ, R is the collision radius of the nucleus, k is 2π times the reciprocal of the neutron wavelength, and J_1 is a Bessell function.

The total cross section σ_t should then be made up of the integrated cross section σ_d for diffraction plus an equal amount to take care of the neutrons that actually strike the nucleus, giving the usually assumed relation:

$$\sigma_t = 2\sigma_d = 2\pi R^2 \tag{2}$$

This should be strictly valid when $kR \gg 1$, if the nucleus can be considered as an opaque obstacle. If the nucleus is partially transparent, as is apparently the case for lighter nuclei at 90 Mev neutron energy, the situation is more complicated, and both the magnitude and angular distribution of the diffraction can be altered. One can however still treat (2) as a definition of R in these cases, and use the diffraction formula (1) as a first approximation, with the understanding that the R so defined may be smaller than the actual nuclear radius.

In the cases to be considered here, we are dealing with 90 Mev neutrons, for which $k = 2.15 \times 10^{13}$ cm^{-1}; the collision radius found for the uranium nucleus is 9.0×10^{-13} cm., giving $kR = 19$. According to eqn. (1) the diffraction pattern for uranium falls to half intensity at $\theta = 0.085$ radian, while the patterns for other elements will be wider.

At small values of θ, the diffraction per unit solid angle is given approximately by the first two terms in the series expansion for the Bessel function, thus:

$$\sigma_d(\theta) \sim 1/4\ k^2 R^4 \left[1 - 1/8\ (kR \sin\theta)^2\right]^2 \tag{3}$$

II. Diffraction Error in an Attenuation Experiment

With the above preliminaries we can estimate the intensity diffracted into the detector in a typical attenuation experiment (3), set up as in Fig. 1. The method of calculation is similar to that of ref. (2), appendix II. The source and detector are treated as points, since they subtend angles small compared to the width of the central diffraction peak.

Let I_o = neutron intensity per unit area at detector, in absence of scatterer. Then the intensity at the position of the scatterer is $I_o (x_1 + x_2)^2 / x_1^2$, from the inverse square law, and the number striking the scatterer between r and r + dr is:

$$dI_s = 2\pi r \, dr \cdot I_o (x_1 + x_2)^2 / x_1^2 \qquad (4)$$

Now the probability that a neutron will pass through the scatterer with no collisions is $e^{-\ell/\lambda}$, where λ is the mean free path. The probability of making just one diffraction collision is $1/2 \, (\ell/\lambda) \, e^{-\ell/\lambda}$, the factor of 1/2 coming from the fact that half the total cross section is due to diffraction. The probability of making just n diffraction collisions is $(\ell/2\lambda)^n \, (1/n!) \, e^{-\ell/\lambda}$, assuming that the paths remain nearly parallel to the axis. This assumption becomes invalid in the present case only for values of n too large to have any importance.

The next step is to compute the intensity directed toward the detector for each number of collisions. For one collision, this is very simple. Combining eqn. (4) with the result of the last paragraph, we get the number of collisions occuring at each value of r;

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(3) BP-122; Phys. Rev. Dec. 15, 1947

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this must be multiplied by the intensity per unit solid angle in the direction of the detector per collision which is equal to $\sigma_d(\theta)/\sigma_d$, and finally by the solid angle of unit area at the detector as seen from the scatterer, which is equal to $1/x_2^2$. Then the intensity diffracted into the detector by single collisions is given by:

$$I_d^{(1)} = \pi I_o (\ell/\lambda) e^{-\ell/\lambda} \left(\frac{1}{x_1} + \frac{1}{x_2}\right)^2 \int_a^o \frac{\sigma_d(\theta)}{\sigma_d} r\, dr \qquad (5)$$

Putting in the approximation (3) for $\sigma_d(\theta)$, and noting that $\sin\theta \sim \left(\frac{1}{x_1} + \frac{1}{x_2}\right) r$, this is easily integrated.

Using the relations (2), the result can be written:

$$I_d^{(1)} = I_o (\ell/\lambda) e^{-\ell/\lambda} \cdot K(1 - K) \qquad (6)$$

where $K = 1/8\; k^2 R^2 a^2 \left(\frac{1}{x} + \frac{1}{x_2}\right)^2 = \frac{1}{16\pi}\; k^2\, \sigma_t\, a^2 \left(\frac{1}{x_1} + \frac{1}{x_2}\right)^2$.
The setup we are interested in has $k = 2.15 \times 10^{13}\ cm^{-1}$, $a = 1.25''$, $x_1 = 110''$, $x_2 = 88''$; for lead $\sigma_t = 4.53 \times 10^{-24}\ cm^2$, giving $K = 0.0273$. Thus K can be neglected in the parenthesis in (6), which means that the angles introduced by the finite width of the scatterer are not important.

The computation of the intensity due to multiple scattering is more involved if carried out to the second order as done above, and we shall content ourselves with a first order computation which will be of sufficient accuracy. The frequency of multiple collisions is as given above; the width of the central peak after n collisions increases about as $n^{1/2}$, and therefore the central intensity varies about as $1/n$. Thus the contribution of intensity due to the various numbers of scatterings are proportional to $(\ell/2\lambda)^n / (n \cdot n!)$ and the total intensity is:

$$I_d = I_d^{(1)} \left[1 + (\ell/2\lambda)/4 + (\ell/2\lambda)^2/18 + \cdots \right] \qquad (7)$$

This series can be summed as an exponential integral, but this is hardly justified since the higher terms are certainly not accurate and are not important in the present case, where $\ell/\lambda \sim 1$.

To find the error which this effect produces in the attenuation measurement, we consider that the cross section is computed from the relation:

$$N \ell \sigma_t = \ln \frac{I_0}{I_0 e^{-\ell/\lambda} + I_d} \qquad (8)$$

$$\sim \frac{\ell}{\lambda} \left[1 - K \left(1 + \frac{\ell}{8\lambda} \right) \right]$$

and that therefore the fractional error is given by the difference of the bracket from unity. This formula was used for computing the corrections applied to the attenuation experiments; the correction is 3.1% in the case of lead, less for lighter elements because of the smaller value of σ_t, and less for U because in this case the radius of the scatterer was only 1".

III. Direct Measurement of Diffracted Intensity

In order to check to some extent the validity of the assumptions used above, an experiment was set up as shown in Fig. 2.

The cyclinder A is of copper, 10" long, and absorbs over 99% of the direct beam. The ring B is lead or copper, with $\ell \sim \lambda$. Three measurements were made, using the same technique as in the attenuation experiments:

I_1 = intensity at detector with A and B away

I_2 = background = intensity with A in place

I_3 = intensity with both A and B in place

Then the ratio of the intensity scattered by B to the initial intensity is given by $(I_3 - I_2)$ $(I_1 - I_2)$. This is to be compared to the theoretical ratio, obtained from equations (6) and (7):

$$\frac{I_d}{I_o} = \frac{\ell}{\lambda} e^{-\frac{\ell}{\lambda}} \left(1 + \frac{1}{8}\frac{\ell}{\lambda}\right) \left[K_2 (1 - K_2) - K_1 (1 - K_1) \right] \qquad (9)$$

where the two values of K correspond to the two radii a_1 and a_2. Since $a_1 = 1.5"$, $a_2 = 3"$, for lead $K_1 = 0.036$, $K_2 = 0.142$, and for Cu $K_1 = 0.018$, $K_2 = 0.070$. The expected ratios are then 0.036 for Pb, 0.019 for Cu.

The measured values, with mean errors from the counting statistics, are:

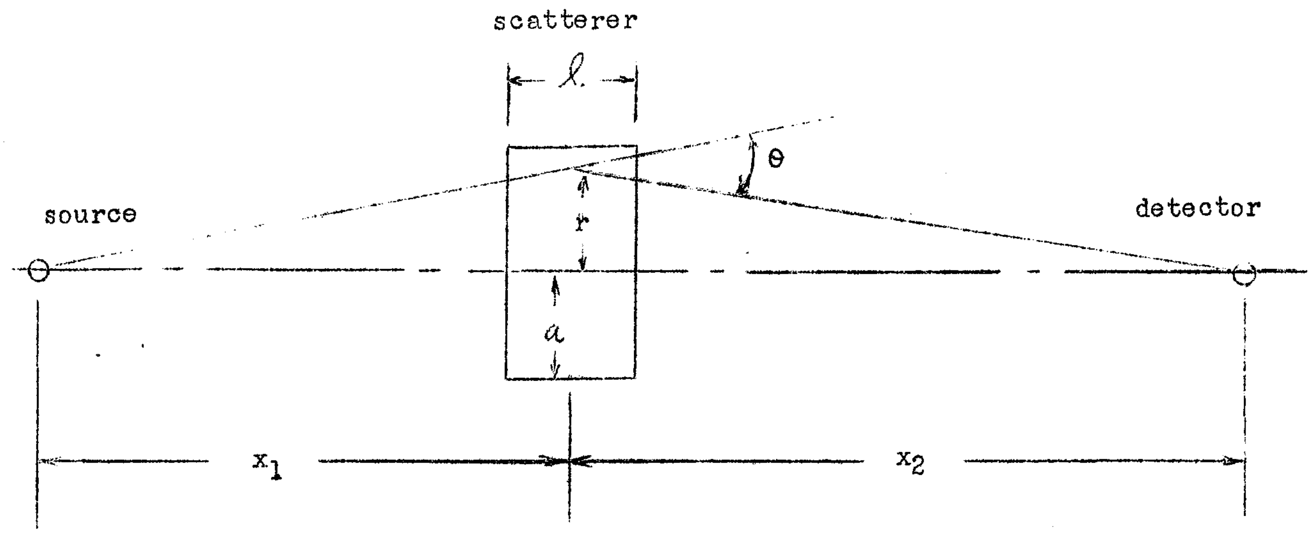
$$I_1 = 2.74 \pm 0.04$$

$$I_2 = 0.217 \pm 0.003$$

$$I_3 \text{ (Pb ring)} = 0.304 \pm 0.004$$

$$I_3 \text{ (Cu ring)} = 0.259 \pm 0.004$$

These give $I_d/I_o = 0.035 \pm 0.002$ for Pb and 0.017 ± 0.002 for Cu, in excellent agreement with the computed values.


scatterer
ℓ
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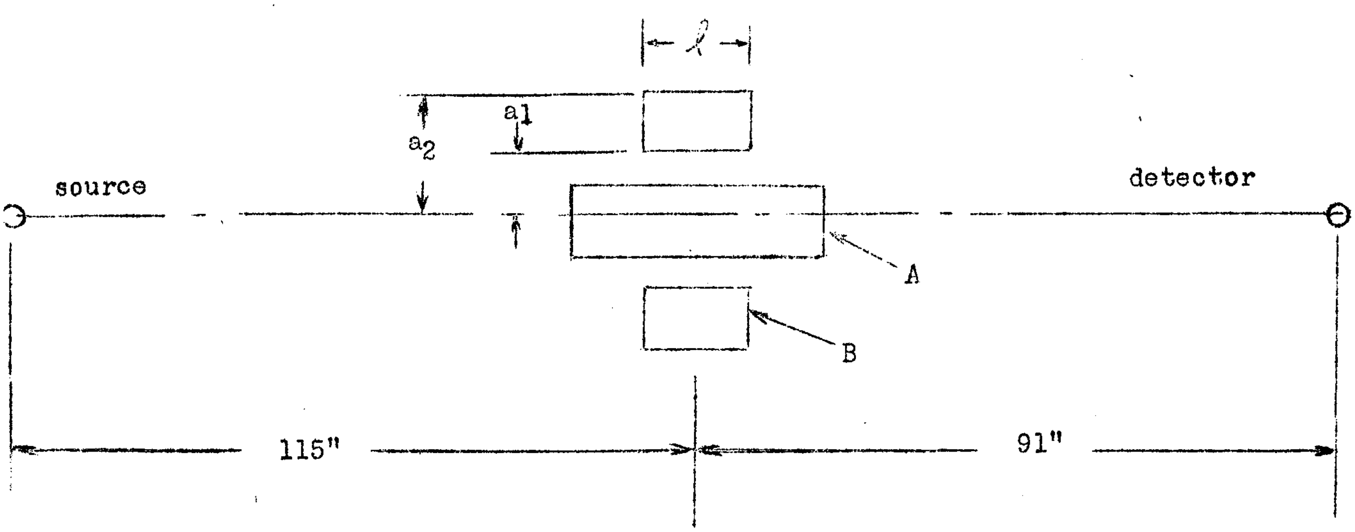
detector
x_1
x_2

FIG. 1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a_1
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detector
A
B
115"
91"

FIG. 2