

# Twist-three Fragmentation Function Contribution to the Single Spin Asymmetry in $pp$ Collisions

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## Abstract

We study the twist-three fragmentation function contribution to the single transverse spin asymmetries in inclusive hadron production in  $pp$  collisions,  $p^\uparrow p \rightarrow h + X$ . In particular, we evaluate the so-called derivative contribution which dominates the spin asymmetry in the forward direction of the polarized proton. With certain parametrizations for the twist-three fragmentation function, we estimate its contribution to the asymmetry of  $\pi^0$  production at RHIC energy. We find that the contribution is sizable and might be responsible for the big difference between the asymmetries in  $\eta$  and  $\pi^0$  productions observed by the STAR collaboration at RHIC.

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Single-transverse spin asymmetries (SSAs) in hadronic processes, such as in the single inclusive hadron production in single transversely-polarized nucleon-nucleon scattering,  $p^\uparrow p \rightarrow hX$ , have attracted much interests from both experimental and theoretical sides in the last few years, and great progress has been made in understanding the underlying physics [1]. Although it is a simple observable, defined as the spin asymmetry when one flips the transverse spin of one of the hadrons involved in the scattering:  $A_N = (d\sigma(S_\perp) - d\sigma(-S_\perp))/(d\sigma(S_\perp) + d\sigma(-S_\perp))$ , it is far more complicated to explain in the fundamental theory of strong interaction. It usually represents a correlation between the transverse polarization vector  $S_\perp$  of one of the hadrons and the transverse momentum  $P_{h\perp}$  of the final-state hadron in the differential cross section. For example, in the process  $p^\uparrow p \rightarrow hX$ , it is the correlation between the polarization vector  $S_\perp$  of the incoming nucleon and the transverse momentum  $P_{h\perp}$  of the final-state hadron that generates the asymmetry. In this paper, we will focus on the SSAs in these processes, especially for the neutral mesons,  $\pi^0$  and  $\eta$  production, motivated by the recent striking experimental observations of large SSAs for them by the STAR collaboration at RHIC experiments [2, 3]. In particular, at forward direction of the polarized proton, it was found that the SSA for  $\eta$  meson is much larger than that for  $\pi^0$  meson. This also confirmed previous experimental observations from the fixed target experiments [4]. In addition to these STAR results, both BRAHMS and PHENIX collaborations at RHIC have observed large single transverse spin asymmetries in charged and neutral meson production in the forward rapidity region [5, 6].

In the QCD framework, there have been mainly two approaches to study the SSAs in high energy scattering processes: the transverse momentum dependent (TMD) parton distribution and fragmentation function approach [7–15] and the twist-three quark-gluon correlation approach in the collinear factorization [16–21]. In the TMD approach, it is required to have an additional hard momentum scale besides the transverse momentum  $P_{h\perp}$  of the hadron, such as  $Q^2$ , the momentum transfer square of the virtual photon in semi-inclusive deep inelastic scattering (SIDIS) or Drell-Yan lepton pair production in  $pp$  collisions, to study the transverse momentum dependence in these processes. On the other hand, the twist-three approach is more appropriate in the processes that all momentum scales are much larger than the nonperturbative scale  $\Lambda_{\text{QCD}}$  [17]. It has been shown that these two approaches are consistent with each other for the SIDIS and Drell-Yan processes in the intermediate transverse momentum region where they both apply [22, 23].

The TMD approach has also been used to calculate the SSA in the single inclusive hadron production in  $p^\uparrow p$  collisions in a model dependent way [10, 24, 25]. However, there has been no factorization argument based on TMD parton distribution for this process [26]. In this paper, we follow the collinear factorization approach to study the single spin asymmetry coming from the twist-three fragmentation function contribution. In this process, there is only one large momentum scale, the transverse momentum  $P_{h\perp}$  of the final-state hadron, and the SSA is naturally suppressed by  $1/P_{h\perp}$  at large  $P_{h\perp}$  as a higher-twist effect, although how large this behavior should manifest is not clear. At low transverse momentum, it will ultimately fail, and a modification has to be made to account for the experimental data [3, 27].

In the twist-three collinear factorization approach, for the SSAs in hadron production in

$$p^\uparrow(A) + p(B) \rightarrow h + X \ , \quad (1)$$

the twist-three effects come from the distribution functions of the incoming polarized nucleon ( $A$ ) with momentum  $P_A$  or the unpolarized nucleon ( $B$ ) with momentum  $P_B$ , or the fragmentation function for the final state hadron ( $h$ ) with momentum  $P_h$ . Therefore, schematically, we can write down the single transverse spin dependent differential cross section in the following way [17, 18],

$$\begin{aligned} d\sigma(S_\perp) = \epsilon_\perp^{\alpha\beta} S_{\perp\alpha} P_{h\beta} \int [dx][dy][dz] \left\{ \phi_{i/A}^{(3)}(x, x') \otimes \phi_{j/B}(y) \otimes D_{h/c}(z) \otimes \mathcal{H}_{ij \rightarrow c}^{(A)}(x, x', y, z) \right. \\ \left. + \phi_{i/A}(x) \otimes \phi_{j/B}^{(3)}(y, y') \otimes D_{h/c}(z) \otimes \mathcal{H}_{ij \rightarrow c}^{(B)}(x, y, y', z) \right. \\ \left. + \phi_{i/A}(x) \otimes \phi_{j/B}(y) \otimes D_{h/c}^{(3)}(z, z') \otimes \mathcal{H}_{ij \rightarrow c}^{(c)}(x, y, z, z') \right\} \ , \quad (2) \end{aligned}$$

where  $\epsilon_\perp^{\alpha\beta}$  is defined as  $\epsilon_\perp^{\alpha\beta} = \epsilon^{\mu\nu\alpha\beta} P_{A\mu} P_{B\nu} / P_A \cdot P_B$  with convention of  $\epsilon^{0123} = 1$ . Here we use  $\perp$  to label the transverse direction in the center of mass frame of  $P_A$  and  $P_B$ . In the above equation,  $\otimes$  stands for the convolution in the longitudinal momentum fractions  $x$ ,  $y$ , and  $z$ . The leading twist parton distributions are labeled by  $\phi_{i/A}(x)$  and  $\phi_{j/B}(y)$ . Because hadron  $A$  is transversely polarized, we immediately see that  $\phi_{i/A}$  represents the leading-twist quark transversity distributions. However, for unpolarized hadron  $B$ ,  $\phi_{j/B}$  represents both leading twist quark and gluon distributions. Leading twist parton fragmentation function is represented by  $D_{h/c}(z)$  where parton  $c$  can be a quark or gluon. In the above equation, the superscript (3) represents the twist-three correlations for the distribution functions or the fragmentation functions. Since these twist-three functions normally involve two variables in

longitudinal momentum fractions, we have made them explicit in the above formula. For a complete analysis, the above three terms have to be taken into account. The first term in the above equation, the contribution from the twist-three parton distributions  $\phi_{i/A}^{(3)}(x, x')$  from the polarized nucleon have been calculated in Refs. [17–19]. The second term from the twist-three effect in the unpolarized nucleon  $\phi_{j/B}^{(3)}(x, x')$  is found very small in the forward region of the polarized nucleon where the largest asymmetry has been found experimentally [28]. The third term is least known in the literature, which we will focus in this paper. There have been attempts to formulate this part of contribution [29]. However, a universality argument for the Collins fragmentation function [30, 31] would indicate that the contribution calculated in Ref. [29] vanishes. This universality has also been extended to  $pp$  collisions [32]. Large azimuthal asymmetries coming from this Collins effect have been observed in semi-inclusive hadron production in deep inelastic scattering and di-hadron production in  $e^+e^-$  annihilation processes by the HERMES [33] and COMPASS [34], and BELLE collaborations [35], respectively. In a recent publication, two of us have re-examined the twist-three fragmentation contribution to the single spin asymmetry [23]. In particular, we have identified the twist-three fragmentation function corresponding to the TMD Collins fragmentation function, and shown that the TMD and collinear factorization approaches are consistent in the intermediate transverse momentum region in the SIDIS process. In this paper, we will extend this formalism to the twist-three fragmentation function contribution to the above process of (1), the single spin asymmetries in inclusive hadron production in  $p^\uparrow p$  scattering.

The twist-three fragmentation function which corresponds to the universal Collins TMD fragmentation function, is defined as <sup>1</sup>

$$\hat{H}(z) = \frac{z^2}{2} \int \frac{d\xi^-}{2\pi} e^{ik^+\xi^-} \frac{1}{2} \left\{ \text{Tr} \sigma^{\alpha+} \langle 0 | \left[ iD_T^\alpha + \int_{\xi^-}^{+\infty} d\zeta^- g F^{\alpha+}(\zeta^-) \right] \psi(\xi) | P_h X \rangle \right. \\ \left. \times \langle P_h X | \bar{\psi}(0) | 0 \rangle + h.c. \right\} , \quad (3)$$

where  $k^+ = P_h^+/z$ . This function can also be written as the transverse momentum moment of the Collins fragmentation function  $H_1^\perp(z, p_T^2)$ ,

$$\hat{H}(z) = \int d^2 p_T \frac{|\vec{p}_T|^2}{2M_h} H_1^\perp(z, p_T^2). \quad (4)$$

In the above definition Eq. (3), we have set the hadron momentum in the  $\hat{z}$  direction. The index  $\alpha$  is in the transverse direction perpendicular to the hadron momentum direction, and

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<sup>1</sup> In the definition of  $\hat{H}(z)$  in Ref. [23], a factor of 1/2 missed.

is not considered to be summed up. In order to uniquely define this transverse component (index), we introduce a light-like vector  $n_h^\mu \propto (P_h^0, -\vec{P}_h)$  with normalization that  $n_h \cdot P_h = 1^2$ . Here and in the following, we use subscript “ $T$ ” to distinguish the transverse direction perpendicular to  $P_h$  from “ $\perp$ ” in Eq. (2) for transverse direction perpendicular to  $P_A$  and  $P_B$ . Therefore, the transverse component for any vector  $p^\mu$  can be defined as  $p_T^\mu = p^\mu - p \cdot n_h P_h^\mu - p \cdot P_h n_h^\mu$ . From this definition, we can immediately see that  $p_T$  is space-like, and perpendicular to  $\vec{P}_h$ :  $\vec{p}_T \cdot \vec{P}_h = 0$ .

As discussed in Ref. [23], the above twist-three fragmentation function belongs to more general two-variable dependent twist-three fragmentation function defined as,

$$\hat{H}_D(z_1, z_2) = \frac{z_1 z_2}{2} \int \frac{d\xi^- d\zeta^-}{(2\pi)^2} e^{ik_2^+ \xi^-} e^{ik_g^+ \zeta^-} \frac{1}{2} \{ \text{Tr} \sigma^{\alpha+} \langle 0 | i D_T^\alpha(\zeta^-) \psi(\xi^-) | P_h X \rangle \times \langle P_h X | \bar{\psi}(0) | 0 \rangle + h.c. \} , \quad (5)$$

where  $k_i^+ = P_h^+ / z_i$  and  $k_g^+ = k_1^+ - k_2^+$ . Similarly, we can define a  $F$ -type fragmentation function  $\hat{H}_F(z_1, z_2)$  by replacing  $D_T^\alpha$  with  $F^{+\alpha}$  in Eq. (5).  $D$ -type and  $F$ -type functions are related to each other,

$$\hat{H}_D(z_1, z_2) = PV \left( \frac{1}{\frac{1}{z_1} - \frac{1}{z_2}} \right) \hat{H}_F(z_1, z_2) + \delta \left( \frac{1}{z_1} - \frac{1}{z_2} \right) \frac{z_2}{z_1} \hat{H}(z_1) . \quad (6)$$

From this equation, we can see that  $\hat{H}_D$  is more singular than  $\hat{H}_F$  at  $z_1 = z_2$ . Moreover, recent study has shown that  $\hat{H}_F(z_1, z_2)$  vanishes when  $z_1 = z_2$  [31]. Therefore, it is more convenient to express the cross section in terms of  $\hat{H}(z)$  and  $\hat{H}_F(z_1, z_2)$ , which we will follow.

In this paper, we are interested in obtaining the phenomenological important contributions in hadron production in the forward region of the polarized beam. Similar to the twist-three distribution function contributions, we find that the twist-three fragmentation function contributions also contain a derivative term which certainly will dominate this part of contribution in the forward rapidity region. In addition, the derivative contribution from the twist-three fragmentation function is associated with  $\hat{H}(z)$  defined in Eq. (3), whereas

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<sup>2</sup> In the low transverse momentum semi-inclusive hadron production in DIS process,  $n_h$  will be proportional to the incoming nucleon momentum as we used in Ref. [23]. In current study, there is no natural available momentum for this choice. We can also perform the calculations without choosing the vector  $n_h$ , but keeping all transverse component in Eq. (3) perpendicular to the hadron momentum. This will lead to the same results.

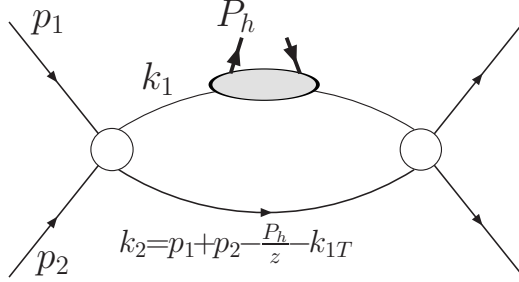


FIG. 1. *Generic Feynman diagrams to calculate the derivative term of the twist-three fragmentation function contributions to the single inclusive hadron production in  $pp$  scattering,  $p^\uparrow p \rightarrow hX$ :  $p_1$  and  $p_2$  are the two incident partons' momenta,  $k_1$  and  $k_2$  are the outgoing partons' momenta, where the quark  $k_1$  fragments into final-state hadron  $P_h$ . The expansion of the scattering amplitude in terms of transverse momentum component of  $k_1 = P_h/z + k_{1T}$  leads to the twist-three contributions from the fragmentation function  $\hat{H}(z)$  defined in Eq. (3).*

the contribution associated with  $\hat{H}_F(z_1, z_2)$  does not contain any derivative contributions. In this paper, we will report the derivative-term results, and carry out the numerical estimates for their contributions to the single spin asymmetries at RHIC. We will leave the detailed derivations, including the derivative and non-derivative terms in a future publication.

We follow the technique developed in the last few years [17–23] to derive the contributions from the twist-three fragmentation functions. First, we calculate the associated scattering amplitudes in terms of various twist-three matrix elements of hadrons. From diagrammatic point of view, these individual contributions are not in a gauge invariant way. However, the final results will be gauge invariant when we sum up all the contributions. In terms of twist-three operators, we have the contributions from the  $\partial_T\psi$ ,  $A_T$ , and  $\partial_T A^+$  ( $\partial^+ A_T$ ). These contributions indeed will form the gauge invariant results in terms of  $\hat{H}(z)$ ,  $\hat{H}_F$  as defined above. In particular,  $\partial_T\psi$  term corresponds to  $\hat{H}(z)$  whereas  $A_T$  term corresponds to  $\hat{H}_F$ . Since the derivative terms are associated with the function  $\hat{H}(z)$ , we will need to calculate the contributions from the  $\partial_T\psi$  part. In order to carry out this calculation, we have to perform the collinear expansion for the hard partonic part associated with this matrix element. In Fig. 1, we illustrate the generic Feynman diagrams contributing to the derivative terms. In this diagram,  $p_1$  and  $p_2$  are the two incoming partons' momenta from the polarized and unpolarized nucleons, respectively;  $k_1$  and  $k_2$  are the two outgoing partons' momenta

where  $k_1$  fragments into the final-state hadron  $P_h$ . The derivative terms come from the transverse momentum expansion of hard partonic scattering amplitudes, in particular, from the on-shell condition for the unobserved particle  $k_2$  in Fig. 1. This momentum can be written as  $k_2 = p_1 + p_2 - k_1$ , where we can parameterize  $k_1$  as  $k_1 = P_h/z + k_{1T}$ . Thus, the on-shell condition will lead to the following expansion,

$$\delta((p_1 + p_2 - k_1)^2) = \delta((p_1 + p_2 - P_h/z)^2) - 2(p_1 + p_2) \cdot k_{1T} \delta'((p_1 + p_2 - P_h/z)^2), \quad (7)$$

where the second term will result into a derivative term contribution to the transverse spin dependent differential cross section. As discussed above, for convenience, we use a light-like vector  $n_h$  to constrain the transverse momentum expansion which is perpendicular to the final state hadron's momentum. Therefore,  $k_{1T}$  can be expressed as  $k_{1T}^\mu = k_1^\mu - k_1 \cdot P_h n_h^\mu - k_1 \cdot n_h P_h^\mu$ . Substituting this expression into the above equation, we will obtain the contribution from the matrix element associated with the operator  $\partial_T \psi$  which will result into the contribution of  $\hat{H}(z)$ . The derivative on the Delta function will be translated into the derivative on the twist-three fragmentation function  $\hat{H}(z)$  by partial integral. This contribution is similar to what has been calculated for the twist-three distribution functions [17]. In particular, the hard part is proportional to that from the relevant twist-two Born diagrams. In our case, it is the transverse spin transfer coefficients calculated in Ref. [36]. However, there exist major difference between these two contributions. The twist-three distribution contributions come from the pole contributions from the initial/final state interactions, and the derivative terms have contributions from the expansion of the on-shell condition for the unobserved particle and the double pole contribution from the final state interaction diagrams [17]. In the twist-three fragmentation function contributions, on contrast, we do not take pole contributions, and the derivative terms only come from the expansion of the on-shell condition of the unobserved particle in the above  $2 \rightarrow 2$  processes. The final result for the derivative contributions of  $\hat{H}(z)$  takes the following form,

$$E_h \frac{d^3 \Delta \sigma(s_\perp)}{d^3 P_h} = \epsilon_{\perp \alpha \beta} S_\perp^\alpha \frac{2\alpha_s^2}{S} \sum_{a,b,c} \int_{x'_{\min}}^1 \frac{dx'}{x'} f_b(x') \frac{1}{x} h_a(x) \int_{z_{\min}}^1 \frac{dz}{z} \left[ -z \frac{\partial}{\partial z} \left( \frac{\hat{H}(z)}{z^2} \right) \right] \\ \times \frac{1}{x' S + T/z} \left\{ \frac{P_h^\beta + z(p_2 \cdot P_h n_h^\beta - p_2 \cdot n_h P_h^\beta)}{-z \hat{u}} \right\} H_{ab \rightarrow c}(\hat{s}, \hat{t}, \hat{u}), \quad (8)$$

where

$$x'_{\min} = \frac{-T/z}{S + U/z}, \quad z_{\min} = -\frac{T + U}{S}, \quad (9)$$

and the hard factors  $H_{ab \rightarrow c}$  are defined as

$$\begin{aligned}
H_{qq' \rightarrow qq'} &= H_{q\bar{q}' \rightarrow q\bar{q}'} = \frac{N_c^2 - 1}{4N_c^2} \frac{4\hat{s}\hat{u}}{-\hat{t}^2}, \\
H_{qq \rightarrow qq} &= \frac{N_c^2 - 1}{4N_c^2} \left[ \frac{4\hat{s}\hat{u}}{-\hat{t}^2} - \frac{1}{N_c} \frac{4\hat{s}}{-\hat{t}} \right], \\
H_{q\bar{q} \rightarrow q\bar{q}} &= H_{\bar{q}q \rightarrow \bar{q}q} = \frac{N_c^2 - 1}{4N_c^2} \left[ \frac{4\hat{s}\hat{u}}{-\hat{t}^2} + \frac{1}{N_c} \frac{4\hat{u}}{\hat{t}} \right], \\
H_{\bar{q}\bar{q} \rightarrow \bar{q}\bar{q}} &= H_{q\bar{q} \rightarrow \bar{q}q} = -\frac{N_c^2 - 1}{N_c^3}, \\
H_{qg \rightarrow qg} &= \frac{N_c^2 - 1}{N_c^2} + \frac{1}{2} \frac{4\hat{s}\hat{u}}{-\hat{t}^2},
\end{aligned} \tag{10}$$

where  $\hat{s}$ ,  $\hat{t}$ , and  $\hat{u}$  are the usual partonic Mandelstam variables. They are the same as the transverse spin transfer hard coefficients calculated in Ref. [36]. Substituting the expression of  $n_h$  into Eq. (8), the factor in the bracket can also be written as,

$$\left\{ \frac{P_h^\beta + z(p_2 \cdot P_h n_h^\beta - p_2 \cdot n_h P_h^\beta)}{-z\hat{u}} \right\} \rightarrow \left( \frac{P_h^\beta}{z} \right) \frac{x - x'}{x(-\hat{u}) + x'(-\hat{t})}. \tag{11}$$

In the forward rapidity region of the polarized nucleon, we have  $x \gg x'$  and  $-\hat{u} \gg -\hat{t}$ , and we can further simplify the transverse spin dependent differential cross section as

$$\begin{aligned}
E_h \frac{d^3 \Delta \sigma(s_\perp)}{d^3 P_h} \Big|_{forward} &= \epsilon_{\perp \alpha \beta} S_\perp^\alpha P_{h\perp}^\beta \frac{2\alpha_s^2}{S} \sum_{a,b,c} \int_{x'_{min}}^1 \frac{dx'}{x'} f_b(x') \frac{1}{x} h_a(x) \int_{z_{min}}^1 \frac{dz}{z} \left[ -z \frac{\partial}{\partial z} \left( \frac{\hat{H}(z)}{z^2} \right) \right] \\
&\times \frac{1}{x'S + T/z} \frac{1}{-z\hat{u}} H_{ab \rightarrow c}(\hat{s}, \hat{t}, \hat{u}).
\end{aligned} \tag{12}$$

This term will be most phenomenological relevant for the single spin asymmetries of hadron production in the forward direction of the polarized nucleon.

In order to demonstrate the twist-three fragmentation function contribution to the SSA in inclusive hadron production in  $p^\uparrow p$  collisions, we need the unknown, but universal, twist-three fragmentation function  $\hat{H}(z)$ . We notice that  $\hat{H}(z)$  can be related to the Collins fragmentation function  $H_1^\perp(z, p_\perp^2)$  as in Eq. (4), which has been studied from the available experimental data [37, 38]. However, we emphasize that the Collins function  $H_1^\perp(z, p_\perp^2)$  are fitted from small transverse momentum region where TMD factorization applies. To obtain the functional form for  $\hat{H}(z)$ , one has to assume a transverse momentum dependence in the Collins fragmentation functions. In principle, the twist-three fragmentation functions  $\hat{H}(z)$  should be extracted from the experimental data of the SSAs at large transverse momentum region where collinear factorization applies, similar to what has been done in [18], or from



the transverse momentum weighted azimuthal asymmetry measurements in which the twist-three fragmentation function  $\hat{H}(z)$  enters directly.

For the purpose of estimating its contribution to the SSAs in the inclusive  $\pi^0$  production in  $p^\uparrow p$  collisions at RHIC energy, we parameterize the twist-three fragmentation function in terms of the leading-twist fragmentation function,

$$\hat{H}(z) = C_f z^a D(z) . \quad (13)$$

We leave a more comprehensive parameterization for these fragmentation functions in a future study, including that for charged mesons as well. The additional  $z$  factor in the parameterization comes from the consideration that this novel fragmentation is mostly a valence behavior. Normally, we would also have a  $(1 - z)$  suppression for the twist-three function. However, for scalar hadron production, the twist-three fragmentation function is not power suppressed in terms of  $(1 - z)$ , similar to the power counting of the Boer-Mulders function of  $\pi$  meson at large- $x$  [39]. We have also adopted the quark transversity distributions from the parameterizations in Ref. [40] and the unpolarized fragmentation function in [41]. In Fig. 2, we show the predictions with the above parameterizations for the  $\pi^0$  production with  $C_f = -0.4$ . The three curves from up to bottom correspond to  $a = 1$  (solid),  $a = 2$  (dashed), and  $a = 4$  (dotted), respectively. With our parametrization of  $\hat{H}(z)$ , the twist-three fragmentation function can generate a sizable SSA in inclusive  $\pi^0$  production at RHIC energy  $\sqrt{s} = 200$  GeV. These contribution are comparable to that of the twist-three distribution from the polarized nucleon which have similar parameterizations [17, 18].

The single transverse spin asymmetry of  $\eta$  meson has also been studied by the STAR collaboration at RHIC recently [2]. A significantly larger asymmetry  $A_N$  has been observed for  $\eta$  meson compared to  $\pi^0$ . As we discussed, in the twist-three collinear factorization, the dominate contribution comes from the twist-three distribution of the polarized nucleon, as shown in the first term of Eq. (2), and the twist-three fragmentation function, as shown in the third term of Eq. (2). The difference between  $\eta$  and  $\pi^0$  from the first term will be mainly due to the difference in the unpolarized meson fragmentation function. However, from the measurements of spin-averaged cross section of  $\eta$  and  $\pi^0$  by PHENIX collaboration at RHIC [42], one finds that for a large range of transverse momentum,  $\eta/\pi^0$  ratio is a constant. This indicates that the unpolarized fragmentation function for  $\eta$  and  $\pi^0$  is very similar, and will

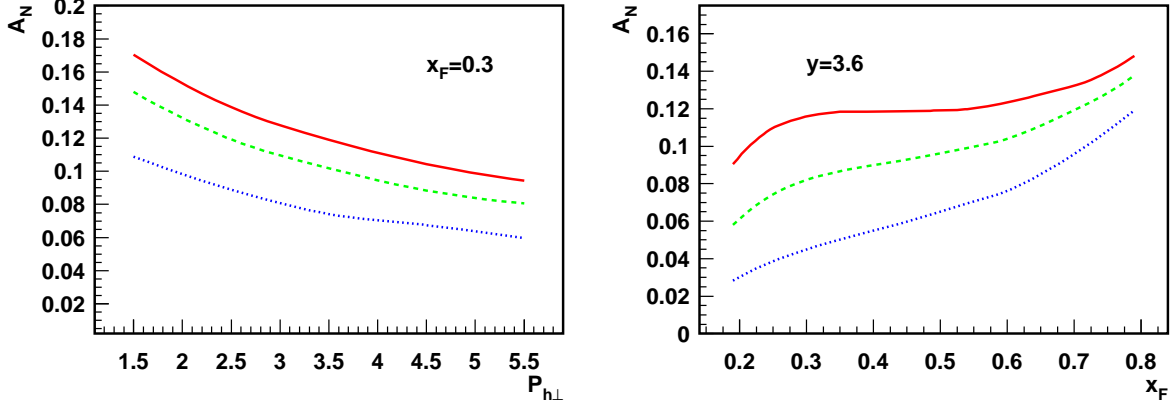


FIG. 2. Twist-three fragmentation contributions to the single spin asymmetries in hadron production in the forward direction of the polarized nucleon at RHIC at  $\sqrt{s} = 200\text{GeV}$ , as functions of  $x_F$ , with the parameterization for the twist-three fragmentation function  $\hat{H}(z)$  in Eq. (13) with parameters  $C_f = -0.4\text{ GeV}$  and  $a = 1, 2, 4$  (from up to bottom), respectively.

lead to the similar SSAs for them if this contribution dominates. In other words, it will be very difficult to explain the large difference between the SSAs of  $\eta$  and  $\pi^0$  mesons from the twist-three distribution contribution from the polarized nucleon<sup>3</sup>. Since the second term in Eq. (2) generates a very small asymmetry, one would expect the large difference between  $\eta$  and  $\pi^0$  would come from the twist-three fragmentation function contribution if one believes the collinear factorization is indeed the right approach to describe the SSA of inclusive hadron production in  $pp$  collisions. Since the twist-three fragmentation function  $\hat{H}(z)$  for  $\eta$  and  $\pi^0$  in general need not to be the same, this might generate the needed difference observed by the experiments if  $\eta$  meson has a much larger twist-three fragmentation function  $\hat{H}(z)$  compared to  $\pi^0$ . To test this scenario, it will be very important to study the associated Collins fragmentation for  $\eta$  meson in  $e^+e^-$  annihilation and/or semi-inclusive DIS processes and compare to that for  $\pi^0$  meson. We hope that, in particular, the BELLE collaboration can carry out this measurement and cross check with the STAR observation. Meanwhile, we emphasize that to better understand the single spin asymmetry for these processes and finally pin down the difference, one needs to take into account the both twist-three contributions

<sup>3</sup> It might still be possible that the strange quark contribution from the polarized nucleon may dominates and leads to a larger SSA in  $\eta$  meson production, which is, however, unlikely in the forward rapidity region (the valence region) of the polarized nucleon.

and perform a global analysis of the experimental data.

In conclusion, in this paper, we have studied the twist-three fragmentation function contribution to the inclusive hadron's SSA in  $pp$  scattering  $p^\uparrow p \rightarrow hX$ . With a simple parametrization for the twist-three fragmentation function, we estimate its contribution to the SSAs of  $\pi^0$  production at RHIC energy. We find that the contribution of the twist-three fragmentation function is comparable to that of the twist-three distribution function from the polarized nucleon. We comment on the possibility to use our approach to describe the large difference of the SSAs between the  $\eta$  and  $\pi^0$  meson. We emphasize that one need to include both contributions from twist-three distribution and fragmentation functions into a global analysis, in order to better understand the single spin asymmetry for the inclusive hadron production.

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