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Pion Interferometric Tests of Transport Models*

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Abstract:
In hadronic reactions, the usual space-time interpretation of pion interferometry often breaks down due to strong correlations between spatial and momentum coordinates. We derive a general interferometry formula based on the Wigner density formalism that allows for arbitrary phase space and multiparticle correlations. Correction terms due to intermediate state pion cascading are derived using semiclassical hadronic transport theory. Finite wavepackets are used to reveal the sensitivity of pion interference effects on the details of the production dynamics. The covariant generalization of the formula is shown to be equivalent to the formula derived via an alternate current ensemble formalism for minimal wavepackets and reduces in the nonrelativistic limit to a formula derived by Pratt. The final expression is ideally suited for pion interferometric tests of Monte Carlo transport models. Examples involving Gaussian and Inside-Outside phase space distributions are considered.

1 Introduction and Summary

Pion interferometry has been used for a long time[1]-[19] to probe the space-time geometry of high energy hadronic reactions (for a comprehensive review, see [20]). It is based on exploiting the constructive interference between identical bosons when their relative momenta are small compared to the inverse of the typical spatial

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dimensions of the reaction volume. Experimentally, the interference pattern is deduced by measuring like pion correlation functions,

$$C_n(\mathbf{k}_1, \cdots, \mathbf{k}_n) = \mathcal{N}_n P_n(\mathbf{k}_1, \cdots, \mathbf{k}_n) / \prod_{i=1}^{n} P_1(\mathbf{k}_i) \quad , \tag{1}$$

where P_n denotes the n (identical) pion inclusive distributions, and $\mathcal{N}_n$ is the inverse of the normalized n^{th} factorial moment of the multiplicity distribution.

Unfortunately, the simple geometrical interpretation of the interference pattern is only valid in the semi-classical limit and in the absence of correlations between the spatial and momentum coordinates[6,7]. In such idealized cases, $C_2(\mathbf{k}_1, \mathbf{k}_2)$ is directly related to the space-time density, $\rho(x)$, of pion emission points through

$$C_2(\mathbf{k}_1, \mathbf{k}_2) = 1 + \lambda |\rho(k_1 - k_2)|^2 \quad , \tag{2}$$

with $\rho(q) = \int d^4x e^{iqx} \rho(x)$ and with the incoherence or chaoticity parameter $\lambda = 1$. In many interesting cases, dynamical effects can lead, however, to strong correlations between $\mathbf{x}$ and $\mathbf{k}$ which can distort the interference pattern and obscure the space-time interpretation of $C(\mathbf{k}_1, \mathbf{k}_2)$. In such cases the analysis of correlation functions necessarily becomes model dependent! Nevertheless, the study of small relative momentum pion correlations is still useful as a unique and complementary test of specific dynamical models since identical pion correlations are sensitive to the *phase space correlations* predicted by transport models, which are otherwise not tested in other inclusive measurements. However, as we show below it is essential in that case to use a more refined formalism to connect transport calculations with interferometry data.

A characteristic symptom of the breakdown of the ideal picture is that $C_2(\mathbf{k}_1, \mathbf{k}_2)$ is found to depend on the mean pion momentum, $\mathbf{K} = (\mathbf{k}_1 + \mathbf{k}_2)/2$, as well as on the relative momentum, $\mathbf{q} = \mathbf{k}_1 - \mathbf{k}_2$ even in the case $\mathbf{K} \cdot \mathbf{q} = 0$ (see e.g. [7,8,10,14]) (A dependence on the component of $\mathbf{K}$ parallel to $\mathbf{q}$ always occurs if there is time dependence of $\rho(\mathbf{x}, t)$.) A second symptom of the breakdown of the ideal picture is a fitted value of $\lambda < 1$, also found often experimentally. While in principle partially coherent fields could be produced[5], the most likely cause of an apparent $\lambda < 1$ is an overly simplified analysis of the complex six-dimensional dependence of $C_2(\mathbf{k}_1, \mathbf{k}_2)$ involving integrations over four or five of the momentum variables and/or neglecting additional important dynamical degrees of freedom such as resonances. These points have been emphasized for example in refs.[14,19].

Present interest in this problem stems from new data on pion interferometry of nuclear collisions at CERN[15] and the development of Monte Carlo transport models[18,24] for high energy reactions. At high energies, Lorentz boost invariance along the beam direction leads to a strong (so called Inside-Outside[23]) correlation between the production points, x^μ, and final momenta, p^μ. The modifications of C_2 due to such phase space correlations have been studied in Refs.[11,12,14,17] using a variety of simplifying assumptions and techniques. There has also been recent progress toward more realistic calculations, taking into account additional dynamical complications predicted by detailed transport models in refs.[18,19]. However, the theoretical basis for those calculations has not been adequately discussed in the literature.

The purpose of this paper is to derive a general interferometry formula applicable to cases where strong phase space and multiparticle correlations are predicted and to broaden the theoretical basis for the formula used in recent pion interferometric analysis[19]. Our formula turns out to be a natural generalization of the one proposed by Pratt[7] and is derived in a more comprehensive way using transport theory and the Wigner density formalism developed by Remler[21,22]. Finite wavepackets are used to expose the sensitivity of the interference effects to the production mechanism.

The Wigner formalism connects the *rate of change* of the n particle phase space distribution, $f_n(\mathbf{x}_1, \mathbf{p}_1, \cdots, \mathbf{x}_n, \mathbf{p}_n, t)$ to asymptotic observables. As emphasized in [21,22], transport theories, such as hydrodynamics or cascade models, can only approximate the rate of change of f_n during the limited time interval when relatively high momentum transfer processes are occuring. At asymptotic times such models break down or predict free streaming. Low momentum transfer final state interactions leading to weakly bound states[22] and subtle Bose interference effects can only be rigorously extracted from transport models using the Wigner formalism. The formalism also allows us to derive a new equation incorporating effects of intermediate time cascading of pions and to study the conditions under which only the final freeze-out coordinates dominate the interference pattern.

The main result of this paper is summarized by the following formula for the Bose-Einstein symmetrized n pion invariant distribution:

$$P_n(\mathbf{k}_1, \cdots, \mathbf{k}_n) \propto \left\langle \sum_\sigma \prod_{j=1}^{n} e^{i(k_j - k_{\sigma_j})\cdot x_j} \delta_\Delta(k_j, k_{\sigma_j}, p_j) \right\rangle \quad , \tag{3}$$

with the smoothed delta function given by

$$\delta_\Delta(k, k', p) = (2\pi\Delta p^2)^{-3/2} \exp(\tfrac{1}{2}(p - \tfrac{1}{2}(k + k'))^2/\Delta p^2 + \tfrac{1}{2}(k - k')^2 \Delta x^2) \ . \tag{4}$$

The brackets $\langle \cdots \rangle$ denote an average over the $8n$ pion *freeze-out* space coordinates $\{x_1, p_1, \cdots, x_n, p_n\}$, as obtained from the output of a specific transport model such as a cascade[18] or LUND model[19]. In this form, Eq.(4) is ideally suited for Monte Carlo computation of pion interference effects. The smoothed delta function results from the use of Gaussian wavepackets with widths Δx and Δp that depend on details of the pion production mechanism. The sum is over $n!$ permutations, $\sigma = (\sigma_1, \cdots, \sigma_n)$, of the indices ($x, k, p, \cdots$ denote four vectors and all momenta are on-shell).

There are several important points to note in conection with (3):

1. The freeze-out coordinates do not correspond in general to the set of coordinates $\{\mathbf{x}_i(t_f), \mathbf{p}_i(t_f)\}$ at any particular "freeze-out" time since the decoupling times, x_i^0, are usually widely distributed[21,22]. In a cascade model, the

freeze-out time for particle i is the time, t_{fi}, when the last binary collision was suffered by that particle and $(x_i^\mu, p_i^\mu) = (x^\mu(t_{fi}), p^\mu(t_{fi}+\epsilon))$. These $8n$ coordinates can be arbitrarily correlated.

2. The wavepacket widths enters because the uncertainty principle permits us to interpret the (x_i^μ, p_i^μ) only as the mean values of the pion wavepackets. In Monte Carlo calculations involving a finite sample of freeze-out coordinates, the interference terms are nonvanishing only if $\Delta p > 0$ since no two p_i are ever the same. However, in the semi-classical limit $(\langle(\mathbf{x}_i - \mathbf{x}_j)^2\rangle \gg \Delta x^2,\ \langle(\mathbf{p}_i - \mathbf{p}_j)^2\rangle \gg \Delta p^2)$, the dependence on the widths drops out.

3. Eq.(3) reduces to the expression derived via a covariant current ensemble formalisms[19] for minimal wavepackets ($\Delta x \Delta p = \frac{1}{2}$). In that case $\Delta p^2 = mT$ in terms of the pion mass and the pseudo-temperature parameter characterizing current elements. Our derivation thus clarifies the interpretation of the current elements in the later formalism.

4. The Pratt[7] formula for interferometry correspond to the nonrelativistic and the $\Delta x = \Delta p = 0$ limits of (3). The hybrid Yano-Koonin formula[4] follows from (3) only if correlations between x_i and p_i can be neglected. In addition the wavepackets provide a physical basis for the numerical smoothing proceedure adopted in [18].

5. In general, corrections terms to (3) appear due to cascading prior to the freeze-out time as we show in section 2.2 but can be neglected in the limit that the mean free path of pions is small compared to the source size (the hydrodynamic limit) or if the momentum transfers are small compared to the pion momenta (the Eikonal limit).

6. Finally, in cases where P_n is found to be sensitive to the wavepacket size, pion interferometry cannot separate production dynamics from the transport dynamics, and Δx and Δp must be treated as addition physical parameters. A similar sensitivity to the form of the current elements in the current ensemble method is possible. As we emphasize in section 3.2, this is the case for the ideal Inside-Outside cascade dynamics[11]-[19], where the rapidity correlation scale, $\delta y \sim (\tau \Delta p)^{-1}$, dependends not only on the mean pion freez-out proper time but also on Δp.

7. Eq.(3) could be further generalized by allowing every packet to vary independently, e.g., via a different $\Delta x_i, \Delta p_i$. Choosing, the coherence length Δx_i to be very large for a fraction of the pions due to some exotic production mechanism, the interference pattern would be similar to that due to partially coherent fields[5].

8. Relative Coulomb and other final state interactions are not considered here but can be included via methods dicussed by Bowler[16].

The remaining sections are organized as follows: In section 2.1 the Wigner density formalism is introduced in connection with pion interferometry. Section 2.2 is the main body of this paper where the interferometry of cascade dynamics is derived based on non-relativistic transport theory. In section 2.3 the covariant generalization of the Wigner density formula is proposed and shown to be equivalent to one derived via the covariant current ensemble method. In section 3.1 we illustrate the formulas first for an uncorrelated Gaussian source, and finally in section 3.2, interferometry of ideal inside-outside dynamics is considered in detail.

2 Pion Interferometry of Cascading Systems

2.1 Wigner Density Formalism

The inclusive number distribution, $P(\alpha)$, of final multipion configurations in states, $|\alpha\rangle$, can be calculated if the the exact density matrix, $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$, for the system were known. Formally, (see, e.g., refs.[21,22])

$$P(\alpha) = \lim_{t\to\infty} Tr\rho_\alpha \rho(t) = \int dt Tr\rho_\alpha \frac{d}{dt}\rho(t) = -i\int dt Tr\rho_\alpha [H_I(t), \rho(t)] \ , \qquad (5)$$

where we used the equation of motion of ρ and split the Hamiltonian, $H = H_0 + H_I$, into a part, H_0, whose eigenstates include, $|\alpha\rangle$, and an interaction part, H_I, including the source currents producing pions and interactions with other particles of the system. The last equality follows as in [22] by use of the cyclic property of commutators in traces and the assumption that $[H_0, \rho_\alpha] = 0$. In this paper we consider noninteracting multipion states, e.g., $|\alpha\rangle = |\mathbf{k}_1, \cdots, \mathbf{k}_n\rangle$ for which H_0 is just the free Hamiltonian.

Evaluating the trace in the Wigner representation, the n-pion inclusive distribution can be evaluated from

$$P(\alpha) = \int dt \int d\phi_1 \cdots d\phi_n W_\alpha(\phi_1, \cdots, \phi_n) \left(\frac{\partial}{\partial t}\right)_I f_n(\phi_1, \cdots, \phi_n, t) \ , \qquad (6)$$

where $\phi_i = (\mathbf{x}_i, \mathbf{p}_i)$ are six dimensional phase space coordinates with integration measure $d\phi_i = d^3\mathbf{x}_i d^3\mathbf{p}_i (2\pi)^{-3}$ in standard ($\hbar = c = 1$) units, and W_α is the Wigner representation of the asymptotic density matrix, $|\alpha\rangle\langle\alpha|$,

$$W_\alpha(\phi_1, \cdots, \phi_n) = \int \left\{ \prod_{i=1}^{n} d^3\mathbf{y}_i e^{-i\mathbf{p}_i\cdot\mathbf{y}_i} \right\} \langle\{\mathbf{x}_i + \tfrac{1}{2}\mathbf{y}_i\}|\alpha\rangle\langle\alpha|\{\mathbf{x}_i - \tfrac{1}{2}\mathbf{y}_i\}\rangle \ , \qquad (7)$$

and the n-pion inclusive phase space density at time t as given by

$$f_n(\phi_1, \cdots, \phi_n, t) = \int \left\{ \prod_{j=1}^{n} d^3\mathbf{y}_j e^{-i\mathbf{p}_j\cdot\mathbf{y}_j} \right\} \langle\{\mathbf{x}_i + \tfrac{1}{2}\mathbf{y}_i\}|\rho(t)|\{\mathbf{x}_i - \tfrac{1}{2}\mathbf{y}_i\}\rangle \ . \qquad (8)$$

It is important to note that the label I on the time derivative in (6) implies that only time variation of f_n due to interactions are to be taken into account. Transport models provide at best an approximation to the dynamical evolution only over a limited space-time region, where short-range interactions due to H_I are important. As emphasized in ref.[21,22], Eq.(6) is the point at which such approximations can be introduced. Hydrodynamics[17], for example, can approximate the dynamics only during the time that local equilibrium is maintained and breaks down beyond some freeze-out time. Monte Carlo event generators[24] and cascade models[18] can approximate the dynamics only to the last large momentum transfer interaction and assume free streaming beyond that. Bose or Fermi symmetrization and multiple soft interactions leading to weakly bound states[22] are obviously beyond the scope of most classical transport models. In that case Eq.(6) provides the only rigorous link between of transport calculations and the asymptotic multiparticle final states observables. The main trick is that quantum effects are included through the use of an exact final state Wigner densities. Of course, it may well be that no classical transport model can adequetly describe $(\partial/\partial t)_I f_m$. However, the point is that (6) allows an experimental test of such models.

For pion interferometry our primary interest is to exploit the Bose symmetry of the asymptotic wave functions. Neglecting final state interactions[5,16] the symmetrized final wavefunction

$$\langle \mathbf{x}_1, \cdots, \mathbf{x}_n | \mathbf{k}_1, \cdots, \mathbf{k}_n \rangle = \frac{(2\pi)^{-3n/2}}{\sqrt{n!}} \sum_\sigma e^{i\mathbf{k}_{\sigma 1}\cdot \mathbf{x}_1} \cdots e^{i\mathbf{k}_{\sigma n}\cdot \mathbf{x}_n} \tag{9}$$

leads to the following Wigner density:

$$W_{\mathbf{k}_1,\cdots,\mathbf{k}_n}(\phi_1, \cdots, \phi_n) = \frac{1}{n!} \sum_{\sigma\tilde{\sigma}} \prod_{i=1}^{n} \left\{ e^{-i\mathbf{q}(\sigma_i,\tilde{\sigma}_i)\cdot \mathbf{x}_i} \delta^3(\mathbf{p}_i - \mathbf{K}(\sigma_i, \tilde{\sigma}_i)) \right\} , \tag{10}$$

where $\sigma = (\sigma_1, \cdots, \sigma_n)$ and $\tilde{\sigma}$ denote permutation vectors of n indices, and the relative and mean momenta are given by

$$\begin{aligned} \mathbf{q}(i,j) &= \mathbf{k}_i - \mathbf{k}_j \\ \mathbf{K}(i,j) &= \tfrac{1}{2}(\mathbf{k}_i + \mathbf{k}_j) . \end{aligned} \tag{11}$$

Denoting the spatial Fourier transform of the inclusive phase space densities by

$$\tilde{f}_n(\tilde{\phi}_1, \cdots, \tilde{\phi}_n, t) \equiv \int \prod \{(2\pi)^{-3} d^3\mathbf{x}_i e^{-i\mathbf{q}_i\cdot \mathbf{x}_i}\} f_n(\phi_1, \cdots, \phi_n, t) , \tag{12}$$

where $\tilde{\phi}_i \equiv (\mathbf{q}_i, \mathbf{p}_i)$, the integration over phase space coordinates in (6) given (10) leads to the following expression for the n-pion inclusive distribution:

$$P_n(\mathbf{k}_1, \cdots, \mathbf{k}_n) = \sum_\sigma \int dt \left(\frac{\partial}{\partial t}\right)_I \tilde{f}_n(\tilde{\phi}(1,\sigma_1), \cdots, \tilde{\phi}(n,\sigma_n), t) , \tag{13}$$

where $\tilde{\phi}(i,j) = (\mathbf{q}(i,j), \mathbf{K}(i,j))$, and the permutation symmetry of the classical distribution $f_n(\phi_1, \cdots, \phi_n, t) = f_n(\phi_{\sigma_1}, \cdots, \phi_{\sigma_n}, t)$ has been employed.

2.2 Semi-classical Cascade Ensemble

In classical transport theory[25], the n-particle phase space distribution can be expressed as the ensemble average of a product of microscopic distributions

$$g_a(\phi, t) = (2\pi)^3 \delta^6(\phi - \phi_a(t))\theta(t - t_{a0}) \ , \tag{14}$$

that describe the phase space distribution of individual test particles, labeled a, which were produced at times, t_{a0}, and which move along classical trajectories

$$\phi_a(t) = (\mathbf{x}_a(t), \mathbf{p}_a(t)) \quad , t \geq t_{a0} \ ,$$

with

$$\frac{d}{dt}\mathbf{x}_a = \mathbf{p}_a / E_a \equiv \mathbf{v}_a \ , \ \ E_a = (p_a^2 + m_a^2)^{1/2} \ .$$

In terms of the microscopic distributions, g_a, the n-particle distribution is given by

$$f_n(\phi_1, \cdots, \phi_n, t) = \left\langle C_{N,n} \prod_{i=1}^{n} g_{a_i}(\phi_i, t) \right\rangle \ , \tag{15}$$

where $\langle \cdots \rangle$ denotes the ensemble average over the trajectories of n-tuples in events with total multiplicty, N, and $C_{N,n} \equiv N!/(N-n)!$.

To incorporate minimal effects due to the uncertainty principle, we must allow for a spread of coordinates around the classical trajectories. The semi-classical generalization of (14) is achieved by the simple substitition

$$(2\pi)^3 \delta^6(\phi - \phi_a) \to \delta^6_\Delta(\phi - \phi_a) = (\Delta x \Delta p)^{-3} e^{-(\mathbf{x}-\mathbf{x}_a)^2/2\Delta x^2} e^{-(\mathbf{p}-\mathbf{p}_a)^2/2\Delta p^2} \ , \tag{16}$$

with the condition that $\Delta x \Delta p \geq \frac{1}{2}$. We note that classical transport theory is generally valid only in cases when the phase space density is low, i.e. $f(x,p)\Delta x \Delta p \ll 1$, and when the wavepackets do not overlap on the average. In the semi-classical limit the final observables should then not be sensitive to the wavepacket size. On the other hand, if a sensitivity to Δx and Δp is found, then this indicates that the interference pattern involves a convolution of effects depending on the quantal aspects of the production dynamics and of effects resulting from the transport dynamics.

We consider here the simplest case, where transport can be described by classical (billiard ball) cascade dynamics. In this case, the momenta of particles are changed discontinuously through momentum transfers, $\Delta \mathbf{p}_{ai}$ at specific "collision" times t_{ai} corresponding to the i^{th} collision of particle a such that

$$\mathbf{p}_a(t) = \mathbf{p}_{a0} + \sum_{i=1}^{f(a)} \Delta \mathbf{p}_{ai} \theta(t - t_{ai}) \ . \tag{17}$$

The particles move along zigzag paths in coordinate space. In (17), $f(a)$ denotes the total number of collisions suffered by particle a. We denote the straight line phase space trajectory between $t_{ai} \leq t \leq t_{a(i+1)}$ by

$$\phi_{ai}(t) = (\mathbf{x}_{ai} + \mathbf{v}_{ai}(t - t_{ai}), \mathbf{p}_{ai}) \ , \tag{18}$$

where $\mathbf{x}_{ai} \equiv \mathbf{x}_a(t_{ai})$, $\mathbf{p}_{ai} \equiv \mathbf{p}_a(t_{ai}+0)$, and $\mathbf{v}_{ai} = \mathbf{v}_a(t_{ai}+0)$. The semi-classical microscopic distributions (14) can be thus expressed as

$$\begin{aligned} g_a(\phi,t) &= \theta(t-t_{af})\delta^6_\Delta(\phi-\phi_{af}(t)) \\ &\quad + \sum_{i=0}^{f(a)-1} \theta(t-t_{ai})\theta(t_{a(i+1)}-t)\delta^6_\Delta(\phi-\phi_{ai}(t)) \ , \end{aligned} \tag{19}$$

where t_{af} denotes the last collision time, i.e., the decoupling time of particle a. Taking the time derivative and using the equations of motion, we see that g_a obeys the classical the transport equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v_p}\cdot\nabla_\mathbf{x}\right) g_a(\phi,t) = s_a(\phi,t) \ , \tag{20}$$

where $\mathbf{v}_p = \mathbf{p}/E_p$, and the "source" phase space density *including* modifications due to final state cascading is given by

$$\begin{aligned} s_a(\phi,t) &= \delta(t-t_{af})\delta^6_\Delta(\phi-\phi_{af}(t)) \\ &\quad + \sum_{i=0}^{f(a)-1} (\delta(t-t_{ai})-\delta(t-t_{a(i+1)}))\delta^6_\Delta(\phi-\phi_{ai}(t)) \ . \end{aligned} \tag{21}$$

In the absence of cascading, the source density reduces to the intuitive form

$$s_{a0}(\phi,t) = \delta(t-t_{a0})\delta^6_\Delta(\phi-\phi_{a0}) \ . \tag{22}$$

With cascading included, s_a includes not only (22) but also a "collision" term

$$c_a(\phi,t) = \sum_{i=1}^{f(a)} \delta(t-t_{ai})(\delta^6_\Delta(\phi-\phi_{ai}) - \delta^6_\Delta(\phi-\phi_{a(i-1)})) \ .$$

With the transport equation (20), the time variation of f_n is given by

$$\left(\frac{\partial}{\partial t} + \sum_{i=1}^{n} \mathbf{v_i}\cdot\nabla_{\mathbf{x_i}}\right) f_n(\phi_1,\cdots,\phi_n,t) = \langle C_{N,n}\sum_{i=1}^{n} s_{a_i}(\phi_i,t)\prod_{j\neq i}^{n} g_{a_j}(\phi_j,t)\rangle \ . \tag{23}$$

The left hand side obviously describes free streaming while the right hand side is the sought after rate of change of f_n due to interactions in this model.

To perform the time integration in (13), we show next that the spatial Fourier transform of the right hand side of (23) can be written as of a total time derivative in the semiclassical limit for the special set of momentum coordinates involved in eq.(13). Using the tilde notation of (12), the transformed transport equation from (20) is

$$(\partial_t + i\mathbf{q}\cdot\mathbf{v_p})\,\tilde{g_a}(\mathbf{q},\mathbf{p},t) = \tilde{s_a}(\mathbf{q},\mathbf{p},t) \ . \tag{24}$$

Since s_a is given by (21), the solution of (24) can be written as

$$\tilde{g_a}(\mathbf{q},\mathbf{p},t) = e^{-i\mathbf{q}\cdot\mathbf{v}_p t} n_a(\mathbf{q},\mathbf{p},t) \ , \tag{25}$$

where

$$n_a(\mathbf{q},\mathbf{p},t) = \int_{-\infty}^{t} dt' e^{i\mathbf{q}\cdot\mathbf{v_p}t'} \tilde{s_a}(\mathbf{q},\mathbf{p},t') \quad . \tag{26}$$

In terms of this solution, the space-time Fourier transform of the interaction rate in (13) can be expressed finally as

$$\left(\frac{\partial}{\partial t}\right)_I \tilde{f}_n(\tilde{\phi}_1,\cdots,\tilde{\phi}_n,t) = e^{-i\theta_n t}\frac{\partial}{\partial t}\left(\prod_i n_{a_i}(\tilde{\phi}_i,t)\right) \quad , \tag{27}$$

where

$$\theta_n(\{\tilde{\phi}_i\}) = \sum_{i=1}^{n} \mathbf{q}_i \cdot \mathbf{v}_{\mathbf{p}_i} \quad . \tag{28}$$

Only the $e^{i\theta_n t}$ phase factor stands in the way of immediate time integration in (13). However, for pion interferometry, a special set of momentum coordinates, $\{\tilde{\phi}_i\}$, are required in eq.(13). For a given permutation, σ, they are

$$\{\tilde{\phi}_i\} = \{\tilde{\phi}(i,\sigma_i)\} = \{\mathbf{k}_1 - \mathbf{k}_{\sigma_1}, \tfrac{1}{2}(\mathbf{k}_1 + \mathbf{k}_{\sigma_1}),\cdots,\mathbf{k}_n - \mathbf{k}_{\sigma_n}, \tfrac{1}{2}(\mathbf{k}_n + \mathbf{k}_{\sigma_n})\} \quad . \tag{29}$$

Evaluating θ_n in this case leads to

$$\theta_n(\{\tilde{\phi}(i,\sigma_i\}) = \sum_{i=1}^{n}(\mathbf{k}_i^2 - \mathbf{k}_{\sigma_i}^2)/E(i,\sigma_i) \approx 0 \quad , \tag{30}$$

where $E(i,\sigma_i) = ((\mathbf{k}_i + \mathbf{k}_{\sigma_i})^2/4 + m^2)^{\frac{1}{2}}$. For two pion interferometry $\theta_2 \equiv 0$ is trivial, but even for $n \geq 3$ pion interferometry, $\theta_n \approx 0$ in the nonrelativistic ($E \longrightarrow m$) limit.

With (30), the time integration in (13) gives

$$P_n(\mathbf{k}_1,\cdots,\mathbf{k}_n) \approx \sum_\sigma \left\langle C_{N,n}\prod_{i=1}^{n} n_{a_i}(\tilde{\phi}(i,\sigma_i),\infty)\right\rangle \quad , \tag{31}$$

where

$$n_a(\tilde{\phi}(i,j),\infty) = (2\pi)^{-3}\int d^4x e^{iq^\mu(i,j)x_\mu} s_a(\mathbf{x},\mathbf{K}(i,j),t) \quad , \tag{32}$$

is the space-time Fourier transform of the source distribution (21) with

$$q_0(i,j) = \mathbf{q}(i,j)\cdot\mathbf{K}(i,j)/E(i,j) \approx (\mathbf{k}_i^2 + m^2)^{\frac{1}{2}} - (\mathbf{k}_j^2 + m^2)^{\frac{1}{2}} \tag{33}$$

corresponding to the approximate pair energy difference.

Taking the space-time Fourier transform of s_a in (21), we therefore find that

$$n_a(\mathbf{q},\mathbf{K},\infty) = \{e^{iqx_{af}}\tilde{\delta}_\Delta(\mathbf{q},\mathbf{K}-\mathbf{p}_{af}) + \sum_{i=0}^{f(a)-1}(e^{iqx_{ai}} - e^{iqx_{a(i+1)}})\tilde{\delta}_\Delta(\mathbf{q},\mathbf{K}-\mathbf{p}_{ai})\} \quad , \tag{34}$$

where this Gaussian smoothed delta function is given by

$$\tilde{\delta}_\Delta(\mathbf{q},\mathbf{p}) = (2\pi\Delta p^2)^{-3/2} e^{-\mathbf{p}^2/2\Delta p^2} e^{-\mathbf{q}^2\Delta x^2/2} \quad . \tag{35}$$

In the limit, $\Delta x, \Delta p \to 0$, obviously $\tilde{\delta}_\Delta(\mathbf{q},\mathbf{p}) \to \delta^3(\mathbf{p})$.

There are three interesting limits of (34). First in the strict classical limit every term in the sum over intermediate scatterings vanishes because classical particles propagate along straight line trajectories between collisions such that

$$\mathbf{v}_{ai} t_{a(i+1)} - \mathbf{x}_{a(i+1)} = \mathbf{v}_{ai} t_{ai} - \mathbf{x}_{ai} \quad . \tag{36}$$

Therefore, in the classical limit only the final decoupling time contributes and (34) reduces to

$$n_a(\mathbf{q},\mathbf{K},\infty) \approx e^{iqx_{af}} \delta^3(\mathbf{K}-\mathbf{p}_{af}) \quad . \tag{37}$$

Second, even for $\Delta p > 0$, the cascade corrections can be neglected if $R \gg \lambda$, where R is the radius and λ the mean free path. In this case the number of terms, $f(a) \sim R/\lambda$ in the sum is large, but the difference of the phases in each term is very small:

$$\langle e^{iqx_{a(i+1)}} - e^{iqx_{ai}} \rangle \sim (\mathbf{q}\cdot\delta\mathbf{v}\delta t)^2 \sim (\lambda/R)^2 \quad . \tag{38}$$

To see this, note that the interesting range of q is $\sim 1/R$, the time interval between collisions, $\delta t \sim \lambda/v_a$, the linear term vanishes on account of $\langle \mathbf{q}\cdot\delta\mathbf{v}\rangle \approx 0$, and finally that the velocity dispersion is $\delta\mathbf{v}^2/\mathbf{v}_a^2 \lesssim 1$. Therefore, in this "thermal" limit

$$n_a(\mathbf{q},\mathbf{K}) \approx e^{iqx_{af}} \tilde{\delta}_\Delta(\mathbf{q},\mathbf{K}-\mathbf{p}_{af}) \quad , \tag{39}$$

with corrections on the order of $\delta n_a/n_a \sim (\lambda/R)$. For the above estimate we assumed implicitly that the typical momentum transfers were on the order of the pion momenta. In the opposite (Eikonal) limit, where $\Delta p_{ai}/p_{ai} \ll 1$ and $p_{af} \approx p_{a0}$, (39) still holds, but in this case, n_a can also be expressed directly in terms of the *initial* production coordinates as

$$n_a(\mathbf{q},\mathbf{K},\infty) \approx e^{iqx_{a0}} \tilde{\delta}_\Delta(\mathbf{q},\mathbf{K}-\mathbf{p}_{a0}) \quad , \tag{40}$$

with corrections appearing to order $\delta n_a/n_a \sim (R/\lambda)\Delta p_a^2/p_a^2$.

For applications where $\delta n_a/n_a \ll 1$, (31) reduces to the following semiclassical expression for the n-pion inclusive distribution

$$P_n(\mathbf{k}_1,\cdots,\mathbf{k}_n) \approx \sum_\sigma \left\langle C_{N,n} \prod_{i=1}^n e^{iq(i,\sigma_i)x_i} \tilde{\delta}_\Delta(\mathbf{q}(i,\sigma_i),\mathbf{K}(i,\sigma_i)-\mathbf{p}_i) \right\rangle \quad , \tag{41}$$

which involves only the phase space coordinates, $(\mathbf{x}_i,\mathbf{p}_i)$, of the pions at the decoupling times, t_i.

It depends also in general on the spread of the wavepacket in phase space. We emphasize that (41) permits arbitrary dynamical correlations between the seven dimensional decoupling phase space coordinates and also arbitrary n-pion dynamical correlations. While intermediate cascade corrections to (41) may be small, such correlations resulting from the cascading can nevertheless modify strongly the final interference pattern, e.g., in the case where collective flow is generated[7]. In the case when only a few collisions occur, eqs. (31,34) provides a way to calculate distortions caused by intermediate state cascading.

If n-pion dynamical correlations are absent, then the ensemble average in (41) can be expressed in terms of the space-time Fourier transform of the pion breakup distribution

$$D(q,\mathbf{p}) = \int d^4x \exp(iqx) D(x,\mathbf{p}) \quad , \tag{42}$$

where

$$D(x,\mathbf{p}) = \langle \delta(t-t_f)\delta^6(\phi-\phi_f)\rangle \quad , \tag{43}$$

and ϕ_f are the breakup phase space coordinates and t_f are the breakup times. Note that D is normalized as $\int d^4x d^3p D(x,\mathbf{p}) = 1$. We emphasize that D does not correspond to the one body phase space density, $f(x,\mathbf{p})$, at any one time, x^0, because it involves the fluctuations in the breakup times. Only in the ideal hydrodynamic model[17] can D be identified with f at the freeze-out time. In terms of D,

$$P_n(\mathbf{k}_1,\cdots,\mathbf{k}_n) \approx \langle C_{N,n}\rangle \sum_\sigma \prod_{i=1}^n D_\Delta(q(i,\sigma_i),\mathbf{K}(i,\sigma_i)) \quad , \tag{44}$$

where D_Δ is the wavepacket smoothed breakup distribution

$$D_\Delta(q,\mathbf{K}) = \int d^3p D(q,\mathbf{p})\tilde{\delta}_\Delta(\mathbf{q},\mathbf{p}-\mathbf{K}) \quad . \tag{45}$$

In particular, the single inclusive distribution is $P_1(\mathbf{k}) = \langle N\rangle D_\Delta(q=0,\mathbf{k})$. The two-pion correlation function is given by

$$C(\mathbf{k}_1,\mathbf{k}_2)-1 = \frac{\langle \cos(\mathbf{q}\cdot(\mathbf{v}_K(t_{af}-t_{bf})-(\mathbf{x}_{af}-\mathbf{x}_{bf}))e^{-\mathbf{q}^2\Delta x^2}\delta^3_{\Delta p}(\mathbf{K}-\mathbf{p}_{af})\delta^3_{\Delta p}(\mathbf{K}-\mathbf{p}_{bf})\rangle}{\langle \delta^3_{\Delta p}(\mathbf{k}_1-\mathbf{p}_{af})\rangle\langle \delta^3_{\Delta p}(\mathbf{k}_2-\mathbf{p}_{bf})\rangle} \quad . \tag{46}$$

with $\mathbf{q} = \mathbf{k}_1 - \mathbf{k}_2$ and $\mathbf{K} = \frac{1}{2}(\mathbf{k}_1+\mathbf{k}_2)$. If the multiplicity distribution is not Poisson, then the ratio binomial moments enters as in (1). In the limit $\Delta x = \Delta p = 0$, (46) reduces to the expression derived by Pratt [7]. In addition to including smearing associated with the production of finite wavepackets, our derivation more general than that of Ref.[7] and show how corrections due to intermediate state cascading can be calculated via (34)

It is interesting to compare (46) with the phenomenological formula proposed in [4] and used in cascade studies in [10]. The Yano-Koonin formula differs from (46) in the replacement of the two $\mathbf{K}$'s in the numerator by $\mathbf{k}_1$ and $\mathbf{k}_2$ and setting $\Delta x = \Delta p = 0$ leading to

$$\begin{aligned} C_{YK}(\mathbf{k}_1,\mathbf{k}_2) - 1 &= Re\{D(q,\mathbf{k}_1)D(-q,\mathbf{k}_2)\}/D(0,\mathbf{k}_1)D(0,\mathbf{k}_2) \\ &= \frac{\langle \cos(q\cdot(x_1-x_2))\delta^3(\mathbf{p}_1-\mathbf{k}_1)\delta^3(\mathbf{p}_2-\mathbf{k}_2)\rangle}{\langle \delta^3(\mathbf{p}_1-\mathbf{k}_1)\delta^3(\mathbf{p}_2-\mathbf{k}_2)\rangle} \quad . \end{aligned} \tag{47}$$

In this expression, q_0, is the exact relativistic energy difference. In general (47) is valid only when the variation of $D(\mathbf{q},\mathbf{k})$ with $\mathbf{k}$ is small in the sense that

$$\tfrac{1}{2}q\cdot\nabla_K D(q,K) \ll D(q,K) \quad . \tag{48}$$

Since $\nabla_K \sim 1/P$ and $q \sim 1/R$, (48) holds in the semiclassical limit when $RP \gg 1$. However, as we shall see in section 3.2, C_{YK} could differ substantially from the true correlation function in situations where $D(x, \mathbf{p})$ involves strong correlations in phase space.

2.3 Covariant Generalization and the Current Ensemble

A major limitation of the above derivation is the lack of Lorentz covariance. This problem can be cured either by turning to the covariant current ensemble formalism[5,12] or by generalizing the above Wigner formalism. We propose here a simple covariant generalization of (41), which reduces to it in the nonrelativistic limit and show that the same result can be obtained with the covariant current ensemble method[19].

The covariant generalization of the breakup distribution (43) is clearly

$$\begin{aligned} \mathcal{D}(x,p) &= \langle \delta^4(x-x_f)\delta^4(p-p_f)\rangle \ , \\ \mathcal{D}(q,p) &= \langle e^{iqx_f}\delta^4(p-p_f)\rangle \ , \end{aligned} \tag{49}$$

Next we note that for $K^\mu = \frac{1}{2}(k_i^\mu + k_j^\mu)$, $q^\mu = k_i - k_j$, and $k_i^2 = p_f^2 = m^2$, in the nonrelativistic limit we have $q^2 \approx -\mathbf{q}^2$, $K^2 \approx m^2 + \mathbf{q}^2/4$, and $Kp \approx m^2 + ((\mathbf{k}_i - \mathbf{p})^2 + (\mathbf{k}_j - \mathbf{p})^2)/4$. A convenient covariant generalization of $\tilde{\delta}(\mathbf{q}, \mathbf{K} - \mathbf{p})$ is the covariant delta function, $\delta_\Delta(k_i, k_j, p)$ given by Eq.(4). With these generalizations we see finally that Eq.(3) in section 1 is a natural covariant generalization of Eq.(41).

For the case without multiparticle dynamical correlation, the genralization of (45) is then

$$\mathcal{D}_\Delta(q,K) = \mathcal{N} e^{q^2\Delta x^2/2} \langle e^{iqx_f} e^{(K-p_f)^2/2\Delta p^2}\rangle \ . \tag{50}$$

It is remarkable to note that, for the *minimal* wavepackets satisfying $\Delta x \Delta p = \frac{1}{2}$, (50) simplifies to same expression as derived via the covariant current ensemble method in [19]:

$$\mathcal{D}_\Delta(q,K) \propto \langle e^{iqx_f} e^{-Kp_f/\Delta p^2}\rangle \ . \tag{51}$$

To see this connection, we recall that in the later formalism[5,12], the source of pions is represented by an ensemble of current elements, each one specified as

$$j_a(x) = j_0(u_a^\mu (x - x_a)_\mu) \ , \tag{52}$$

where u_a^μ is the boost velocity of the emitting source and x_a is the space-time origin of current element a, and $j_0(x)$ refers to each current element in its rest frame. The Fourier transform of the total source current is then

$$j(k) = \sum_a j_0(u_a^\mu k_\mu) e^{ik_\mu x_a^\mu} e^{i\phi_a} \ , \tag{53}$$

with random phase factors $e^{i\phi_a}$ introduced to describe completely chaotic sources. The n-pion inclusive distribution function is then given by

$$P_n(k_1, \cdots, k_n) = \langle |\, j(k_1)\,|^2 \cdots |\, j(k_n)\,|^2 \rangle \ , \tag{54}$$

where $\langle\cdots\rangle$ denotes the ensemble average over the space-time coordinates x_a, four-velocities u_a, and random phases ϕ_a. In the absence of dynamical multi-pion correlations, that ensemble average can also be expressed in terms of a "freeze-out" phase-space distribution, $\mathcal{D}(x,p)$, where $p_f^\mu \equiv mu_f^\mu$. Then, the n pion inclusive distribution function can be written as

$$P_n(k_1,\cdots,k_n) = \sum_\sigma \left\{\prod_{i=1}^n G(k_i,k_{\sigma_i})\right\} \quad , \tag{55}$$

with a complex amplitude $G(k_i,k_j)$ given by the convolution of the freeze-out distribution and two currents elements that specify the production dynamics:

$$G(k_i,k_j) = \int d^4pD(k_i-k_j,p)j_0^*(pk_i/m)j_0(pk_j/m) = \langle e^{i(k_i-k_j)x_a}j_0^*(p_ak_i/m)j_0(p_ak_j/m)\rangle \quad . \tag{56}$$

As emphasized in [12,19] the dynamical model dependence therefore enters, not only through the distribution , $\mathcal{D}(x,p)$, of the freeze-out phase-space coordinates but also through the the specific form of the current elements j_0.

Finally, we recall that for a pseudo-thermal parametrization[12,19] for on shell Fourier transform of the current elements, with

$$j_0(pk/m) = e^{-p^\mu k_\mu/(2mT)} \quad , \tag{57}$$

the amplitude $G(k_i,k_j)$ is given by

$$G(k_i,k_j) = \langle e^{i(k_i-k_j)x_{af}}e^{-\frac{1}{2}(k_i+k_j)p_{af}/(mT)}\rangle \quad . \tag{58}$$

We therefore see that the pseudo-thermal current ensemble is completely equivalent to the minimum Gaussian packet Wigner formulation in Eq.(51), if $\Delta p^2 \sim mT$. In this formalism, the current elements play the same role as wavepackets do in the Wigner density formalism described above.

3 Examples

3.1 Uncorrelated Gaussian

Consider first the nonrelativistic and uncorrelated Gaussian breakup distribution

$$D(x,\mathbf{p}) = (2\pi\tau^2)^{-1/2}(2\pi R^2)^{-3/2}(2\pi P^2)^{-3/2}\exp(-t^2/2\tau^2 - \mathbf{x}^2/2R^2 - \mathbf{p}^2/2P^2) \quad , \tag{59}$$

where $\tau^2 = \langle t^2\rangle$, $R^2 = \langle x_i^2\rangle$, and $P^2 = \langle p_i^2\rangle$. Evaluating the wavepacket smeared Fourier transform (45) leads to

$$\begin{aligned} P_1(\mathbf{k}) &\propto e^{-k^2/2(P^2+\Delta p^2)} \quad , \\ C(\mathbf{q},\mathbf{K}) &= 1 + e^{-q_0^2\tau^2 - \mathbf{q}^2R_\Delta^2} \quad , \end{aligned} \tag{60}$$

where q_0 is given by (33), and the effective root mean square radius in each spatial direction is

$$R_\Delta^2 = R^2 + \Delta x^2 - \frac{1}{4(P^2 + \Delta p^2)} \quad . \tag{61}$$

In the semiclassical limit, $RP \gg 1$, and thus the last term is negligible.

For this Gaussian ensemble, C_{YK} from (47) is also given by (60) but with $\Delta p = 0$ and $R_\Delta \to 0$. For $\Delta x \Delta p \geq \frac{1}{2}$, $R_\Delta \geq R$, and thus the correlation function is always narrower than naively expected for a given R.

This simple Gaussian ensemble show clearly how breakup time fluctuations can distort the geometrical interpretation of the correlation function. Time fluctuations ($\tau > 0$) induce a $\mathbf{K}$ dependence of $C(k_1, k_2) = C(\mathbf{K}, \mathbf{q})$ through its dependence on the relative energy: $q_0 \approx \mathbf{q} \cdot \mathbf{v}_K$. If we try to extract the geometrical information from the small $\mathbf{q}$ variation of C using

$$C(\mathbf{K}, \mathbf{q}) \approx 1 - (\mathbf{q} \cdot \mathbf{v}_K)^2 \tau^2 - q_z^2 R_{z,eff}^2 - \tfrac{1}{2} q_\perp^2 R_{\perp,eff}^2 \quad , \tag{62}$$

then we see that

$$\begin{aligned} R_{z,eff}^2 &\approx R_\Delta^2 + \tau^2 (K_z/E_K)^2 \\ R_{\perp,eff}^2 &\approx 2(R_\Delta^2 + \tau^2 (K_\perp/E_K)^2 \cos^2\theta_\perp) \quad . \end{aligned} \tag{63}$$

Thus if we integrate over all orientations of $\mathbf{K}_\perp$ with respect to $\mathbf{q}_\perp$, then the effective rms transverse radius would be

$$R_{\perp,eff}^2 \approx 2R_\Delta^2 + \tau^2 (K_\perp/E_K)^2 \quad . \tag{64}$$

Of course, experiments always have a finite resolution for longitudinal momentum differences. Therefore, the experimental correlation function plotted as a function of one variable, say the transverse momentum, always has an effective intercept in the lowest $q_\perp$ bin less than 2. For correlations projected on a single variable, say $q_\perp$, with other variables integrated over certain cuts the intercept can differ dramatically from 2 and has often led to erroneous conclusions (see [14] for further discussion).

For $K = 0$ pairs of course no problem occurs. However, experimentally pairs are analyzed by binning in finite rapidity intervals in which $K/E_K \sim 1$. This simple analytic case shows that R_z^{eff} would then measure roughly the sum of the longitudinal radius and source time with relatively small dependence on the rapidity interval under consideration, while $R_\perp^{eff}$ would show a maximum value exceeding the correct transverse radius in the rapidity interval nearest the rapidity of the source. This behavior of $R_\perp^{eff}$ would occur because $K_\perp$ is limited emiprically while K_z is not. Note that $R_\perp^{eff}$ would tend to the correct transverse radius in rapidity bins where $K_\perp/E_K \to 0$. Therefore, we see that large fluctuations of the emission times of the pions can result in significant narrowing of correlation functions and that narrowing could be misinterpreted as evidence for anomalously large radii and less than chaotic fields.

3.2 Ideal Inside-Outside Cascade

A more interesting example involving relativistic and correlated distributions is given by the ideal inside-outside distribution[23,7,12,14,17] . The freeze-out phase-space distribution is in this case

$$\mathcal{D}(x,p) \propto \delta(\tau(t,z)-\tau)\delta(\eta-y)\delta(E-E_{\mathbf{p}})\delta^2(\mathbf{p}_\perp)e^{-x_\perp^2/R_\perp^2} \ , \qquad (65)$$

where $E_{\mathbf{p}} = \sqrt{\mathbf{p}^2+m^2}$, $\tau(t,z) = (t^2-z^2)^{\frac{1}{2}}$ is fixed to the freeze-out proper time, τ, and the space-time and momentum rapidity variables,

$$\eta = \tfrac{1}{2}\log((t+z)/(t-z)) \ , \ \ y = \tfrac{1}{2}\log((E+p_z)/(E-p_z)) \ , \qquad (66)$$

are assumed to be perfectly correlated. A uniform rapidity distribution is assumed. The rms transverse radius of the system is $R_\perp$.

The covariant Wigner distribution (50) can be evaluated as in [12] and we find that

$$D_\Delta(q,K) \propto e^{\frac{1}{2}q^2\epsilon^2} e^{-\mathbf{q}_\perp^2 R_\perp^2/4} K_0(z) \ , \qquad (67)$$

where

$$\epsilon^2 = \Delta x^2 - 1/(4\Delta p^2) \geq 0 \qquad (68)$$

measures deviations from a minimal packet, and

$$z^2 = (\frac{m^2}{4\Delta p^4}-\tau^2)(m_{1\perp}^2+m_{2\perp}^2)-\frac{im\tau}{\Delta p^2}(m_{1\perp}^2-m_{2\perp}^2)+2(\tau^2+\frac{m^2}{4\Delta p^4})m_{1\perp}m_{2\perp}\cosh(y_1-y_2) \ . \qquad (69)$$

Here $m_\perp^2 = m^2 + \mathbf{k}_\perp^2$. In the limit that $\epsilon = 0$ and $\Delta p^2 = mT$, we thus recover the formula in Ref.[12].

It is interesting to study the rapidity correlation scale by restricting to $m_{1\perp} = m_{2\perp} = m_\perp$ and small $\Delta y = y_1 - y_2$. Furthermore assume that $\Delta p \ll m$. In that case,

$$z \approx \frac{mm_\perp}{\Delta p^2}[1+\frac{1}{2}(\frac{\tau^2\Delta p^4}{m^2}+\frac{1}{4})\Delta y^2] \ , \qquad (70)$$

and we can use the asymptotic expansion of the Bessel function,

$$K_0(z) \approx (\frac{\pi}{2z})^{\frac{1}{2}} e^{-z} \ . \qquad (71)$$

With this approximations, (67) becomes

$$D_\Delta(q,K) \propto 1 - [\tfrac{1}{2}\epsilon^2 + \frac{1}{8\Delta p^2}\frac{m}{m_\perp} + \frac{\tau^2\Delta p^2}{2mm_\perp}(1+\frac{\Delta p^2}{2mm_\perp}) + \frac{1}{4m_\perp^2}]m_\perp^2\Delta y^2 \ , \qquad (72)$$

The single inclusive distribution is therefore

$$P_1(k) \propto \mathcal{D}_\Delta(0,k) \propto e^{-mm_\perp/\Delta p^2}/\sqrt{m_\perp} \ , \qquad (73)$$

and the correlation function is given by

$$C_2(k_1,k_2) \approx 2[1-[\tfrac{1}{2}\epsilon^2+\frac{1}{8\Delta p^2}\frac{m}{m_\perp}+\frac{\tau^2\Delta p^2}{2mm_\perp}(1+\frac{\Delta p^2}{2mm_\perp})+\frac{1}{16m_\perp^2}]m_\perp^2\Delta y^2] \ . \quad (74)$$

For $\epsilon = 0$ and $\Delta p^2 = mT$,

$$C_2(k_1,k_2) \approx 2[1-(\frac{m_\perp}{8T}+\tfrac{1}{2}\tau^2 m_\perp T(1+\frac{T}{2m_\perp})+\frac{1}{16})\Delta y^2] \ . \quad (75)$$

What is most important to observe in the above is that the rapidity correlation scale is strongly influenced by the width of the wavepacket in momentum space. In this model that width determines the average transverse momentum of the final pions through (73) and thus is constrained experimentally. However, even at large freeze-out times it is thus clear that the rapidity correlations in the ideal inside-outside cascade picture depend on the production dynamics.

Another way to emphasize the above point is to compare the above results to those obtained via the Yano-Koonin ansatz[4]. That ansatz can be recovered from (3) by replacing in Eq. (4)

$$\delta_\Delta(k,k',p) \to \delta^3(\mathbf{k}-\mathbf{p}) \ . \quad (76)$$

With this substitution, the ensemble average with (65) is simply proportional to a phase (for $\mathbf{q}_\perp = 0$),

$$D(q,k) \propto e^{i\tau(q_0 \cosh y - q_z \sinh y)} \ , \quad (77)$$

and the correlation function is

$$C_{YK}(k_1,k_2) = 1 + \frac{Re[D(q,k_1)D(-q,k_2)]}{D(0,k_1)D(0,k_2)} \ . \quad (78)$$

This gives

$$C_{YK}(k_1,k_2) = 1+\cos(\tau(m_{1\perp}+m_{2\perp})[1-cosh(y_1-y_2)]) \ , \quad (79)$$

which in the $\Delta y \ll 1$ limit leads to

$$C_{YK}(k_1,k_2) \approx 2[1-\frac{1}{4}\tau^2 m_\perp^2 \Delta y^4] \ . \quad (80)$$

Therefore, without the use of finite wavepackets, the dependence on Δy is only of 4^{th} order. This shows that Eq.(78) is not adequate in general to deal with correlated dynamics.

In the general case, it is necessary to use Eq.(3) as summarized in section 1.

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