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Nuclear Models and Their Application to the Heavy Elements

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Author's note: This report was prepared as a part of a comprehensive review of the radioactivity and nuclear properties of the heavy elements. It will ultimately be published together with the rest of that comprehensive review. This accounts for some of the peculiarities in numbering of figures, tables, etc. The purpose of this treatment of nuclear models is to provide an elementary outline of those which are most often used in the discussion of heavy nuclei. The second half of the report summarizes much of the experimental data from the heavy elements which relate to various features of the nuclear models such as rotational excitations, vibrational excitations, and Nilsson wave function assignments.

NUCLEAR MODELS AND THEIR APPLICATION TO THE HEAVY ELEMENTS

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In other reports the author has summarized the radiations emitted by individual isotopes in the heavy element group of elements. In order to bring about some systemization of what might otherwise be a confusing mass of data it is useful to try to correlate the data in terms of some theory of nuclear structure. It must be stated at the start, however, that no complete theory of nuclear structure exists. Any complete theory for a complex nucleus has to start from a detailed knowledge of the forces exerted between free nucleons. Our present knowledge of such forces are detailed but there are many matters on which there is still considerable doubt. More importantly, it is known that the forces between two free nucleons are quite complex so that it is a formidable task to build up a theory of the structure of nuclei containing many particles and still retain all the known elements of the complex interparticle forces. In the face of such difficulties the usual approach is to construct a limited theory based on simplified assumptions, that is, to construct a theoretical model. If the consequences and predictions of that model are in reasonably good agreement with experimental facts that model is considered useful and is retained until a better one is formulated. If the predictions of the model are in gross disagreement with the facts then the model is reformulated or abandoned.

In discussing the heavy elements we shall often find it desirable to refer chiefly to two related nuclear models. It is the purpose of this chapter to give a description of the assumptions, the development, and the predictions of these models adequate for the purposes of the rest of this book. The first of these models is the independent particle or shell model. The second is a generalization of the shell model which we shall refer to as the unified model or as the Bohr-Mottelson model of the nucleus.

3.1 The Shell Model of the Nucleus

3.1.1 Introductory Remarks. The basic assumption of the shell model of the nucleus is that a single nucleon travels within a complex nucleus in a smoothly-varying average field of force generated by all the other nucleons in the nucleus and that each particle moves essentially undisturbed in its own closed orbit.

This assumption leads to a theory with many analogies to the theory of the atom which describes the motion of electrons in the central Coulomb field of force generated by the atomic nucleus. The quantum theory of the atom leads, through the operation of the Pauli principle, to results which explain the abrupt changes in chemical properties familiar to us in the periodic system of the elements. The development of the independent particle model of the nucleus was stimulated by the observation of many abrupt changes in nuclear characteristics which could be empirically correlated with numbers of neutrons or protons in the nucleus. Such fluctuations were associated with closure of nucleon configurations analogous to the completion of electron configurations in the electronic structure of the atom. The nuclear fluctuations with nucleon numbers were noted as far back as the 1920's and in that sense the shell-model had its origin at that time. Elsasser in 1933 and 1934, to cite one author, summarized the early observations in favor of a nuclear shell structure.

However, up until 1948 the independent particle description of nucleonic motion within the nucleus was not taken seriously because it had no apparent theoretical basis. There seemed to be no way to derive from the very short-range interparticle forces a smoothly varying nuclear potential operating over nuclear dimensions. Presumably the force experienced by a nucleon in its travels through the nucleus should have strong local fluctuations. Hence the nucleon should undergo many collisions in a distance of the order of a nuclear diameter whereas the basic assumption of the model implies a mean free path long compared to the nuclear diameter. The independent particle model would still be rejected because of its shaky theoretical underpinnings if in the past 20 years it had not proved extraordinarily successful in predicting nuclear properties. This great success has stimulated renewed attempts on the part of theoreticians to answer the question, "Why does the shell model work?" In recent years it has come to be realized that the Pauli principle plays a vital role in smoothing out the path of a nucleon traveling in nuclear matter. No attempt will be made to go into this matter in this chapter. The interested reader can turn to a short paper by WEISSKOPF[1] or to a longer article by ELLIOTT and LANE.[2]

The real success of the shell model dates from the year 1948. MARIA GOEPPORT MAYER[3,5] published a paper in which she summarized evidence that nuclei with the following "magic numbers" of neutrons or protons were of exceptional stability: 2, 8, 20, 28, 50, 82 and 126. In a companion paper she showed how an independent particle model could lead to the prediction of these "magic numbers" provided a strong spin-orbit force was included. This modification implies that the important quantum numbers associated with each nucleon orbit are not only the principal quantum number n and the orbital angular momentum ℓ but also a total angular momentum $j = \ell + 1/2$. With this modification the shell model soon had many striking successes. HAXEL, JENSEN and SUESS[4,5] independently published a similar analysis and theory at the same time.

We now proceed to outline briefly the independent particle model in its simplest form.

3.1.2 The Single Particle Form of the Shell Model. We consider the motion of a single nucleon in a central nuclear field, V(r). We wish to solve the Schrödinger equation,

$$\left(- \frac{\hbar^2}{2M} \nabla^2 + V(r)\right)\psi = E\psi \quad . \tag{3.1}$$

Before we can proceed we have to make some decision about the form of V(r). It is convenient to assume a static, spherically-symmetric potential with the form either of a three-dimensional harmonic oscillator or of a square well:

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1. V. Weisskopf, Paper P2399, Vol. 15, Proceedings of the Second U.N. Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958.

2. J. P. Elliott and A. M. Lane, The Nuclear Shell Model, Handbuch der Physik, Vol. XXXIX (Springer-Verlag, Berlin, 1957).

3. M. G. Mayer, Phys. Rev. 74, 235 (1948); ibid. 75, 1969 (1949); ibid. 78, 16 (1950).

4. O. Haxel, J. H. D. Jensen, and H. E. Suess, Phys. Rev. 75, 1766 (1949); Z. Physik 128, 295 (1950).

5. M. G. Mayer and J. H. D. Jensen, Elementary Theory of Nuclear Shell Structure (John Wiley and Sons, Inc., New York, 1955).

Harmonic Oscillator Potential	Square Well Potential
$V_r = 1/2\ M\ \omega_o^2\ r^2$	$V(r) = -V_o =$ constant for $r \leq R$
	$V_r = \infty$ for $r > R$

These choices have the virtue that they lead to easy solutions of the Schrödinger equation and that the true nuclear potential may lie between them. These potentials are inserted in the Schrödinger equation and the equation is solved leading to a complete orthonormal set of single-particle wave functions $\psi^{n\ell m}_{(r_i)}$ where the r_i indicate position variables of the ith nucleon and n, ℓ, m denote the quantum numbers of the states.

For the harmonic oscillator the eigenvalues of the equation are given by the expression

$$E = \hbar\ \omega_o(3/2 + N) \qquad (3.2)$$

where N, the principal quantum number is given by the expression

$$N = 2n - 2 + \ell \ .$$

The quantity n is equal to the number of nodes in the radial wave function including those at the origin minus ℓ. The orbital angular momentum quantum number, ℓ, can have the value 0 or any integral number up to a maximum given by Eq. (3.2). Each energy state as given by Eq. (3.2) then consists of a band of single particle levels degenerate in n and ℓ. There is a further degeneracy in the magnetic quantum number m_ℓ and spin quantum number s. The total degeneracy of each energy state for each type of particle (neutrons or protons) is $(N + 1)(N + 2)$. This product should tell the total number of particles within each shell. These numbers are summarized in Table 3.1. From Table 3.1 we see that the predicted "magic numbers" corresponding to shell closures should be 2, 8, 20, 40, 70 and 168.

The levels for the isotropic harmonic oscillator are also shown on the left side of Fig. 3.1. On the right hand side of the same figure are shown the corresponding levels for a square well with infinite walls. In the center of the figure the corresponding levels are shown as the average of the two treatments. This average is expected to fall closer to the situation in real nuclei. We

Table 3.1 Single Particle States of the Harmonic Oscillator Well

N	$E_N/\hbar\omega$	Degenerate orbitals labeled in N and ℓ	$No_N = \sum_\ell 2(2\ell+1)$	Total particles through end of shell*
0	3/2	1s	2	2
1	5/2	1p	6	8
2	7/2	2s 1d	12	20
3	9/2	2p 1f	20	40
4	11/2	3s 2d 1g	30	70
5	13/2	3p 2f 1h	42	112
6	15/2	4s 3d 2g 1i	56	168

*The fourth column gives the number of particles allowed by the Pauli principle in each degenerate N-shell. The last column gives the total number of particles summed over the all N-shells up through the last considered.

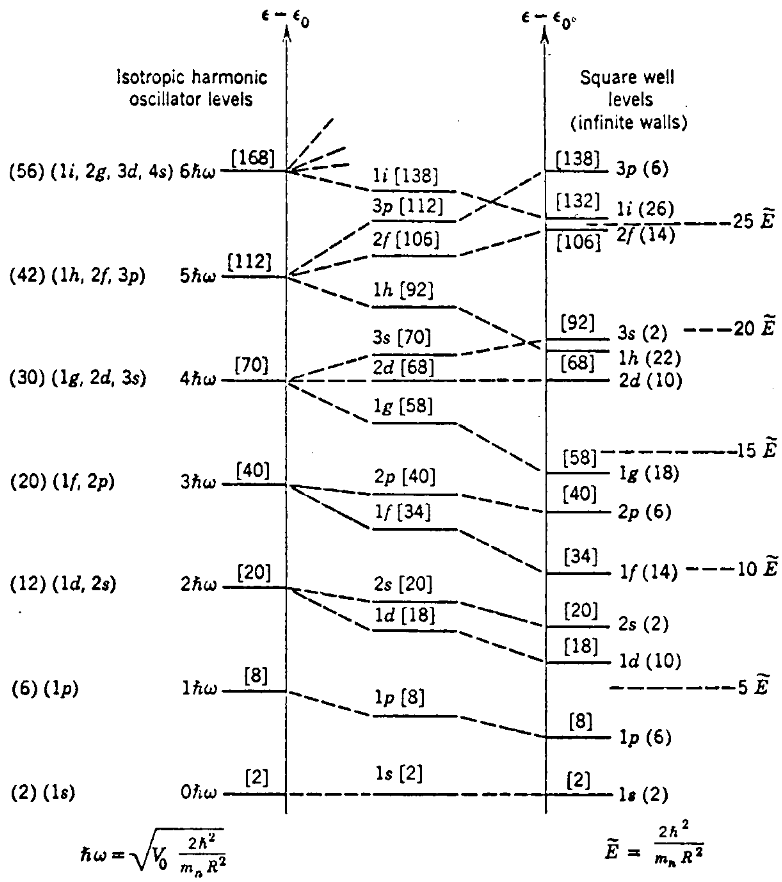

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Fig. 3.1 Level system of the three-dimensional isotropic harmonic oscillator (left) and the square well with infinitely high walls (right). The levels in the center represent an intermediate situation. Spin-orbit coupling is neglected. Figure reproduced from Mayer and Jensen, Elementary Theory of Nuclear Shell Structure (John Wiley and Sons, New York, 1955).

note that the square well gives the same numbers of particles in the major shells but within each shell a degeneracy in ℓ is removed and some sub-shell structure appears.

This treatment gives approximately the correct level ordering for the light nuclei but the predicted shell closures for the heavier nuclei are very different from those observed experimentally. This suggests that some fundamental feature has been left out; the missing feature is spin-orbit coupling. The recognition of the importance of spin-orbit coupling by MAYER[3] and by HAXEL, JENSEN and SUESS[4] established the independent particle model as a major theory for the correlation and prediction of nuclear phenomena. Spin-orbit coupling plays a role in electronic energy states (hyperfine structure) but the effect is small and does not disturb the order of filling of atomic shells. In the nuclear case spin-orbit coupling is large and is of the opposite sign to the atomic case; states with $\ell + 1/2$ are more tightly bound than states with $\ell - 1/2$. The interaction is largest for states of high angular momentum. The effect of strong spin-orbit coupling is to depress certain high-spin levels from one shell to a lower shell and hence to give different occupation numbers for the closed shells. This is made clear in Fig. 3.2.

For example, we note that the $g_{9/2}$ level of the 4th oscillator shell is depressed until it lies close to the preceding oscillator shell group and a wide energy gap is produced after occupation number 50. Similarly the $h_{11/2}$ level of the 5th oscillator shell is pushed down into close proximity with the top level in the 4th oscillator shell leading to a major shell closure at occupation number 82. The important neutron shell number 126 is a result of the depression of the $i_{13/2}$ level from the 6th oscillator shell. Thus the occurrence of all magic numbers can be understood immediately from the spin-orbit interaction without any additional arbitrary assumptions.

The precise order of the levels within the shell cannot be definitely predicted since it depends sensitively on the form of the potential well and on the magnitude of the spin-orbit coupling assumed. The order of filling of the levels has been determined empirically from nuclear decay schemes, the occurrence of isomerism, and other data.[6] It is not possible to secure a proper order of filling or even of major shell closures for a pure square well potential or a pure harmonic oscillator potential. Many authors have introduced a potential of some intermediate "diffuse well" type to get better agreement with the empirical results over the whole range of nucleon numbers. We shall mention briefly two such approaches.

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6. See for example, P. F. A. Klinkenberg, Rev. Mod. Phys. 24, 63 (1952).

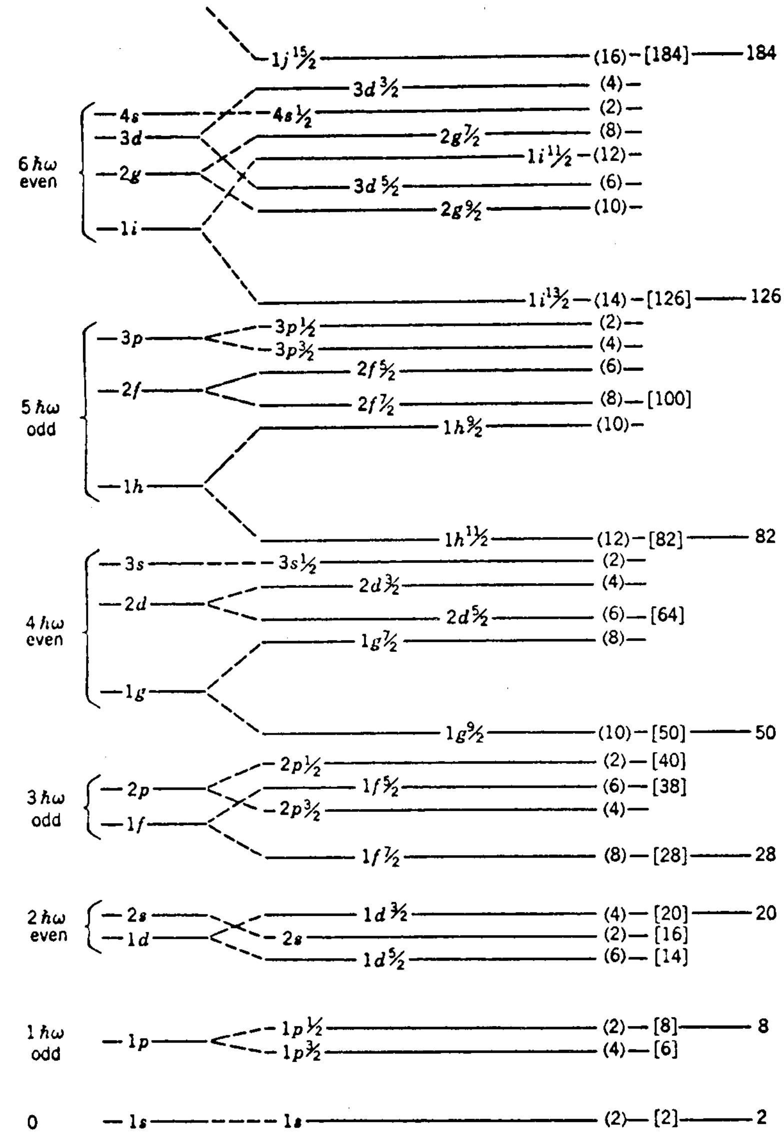

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Fig. 3.2 Reproduced from Mayer and Jensen, Elementary Theory of Nuclear Shell Structure (John Wiley and Sons, New York, 1955). Schematic diagram of nuclear level systems with spin-orbit coupling. Isotropic harmonic oscillator states are given at extreme left. The states resulting from degeneracy removals which occur when a potential intermediate between the harmonic oscillator and a square well (infinite walls) are given immediately to the right of the brackets. The dotted lines lead to the pairs of (n ℓ j) levels resulting from spin-orbit interaction. The numbers on the right hand side give the number of particles in each level, the total occupation number through each (n ℓ j) level, and finally the major shell occupation numbers.

The first approach is that of NILSSON[7] who used a modified harmonic oscillator potential. In going from an oscillator to a square well the potential is raised near the center and lowered just inside the nuclear edge. This has the effect of lowering the energies of those states which have large wave functions at the edge, i.e., of states of large ℓ relative to states of small ℓ. NILSSON[7] simulates this effect by introducing a term in $\overline{\ell}\cdot\overline{\ell}$ in the potential. His choice for a realistic simple nuclear potential is

$$V = 1/2\; m\, \omega_0^2\, r^2 + \hbar\, \omega_0 [D\; \overline{\ell}\cdot\overline{\ell} + C\; \overline{\ell}\cdot\overline{s}] \tag{3.3}$$

where D and C are negative constants whose values are adjusted to fit the empirical data, and s is the spin of the nucleon. The spin-orbit coupling is given by $\hbar\omega_0 C(\overline{\ell}\cdot\overline{s})$ term. Values of C and D can be evaluated separately for each N-shell in the neutron and proton case. This empirical perturbation approach is able to reproduce the major shells and even the order of level filling reasonably well as shown in Fig. 3.3.

A second approach to a realistic diffuse potential is given by ROSS, MARK and LAWSON.[8] These authors used a nuclear potential of the type commonly employed in the optical model of the nucleus. The optical model was developed to explain nuclear cross section data and particularly elastic scattering data. A form of the nuclear potential which has been used by numerous authors is the following:[9]

$$V = \frac{V_0 + i\,\omega_0}{1 + \exp \alpha(r - a)} \quad . \tag{3.4}$$

The imaginary part of the potential is necessary to account for scattering data.

ROSS, MARK and LAWSON carried out shell model calculations using a potential of this type. Since they were interested in calculating bound-state properties it was not necessary to include the imaginary part of the potential. Figure 3.4 shows the shape of the potential adopted by them for the neutron case. Spin-orbit interaction was included in the calculation by introducing the usual Thomas term multiplied by an appropriate constant λ. Therefore in dealing with neutrons the radial Schrödinger equation which must be solved is:

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7. S. G. Nilsson, Dan. Mat.-fys. Medd. 29, No. 16 (1955).

8. A. A. Ross, H. Mark and R. D. Lawson, Phys. Rev. 102, 1613 (1956).

9. See for example the review articles of Ford and Hill, Ann. Rev. Nuclear Science 5, 25 (1955), and of Feshbach, Ann. Rev. Nuclear Science 8, 49 (1958).

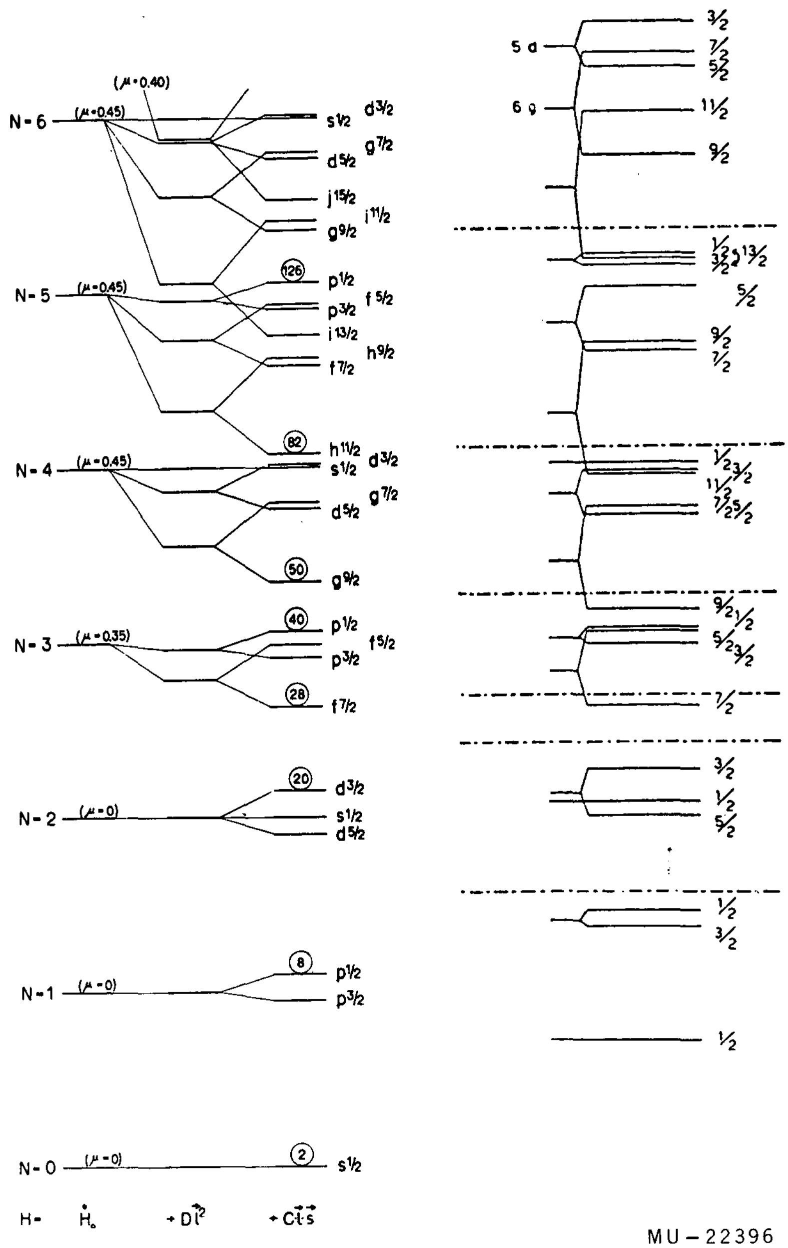

Fig. 3.3 Level order of neutron orbitals as given by NILSSON'S treatment compared to empirical level order. [Klinkenberg, Rev. Modern Phys. 24, 63 (1952).] The oscillator levels are given on the left and the effect of the $\overline{\ell}\cdot\overline{\ell}$ and $\overline{\ell}\cdot\overline{s}$ terms is shown. The quantity μ equals - 20 D as defined in Eq. 3.3 Figure reproduced from article by S. G. NILSSON.[7]

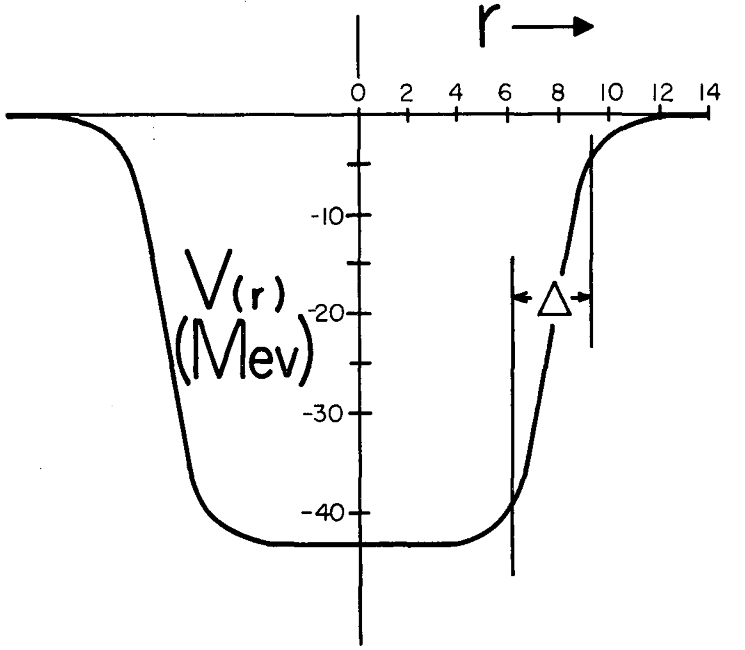

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Fig. 3.4 The potential $V = \frac{V_o}{1 + \exp \alpha(r - a)}$ used by ROSS, MARK and LAWSON shown for a nucleus of mass 208. The optimum parameters for neutrons were $V_o = -42.8$ Mev, $\alpha = 1.45 \times 10^{13}$ cm^{-1} and $a = r_o A^{1/3} = 1.3 \times 10^{-13} A^{1/3}$. The surface layer, Δ, is defined as the distance from the point where the potential has 90% of its maximum value to the point where it has 10%.

$$- \frac{\hbar^2}{2m} \frac{1}{r^2} \frac{d}{dr}\left(r^2 \frac{dR}{dr}\right) + \left(\frac{-V_o}{1 + \exp[\alpha(r - a)]} + \frac{\hbar^2}{2m} \frac{\ell(\ell + 1)}{r^2} \right.$$

$$\left. - \frac{\lambda\hbar^2}{4m^2C^2} \frac{\alpha V_o \exp[\alpha(r - a)]}{\{1 + \exp[\alpha(r - a)]\}^2 \; r} \begin{Bmatrix} \ell \\ -(\ell + 1) \end{Bmatrix} \right) R = ER \quad . \tag{3.5}$$

The operator $\overline{\sigma}\cdot\overline{\ell}$ in the spin-orbit coupling term has been replaced by ℓ and $-(\ell + 1)$, its eigenvalues when operating on a state with $j = \ell + 1/2$ and $j = \ell - 1/2$, respectively.

This equation was solved by a differential analyzer after suitable boundary conditions were set. One of the important boundary conditions was the requirement that the binding energy of the last neutron be made to correspond with the experimental binding energy. The top levels of various representative nuclei were investigated with different values of the parameters. An optimum set proved to be: $V_o = -42.8$ Mev, $\lambda = 39.5$, $\alpha = 1.45 \times 10^{13}$ cm^{-1} and $a = 1.3 \times 10^{-13} A^{1/3}$. The potential V_o could be left constant over the mass range 90 to 208. The top neutron levels for Pb^{208}, Ce^{146}, Zr^{90} and K^{39} are shown in Fig. 3.5.

The level systems shown in this diagram show much more satisfactory agreement with experimental data than do the levels calculated for a harmonic oscillator or square well potential. For example, in the Pb^{208} case we note that the 1 $i_{11/2}$ level lies below the 3 $p_{1/2}$ level in the square well calculation which would lead to a shell closure at 138 neutrons instead of 126. In the diffuse well, however, this level is pushed well above the 3 $p_{1/2}$ state.

When proton single particle states are calculated with the diffuse potential it is necessary to introduce a term in the Schrödinger equation for Coulombic repulsion. ROSS, MARK and LAWSON[8] did this by assuming a repulsion equivalent to that for a uniform charge distribution which extends out to a nuclear radius a. It is also necessary to use a stronger V_o potential for protons in order to avoid a large neutron excess. V_o was taken to be about 52 Mev but had to be varied slightly from nucleus to nucleus to give satisfactory agreement with proton binding energies. The constants α, λ and r_o were the same as in the neutron case.

We shall make no attempt to review the many other publications in which various forms of a central potential and of a spin-orbit force are applied in an attempt to get the best agreement of the shell model with experiment over a wide range of nuclei or over a more restricted range. The treatment we have

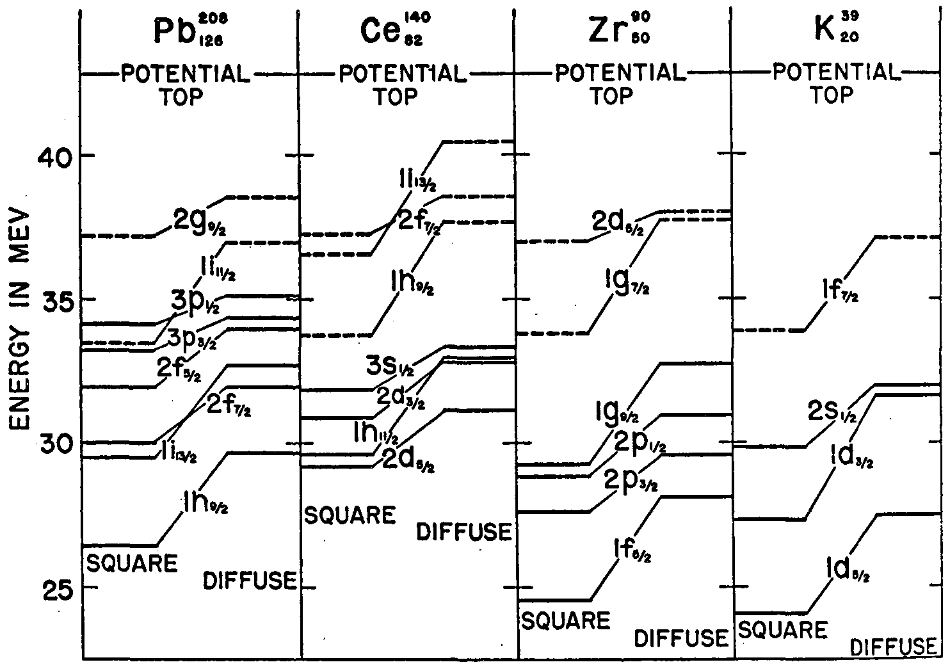

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Fig. 3.5 Top neutron levels for N = 20, 28, 50, 82 and 126 as calculated by ROSS, MARK and LAWSON[8] for a diffuse potential

$$V = \frac{V_o}{1 + \exp \alpha(r - a)} .$$

The shift in levels from a square well is shown. The dotted lines indicate unfilled levels.

given to this point satisfies one of the main purposes of this chapter. We have seen that a nuclear model based on the motion of a nucleon in an average isotropic field of quite simple form generated by the other nuclear constituents leads quite naturally to magic number variations in nuclear properties. To get agreement with empirical facts it is necessary to include strong spin-orbit interaction and to use a nuclear potential with a diffuse edge.

The theory in this simple form is quite useful in the heavy element region for explaining general phenomena such as the great nuclear stability of Pb^{208} and the sudden changes in nuclear binding and in available energy for radioactive decay for nuclei in the neighborhood of Pb^{208}. Repeated use is made of the model in explaining these general trends in other chapters of this book.

For nuclei in the neighborhood of Pb^{208} the single particle form of the shell model can be used with some success to predict the spin and parity of the ground state and the low-lying excited states. In the strictest sense the single-particle model should only be applied to those nuclei with a single proton or a single neutron in excess of a closed shell core. (This can be broadened to include nuclei in which the neutron number or proton number is one less than a closed configuration.) Since this book is concerned only with nuclei of lead or of heavier elements the only nuclei which fit these restrictions are Pb^{207}, Pb^{209}, Tl^{207}, and Bi^{209}. However, the single particle shell model is often extended to cover all nuclei of odd-mass not-too-far removed from a closed shell configuration by making the additional assumption that all particles except the last odd particle are paired off to form an "inert core" of angular momentum zero. According to this assumption, those "loose" nucleons beyond the closed shell which are paired off contribute nothing to the observed properties but only contribute to the average potential in which the odd particle moves. When the model in this form is applied to even-even nuclei the only prediction is that the ground state has spin 0 and even parity. The prinicpal prediction of the model for odd-odd nuclei is that the properties will be determined solely by the last odd neutron and the last odd proton.

The independent-particle form of the shell model has had many successes in explaining the properties of nuclei lying close to Pb^{208}. Bismuth-209 has a measured ground-state spin of 9/2. This is in agreement with the model prediction that the 83rd proton should be in the $1\ h_{9/2}$ orbital. Lead-207 contains one neutron hole in the 126 neutron configuration. It has a measured ground state spin of 1/2 which is in agreement with the predicted assignment of $3\ p_{1/2}$

for one of the levels available just below the 126 neutron shell. Excited levels are known to exist at 569, 900, 1633 and 2350 kev. These are listed in Table 3.2 together with the experimental spin and parity assignments. These assignments are based firmly on a variety of experimental evidence reviewed in Chapter 10. A particularly strong piece of evidence is the occurrence of an isomeric form of Pb^{207} which decays to the ground state with a cascade of gamma rays consisting of a 1.065 MeV M4 transition and a 569 kev E2 transition. This cascade identifies without much question a 13/2 state at 1633 kev. It is quite satisfying that 5 of the 6 neutron orbitals predicted by the shell model between N = 82 and N = 126 correspond in spin and parity with these experimental levels. This correspondence is shown in Table 3.2. Furthermore, there are no additional experimental levels.

In the case of the lighter odd-mass isotopes of lead with three, five, seven and nine neutrons less than a closed configuration, the simple form of the shell model predicts that the low-lying states will be the same as in Pb^{207}. This is in agreement with experiment to the extent that states of the same spin and parity assignment are identified. One striking feature is the systematic occurrence of an isomer I = 13/2 state decaying by an M4 transition to a 5/2 state. This is shown in Fig. 3.6A.

The simple form of the shell model cannot explain the shifts in the spacings of the levels as the neutron deficiency from the closed shell configuration is increased. Nor can it explain the curious absence of an isomeric form of Pb^{205} with a half-life greater than one second, analogous to those found in the other odd-mass isotopes.

The simple form of the independent particle model makes many other detailed predictions of properties which can be compared with experiment. Some of these other properties provide more sensitive tests of the success of the model than do the correct assignments of spin and parity to the observed levels. For example, by use of explicit single-particle wave functions it is possible to calculate such quantities as the magnetic moment, the electric quadrupole moment, and the transition probability for a transition between two single particle states of a given nucleus. Expressions for these quantities are given by ELLIOTT and LANE.[9a]

9a. J. P. Elliott and A. M. Lane, see pp. 252-257 of Ref. 2.

Table 3.2 Experimental Levels of Pb^{207} and Shell Model Assignments

Energy above ground (kev)	Spin and parity	Shell model hole assignment
0	1/2-	$p_{1/2}$
569	5/2-	$f_{5/2}$
900	3/2-	$p_{3/2}$
1633	13/2+	$i_{13/2}$
2350	7/2-	$f_{7/2}$

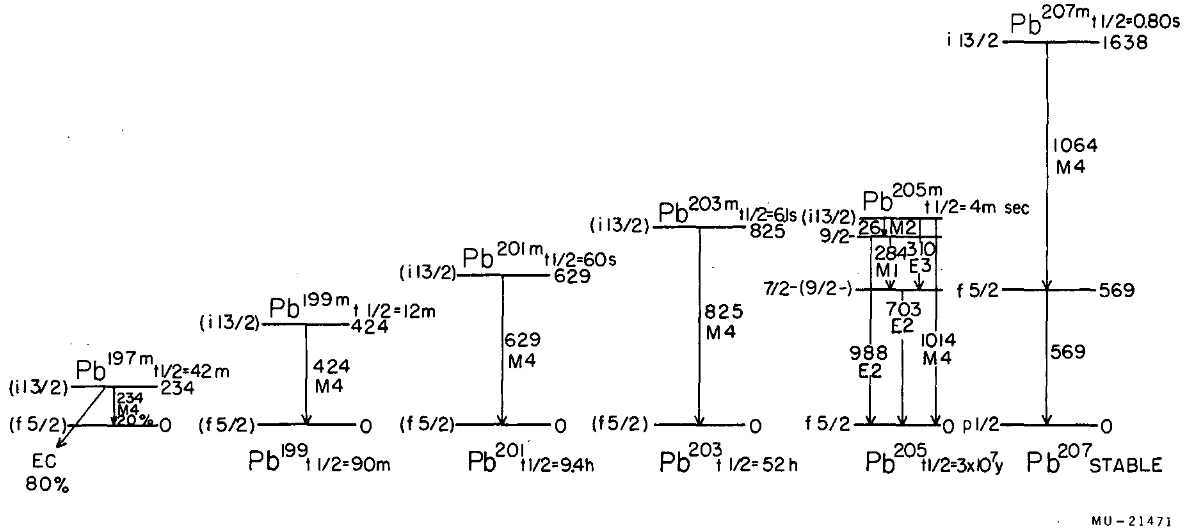

Fig. 3.6A Isomeric states of odd mass lead isotopes showing systematic occurrence of $i_{13/2}$ state.

3.1.3 The Many-Particle Shell Model. In more sophisticated treatments of the shell model the existence of all the particles beyond the closed shell core is explicitly acknowledged and an attempt is made to describe the way in which these particles couple together. It is necessary also to consider the effects of the small residual interparticle forces on the states resulting from this coupling of the "loose" particles. If we consider a group of k particles with quantum numbers n, ℓ and j where n refers to the oscillator shell and ℓ and j have their usual meanings, then the configuration

$$(n,\ \ell,\ j)^k$$

will correspond to a degenerate group of levels if residual interparticle forces are absent. In the presence of these forces these degeneracies will be lifted.

Because of the complexity of the interparticle forces all attempts to calculate a spectrum of levels for a collection of two or more shell model particles have been based on simplified formulations of these forces. Let us outline in a few words some of the treatments which have been applied to this problem for nuclei in the heavy region (chiefly nuclei near Pb^{208}).

Most of these treatments have been tested out on Pb^{206} which is only two particles removed from a double-closed shell. This level structure of Pb^{207} is well known, as mentioned above, and these level energies can be used to set the unperturbed values of the single-particle states. To get the levels of the neighboring nucleus Pb^{206} one adds the energies of the unperturbed single-particle states, calculates a correction from the action of an assumed interparticle force, and compares the resulting spectrum of states with the observed spectrum. A big advantage of the Pb^{206} case is that there exists a detailed knowledge of its level scheme from a study of the complex decay of Bi^{206}.

ALBURGER and PRYCE[10] treated this problem by assuming that the interaction between the two neutron hole states could be satisfactorily represented by a pure singlet zero-range force. They estimated interaction parameters by a rough empirical method. Configuration interaction was not allowed for in this treatment. However, in the few cases where the configuration interaction of levels of the same spin and parity was believed to be important this effect was allowed for by an additional empirical adjustment. Despite the approximations of this method the results were highly encouraging. Nearly all of the many experimental

10. D. Alburger and M. H. L. Pryce, Phys. Rev. 95, 1482 (1954).

levels of Pb^{206} were in agreement in energy, spin and parity with the predictions of this semi-empirical model. Particularly impressive was the predictions of the existence of an isomeric state at 2.2 Mev with a spin assignment 7, negative parity. This provided encouragement that a short-range force could account well for the main features of a many-particle spectrum.

This semi-empirical approach was extended by TRUE[11] with some success to cover the case of Pb^{204} which has a 4-neutron deficiency in the neutron shell. It was also applied by PRYCE[12] and by TRUE[13] to the case of Pb^{205}.

A more rigorous calculation of the Pb^{206} spectrum was carried out by KEARSLEY[14] and by TRUE and FORD.[15] The former assumed singlet-even and triplet-odd forces in the ratio - 1 to + 0.559 and a two-body potential shape as given by Yukawa. The latter authors assumed pure singlet forces and a Gaussian form for the potential shape. Both studies resulted in predictions of spins, parities and energies of the numerous levels lying below 4 Mev. Configuration interaction was computed and exact wave functions were given for all levels. From these exact wave functions it was possible to compute other quantities such as gamma transition rates. Discrepancies in these comparisons were in a direction to indicate that some collective effects were contributing to the observed levels.

The chief type of collective motion which needs to be considered is a quadrupole vibration about an equilibrium spherical shape, i.e., a motion which involves excursions into a spheroidally shaped nucleus. In nuclei which are many nucleons removed from the closed shells the nucleus is rather "soft" toward such vibrations and the quantum states corresponding to them are a prominent part of the low energy spectra in even-even nuclei as is shown later on in this chapter. The nuclei of odd mass which are neighbors to this group of even-even nuclei show clearly the effects of a weak coupling of the surface oscillations with the single particle states of the odd particle;[16] these effects are apparent in the shift in energy of the single particle states and the appearance of new states.

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11. W. W. True, Phys. Rev. 101, 1342 (1956).
12. M. H. L. Pryce, Nuclear Phys. 2, 226 (1956).
13. W. W. True, Nuclear Phys. 25, 155 (1961).
14. M. J. Kearsley, Nuclear Phys. 4, 157 (1957).
15. W. W. True and K. W. Ford, Phys. Rev. 109, 1675 (1958).
16. An example of this behavior in the isotopes of xenon and tellurium is discussed by N. K. Glendenning, Phys. Rev. 119, 213 (1960).

In nuclei close to a closed shell configuration, such as the lead nuclei, these vibrational states are expected to lie somewhat higher and are not so easily identified among the low-lying states. It is still to be expected that there will be some weak coupling between the single particle states and the first quantum (phonon) of vibrational excitation. This coupling will cause some shift in the energy of the low-lying single particle states, some enhancement of electric quadrupole transition probabilities, and some increase in the quadrupole moment. This will be true both in odd-A and even-A nuclei.

Evidence for such effects has been reported in the lead isotopes. For example, in Pb^{207} the transition probability of the electric quadrupole transition $f_{5/2} \rightarrow p_{1/2}$ has been determined by measuring the Coulomb excitation cross section of the first excited state.[17] The measured transition probability is many orders of magnitude larger than would be expected for a neutron transition; the probability of the neutron transition is close to zero. Even a pure proton transition would not go as fast as the measured rate. The conclusion is that collective effects account for the observed E2 transition probability.

Similarly in Pb^{206} the transition probability for the transition between the first excited state (a 2+ state) to the ground state has been found to be much larger than was expected from the single particle composition of these two states as determined by the analysis of PRYCE[10] or of KEARSLEY[14] or TRUE and FORD[15] based on pure short-range interactions of the neutron hole states. Other electric transitions which occur in the decay of Bi^{206} (Pb^{206} levels) also show enhanced transition rates.

TRUE and FORD[15] used the observed rate of the E2 transition between the first excited state and the ground state of Pb^{206} to estimate the strength of the particle-core coupling. They then recalculated all the low-lying states of Pb^{206} for a model consisting of two interacting neutron states, with unperturbed energies identical to the states identified in the spectrum of Pb^{207}, interacting with each other through a short-range singlet force and in addition interacting with the collective vibrational motion of the core. By adjusting the strength of the singlet forces to a value 75 per cent of that used in their previous calculations, in which the particle-core coupling was ignored, they were able to compute spins, parities, and energy values for a large number of levels. The predictions with particle-core coupling included were in better agreement with

17. P. H. Stelson and F. K. McGowan, Phys. Rev. 99, 112 (1955).

the experimental levels than were the predictions which ignored this feature. The eigenfunctions showed a somewhat different mixing of configurations in the predicted levels in the two cases.

Even in Pb^{208} ELLIOTT and CO-WORKERS[18] found evidence for collective motions in the unexpectedly high transition rate of the E3 decay of the 3- level.

Special mention may be made of the anomalous value of the magnetic moment of Bi^{209}. The experimental value of this quantity is + 4.08 nuclear magnetons which is 1.5 units larger than predicted by the single-particle shell model. This is a surprising result since Bi^{209} has only one proton past Pb^{208} and one would predict for its ground state a pure $h_{9/2}$ proton configuration, which prediction is in agreement with the measured value of 9/2 for the spin. More detailed analysis[19] has shown that one cannot entirely neglect the interaction of particles beyond the core with particles in the closed shell core. In this case the wave function of the ground state includes a few per cent admixture of a configuration which represents the interaction of the $h_{9/2}$ proton with the protons in the core resulting in the elevation of an $h_{11/2}$ proton from the core to the $h_{9/2}$ level. While the resulting impurity of the ground state of Bi^{209} is not large it can induce a large magnetic moment in the core.

At a later date a newer theoretical technique for the treatment of residual interparticle forces in a many-particle shell model was developed which has the promising feature of permitting easier computation of the n-particle problem when n is greater than 2. The new model is patterned on the theory of superconductivity introduced by BARDEEN, COOPER and SCHRIEFFER.[20] It makes use of a simplified short-range interaction known as the "pairing" force which can be roughly described by the statement that nucleons prefer to be paired off in states of opposite angular momentum projection. The pairing force can be thought of as a generalization of an interaction operator introduced by RACAH,[21] which was shown to lead to the seniority coupling scheme for degenerate $(j)^n$

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18. Elliott, Graham, Walker and Wolfson, Phys. Rev. 93, 356 (1954).

19. R. J. Blin-Stoyle, M. A. Perks, Proc. Phys. Soc. A67, 885 (1954).

20. J. Bardeen, L. N. Cooper and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957).

21. G. Racah, Phys. Rev. 63, 367 (1943).

configurations. The new scheme generalizes the seniority concept in terms of "quasi particles." The calculation involves a canonical transformation from the original system of interacting nucleons to a new system of non-interacting quasi-particles.

The application of the new theory to nuclei in general has been discussed by BELYAEV[22] and by MOTTELSON[23] among others. A specific application of this model to the mass number range of interest to us here has been made by KISSLINGER and SORENSEN[24] who applied it to all the lead isotopes below Pb^{208}.

This is all that will be said in this chapter concerning the application of many-particle shell model calculations to nuclei in the heavy element region. The decay schemes of lead and bismuth isotopes are discussed rather completely in Chapter 10. In that chapter some of the detailed results of such calculations are compared with experimental data.

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22. S. T. Belyaev, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 31, No. 11 (1959).

23. B. R. Mottelson, Lecture course, University of California, Berkeley (1959), unpublished.

24. L. S. Kisslinger and R. A. Sorensen, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 32, No. 9 (1960).

3.2 Introduction to the Unified Nuclear Model

3.2.1 The Unified Model as an Extension of the Shell Model. The shell model of the nucleus which was reviewed in the earlier part of this chapter considers the motions of individual nucleons in an isotropic average nuclear field generated by the other nucleons in the nucleus. The shell model, in many respects, resembles the model of atomic structure which accounts for the periodic variation in properties of the chemical elements; but the analogy to atomic structure is not complete. The atomic field is governed by the electrical attraction of the heavy central nucleus to the light extra-nuclear electrons. Because of the vast difference in the masses of the two attracting bodies the atomic field can be treated as static. In the nuclear case the attractive potential is generated by the nucleons themselves, many of which travel in orbitals which deviate markedly from spherical symmetry. In many nuclei the existing combinations of nuclear orbitals give rise to a spherical overall distribution of nuclear matter, but in other nuclei the overall distribution may be expected to deviate from spherical symmetry. In this latter case the average field seen by any individual nucleon will not be correctly given by an isotropic average nuclear potential. Furthermore, the cooperative motions of many nucleons may result in collective oscillations of the nucleus as a whole about some equilibrium shape and these collective oscillations may be expected to play an essential role in the low energy spectra of the nucleus.

It was the recognition that collective motions must play an essential role in the theory of the nucleus that led N. BOHR and F. KALCKAR[25] in 1937 to the development of the liquid drop model, a model which BOHR and WHEELER[26] in 1940 were able to use so successfully in explaining many features of the fission reaction.

The shell model and the liquid drop model represent quite distinct approaches to an explanation of nuclear structure. Both models explain certain types of nuclear phenomena quite well, but both have distinct limits on their areas of application. In recent years several attempts have been made to develop a model retaining the main features of both approaches. The description of the nucleus as a shell structure was retained but the assumption of a static, isotropic nuclear field in which the outer nucleons move was replaced by a nuclear potential which allows for variations in the nuclear field caused by oscillations in the

25. N. Bohr and F. Kalckar, Dan. Mat.-fys. Medd. 14, No. 10 (1937).

26. N. Bohr and J. Wheeler, Phys. Rev. 56, 426 (1940).

shape and changes in the spatial orientation of the nucleus. The particle motion is affected by the nuclear oscillations and the nuclear oscillations in turn are affected by the particle motion. The mathematical problem is the calculation of the interaction of these two intrinsically different types of motion and the effects which these interactions have on such nuclear properties as spin, parities, quadrupole moments, magnetic moments, etc. We shall refer to this nuclear model as the Unified Nuclear Model.[27] This system has many features analogous to molecular structure with its interplay between electronic and nuclear motion.

One of the first physicists to recognize the importance of the interaction of individual particle and collective motions was RAINWATER[28] who concluded from a study of the quadrupole moments that the particle structure was capable of deforming the nuclear surface. HILL and WHEELER[29] have made an extensive theoretical treatment of a unified model of the nucleus. A pioneering treatment of the interactions of intrinsic and collective motions of the nucleus was carried out by A. BOHR.[30,31] Bohr's treatment has been developed and exploited with great success by A. BOHR and B. R. MOTTELSON[32-34] and by their many collaborators working at the Institute for Theoretical Physics at Copenhagen. Many of the important papers are listed in the bibliography at the end of this chapter.

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27. The term "rotational model" has also been used when the model is used exclusively for the description of spheroidal nuclei which exhibit characteristic rotational spectra. Also, the term "collective model" has been used when only collective motions involving the nucleus as a whole are considered.

28. J. Rainwater, Phys. Rev. 79, 432 (1950).

29. D. L. Hill and J. A. Wheeler, Phys. Rev. 89, 1102 (1953).

30. A. Bohr, The Coupling of Nuclear Surface Oscillations to the Motion of Individual Nucleons, Dan. Mat.-fys. Medd. 26, No. 14 (1952).

31. A. Bohr, Rotational States of Atomic Nuclei, Eigner Munksgaards, Copenhagen, 1954.

32. A. Bohr and B. R. Mottelson, Collective and Individual-Particle Aspects of Nuclear Structure, Dan. Mat.-fys. Medd. 27, No. 16 (1953).

33. A. Bohr and B. R. Mottelson, Collective Nuclear Motion and the Unified Model, Chapter XVII, Beta and Gamma Ray Spectroscopy, North Holland Publishing Co., Amsterdam, 1955, Kai Siegbahn, Editor. This is a short review of the model.

34. K. Alder, A. Bohr, T. Huus, B. R. Mottelson, and A. Winther, Rev. Modern Phys. 28, No. 4, 523-542 (1956). See especially Chapter V of this paper for a concise review of the model.

In this chapter we shall present a brief outline of those conclusions from the theory as developed by the Bohr-Mottelson group which are of particular importance for a description of the properties of the heaviest elements. The unified model has had many striking successes in the heavy element region and a knowledge of its chief features is essential for an understanding of the decay characteristics of isotopes of these elements.

3.2.2 General Considerations of Collective Motions. The shell model in its simple form is an approximation in that it ignores important nucleonic interactions which are not contained in the average field. If the nucleus had the amorphous structure implied by the term liquid drop, the nucleus would always have its lowest energy for a spherical shape. But the individual nucleons have the characteristic properties given by the shell model which implies a systematic tendency for distortion of the nuclear shape. The basic mechanism for this distortion lies in the effect first pointed out by Rainwater. A single nucleon moving within the nucleus exerts a centrifugal pressure against the walls of the nucleus in its orbital plane and tends to produce an oblate deformation in the nuclear surface. When the nucleus has a closed shell configuration the deforming effects of many nucleons cancel out because the orbitals are oriented equally in all directions. When there are particles not in filled shells the tendency is for the nucleus to adjust its surface to coincide with the density distributions of these particles. If there were no opposing force this centrifugal pressure would result in a nucleus with a space distribution equivalent to the nucleons in unfilled shells. There are, however, two effects working in the opposite direction. One is the difficulty of polarizing the closed shell core which strongly prefers spherical symmetry. The second is the pairing forces of the extra-core nucleons. When a nucleus has only a few nucleons beyond a closed shell these effects overbalance the distorting effect of the last odd nucleon and the nuclear equilibrium shape remains spherical. However, the nucleus does become less resistant to shape changes. This softening becomes evident in the decrease in the energy involved in collective vibrations about the spherical equilibrium shape.

When sufficiently many particles are added outside closed shells the spherical shape becomes unstable and the nucleus assumes a spheroidal equilibrium shape. When this occurs, the collective motions of the nucleus will be of two types: rotational and vibrational.

It is possible to approximate these effects by replacing the spherically symmetric binding potential of the simple shell model with an adjustable anisotropic binding potential. We shall discuss later in this chapter some single particle wave functions calculated by Nilsson for a specific choice of an anisotropic potential.

There remain, however, some significant residual interactions. These are the pairing forces between the nucleons outside the closed shells. These forces tend to couple two equivalent nucleons to a state of zero angular momentum and thus counteract the tendency of the individual nucleons to deform the nuclear shape.

The most important collective degrees of freedom for the low energy nuclear properties are expected to be those associated with oscillations in shape with approximate preservation of the nuclear volume. The nuclear shape can be expressed in spherical harmonics as follows:

$$R(\theta\varphi) = R_o \{1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta\varphi)\} \quad (3.6)$$

where R_o is the equilibrium radius and Y_λ is the normalized spherical harmonic of order λ_μ.

If we make the assumptions that the constants α_λ are small and that the frequencies of single particle excitations are much greater than those involved in collective motions, we can separate the total wave function into a part describing the particle motion and a part describing the collective motion. An approximate expression for the Hamiltonian specifying the collective motion is of the following form:

$$H_{collective} = V_n(\alpha) + T_n(\dot{\alpha}) \quad . \quad (3.7)$$

Here $V_n(\alpha)$ refers to the potential energy of the nucleus as a function of the shape defined by the coefficients α. The subscript n refers to the group of quantum numbers specifying the motions of all the particles in a nucleus with shape defined by the $\alpha_{\lambda\mu}$. The second term gives the kinetic energy involved in small oscillations of the nuclear shape. The $\dot{\alpha}$ is the time derivative of α. The predominant term is quadratic in the α and the normal modes of vibration are of the harmonic oscillator type. In general, the oscillations in shape of lowest order, $\lambda = 2$, are of primary importance. A rigorous calculation of the frequencies of oscillation is difficult but order-of-magnitude estimates indicate that the energy of excitation is of the order of one Mev.

The model depends in an important way on the potential energy function $V_n(\alpha)$. The variation of $V_n(\alpha)$ with nuclear shape is determined by the number of particles outside the closed shell, the particular orbitals which they fill and the residual nucleon-nucleon interactions of these nucleons. It is instructive to note how this function varies as one moves away from a closed shell configuration. A schematic representation of this is given in Fig. 3.6B. From the figure we see that for a nucleus at a closed shell the interparticle forces of the core nucleons result in a strong preference for spherical symmetry, shape changes are firmly resisted, and the potential energy curve rises steeply as a function of small deformations. When a few additional particles are added these tend to polarize the nucleus and make deformations from the spherical shape become more favorable energetically. However, as has been stated above, the resistance of the closed shell core to polarization and the pairing forces between the extra-core nucleus restrain the nucleus to a spherical ground state and the main effect of the polarizing tendency is to reduce the stiffness toward shape changes.

As more and more particles are added beyond the closed shell configuration, the coherent effects of many particles ultimately bring about a stabilized deformation of the nucleus in which a potential energy minimum exists for a non-spherical shape. The motion of the loose particles in unfilled shells is strongly coupled to the nuclear surface. A stabilized non-spherical shape can be considered to be achieved when the shape changes associated with zero point vibrations are small compared to the equilibrium deformation. For non-spherical nuclei the collective excitations include not only vibrational oscillations but changes in orientation without change in shape--that is to say, rotational excitation. Most heavy nuclei of mass number greater than 225 are non-spherical in their ground state. Most of the success of the unified nuclear model has been in the description of the nuclear properties of such deformed nuclei.

The gain in binding energy due to deformation from the spherical shape is very substantial. For the most strongly deformed nuclei in the rare earth elements this binding energy gain is estimated as about 20 Mev.[35] An example of a quantitative calculation of the energy of stabilization of the spheroidal over the spherical nucleus is given in Fig. 3.7.

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35. Private communication with S. G. Nilsson; see also B. R. Mottelson and S. G. Nilsson, Mat. Fys. Skr. Dan. Vid. Selsk. 1, No. 8 (1959).

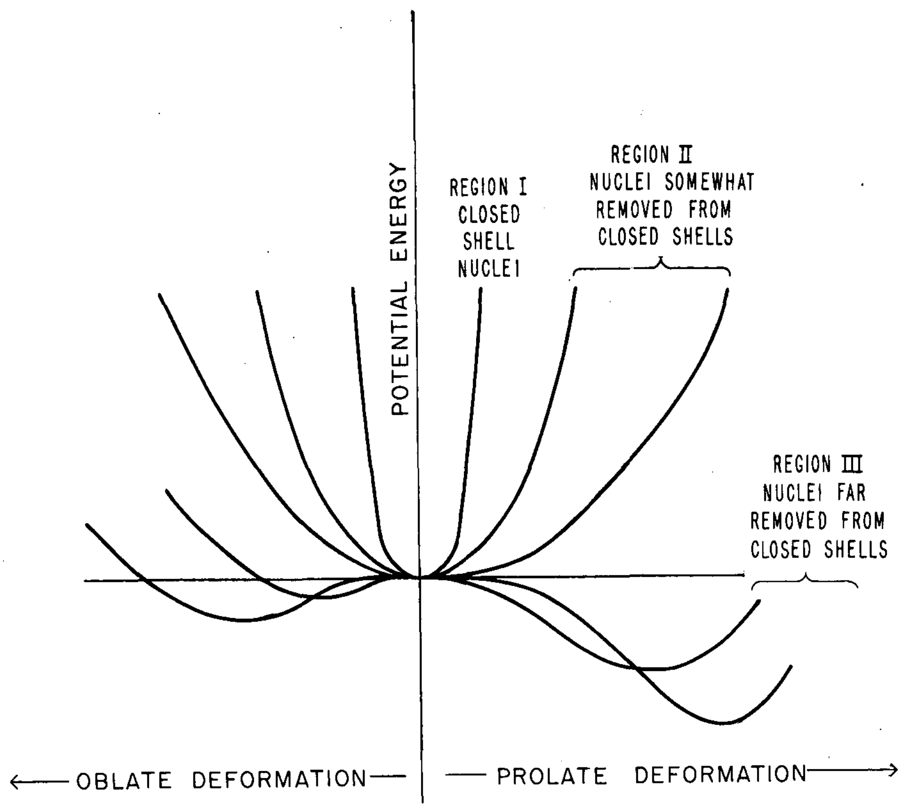

Fig. 3.6B Potential energy V(α) as a function of deformation for even-even nuclei near closed shells, a few nucleons removed from closed shells, and far removed from closed shells where the nucleus is stabilized in a non-spherical shape. The curves are schematic only and all refer to the potential energy of the ground state.

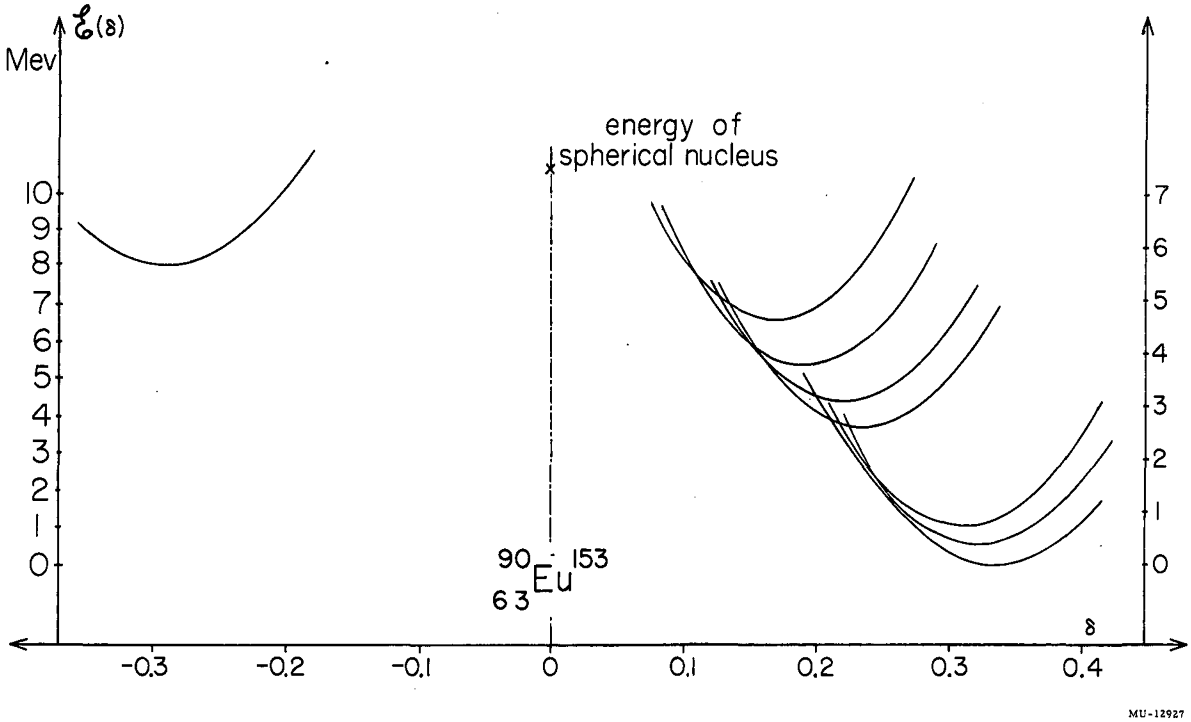

Fig. 3.7 Calculation of energy stabilization of Eu^{153} in a non-spherical equilibrium shape. This drawing was prepared by S. G. NILSSON and is based on the NILSSON wave functions discussed in Sec. 3.4.5 below. The different curves represent different choices from the available single particle configurations. The curves to the left of sphericity refer to oblate spheroidal deformation while those to the right refer to prolate deformation. Prolate deformation is strongly preferred. The deformation parameter δ is defined as the quotient of the difference of the major and minor axes of the spheroid and the average value of the nuclear radius.

3.3 The Unified Model in the Region of Spherical Nuclei

3.3.1 Empirical Evidence for Collective Effects. For nuclei very close to closed shells it is satisfactory for many purposes to apply the isotropic shell models and ignore collective effects. But as has been mentioned above in Sec. 3.1.3, there are many pieces of experimental information in the lead isotope which show that collective effects are of some importance even for such nuclei. Coupling of particle motion to motion of the core leads to:

1) induction of quadrupole motion and enhancement of quadrupole transitions.

2) induction of effective forces between two nucleons resulting from the excitation of the core by one nucleon, which excitation then operates as the second nucleon.

As more nucleons are added to the closed shell configurations we reach an intermediate group of nuclei corresponding to the group labeled region II in Fig. 3.6B. These nuclei differ from those lying close to the closed shells in that the favored spherical shape is relatively easily deformed and collective excitations involving modes of vibration about the spherical shape can occur. On the other hand, they also differ from the nuclei to be discussed below in Sec. 3.4 which are stabilized in a non-spherical ground state and exhibit rotational spectra for their lowest-lying states.

Only a beginning has been made on the classification of single particle states for nuclei in this region either experimentally or theoretically. For odd A nuclei one might expect very strong interaction of intrinsic single particle motions and collective motions of the entire nuclear core but it is hard to estimate the extent of this interaction and to predict energy levels or patterns of energy levels. Hence we shall make no attempt in this chapter to discuss energy levels of odd A nuclei in the intermediate group of nuclei which in the heavy element region would include those with mass number ~ 211 to ~ 223.

In the case of the even-even nuclei there is rather good evidence that the lowest-lying excited levels correspond to collective oscillations about the spherical equilibrium shape. In the heavy element regions the most clearly delineated cases occur in the even-even nuclei of element 86. The energy levels observed in these nuclei fit the predictions of a harmonic oscillator model of such oscillations.

It will be useful at this point to review briefly the experimental data on the low-lying states of even-even nuclei. Various investigators[36,37] have analyzed the experimental data on the lowest states of excitation in the complete system of even-even nuclei and have noted several striking regularities.

(1) The ground state invariably has the designation 0+. The first excited state has character 2+ with very few exceptions. The second excited state has the designation 0+, 2+, 4+, or, in a very few cases, has odd spin odd parity.

(2) The energy of the first excited state is correlated with proton and (especially) neutron number and passes through distinct maxima at closed shells. See Fig. 3.8.

(3) Particular classes of even-even nuclei exhibit further characteristic regularities. Nuclei which are located very far from closed shells have particularly low-lying first excited states. Furthermore for such nuclei the ground state and first few excited states have the character 0+, 2+, 4+, ... and the energies follow the rotational spectrum law, $E = \text{constant}\ I(I + 1)$. The ratio of the energy of the second excited state to the first is 3.33. These nuclei fall in the group of non-spherical nuclei to be discussed in Sec. 3.4 below.

(4) Nuclei immediately adjacent to the non-spherical nuclei, i.e., the intermediate group of nuclei we are primarily concerned with in this section, display somewhat higher energies for their first excited states. The character of the low-lying states commonly follows the sequence 0+, 2+, 2+.

(5) Furthermore, the ratio of the energies of the second excited states (E_2') to that of the first (E_2) ranges between 2 and 2.5. GOLDHABER AND WENESER[38] emphasized the separation of the even-even nuclei into two groups by the figure shown here as Fig. 3.9. The group b in this figure corresponds to our intermediate group. Another plot which emphasizes this separation is shown in Fig. 3.10.

(6) The first and second excited states decay predominantly be E2 radiation. The E2 transition probabilities are much greater than those expected for single particle transitions which emphasizes the cooperative nature of these excitations.

(7) When the 0+, 2+, 2+ sequence appears, the E2 crossover transition (second excited 2+ to 0+ ground state) occurs with much smaller probability than the upper transition (second 2+ to first 2+ state). The transition (first 2+ $\longrightarrow$ 0+) proceeds to E2 radiation while the upper transition (2+ $\longrightarrow$ 2+) proceeds by E2 with a small admixture of M1.

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36. M. Goldhaber and Sunyar, Phys. Rev. 83, 906 (1951).

37. G. Scharff-Goldhaber, Phys. Rev. 90, 587 (1953).

38. Scharff-Goldhaber and Weneser, Phys. Rev. 98, 212 (1955).

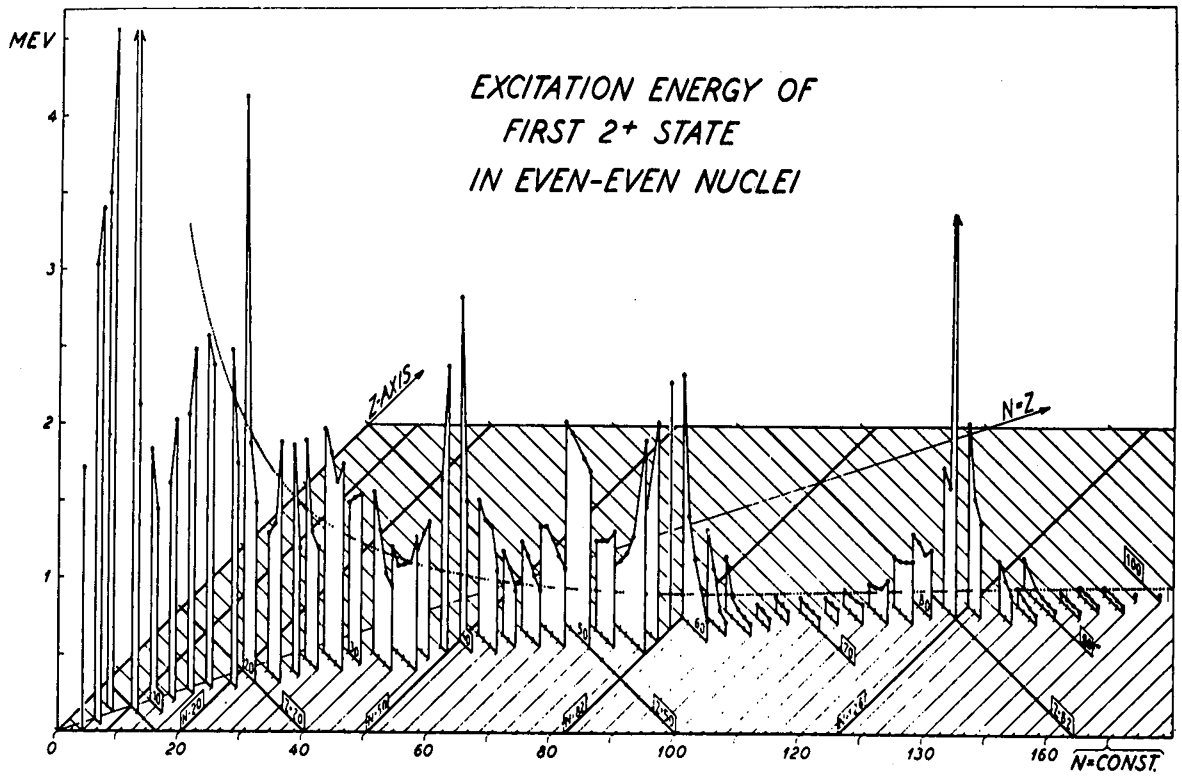

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Fig. 3.8 Energy systematics of first excited (2+) states in even-even nuclei. The energies of the first excited (2+) states are plotted as a function of neutron number N and proton number Z. Note the strong maxima at the double closed-shell positions such as at Pb^{208} (N = 126, Z = 82). The rotational spectra (spheroidal nuclei) occur in the regions furthest from closed shells where the excitation energies are lowest, in other regions the excitations have the character of collective vibrations. The separation between the two regions is given by a rough theoretical criterion (not discussed) whose result is represented by the dotted curve following the stable mass region. Thus the rotational spectra are found in the regions where the observed first excited states have energies less than this separation line. This figure is taken from ALDER et al.[44]

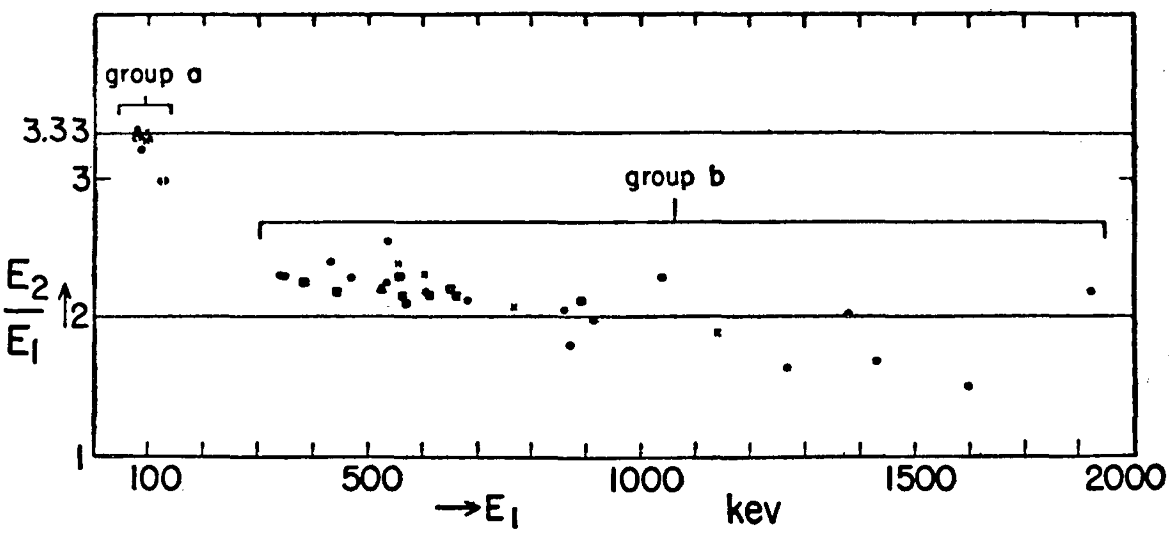

Fig. 3.9 Ratio of energies of the second and first excited states of even-even nuclei as a function of the energy of the first excited state for $36 \leq N \leq 108$. The character of the second excited state is denoted by ▲ (0+), ■ (2+), × (4+), and • (not known). Group a corresponds to a spheroidal nuclei with rotational spectra. Group b probably corresponds to vibrational excitation of spherical nuclei in the intermediate (near-harmonic) region. Figure taken from SCHARFF-GOLDHABER and WENESER.[38]

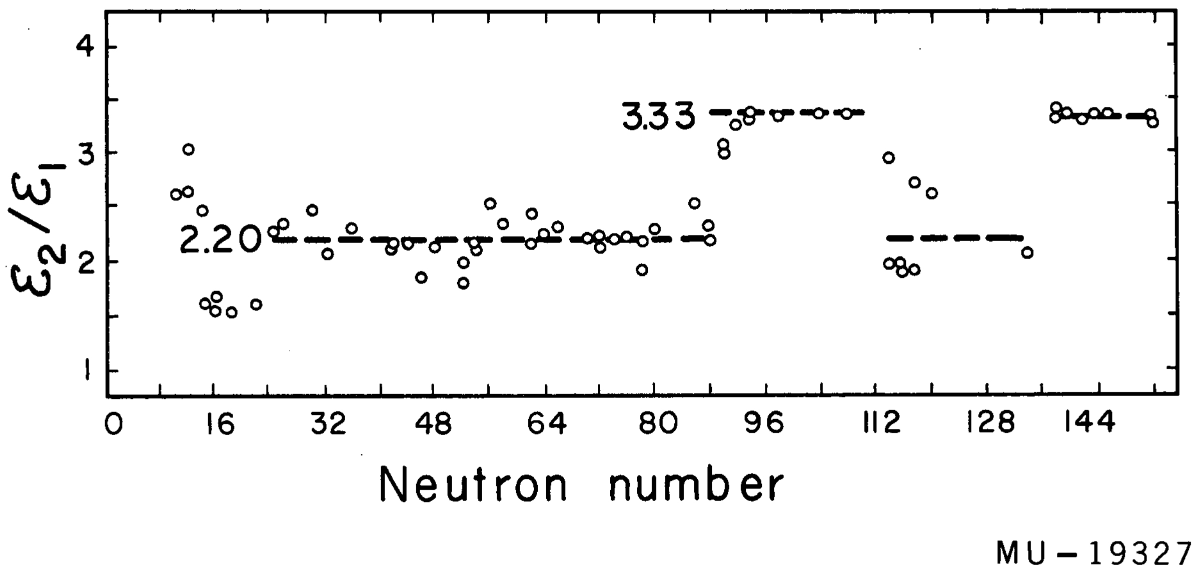

Fig. 3.10 Ratio of the energy of the second excited state to the first excited state as a function of the number of neutrons in even-even nuclei. The ratio 3.33 corresponds to rotational excitation of deformed nuclei. Figure prepared by H. E. Bosch and patterned after an earlier figure by Way et al., Ann. Rev. Nucl. Sci. 6, 129 (1956).

It is natural to associate this systematically occurring pattern of 0+, 2+, 2+ states with a collective oscillation of the nuclear shape around a spherical or nearly spherical ground state. From very simple assumptions one would predict harmonic oscillator-type spectra.

3.3.2 Classical Theory of Collective Shape Oscillations.[39] In the idealized case of a nucleus with constant density and a sharp surface the nuclear surface would be defined by the $\alpha_{\lambda\mu}$ in the equation,

$$R(\theta\varphi) = R_o \{1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta\varphi)\} \quad . \tag{3.8}$$

The $Y(\theta\varphi)$ are spherical harmonics of order λ, while μ is the projection of λ on a space fixed axis and the $\alpha_{\lambda\mu}$ are coefficients. For small amplitudes of oscillation the energy may be expanded in powers of $\alpha_{\lambda\mu}$ and of the time derivatives $\dot{\alpha}_{\lambda\mu}$ and one obtains to a first approximation[39]

$$H_{coll} = \sum_{\lambda\mu} (1/2\, B_\lambda |\dot{\alpha}_{\lambda\mu}|^2 + 1/2\, C_\lambda |\alpha_{\lambda\mu}|^2) \tag{3.9}$$

corresponding to a set of independent harmonic oscillators with energy quanta

$$\hbar\omega_\lambda = \hbar\left(\frac{C_\lambda}{B_\lambda}\right)^{1/2} \quad . \tag{3.10}$$

The parameters B_λ and C_λ can be calculated only if some detailed assumptions are made concerning the structure of the nucleus or they can be left as parameters to be evaluated empirically. The B_λ represents the mass transport associated with the vibration and a theoretical estimate based on the surface oscillations of an irrotational and incompressible liquid drop would yield

$$(B_\lambda)_{irrot} = \frac{1}{\lambda}\,\frac{3}{4\pi}\, AMR_o^{\,2} \quad . \tag{3.11}$$

This estimate, however, does not agree with the empirical value. The parameter C_λ represents an effective surface tension which may be obtained from the surface energy term appearing in the semi-empirical mass formula.

39. A. Bohr, Danske Mat.-fys. Medd. 26, No. 14 (1952).

The shape oscillations may be classified according to their multipole order λ. The excitation quanta, called phonons, have total angular momentum λ, and parity $(-1)^{\lambda}$ and may be further characterized by their component of angular momentum μ along a space fixed axis. The lowest frequencies of collective vibration are expected to be of quadrupole type ($\lambda = 2$) since a surface deformation with $\lambda = 1$ corresponds to a simple translational movement. (An exception to this is the collective dipole oscillations of the neutrons with respect to the protons which are of significance in the nuclear photoreactions discussed in Chapter 12. We do not consider such oscillations here because their energies fall in the 10 to 20 Mev range, far above the energy range of interest to us in the present discussion.)

Let us now consider in slightly more detail the possible types of surface vibration, again restricting our interest to those of quadrupole type ($\lambda = 2$). It is helpful to choose the axes of our coordinate system to coincide with the principal axes of an ellipsoidal nucleus. With this assumption the $\alpha_{\lambda\mu}$ coefficients simplify as follows:

$$\alpha_{21} = \alpha_{2,-1} = 0$$

$$\alpha_{22} = \alpha_{2,-2} \quad .$$

Hence, instead of characterizing a randomly oriented nucleus with five $\alpha_{\lambda\mu}$ coefficients, we characterize it with two shape parameters α_{20} and α_{22} plus three angular coordinates specifying the orientation of the ellipsoid in space. These latter will not be of further interest here.

It is convenient to make the further substitutions

$$\alpha_{20} = \beta \cos \gamma$$

and $$\alpha_{22} = \frac{1}{\sqrt{2}} \beta \sin \gamma \quad .$$

β is a measure of the total deformation from a sphere, while γ describes the deviation from rotational symmetry about the principal axis of the ellipsoid. This can be seen by examining the expansion of the radius expression with explicit evaluation of the spherical harmonics.

$$R = R_o \, [1 + \alpha_{20} Y_{20} + \alpha_{22} Y_{22}] \tag{3.12}$$

$$= R_o \, [1 + \beta \cos \gamma \, Y_{20} + \frac{1}{\sqrt{2}} \beta \sin \gamma \, Y_{22}] \tag{3.13}$$

$$= R_o \, [1 + \beta \cos \gamma \sqrt{\frac{5}{16\pi}} \, (3 \cos^2 \theta - 1) + \frac{1}{\sqrt{2}} \beta \sin \gamma \sqrt{\frac{15}{32\gamma}} \, \sin^2 \theta e^{2i\varphi}] \, . \tag{3.14}$$

If $\gamma = 0^\circ$ the last term drops out. In this case if β is positive the nuclear shape is a prolate spheroid (football); if β is negative it is an oblate spheroid (doorknob). See Fig. 3.11.

The term β vibration refers to an oscillation in the shape parameter β. If the equilibrium shape is spherical a beta vibration represents an excursion into spheroidal shapes slightly different from a sphere. See part (b) of Fig. 3.11. If the equilibrium shape is a spheroid the β-vibration represents an excursion into shapes of slightly greater and lesser eccentricity as indicated in part (c) of the figure. One could also picture β-vibrations superimposed on other equilibrium shapes such as, for example, a shape with an equilibrium γ-distortion.

If the parameter γ differs from 0° the circular cross section of the nucleus perpendicular to the main axis is changed to an ellipse; i.e., the spheroid is changed to ellipsoid. This is a result of the operation of the $e^{2i\varphi}$ term in Eq. 3.14. The shape of the cross section in the other two principal planes, i.e., the XZ and YZ planes is also affected. The nuclear shape for a value of γ between 0° and $2\pi/3$ has the appearance indicated in Fig. 3.12. The term γ-vibration may refer to oscillations around a spherical equilibrium shape, about a spheroidal equilibrium shape, or about an ellipsoidal equilibrium shape. The common feature of these three types is that the nuclear shape is ellipsoidal at the extremes of the oscillatory motion.

It is convenient to introduce a two dimensional polar diagram in which any nuclear shape of order $2 (\lambda = 2)$ can be characterized by a point. Such a diagram is shown in Fig. 3.13. The distance from the origin to any point A equals the deformation parameter β while the polar angle corresponds to the shape parameter γ. A spherical shape is represented on this diagram by a point at the origin. Following HILL and WHEELER[40] several authors have drawn potential energy contour lines on such a $\beta\gamma$ polar plot. Such plots may reveal energy-favored paths of nuclear deformation that are difficult to visualize otherwise.

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40. D. L. Hill and J. A. Wheeler, Phys. Rev. 89, 1102 (1953).

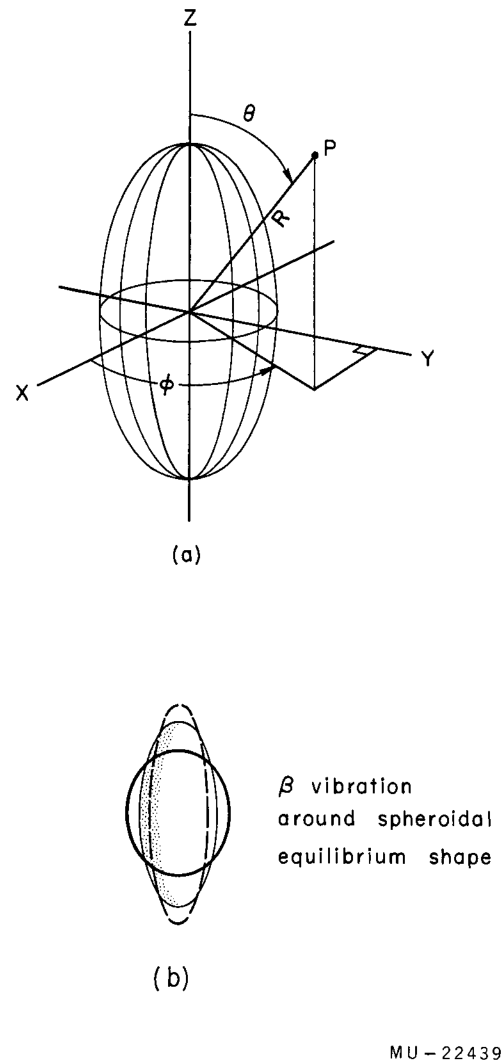

Fig. 3.11 (a) Shape given by Eq. 3.14 for quadrupole deformation with β positive and $\gamma = 0$. (b) Shows β-vibration.

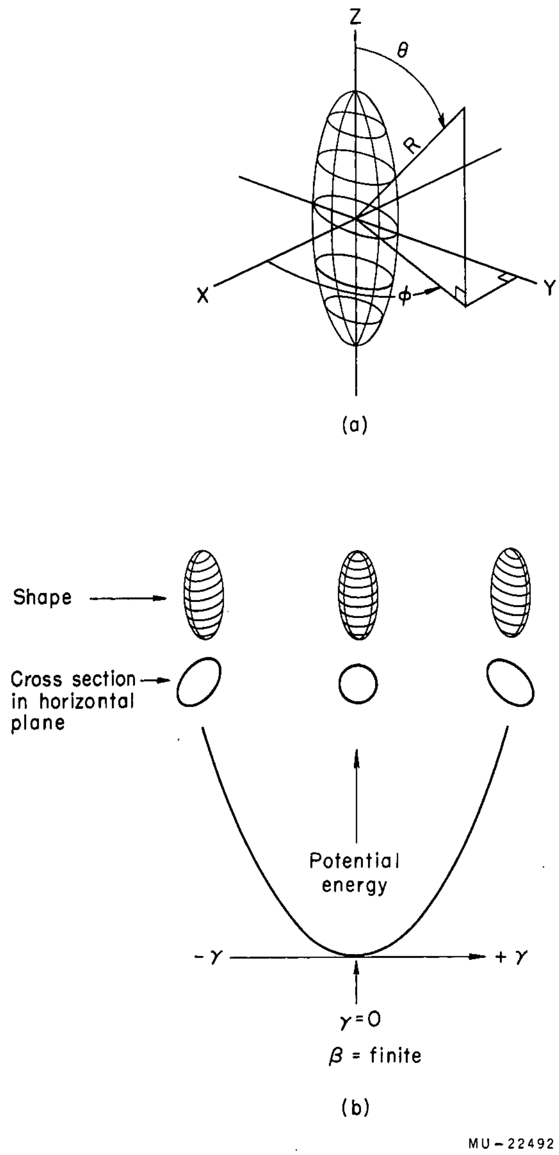

Fig. 3.12 (a) Shape given by Eq. 3.14 for quadrupole deformation with β and γ positive. Cross section is elliptical in any horizontal plane. Part (b) illustrates γ-vibration around a spheroidal equilibrium shape.

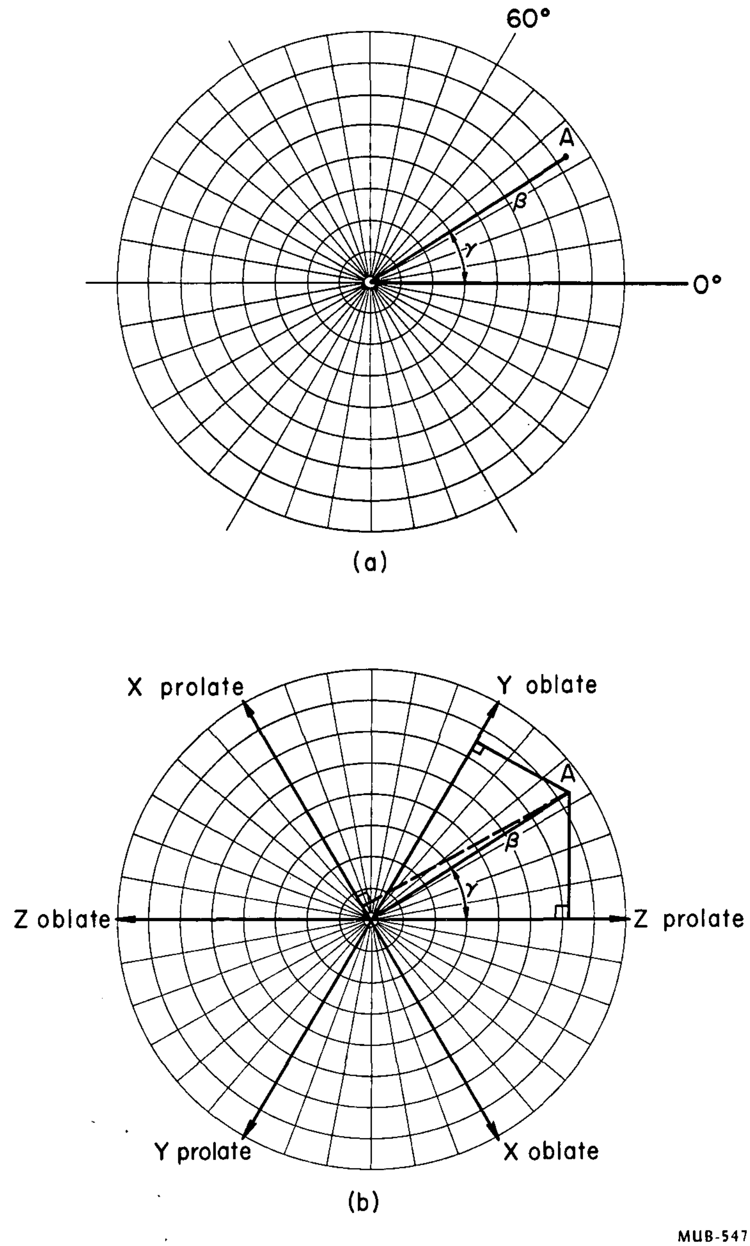

Fig. 3.13 Nuclear deformation expressed in a βγ polar plot. Each shape of quadrupole deformation is characterized by a point A. The radius vector is equal to the parameter β. The shape parameter γ is given by the polar angle. The lower part of the figure has three axes superimposed on the simple polar diagram at 60° to each other. These represent the chief axes of the ellipsoid. The projections of the point A on these three axes are line segments proportional to the three eccentricities in the equation.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

If A falls on any of the three axes the nucleus possesses rotational symmetry with respect to that axis.

The polar diagram can be made somewhat more complex by adding three lines at 60° to represent the coordinate axes x, y, z as in Fig. 3.13 (lower part).

The expected vibrational spectrum for quadrupole vibrations about a spherical equilibrium shape is as follows (neglecting zero point energy).

Energy		Spin and Parity
$3\ \hbar\omega_2$	————	0,2,3,4,6,+
$2\ \hbar\omega_2$	————	0,2,4,+
$\hbar\omega_2$	————	2+
0	————	0+

The term phonons is applied to these quanta of vibration.

In the decay of pure vibrational states M1 radiation is forbidden even when $\Delta I = 0$ or 1. The upper states are expected to decay by a cascade of E2 radiations with no crossover transitions, since transitions must go by one phonon jump. Direct decay to the ground state from the second and higher excited states is forbidden.

3.3.3 Coupling of Particle Motion to Surface Vibrations. There is an approximate correlation of the predictions of this classical theory with the empirical excitation spectra outlined in the introductory Sec. 3.3.1. In the heavy element region the even-even nuclei of element 86 can be taken as the chief examples of this type of spectra. The studies of HARBOTTLE, McKEOWN and SCHARFF-GOLDHABER[41] and particularly those of STEPHENS, ASARO and PERLMAN[42] have led to the results summarized in Fig. 3.14A. It is evident that the 0+, 2+, 2+ sequence of levels seen in the emanation isotopes shows the chief features expected from the harmonic oscillator model, which leads us to conclude that the model has some validity. However, the exact equality of the energy separations is not followed nor is the expected triplet degeneracy of the second excited level observed. As another example of a heavy nucleus in which evidence for harmonic vibrational excitation has been found we may cite the nucleus Po^{214} which has been carefully studied by C. MAYER-BÖRICKE.[43]

41. Harbottle, McKeown and Scharff-Goldhaber, Phys. Rev. 103, 1776 (1956).

42. F. S. Stephens, F. Asaro and I. Perlman, Phys. Rev. 119, 796 (1960).

43. C. Mayer-Böricke, Z. F. Naturforsch 14a, 609 (1959); Lührs and Mayer-Böricke, Z. F. Naturforsch 15a, 103 (1960).

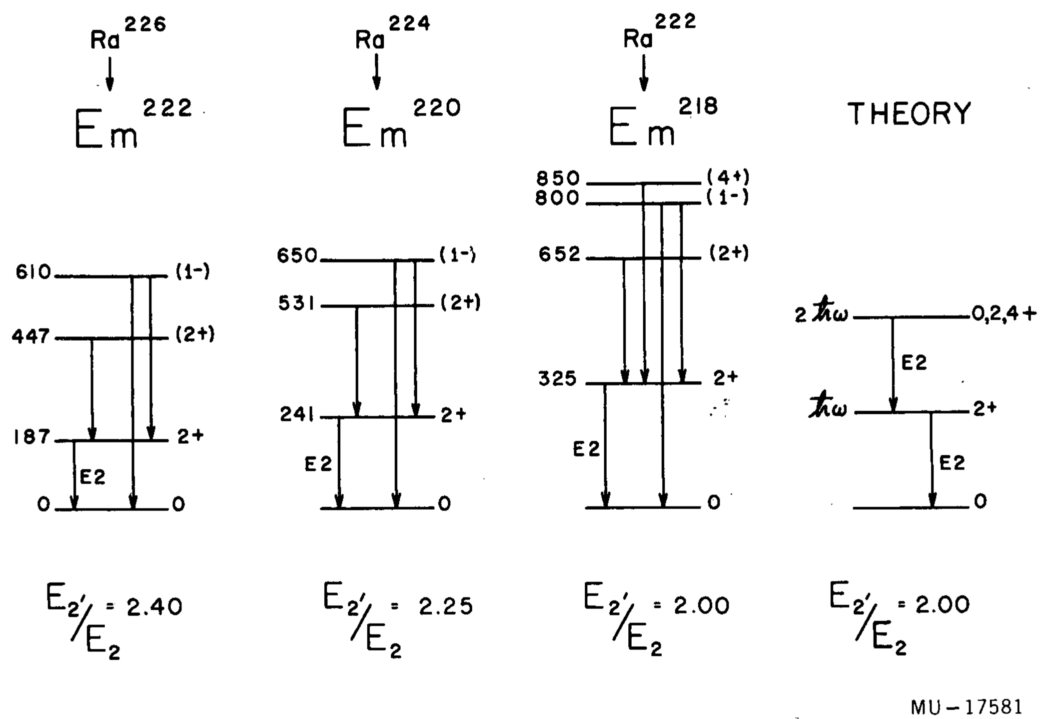

Fig. 3.14A Excited levels in the even-even nuclei of emanation (element 86). The 0+, 2+, 2+ sequence forms a set of states of collective vibration of the type to which attention was first called by SCHARFF-GOLDHABER and WENESER.[38] The ratios E_2'/E_2 are 2.4, 2.2, and 2.0, respectively. The states de-excite by E2 transitions. No cross over transition from the second excited state to ground is observed. The 1- states are believed to represent octupole shape vibration as described in Sec. 3.4.4.

Among lighter nuclei there are many other examples of vibrational-type spectra. See for example the summary of ALDER and CO-WORKERS.[44] In almost all cases only one or two members of the predicted 0+, 2+, 4+ triplet for two phonons of excitation are reported. Some typical examples are shown in Fig. 3.14B prepared by STELSON.[45]

Various authors have studied theoretical models which might account for the deviations of the empirical data from the classical theory. Most of these are concerned directly or indirectly with the effects of the particles beyond the closed shell core. Strong or weak coupling of the single particle motion to the motion of the nuclear surface and residual interparticle (pairing) forces are some of the elements of these theoretical studies. We briefly mention some of these theoretical treatments and their chief conclusions.

SCHARFF-GOLDHABER and WENESER[38] explored the influence of a weak or moderate coupling of particle motion to a core with free surface vibrations. In an illustrative calculation they considered the case of four $f_{7/2}$ particles coupled to $J = 0$ under the action of the residual interparticle forces. This particle state was then coupled to the core vibrations. The resulting spectrum remained essentially a vibrational one but the energy ratio of the second excited 2+ state to the first increased somewhat and the degeneracy of the 0+, 2+, 4+ triplet was removed. According to their sample calculation these levels remain as a close-lying triplet with the 4+ level lying lowest. The E2 transition probabilities were not much affected but a weak M1 component was permitted in the transition de-exciting the second 2+ level.

WILETS and JEAN[46] treated the problem on the basis of a strong coupling approximation. The effects of the nucleons beyond the closed shells were treated as contributing only to an effective potential for collective deformations. General arguments were used to suggest that nuclei some distance from closed shells might be unstable toward deformations of the γ-type. That is, one of

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44. Alder, Bohr, Huus, Mottelson and Winther, Rev. Modern Phys. 28, 432 (1956); see Table V-6, p. 536.

45. P. H. Stelson, Proceedings of the Intl. Conf. on Nuclear Structure, Kingston, Canada, Univ. of Toronto Press, Toronto (1960); see also P. H. Stelson and F. K. McGowan, Phys. Rev. 121, 209 (1961).

46. L. Wilets and M. Jean, Phys. Rev. 102, 788 (1956); Compt. rend. 241, 1108 (1955).

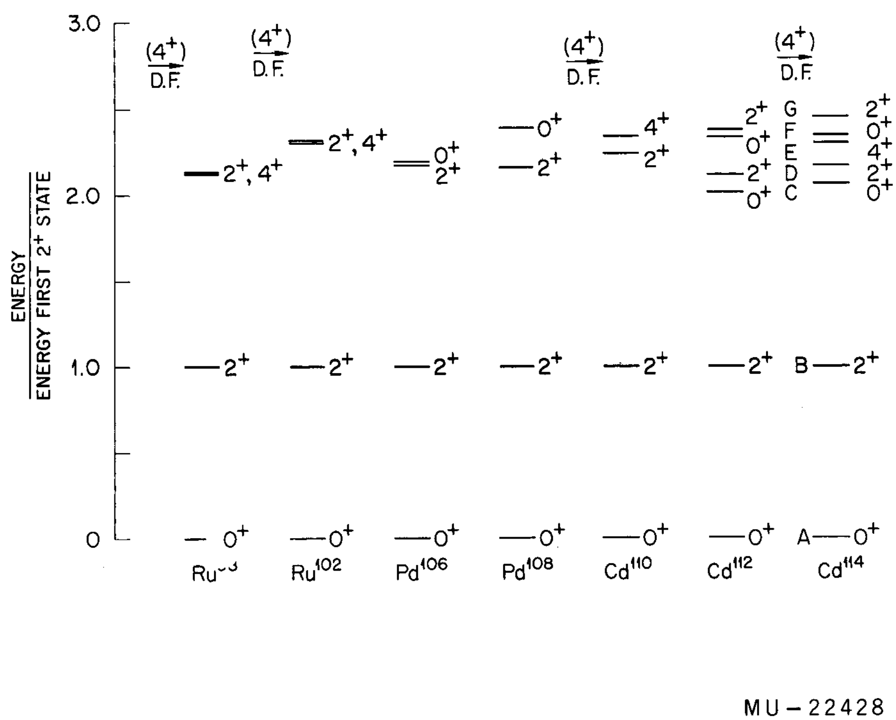

Fig. 3.14B Systematic occurrence of levels in medium weight nuclei showing characteristics of vibrational excitation. From STELSON.[45]

the effects of the extra-core particles is to make the nucleus soft toward those deformations which destroy axial symmetry. WILETS and JEAN explored two types of γ-unstable potentials which yielded an excitation spectrum similar to the experimental observations. The first of this was an anharmonic oscillator; a limiting example of such a potential is the infinite square well:

$$V(\beta) = \begin{cases} \text{constant}, & \beta < b \\ \infty, & \beta > b \end{cases} \quad . \tag{3.15}$$

The second of these was a displaced harmonic oscillator. An example of such an oscillator would be one governed by the potential

$$V(\beta) = 1/2\ C(\beta - \beta_o)^2 \tag{3.16}$$

where β_o is a small but finite quantity. This potential leads to a spectrum with a 0+ ground state, a 2+ first excited state and then a close triplet 4, 2+ and 0+. In the triplet the 0+ level lies considerably higher than the 4+, 2+ pair. The ratio of the energy of the second to the first excited state can vary from 2.0 to 2.5. The E2 transition between the second 2+ state and the ground state remains forbidden.

RAZ[47] examined the results when a weak or intermediate surface interaction is added to a typical two-particle residual interaction for particles beyond the core. He systematically explored the effects of increasing (a) the strength of the surface interaction characterized by a parameter D and (b) the strength of the two particle interaction characterized by a parameter x. Energy levels, wave functions, and γ-ray transition rates were calculated for the explicit configuration $f^2_{7/2}$ with J = 0, 2, 4, and 6. The results were considered to be applicable in a qualitative way to a wide range of nuclei.

The results show that for small deformation the two particle interaction plays a vital role in determining the spectrum. For almost all choices of parameters the spectrum has features like a pure vibrational one. In detail the results are as follows:

(1) the ground state is always 0+.

(2) the first excited state is always 2+.

(3) the second excited state is almost always composed of a level of

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47. B. J. Raz, Phys. Rev. 114, 1116 (1959).

spin 2+ and another of spin 4+ in a close doublet, lying at about twice the energy of the first excited state.

(4) the second 0+ level lies higher in energy than the second excited state.

(5) for a reasonable strength of the two body interaction
   (a) the second 2+ level lies below the 4+ level.
   (b) the reduced γ-ray transition probability B(E2) is much larger for the direct transitions $2' \rightarrow 2$ and $2 \rightarrow 0$ than for the cross-over transition.
   (c) the M1/E2 transition rate ratio is less than 1 for the $2' \rightarrow 2$ transition.

Thus RAZ's[47] calculations indicate that many of the qualitative regularities observed in even-even nuclei in the so-called "vibrational" region can be obtained with an appropriate choice of two-body interaction coupled with a small amount of surface interaction. The agreement is better than that obtained with a pure vibrational spectrum and explains the common observation of 2+ character for the second excited state.

A somewhat different description of the collective oscillations in terms of a pairing force and a long range (P_2) force between particles has been formulated by the Copenhagen school of theorists.[47a] This description gives promise of a much richer description of many characteristics of these vibrations, but published results for heavy nuclei are somewhat limited.

Another model which may have validity for the group of nuclei we are now discussing is that of DAVYDOV and FILIPPOV[48] who considered in some detail what is to be expected for the rotational states of a deformed nucleus of fixed β and γ particularly when γ is taken definitely greater than 0°. If one assumes that such deformed equilibrium shapes exist for the nuclei now under discussion (nuclei of element 86, for example) then one would regard the observed levels as rotational rather than as vibrational states. This model would predict a 2+ first excited state and a widely separated 2+ and 4+ doublet as the next set

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47a. See remarks of B. M. Mottelson, Proceedings of the Intl. Conference on Nuclear Structure, Kingston, Canada, University of Toronto Press (1960), pp. 531-537. See also L. S. Kisslinger and R. A. Sorenson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 32, No. 9 (1960).

48. A. S. Davydov and G. F. Filippov, Nuclear Phys. 8, 237 (1958).

of levels. No 0+ level should appear. A brief discussion of the Davydov-Filippov model is given later in this chapter.

It may be instructive to summarize the predictions of several of these models. In Fig. 3.15 we show possible theoretical spectra for the case of an energy ratio of 2.2 for the 2+ states. In Fig. 3.16 we indicate schematically the potential energy curves applicable to several models. Both figures are patterned after drawings prepared by STELSON.[45]

It is not possible to make any clear choices between these theoretical descriptions on the basis of presently available experimental data in the heavy element group of nuclei--or for that matter in any lighter group of nuclei. This brief treatment is meant to call attention to the fact that further experimental and theoretical investigation of heavy nuclei in the group of nuclei lying between the closed shell nuclei around Pb^{208} and the "rotational" nuclei beyond mass number ~ 225 is certain to be of considerable significance for the understanding of nuclear structure. It seems clear that there are systematic features in the energy level spacings and character and in their γ-ray decay characteristics which are peculiar from the shell model point of view and which suggest strongly that some unified description of particle motion and collective motion is necessary. Just what is the proper unified description must be decided by future research.

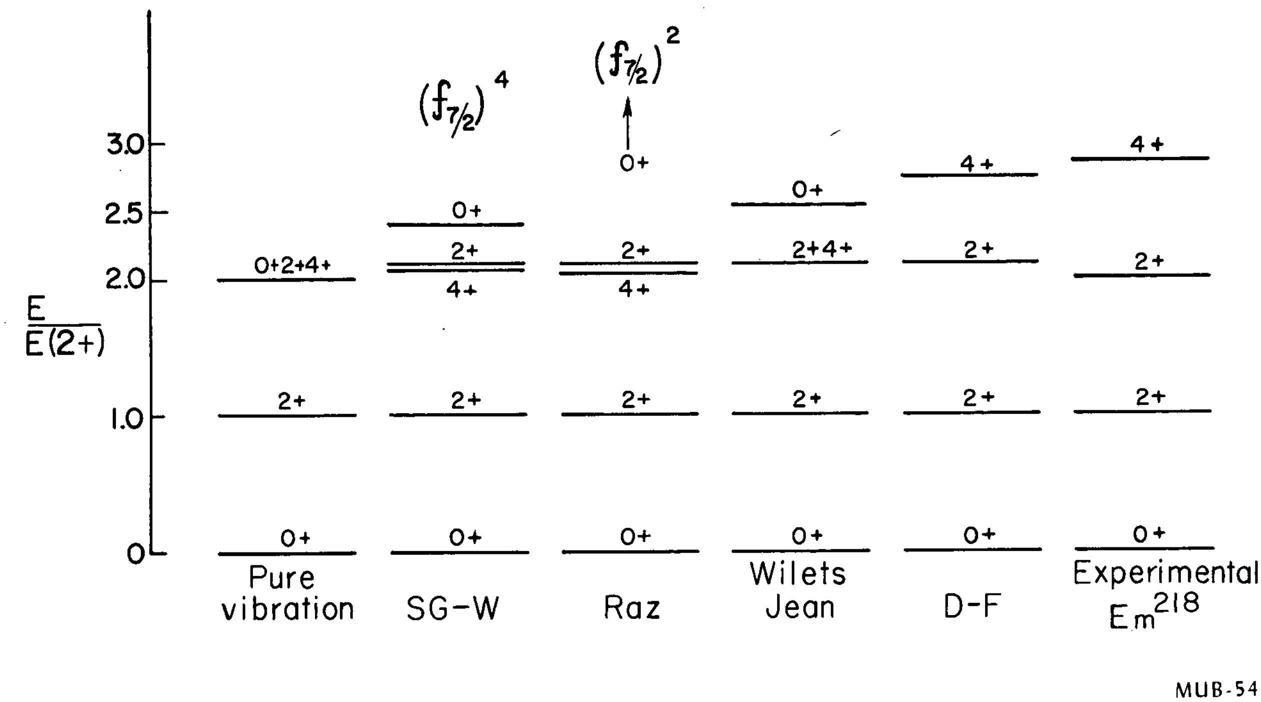

Fig. 3.15 Comparison of possible theoretical spectra for vibrational excitation. Reading from left to right are the predictions of the pure phonon model, the Scharff-Goldhaber calculation of weak coupling of four $f_{7/2}$ particles to the phonon states, the Raz calculation of two $f_{7/2}$ particles coupled to the phonon states, the Wilets-Jean model of a displaced harmonic oscillator with γ instability, and the Davydov-Filippov rotational model for an asymmetric rotor. In all cases the model parameters were adjusted to fit a ratio of 2.2 for the energy of the second 2+ state to the first 2+ state. Finally, the spectrum of Em^{218} is shown as a representative of a heavy nucleus with this type of excitation. Patterned after a figure by STELSON.[45]

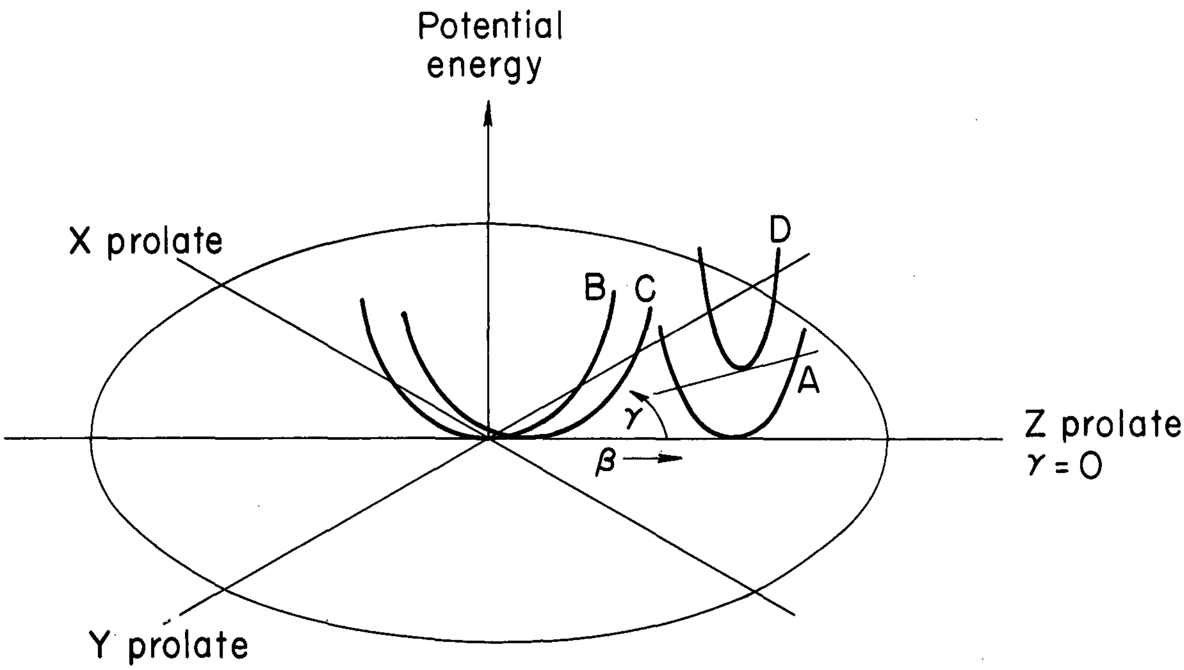

Fig. 3.16 Schematic potential energy surfaces. The horizontal plane is a $\beta\gamma$ diagram of the type shown in Fig. 3.13. The distance of any point from the origin represents β, or the magnitude of the distortion from a sphere. The angle γ measures the extent of distortion from rotational symmetry. Potential energy is shown in the vertical direction. Spheroidal nuclei are represented by curve A. γ values are confined to near 0°. Pure phonon vibrations are represented by curve B. This curve may be rotated through all values of γ as it has no γ dependence. Curve C is meant to represent a possible displaced harmonic oscillator of the type considered by Wilets and Jean. It also is independent of γ. Curve D is a representation of a Davydov and Filippov potential. The nucleus has a fixed β and γ. Since all observed states up to a rather high energy are interpreted according to this model as rotational states, the vibrational potential curve D is shown with steeply rising sides. The potential energy is strongly dependent on both β and γ. This figure is patterned after one by STELSON.[45]

3.4 The Unified Model in the Region of Deformed Nuclei

3.4.1 Separability of Nuclear Wave Functions. At some considerable distance from closed shells the nucleus becomes stabilized in a non-spherical shape under the influence of the coherent effects of many particles in unfilled shells. It is important to note that it is necessary to be far removed from closed shells for both neutrons and protons if conditions are to be favorable for stabilization of the nucleus in a spheroidal shape. Such a location is found among the heavy elements since the 126-neutron shell and 82-proton shell meet in Pb^{208}. Hence, nuclides located well above Pb^{208} are simultaneously remote from closed shells of neutrons and protons.

Similarly, among the heavy rare earth elements located between the mass numbers 150-195 many nuclides are intermediate between the neutron shells at 82 and 126 and the proton shells at 50 and 82. Figure 3.17 indicates the main groups of nuclides which may be expected to have deformed nuclei. In these special regions the nucleus acquires a spheroidal shape which on the basis of simple theoretical arguments may be oblate or prolate. The empirical evidence all throughout the rare-earth region as well as in the actinide element region beyond Ac^{227} shows that prolate deformations are consistently preferred.*

For the strongly deformed nucleus the frequencies of motion which have to do with vibrational and particularly with rotational excitation are lower in general than those related to intrinsic particle excitation. In this case one can obtain an approximate separation of the motion of the individual particles in the potential field defined by equilibrium shape of the core and the relatively slow collective rotation and vibration of the entire system.

In other words, the complete wave function for the nucleus can be assumed to be of the product type:

$$\psi = \chi_{part} \cdot \varphi_{vib} \cdot D_{rot} \quad . \tag{3.17}$$

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* The preference for prolate deformation in these regions has been borne out by calculations of equilibrium deformation by Mottelson and Nilsson [Phys. Rev. 99, 1615 (1955) and Mat. Fys. Skr. Dan. Vid. Selsk. 1, No. 8 (1959)], based on the wave functions of Nilsson to be described later in this chapter. The $\ell \cdot s$ coupling term which in prolate deformation lowers the energy of certain orbitals from higher-lying shells seems chiefly responsible for this effect.

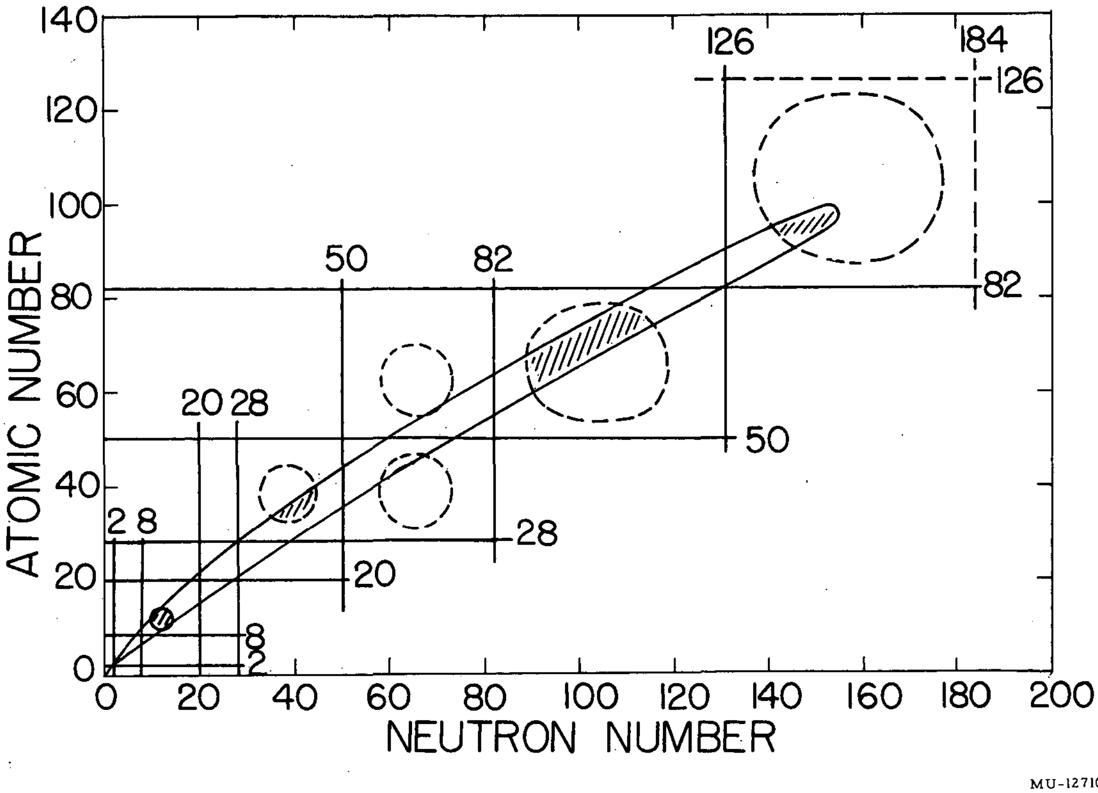

Fig. 3.17 A schematic representation of a Z versus N chart of the nuclides showing the neutron (vertical lines) and proton (horizontal lines) closed shell lines. The irregular boundary line encloses those nuclides having a half-life longer than one minute. The line of beta-stability runs approximately down the center of this area. The groups of nuclides where rotational spectra have been observed are indicated in bold cross hatching. Additional regions where it may be possible to find such nuclei are also indicated by dotted lines. Figure 3.8 presented earlier in this chapter presents data on the first excited (2+) states of even-even nuclei superimposed on a nuclide chart of this type. In that figure the regions of spheroidal nuclei are identified by particularly low values for the first excited state.

Here X represents the intrinsic motion of the nucleons which can be expressed in terms of the independent motion of the individual particles in the deformed field. The second factor φ_{vib} describes the vibrations of the nucleus around the equilibrium shape, while D_{rot} represents the collective rotational motion of the system as a whole. This separation of the wave function into three parts is analogous to that found in theoretical treatment of linear molecules.

3.4.2 The Coupling Scheme. The coupling scheme for deformed nuclei is illustrated in Fig. 3.18. The three important constants of motion are I, K and M where I is the total angular momentum of the nucleus, K is the projection of I on the axis of symmetry and M is its projection on a space-fixed Z axis. For the ground state and for low-lying excited states in which there is no collective rotation about Z', K can be taken equal to Ω defined as the projection of the total particle angular momentum on the nuclear symmetry axis. The total angular momentum $\sum_i j_i$ of the particle system is not, in general, a constant of the motion. The value of Ω for a single particle, Ω_p, takes on half-integral values positive or negative. States differing only in the sign of Ω_p are degenerate since they are identical except for the opposite sense of rotation. The particles fill pairwise into states of opposite Ω_p with no net contribution to K from the pairs. Thus, for an even-even nucleus in the lowest state K = 0. For an odd A nucleus, the last nucleon occupies an unpaired orbit and K equals the Ω_p of the unpaired particle. In an odd-odd nucleus the last proton as well as the last neutron contribute angular momentum along the nuclear axis. In the ground state, K may be the sum or the difference of the Ω_p of these two odd particles.

K will be different from Ω for certain types of vibrational excitations (the γ-vibrations discussed later) in which collective angular momentum is contributed along the nuclear axis. The K value of an excited state can also be different from that of the ground state if the particle structure is changed.

3.4.3 Rotational Excitations. The deformed nucleus may rotate with preservation of shape and internal structure. Since the rotational angular momentum $\vec{R}$ is always perpendicular to the symmetry axis, all members of a rotational band have the same quantum number K.

The energy of the rotational states is given by the following general equation analogous to that describing molecular rotations.

$$E_I = E_o + \frac{\hbar^2}{2\Im} I(I + 1) \qquad K \neq 1/2 \quad . \qquad (3.18)$$

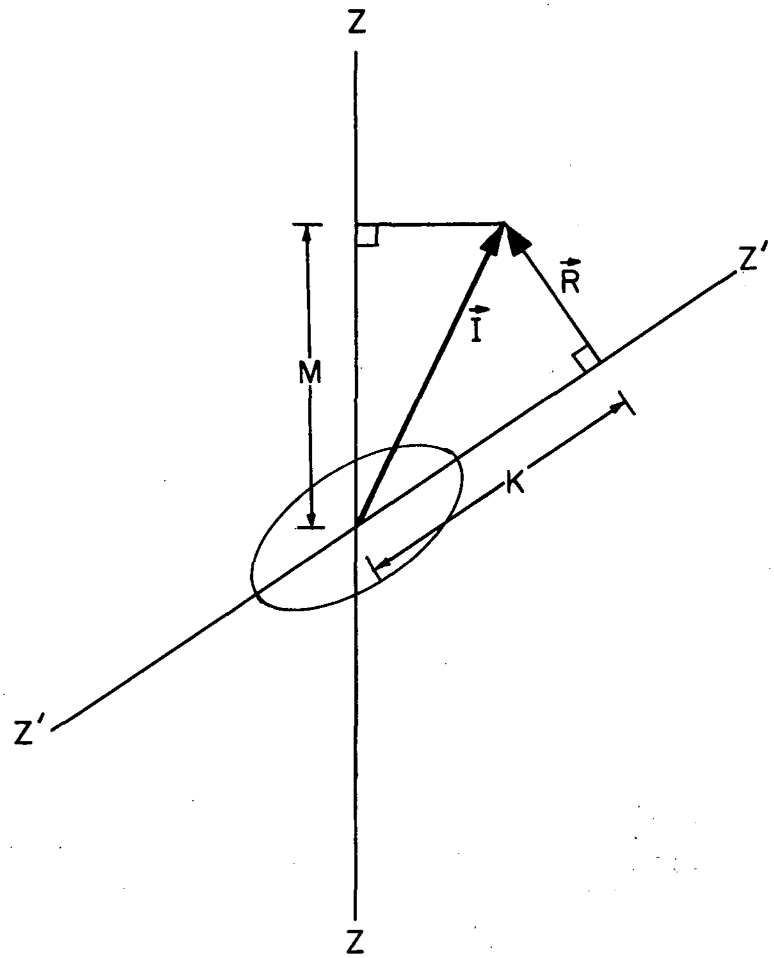

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Fig. 3.18 Coupling scheme appropriate for deformed nuclei. $\vec{I}$ is the total angular momentum of the nucleus. K and M are the projections of $\vec{I}$ on the axis of symmetry Z' and a space fixed axis Z, respectively. $\vec{R}$ is the collective rotational angular momentum. This vector is always perpendicular to Z'. For the ground state and for many low-lying states K is equal to Ω where Ω is defined as the projection of the total angular momentum of the intrinsic particle motion on the symmetry axis.

Here, E_o is a constant depending on the intrinsic structure. $\Im$ represents the effective moment of inertia about the axis of rotation perpendicular to the nuclear symmetry axis. In the special case of K = 1/2 a correction term must be added as follows:

$$E_I = E_o + \frac{\hbar^2}{2\Im} [I(I + 1) + a(- 1)^{I+1/2}(I + 1/2)\delta_{K,1/2}] \quad . \quad (3.19)$$

This term represents a decoupling of the spin angular momentum from the rotational motion; the constant, a, is called the decoupling constant. Since the presence of this term for rotational bands based on a K = 1/2 state alters the spacing, and in several cases, even the order of the levels, such bands are referred to as anomalous rotational bands.

The collective motion which gives rise to the nuclear rotation is essentially different from that of a rigid body and the effective moment of inertia, $\Im$, is somewhat less than the rigid moment, $\Im_{rigid}$, given by the following expression.

$$\Im_{rigid} = \frac{2}{5} MAR_o^2 (1 + 0.31 \beta + 0.44 \beta^2 \ldots) \quad .$$

In the early development of the collective model, estimates of $\Im$ were based on a hydrodynamical model of the nuclear motion. For irrotational flow of an incompressible fluid, this model estimates moments of inertia by the following equation:

$$\Im_{irrot} = \frac{2}{5} AMR_o^2 \beta^2 \quad (3.20)$$

where β is a deformation parameter of a spheroid given by:

$$\beta = 4/3(\pi/5)^{1/2} \Delta R/R_o \approx 1.06 \Delta R/R_o \quad . \quad (3.21)$$

Here R_o is the mean nuclear radius and ΔR is the difference between the major and minor semi-axis of the spheroid. However, the values of $\Im$ found experimentally are usually two or three times larger than $\Im_{irrot}$ and much closer than predicted to the rigid moment as shown in Fig. 3.19.

BOHR and MOTTELSON[49] discussed the possible fundamental nature of the moments of inertia of rotating nuclei in an important paper published in 1955.

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49. A. Bohr and B. R. Mottelson, Dan. Mat.-fys. Medd. 30, No. 1 (1955).

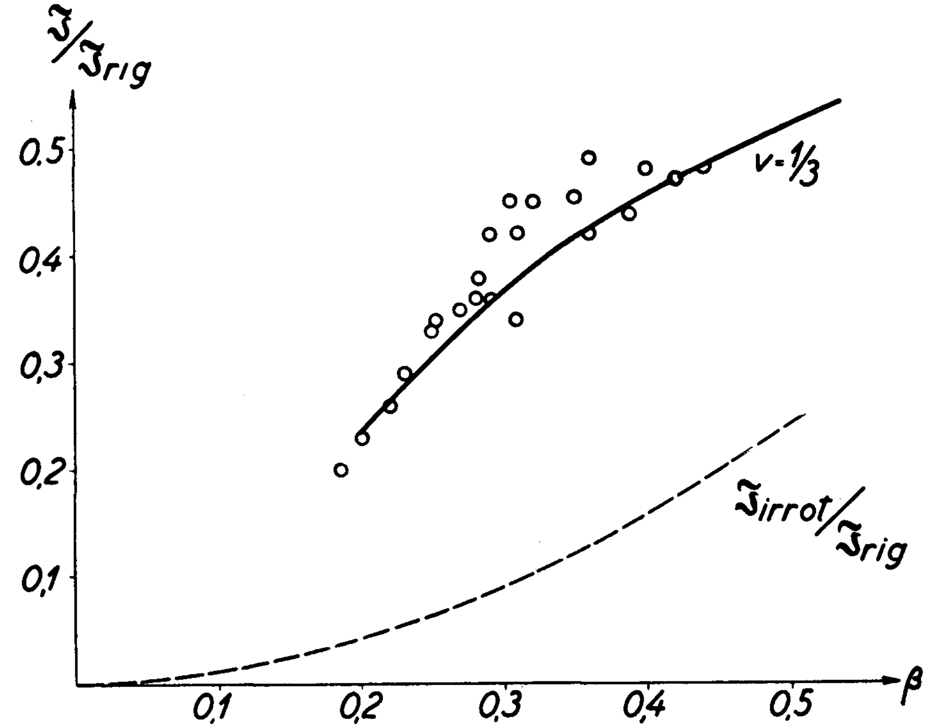

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Fig. 3.19 Dependence of moments of inertia on the nuclear deformation. The empirical moments of inertia determined from observed rotational level spacings in the region $150 < A < 188$ using Eq. 3.18 are plotted in units of $\Im_{rigid}$ versus the deformation parameter β (Eq. 3.21). The moment of inertia corresponding to irrotational flow (Eq. 3.20) is shown by the dotted line. The full drawn curve represents a theoretical estimate based on a simplified model. The parameter V appearing in this estimate is a measure of the strength of the residual interactions between nucleons in unfilled shells and the value chosen has been adjusted to fit the experimental data. See BOHR and MOTTELSON.[49]

In the case of the limiting situation in which the intrinsic nuclear structure can be described in terms of the independent motion of the nucleons in an average potential, the moment of inertia for a many-particle configuration consisting entirely of closed shells has the value expected of irrotational flow. In this case, however, the nuclear equilibrium shape is spherical and the moment of inertia for rotational motion vanishes. In the limit of many nucleons moving independently in the nucleus the moment of inertia approaches the value of an equivalent rigid rotator if the shape of the nucleus is taken to be the equilibrium spheroidal shape; cf., minimum of the Region III curve shown in Fig. 3.6B. Thus, if the intrinsic nuclear structure of strongly deformed nuclei could be described in terms of undisturbed particle motion, calculation would lead one to expect essentially the rigid moment of inertia.

However, if one includes the correlation in the nucleonic motions of the particles in unfilled shells resulting from residual interparticle forces not included in the central potential, significant changes occur in the collective motion and in the resulting moment of inertia. These changes are in the direction of reducing the moment of inertia from that of a rigid rotator. The importance of the residual interparticle forces is expected to be greatest near closed shells and to reduce progressively as nuclear deformation gets larger. It is difficult to estimate the course of the transition between the two limits. BOHR and MOTTELSON[49] demonstrated the nature of the change by considering a greatly simplified model in which the whole effect of nucleons outside of closed shells is represented by two interacting nucleons in p-states. A curve based on this simplified model is given in Fig. 3.19 and is seen to reproduce the trends in the moment of inertia with deformation reasonably well. Since the model is a gross simplification and since an arbitrary adjustment of the strength of the interaction had to be made, the agreement with the experimental data is regarded as significant only in indicating that the key to a real understanding of the moments of inertia of rotating deformed nuclei probably lies in a proper consideration of the residual interparticle forces.

Since this 1955 paper of BOHR and MOTTELSON[49] a new theoretical description of the residual interparticle forces has been introduced. This description known as the "pairing force" or as the "superfluid" model is patterned after the BARDEEN, COOPER and SCHRIEFFER[50] theory of the superconducting states of

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50. J. Bardeen, L. N. Cooper and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957).

matter. Many authors[51] have considered the pairing interaction of particles in shell model states appropriate for the deformed nucleus and have calculated the influence of this interaction on the moment of inertia. These calculations indicate that the moments of inertia are correctly reproduced. Some idea of the extent of agreement of theory and experiment can be obtained from Fig. 3.20.

We shall not go into the details of such calculations or into any further discussion of the fundamental nature of the moment of inertia. From this point on we shall regard $\Im$ only as a constant to be evaluated from Eq. 3.18 using experimentally determined level spacings.

For the rotational levels associated with the ground state of even-even nuclei, Eq. 3.18 takes on a very simple form:

$$E_I = \frac{\hbar^2}{2\Im} I(I + 1) \qquad I = 0,2,4 \ldots \text{ even parity.} \qquad (3.22)$$

The possible quantum states of the nucleus are restricted by the reflection symmetry of the deformation, which implies that states labeled by (K,Ω) must be combined in a definite way with those labeled by $(-K, -\Omega)$. Moreover, the reflection symmetry about the plane passing through the center of the nucleus and perpendicular to the axis of symmetry implies that the collective motion has even parity and that the parity of a nuclear state is therefore determined by that of the particle structure. This symmetry condition limits the acceptable states in the ground state rotational band of even-even nuclei to $I = 0,2,4,6,\ldots$ even parity as written in Eq. 3.22.

In an odd-A or an odd-odd nucleus (or in an even-even nucleus with an excited particle configuration leading to $\Omega \neq 0$) the ground state has normally $I_o = K = \Omega$ and a rotational spectrum given by Eq. 3.18 or 3.19 with $I = I_o, I_o + I, I_o + 2 \ldots$ same parity as ground state.

The rotational formulas given above can give precisely correct values for the spacing of rotational levels only if there is complete separation of rotational from vibrational and intrinsic motions. For even-even nuclei the rotational-particle coupling is insignificant but a small correction for rotational-vibrational interaction can be introduced as shown by the second term in the corrected formula:

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51. Some of the authors who have contributed to such calculations are: J. J. Griffen and M. Rich, Phys. Rev. Letters 3, 342 (1959) and Phys. Rev. 118, 850 (1960); A. B. Migdal, Nuclear Phys. 13, 655 (1959) and J.E.T.P. (USSR) 37, 249 (1959); S. G. Nilsson and G. Prior, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 32, No. 16 (1960).

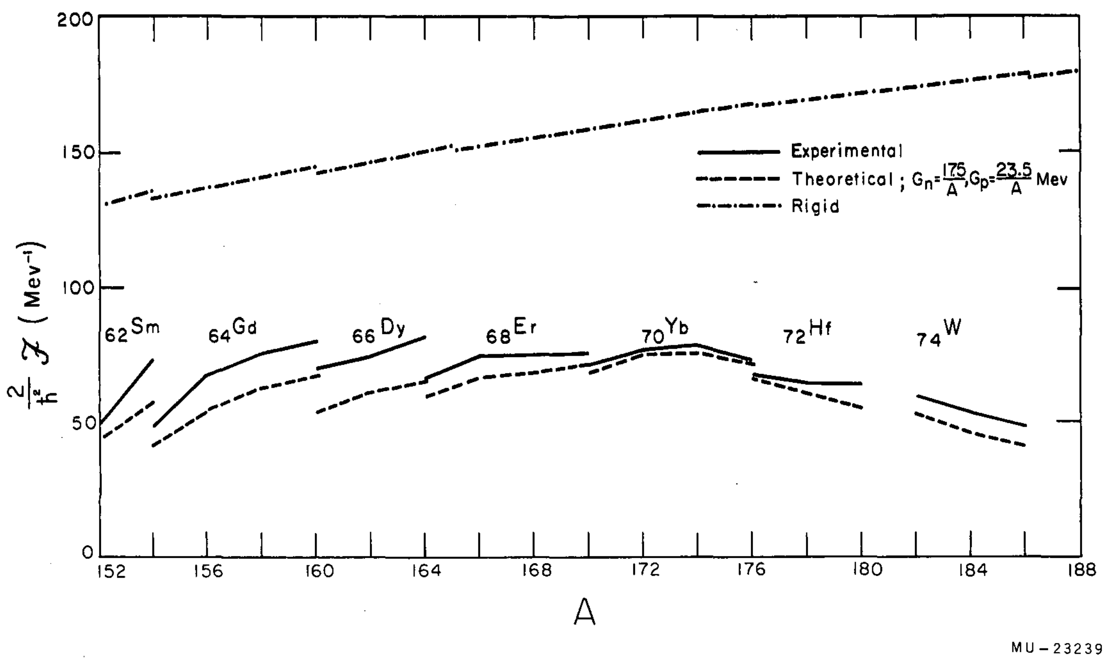

Fig. 3.20 Experimental values of moments of inertia compared to values computed by NILSSON and PRIOR[51] using a pairing correlation between the extra-core nucleons.

$$E_I = \frac{\hbar^2}{2\Im} I(I + 1) + BI^2(I + 1)^2 \quad . \tag{3.23}$$

The constant term B can be regarded as a constant to be evaluated empirically from the spacing of the first three members of the rotational spectrum. B is always negative for ground state rotational bands of even-even nuclei. In heavy element even-even nuclei above mass 230 the energy correction to higher-lying rotational terms is less than one per cent.

This rotational-vibrational correction also applies to rotational levels in odd-A nuclei but is not usually considered since other correction terms such as the interaction of nearby particle states (rotational-particle coupling) is likely to cause greater shifts in the level energy. The nature and extent of the rotational-particle coupling in odd-A nuclei has been discussed by KERMAN.[52]

Rotational systems of levels are based not only on ground states but also on states of vibrational excitation or on excited particle states. Figure 3.22 shows rotational bands in an odd-A nucleus based on the ground state K and on two levels of intrinsic excitation, K' and K".

3.4.4 Vibrational Excitations. We next consider the oscillations around the equilibrium shape, i.e., the φ_{vib} part of the separated wave function (Eq. 3.17). A necessary condition for the validity of the solution is that the amplitudes of these vibrations be small compared to the total magnitude of the deformation. The most prominent vibrations are of the quadrupole type of which there are two types. The first preserves cylindrical symmetry and only involves variations in the eccentricity of the spheroidal shape; the term β-vibration is applied to this type. See Sec. 3.3.2 and especially the discussion of Fig. 3.11. The second leads to deviations from cylindrical symmetry and is referred to under the term, γ-vibration. See the discussion of Fig. 3.12 in Sec. 3.3.2.

For small oscillations about the equilibrium shape these modes are expected to be approximately harmonic and we describe these modes of motion by the quantum numbers n_β and n_γ, respectively.

The energy associated with the excitation of the lowest vibrational states is of the order of one Mev. In a number of heavy element nuclei excited levels have been identified as beta or gamma vibrational states, as is described in Sec. 3.5.2, but the information on such states is more limited than that for the rotational states.

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52. A. Kerman, Dan. Mat.-fys. Medd. 30, No. 15 (1955).

The characteristics to be expected of the lowest-lying vibrational levels (corresponding to one phonon of excitation) in even-even nuclei are the following:

Quadrupole β-vibrations. The quantum numbers (K, I, π) are (0, 0, +) in this case. This level may be the base level of a rotational family given by the equation below:

$$E_{rot} = E_o + \frac{\hbar^2}{2\Im} I(I + 1) \quad . \tag{3.24}$$

The energy of the base level in this equation is E_o. The constant $\frac{\hbar^2}{2\Im}$ has the same value as does the ground state rotational band. The allowed values of I are restricted to 0+, 2+, 4+ ... because of the same symmetry considerations as in the ground state rotational band.

The base level (0, 0+) is expected to de-excite by an E2 transition to the 2+ level of the ground state rotational band and by an E0 transition (characterized by a complete absence of photons) to the ground state. The energy of these two transitions will differ, of course, by just the energy of the 2+ state of the ground state rotational family.

The (0, 2+) first rotational level based on the (0, 0+) vibrational level just discussed will also de-excite by a characteristic triplet of E2 gamma rays differing by just the energy of the 0+, 2+ and 4+ states of the ground state rotational band. The pattern of gamma de-excitation is shown in the schematic drawing Fig. 3.21A.

Quadrupole γ-vibrations. The quantum numbers (K, I, π) are (2, 2+) in this case. This level may be the base level of a rotational band given by Eq. 3.18. In this case, the allowed values of I are 2+, 3+, 4+, etc.

The base state (2, 2+) will de-excite to the 0+ and 2+ levels of the ground state rotational band by pure E2 transitions. M1 contribution is forbidden by the K-selection rule (Eq. 3.41 in Sec. 3.4.7) which states that the multipolarity must equal or exceed ΔK of the transition, which in this case is 2. The E2 transition to the 4+ level of the ground state rotational band is permitted by the K-selection rules but the reduced transition probability calculation (relative rates given by Clebsch-Gordan coefficient ratios) shows that it will be weak. Hence, the characteristic observed pattern of gamma de-excitation of the lowest γ-vibrational state is a triplet of pure E2 gamma rays of which the two most energetic ones are prominent. These two differ in energy by the energy (about 45 kev in transuranium nuclei) of the I = 2+ level of the ground state rotational band. This pattern is illustrated by the schematic drawing Fig. 3.21A.

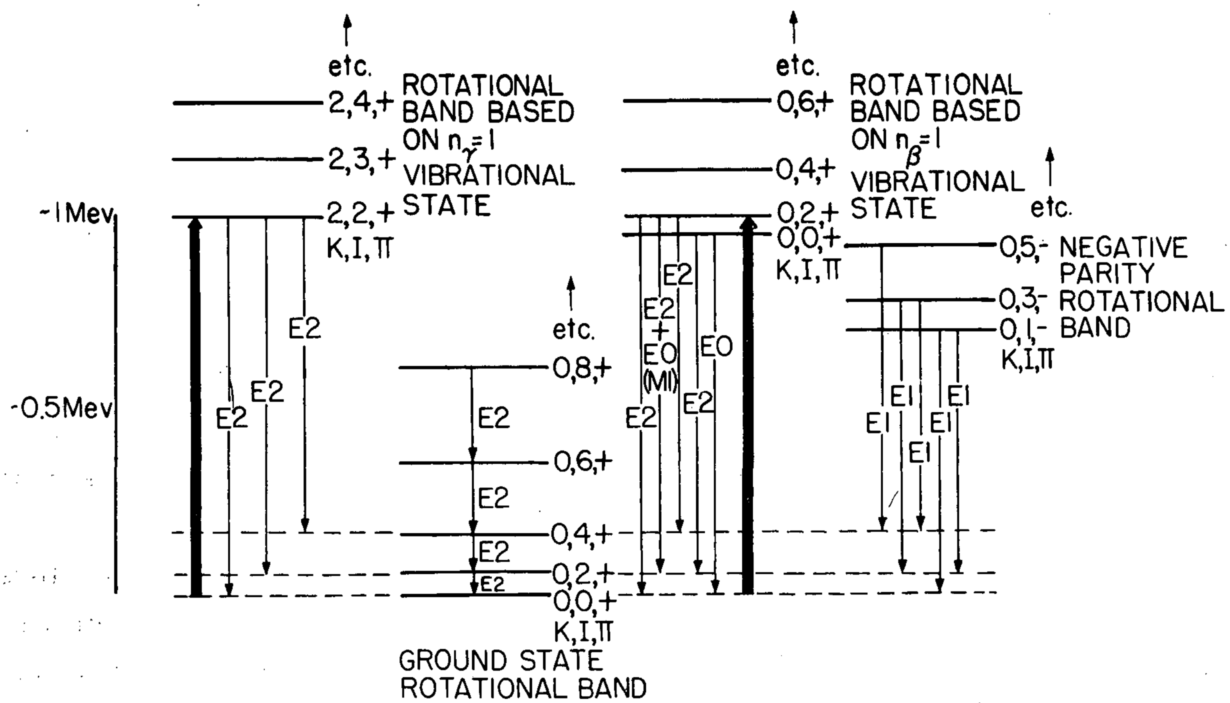

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Fig. 3.21A Pattern of rotational and vibrational levels for an idealized even-even nucleus in the region of deformed nuclei. The expected gamma ray pattern is also shown.

Octupole Vibrations. Experimental proof has been presented for the existence of states of odd spin and negative parity lying below one Mev of excitation in the even-even nuclei in the region of permanently deformed nuclei. The experimental results are reviewed in Sec. 3.5.3. The systematic existence of these states at this low excitation makes it logical to assign them to collective motions of some type. Since negative parity states are forbidden in even-even nuclei if shape changes are restricted to quadrupole terms, it is necessary to consider the possibility that higher order terms are involved in the collective motions of real nuclei. CHRISTY[53] suggested that the negative parity states might be associated with distortions of the nucleus into a pear shape. Referring back to Eq. 3.8 we note that octupole terms ($\lambda = 3$) are the lowest order terms which would lead a pear shaped nucleus, i.e., a nucleus with an axis of symmetry but without inversion symmetry.

Not enough is known about the response of the nucleus to shape distortions of the octupole type to make it possible to give more than a very general description of the spectra to be associated with such distortions. We have tried to treat two possibilities in a very general schematic way in Fig. 3.23. We start with the assumption of a nucleus containing appreciable $\alpha_{20}Y_{20}$ distortion in its ground state and ask how the potential energy of the nucleus might change if we introduced $\alpha_{30}Y_{30}$ distortion. In one case we assume that α_3 distortion is favored so that the nucleus in its ground state is pear shaped. The potential energy for such a case in a real nucleus might have a small hump at zero α_3 distortion. In a second case which might seem more in accord with our intuition based on the experimental spectra so far observed, the ground state of the nucleus contains no α_3 deformation but the nucleus is soft toward excursions into pear shapes. The figure shows the type of spectra one might expect from the two cases. Each quantum state of α_3 vibrational excitation can have rotational excitation superimposed on it and we have shown the predicted rotational bands. If the potential energy curve for α_3 distortion is really of the harmonic oscillator type, we should expect to see a positive parity band with the sequence 0+, 2+, 4+ at twice the excitation of the 1-, 3-, 5- negative parity band. This would provide a crucial test of the theory.

53. R. F. Christy, private communication quoted by Alder et al. in Rev. Modern Phys. 28, 432 (1956).

We might emphasize here again that the old liquid drop model would predict a great resistance to deformations of this type so that about 3 Mev of excitation might be required to reach the first quantum state. But in the unified model we can expect that the interaction of the particle structure beyond the closed shell core with the nuclear surface might decrease greatly the energy involved in such shape changes.

The literature on the theoretical interpretation of octupole vibrations of nuclei is not extensive[54,55] but it is certain that more theoretical work will be done as more experimental evidence is accumulated on negative parity states in even-even nuclei. It is also quite likely that octupole states play an important role in fission and fission asymmetry.[56,57,57a]

Figure 3.21B summarizes the pattern of vibrational and rotational excitations in an idealized even-even nucleus.

3.4.5 Single Particle Wave Functions in Deformed Nuclei. It is our purpose now to review the evaluation of the X function of Eq. 3.17; the X function is that part of the total wave function which describes the motion of the last nucleon in the spheroidal field generated by the core nucleons.

Detailed calculations of these functions have been made by GOTTFRIED,[58] MOSZKOWSKI,[59] RICH,[60] RASSEY,[61] and NILSSON.[62] We shall follow the treatment of Nilsson since it has been the most widely applied.

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54. Alder, Bohr, Huus, Mottelson and Winther, Rev. Modern Phys. 28, 432 (1956); see expecially p. 541.

55. S. Moszkowski, General Survey of Nuclear Models, Handbuch der Physik, Vol. 39, S. Flügge, Editor (Springer-Verlag, Berlin, 1957).

56. A. Bohr, Paper P/911, Proceedings of the First Intl. Conference on the Peaceful Uses of Atomic Energy, United Nations, 1955; also unpublished work of A. Bohr and B. Mottelson.

57. L. Wilets, Proceedings of the Rehovoth Conference on Nuclear Structure, H. J. Lipkin, Editor, North-Holland Publishing Co., Amsterdam, 1958, pp. 122-133; also unpublished work.

57a. S. A. E. Johansson, Nuclear Phys. 22, 529 (1961).

58. K. Gottfried, Phys. Rev. 103, 1017 (1956).

59. S. Moszkowski, Phys. Rev. 99, 803 (1955).

60. M. Rich, Univ. of California Radiation Laboratory Report UCRL-3587.

61. A. J. Rassey, Phys. Rev. 109, 949 (1958).

62. S. G. Nilsson, Binding States of Individual Nucleons in Strongly Deformed Nuclei, Dan. Mat.-fys. Medd. 29, No. 16 (1955).

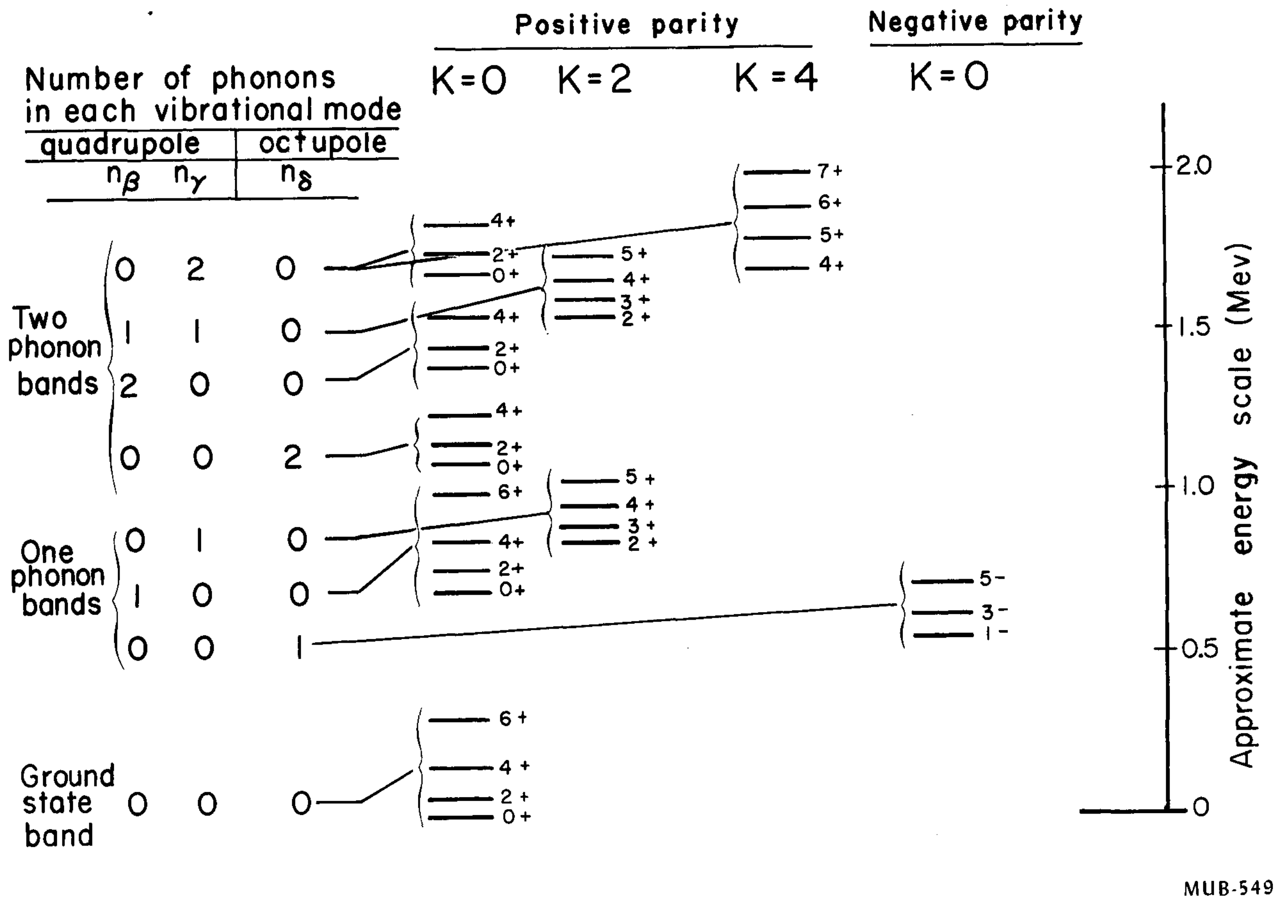

Fig. 3.21B Schematic diagram showing idealized rotational bands corresponding to different types of vibrational excitation. The energy scale is approximately correct for a heavy nucleus and is meant only to show order of magnitude of vibrational energies. Experimental spectra include many examples of the one-phonon band type. In only a few cases have experimental levels been identified with the two-phonon states. Not shown in the figure are additional types of two-phonon excitation in which one phonon of β or γ vibration is combined with one phonon of octupole type; nor is reference made in the text or in the diagram to a second type of octupole vibration corresponding to the Y_{32} spherical harmonic which by itself or in combination with β- or γ-vibrations could give rise to additional rotational bands.

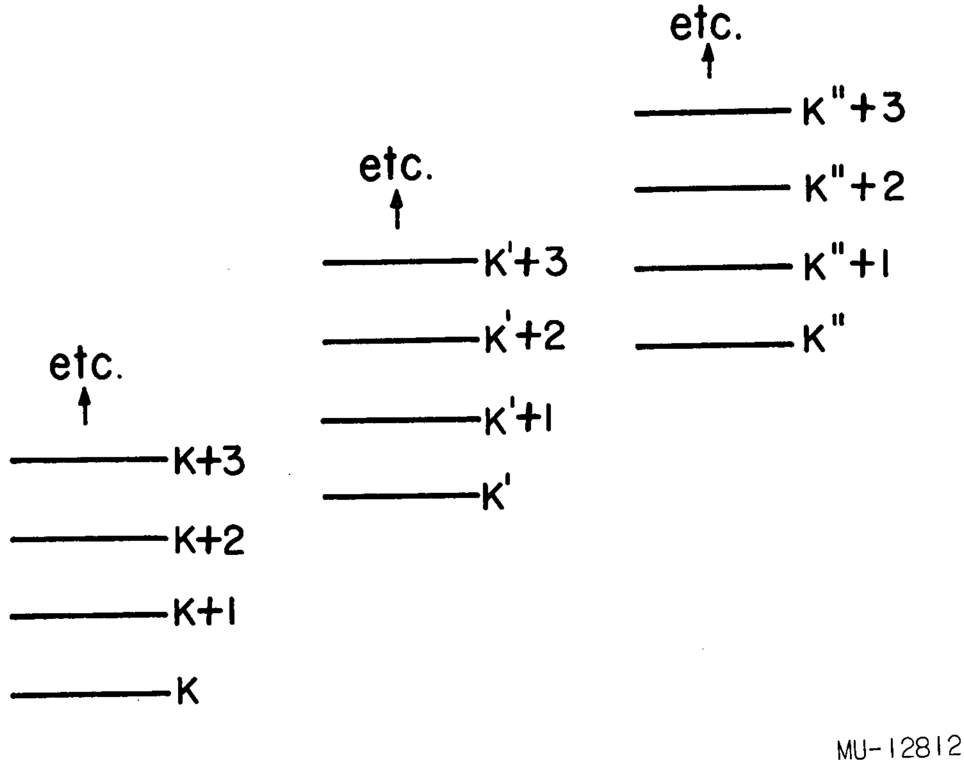

Fig. 3.22 Pattern of levels for an idealized odd-A nucleus in the region of deformed nuclei. The level spacings for the three rotational bands will depend on the K-value of the band. The basic nature of the K' and K" base-levels is unspecified and may represent particle or vibrational excitation. The level patterns for odd-A nuclei are much less regular than those for even-A nuclei.

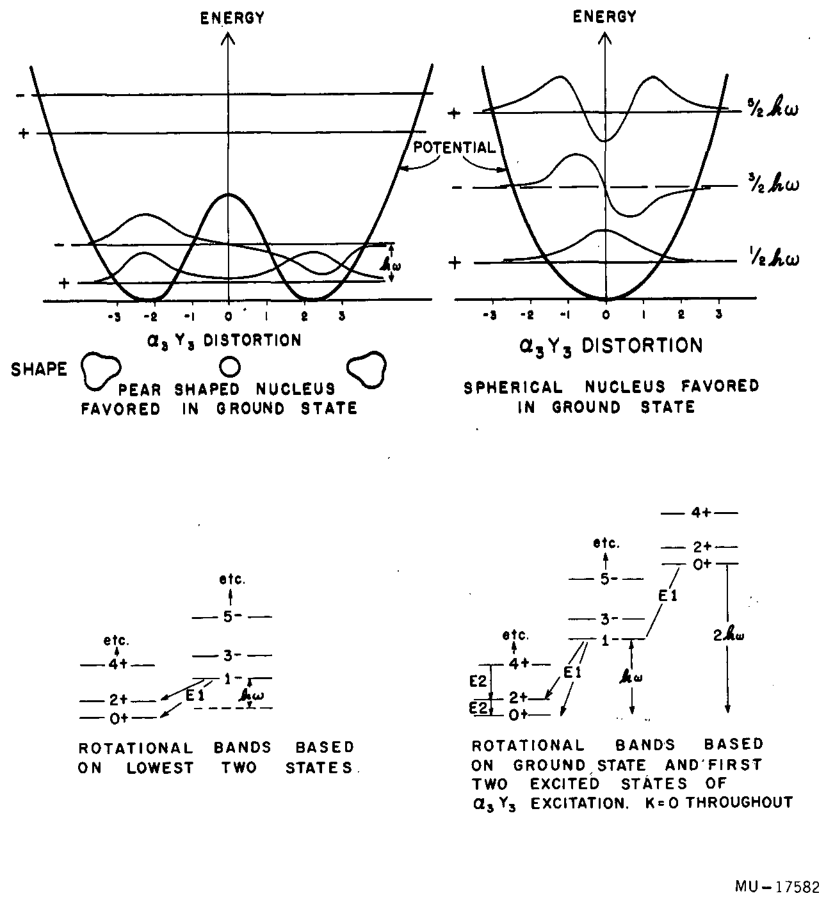

Fig. 3.23 Octupole vibrations in nuclei. Potential energy curves are shown under the assumption that α_3 (pear-shaped) deformations are favored and under the assumption that they are not favored in the ground state. The possible quantum states and their wave functions are shown schematically. Under the further assumption that the nucleus contains an appreciable quadrupole deformation the predicted rotational band spectra are shown in the lower part of the diagram.

NILSSON[62] used as his basic model an anisotropic harmonic oscillator potential with cylindrical symmetry. The anharmonicity in the oscillator potential is derived from the strong coupling of the particles in unfilled shells with quadrupole deformations of the surface. The Hamiltonian for the particle motion furthermore contains a spin-orbit term and a term proportional to $\bar{\ell}^2$ which gives a correction with some features of a truncation of the oscillator bottom. The Hamiltonian has the form

$$H = H_o + c\ \bar{\ell}\cdot\bar{s} + D\ \bar{\ell}^2 \qquad (3.26)$$

$$H_o = \frac{\hbar^2}{2M}\Delta + \frac{M}{2}\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right) \quad . \qquad (3.27)$$

M is the nucleon mass. The frequencies ω_x and ω_y and ω_z are related by the expression

$$\omega_x = \omega_y \neq \omega_z \quad . \qquad (3.28)$$

The relative contributions of the spin-orbit term, $\bar{\ell}\cdot\bar{s}$, and the $\bar{\ell}^2$ terms were determined by requiring that the Hamiltonian yield, in the case of spherical symmetry, the empirically known sequence of single-particle levels.

Using a suitable transformation and representation of an isotropic harmonic oscillator NILSSON[62] diagonalized the Hamiltonian (Eq. 3.26) and evaluated the eigenvalues and the eigenfunctions of the Hamiltonian as a function of the deformation with the aid of an electronic computer.

It is beyond the scope of our interest here to present the mathematical details of the calculation and of the resulting eigenvalues and eigenfunctions. We shall instead merely summarize the most significant results in graphical form and show how these results can be very useful in a qualitative way.

As an indication of the nature of the calculations, consider Fig. 3.24 which shows the behavior of the $f_{7/2}$ neutron state of the 5th oscillator shell which is a degenerate state in a spherical nucleus. As the spherical nucleus is changed to a prolate or an oblate shape, the level with spin j (in this case j = 7/2) splits into $\frac{2j+1}{2}$ levels labeled with the new quantum number Ω, defined as the projection of the particle angular momentum on the nuclear symmetry axis.

In the spherical nucleus the particle state labeled by ℓ and j is degenerate. A small nuclear deformation essentially removes the degeneracy and splits the levels in such a way that for prolate deformation the state with the highest Ω

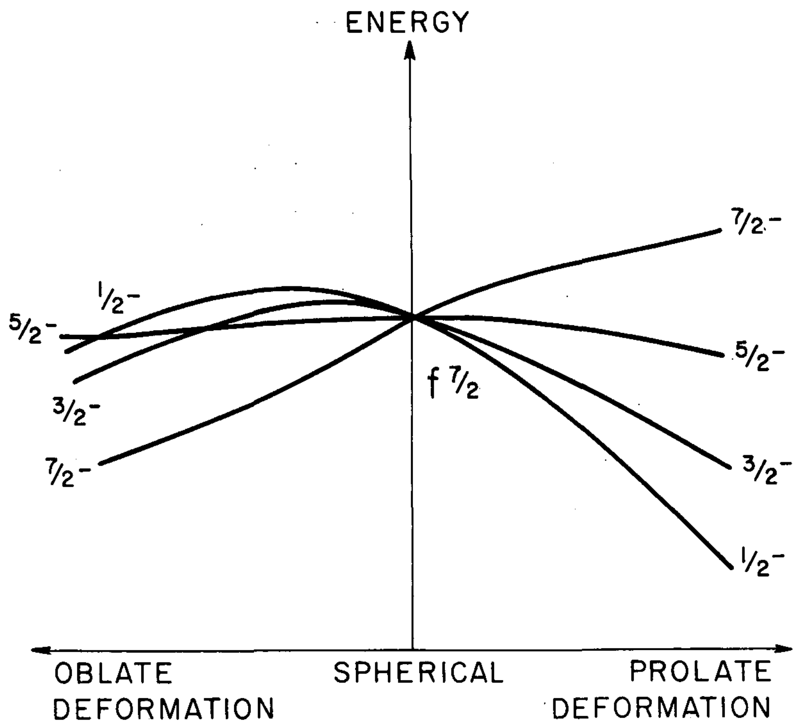

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Fig. 3.24 A degenerate $f_{7/2}$ neutron state is shown for a spherical potential. The diagram indicates schematically the removal of the degeneracy in a deformed nuclear field upon application of the Nilsson Hamiltonian. As soon as the deformation becomes appreciable the purity of the $f_{7/2}$ character of the degenerate levels is destroyed. The important quantum number becomes Ω which is shown at the left and right borders. In the Nilsson calculations of these levels the influence of all neighboring shell model states is included.

(corresponding to an orbital lying mainly in a plane perpendicular to the axis of elongation) rises highest. For larger deformations, states of different ℓ and j couple in a way to destroy even the approximate character of ℓ and j as constants of the motion. When this occurs, it is useful to describe the wave functions in terms of a new set of "asymptotic" quantum numbers which are "good" quantum numbers in the limit of strong ellipsoidal deformation. In addition to Ω and parity these quantum numbers include the following: N, the total oscillator quantum number, n_z, the number of oscillator quanta along the symmetry axis; Λ, the component of the total orbital angular momentum along the symmetry axis; Σ, the component of the intrinsic spin s on the symmetry axis, limited to the values $\pm 1/2$.

The much more complex Figs. 3.25 and 3.26 show the behavior of all orbitals above the 82-particle shell closures. The ordinate in these figures represents an energy scale. The abscissa represents a deformation parameter. This parameter δ is related to the parameter β of Eq. 3.21 and to first order the relationship is:

$$\delta = 0.95\ \beta \quad . \tag{3.29}$$

In the upper right hand corner of the Fig. 3.26 there appears a gap in the energy level spacings at large nuclear deformation. This gap occurs at 152 neutrons (indicated by encirclement) and may be the explanation for the "subshell" at 152 neutrons which has been detected as a result of the interpretation of spontaneous fission and alpha decay data. (Discussed in Chapters 4 and 11.)

Along the left hand edge of the figure are shown the energy levels derived from the ordinary shell model in the limit of an isotropic potential.

3.4.6 Use of Nilsson Orbitals. One chief purpose to which the Nilsson diagram has been put is the prediction of spins of ground state and low-lying states for odd-A nuclei. If one knows the deformation of the nucleus one merely moves out to the abscissa corresponding to this deformation and then moves vertically through the diagram counting each state twice (each state is doubly degenerate corresponding to the two possible signs of Ω_p) until the state corresponding to the last particle is reached. This state should then correspond to the observed ground state in spin and parity. If one or more of the Nilsson state lines happen to be quite close together it may be that the ground state will actually correspond to one of the other states because of the approximations

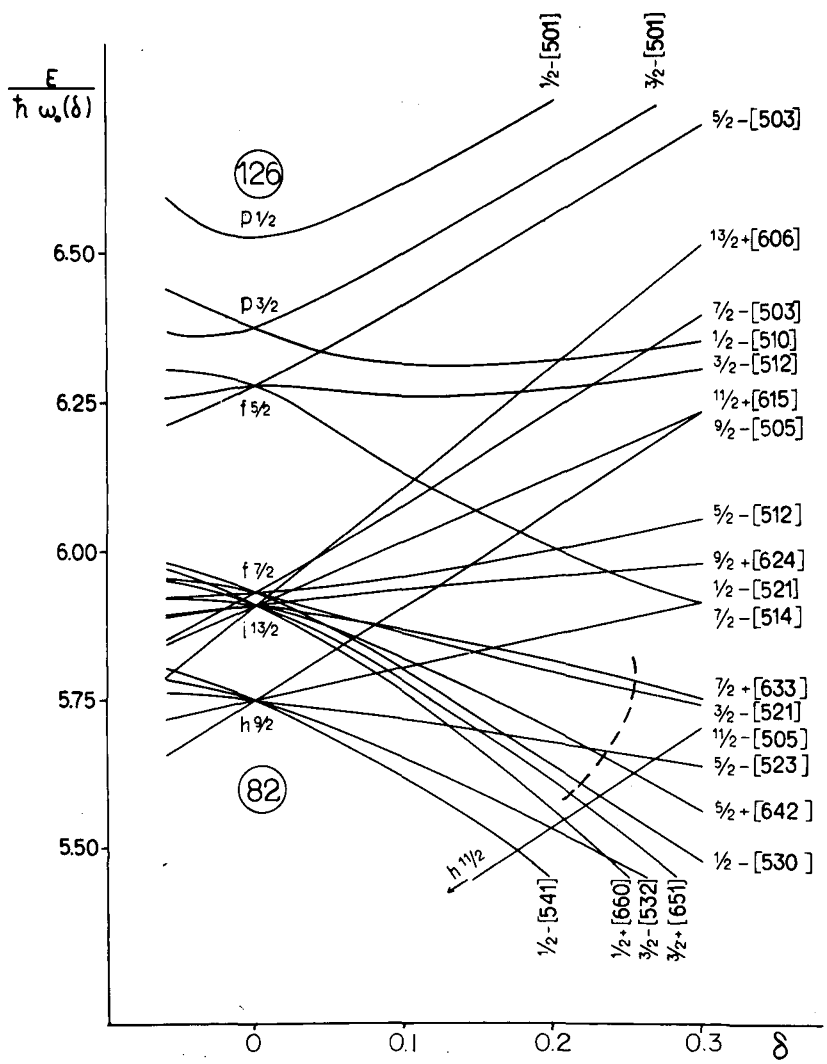

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Fig. 3.25 Energy level diagram for protons in the 82-126 shell as a function of prolate nuclear deformation given in terms of the parameter δ. Shell model assignments are given for the spherical case (δ = 0). For strong deformation each level is labeled with Ω and parity quantum numbers. In brackets the asymptotic quantum numbers [N, n_z, Λ] also are given. Reproduced from a figure obtained from Mottelson and Nilsson.

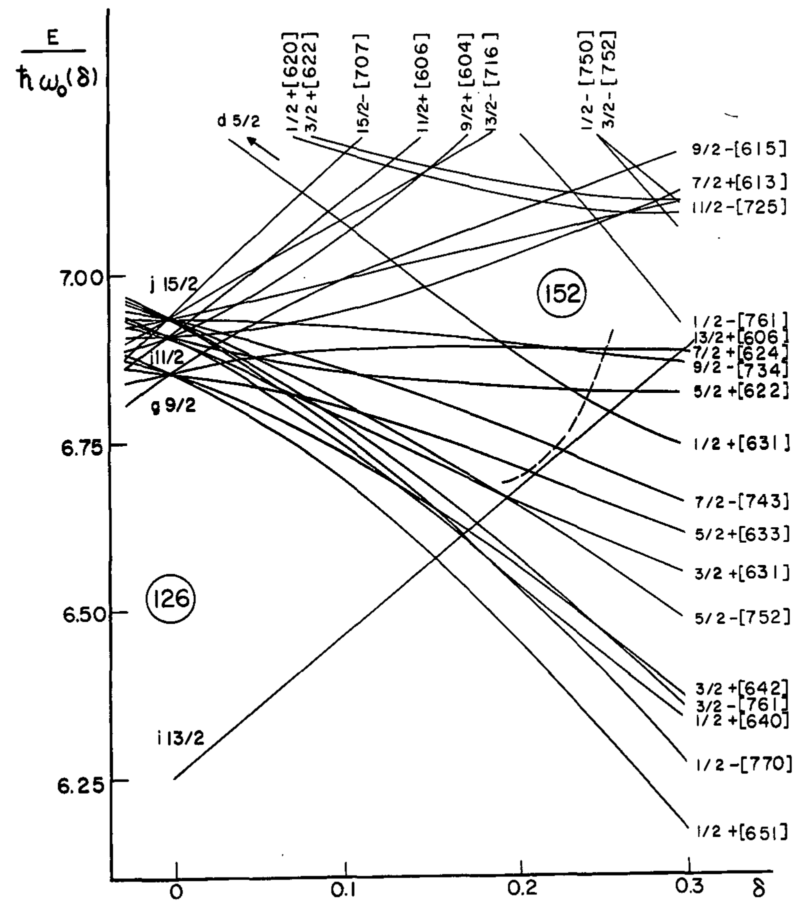

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Fig. 3.26 Energy level diagram for neutrons in the region $126 \leq N \leq 160$ as a function of prolate nuclear deformation. Shell model assignments are given for the degenerate levels in a spherical nucleus. For strong deformation each level is labeled with Ω, π and with the asymptotic quantum numbers $[N, n_z, \Lambda]$. The ordinate indicates the energy of the various levels. The dashed line indicates very roughly the deformations empirically found for nuclei with certain numbers of neutrons. Reproduced from a figure obtained from Mottelson and Nilsson.

in the treatment. In the case of such states lying close together all should be observed among the low-lying levels of the nucleus.

As an example we can use the nucleus Np^{237} which has 93 protons (11 past the shell closure at 82) and a deformation parameter $\delta = 0.20 - 0.25$. The 11 protons past the closed shell should fill in pairs into the lowest unoccupied levels of Fig. 3.25 until a single proton remains. This proton is expected to be in the next lowest available level and the characteristics of this level should determine the ground state characteristics of Np^{237}. This procedure leads us to the choice of the level indicated by the designation 5/2+ [642]. A spin and parity of 5/2+ are thus expected for Np^{237} in agreement with the facts. In addition, in the case of Np^{237}, there is a well-known excited level at 59.57 kev which has spin 5/2 and opposite parity to that of the ground state. There is strong evidence that this state is correctly assigned to the 5/2- [523] level which lies immediately above the 5/2+ [642] level for a δ value of 0.25.

In order to carry through this procedure for the assignment of ground state and low-lying levels, we must have some way to estimate the deformation of the nuclei involved. One way in which this can be done is by measurement of the electric quadrupole moment of the nucleus since this quantity is intimately related to the shape of the nucleus. The intrinsic quadrupole moment, Q_o, is related to the deformation parameter δ by the relation:

$$Q_o = \frac{4}{5} Z R_o^2 \delta (1 + \frac{1}{2}\delta + \dots) \qquad (3.31)$$

where Z is the nuclear charge number, δ is the deformation parameter and R_o is the mean radius for the nuclear charge distribution.

The intrinsic quadrupole moment Q_o is not identical with the spectroscopic quadrupole moment but is related to it by the following expression

$$q = \frac{3 K^2 - I(I + 1)}{(I + 1)(2I + 3)} Q_o \quad . \qquad (3.32)$$

Another way in which the intrinsic quadrupole moment, and indirectly the deformation, is often determined is by measurement of the reduced transition probability of an E2 transition within a rotational band. Such information is available, for example, from Coulomb excitation experiments. The pertinent relationship is given in Eqs. 3.40 and 3.41 in Sec. 3.4.7.

Still another independent estimate of δ may be obtained from a calculation of the equilibrium deformation of the nucleus on the basis of the single particle wave functions.[63,64,64a] The energy contributions of all particles beyond closed shells are added and the minimum value of this sum is determined as a function of the deformation. Such calculations are tedious but have been rewarding because of the agreement of calculated and experimental values of δ and Q_o particularly in the rare earth region. The results of such calculations by MOTTELSON and NILSSON[64] are shown in Fig. 3.27.

The Nilsson diagrams in Figs. 3.25 and 3.26 have been applied with remarkable success to correlation of measured ground state spins in the mass region $150 < A < 190$. Table 3.3 lists the measured spins together with the Nilsson predictions for nuclei whose deformation is known or can be interpolated. The Nilsson wave function assignments take into account all the data available on the decay scheme and level system of the nucleus. The assignment in most cases is precisely that given by the diagram, but in a few cases there are minor changes in the order of filling. The agreement of the observed spins with the small range of choices provided by the theory is excellent and thus supports the coupling scheme employed in the description of the nucleonic motion. It should be noted that the observed spin in several cases is distinctly different from the prediction of the simple shell model for a spherical nucleus. Later in this chapter (Sec. 3.5.5) we show how the Nilsson diagram has been strikingly successful in the classification of particle states of odd-A nuclei in actinide elements.

It will be noted that the low-lying levels predicted and found for deformed nuclei contain many deviations from the usual shell model rules as to the relation of spin and parity in a given shell. One may use the Nilsson diagram to predict or interpret isomerism but it is worth mentioning that chances of isomerism are reduced because of the many states of varying Ω which are produced from each shell model state.

3.4.7 Gamma Transition Rates in the Region of Deformed Nuclei. Collective motions of the nucleus and the interaction of collective and single particle motions have important implications for gamma ray transition probabilities. There are many deviations from the values expected for transitions between

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63. B. R. Mottelson and S. G. Nilsson, Phys. Rev. 99, 1615 (1955).

64. B. R. Mottelson and S. G. Nilsson, The Intrinsic States of Odd-A Nuclei Having Ellipsoidal Equilibrium Shape. Mat. Fys. Skr. Dan. Vid. Selsk. 1, No. 8 (1959).

64a. Z. L. Szymanski and D. Bes, to be published, 1962. This calculation includes pairing effects and covers the rare earth elements. The results show that Eq. 3.31 gives a correct value within a few per cent.

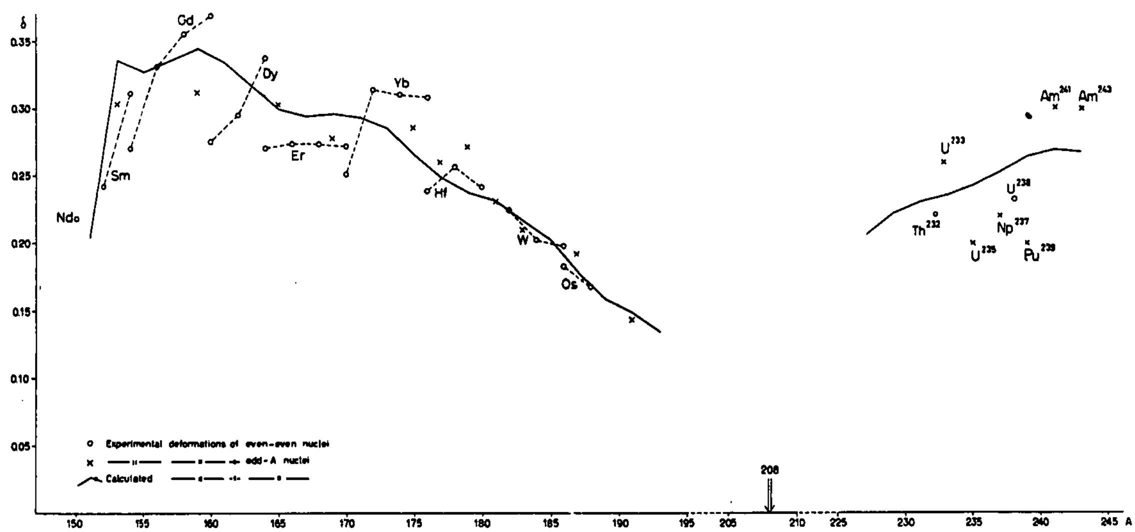

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Fig. 3.27 Comparison of equilibrium ground-state nuclear deformation calculated from Nilsson wave functions with experimental values. The solid line represents calculated values of the equilibrium deformation for the odd-A nuclei along the valley of beta stability. The experimental data correspond to δ values obtained by means of the equation,

$$Q_o = \frac{4}{5}\,\delta\, Z\, R_o^2\,(1 + 1/2\,\delta + \ldots)$$

by use of measured Q_o-values. These latter are based on observed E2 transition probabilities and their experimental uncertainty is usually of the order of 10-20%. Values of δ corresponding to odd-A nuclides are denoted by crosses. Even-even nuclei, denoted by circles, are included for completeness. The deformations of the americium isotopes are obtained from hfs measurements. This figure is reproduced from the paper of MOTTELSON and NILSSON.[64]

Table 3.3 Nilsson Wave Function Assignments to Ground States of Odd Mass Nuclei in the Rare Earth Region

Nucleus	Deformation in δ units	Nilsson orbital assignment to ground state	Observed spin
Odd-Proton Nuclei			
${}^{90}_{63}Eu^{153}$	0.30	5/2+ [413]	5/2
${}^{92}_{65}Tb^{157}$	0.31	3/2+ [411]	
${}^{94}_{65}Tb^{159}$	0.31	3/2+ [411]	3/2
${}^{96}_{65}Tb^{161}$		3/2+ [411]	
${}^{98}_{67}Ho^{165}$	0.30	7/2- [523]	7/2
${}^{100}_{69}Tm^{169}$	0.28	1/2+ [411]	1/2
${}^{102}_{69}Tm^{171}$	0.28	1/2+ [411]	1/2
${}^{104}_{71}Lu^{175}$	0.28	7/2+ [404]	7/2
${}^{106}_{71}Lu^{177}$	0.26	7/2+ [404]	7/2
${}^{108}_{73}Ta^{181}$	0.23	7/2+ [404]	7/2
${}^{108}_{75}Re^{183}$	0.21	5/2+ [402]	
${}^{110}_{75}Re^{185}$	0.19	5/2+ [402]	5/2
${}^{112}_{75}Re^{187}$	0.19	5/2+ [402]	
Odd-Neutron Nuclei			
${}^{91}_{64}Gd^{155}$ + ${}^{91}_{62}Sm^{153}$	0.31	3/2- [521]	3/2, 3/2
${}^{93}_{64}Gd^{157}$	0.31	3/2- [521]	3/2
${}^{95}_{66}Dy^{161}$	0.30	5/2+ [642]	5/2
${}^{97}_{66}Dy^{163}$	0.30	5/2- [523]	5/2
${}^{99}_{68}Er^{167}$ + ${}^{99}_{66}Dy^{165}$	0.29	7/2+ [633]	7/2, 7/2
${}^{101}_{70}Yb^{171}$ + ${}^{101}_{68}Er^{169}$	0.28	1/2- [521]	1/2, 1/2
${}^{103}_{70}Yb^{173}$ + ${}^{103}_{68}Er^{171}$	0.28	5/2- [512]	5/2, 5/2
${}^{105}_{72}Hf^{177}$	0.27	7/2- [514]	7/2
${}^{107}_{72}Hf^{179}$	0.26	9/2+ [624]	9/2
${}^{109}_{74}W^{183}$	0.21	1/2- [510]	1/2
${}^{109}_{76}Os^{185}$		1/2- [510]	

This table was prepared from a paper of MOTTELSON and NILSSON.[64]

intrinsic particle states as estimated from the simple spherical shell model.≠

Gamma transitions are classified according to the angular momentum λ, and the parity π, of the emitted radiation. The name multipole order is given to the quantity λ. If I_i and I_f are the angular momenta of the initial and final states, respectively, of the radiating nucleus then the conservation of angular momentum and parity impose the selection rules

$$|I_i - I_f| \leq \lambda \leq |I_i + I_f| \tag{3.33}$$

$$\pi = \pi_i \pi_f \quad . \tag{3.34}$$

The gamma transitions are classified as electric 2^λ pole and magnetic 2^λ pole depending on which of the following relations is followed in the parity change:

$$\pi = (-)^\lambda \text{ for electric } 2^L \text{ pole radiation}$$

$$\pi = (-)^{\lambda+1} \text{ for magnetic } 2^L \text{ pole radiation} \quad .$$

≠ The single particle gamma transition rates are usually expressed by the relationship presented orginally by Weisskopf [Phys. Rev. 83, 1073 (1951)] or in some modified form of this equation such as that given by Moszkowski [Beta and Gamma Ray Spectroscopy, K. Siegbahn, Editor, Chapter XIII (Interscience, 1955)]. For example, Moszkowski gives the following approximate expressions:

Multipolarity	T_{sp} in sec^{-1}
E1	$1.5 \times 10^{14}\ A^{2/3}\ E_\gamma^3$
M1	$2.8 \times 10^{13}\ E_\gamma^3$
E2	$1.6 \times 10^{8}\ A^{4/3}\ E_\gamma^5$
M2	$1.2 \times 10^{8}\ A^{2/3}\ E_\gamma^5$
E3	$1.1 \times 10^{2}\ A^{2}\ E_\gamma^7$
M3	$1.8 \times 10^{2}\ A^{4/3}\ E_\gamma^7$
E4	$5.0 \times 10^{-5}\ A^{8/3}\ E_\gamma^9$
M4	$1.5 \times 10^{-4}\ A^{2}\ E_\gamma^9$
E5	$1.6 \times 10^{-11} A^{10/3}\ E_\gamma^{11}$
M5	$7.5 \times 10^{-11} A^{8/3}\ E_\gamma^{11}$

These estimates do not include the statistical factors. E_γ is in Mev. Montalbetti [Can. J. Phys. 30, 660 (1952)] has also presented a useful nomagram for evaluating the Weisskopf formula.

The term multipolarity specifies the kind of radiation both as to class (electric or magnetic) and multipole order. The most commonly encountered types of radiation are listed in Table 3.4.

The transition probability depends on the multipolarity λ, the energy of the gamma radiation, ΔE, and the wave functions of the nuclear states involved in the transition.

A very general expression for the rate of gamma emission is the following:

$$T(\lambda) = \frac{8\pi(\lambda + 1)}{\lambda[(2\lambda + 1)\,!!]^2} \frac{1}{\hbar} \left(\frac{\Delta E}{\hbar c}\right)^{2\lambda+1} B(\lambda) \quad . \qquad (3.35)$$

Here T is the transition probability expressed in $\sec^{-1}$

λ is the multipole order

$\hbar$ is Planck's constant, h, divided by 2π

E is the energy of the transition

c is the speed of light

the notation $(2\lambda + 1)\,!! \equiv 1 \times 3 \times 5 \ldots \times (2\lambda + 1)$

and $B(\lambda)$ is the "reduced transition probability" .

The quantity $B(\lambda)$ contains within it all the dependence of the transition rates on the details of the nuclear structure.

In the case of the M1 and E2 radiations, which are most frequently encountered, the above equation reduces to

$$T(M1) = \frac{16}{9} \frac{\pi}{\hbar} \left(\frac{\Delta E}{\hbar c}\right)^3 B(M1) \qquad (3.36)$$

$$T(E2) = \frac{4}{75} \frac{\pi}{\hbar} \left(\frac{\Delta E}{\hbar c}\right)^5 B(E2) \quad . \qquad (3.37)$$

We consider first transitions within the same rotational band. Only M1 and E2 transitions are observed. If $|\Delta I| = 2$ the radiation is pure E2; for $|\Delta I| = 1$ both M1 and E2 can contribute. On the basis of single particle estimates of transition rates one would expect the M1 radiation to dominate. However, in the deformed nuclei the E2 transitions involve the collective effects of many nucleons and the transitions are greatly enhanced with respect to the single particle estimates. This enhancement can be of the order of a factor of 100. On the other hand M1 transitions are not enhanced significantly over the single particle estimates. This favoring of E2 over M1 transitions has frequently been a valuable piece of evidence in the assignment of nuclear levels and transitions to collective effects.

Table 3.4 Classification of Gamma Transitions

Class	Multipole Order and Parity Change					
	0	1	2	3	4	5
Electric	E0 no	E1 yes	E2 no	E3 yes	E4 no	E5 yes
Magnetic	-	M1 no	M2 yes	M3 no	M4 yes	M5 no

For transitions within a rotational band the initial and final states have the same intrinsic wave function and the reduced transition probability will depend only on such parameters as the intrinsic quadrupole moment Q_o, the K values, g_r and g_Ω, and statistical factors but not on the details of the intrinsic motion.

Transitions which connect the 0+, 2+, 4+ ..., levels of the ground state rotational band of an even-even nucleus must go by emission of electric quadrupole radiation since M1 radiation is forbidden. The reduced transition probability for I + 2 → I is given by the expression:

$$B(E2) = \frac{15}{32\pi} e^2 Q_o^2 \frac{(I+1)(I+2)}{(2I+3)(2I+5)} \qquad (3.38)$$

where e is electronic charge, I is the spin of the final state and Q_o is the intrinsic quadrupole moment. B(E2) of the first 2+ state can be obtained experimentally from the cross section for its Coulomb excitation.

In odd-A nuclei the reduced transition probabilities of M1 transitions within a rotational band are given by the equation:

$$\underset{I+1\to I}{B(M1)} = \frac{3}{4\pi}\left(\frac{e\hbar}{2Mc}\right)^2 (g_\Omega - g_r)^2 \frac{K^2(I+1-K)(I+1+K)}{(I+1)(2I+3)} \qquad (3.39)$$

where $K \neq 1/2$.

In this equation the symbols g_Ω and g_r refer to the gyromagnetic ratios of the intrinsic motion and the collective rotational motion, respectively. This equation holds only when $K \neq 1/2$; when $K = 1/2$ a second term must be added which allows for the partial decoupling of the intrinsic spin of the last odd nucleon from the rotational motion.

For E2 transitions within a rotational band of an odd-A nucleus,

$$\underset{I+1\to I}{B(E2)} = \frac{15}{16\pi} e^2 Q_o^2 \frac{K^2(I+1-K)(I+1+K)}{I(I+1)(2I+3)(I+2)} \quad . \qquad (3.40)$$

This refers to E2 transitions of the type I + 1 → I. For cross-over transitions of the type I + 2 → I, M1 radiation is forbidden and E2 goes via the relation

$$\underset{I+2\to I}{B(E2)} = \frac{15}{32\pi} e^2 Q_o^2 \frac{(I+1-K)(I+1+K)(I+2-K)(I+2+K)}{(I+1)(2I+3)(I+2)(2I+5)} \quad . \qquad (3.41)$$

The above equations have been taken from the work of BOHR and MOTTELSON.[65] The competition of M1 and E2 radiations in the de-excitation of rotational levels in an odd-A nucleus is strikingly different from the pattern of single particle transitions. In the latter case de-excitation would proceed by a cascade of almost pure M1 radiation since the transition probability for an M1 gamma ray is 10^3 or 10^4 fold greater than for an E2. For collective transitions the effect of the quadrupole moment enhances very considerably the E2 component and also permits E2 crossovers of the type given by Eq. 3.41.

We now turn to a brief consideration of transitions between different rotational bands. In the general case the intrinsic states are different. The transitions should chiefly involve only one nucleon and the transition rates should not exceed the single particle values. A possible exception is the special case of transitions between rotational bands based on different vibrational states of the same intrinsic particle configuration. In this special case the nuclear shape may be slightly different for the initial and final states so that the wave functions of all nucleons must change somewhat with a resultant very considerable reduction in the transition probability. Such transitions may go slower than expected from the single particle model.

An important selection rule in the K-quantum number must be considered in connection with transitions between two rotational bands. Insofar as the intrinsic particle motion is able to adjust to changes in nuclear orientation, i.e., insofar as the adiabatic approximation holds, K is a good quantum number and gamma transitions are restricted by the relation

$$|K_i - K_f| \equiv \Delta K \leq \lambda \tag{3.42}$$

where λ again is the multipole order of the transition.

In fact, it is sometimes observed that transitions do occur in apparent violation of the rule. This violation of the K-selection rule can be attributed to small K-admixtures in the initial and final state wave functions. Since only small parts of the wave function can contribute to the transition probability, the rate is greatly reduced from single particle estimates. A spectacular example of the operation of the K-selection rule occurs in the decay of Hf^{180m} where an E1 transition involves a ΔK of 8 and the transition probability is only about 10^{-15} of the single particle value.

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65. A. Bohr and B. R. Mottelson, Dan. Mat.-fys. Medd. 27, No. 16 (1953).

Several examples of K-forbiddeness have been noted among the heavy element nuclei. Consider for example the 5/2+ level 286 kev above the ground state in Pu^{239} (see the detailed discussion of the decay of Np^{239} in Sec. 9.1.11) which de-excites to three members of the ground state rotational band by well-characterized M1 transitions. The 286-kev level has a measured half-life of 1.1×10^{-9} seconds, a factor of 10^4 greater than predicted from the single particle formula. Since the K-quantum number of the ground state band is 1/2 and that of the 5/2+ level is 5/2 the retardation is readily explained by the K-selection rule. Another example is the triplet of electric quadrupole transitions from a spin 6 state at 1042 kev in Cm^{244} to the 4+, 6+, and 8+ members of the ground state rotational band in that nucleus. These E2 transitions are delayed by a factor of 10^7 compared to the single particle estimate. The K quantum number of the 1042-kev level is 6 while that of the ground state band is 0.

Another general consequence of the rotational spectra observed in the heavy elements is that the branching ratios of the reduced transition probabilities of transitions leading to several members of a rotational band can be readily evaluated. If one compares the reduced transition probabilities for the emission of radiation of a given multipolarity from an initial state i to several members f, f', f" ... of a rotational family the factors in the full expressions which involve the intrinsic details of the nucleus are the same. Hence when the ratio is formed these factors cancel out and one obtains a ratio which depends only on the geometrical factors; thus,

$$\frac{B(\lambda, I_i \rightarrow I_f)}{B(\lambda, I_i \rightarrow I_{f'})} = \frac{\langle I_i \lambda K_i (K_f - K_i) | I_i \lambda I_f K_f \rangle^2}{\langle I_i \lambda K_i (K_f - K_i) | I_i \lambda I_{f'} K_f \rangle^2} \qquad (3.43)$$

(valid when $|K_i - K_f| \leq \lambda$).

In this expression $B(\lambda, I_i \rightarrow I_f)$ is the symbol of the reduced transition probability for a transition of multipole order λ from a state of initial angular momentum I_i to one of final angular momentum I_f. The symbol $\langle I_i \lambda K_i (K_f - K_i) | I_i \lambda I_f K_f \rangle$ is the vector addition coefficient for the combination of the angular momenta I_i and λ to form the resultant I_f. A description of these geometrical factors is given by CONDON and SHORTLEY.[66] These

66. E. U. Condon and G. H. Shortley, The Theory of Atomic Spectra (University Press, Cambridge, 1953).

vector addition coefficients are often referred to as Clebsch-Gordan coefficients. Useful tables of these coefficients have been prepared.[67-71]

Equation 3.43 also holds when the initial state i is a higher member of the rotational family containing states f and f' as lower members.

To evaluate the ratio it is necessary only to know the multipole order, the I and K values of the states involved and to look up the Clebsch-Gordan coefficients in a table. If the transition can go by mixed multiples it is necessary to correct for this. Equation 3.43 has been helpful in classifying and interpreting nuclear states. For example, the identification of negative parity states in even-even nuclei is aided by an evaluation of this ratio for decay to two members of the ground state rotational family (see Sec. 3.5.3).

The cross section for Coulombic excitation of particular states is proportional to $B(\lambda, I_i \rightarrow I_f)$. Hence it is possible to check the reliability of the ratio Eq. 3.43 by the ratio of the cross sections for the excitation of two members of a rotational band.

Equation 3.43 is quite general and is not limited to gamma transitions. It is also applicable to beta decay from an initial nucleus of spin I_i to two members of a rotational band with spins I_f and $I_{f'}$. Furthermore, it is very useful in an analysis of alpha decay intensities to several members of a rotational series in a daughter nucleus.[72] For an application of this see the discussion of Cm^{243} in Sec. 9.4.7 of Chapter 9.

In addition to the K-selection rule gamma transitions are subject to some selection rules in the asymptotic quantum numbers which are intrinsic properties of wave functions of an anisotropic harmonic oscillator. ALAGA,[73] in particular,

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67. A. Simon, Oak Ridge National Laboratory Report ORNL-1718 (June 1954).

68. B. J. Sears and M. G. Radtke, Algebraic Table of Clebsch-Gordan Coefficients, Chalk River Laboratory Report TPL-75 (August 1954).

69. J. K. Lum, J. H. Light, and F. Asaro, Lawrence Radiation Laboratory Report UCRL-9679 (1961), unpublished. This report evaluates the squared brackets of Eq. 3.43 for all transitions of interest connecting half-integral values of K.

70. M. Simmons, University of California Radiation Laboratory Report UCRL-8554 (1958).

71. E. R. Cohen, Tables of the Clebsch-Gordan Coefficients, NAA-SR-2123 (1958).

72. A. Bohr, P. Froman and B. Mottelson, Dan. Mat.-fys. Medd. 29, 10 (1955).

73. G. Alaga, Nuclear Phys. 4, 625 (1957).

has considered the influence of the Nilsson quantum numbers N, n_z and Λ. In the limit of very strong deformations N, the principal oscillator shell, n_z, the number of oscillator quanta along the symmetry axis of the spheroidal nucleus, and Λ, the projection of the orbital angular momentum of the odd particle along the symmetry axis, become constants of the particle motion. Because of the approximations of the treatment these constants of the motion are only approximate and the selection rules associated with these asymptotic quantum numbers will hinder rather than forbid the transitions. This hindrance is not as strong as in the case of violation of the K-selection rule and ordinarily will amount to about a factor of 10.

We quote here from the tables published by ALAGA.[73] Similar tables with slight differences in details have been developed by others.[74-76]

3.4.8 Beta Transition Rates in the Region of Deformed Nuclei. Beta decay transition rates are subject to selection rules in spin and parity change. The usual classification of beta transformations by log-ft values is summarized in Table 3.6.

ALAGA[73,77] has discussed a number of other selection rules in the asymptotic quantum numbers. These rules, applicable for transitions between non-spherical nuclei, are summarized in Table 3.7.

The analysis by ALAGA[77] of beta decay log-ft values in the rare-earth element region has indicated that those beta transitions which violate the asymptotic selection rules are in general retarded by about a factor of ten. These selection rules are applicable also in the heavy element region and can be used to interpret observed facts providing that the initial and final states in a given transition can be identified with one of the Nilsson states. As an example, HOLLANDER[78] has shown the application of these asymptotic selection rules in the beta decays of Np^{239} and Am^{239} leading to various levels of Pu^{239}.

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74. B. R. Mottelson and S. G. Nilsson, The Intrinsic States of Odd-A Nuclei Having Ellipsoidal Equilibrium Shape, Mat. Fys. Skr. Dan. Vid. Selsk. 1, No. 8 (1959).

75. D. Strominger and J. O. Rasmussen, Nuclear Phys. 3, 197 (1957).

76. Chasman and Rasmussen as quoted by Rasmussen, Canavan and Hollander, Table XI, Phys. Rev. 107, 141 (1957).

77. G. Alaga, Preliminary note in Phys. Rev. 100, 432 (1955). For fuller treatment see Ref. 73.

78. J. M. Hollander, Phys. Rev. 105, 518 (1957).

Table 3.5 Selection Rules for Electromagnetic Transitions
(Alaga, Nuclear Phys. 4, 625 (1957))

Multipole	Operator	Selection rules			
E1	rY_1	$\Delta\Omega=0$	$\Delta\Lambda=0$	$\Delta n_z=$	$\Delta N=\pm1$
		$\Delta\Omega=$	$\Delta\Lambda=\pm1$	$\Delta n_z=0$	$\Delta N=\pm1,\mp1$
E'1 (see note)	$(\bar{\sigma}\times\bar{r})_1 \approx T_{11}(\bar{\sigma},\bar{r})$	$\Delta\Omega=0,\pm1$	$\Delta\Lambda=\pm1$	$\Delta n_z=0$	$\Delta N=\pm1,\mp1$
		$\Delta\Omega=\pm1$	$\Delta\Lambda=0$	$\Delta n_z=$	$\Delta N=\pm1,\mp1$
E2	r^2Y_2	$\Delta\Omega=0$	$\Delta\Lambda=0$	$\Delta n_z=$	$\Delta N=\pm2,\mp2$
		$\Delta\Omega=$	$\Delta\Lambda=0,\pm2$	$\Delta n_z=0$	$\Delta N=0,\pm2,\mp2$
		$\Delta\Omega=$	$\Delta\Lambda=\pm1$	$\Delta n_z=^{\pm1}_{\mp1}$	$\Delta N=0^{\pm2}_{\mp2}$
E3	r^3Y_3	$\Delta\Omega=$	$\Delta\Lambda=\pm3$	$\Delta n_z=0$	$\Delta N=\pm1,\mp1,\pm3,\mp3$
		$\Delta\Omega=$	$\Delta\Lambda=\pm2$	$\Delta n_z=^{\pm1}_{\mp1}$	$\Delta N=\pm1,\mp1\,^{\pm3}_{\mp3}$
M1	σ_1	$\Delta\Omega=0,\pm1$	$\Delta\Lambda=0$	$\Delta n_z=0$	$\Delta N=0$
	$L_1 \approx T_{11}(\bar{r},\bar{\nabla})$	$\Delta\Omega=0$	$\Delta\Lambda=0$	$\Delta n_z=0$	$\Delta N=0$
		$\Delta\Omega=$	$\Delta\Lambda=\pm1$	$\Delta n_z=^{\pm1}_{\mp1}$	$\Delta N=0^{\pm2}_{\mp2}$
M2	$T_{21}(\bar{\sigma},\bar{r})$	$\Delta\Omega=0,\pm1,\pm2$	$\Delta\Lambda=\pm1$	$\Delta n_z=0$	$\Delta N=\pm1,\mp1$
		$\Delta\Omega=0,\pm1$	$\Delta\Lambda=0$	$\Delta n_z=$	$\Delta N=\pm1,\mp1$
	$T_{21}(\bar{r},\bar{L})$	$\Delta\Omega=$	$\Delta\Lambda=0,\pm2$	$\Delta n_z=^{\pm1}_{\mp1}$	$\Delta N=^{\pm3}_{\mp3}\pm1,\mp1$
		$\Delta\Omega=$	$\Delta\Lambda=\pm1$	$\Delta n_z=0$	$\Delta N=\pm1,\mp1$
		$\Delta\Omega=$	$\Delta\Lambda=\pm1$	$\Delta n_z=^{\pm2}_{\mp2}$	$\Delta N=^{\pm3}_{\mp3}\pm1,\mp1$
M3	$T_{32}(\bar{\sigma},\bar{r})$	$\Delta\Omega=\pm2$	$\Delta\Lambda=\pm1$	$\Delta n_z=^{\pm1}_{\mp1}$	$\Delta N=0^{\pm2}_{\mp2}$
		$\Delta\Omega=\pm2,\pm3$	$\Delta\Lambda=\pm2$	$\Delta n_z=0$	$\Delta N=\pm0,\pm2,\mp2$
M3	$T_{32}(\bar{r},\bar{L})$	$\Delta\Omega=$	$\Delta\Lambda=\pm2$	$\Delta n_z=^{\pm2}_{\mp2}$	$\Delta N=0^{\pm4}_{\mp4},\pm2,\mp2$
		$\Delta\Omega=$	$\Delta\Lambda=\pm2$	$\Delta n_z=0$	$\Delta N=0,\pm2,\mp2$
		$\Delta\Omega=$	$\Delta\Lambda=\pm3$	$\Delta n_z=^{\pm1}_{\mp1}$	$\Delta N=0^{\pm4}_{\mp4},\pm2,\mp2$

NOTE: The magnitude of the matrix elements is smaller in the E'1 case than in the E1 case for energies ~ 100 kev by a factor of ~ 10^3.

Table 3.6 Selection Rules for Beta Decay

Type	Parity change	ΔI	log ft
Allowed (favored)	no	0 or 1	3 to 4
Allowed (unfavored)	no	0 or 1	4 to 6
First forbidden	yes	0 or 1	6 to 8
First forbidden (unique)	yes	2	(8 to 9)
Second forbidden	no	2	10.3 to 13.3
Second forbidden (unique)	no	3	12
Third forbidden	yes	3	18
Third forbidden (unique)	yes	4	15 to 16

Table 3.7 Selection Rules in the Asymptotic Quantum Numbers for Beta Decay
(Alaga, Nuclear Phys. 4, 625 (1957))

Type of Transition	Operators	Selection Rules			
Allowed	1	$\Delta\Omega = 0$	$\Delta\Lambda = 0$	$\Delta n_z = 0$	$\Delta N = 0$
	$\bar{\sigma}$	$\Delta\Omega = 0, \pm 1$	$\Delta\Lambda = 0$	$\Delta n_z = 0$	$\Delta N = 0$
First forbidden	$\bar{\sigma}\cdot\bar{r},\ \bar{\sigma}\cdot\bar{\nabla}$	$\Delta\Omega = 0$	$\Delta\Lambda = \pm 1$	$\Delta n_z = 0$	$\Delta N = \pm 1, \mp 1$
		$\Delta\Omega = 0$	$\Delta\Lambda = 0$	$\Delta n_z =$	$\Delta N = \pm 1$
	$\bar{\sigma}\times\bar{r},\ \bar{\sigma}\times\bar{\nabla}$	$\Delta\Omega = 0, \pm 1$	$\Delta\Lambda = \pm 1$	$\Delta n_z = 0$	$\Delta N = \pm 1, \mp 1$
		$\Delta\Omega = \pm 1$	$\Delta\Lambda = 0$	$\Delta n_z =$	$\Delta N = \pm 1, \mp 1$
	$\bar{r},\ \bar{\nabla}$	$\Delta\Omega = 0$	$\Delta\Lambda = 0$	$\Delta n_z =$	$\Delta N = \pm 1$
		$\Delta\Omega =$	$\Delta\Lambda = \pm 1$	$\Delta n_z = 0$	$\Delta N = \pm 1, \mp 1$
	$T_{21}(\bar{\sigma}, \bar{r})$	$\Delta\Omega = 0, \pm 1, \pm 2$	$\Delta\Lambda = \pm 1$	$\Delta n_z = 0$	$\Delta N = \pm 1, \mp 1$
		$\Delta\Omega = 0, \pm 1$	$\Delta\Lambda = 0$	$\Delta n_z =$	$\Delta N = \pm 1, \mp 1$
Second forbidden	$r^2 Y_2$	$\Delta\Omega = \pm 2$		$\Delta n_z = 0$	$\Delta N = 0, \pm 2, \mp 2$
	$T_{22}(\bar{\sigma}, \bar{r})$	$\Delta\Omega = \pm 2$	$\Delta\Lambda = \pm 1$	$\Delta n_z = \pm 1$ ∓ 1	$\Delta N = \pm 2$ 0 ∓ 2
		$\Delta\Omega = \pm 2$	$\Delta\Lambda = \pm 2$	$\Delta n_z = 0$	$\Delta N = 0, \pm 2, \mp 2$
	$T_{21}(\bar{r}, \bar{\nabla})$	$\Delta\Omega =$	$\Delta\Lambda = \pm 2$	$\Delta n_z = 0$	$\Delta N = 0, \pm 2, \mp 2$
	$T_{21}(\bar{r}, \bar{\sigma}\times\bar{\nabla})$	$\Delta\Omega =$	$\Delta\Lambda = \pm 2$	$\Delta n_z = 0$	$\Delta N = 0, \pm 2, \mp 2$
		$\Delta\Omega = \pm 2$	$\Delta\Lambda = \pm 1$	$\Delta n_z = \pm 1$ ∓ 1	$\Delta N = \pm 2$ 0 ∓ 2
	$T_{32}(\bar{\sigma}, \bar{r})$	$\Delta\Omega = \pm 2$	$\Delta\Lambda = \pm 1$	$\Delta n_z = \pm 1$ ∓ 1	$\Delta N = \pm 2$ 0 ∓ 2
		$\Delta\Omega = \pm 2, \pm 3$	$\Delta\Lambda = \pm 2$	$\Delta n_z = 0$	$\Delta N = 0, \pm 2, \mp 2$

$T_{21} = B_{ij}$ $\quad r^2 Y_2 = R_{ij}$ $\quad T_{22} = T_{ij}$ $\quad T_{23} = S_{ijk}$

Beta transitions are also subject to the K-selection rule expressed by Eq. 3.42. The K-selection rules governing beta transitions are of a more complicated type than those of the γ-transitions due to the different types of β-operators involved in the same transition. Thus, for allowed β-transitions the Fermi transitions are limited to $\Delta K = 0$, while Gamow-Teller transitions allow $\Delta K = 0, 1$. For unique transitions of nth order of forbiddeness the selection rules are

$$|I_i - I_f| \leq n + 1 \leq |I_i + I_f|$$
$$\Delta K \leq n + 1 \quad . \tag{3.44}$$

For beta transitions to several members of a rotational family of levels the ratio of reduced transition probabilities can be evaluated from vector addition coefficients as mentioned above in connection with the discussion of Eq. 3.43 to the extent that the transition involves only one type of beta operator.

3.4.9 Alpha Transition Rates and the Unified Model. The special subject of alpha decay is discussed in considerable detail in the next chapter. Since the majority of alpha-particle emitters have highly deformed nuclei, it is natural that many features of alpha processes can be discussed with reference to unified model characteristics of nuclei. We wish to call attention only to a few points here.

Alpha decay transition probabilities are usually thought of as consisting of two parts, the first being the alpha formation probability which depends on the details of the nuclear interior and the second being the barrier penetrability through the nuclear surface region. We can appreciate at once that the theory of barrier penetrability for a spherical and a markedly nonspherical nuclear surface must be quite different. A complete theory must consider not only the penetration of the alpha particle through the anisotropic Coulomb barrier, but also the exchange of angular momentum between the alpha particle and the residual nucleus as the alpha particle passes through the barrier; this exchange is caused by the large electric quadrupole moment of the daughter nucleus.

Insofar as the interior of the nucleus is correctly described by an independent particle system with individual wave functions given by the Nilsson orbitals, say, it might be anticipated that alpha particle formation from individual nucleons might most readily occur from two pair of protons and neutrons occupying states differing only in the sign of Ω. This type of alpha decay follows the selection rules $\Delta K = 0$ and no change in parity and is classified as favored. In an even-even nucleus the alpha decay takes place primarily to the ground state of the daughter and to the levels of rotational excitation of this state.

The concept of favored alpha decay is especially meaningful in odd-A nuclei. In the Nilsson model the ground state of an odd-A nucleus is largely characterized by the orbital of the last odd particle. Since this particle is unpaired, it would not be incorporated into the newly-formed alpha particle and the daughter nucleus would be produced in a state in which the last odd particle is moving in the same orbital as it was in the parent. Hence the favored alpha transition will connect parent-daughter states with the same Ω and Nilsson wave function. This state in some cases will be the ground state of the daughter nucleus, but in general, will be an excited state of the daughter. This concept of favored alpha decay[72] in odd-A nuclei makes it possible to discern much regularity in the otherwise highly irregular complex structure of such alpha-emitters.

The last comment we wish to make about alpha decay here concerns the application of Eq. 3.43 when alpha decay proceeds to several members of a rotational band in an odd-A nucleus. If the angular momentum of the alpha wave is 2, 4, 6 or more, Eq. 3.43 expresses the reduced transition probability ratios for population of all daughter states which can be reached by an $\ell = 2$ wave, or an $\ell = 4$ wave, etc. This regularity has proved useful in the analysis of complex structure patterns in odd-A alpha emitters.

3.4.10 Particle Excitation in Even-Even Nuclei. All of our discussion of the application of Nilsson intrinsic states of particle motion has pertained to odd-A nuclei. It is natural to inquire about the application of the independent particle model to even-even nuclei having non-spherical shape. MOTTELSON and NILSSON[79] and BOHR, MOTTELSON and PINES[80] have noted that when one does this he immediately finds that there is a qualitative difference between the observed

79. B. R. Mottelson and S. G. Nilsson, The Intrinsic States of Odd-A Nuclei Having Ellipsoidal Equilibrium Shape, Mat. Fys. Skr. Dan. Vid. Selsk. 1, No. 8 (1959).

80. A. Bohr, B. R. Mottelson and D. Pines, Phys. Rev. 110, 936 (1958).

spectra of these nuclei and what would be predicted by a simple application of the model. If the nucleons were moving quite independently, there should be low-lying excited states in even-even nuclei corresponding to the breaking of pairs or the pairwise excitation of nucleons. The excitation energy of this state would be expected to be of the same order of magnitude as that of the low-lying intrinsic states of neighboring odd-A nuclei. In actuality, states of intrinsic excitation are not found in even-even nuclei until energies of several fold greater are reached. The lowest levels of intrinsic excitation are usually not found below 1 or 2 Mev. This energy gap in the even-even nuclei must be attributed in some way to an essential correlation in the motion of a pair of nucleons which fill the degenerate orbits labeled by +K and -K, i.e., the effect must be attributed to residual forces between the nucleons.

It is beyond the scope of our discussion to review the many theoretical papers which have contributed to the attack on this problem. Various types of short range forces have been tried with 2 particle or multi-particle systems to show that the main features of the particle excitation spectra of even nuclei--namely, the energy gap--can be accounted for. Perhaps the most generally useful of these theoretical approaches has been the "pairing" force correlation whose development was stimulated by the BARDEEN, COOPER and SCHRIEFFER[20] theory of conductivity. The pairing force concept was formulated for the nuclear problem by BELYAEV.[22] An excellent summary of the superfluid model of nuclear structure is given by NILSSON and PRIOR.[80a]

One of the fundamental approximations of this model is that the particles beyond a closed shell exert no force on each other unless the pairs of nucleons which are identical in all respects except for the opposite direction of their angular momenta components K along the nuclear symmetry axis are paired off to total angular momentum zero. Such pairs are subject to a force which scatters pairs of particles into higher-lying levels which would be unoccupied if this pairing force were not operating. The new wave function for the even nucleus then represents the ground state as one with a diffuse energy surface; the amplitude of the last few pairs of particles is spread over several orbitals near the Fermi surface.

80a. S. G. Nilsson and O. Prior, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 32, No. 16 (1961).

Any excited state which involves the promotion of one or a pair of particles to low-lying orbitals above the Fermi energy makes these orbitals unavailable for the scattered particles and necessarily spoils some of the pair correlation. The extra energy which must be supplied to make up that lost by disturbance of the pair correlation energy is about equivalent to the gap energy.

Two other important characteristics of nuclei which are "explained" by the superfluid model are the even-odd mass difference and the magnitude of the moment of inertia associated with collective rotation.[80a]

Above the energy gap we may expect to find levels of various types of particle excitation. Below about 2.5 Mev in heavy nuclei these should be chiefly of the two following types.

1) Lifting of a pair of like particles from a lower to a higher orbital leaving them coupled to total angular momentum zero.
2) Uncoupling of a pair of like nucleons and promotion of one or both nucleons to different orbitals labeled by Ω_1 and Ω_2. Recoupling of these particles to $\Omega = \Omega_1 \pm \Omega_2$.

We reserve to Sec. 3.5.6 a discussion of the few data and analyses bearing on the interpretation of such particle excitations in heavy nuclei.

3.4.11 The Asymmetric Rotor Model. In the discussion up to this point, we have assumed throughout that the shape of the nucleus in its ground state possesses axial symmetry. We have considered vibrational states called γ-vibrations in which this axial symmetry is destroyed, but the nuclear ground state has been assumed to be of spherical or spheroidal configuration. It is of interest to consider the consequences of dropping this assumption of axial symmetry. This has been done in a series of papers by DAVYDOV and his co-workers.[81-83]

In the beginning, DAVYDOV and FILIPPOV[81] considered the rotational states of a nucleus with an ellipsoidal shape under the adiabatic approximation that

81. A. S. Davydov and G. F. Filippov, Nuclear Phys. 8, 237 (1958).

82. A. S. Davydov and V. S. Rostovsky, Nuclear Phys. 12, 58 (1959).

83. A. S. Davydov and A. A. Chaban, Nuclear Phys. 20, 499 (1960).

83a. A. S. Davydov, Nuclear Phys. 24, 682 (1961).

83b. A. S. Davydov, V. S. Rostovsky and A. A. Chaban, Nuclear Phys. 27, 134 (1961).

there was no interchange of energy with intrinsic or vibrational states during rotation. The Hamiltonian operator for such a rotation is

$$H = \sum_{\lambda=1}^{3} \frac{A \bar{J}_\lambda^2}{2 \sin^2(\gamma - \frac{2\pi}{3} \lambda)} \qquad (3.45)$$

where $A = \frac{\hbar^2}{4B \beta^2}$.

In these equations β and γ are the shape parameters discussed in Sec. 3.3.2 above (see Fig. 3.13, for example). B is a mass parameter. The $\bar{J}_\lambda$ are operators of the projection of the nuclear angular momentum J on the axes of the coordinate system which are chosen to correspond to the principle axes of the ellipsoid. For $\gamma \neq 0$ or $\pi/3$, Eq. 3.45 represents the rotation of an asymmetric top. The authors solved the Schrödinger equation and derived analytical expressions for the allowed states as a function of γ. A summary of the rotational states is shown in Fig. 3.28.

We note that at $\gamma = 0$ we have a 2+, 4+, 6+ series of states which is identical in all respects with the ground state band of the deformed nucleus discussed previously in this chapter. At the $\gamma = 0$ limit the energy spacings of these levels follow the $I(I + 1)$ rule. As deviations from axial symmetry increase the energies of these levels increase slightly. The minimum value for the ratio of the 4+ to the 2+ level is 8/3. As γ increases a second series of levels with the spin sequence 2+, 3+, 4+, 5+, etc., drops rapidly to low energy.

In experimental spectra of heavy nuclei it is not uncommon to find a 2+, 3+, 4+ series of levels at about 1 Mev of excitation. According to the earlier discussion in this section the 2+ state would probably be identified with the γ-vibration of a nucleus about a $\gamma = 0$ equilibrium value. The higher spin values would be pictured as rotational excitation superimposed on this vibrational state. In the asymmetric rotor model this 2+ state is interpreted instead as a rotational state. In the interpretation of experimental spectra one may use the observed ratio of the second 2+ to the first 2+ state to define the value of the parameter γ either by use of the figure or of the analytical expression,

$$\frac{(E2+)\ \text{second}}{(E2+)\ \text{first}} = \frac{(1 + \sqrt{1 - 8/9 \sin^2 (3\gamma)}\ )}{(1 - \sqrt{1 - 9/2 \sin^2 (3\gamma)}\ )} \ . \qquad (3.46)$$

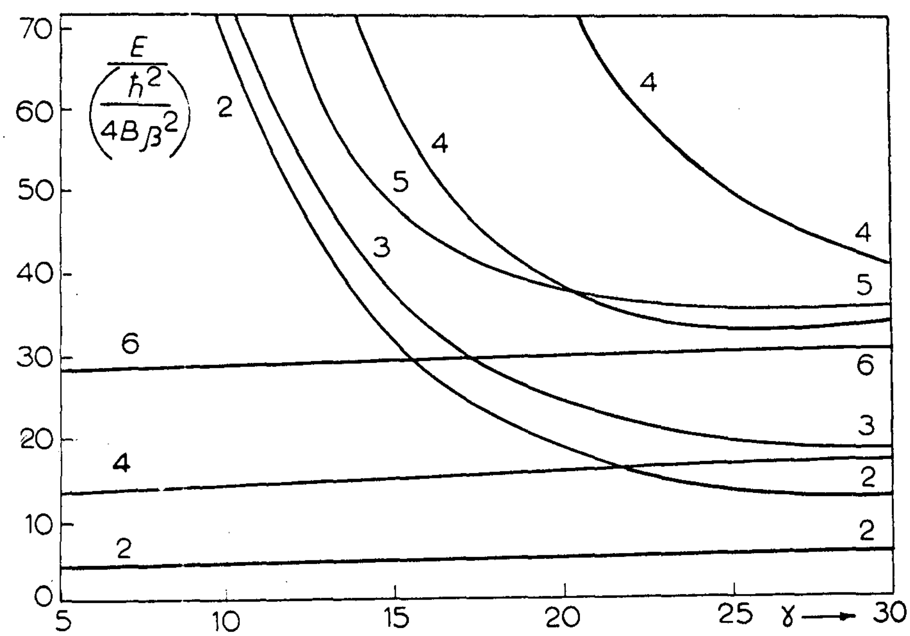

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Fig. 3.28 Prediction of the rotational spectrum by Davydov and Filippov for a rigid asymmetric rotor as a function of the deformation parameter γ, which measures the deviation of an ellipsoid from axial symmetry.

With the value of γ so determined one can calculate the energies of the other states. Expressions for the transition probabilities of gamma transitions between the states are also dependent upon the parameter γ.

The fact that most of the quantities predicted by this model are closely related to the parameter γ which in turn is related to the ratio of the first and second 2+ states has encouraged a number of comparisons[84,85] of experimental data as a function of this energy ratio. Figure 3.29 is an example of such a correlation. One thing such attempts at data correlation have revealed is a rather smooth trend in the excited states over a wide range of nuclear mass numbers stretching from the two extremes of spherical nuclei near closed shells to strongly deformed nuclei far from closed shells. This has encouraged the further development of the model.

The asymmetric rotor model has been extended to include the effects of interaction of rotational and vibrational modes by DAVYDOV and CHABAN.[83] It is assumed that the nucleus has an equilibrium value of β and of γ about which it can execute vibrations. The problem is to determine the coupling of these vibrations to the rotation because of centrifugal forces. In the paper cited account was taken only of the β vibrations.

The classical energy corresponding to β vibrations of the nuclear surface and to rotation of the nucleus can be written in the form

$$E = \frac{1}{2} B\left(\frac{d\beta}{dt}\right)^2 + V(\beta) + \frac{1}{2} \sum_{\lambda=1}^{3} \frac{J_\lambda^2}{4B\,\beta^2 \sin^2(\gamma - 2/3\,\pi\lambda)} \tag{3.47}$$

where β and γ are the deformation parameters previously defined (see Fig. 3.13)

B is a mass parameter

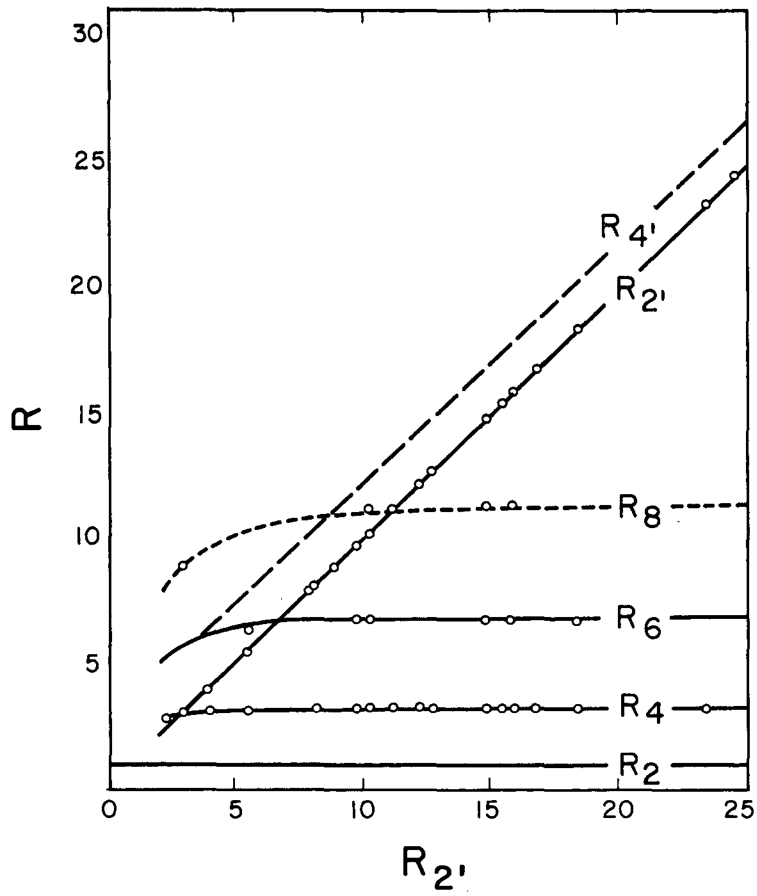
$V(\beta)$ is the potential energy of the β vibrations

$4B\,\beta^2 \sin^2(\gamma - 2/3\,\pi\lambda)$ are the moments of inertia on the three principle axes

J_λ is the projection of the nuclear angular momentum on the principal axes.

In its quantum mechanical form Eq. 3.47 is transformed to

84. C. A. Mallmann, Phys. Rev. Letters 2, 507 (1959).
85. C. A. Mallmann and A. K. Kerman, Nuclear Phys. 16, 105 (1960).



MU-19287

Fig. 3.29 Relative energy of excited states in even-even nuclei as a function of $R_{2'}$ which is the ratio of the energies of the second and the first 2+ states. R is the energy ratio of a given state to the first 2+ state. Figure prepared by H. E. Bosch from more detailed figure of MALLMANN and KERMAN.[85] Points plotted represent nuclei of osmium, tungsten, several rare-earth elements, thorium, uranium, plutonium and californium.

$$H = \frac{\hbar^2}{2B} \left\{ \frac{1}{\beta^3} \frac{\partial}{\partial\beta} \left(\beta^3 \frac{\partial}{\partial\beta} \right) - \sum_\lambda \frac{J_\lambda^2}{4\, \beta^2 \sin^2(\gamma - 2/3\, \pi\lambda)} \right\} + V(\beta) \quad . \qquad (3.48)$$

DAVYDOV and CHABAN[83] determined the eigenfunctions and eigenvalues of this Hamiltonian. In the general case collective excitation corresponding to a definite value of the total angular momentum J is a complicated combination of vibrational and rotational motions, but under certain conditions there is an approximate separation of the two. The authors introduce a parameter μ whose value serves as a guide to the extent that the rotational and vibrational modes are coupled. With the aid of this new parameter and the older parameter γ the authors were able to make many significant correlations of experimental spectra for a rather wide range of nuclei. MALLMANN and KERMAN[85] independently considered the interactions of beta vibrations with the rotations of an asymmetric quadrupole rotor.

We shall not discuss the asymmetric rotor model or its correlations with experimental data in any more detail in this chapter because the published analyses of the excitation spectra of the trans-lead nuclei have been almost exclusively related to the models described earlier which presuppose axial symmetry. It is necessary to keep in mind, however, that the Davydov model or models may provide alternate interpretations of many of the results.

3.4.12 Summary of Essential Features of Unified Model. Table 3.8 provides a brief overall summary of the essential features of the unified description of nuclear spectra. Nuclei are classified in three broad groups: the closed shell group, the strongly deformed group and an intermediate group easily deformed from a spherical ground state.

Table 3.8 General Characteristics of Nuclides of the Heaviest Elements as Classified by the Bohr-Mottelson Unified Model of the Nucleus

Region	Region of applicability	Basic description	Role of		
			Intrinsic particle states	Vibrational motion	Rotational motion
I	Nuclides near Pb^{208}	Nucleus strongly prefers spherical symmetry. Stiff toward shape changes. Very weak coupling of nucleonic and collective motion. Simple shell model is applicable to nuclides one nucleon removed from Pb^{208}. Simple shell model, corrected for residual interparticle effects, gives good detailed description for several other nuclides near Pb^{208}.	In odd-A nuclei all or nearly all low-lying levels assigned to excitation of intrinsic states.	Each intrinsic state may have vibrational structure but the energy of excitation of these will be high and interaction with particle states will be severe.	No rotational structure of levels expected.
II	Nuclides removed by several mass numbers from Pb^{208}. Above A = 208, the region extends up to about A = 222.	Nucleus prefers spherical symmetry but is soft to shape changes. Weak to moderate coupling of nucleonic and collective motion. Equations of motion are complex. Theoretical treatment is most uncertain in this region for odd-A nuclei. In even-even nuclei expect to see collective oscillations of harmonic oscillator or slightly anharmonic oscillator type.	Intrinsic particle states important in lower-lying levels of odd-A nuclei but theoretical predictions for these states are uncertain.	Expect to see states corresponding to vibrations about spherical equilibrium. In even-even nuclei the level pattern will be 0+, 2+ (0+, 2+, 4+) etc.	No rotational structure of levels is expected.

Table 3.8 (cont'd.)

Region	Region of applicability	Basic description	Role of: Intrinsic particle states	Role of: Vibrational motion	Role of: Rotational motion
III	Nuclides with mass above A = 225. Exact boundary uncertain. Extends up to highest mass numbers yet studied; i.e., A = 256.	Nucleus is stabilized in non-spherical shape. Prolate spheroid with quadrupole symmetry preferred. Shell model nuclear states re-calculated for non-spherical nuclear field. Nuclear motions are separable into intrinsic, vibrational and rotational parts. Strong coupling of intrinsic particle motions to surface oscillations.	Intrinsic states are described by Nilsson diagrams. Most of the low-lying levels in even-even nuclei, represent rotational or vibrational structure based on ground intrinsic state. Energy gap of 1-2 Mev in particle excitation in even nuclei caused by residual short-range pairing forces. Excited particle states expected in low-lying levels for odd-A or odd-odd nuclei.	Vibrational motions classified in two main types. Vibrations with retention of spheroidal symmetry characterized by quantum number, n_β. Vibrations with change from spheroidal symmetry characterized by quantum number, n_γ. These excitations expected to require ~1 Mev. Each vibrational level may have rotational structure superimposed on it.	Rotational motions easily excited. Described by $\hbar^2/2\mathfrak{I} \times I(I+1)$ formula. Typical value of $\hbar^2/2\mathfrak{I}$ is 7.5 kev corresponding to ~ 45 kev for first rotational level in even-even nucleus. Every even-even nucleus has rotational sequence 0+, 2+, 4+, 6+ ... based on 0+ intrinsic ground state. Rotational levels may be based also on vibrational levels or excited nucleonic levels. Applicable quantum numbers are I, K and M. Odd-A nuclei have rotational bands with spin sequence K, K+1, K+2, etc.

3.5 Application of the Unified Model to Nuclides of the Heaviest Elements

The principal reason for devoting the earlier sections of this chapter to the unified model is the outstanding success of this model in explaining a variety of experimental data on heavy element nuclei particularly for those nuclei in which the stable equilibrium state is non-spherical. A number of the more important applications are summarized in this section. Spherical nuclei in lead and elements near lead are discussed in Chapter 10.

3.5.1 Rotational Bands in Even-Even Nuclei. Extensive data exist to prove that every even-even nucleus in the mass region above A = 226 has a series of low-lying states with the energy spacings of the rotational formula, Eq. 3.22. These states constitute a rotational family of levels based on the 0+ ground state. They are de-excited by a series of E2 gamma rays. Most of the information comes from a study of alpha emitters; the uniform pattern of alpha decay to the ground state rotational band of the daughter nucleus is evident from a inspection of Figs. 4.1 and 4.2 (see Chapter 4). Supplementary information comes from a study of beta emitters and orbital electron capturing isotopes. Additional information comes from the Coulombic excitation of rotational levels with charged particle beams, particularly heavy ions. Table 3.9 lists the known ground-state rotational levels for all even-even nuclei above Pb^{208}. For nuclei above 232 the energy of the 2+ state drops to ~ 45 kev and levels off at this value. Above mass 232 the unified model is a very good description of the nuclear dynamics and Eq. 3.22 is closely followed. Values of $\hbar^2/2\mathfrak{J}$ are about 7.5 kev. The second order correction term given in Eq. 3.23 is small in this region as can be shown by a consideration of the observed levels in Pu^{238}. In this nucleus the energies of all levels up through 8+ have been measured and it is possible to evaluate carefully the extent of the second order corrections as is done in Table 3.10.

A strong confirmation of the rotational band nature of these transitions is the identification of the gamma ray pattern. The expected pattern is a series of E2 cascade transitions with no crossover transitions. This pattern is fully confirmed. The E2 nature has been fully established in many instances by sub-shell conversion ratios, by conversion coefficients and by alpha-gamma angular correlation experiments. The details for every case listed in Table 3.9 are given in the sections of Chapters 8 and 9 which describe the individual nuclides.

Table 3.9 Ground State Rotational Bands in Even-Even Nuclei

Parent emitter*			Daughter nucleus	Energy of levels				$\frac{\hbar^2}{2\mathfrak{J}}$
α	β	EC		2+	4+	6+	8+	
Th^{226}			Ra^{222}	111.1	300			18.67
Th^{228}		Ac^{224}	Ra^{224}	84.47	253			14.08
Th^{230}		Ac^{226}	Ra^{226}	67.62	210	416		11.27
Th^{232}			Ra^{228}	59	185			9.7
U^{230}			Th^{226}	72.13	226.4			12.02
U^{232}	Ac^{228}	Pa^{228}	Th^{228}	57.5	186.1			9.58
U^{234}		Pa^{230}	Th^{230}	53	171			8.73
U^{236}			Th^{232} †	49.8	163	333	555	8.33
U^{238}			Th^{234}	48				8.00
Pu^{234}	Pa^{230}		U^{230}	51.7				7.83
Pu^{236}	Pa^{232}		U^{232}	47.5	156	321		7.917
Pu^{238}	Pa^{234}(?)	Np^{234}(?)	U^{234}	43.50	143.31	296.4	499	7.25
Pu^{240}		Np^{236}	U^{236}	44.2				7.547
Pu^{242}			U^{238}	44.7	148	309	523	7.50
	Np^{236}		Pu^{236}	44.63				7.2
Cm^{242}	Np^{238}	Am^{238}	Pu^{238}	44.11	145.8	303.7	514	7.37
Cm^{244}	Np^{240}(?)	Am^{240}	Pu^{240}	42.88	141.8	292		7.16
		Am^{242}	Pu^{242}	44.6				7.43
Cf^{246}	Am^{242}		Cm^{242}	42.12	138	284		7.02
Cf^{248}	Am^{244}		Cm^{244}	42.9	142.3	296	502	7.16
Cf^{250}		Bk^{246}	Cm^{246}	42.9	140			7.15
Cf^{252}			Cm^{248}	43.4	143			7.23
Fm^{254}		Bk^{250}	Cf^{250}	42.2	140.4			7.03
	E^{254}		Fm^{254}	44.93	149.9			7.49

* The parent emitter is listed if the rotational levels are observed in its decay.

† In the case of Th^{232} the 6+ and 8+ states and an unlisted 10+ state at 828 kev are known from Coulomb excitation with heavy ions. In the case of U^{238} the 4+, 6+ and 8+ states and a 10+ state at 787 kev and a 12+ state at 1098 kev are known from Coulomb excitation with heavy ions. See Table 3.10. Also, Cranberg has reported that the 4+ and 6+ levels of U^{238} were observed by inelastic neutron scattering. See Report TID-7547, International Conference on the Neutron Interactions with the Nucleus, Columbia University.

Table 3.10 Energies of Rotational Band Members in Pu^{238}

Member of band ($\underline{I}$, π)	0+	2+	4+	6+	8+
Measured energy	0	44.11	145.8	303.7	514*
Calculated energies based upon $E = \hbar^2/2\mathfrak{I}\ I(I + 1)$; $\hbar^2/2\mathfrak{I}$ evaluated from 44.11-kev 2+ state.		(44.11)	147.0	308.8	529.3
Calculated energies including $I^2(I + 1)^2$ term; $\hbar^2/2\mathfrak{I} = 7.37$ kev, $\underline{B} = 0.0034$ from measured energies of 2+ and 4+ states.		(44.11)	(146.0)	303.4	513.0

* This energy is based upon a very weak gamma ray whose energy is known only to ± 10 kev. The close agreement with the calculated energy is therefore fortuitous.

It will be difficult to obtain detailed information on the 6+, 8+, and higher levels of the ground state rotational band for most of the even-even nuclei by additional study of radioactive nuclei. Alpha decay to the higher lying states is subject to a severe energy dependence and, over and above this, to a large hindrance factor. For example, alpha decay of Cm^{242} gives rise to the 8+ level in Pu^{238} only once in 10^5 disintegrations. Only in very favorable cases (suitable half-life, high isotopic purity, etc.) is it possible to identify alpha and gamma transitions which occur in this low abundance. However, the method of Coulomb excitation with heavy ions offers considerable promise for those nuclei which can be prepared in sufficient quantity for use as targets in such experiments.[86] See Table 3.11.

Another strong proof of the identity of these states with rotational excitation is the agreement of the level spacings with the I(I + 1) formula. For the spin sequence 0, 2, 4, 6 ... the theoretical energy ratios are

$$\frac{E_4}{E_2} = \frac{10}{3} = 3.33$$

$$\frac{E_6}{E_2} = 7$$

$$\frac{E_8}{E_2} = 12$$

The empirical energy ratios are shown in Fig. 3.30 and are seen to be in remarkably good agreement with the simple rotational model above a neutron number of about 138. At lower neutron numbers the ratios drop away from the theoretical values. This drop corresponds to a shift into a region of spherical nuclei with low-lying vibrational states.

3.5.2 Vibrational Levels in Even-Even Nuclei. The characteristics of beta- and gamma-vibrational collective excitation of a nucleus are discussed in Sec. 3.4.4. Levels of excitation having many of these characteristics have been found in the excitation spectra of heavy even-mass nuclei. A classification of such levels is presented in Figs. 3.31 through 3.33 and in Table 3.12.

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86. F. S. Stephens, Jr., R. M. Diamond and I. Perlman, Phys. Rev. Letters 3, 435 (1959).

Table 3.11 Gamma Transition Energies and Excited State Energies from the Coulomb Excitation of Rotational Levels in Th^{232} and U^{238} with High Energy Argon (Stephens, Diamond and Perlman[86])

		Transition Energies (kev)					
		$2+ \rightarrow 0+$	$4+ \rightarrow 2+$	$6+ \rightarrow 4+$	$8+ \rightarrow 6+$	$10+ \rightarrow 8+$	$12+ \rightarrow 10+$
U^{238}	Experimental			161	214	264	311
	From $AI(I + 1)$ formula with $A = 7.45$ kev	44.7	104	164	224	283	343
	From $AI(I + 1) - BI^2(I + 1)^2$ formula with $A = 7.47$, $B = 2.9 \times 10^{-3}$	44.7	103.5	162	215	265	310
		Energies of Excited States (kev)					
		2+	4+	6+	8+	10+	12+
	Experimental			309	523	787	1098
		Transition Energies (kev)					
			$2+ \rightarrow 0+$	$4+ \rightarrow 2+$	$8+ \rightarrow 6+$	$10+ \rightarrow 8+$	
Th^{232}	Experimental		113	170	222	273	
	From $AI(I + 1)$ formula $A = 8.33$ kev	50	117	183	250	317	
	From $AI(I + 1) - BI^2(I + 1)^2$ $A = 8.38$, $B = 8.5 \times 10^{-3}$	50	114	172.5	222	259	
		Energies of Excited States (kev)					
		2+	4+	6+	8+	10+	
			163	333	555	828	

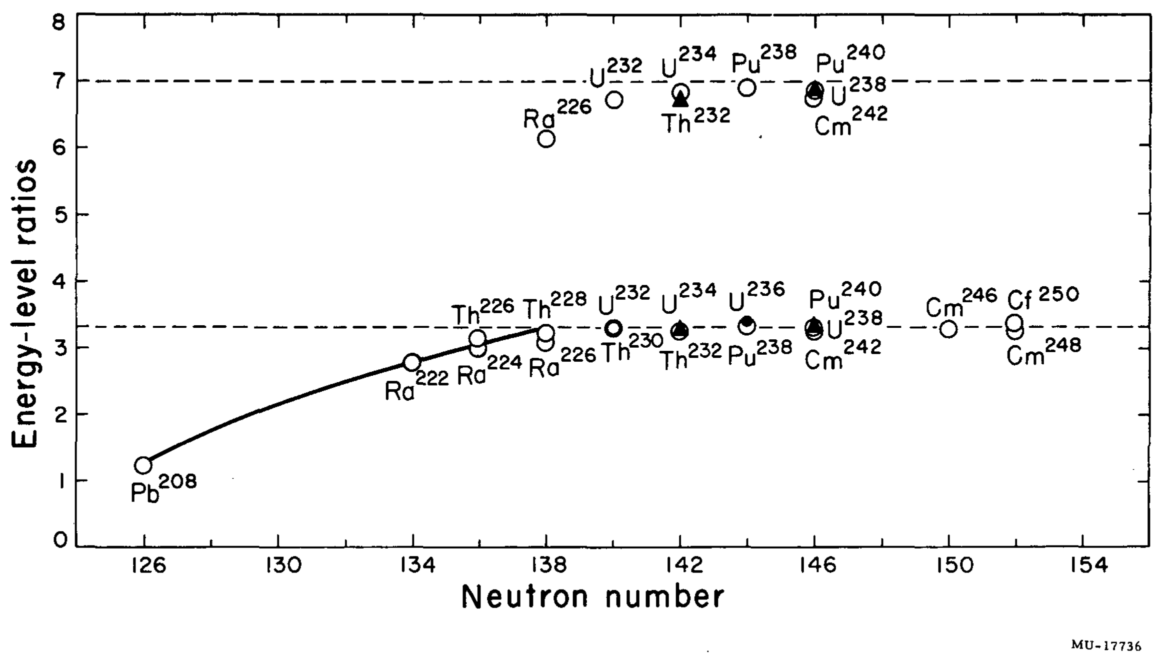

Fig. 3.30 Empirical ratios of rotational levels in even-even nuclei compared to the theoretical ratios of the I(I + 1) formula.

○ From gamma-ray data

● From alpha-particle data

▲ From Coulomb-excitation data

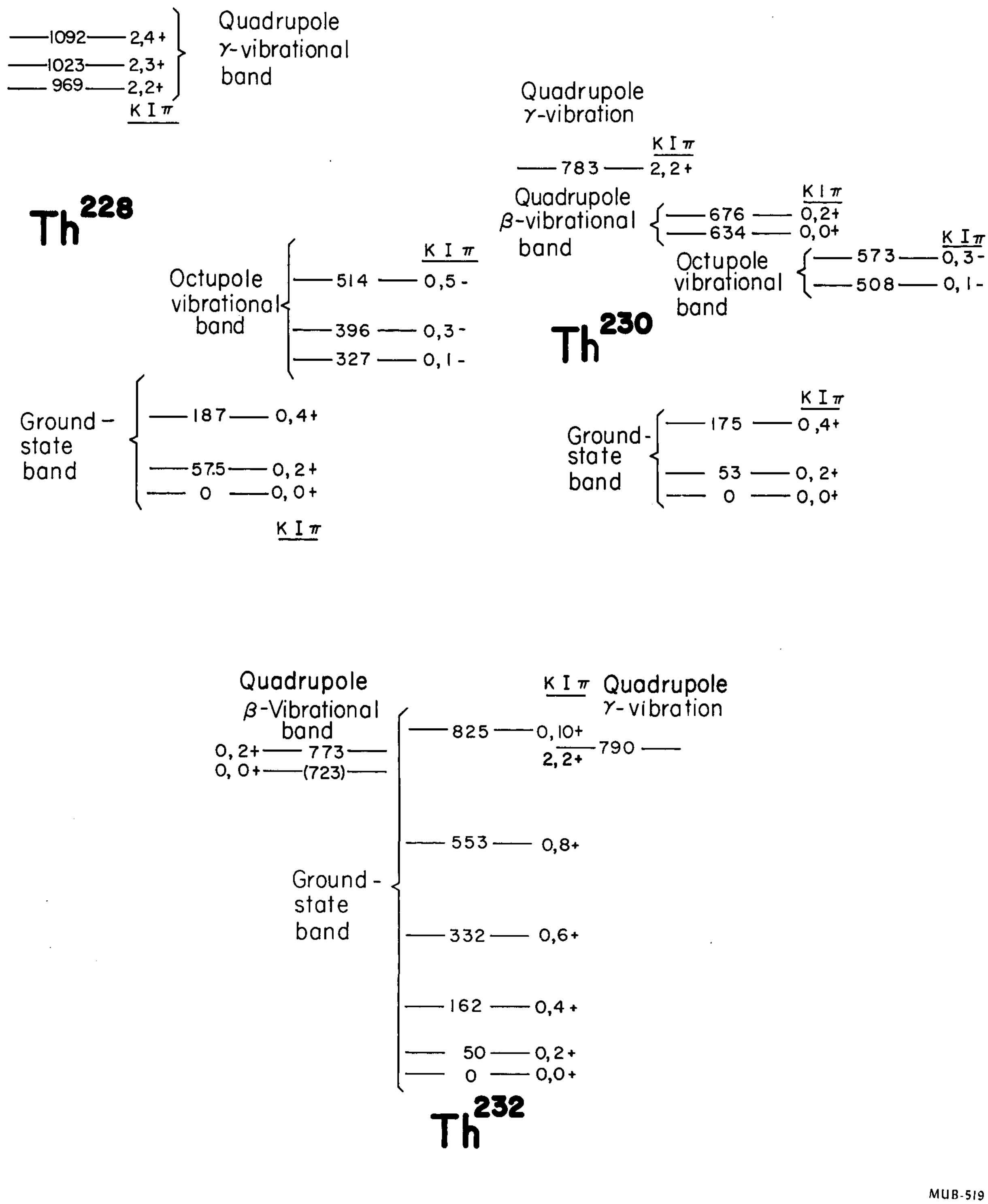

Fig. 3.31 Rotational bands based on beta and gamma vibrational states in even nuclei of thorium.

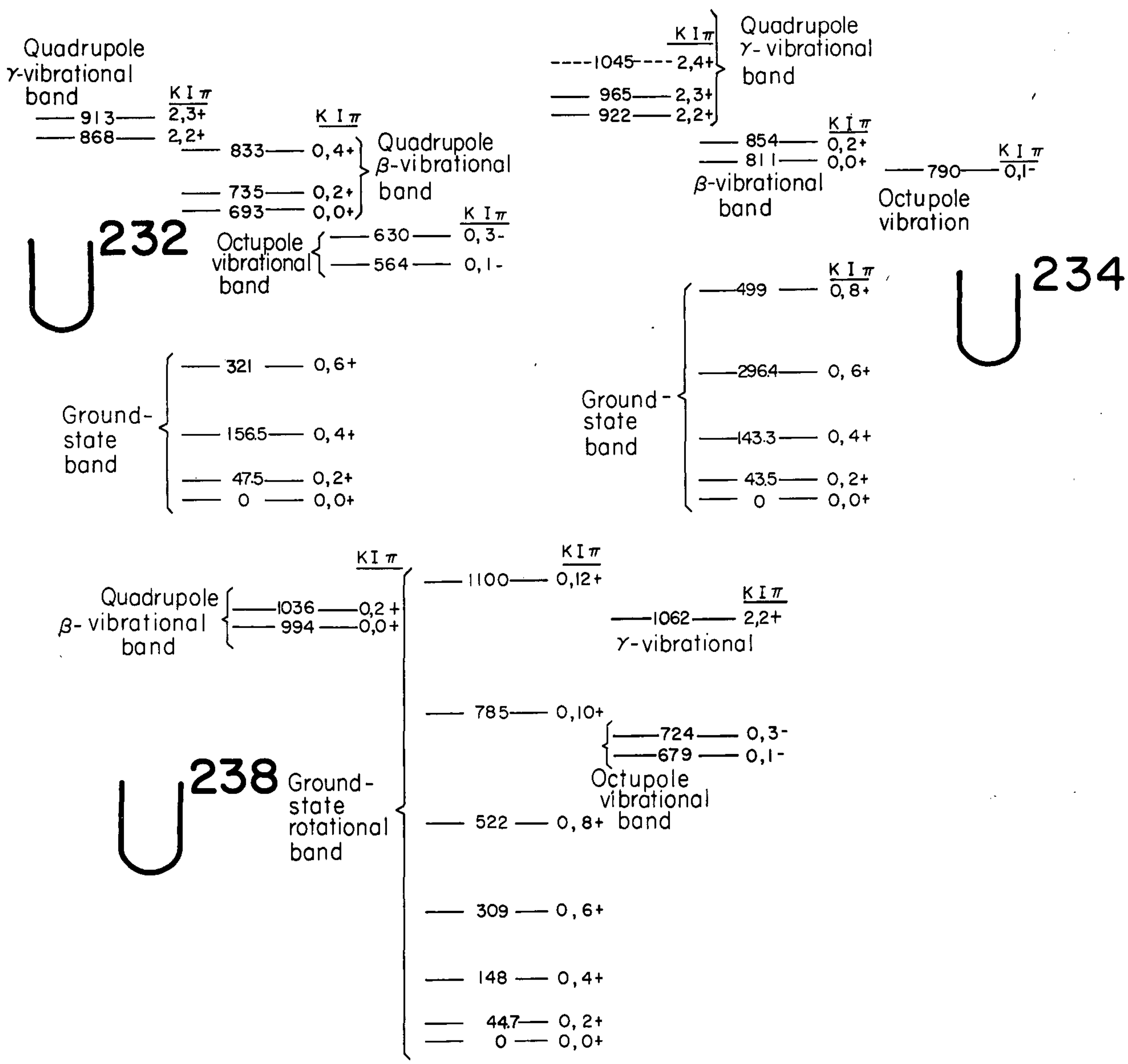

Fig. 3.32 Rotational bands based on beta and gamma vibrational states of even nuclei of uranium.

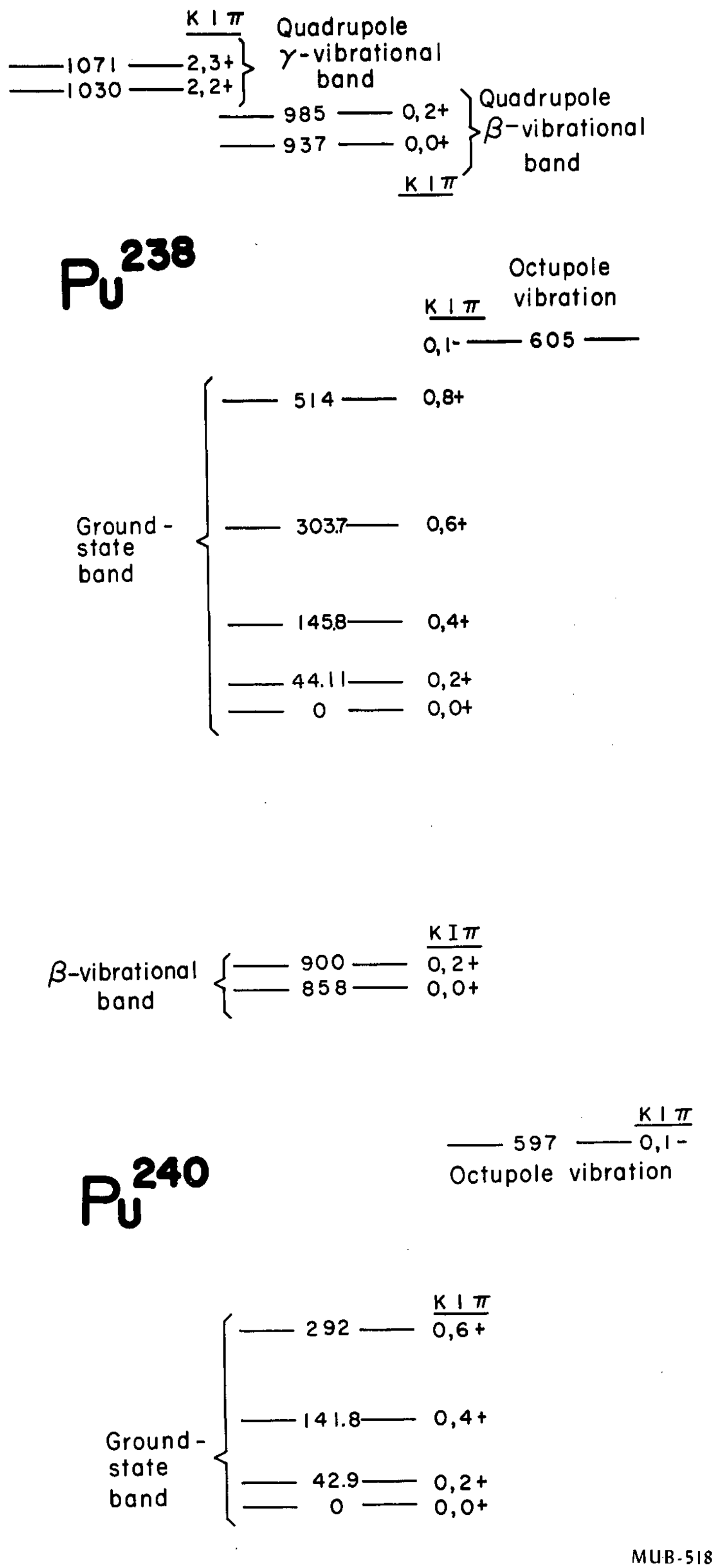

Fig. 3.33 Rotational bands based on beta and gamma vibrational states of even nuclei of plutonium.

Table 3.12 Beta and Gamma Vibrational Levels in Heavy Nuclei (in kev)

Nucleus	Beta Vibration			Gamma Vibration		
	Base State	First Rotational Level	Higher Rotational Levels	Base State	First Rotational Level	Second Rotational Level
	K,I,π = 0,0+	K,I,π = 0,2+		K,I,π = 2,2+	K,I,π = 2,3+	2,4+
Th^{228}				969	1023	1092
Th^{230}	634	676		783		
Th^{232}	730	774	873(4+); 1023(6+)	708		
U^{230}						
U^{232}	693	735	833(4+)	868	913	
U^{234}	811	854		922	965	(1045)
U^{236}	---	954				
U^{238}	993	1035	1127(4+); 1270(6+)	1058		
Pu^{238}	937	985		1030	1071	
Pu^{240}	858	900		(942)		
Cf^{250}				(1032)	1074	
Fm^{254}				693	733	

The labeling of these levels as states of beta- and gamma-vibration has been justified on the following grounds. 1) They occur at about the right energy, 600-1200 kev. In particular many of them occur below the "energy gap," i.e., below the energy at which the first level of particle excitation is expected in even nuclei. 2) The pattern of gamma ray de-excitation of these levels follows that shown schematically in Fig. 3.21. 3) The relative intensities of the E2 radiations connecting a gamma level to several members of the ground state rotational band follow the expected pattern reasonably close, i.e., the ratios of the reduced transition probabilities are nearly equal to the appropriate ratios of squared Clebsch-Gordan coefficients as expressed in Eq. 3.43. However, we shall have a further comment on this point below. 4) Many of these levels are excited by the Coulombic excitation process as discussed more fully in Sec. 3.5.8 below. The probability for this excitation gives a measure of the transition probability for the electric quadrupole transition to the ground state. These transition probabilities are about five-fold greater than those of pure single particle excitations. This enhancement is indicative of a collective excitation. 5) These levels occur in a whole series of even-mass nuclei. There are systematic trends in the energy of the states and in the hindrance in the alpha decay which populates many of them. 6) Electric monopole transitions from the β-band to the ground state band are observed. Characterization of such transitions is unambiguous since they can proceed only by emission of conversion electrons. The most prominent E0 transition is that between the K = 0 I = 0 level of the β-vibrational band to the K = 0 I = 0 ground state. One example occurs in the Pu^{238} nucleus. In the alpha decay of Cm^{242} a 0+ level at 935 kev is populated in 6×10^{-5} of the events. This level decays to the 0+ ground state solely by the emission of conversion electrons. Another example of an electric monopole transition is the well-studied 803-kev transition observed[87-89] in the beta decay of UX_2 and in the electron capture decay[90] of Np^{234}, both to the daughter nucleus U^{234}. Several other examples could be cited.

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87. S. A. E. Johansson, Phys. Rev. 96, 1075 (1954).

88. Ong Ping Hok, Phys. Rev. 99, 1613 (1955).

89. Ong Ping Hok, J. T. Verschoor and P. Born, Physica 22, 465 (1956).

90. C. Gallagher and T. D. Thomas, Nuclear Phys. 14, 1 (1959).

There are also many cases in which an electric monopole transition occurs in competition with electric quadrupole transitions when the latter are allowed by the spin selection rules. Such cases occur for example in the transition from the 2+ state of the β-vibrational band to the 2+ level of the ground state band. DURHAM, RESTER and CLASS[90a] have collected interesting data for such cases by studying the conversion electron spectra of even nuclei of thorium and uranium excited by the Coulombic fields of 5 Mev protons.

This completes our summary of the evidence for the identification of certain excited states as beta or gamma vibrational states. We wish now to enter the word of caution that while the evidence makes it rather clear that some sort of collective states are occurring systematically in a range of even mass nuclei there is no clear proof that these states are in fact caused by the vibration of the nucleus into the shapes implied by the labels β- and γ-vibration. It is quite possible that later developments in theory will provide alternate descriptions of nuclear excitations with the same general properties as those observed in the cases here summarized. In particular it is not excluded that the excitation of small numbers of particles from the ground state configuration might produce, through the operation of residual interparticle forces, a series of levels with the spin, parity and de-excitation characteristics of the observed levels.

We wish also to call attention to the fact that there exist in the experimental data certain facts which do not follow exactly the predictions of the model in the pure classical form.

A general discrepancy is a deviation in the pattern of transition intensities for the E2 transitions connecting levels in the γ-band with levels in the ground state band from the pattern predicted by squares of Clebsch-Gordan coefficient ratios. The observed ratios differ from the theoretical ones in a way to suggest some small mixing of the two bands into each other. A small admixture of the ground state wave function into the γ-band can change the intensity ratios considerably because transitions between states in a rotational band proceed at a rate 20 to 40 times greater than transitions between a vibrational and a rotational band. This mixing has been treated[90b] in terms

90a. F. E. Durham, D. H. Rester and C. M. Class, Proceedings of the International Conference on Nuclear Structure at Kingston, Ontario, August 29 to September 3, 1960, University of Toronto Press; also Phys. Rev. Letters 5, 202 (1960).

90b. P. G. Hansen, O. B. Nielsen and R. K. Sheline, Nuclear Phys. 12, 389 (1959).

of a parameter $Z \approx \epsilon\sqrt{24}\ Q_{00}/Q_{20}$ where $\epsilon\sqrt{24}$ is equal to the admixed amplitude, Q_{00} is the mean quadrupole moment of the two bands and Q_{20} is the E2 transition amplitude from the K = 2 to the K = 0 band. This Z-parameter has been evaluated in several cases[90c] in the heavy nuclei and found to range from 0.005 to 0.07.

In the Th^{232} nucleus there is a more complex case of band mixing as deduced by STEPHENS and DIAMOND[90d] from their Coulombic excitation studies of electron energies and intensities. They find a small admixture of the ground state band into the gamma band. In addition, because the 2+ states of the gamma and beta vibrational bands are only 11 kev apart, there is a strong mixing (of the order of 25 per cent) of these two bands. One piece of evidence for this deduction is the occurrence of a prominent E0 contribution to the decay of the 2+ gamma base-state to the 2+ level of the ground state rotational band. If the gamma band were pure K = 2 this E0 transition would be forbidden.

Another anomaly appears in a comparison of Pu^{238} and Pu^{240}. These nuclei have very similar level systems. Nevertheless there is a striking difference in the de-excitation of the 0+ β-vibrational levels lying at 940 and 870 kev respectively in these nuclei. In both nuclei there is an E0 transition to the ground state, an E2 to the 2+ level of the ground state rotational band and an E1 to a negative parity spin 1 level lying at about 600 kev. One would expect the relative intensities of these three transitions to be nearly the same in both nuclei; but BJORNHOLM et al.[90e] found a striking change in these relative intensities. There was a factor of 10 difference in the ratio of E0 to E2 and a factor of > 20 in the ratio of E1 to E0.

90c. E. Arbman, S. Bjornholm and O. B. Nielsen, Nuclear Phys. 21, 406 (1960).
G. T. Ewan, R. L. Graham, and J. S. Geiger, Nuclear Phys. 22, 610 (1961).
J. Borggren, O. B. Nielsen and H. Nordby, Nuclear Phys. 29, 515 (1962).
J. M. Hollander, C. L. Nordling and K. Siegbahn, Arkiv Fysik 23, 35 (1962).
O. B. Nielsen, Proceedings of Rutherford Jubilee International Conference, Manchester, September 1961.

90d. F. Stephens and R. Diamond, unpublished results, 1962.

90e. S. Bjornholm, M. Lederer, F. Asaro and I. Perlman, Phys. Rev., to be published (1963).

These are some of the experimental facts which are in conflict with the simple classical model of adiabatic shape vibrations. Future experimental data and theoretical development should provide us with a much more detailed idea of the true nature of these systematically-occurring excitations.

3.5.3 States of Negative Parity in Even-Even Nuclei. In the region of the deformed nuclei the predominant feature of the low-lying states, and in most cases the only observed feature of the low-lying states, is the occurrence of the even-parity members of the ground state rotational band. In several instances, however, the occurrence of a low-lying state of negative parity and spin 1 has been noted.[91-94] Since states of negative parity are forbidden on theoretical grounds in the rotational spectra of an even-even nucleus with spheroidal symmetry it was necessary to marshal considerable experimental evidence to establish carefully the spin and parity of these levels. The first two such cases occurred in the alpha decay of Th^{228} and of Th^{230} to the daughter nuclei Ra^{224} and Ra^{226}, respectively. In both cases the 1- designations were securely determined by gamma x-ray coincidence experiments, by the K-shell conversion coefficient, and by the angular correlation of the alpha particles with the gamma rays de-exciting the 1- state. The alpha-gamma angular correlation is particularly strong evidence since the $(0+) \rightarrow (1-) \rightarrow (0+)$ correlation is strikingly different from the $(0+) \rightarrow 2+ \rightarrow 0+$ correlation.

After the discovery of the 1- states in Ra^{224} and Ra^{226} a search was conducted throughout the entire range of heavy elements and a considerable number of 1- states were found. These are summarized in Fig. 3.34 and Table 3.13. Such states probably exist in all heavy even nuclei. It must be emphasized that it usually is not an easy matter experimentally to observe and characterize them because of the low abundances in which they occur in the decay of heavy element nuclei. Most of them must of necessity be sought in the decay of alpha-emitting nuclides. Since alpha decay is subject to an exponential energy dependence the decay to these high-lying 1- states is slight even if no additional hindrance is associated with the decay. In addition, it has been observed that alpha decay to the negative parity states is in most cases subject to a

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91. F. Asaro, F. S. Stephens, Jr. and I Perlman, Phys. Rev. 92, 1495 (1953).
92. F. S. Stephens, Jr., F. Asaro and I. Perlman, Phys. Rev. 96, 1568 (1954).
93. F. S. Stephens, Jr., F. Asaro and I. Perlman, Phys. Rev. 100, 1543 (1955).
94. F. S. Stephens, Jr., F. Asaro and I. Perlman, Phys. Rev. 107, 1091 (1957).

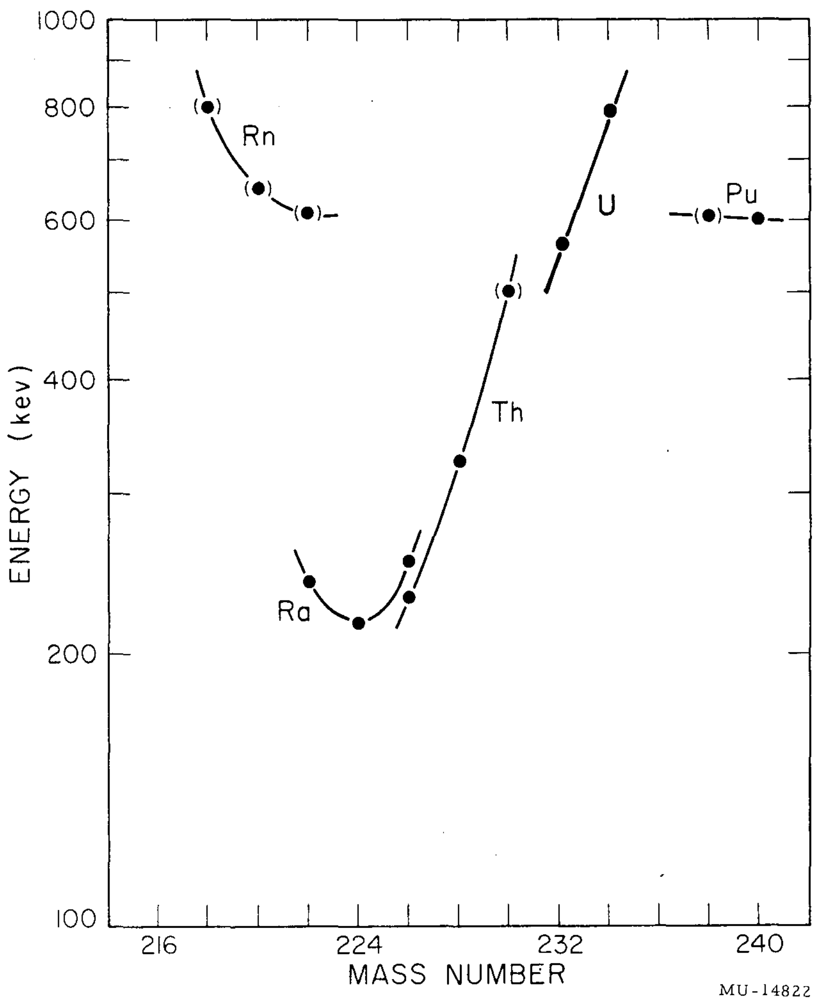

Fig. 3.34 The energy of the 1- states of even-even nuclei plotted against mass number.

Table 3.13 Summary of Octupole Vibrational States in Even Nuclei

Summary of 1- States					Summary of 3- States				Summary of 5- States			
Parent emitter*	Daughter nucleus	Gamma energy** (kev) γ_0	γ_2	Reduced transition probability γ_0/γ_2	Gamma energy*** (kev) γ_2	γ_4	Reduced transition probability γ_2/γ_4	Energy of 3- state	Gamma energy**** (kev) γ_4	γ_6	Reduced transition probability	Energy of 5- state
Np^{240}, Cm^{244}	Pu^{240}	599.5	557.0	0.47 ± 0.08								
Cm^{242}	Pu^{238}	605	562	0.60 ± 0.15								
Coulombic exc.	U^{238}	679	632		680	576		724				
Np^{234}, Pa^{234}	U^{234}	788										
Pa^{232}	U^{232}	564	516	0.53 ± 0.06	584	473	0.81 ± 0.04	630				
(U^{234}) Pa^{230}	Th^{230}	508	455	0.4 ± 0.2	520	398	---	573				
U^{232}, Ac^{228}, Pa^{228}	Th^{228}	327.5	268	0.43 ± 0.08	338	209	0.53	396	327.5	136	---	514
Ac^{226}, U^{230}	Th^{226}	230.31	158.1	0.49 ± 0.05								
Ac^{226}, Th^{230}	Ra^{226}	253	185	0.50 ± 0.05	253	110	0.7	320	235		---	445
Ac^{224}, Th^{228}	Ra^{224}	217	133	0.43 ± 0.05	167	---	---	289				
Th^{226}	Ra^{222}	242	130	0.48 ± 0.15								
(Ra^{222})	Em^{218}	794	475	0.7 ± 0.2								
(Ra^{224})	Em^{220}	650	410	0.4 ± 0.2								
(Ra^{226})	Em^{222}	610	420	0.5 ± 0.2								

* Underlining indicates that the 1- character of the state has been completely established. Parenthesis indicates that the proof of the 1- character is incomplete.

** γ_0 refers to the energy of the transition from the 1- state to the ground state; thus this energy is the energy of the 1- state. γ_2 is the energy of the transition from the 1- state to the 2+ level of the ground state rotational band.

*** Here γ_2 refers to the transition 3- → 2+ while γ_4 refers to the transition 3- → 4+.

**** Here γ_4 refers to the transition 5- → 4+ while γ_6 refers to the transition 5- → 6+.

sizeable hindrance.* This hindrance is shown explicitly in Fig. 3.35. The net result of the energy dependence of alpha decay plus this additional hindrance is that in many cases the negative parity states may be populated only to a very slight extent. In a few cases it has been possible to study negative parity states formed during the beta decay or electron capture decay of suitable nuclides. A study of the decay of Ac^{224}, Ac^{226}, Ac^{228}, Pa^{228}, Pa^{230}, Pa^{234}, and of Np^{240} and Np^{234} has proved helpful in this respect.

The systematic occurrence of the low-lying 1- states and the systematic shifts in their properties such as in the alpha-decay hindrance factor support the hypothesis that these states represent some sort of collective motion. From symmetry considerations the 1- states can arise only if the spheroidal symmetry of the nucleus is destroyed. Nuclear octupole vibrations in which the nucleus is distorted into a pear-shape or other shape having one less plane of symmetry than the spheroid could account for the odd parity states.** We have discussed this matter in Sec. 3.4.4 earlier in this chapter.

If the interpretation of the 1- states in terms of an octupole vibrational state is correct one might expect to see a rotational band of levels 1-, 3-, 5-, ... with a spacing following the $I(I + 1)$ law of Eq. 3.18 and with a moment of inertia parameter roughly equal to that of the even-parity rotational band in the same nucleus. STEPHENS, ASARO and PERLMAN[93,94] found two cases in which the 1- level is the base level of a 1-, 3-, 5- series. In the case of the Ra^{224} nucleus a careful study of the rare gamma transitions in the alpha decay of Th^{228} shows the presence of a 3- level as shown in Fig. 3.36. A similar study of the alpha decay of ionium showed that a 3- and a 5- level were present in the daughter nucleus, Ra^{226}, and that the spacings of these levels followed the $I(I + 1)$ relation quite closely. The value of $\hbar^2/2\mathfrak{J}$ evaluated from the level spacings is 6.7 kev, which can be compared to the 11.3 kev observed for the even-parity band. It is not known why the moment of inertia term should be so different. The odd parity rotational band in the decay of ionium to Ra^{226} is shown in Fig. 3.37.

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* Hindrance factor refers to the actual alpha decay rate compared to the alpha decay rate calculated from simple alpha decay theory in which the major controlling factor is the alpha disintegration energy. Refer to Sec. 4.2 of Chapter 4.

** R. F. Christy, unpublished communication cited by Alder et al., Rev. Modern Phys. 28, 541 (1956).

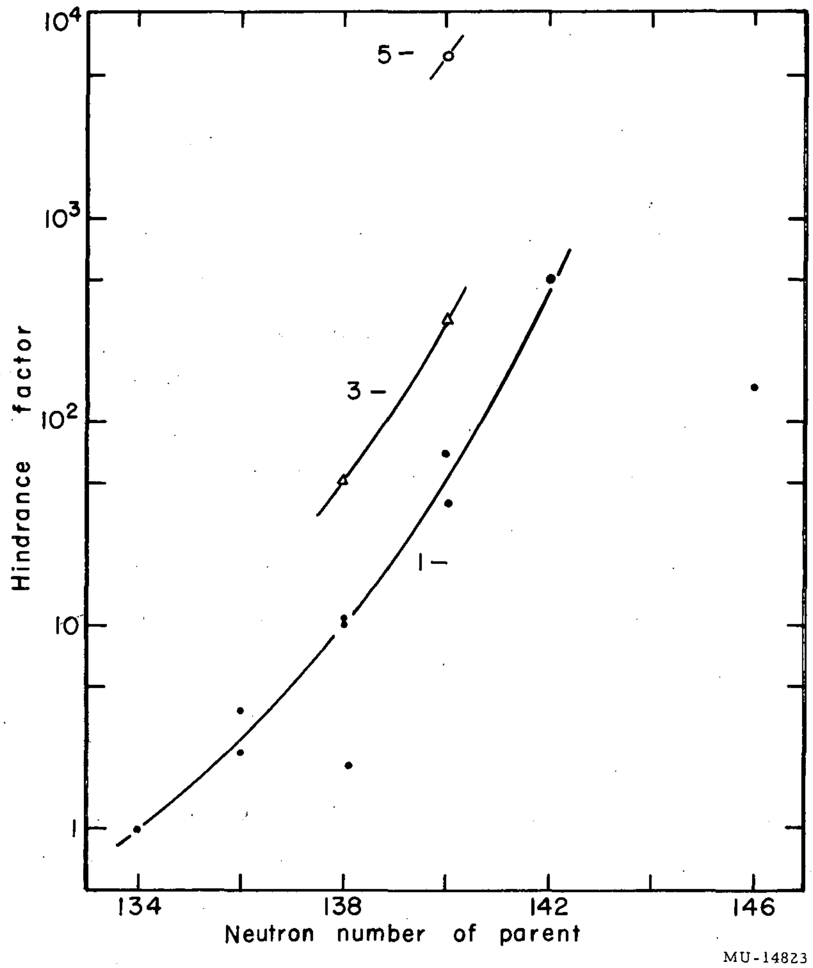

Fig. 3.35 Hindrance factors for alpha decay to the members of the odd parity rotational band in even-even nuclei. The point (corresponding to Cm^{242} alpha decay) at 146 neutrons makes it seem that the hindrance factors for decay to the 1- state may go through a maximum around 144 neutrons.

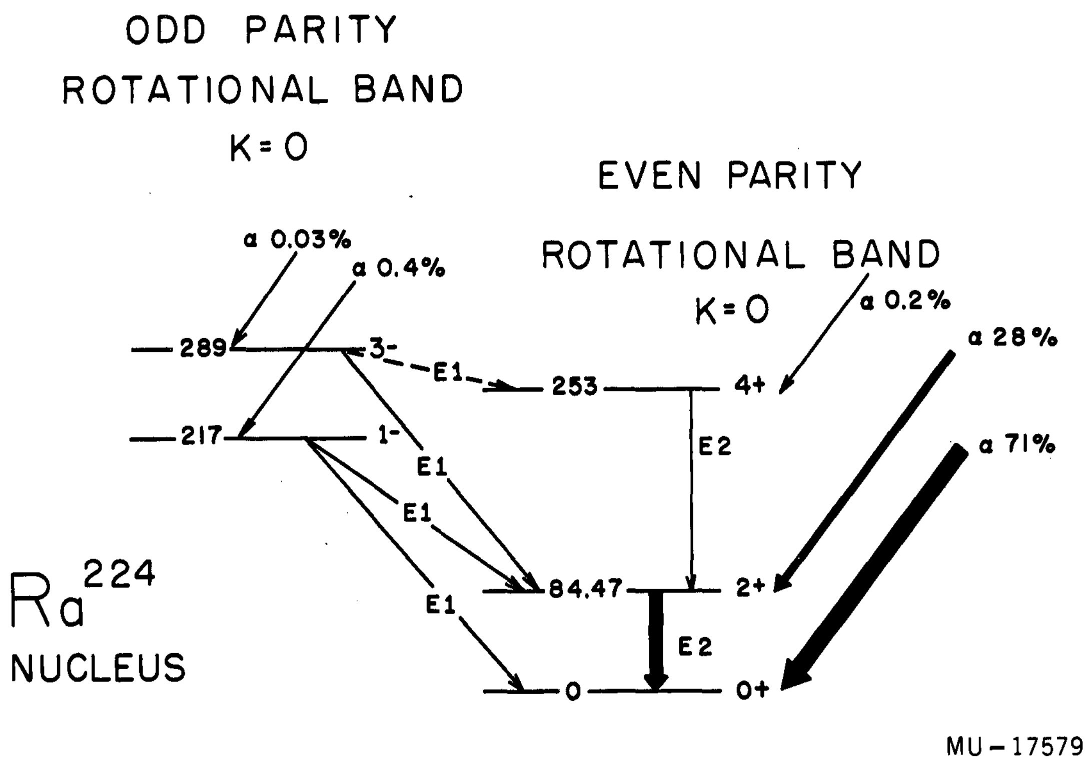

Fig. 3.36 Odd parity and even parity states of Ra^{224} grouped so as to emphasize their interpretation as even parity and odd parity rotational bands. Note the characteristic patterns of gamma de-excitation. The even states de-excite by a cascade of E2 transitions within the even-parity band. The odd parity states each decay by a pair of E1 transitions to two members of the even-parity band. These levels were observed in a study of the alpha-decay of Th^{228}. A more conventional and more detailed decay scheme and a fuller discussion of the experimental details are given in Sec. 8.2.6 of Chapter 8.

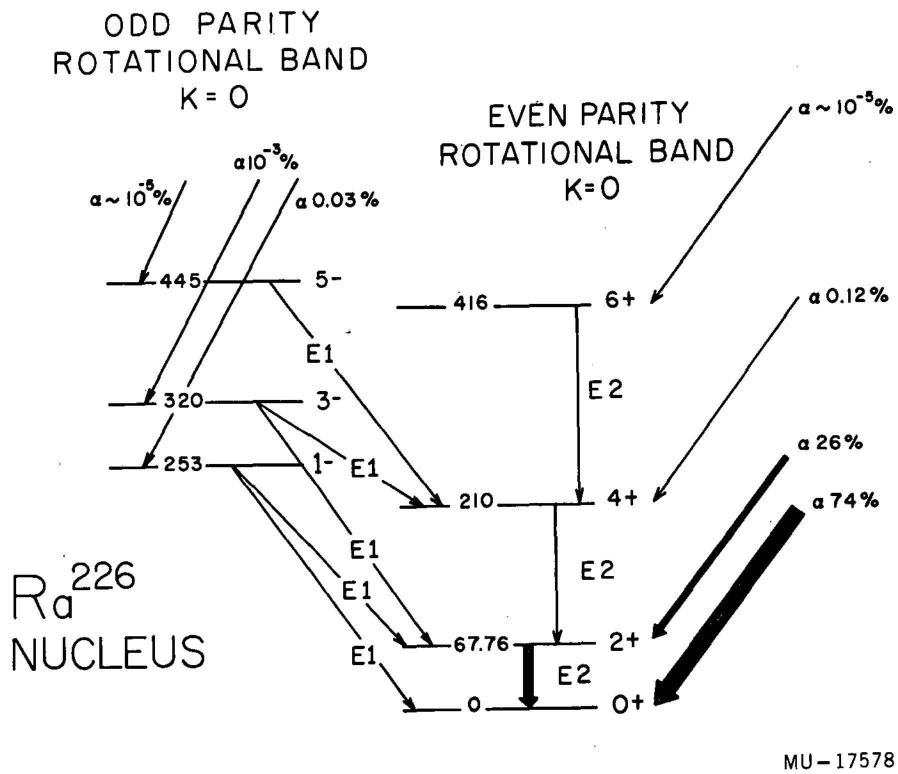

Fig. 3.37 Odd parity and even parity states of Ra^{226} grouped so as to emphasize their interpretation as even parity and odd parity rotational bands both with K-quantum number = 0. The characteristic pattern of gamma ray de-excitation is the same as in Fig. 3.36. This level scheme was deduced by the investigation of the decay of ionium. A fuller account of the experimental data is given in Sec. 8.2.8 of Chapter 8.

Note in Fig. 3.36 and 3.37 that there is a characteristic pattern of de-excitation of the negative parity states by E1 transitions to the even parity states with spin ± 1 spin unit away. A further important bit of information bearing on the interpretation of these negative parity states as collective excitations is the value of the K quantum number deduced from the relative abundances of the E1 transitions originating with these states.

According to the discussion of Sec. 3.4.7 we know that transitions leading to two members of a given rotational band should occur with the reduced transition probability ratios expressed in Eq. 3.43. We can use that equation in connection with the observed energies and abundances of the E1 transitions originating in any of the 1- states to evaluate the K quantum number of the state. To do this we first correct the ratio of the observed transition abundances to allow for the $(\Delta E)^3$ energy dependence of Eq. 3.35 and thus to determine the ratio of reduced transition probabilities $\frac{B(E1, I_i \rightarrow I_f)}{B(E1, I_i \rightarrow I_{f'})}$ of Eq. 3.43. We compare this ratio with that predicted by the ratio of the squares of the Clebsch-Gordan coefficients for the two possible values of K_i; i.e., K = 0 or 1. In every instance where the data are sufficient to carry out this procedure the ratio 0.5 is found, which is that predicted for a K_i value of 0. The calculations are summarized in Table 3.13.* If K_i had the value 1 the Clebsch-Gordan coefficients would lead to a value of 2.0 for the above ratio. Hence the K assignment of 0 to the negative parity rotational bands seems well established. The consistency of this ratio in all the cases listed in Table 3.13 indicates the systematic occurrence of some type of excitation and suggests that this excitation is of collective nature. The K value of 0 is consistent with the interpretation of the negative parity states as octupole vibrations.

There are a number of puzzling features about the negative parity states which any complete theory must take into account. One of these is the strong variation in the energy of the 1- states as a function of mass or neutron number (the variation shown in Fig. 3.34). Another is the marked increase in the hindrance factor for alpha decay events leading to the 1- states. We note in Fig. 3.35 a variation from essentially no hindrance to a factor of 500. Still

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* A similar evaluation has been made for the de-excitation of the 3- state in Ra^{226}. If the 3- state is in truth a member of the rotational band based on the 1- level we would expect K to be 0 for the 3- level as well. Under these conditions the ratio of reduced transition probabilities for the E1 transitions to the 2+ and 4+ states should be 0.75. The experimental ratio of 0.7 is in excellent agreement with this value.[94] Table 3.13 lists other 3- states to which the criterion for a K = 0 assignment has been applied.

another is the fact that the moment of inertia for the motion giving rise to the negative parity rotational spectrum in Ra^{226} has a value about twice that for the motion producing the even-parity spectrum. In the same nucleus it is also puzzling that the negative parity states follow closely the simple $I(I + 1)$ formula while the even parity states require second or third order correction terms of the type given in Eq. 3.23 for a good fit to the observed levels.

3.5.4 Rotational Bands in Odd-A Nuclei. A number of rotational bands have been identified in heavy element odd-A nuclei in spite of the fact that the levels systems of odd-A nuclei are much less regular than those of even-even nuclei. States of excitation of the intrinsic particle structure may lie within a few hundred kilovolts of ground making it necessary to distinguish carefully whether an experimental level represents collective or single particle excitation. Furthermore, the Coriolis interaction of particle motion with the nuclear rotational motion may perturb the energy spacings of the rotational bands and cause more deviation from the $I(I + 1)$ rule.

We list in Table 3.14 a number of rotational bands which have been identified in odd-A nuclei among the spheroidal nuclei of mass number greater than 228. These assignments are based on an analysis of the pertinent decay schemes which are described in Chapters 8 and 9 and on Coulombic excitation experiments. A few points are worthy of special note. The table lists several examples of a $K = 1/2$ band with the "anomalous" spacings predicted by Eq. 3.19. In the cases of Pa^{231}, Pa^{233} and probably of Ac^{227} (not listed) the effect of the correction term "a" is to reverse the order of the 1/2, 3/2 members of the band so that the 3/2 state actually lies lower. Another noteworthy point is that the rotational constant $\hbar^2/2\mathfrak{I}$ is systematically lower for odd-A nuclei than it is for the neighboring even-even nuclei.* (See Table 3.9)

A few of the rotational bands of the odd-A nuclei are displayed in Figs. 3.38 through 3.41.

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* O. Prior (Ark. Fysik 14, 451 (1959)) was able to account quantitatively for this difference by computing the orbital contribution of the last particle and the contribution of the even-even core due to a change in shape induced by the single particle. The calculations used Nilsson wave functions for the odd particle.

Table 3.14 Rotational Bands in Odd-A Nuclei

Nucleus	K-value and parity of band	Levels observed*	Nilsson orbital assignment of base level Kπ[N n_z Λ]	ħ/2ℑ (kev)	Decoupling constant a for anomalous K = 1/2 bands	Evidence for band obtained: From decay of these nuclides	Evidence for band obtained: From Coulombic excitation	More complete description given in these book sections
Ac^{225}	5/2-(ground)	5/2,7/2,9/2,11/2	5/2-[523]			Pa^{229}		8.3.5
Th^{229}	5/2+(ground)	5/2,7/2,9/2,11/2	5/2+[633]			U^{233}, Pa^{229}		8.4.6
Th^{231}	5/2+	5/2,7/2,9/2	5/2+[633]			U^{235}		8.4.8
U^{233}	5/2+(ground)	5/2,7/2,9/2	5/2+[633]	5.76		Pa^{233}	yes	8.3.9
Pa^{231}	1/2-(ground)	3/2,1/2,7/2,5/2	1/2-[530]	6.35		Th^{231}, U^{231}, Np^{235}	yes	8.2.9,8.4.5,9.1.6
Pa^{233}	1/2-(ground)	3/2,1/2,7/2,5/2	1/2-[530]		- 1.38	Th^{233}, Np^{237}		8.2.11,9.1.9
U^{235}	7/2-(ground)	7/2,9/2,11/2	7/2-[743]				yes }	8.4.8,9.2.9
U^{235m}	1/2+	1/2,3/2,(5/2)(7/2)(9/2)	1/2+[631]	6.1	- 0.276	Pu^{239}	}	
Np^{237}	5/2+(ground) 5/2-(60 kev) 1/2+(332 kev)	5/2,7/2,9/2 5/2,7/2,9/2,(11/2)(13/2) 1/2,(5/2)(3/2)	5/2+[642] 5/2-[523]	4.75 6.21 6.2	 + 1.08	Am^{241}, U^{237}, Pu^{237}	yes	9.3.6,8.4.11 9.2.7
Np^{239}	5/2+(ground) 5/2-(74 kev)	5/2,(7/2?) 5/2,7/2,9/2	5/2+[642] 5/2-[523]			Am^{243}		9.3.8
Pu^{239}	1/2+(ground) 5/2+(286 kev) 7/2-(392 kev)	1/2,3/2,5/2,7/2,9/2,11/2 5/2,7/2,9/2,11/2 7/2,9/2,11/2	1/2+[631] 5/2+[622]	6.25	- 0.58	Cm^{243}, Np^{239}, Am^{239}	yes	9.4.7,9.1.11,9.3.4
Pu^{241}	7/2+(172 kev)	7/2,9/2	7/2+[624]			Cm^{245}		9.4.9
Cm^{245}	7/2+(ground) 5/2+(255 kev) 9/2-(394 kev)	7/2,9/2,(11/2)(13/2) 5/2,7/2,9/2,11/2 (9/2)(11/2)(13/2)	7/2+[624] 5/2+[622] 9/2-[734]			Cf^{249}, Bk^{245}, Am^{245}		9.6.7
Bk^{249}	7/2+(ground) 3/2-(8.8 kev) 5/2+(393 kev)	7/2,9/2,11/2,(13/2) 3/2,5/2,7/2,9/2 5/2,7/2,9/2	7/2+[633] 3/2-[521] 5/2+[642]	small		E^{253}		9.7.9

* Parentheses on this column indicate speculative assignments.

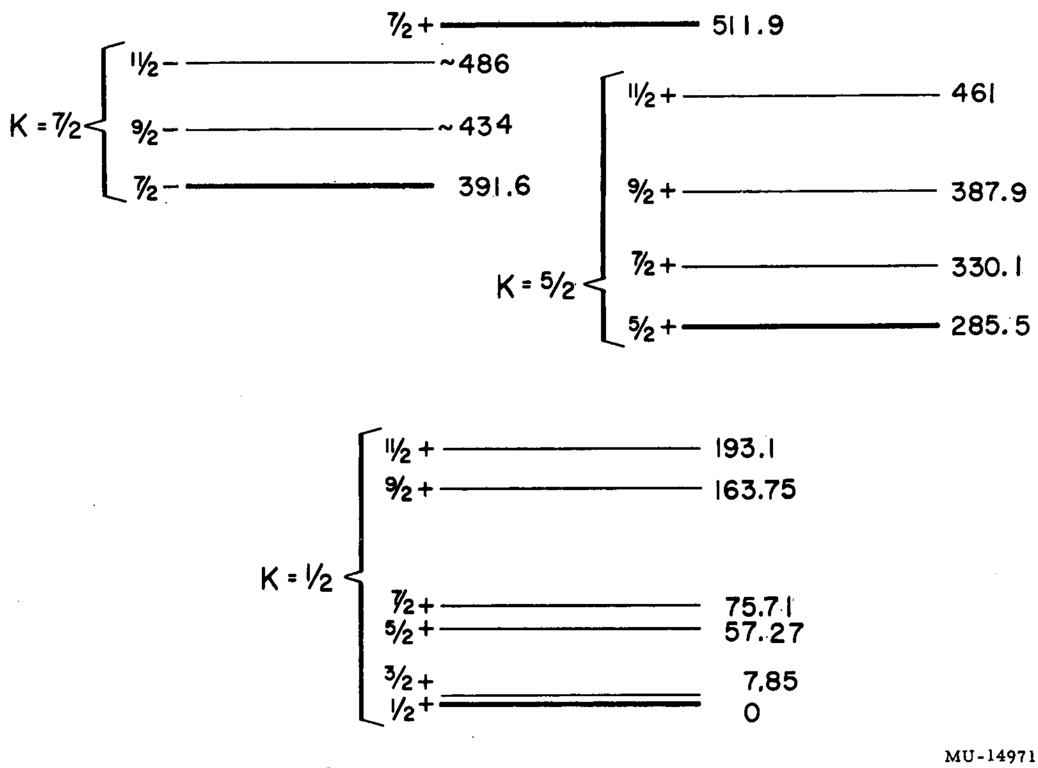

Fig. 3.38 Energy levels of Pu^{239} with associated levels of rotational excitation. Energies are given in kev at the right of each level. Spin values are given at the left. Note the characteristic grouping of the levels into pairs in the K = 1/2 "anomalous" rotational band as a result of the a-term in Eq. 3.19.

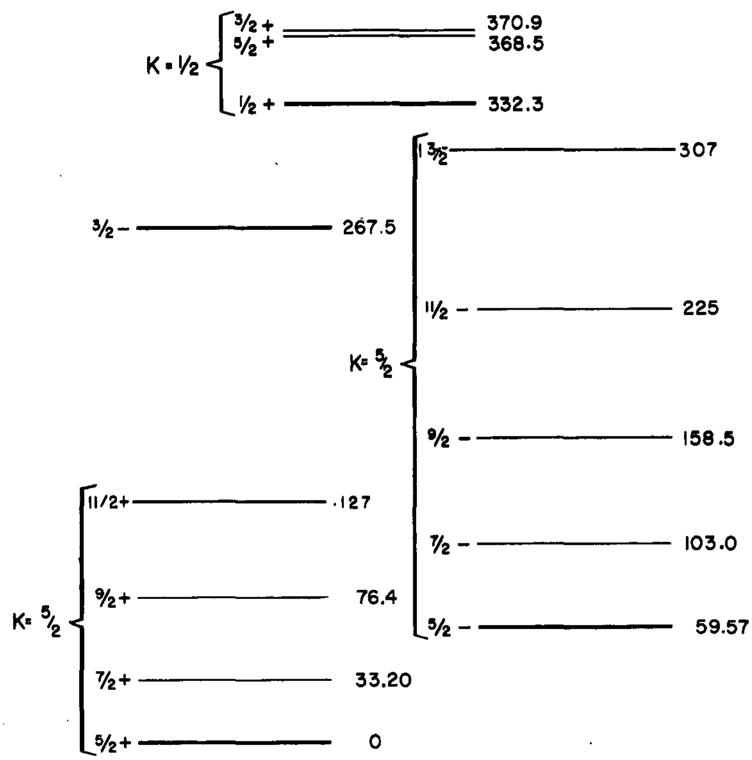

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Fig. 3.39 Energy levels of Np^{237} with associated levels of rotational excitation. Energies are given in kev at the right of each level. Spin values are given at the left.

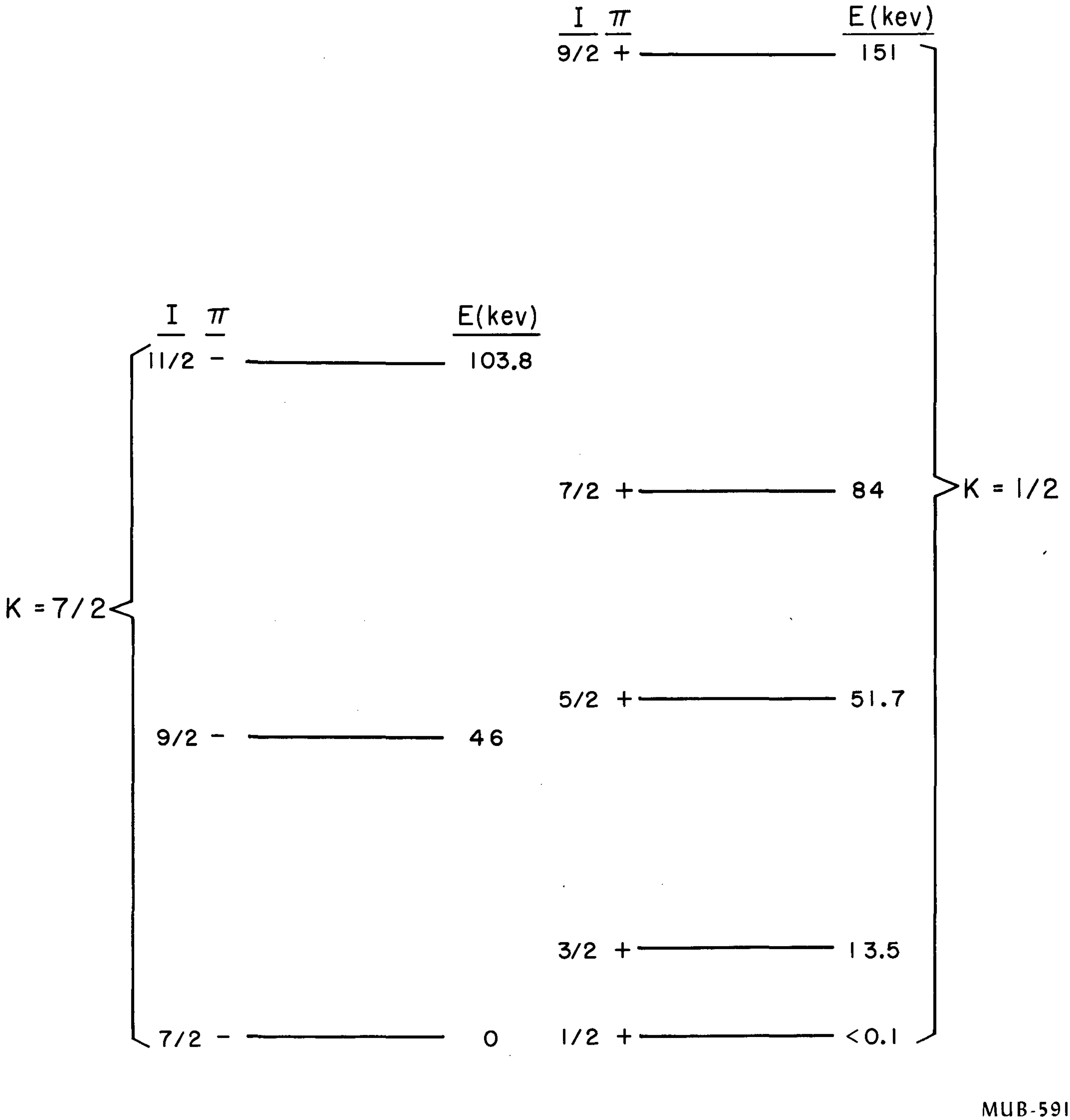

Fig. 3.40 The two lowest states of intrinsic excitation of U^{235} with associated levels of rotational excitation.

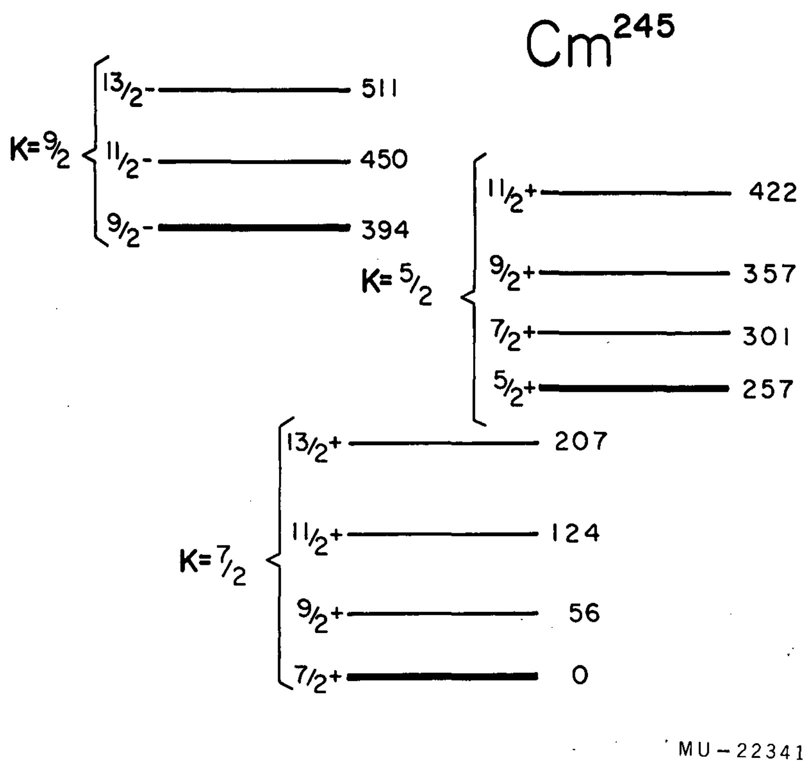

Fig. 3.41 Energy levels of Cm^{245} with associated levels of rotational excitation. Energies are given in kev to the right of each level, spin values to the left.

3.5.5 Identification of Nilsson Levels in Odd-Mass Nuclei. One of the strong pieces of evidence demonstrating the validity of the Unified Model approach to nuclear structure has been the possibility of correlating energy levels in odd mass nuclei with the nucleonic binding states calculated for a spheroidal nucleus. MOTTELSON and NILSSON[95,96] have shown how detailed assignments of ground states and of low-lying intrinsic states are possible for numerous nuclei lying in the mass region where spheroidally deformed nuclei have definitely been established. It is the purpose of this section to summarize in terms of the Nilsson wave functions the classification of nuclear energy levels in odd-A nuclei in the mass region $A > 228$. In making this summary we draw freely on the excellent papers of MOTTELSON and NILSSON[96] and of STEPHENS, ASARO and PERLMAN.[97]

The Nilsson wave functions are described in Sec. 3.4.5 and the particular group of Nilsson states with which we are concerned are shown in Figs. 3.25 and 3.26. The general procedure by which these Nilsson diagrams are employed to decide which theoretical states are available for possible identification with the ground and low-lying states of odd-A nuclei is described in Sec. 3.4.6. To make exact assignments it is necessary to consider all the evidence available from radioactive nuclei such as log-ft values, gamma ray de-excitation patterns, spacings of the levels in rotational bands, Coulombic excitation of specific levels, etc. From a detailed consideration of all types of evidence the assignments listed in Tables 3.14, 3.15 and 3.16 were made. In the succeeding pages we outline briefly the evidence upon which these choices were based in each case. In many instances these assignments are covered in more detail in other chapters of this book where the decay schemes of individual nuclides are discussed. Also more details and references to research papers can be found in the references[96,97] from which our discussion is taken. Our main purpose here is to show that the Nilsson treatment has been of very great usefulness in the classification and understanding of intrinsic levels in odd-A nuclei. The general reliability of

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95. B. R. Mottelson and S. G. Nilsson, Phys. Rev. 99, 1615 (1955).

96. B. R. Mottelson and S. G. Nilsson, The Intrinsic States of Odd-A Nuclei Having Ellipsoidal Equilibrium Shape, Mat. Fys. Skr. Dan. Vid. Selsk. 1, No. 8 (1959).

97. F. S. Stephens, Jr., F. Asaro and I. Perlman, Phys. Rev. 113, 212 (1959).

Table 3.15 Odd-Neutron Level Assignments

Isotope	3/2+ [631]	5/2- [752]	5/2+ [633]	7/2- [743]	1/2+ [631]	5/2+ [622]	7/2+ [624]	9/2- [734]	Neutron number
Ra^{225}									137
Th^{227}									
U^{229}									
Ra^{227}									
Th^{229}			0						139
U^{231}		((0))							
Pu^{233}									
Th^{231}		185	0	390					
U^{233}	(311.9)		0		((398.5))				141
Pu^{235}									
Th^{233}									
U^{235}				0	~ 0.1				143
Pu^{237}				0	145				
Cm^{239}									
U^{237}					0	(145)			
Pu^{239}				392	0	286	(512)		145
Cm^{241}					0				
U^{239}						((0))			
Pu^{241}						0	172		147
Cm^{243}						0			
Cf^{245}									
Pu^{243}							(0)		
Cm^{245}					((631))	255	0	394	
Cf^{247}									149
Fm^{249}									
Pu^{245}									
Cm^{247}									151
Cf^{249}								0	
Fm^{251}									

Table 3.16 Odd-Proton Level Assignments

Isotope	3/2- [532]	3/2+ [651]	1/2- [530]	5/2+ [642]	5/2- [523]	3/2- [521]	7/2+ [633]	7/2- [514]	Proton number
Ac^{225}									
Ac^{227}		($\underline{0}$)	(330)						89
Ac^{229}									
Pa^{227}									
Pa^{229}									
Pa^{231}		(166)	$\underline{0}$	(84)					91
Pa^{233}		(~200)	0	(86)					
Pa^{235}									
Np^{231}									
Np^{233}									
Np^{235}				0	48				93
Np^{237}			((267))	$\underline{0}$	60				
Np^{239}				0	75				
Am^{237}									
Am^{239}			((540))	(187)	0				
Am^{241}			((480))	(206)	$\underline{0}$	((630))			95
Am^{243}			((265))	84	0		465		
Am^{245}									
Bk^{243}						(0)			
Bk^{245}						0			97
Bk^{247}						(0)			
Bk^{249}				(393)		(8.8)	0		
E^{249}									
E^{251}									
E^{253}							0		99
E^{255}									
Mv									101

the model can be gauged both by the orderly sequence of assignments and by the fact that in no case where accurate data are available is an assignment indicated which is significantly at variance with the expectations of the theory. It should be kept in mind that many of the known excited states in odd-A nuclei have not yet been characterized.

In Tables 3.15 and 3.16 the Nilsson levels are listed across the top in order of increasing energy (appropriate to the nuclear deformations shown by the broken lines in Figs. 3.25 and 3.26) and the numbers appearing in the tables are the energies of the states (in kev) relative to their ground states. One might expect that a given Nilsson state (neutron or proton) would appear as the ground state in only a single nucleus but that the same state would be expected to appear as excited levels for neighboring nuclei. An inspection of the tables shows that the second part of this statement is true but that there are instances where the same Nilsson state appears as the ground state in neighboring nuclei. This is not a serious discrepancy since an appreciable change in nuclear deformation in adding two nucleons may cause shifts in the order of filling and result in a certain orbital being the ground state for the two neighboring nuclei. (Also residual interparticle forces, neglected in the model quoted, may cause shifts of the order of 100 kev in level positions.) In the tables the states are labeled by Ω and π and by the asymptotic quantum numbers $[N\ n_z\ \Lambda]$ defined previously. If we use as an example the proton state 5/2+[642] we can interpret the state designation as follows: The projection of the total angular momentum on the symmetry axis is 5/2 units and the parity of the state is positive. The level comes from the 6th oscillator shell as can be seen in Fig. 3.25 by its connection to the $i_{13/2}$ level in the spherically symmetric potential. The wave function has 4 nodal planes ($n_z = 4$) perpendicular to the axis of symmetry of the deformed nucleus. The final quantum number Λ can differ from Ω only by $\pm 1/2$ depending on the orientation of the intrinsic spin of the nucleon with respect to the symmetry axis. In the example cited, the intrinsic spin is anti-parallel to Ω.

The method of indicating the reliability of the assignments in the tables is by means of parentheses enclosing the energies of the states. Absence of parentheses indicates rather conclusive evidence; a single set implies a tentative assignment based on substantial evidence; and double parentheses are used to signify that some evidence is available but it is far from conclusive. For those entries which are underlined the corresponding spin has been determined directly, but it should be pointed out that this in itself does not constitute proof that the assignment is correct.

The assignment of experimental levels in Tables 3.15 and 3.16 to specific Nilsson wave functions does not inform us of the "purity" of the state. The asymptotic quantum numbers are fully descriptive only in the limit of high deformation and in real nuclei we must expect some admixture of theoretical states in each of the observed levels. The "impurity" of the states have some consequences principally in transition probabilities and in the applicability of transition selection rules.

In the discussions which follow, the odd-neutron cases are considered first and are grouped according to element. Following these are the odd-proton nuclei, again grouped by element. The tables summarize the data in terms of increasing neutron or proton numbers.

ODD-NEUTRON NUCLEI

Th^{229} (neutron number 139). The ground state of Th^{229} is assigned to the 5/2+[633] level which comes in at about the expected place for neutron number 139. This assignment will be seen to be the same as that for the ground state of U^{233} and was arrived at on the basis that the "favored alpha decay" of U^{233} leads to the ground state of Th^{229}.[98-100] BOHR, FRÖMAN and MOTTELSON[99] were the first to give theoretical grounds for assigning identical structures to states that are connected by an alpha emission process whose rate obeys simple one-body alpha-decay theory. The ground-state transitions of even-even alpha emitters (both states 0+) provide the empirical basis for defining "favored" or "unhindered" alpha emission.

The reader is referred to the section below on U^{233} for the assignment of 5/2+[633] for its ground state. The fact that Th^{229} and U^{233} both have the same ground-state configurations implies that the odd-particle state filled for Th^{229} is vacated when the particle becomes paired. Further information on low-lying states around this neutron number should help explain this repetition in configuration.

The states associated with neutron number 139 are discussed further under U^{231}.

98. I. Perlman and F. Asaro, Ann. Rev. Nuclear Sci. 4, 157 (1954).
99. Bohr, Fröman, and Mottelson, Dan. Mat.-fys. Medd. 29, No. 10 (1955).
100. Gol'din, Novikova, and Tretyakov, Phys. Rev. 103, 1004 (1956).

Th^{231} (neutron number 141). The assignments of the levels of Th^{231} come from an analysis of the alpha and gamma spectrum of U^{235} and are fully discussed in Sec. 8.4.8 of Chapter 8.

Th^{233} (neutron number 143). Th^{233} is a short-lived beta emitter about which very little is known. Nothing is known about its excited states, and the only information available concerning its ground state is that derived from its beta-decay properties. By analogy with U^{235} and Pu^{237} one might guess that the ground state is either 7/2-[743] or 1/2+[631]. The data of FREEDMAN and CO-WORKERS[101] indicate that an appreciable proportion of the decay of Th^{233} goes to the ground-state rotational band of Pa^{233} which is believed to have K = 1/2 as is discussed below. This information would seem to favor the 1/2+[631] assignment for Th^{233} if the choice rests between the two states mentioned. Possible difficulty exists, however, because there also seems to be direct population of the 5/2+ band of Pa^{233}, although this is by no means certain. In view of the paucity of information, no assignment is entered in Table 3.14. Still, it is interesting to note the relative positions of the above-mentioned two states in other nuclei with 143 neutrons. In Pu^{237}, the state 1/2+[631] is 145 kev above the 7/2-[743] ground state; in U^{235} the two states lie within 100 ev of each other; and if the above-mentioned evidence is significant, the 1/2+ state becomes the ground state in Th^{233}.

U^{231} (neutron number 139). At present very little is known about the energy levels of U^{231} because information comes only from its electron-capture decay properties. Something possibly may be learned about its excited states when its alpha-emitting parent, Pu^{235}, can be studied. The ground state would be expected to be 5/2-[752] (185-kev state in Th^{231}), 5/2+[633] (ground state of Th^{231} and U^{233}), or 3/2+[631] (an excited state in U^{233}). The 5/2+ assignment can probably be ruled out because Th^{231} and U^{231} both decay to Pa^{231}, and there are selective differences in the levels occupied. Of the two remaining assignments, 5/2-[752] is slightly preferred on the basis that no population to the ground state, K = 1/2 band of Pa^{231}, could be detected in the U^{231} electron-capture decay. From a consideration of the selection rules in asymptotic quantum numbers it would seem that U^{231} should have an appreciable decay to the K = 1/2 band of Pa^{231} if the ground state of U^{231} were 3/2+[631].

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101. Freedman, Engelkemeir, Porter, Wagner, Jr., and Day, Argonne National Laboratory, private communication to J. M. Hollander (1957).

U^{233} (neutron number 141). The energy levels of U^{233} were classified by J. O. Newton[102] from a consideration of his own data on the Coulombic excitation of rotational levels in U^{233}, from other physical data such as spin and magnetic moment and from a consideration of the available data on the beta decay of Pa^{233}. Details are given in Sec. 8.3.9 of Chapter 8. We list the assignments here without comment.

U^{235} (neutron number 143). A long-standing puzzle concerning the low-lying levels of U^{235} has been solved. The principal (unhindered) alpha group of Pu^{239}, (a nucleus which has spin 1/2, apparently led to the ground state of U^{235} (with measured spin of 7/2) in contradiction to major selection rules, but it was found that this favored alpha group actually went to a 26-minute isomeric state of spin 1/2 which had not previously been observed because of its extremely soft radiations. The isomeric level lies less than 0.1 kev above the ground state. This puzzle, its solution and the assignment of Nilsson levels to the ground state and the isomeric state of U^{235} are fully discussed in Sec. 9.2.9. The ground state of U^{235} is 7/2-[743]; the low-lying isomeric state is 1/2+[631].

U^{237} (neutron number 145). The ground-state spin of U^{237} has not been measured, but it is almost certainly 1/2 or 3/2 because in its beta decay to Np^{237} no state with spin higher than 3/2 is populated. (The decay scheme of U^{237} is covered in Sec. 8.4.10 in Chapter 8.) RASMUSSEN et al.[103] have suggested possible assignments 1/2-[501] and 1/2+[631]. Because there is no evidence for the 1/2-[501] state in other nuclei in this vicinity, and since the log-ft values can be reasonably well explained on the basis of the 1/2+[631] assignment, this latter one is preferred. It will be seen that two other nuclei with the same neutron number, Pu^{239} and Cm^{241}, also have this configuration in their ground states.

The alpha decay of Pu^{241} has been investigated[104,105] and the favored group goes to a level in U^{237} at 145 kev. The ground state of Pu^{241} has been assigned 5/2+[622] as will be discussed below, hence the U^{237} level at 145 kev is ascribed to the same orbital. The gamma-ray de-excitation properties of this level support the idea that it has the same parity as the ground state. The transitions to the

102. J. O. Newton, Nuclear Phys. 5, 218 (1958).

103. Rasmussen, Canavan, and Hollander, Phys. Rev. 107, 141 (1957).

104. Asaro, Stephens, and Perlman, unpublished data (1957).

105. Freedman, Wagner, and Engelkemeir, Phys. Rev. 88, 1155 (1952).

ground-state rotational band appear to be largely M1 (presumably K-forbidden). It will be seen that the same situation is found in Pu^{239} in which the 5/2+ state appears at a somewhat higher energy. As has been mentioned, this state 5/2+[622] shows up as the ground state for neutron number 147.

U^{239} (neutron number 147). There is little experimental information on this short-lived beta emitter, but in analogy to Pu^{241} the ground state may be 5/2+[622]. This assignment is consistent with the beta-decay properties of U^{239} in which a 5/2- state in Np^{239} of 74 kev receives most of the beta population.

Pu^{237} (neutron number 143). Although the spin of Pu^{237} has not been measured directly, the assignments of the ground state to 7/2-[743] and a state at 145 kev to 1/2+[631] are considered to be on rather firm ground. These states occur very close together in U^{235} and result in an isomeric transition with a half life of 26.5 min. On the supposition that the structure would repeat in Pu^{237}, STEPHENS et al.[106] searched for and found an E3 isomeric transition, with a 0.18-sec half life and energy of 145 kev resulting from an alpha decay of Cm^{241}. (Refer to Sec. 9.4.5.) The ground-state assignment 7/2-[743] has also been made by HOFFMAN and DROPESKY[107] from a study of the electron-capture decay of Pu^{237} to Np^{237}. (See Sec. 9.2.7.)

Pu^{239} (neutron number 145). Several levels of Pu^{239} are populated in the decay of Np^{239}, Am^{239}, and Cm^{243} and by Coulombic excitation of Pu^{239} itself.[108] The study of these decay schemes has provided us with definite information on the Pu^{239} level scheme and on the Nilsson orbital assignments. Detailed discussion can be found in the following references: Np^{239}, Ref. 109 and Sec. 9.1.11; Am^{239}, Ref. 110 and Sec. 9.3.4; and Cm^{243}, Refs. 111, 112 and Sec. 9.4.7. We summarize here briefly what the assignments are. See also Fig. 3.38.

There is a well-defined rotational band based upon the K = 1/2 ground state (1/2+[631]) which orbital appears as the first excited state in Pu^{237} and U^{235} as already discussed. The 7/2-[743] configuration which is the ground state for U^{235} and Pu^{237} occurs at 392 kev and the 5/2+[622] configuration which will appear as the ground state for neutron number 147, occurs here at 286 kev. These assignments have been discussed in detail, but additional comment seems

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106. Stephens, Asaro, Amiel, and Perlman, Phys. Rev. 107, 1456 (1957).
107. D. C. Hoffman and B. J. Dropesky, Phys. Rev. 109, 1282 (1958).
108. J. O. Newton, Nuclear Phys. 3, 345 (1957).
109. Hollander, Smith, and Mihelich, Phys. Rev. 102, 740 (1956).
110. Smith, Gibson, and Hollander, Phys. Rev. 105, 1514 (1957).
111. Asaro, Thompson, and Perlman, Phys. Rev. 92, 694 (1953).
112. Asaro, Thompson, Stephens, Jr., and Perlman, Bull. Am. Phys. Soc. 2, No. 8, Paper R1 (1957).

to be in order concerning a level at 512 kev assigned by HOLLANDER et al.[109] as either 5/2+ or 7/2+. The only two assignments that seem reasonable are 5/2+[633] and 7/2+[624]. The state 5/2+[633] would result from opening a filled level that appeared as the ground state for neutron number 141, and the 7/2+[624] would be a new level to appear for higher neutron numbers as the ground state. Of these, the choice for the 7/2+ assignment seems preferable to STEPHENS, ASARO and PERLMAN[97] but MOTTELSON and NILSSON[96] favor the alternate choice 5/2+[633].

Pu^{241} (neutron number 147). The ground-state spin of Pu^{241} has been measured as 5/2.[113] The 5/2+[633] orbital was the ground state for neutron number 141, so the ground state of Pu^{241} is almost surely 5/2+[622]. The only information on excited states is that obtained from the alpha decay of Cm^{245} (Refs. 114 and 115 and Sec. 9.4.9). The favored alpha group leads to a state 172 kev above ground. Another state of 58 kev higher energy has been observed and interpreted as the first member of the rotational band based upon the 172-kev state.[115] This spacing suggests that the band has K = 7/2 or higher. The parity is fixed as even from the observation that the 172-kev state decays to the ground-state band by M1 transitions. In particular, the transition to the 5/2+ ground state is definitely M1. This fact not only fixes the parity of the 172-kev state, but also is consistent with the spin assignment of 7/2. The only Nilsson level in this region with these properties is 7/2+[624], and the assignment is considered to be reasonably certain.

Pu^{243} (neutron number 149). Information is available only on the ground state of Pu^{243} and is derived from the decay of this isotope to Am^{243}. See Sec. 9.2.13. States in Am^{243} having spins 5/2, 7/2, and probably 9/2 seem to receive direct beta population from Pu^{243} so that a spin of 7/2 for Pu^{243} seems most reasonable. Because this coincides with the expected Nilsson level, 7/2+[624], this assignment is given to Pu^{243}.

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113. Bleaney, Llewellyn, Pryce, and Hall, Phil. Mag. 45, 773, 991 (1954).

114. Hulet, Thompson, and Ghiorso, Phys. Rev. 95, 1703 (1954).

115. Asaro, Thompson, and Perlman, unpublished data (1954) reported by I. Perlman and J. O. Rasmussen in Handbuch der Physik, Vol. 42 (Springer-Verlag, Berlin, 1957).

Cm^{241} (neutron number 145). The ground state of Cm^{241} is almost surely the same as that of Pu^{239} (also with 145 neutrons), 1/2+[631]. This assignment was made by STEPHENS et al.[106] whose arguments will be reviewed briefly here. See also Sec. 9.4.5. Cm^{241} decays by electron capture to Am^{241} and although there are low-lying 5/2+ and 5/2- states, neither of these is directly populated. Instead decay takes place to states that seem to have spins of 3/2 or 1/2.

As already mentioned under Pu^{237}, the favored alpha decay of Cm^{241} populates a 0.18-sec isomeric state of Pu^{237} which drops to the ground state by an E3 transition. The evidence is excellent that the isomeric state has the assignment 1/2+[631], hence the ground state of Cm^{241} is almost surely the same.

There is no information at present on the excited states of Cm^{241}, although applicable data might be obtained from studies of the alpha decay of Cf^{245}.

Cm^{243} (neutron number 147). The ground state of Cm^{243} might be expected to be 5/2+[622], the same as Pu^{241}. There is some experimental evidence for this assignment from the study of the alpha spectrum of Cm^{243}. (See Sec. 9.4.7.) The favored alpha group populates a level in Pu^{239} at 286 kev and the assignment of 5/2+[622] has been made for this state.[111,112]

Excited states of Cm^{243} are known from the electron-capture decay of Bk^{243}, but no information is available that is suitable for making assignments.

Cm^{245} (neutron number 149). An inspection of the Nilsson diagram Fig. 3.26 leads us to expect for neutron number 149 either 7/2+[624] or 9/2-[734]. Information pertaining to the levels of Cm^{245} and the Nilsson assignments of these levels comes from a study of the following nuclides: Cf^{249}, see Sec. 9.6.7; Bk^{245}, see Sec. 9.5.4; Am^{245}, see Sec. 9.3.10 and Cm^{245} itself, see Sec. 9.4.9. The chief discussion of the Nilsson assignments of the Cm^{245} levels is given under Cf^{249} (see Sec. 9.6.7) and will only be outlined here. See also the Fig. 3.41.

The ground state is 7/2+[624]. A level at 257 kev is populated by the beta decay of Bk^{245} which is believed to have spin 3/2. This 257-kev level decays only to the ground state of Cm^{245} (not to higher members of the ground-state rotational band) by an M1 transition. Furthermore, four members of the rotational band based on the 257-kev level have been observed in the alpha decay of Cf^{249} and the spacing and alpha population of these states suggest a spin 5/2 for the 257-kev level. All the data strongly suggest a spin and parity of 5/2+ for the 257-kev level and its assignment as the 5/2+[622] Nilsson level seems almost certain.

The favored alpha decay of Cf^{249} populates a rotational band whose base level is 394 kev above the ground state of Cm^{245}. This level decays by transitions that appear to be E1 to the 7/2 and 9/2 members of the ground-state rotational band, with no detectable branching to the 5/2+[622] band at 257 kev. No branching to this (394-kev) level was observed in the decay of either Am^{245} or Bk^{245} (spins 5/2 and 3/2). From these data it is concluded that the spin and parity of the 394-kev level is 7/2- or 9/2-, with 9/2- somewhat more likely. An alpha-gamma angular-distribution measurement was made to distinguish between these two choices,[112] and the results, while not absolutely definitive, also favored the 9/2- spin. It is thus concluded that the 394-kev level (and hence the ground state of Cf^{249}) is very likely the 9/2-[734] Nilsson level. The only other level observed in Cm^{245} is one at about 630 kev populated in the decay of Bk^{245}. This level decays by a single gamma ray to the 5/2+[622] state and therefore presumably has low spin. STEPHENS, ASARO and PERLMAN[97] have very tentatively assigned it as the 1/2+[631] state which comes in as the ground state for neutron number 145.

Cf^{245} (neutron number 147). CHETHAM-STRODE and co-workers[116] have obtained evidence that the alpha-decay of Cf^{245} leads predominantly to the ground state of Cm^{241}, and we can say that the transition is unhindered or only slightly hindered. In the absence of other information, it might be inferred that the ground state of Cf^{245} is the same as Cm^{241}, 1/2+[631]. This assignment is possible but not expected. Because there is no evidence bearing on this assignment other than the observation cited, we have not made an entry in Table 3.16.

Cf^{249} (neutron number 151). It has already been suggested under the Cm^{245} discussion that Cf^{249} ground state has the assignment 9/2-[734]. Consistent with this assignment is the value log ft = 7.0 for the beta decay of Bk^{249},[117] since Bk^{249} is thought to have spin and parity 7/2+.

ODD-PROTON NUCLEI

It has been seen (Table 3.15) that the same level repeats for ground states of nuclei having the same (odd) neutron number. This is a reflection of the adequacy of the model employed. Similar behavior for unpaired protons allows grouping of isotopes of each odd element for discussion. The applicable Nilsson diagram is shown in Fig. 3.25, and the summary of assignments in Table 3.16.

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116. Chetham-Strode, Choppin, and Harvey, Phys. Rev. 102, 747 (1956).
117. Magnusson, Studier, Fields, Stephens, Meek, Friedman, Diamond, and Huizenga, Phys. Rev. 96, 1576 (1954).

Actinium (proton number 89). It should perhaps be pointed out at the start that the lack of apparent rotational structure in the actinium (and, for that matter, the protactinium) isotopes has for some time been noted, and initially it seemed that these isotopes were outside the region of stable spheroidal deformation and could not be described by the strong-coupling approximation of the unified nuclear model. STEPHENS, ASARO and PERLMAN[97] concluded that this absence of simple rotational bands is probably due rather to the many anomalous K = 1/2 rotational bands in this region, and the influence of these bands through rotational-particle coupling on the K = 3/2 and even in some cases, the K = 5/2 bands in this vicinity. The assignments made for the actinium (and protactinium) isotopes are based on this conclusion, and therefore are perhaps somewhat less certain than those made in regions where the unified nuclear model is certainly applicable.

The energy levels of Ac^{227} have been studied following the beta decay of Ra^{227},[118,119] and also rather thoroughly following the alpha decay of Pa^{231}. (See Sec. 8.3.7.) The levels of Ac^{225} have been studied only as populated by the beta decay of Ra^{225},[120,121] although some information from the alpha decay of Pa^{229} has been obtained.[122] For this reason considerably more is known about the energy levels in Ac^{227}, and this isotope will be discussed first.

The favored alpha decay of Pa^{231} seems to populate a level in Ac^{227} at 330 kev. Because the ground state of Pa^{231} is believed to be the 3/2 member of the K = 1/2 rotational band based on the state, 1/2-[530], this assignment is also given to the 330-kev level in Ac^{227}. It should be emphasized that the state at 330 kev has spin 3/2 according to this assignment but is properly designated by the K quantum number of the orbital which is 1/2. An unambiguous designation would be 3/2, 1/2-[530], showing that it is the I = 3/2 number of the K = 1/2 band. The I = 1/2 and I = 7/2 members of this band are also presumably seen in Ac^{227} at energies of 356 and 386 kev, respectively. The alpha populations from Pa^{231} are in good agreement with those expected for favored decay to this band.

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118. J. P. Butler and J. S. Adam, Phys. Rev. 91, 1219 (1953).

119. Asaro, Stephens, and Perlman, unpublished data (1956).

120. Magnusson, Wagner, Engelkemeir, and Freedman, Argonne National Laboratory Report ANL-5386 (January 1955).

121. Perlman, Stephens, and Asaro, Phys. Rev. 98, 262A (1955).

122. Max Hill, University of California Radiation Laboratory Report UCRL-8423, (1958) and private communication.

The ground state of Ac^{227} has a measured spin of 3/2 and is probably connected with the 330-kev level by what appears to be an M2 transition. The ground state must therefore have even parity, and because the two states both have spin 3/2 there is implied a strong retardation of the permitted E1 transition. The most likely 3/2+ assignment is 3/2+[651].

Between about 25 and 125 kev above the ground state of Ac^{227} there are at least six levels observed, most of which can be shown to have odd parity if the two previous assignments are correct. There is no apparent rotational structure among these levels, but, as will be pointed out, this does not necessarily mean that these states all have different intrinsic configurations. Under conditions which could apply here, there can be severe distortions in level spacings in a rotational band.

Returning to the assignments of these levels, we note from Fig. 3.25 that aside from 1/2-[530] the only odd-parity states in the vicinity of 3/2+[651] are those connected with the $h_{9/2}$ orbital: 1/2-[541], 3/2-[532], and 5/2-[523]. The positions of these levels for proton number 89 are not known, but it will be seen that the state 5/2-[523] comes in as shown in Fig. 3.25 as the ground state for proton number 95. In order for it to lie below 1/2-[530] (at 330 kev in Ac^{227}) the nuclear deformation would have to be considerably less for this state in Ac^{227} than that indicated by the dashed line in Fig. 3.25. In this regard, the spin 5/2 for Pa^{229} suggested by HILL[122] (see section on protactinium) would make this assumption reasonable. From the foregoing arguments we might expect the 3/2-[532] and 5/2-[523] levels to lie closest to the 3/2+[651] ground state of Ac^{227} and suggest tentatively that a group of levels around 27 kev be assigned the orbital 3/2-[532] and those around 110 kev, 5/2-[532]. The apparent absence of rotational structure is attributed to the displacement of levels by the mechanism discussed by KERMAN.[123] However, these assignments are too speculative for inclusion in Table 3.16.

Very little is known about Ac^{225} but there is one piece of information which suggests similarity to Ac^{227}. In Ac^{227} there is a 27-kev E1 transition between an excited state of 27 kev and ground. In Ac^{225} a 40-kev E1 transition is found and is the only prominent gamma transition following Ra^{225} decay. This implies that there is a level in Ac^{225} at 40 kev bearing the same relation to the ground state as the 27-kev level in Ac^{227}.

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123. A. K. Kerman, Dan. Mat.-fys. Medd. 30, No. 15 (1956).

Protactinium (proton number 91). A considerable number of protactinium isotopes are known but substantial data are available for only Pa^{231} and Pa^{233}. The energy levels of Pa^{231} have been studied from the beta decay of Th^{231} (Sec. 8.2.9), U^{231} electron capture (Sec. 8.4.5), alpha decay of Np^{235} (Sec. 9.1.6), and Coulomb excitation of Pa^{231} itself.[124] The states of Pa^{233} have been studied in conjunction with Th^{233} beta decay (Ref. 101 and Sec. 8.2.11), and Np^{237} alpha decay (Sec. 9.1.9).

The ground-state spin of Pa^{231} has been measured as 3/2 and that for Pa^{233} is deduced to be the same from the decay properties. Each is probably the I = 3/2 member of the K = 1/2 band, 1/2-[530], which has already been mentioned in the discussion of Ac^{227} where this state appears at 330 kev above ground. In Pa^{233}, the I = 1/2, 3/2, 5/2, and 7/2 members have been observed, and these lie at $\sim$ 6, 0, 69 and 56 kev, respectively. The structure and spacings in this anomalous rotational band are probably much the same in Pa^{231}.

In both Pa^{231} and Pa^{233} there is another intrinsic state at about 85 kev. Each drops to the 3/2 and 7/2 members of the ground state band by E1 transitions and has been assigned 5/2+[642] partly on the basis of these gamma transitions and also because this level in Pa^{233} receives the favored alpha transition from Np^{237} decay and 5/2+[642] is almost surely the ground state of Np^{237}.

A somewhat uncertain assignment of 3/2+[651] has been made for levels in Pa^{231} and Pa^{233} at 166 kev and $\sim$ 200 kev, respectively. There is no obvious rotational-band structure based on either this state or the 5/2+[642] state. However, similar states lying so close to each other might be expected to interact in such a way as to distort the normal rotational-level spacings.

There is also some information available for assigning the ground states of Pa^{235} and Pa^{229}. Pa^{235} might be expected to have the 1/2-[530] band as its ground state in analogy to Pa^{231} and Pa^{233}. This would be consistent with the observed decay of Pa^{235} without gamma-ray emission to U^{235}, since a 1/2+ state lies within 0.1 kev of the ground state of U^{235}. Also the log-ft value for this decay is very similar to that observed for decay between these two states in other nuclei. Nevertheless, because the ground state of U^{235} has a spin of 7/2-, almost any spin up to 9/2 would be possible for Pa^{235}. An enlightening experiment would be a determination of whether the 26-min isomeric state (spin 1/2) of U^{235} is populated in the decay of Pa^{235}.

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124. J. O. Newton, Nuclear Phys. 3, 345 (1957); and private communication (1957).

HILL[122] has suggested that the ground state of Pa^{229} has the spin 5/2 on the grounds that states having spin of 7/2 and probably 5/2 are populated in the decay of this isotope to Th^{229}, and also on preliminary evidence that a 5/2 rotational band in Ac^{225} receives the favored alpha decay of Pa^{229}. If this is the case, the only two reasonable assignments would be 5/2+[642] or 5/2-[523]. It is not possible to make a clear choice between these assignments but a slight preference can be given the latter, however, because it seems less likely (see Fig. 3.25) that the 1/2-[530] state (ground states of Pa^{231} and Pa^{233}) would cross the 5/2+[642] state than it would cross the 5/2-[523] state. Further information on protactinium levels is given in a thesis study by MARSHALL.[125]

Neptunium (proton number 93). At several places in the previous discussions, special use has been made of observed E1 transitions in identifying states (see, for example, protactinium) because such transitions limit considerably the possible choices. For low energies, < 100 kev, E1 transitions are particularly easy to identify because the conversion coefficients are small and unique. In fact, very often a rough intensity measurement of the photon is sufficient for identifying the transition unambiguously as E1. Three isotopes of neptunium, Np^{235}, Np^{237}, and Np^{239}, all have prominent E1 transitions following alpha decay of their respective parents,[126] (see also the discussion of Am^{239}, Am^{241} and Am^{243} in Secs. 9.3.4, 9.3.6 and 9.3.8) and the assignments of the states involved for two of these (Np^{237} and Np^{239}) have been previously made.[127] These are 5/2+[642] for the ground states and 5/2-[523] for low-lying excited states. The arguments for these assignments are considered sound and will not be discussed further here. The energies may be seen in Table 3.16. The supporting evidence in Np^{235} is not so extensive but the analogy in energy and intensity of the E1 transition is so close that these same assignments may be made with confidence. It may be mentioned that the electron-capture decay characteristics of Np^{235} are consistent with the ground-state assignment.[128]

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125. Thomas Marshall, University of California Ph.D. thesis, 1959; see also Lawrence Radiation Laboratory Report UCRL-8740.

126. Asaro, Stephens, Gibson, Glass, and Perlman, Phys. Rev. 100, 1541 (1955).

127. Hollander, Smith, and Rasmussen, Phys. Rev. 102, 1372 (1956).

128. Hoff, Olsen, and Mann, Phys. Rev. 102, 805 (1956).

Rotational states based upon these two intrinsic states have been identified in Np^{237} and Np^{239}, but the only other well-studied intrinsic state in an odd-mass neptunium isotope is the 268-kev level of Np^{237}, which very likely has spin and parity 3/2-, and was assigned as the 3/2-[521] state by RASMUSSEN, CANAVAN and HOLLANDER.[103] STEPHENS, ASARO and PERLMAN[97] preferred the assignment 1/2-[530] with the 3/2- member of this band lying lowest, the same assignment made for the ground states of protactinium isotopes. This assignment is preferred by them for the following reasons: (1) an energy of 268 kev is already somewhat higher than might be expected for the 1/2-[530] band on the basis of its position relative to the 5/2+[642] state in the protactinium isotopes, yet it is quite unlikely that the 1/2-[530] band could lie at lower energies in Np^{237} and not have been detected from the beta decay of U^{237}. On the other hand, from the assignments made below for the americium and berkelium isotopes, it would be expected that the 3/2-[521] level would lie at energies somewhat higher than 268 kev in Np^{237}. (2) De-excitation of the 267-kev level has been shown to have M2 decay in competition with E1, and E2 with M1,[103] which can be best explained by the 1/2-[530] assignment, since this would involve K-forbidden restrictions for the dipole transitions. Neither of these arguments is very conclusive, however, so the preference for 1/2-[530] over 3/2-[521] is slight.

A Np^{237} level at 332.3 kev was seen in the β decay of U^{237} and given the tentative assignment 1/2+[400] by RASMUSSEN, CANAVAN and HOLLANDER.[128a] Levels at 368.5 and 370.9 are believed to be the I = 5/2+ and I = 3/2+ members of its rotational band. The 9/2 and 7/2 levels of this band should fall at 458 and 464 kev; BARANOV et al.[128b] found that rare alpha groups of Am^{241} populated these levels.

<u>Americium (proton number 95)</u>. The data on the americium isotopes come from the alpha decay of the berkelium isotopes (see Sec. 9.5), the electron capture decay of Cm^{241} (see Sec. 9.4.5), the beta decay of Pu^{243} (see Sec. 9.2.13), and the alpha decay of the americium isotopes, themselves (Sec. 9.3). Americium-241 and Am^{243} have measured spins of 5/2 and from studies of their alpha decay to Np^{237} and Np^{239}, it is reasonably certain that their ground state is the level, 5/2-[523]. The ground state of Am^{239} is assigned as this same Nilsson state because its pattern of alpha decay seems to be quite similar to that of Am^{241} and Am^{243}.[126]

128a. J. O. Rasmussen, F. L. Canavan and J. M. Hollander, Phys. Rev. <u>107</u>, 141 (1957).

128b. S. A. Baranov, V. M. Koulakov, A. G. Zelenkov and V. M. Chatinski, Zhur. Eksp. Teor. Fiz. <u>43</u>, No. 3, 795 (1962).

We will next turn to two excited states in Am^{243}, at 84 and 465 kev, observed following the beta decay of Pu^{243}. The 84-kev level decays by a prominent E1 transition to the ground state of Am^{243} and by a very weak E1 transition to an ~ 40-kev level--presumably the first member of the ground-state rotational band. This fixes the spin and parity of the 84-kev level at 5/2+ or 7/2+. Assignment is made to K = 5/2, in particular 5/2+[642], because the energy spacing with respect to the ground state, 5/2-[523], is similar to that seen in neptunium isotopes, except that the states are reversed. The 465-kev level decays by a predominantly M1 transition to the 84-kev level and to one, possibly two, higher members of the rotational band based on the 84-kev level. Thus the parity of the 465-kev band is even, and the spin is probably 7/2 or 9/2, although 5/2 is also a possibility. The assignment 7/2+[633] is consistent with these data and with the proposed 7/2+ spin of Pu^{243}. There is no other Nilsson level in the vicinity that seems to be satisfactory for this state.

The alpha decay properties of Bk^{243}, Bk^{245}, and Bk^{247} have all been studied, and provide an interesting comparison of the levels in Am^{239}, Am^{241}, and Am^{243}. In each case the ground-state transition is highly hindered, with hindrance factors of 660, 450, and $\gtrsim$ 500, respectively. The alpha decay transitions to the lowest intrinsic excited states observed in each of the americium isotopes also have rather similar alpha-hindrance factors. These states lie at energies of 187, 206, and 84 kev (in order of increasing mass number) and their alpha-hindrance factors are 54, 37, and 68, respectively. This suggests the same Nilsson level is involved for each isotope, and because the 84-kev level in Am^{243} has been assigned as the 5/2+[642] state, we suggest this assignment for the other two levels as well. There is additional evidence in favor of these assignments. In both Am^{239} and Am^{241} the levels (187 and 206 kev) decay to the I = 5/2 and I = 7/2 members of the ground-state (5/2-[523]) rotational band by transitions which are probably E1 (although E2 cannot be ruled out). These are just the two levels to which we would expect decay from the 5/2+[642] state. This argument, of course, hinges on whether or not the radiations observed are indeed E1. Furthermore, Cm^{241} (spin 1/2+) has no observed population of decay to the 206-kev level in Am^{241}, which is to be expected if the 5/2+ assignment of this level is correct. The differences in spacing of the 5/2+[642] levels relative to the ground states in the three americium isotopes are rather large and will be discussed later.

A third state which seems to be populated systematically in the alpha decay of the berkelium isotopes lies at energies of 540, 480, and 265 kev in Am^{239}, Am^{241}, and Am^{243}, respectively. The hindrance factors for alpha decay to these states are 3.6, 2.3, and 4.2, respectively. In Am^{239} the 540-kev state decays to the ground state by a transition whose multipolarity is uncertain. In Am^{241}, however, the 480-kev level is also heavily populated in the electron-capture decay of Cm^{241}, and decays to the ground state of Am^{241} by a predominantly M1 transition (with apparently some E2 admixture). These data, together with the 1/2+ spin of Cm^{241} and the 5/2- ground-state spin of Am^{241}, fix the spin and parity of the 480-kev level at 3/2-. In Am^{243} the 265-kev level also decays to the ground state by a transition that has been shown to be predominantly M1. Thus the 540-, 480-, and 265-kev levels in the three americium isotopes seem to be quite similar, and probably all have spin and parity 3/2-. On this basis the Nilsson assignment for these levels is almost certainly either 1/2-[530] (3/2- member lying lowest) or 3/2-[521]. STEPHENS, ASARO and PERLMAN[97] slightly prefer the former assignment because the level seems to vary in energy relative to the 5/2-[523] state in about the same manner as the 5/2+[642] state. From the slopes of the levels on the Nilsson diagram this is more reasonable for the 1/2-[530] than the 3/2-[521] assignment. However, MOTTELSON and NILSSON[96] prefer the former.

The large variation in spacing between the 5/2+[642] and 1/2-[530] states in the three americium isotopes relative to the 5/2-[523] ground state is somewhat puzzling. This is possibly due to a larger nuclear deformation in Am^{239} and Am^{241} than in Am^{243}. The reason for larger nuclear deformations in Am^{239} and Am^{241} might be found in the neutron levels filling in this vicinity, which are the steeply down-sloping (deforming) 7/2-[743] and 1/2+[631] orbitals. On the other hand, around Am^{243} and Am^{245} the much flatter (less deforming) neutron levels, 5/2+[622] and 7/2+[624], are filling. Another explanation might be based on Coriolis forces.

The only other level observed in an americium isotope is the 630-kev level populated in Am^{241} by Cm^{241} electron-capture decay. This state decays both to the ground and the 480-kev states and is tentatively given the assignment, 3/2-[521]. The assignment 3/2+[651] is also possible but seems less likely because this state would be expected to decay to the 5/2+[642] state, and no such gamma transition has been observed.

Berkelium (proton number 97). The similarity of the alpha decay schemes of Bk^{243}, Bk^{245}, and Bk^{247} has already been described, and thus we believe these three isotopes all have the same Nilsson level for their ground states. Information as to the identity of this level may be derived from the electron-capture decay of Bk^{245} to Cm^{245} (Sec. 9.5.4). This decay goes predominantly to a level assigned spin and parity 5/2+ in Cm^{245}, with no detectable branching to the 7/2+ (ground) or 9/2- (394 kev) states. This suggests a spin of 3/2 for Bk^{245}, although 5/2 or 7/2 would also be possible. From the Nilsson diagrams, either the 3/2-[521] or the 7/2+[633] level would be expected as the ground state of the berkelium isotopes, and on the basis of the above data, we definitely prefer the 3/2- state.

Considerably more information is available about the energy levels of Bk^{249} mostly from the alpha decay of E^{253}. These assignments are based on the interpretation of ASARO, THOMPSON, STEPHENS and PERLMAN[129] which is discussed in Sec. 9.7.9 and summarized in the figure accompanying that description. Three rotational bands of levels are seen. The ground state assignment is 7/2+[633]. Since the parent alpha emitter populates this rotational band with greatest intensity the ground state assignment of E^{253} is also 7/2+[633]. A second Nilsson state with quantum numbers 3/2-[521] occurs at 8.8 kev in Bk^{249}. A third state occurs at 393 kev and is believed to be 5/2+[642]. The spacings of the rotational bands based on this state and on the ground state are abnormal because of a strong Coriolis interaction of these two rotational bands.

Einsteinium (proton number 99). The only einsteinium isotope for which sufficient data are available to make an assignment is E^{253}. This isotope very likely has a ground-state assignment of 7/2+[633] because favored alpha decay was observed to a state in Bk^{249} that was given this assignment. See Sec. 9.7.9.

3.5.6 Identification of Nilsson States in Even-Mass Nuclei. The success achieved in the description of odd-mass nuclei with Nilsson wave functions stimulated a re-examination of nuclei of odd-odd type to decide whether the properties of such nuclei could be explained by the combination of a proton and a neutron in specific Nilsson states. It was found that this is indeed the case. Already in 1953 BOHR and MOTTELSON[130] noted that the ground-state

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129. F. Asaro, S. G. Thompson, F. S. Stephens, Jr. and I. Perlman, Lawrence Radiation Laboratory Report UCRL-8783 (1960).

130. A. Bohr and B. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 27, No. 16 (1953).

spins of many odd-odd nuclei could be accounted for on the basis of the coupling of the wave functions of the last neutron and proton. PEKER[131] also contributed to this discussion. GALLAGHER and MOSZKOWSKI[132] made a general survey of odd-odd nuclei in the deformed nuclei of rare earth and actinide elements and formulated coupling rules.

From an examination of the experimental data it is apparent that in deformed odd-odd nuclei the unpaired particles are individually strongly coupled to the deformed core and that the total spin I is equal to $\Omega_p \pm \Omega_n$ where the Ω_p and Ω_n Nilsson states are identical with those found in neighboring odd-proton or odd-neutron nuclei. The double degeneracy implied here is broken by a weak np interaction which is dominated by a spin-spin force which is a maximum when the intrinsic spins of these nucleons are parallel. This is expressed in the GALLAGHER-MOSZKOWSKI[132] rules for the spin of the lower state of the doublet. These rules are:

$$I = \Omega_p + \Omega_n \quad \text{if} \quad \Omega_p = \Lambda_p \pm 1/2 \quad \text{and} \quad \Omega_n = \Lambda_n \pm 1/2$$

$$I = |\Omega_p - \Omega_n| \quad \text{if} \quad \Omega_p = \Lambda_p \pm 1/2 \quad \text{and} \quad \Omega_n = \Lambda_n \mp 1/2 \quad .$$

Several examples showing the application of these rules to odd-odd nuclei in the heavy elements are collected in Table 3.17. In each case the Nilsson orbital assignments are logical ones on the basis of the occurrence of those orbitals in neighboring odd-A nuclei, but the assignments are not conclusively proved. Nonetheless there is impressive agreement between prediction and experiment on the spins of ground states and isomeric states. Even the Am^{242} case, where there appears to be disagreement on the ground state assignment, is not really an exception. For reasons[133] which are detailed in the Am^{242} section of Chapter 9 the observed ground state of Am^{242} is believed to be the I = 1 rotational level of a K = Ω = 0 ground state level. For special reasons the I = 1 rotational state is depressed below the I = 0 state.

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131. L. K. Peker, Izvestiia, Akad. Nauk SSSR, Ser. Fiz. 21, 1029 (1957).

132. C. J. Gallagher, Jr. and S. A. Moszkowski, Phys. Rev. 111, 1282 (1958).

133. F. Asaro, I. Perlman, J. O. Rasmussen and S. G. Thompson, Phys. Rev. 120, 934 (1960).

Table 3.17 Nilsson Assignments in Odd-Odd Nuclei

Nucleus	Proton orbital	Neutron orbital	Ground State: Spin and parity by GM rule	Ground State: Observed spin and parity	Isomer: Spin and parity by GM rule	Isomer: Observed spin and parity
Np^{236}	5/2+[642]	7/2-[743]	6-	6(?)	1-	1-
Np^{238}	5/2+[642]	1/2+[631]	2+	2+	--	--
Np^{240}	5/2+[642]	7/2+[624]	1+	1+	6+	high
Am^{242}	5/2-[523]	5/2+[622]	0-	1-	5-	5-
Am^{244}	5/2-[523]	7/2+[624]	6-	6-	1-	1-
E^{254}	7/2+[633]	11/2-[425]	9-	high	2-	2-

Note: Detailed discussions of these assignments and literature citations are provided in Chapter 9.

The analysis of odd-odd nuclei may be extended by considering the possibility that some of the excited levels of such nuclei may result from the excitation of one or both of the odd nucleons to different Nilsson orbitals and the recoupling of the odd particles in these new states according to the GALLAGHER-MOSZKOWSKI[132] rules. We discuss one example.

ASARO, MICHEL, THOMPSON and PERLMAN[134] studied the alpha and gamma radiations of the 152-year isomer of Am^{242}. The eleven levels of Np^{238} populated in this decay could be grouped into several rotational bands associated with levels of particle excitation. These groupings were based on the level spacings, the pattern of alpha intensities, and the multipolarities, energies and intensities of gamma transitions. The levels of intrinsic excitation in Np^{238} could be identified in a natural way with combinations of neutron and proton Nilsson orbitals taken from the two lowest-lying proton and four lowest-lying neutron orbitals in neighboring odd-A nuclei. A key point in the analysis was the identification of a level at 337 kev as a 5/2-[523] proton coupled with a 5/2+[622] neutron to total spin and parity 5-. This is identical with the ground state of the alpha parent Am^{242m}. This identification is based on the observation that the predominant (favored) alpha decay transition populated the 337-kev level and its rotational members. Further details are given in Chapter 9.

Such considerations can also be extended to even-even nuclei. That is, given the assurance that the Nilsson states of odd nucleons retain their identity in odd-odd nuclei, it is possible that some of the excited states of even nuclei may represent the breaking of a nucleon pair, the excitation of one member of the pair to a different Nilsson state and the recoupling of the two nucleons (now with different Nilsson assignments). GALLAGHER[135] considered this possibility in the analysis of spectra of even nuclei of hafnium and formulated the following rule for the coupling of like particles.

$$\Omega = |\Omega_1 - \Omega_2| \quad \text{if} \quad \Omega_1 = \Lambda_1 \pm 1/2 \;, \quad \Omega_2 = \Lambda_2 \pm 1/2$$
$$\Omega = \Omega_1 + \Omega_2 \quad \text{if} \quad \Omega = \Lambda_1 \pm 1/2 \;, \quad \Omega_2 = \Lambda_2 \mp 1/2 \;.$$

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134. F. Asaro, M. C. Michel, S. G. Thompson and I. Perlman, Proceedings of the Kingston Conference, University of Toronto Press, Toronto, p. 547 (1960).

135. C. J. Gallagher, Jr., Phys. Rev. 126, 1525 (1962).

These relations give the lower-lying of the pair of levels formed by the coupling of Ω_1 and Ω_2. We note that strongest binding occurs when intrinsic spins are antiparallel, which is the opposite of the case when a neutron-proton pair is considered. This is a reflection of the fact that the singlet interaction (spins coupled to zero) is known to be stronger than the triplet interaction (spins coupled to unity) in the case of like particles.

As an example of these considerations we can cite the comments of VANDENBOSCH and DAY[136] on the 1042-kev level of Cm^{244} prominently populated in the decay of the 10.1-hour isomer of Am^{244}. This level has spin 6+ and decays by a triplet of E2 transitions to 4+, 6+ and 8+ members of the ground state K = 0 band of Cm^{244}. But the lifetime of the 1042-kev level is > 10 μsec which is a retardation of 10^7 compared to the single particle estimate. This retardation suggests a high K-quantum number for the state. The authors suggest a K assignment of 6 resulting from the coupling of a 5/2+[622] and a 7/2+[624] neutron. The parent Am^{244} contains a 5/2-[523] proton and a 7/2+[624] neutron and can easily produce the suggested structure in the Cm^{244} daughter by converting the proton into a 5/2+[622] neutron. See Chapter 9 for details.

GALLAGHER and SOLOVIEV[137] found in a general survey of all even nuclei in the 150 < A < 190 region that many excited levels in these nuclei could be identified as two neutrons or two neutrons in different Nilsson states. They calculated energy values of the possible two-particle states using "quasi-particle" energies derived from the superfluid model of the nucleus. This method of identification of excited levels in even-even nuclei makes no provision for collective excitations. Hence problems do arise in deciding whether an experimental 0+ or 2+ level is β or γ collective vibration or a state of particle excitation if the particle excitation spectrum is expected to include a 0+ or 2+ level. Other criteria must be applied to select the correct assignment in such cases.

3.5.7 Transition Rates for Gamma and Beta Processes. The transition rate formulas and the special selection rules for gamma and beta transitions which are reviewed in Secs. 3.4.7 and 3.4.8 have many concrete applications in the heavy element region. One of the most characteristic features of heavy deformed

136. S. E. Vandenbosch and P. Day, Nuclear Phys. 30, 177 (1962).

137. C. J. Gallagher and V. G. Soloviev, Kgl. Danske Videnskab. Selskab, Mat.-fys. Skrifter 2, No. 2 (1962).

nuclei is the widespread occurrence of electric quadrupole transitions with transition probabilities strongly enhanced over estimates based on changes in single particle states. This enhancement is strong evidence for the collective nature of such transitions. A good example is the survey made by BELL, BJØRNHOLM and SEVERIENS[138] of the half life for the electric quadrupole transition connecting the first 2+ state to the ground state in eighteen even nuclei of emanation, radium, thorium, uranium and plutonium. The experimental values of the half lives and the reduced transition probabilities calculated by them with the aid of Eq. 3.37 are listed in Table 3.18. In all the cases shown, with the exception of the emanation isotopes, there is evidence from the study of decay schemes that the first excited state represents rotation of a non-spherical nucleus. In agreement with this evidence the reduced transition probabilities listed in Table 3.18 are all much higher, in many cases more than a factor of 100 higher, than the single particle estimate. If one assumes that such nuclei are correctly described as spheroidal the B(E2)'s may be used in connection with Eq. 3.38 to calculate the intrinsic quadrupole moment, Q_o. The values so calculated are large as shown in the table. The deformation parameter β (see Eq. 3.21) was evaluated from the following expression

$$Q_o = \frac{3}{\sqrt{5\pi}} ZR_o^2\beta(1 + 0.16\,\beta + \ldots) \quad .$$

The emanation nuclei which were included in this study lie outside the region of spheroidally deformed nuclei. Their first states of excitation probably represent vibrations around a spherical or nearly spherical ground state as discussed in Sec. 3.3 above. See Fig. 3.14A. Hence it is not surprising that the B(E2) and Q_o values are relatively small compared to the heavier nuclei.

Many other examples of the application of transition rate formulas and selection rules are given in the detailed discussion of individual decay schemes particularly in Chapters 8 and 9. Some good examples of the early detailed application of these considerations to a heavy nucleus are in the work of HOLLANDER, SMITH and MIHELICH[109,139] on Pu^{239} and of RASMUSSEN, CANAVAN and HOLLANDER[103] on Np^{237}.

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138. R. E. Bell, S. Bjørnholm and J. C. Severiens, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 32, No. 12 (1960).

139. J. M. Hollander, Phys. Rev. 105, 1518 (1957).

Table 3.18 Half Lives of 2+ States in Even Nuclei
and Quantities Derived Therefrom

Nuclide	Eγ (kev)	$T_{1/2}$ (10^{-10} sec)	B(E2) 0 → 2 ($e^2\ 10^{-48}\ cm^4$)	Q_o ($10^{-24}\ cm^2$)	β
Em^{218}	325	0.8	> 0.875	> 2.97	> 0.085
Em^{220}	241	1.50 ± 0.1	1.8 ± 0.12	(4.26 ± 0.14)	(0.121)
Em^{222}	187	3.2 ± 0.2	2.27 ± 0.14	(4.79 ± 0.15)	(0.136)
Ra^{222}	111	5.2 ± 0.4	4.37 ± 0.34	6.63 ± 0.26	0.184
Ra^{224}	84.4	7.6 ± 0.3	3.84 ± 0.15	6.21 ± 0.13	0.171
Ra^{226}	68	6.3 ± 0.2	5.17 ± 0.17	7.22 ± 0.12	0.197
Ra^{228}	59	5.5 ± 0.4	6.03 ± 0.44	7.79 ± 0.28	0.212
Th^{226}	72	3.95 ± 0.2	6.77 ± 0.34	8.24 ± 0.20	0.220
Th^{228}	57.8	4.0 ± 0.3	7.14 ± 0.54	8.47 ± 0.32	0.225
Th^{230}	53	3.7 ± 0.2	7.70 ± 0.63	8.80 ± 0.35	0.233
	53.4		a) 11.1 ± 1.7	10.5	
Th^{232}	50	3.45 ± 0.15	8.50 ± 0.37	9.25 ± 0.23	0.243
	50.1		a) 11.5 ± 1.7	10.8	
Th^{234}	48	3.7 ± 0.3	7.93 ± 0.64	8.93 ± 0.35	0.233
U^{230}	51.7	2.6 ± 0.3	8.90 ± 1.00	9.46 ± 0.52	0.245
U^{232}	47	2.54 ± 0.2	9.90 ± 0.78	9.98 ± 0.28	0.247
U^{234}	43	2.66 ± 0.2	9.50 ± 0.72	9.77 ± 0.38	0.251
	43.5		a) 11.4 ± 1.7	10.7	
U^{236}	45	2.32 ± 0.2	10.70 ± 0.92	10.35 ± 0.44	0.263
	44.9		a) 13.1 ± 2.0	11.5	
U^{238}	45	2.25 ± 0.2	11.05 ± 0.98	10.35 ± 0.44	0.268
			a) 13.2 ± 2.0	11.5	
Pu^{238}	44	1.83 ± 0.15	11.9 ± 1.0	10.52 ± 0.48	0.271
	44.7				
Pu^{240}	43	1.73 ± 0.15	12.60 ± 1.1	11.26 ± 0.48	0.278
Cm^{244}	42.9	0.97 ± 0.05	18.1 ± 0.9	13.5 ± 0.3	0.32

From BELL, BJØRNHOLM and SEVERIENS[139] except for values marked a) which are results of Rester, Moore and Class, Nuclear Phys. 22, 104 (1961) and the Cm^{244} values which are those of Christensen, Nuclear Phys. 37, 482 (1962).

MOTTELSON and NILSSON[140] tabulate log-ft values of beta transitions throughout the heavy element region and discuss the correlation of the results with the selection rules. STEPHENS, ASARO and PERLMAN[141] give a similar table which we reproduce here as Table 3.19. Here the log-ft value and the parent nucleus are listed at the intersection of the column and row corresponding to the two levels that the beta transition connects. The log-ft values are only approximate, and in a few cases the value given very likely represents that for decay to more than one member of the rotational band, rather than just the base level. These errors are probably not larger than a factor of 2 or 3 in the ft value, however. The classification of each beta transition is given at the top of each group. These classifications are allowed (a) or first or second forbidden (1f or 2f), according to the usual selection rules, and hindered (h) or unhindered (u) in the asymptotic quantum numbers of deformed nuclei according to the rules given by ALAGA[142] in Table 3.7. The parentheses around the transitions indicate uncertainty in orbital assignment as in Tables 3.15 and 3.16 and in this case the notation on the beta transition is that of the least certain of the levels which it connects.

The range of log-ft values for the various degrees of hindrance may be summarized as follows.[140]

$4.5 < \log ft < 5.0$	au
$6.0 < \log ft < 7.5$	ah
$5.5 < \log ft < 7.5$	lu
$7.5 < \log ft < 8.5$	lh

We have shown in the preceding section that some success has been achieved in the identification of Nilsson proton and neutron states in odd-odd nuclei. In beta decay of nuclei in which such an analysis has been carried out it is possible to specify the Nilsson states involved in the beta transformation. It is interesting to see whether the log-ft value is the same as that for the identical transformation in a neighboring nucleus of odd A. We present data on several cases in Table 3.20. The allowed hindered transformation in the decay of

140. B. R. Mottelson and S. G. Nilsson, The Intrinsic States of Odd-A Nuclei Having Ellipsoidal Equilibrium Shape, Mat.-fys. Skr. Dan. Vid. Selsk. 1, No. 8 (1959).

141. F. S. Stephens, F. Asaro and I. Perlman, Phys. Rev. 113, 212 (1959).

142. G. Alaga, Phys. Rev. 100, 432 (1955).

Table 3.19 Beta Decay Log-Ft Values

Neutron state	Proton state 3/2+[651]	1/2-[530]	5/2+[642]	5/2-[523]	3/2-[521]	7/2+[633]
3/2+[631]		1f, u (Pa^{233} 7.1)				
5/2-[752]		2f ((U^{231}> 7.3))	1f, u ((U^{231} 5.9))			
5/2+[633]	a, h (Th^{231}~ 5.8)	1fΔl = 2, h Th^{231}> 7 Pa^{233} 9.3	a, h (Th^{231} 5.7)			
7/2-[743]			1f, u Np^{235} 6.6 Np^{239} 6.5 Pu^{237}~ 6.8	a, hΔN = 2 Am^{239}> 8		
1/2+[631]		1f, u ((Pa^{233}~ 6.5)) ((U^{237} 6.2)) ((Cm^{241} 7.1))	2f U^{237}> 10 Np^{239}> 9.1 (Cm^{241}> 9)	1fΔI = 2, h U^{237}> 8.9 Am^{239}> 7 Cm^{241}> 9	1f, u ((Cm^{241} 7.3)) (Bk^{245} 7.0)	
5/2+[622]			a, h Np^{239} 7.0	1f, u ((U^{239}~ 5.8)) Pu^{241} 5.7 Am^{239} 5.9	1f, u Bk^{245} 7.0	
7/2+[624]			a, h (Np^{239} 6.8) (Pu^{243} 5.9)	1f, u (Pu^{243} 6.1) (Am^{239} 6.1)	1fΔI = 2, h Bk^{245}> 8.3	a, h (Pu^{243} 5.5)
9/2-[734]						1f, u Bk^{249} 7.0

Table 3.20 Beta Decay Log-Ft Values in Even Mass Nuclei

Beta Transformation	Nucleonic Transformation	Classification	Log Ft	Log Ft in Odd-A Neighbor
$Np^{236} \rightarrow U^{236}$	5/2-[523] → 7/2-[743]	ah	7.0	
$Np^{236} \rightarrow Pu^{236}$	7/2-[743] → 5/2-[523]	ah	6.6	
[a] $Am^{242} \rightarrow Pu^{242}$	5/2+[622] → 5/2-[523]	lu	7.1	5.8
$Am^{242} \rightarrow Cm^{242}$	5/2-[523] → 5/2+[622]	lu	6.8	5.8
[b] $Am^{244} \rightarrow Cm^{244}$	5/2+[622] → 5/2-[523]	lu	5.7	5.8
$Am^{244m} \rightarrow Cm^{244}$	7/2+[624] → 5/2-[523]	lu	6.3	6.1

a) F. Asaro et al., Phys. Rev. 120, 934 (1960).

b) S. E. Vandenbosch and P. Day, Nuclear Phys. 30, 177 (1962).

Np^{236} and the first-forbidden unhindered transitions in the decay of Am^{244} fall right in the middle of the range for transitions of these types. In the case of the decay of Am^{242} the log-ft value falls in the range but is definitely higher than for the identical transition in U^{239}, Pu^{241} and Am^{239}. This may be caused by the peculiarity in Am^{242} related to the occurrence of the I = 1 rotational level of the K = 0- state as the ground state, as discussed by ASARO, PERLMAN, RASMUSSEN and THOMPSON.[133]

GALLAGHER[143] reviewed all data relating to the classification of beta and gamma transitions between Nilsson intrinsic states in deformed even-mass nuclei in the lanthanide and actinide elements.

VOROS, SOLOVIEV and SIKLOS[143a] have also systematically analyzed all data on beta transformations in nuclei of odd and even mass in the transuranium element group. They classified all transitions in terms of Nilsson orbitals treated in the formalism of the superfluid model of the nucleus.

3.5.8 Lifetime and Conversion Anomalies of E1 Gamma Transitions in Odd Mass Spheroidal Nuclei.[144] A large number of electric dipole transitions have been identified between low-lying states of odd-mass nuclei in the regions of spheroidal nuclei. The mere existence of low-lying E1 transitions in odd-mass nuclei is not explainable in terms of transitions between states of lowest seniority (all nucleons paired except the odd one) in the spherical shell model since the required parity change and the spin change of one or zero are not to be found between orbitals within a given shell; $\Delta I_{minimum} \geq 2$ for transitions with parity change. On the other hand the occurrence of low energy E1 transitions is a natural consequence of the unified model for deformed nuclei.

An inspection of the Nilsson chart in Figs. 3.25 and 3.26 along the dashed lines which indicate the expected spheroidal deformation will show several places where the spin and parity values of near-lying states would permit E1 transitions if only the simple selection rule, $\Delta\Omega = 0, 1$ parity change, is considered.

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143. C. J. Gallagher, Nuclear Phys. 16, 215 (1960).

143a. T. Voros, V. G. Soloviev and T. Siklos, United Institute of Nuclear Studies, Dubna, USSR, Report Dubna-932 (1962).

144. In this section we follow closely the treatment of D. Strominger and J. O. Rasmussen in Nuclear Phys. 3, 197 (1957).

The El transitions under discussion have been of considerable interest ever since BELING, NEWTON and ROSE[145] discovered the measurable half life of the 59.7-kev El transition following the alpha decay of Am^{241} to Np^{237}.

Table 3.21 lists electric dipole transitions of odd-proton nuclei in the actinide element region. Most of these are of measured lifetime. These measured lifetimes are compared in column 4 with lifetime estimates based on MOSZKOWSKI'S[146] version of the Weisskopf "single proton" transition rate formula. It is evident that a large retardation of transition probability from the value calculated by the use of the single-proton formula is the general rule.

These electric dipole transitions with anomalously long half lives are also anomalous with respect to their conversion coefficients. ASARO, STEPHENS, HOLLANDER and PERLMAN[147] summarized all experimental data on the transitions listed in Table 3.21 and showed that whereas the conversion coefficients in the L_{III} shell agree with the theoretical coefficients, the L_I and L_{II} coefficients are substantially larger than the theoretical values. The most striking anomaly occurs in the 84.2-kev transition in Pa^{231}, where the L_I and L_{II} coefficients are 21 and 15 times larger than the theoretical values, respectively. The experimental L_I and L_{II} coefficients are correlated with the lifetimes of the transitions; the magnitude of the anomaly $(L_I + L_{II})$ is proportional to the retardation in gamma ray lifetime over that calculated from the single-proton formula.

Some progress toward an explanation of these conversion anomalies has been made. It is recognized that the infrequently observed deviations can be attributed to the influence of the nucleus on those electrons whose wave functions show an appreciable penetration of the nuclear volume. The overwhelming contribution to the normal internal-conversion process comes from regions outside the nuclear volume. Hence theoretical treatments which treat the nucleus as a point or which allow for the finite extension of the nucleus in a simplified

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145. J. K. Beling, J. O. Newton, and B. Rose, Phys. Rev. 87, 670 (1952).

146. S. A. Moszkowski, Theory of Multipole Radiation in Beta- and Gamma-Ray Spectroscopy, edited by K. Siegbahn (North Holland Publishing Company, 1955), pp. 373-395.

147. F. Asaro, F. S. Stephens, J. M. Hollander and I. Perlman, Phys. Rev. 117, 492 (1960).

Table 3.21 Lifetimes of E1 Transitions in Odd-Z Heavy Element Isotopes

Daughter	Energy of gamma (kev)	Half-life of parent level (seconds)	Partial half-life of photon (seconds)	Retardation factor from single proton formula	Reference
Am^{243}	85	$< 4 \times 10^{-9}$	$< 5 \times 10^{-9}$	$< 2.5 \times 10^{4}$	1
Np^{239}	75	$< 1.6 \times 10^{-9}$	$< 2 \times 10^{-9}$	$< 1.1 \times 10^{4}$	2
Np^{237}	59.6	6.3×10^{-8}	1.6×10^{-7}	2.8×10^{5}	3
Np^{237}	26.4	6.3×10^{-8}	2.25×10^{-6}	3.5×10^{5}	3
Pa^{233}	86.9	3.7×10^{-8}	1.5×10^{-7}	8.2×10^{5}	4, 5
Pa^{233}	30.1	3.7×10^{-8}	2.2×10^{-7}	5.0×10^{4}	4, 5
Pa^{231}	84	4.1×10^{-8}	1.4×10^{-7}	6.7×10^{5}	4, 6, 7
Pa^{231}	25.5	4.1×10^{-8}	1.5×10^{-7}	2.0×10^{4}	4, 6, 7
Ac^{227}	27	4×10^{-8}	3.6×10^{-7}	5.7×10^{4}	6
Ac^{225}	40	$< 2 \times 10^{-9}$	$< 4.2 \times 10^{-9}$	$< 2.2 \times 10^{3}$	4

1. D. W. Engelkemeir, P. R. Fields, and J. R. Huizenga, Phys. Rev. 90, 6 (1953).

2. F. Asaro and I. Perlman, Phys. Rev. 93, 1423 (1954).

3. J. R. Beling, J. O. Newton, and B. Rose, Phys. Rev. 87, 670 (1952).

4. F. S. Stephens, Jr., University of California Radiation Laboratory Report UCRL-2970 (June 1955).

5. D. W. Engelkemeir and L. B. Magnusson, Phys. Rev. 94, 1395 (1954).

6. J. Teillac, M. Riou and P. Desneiges, Compt. rend. 237, 41 (1953).

7. D. Strominger and J. O. Rasmussen, Phys. Rev. 100, 844 (1955).

way result in highly satisfactory theoretical coefficients for most gamma transitions. The electric dipole transitions under consideration, however, apparently involve in an important way electronic wave functions which penetrate the nuclear volume. Hence a detailed nuclear model is required for the proper calculation of their coefficients. NILSSON and RASMUSSEN[148] develop such a model-dependent theory appropriate for a spheroidally-deformed nucleus. This theory was extended by KRAMER and NILSSON[148a].

The question of the great retardation in transition rates of these E1 transitions has also been an intriguing one. The spectacularly hindered (factor of 10^{15}) E1 isomer of 5.5-hour half life in Hf^{180} finds good qualitative explanation in terms of a large K-forbiddenness. However, for all cases of Table 3.21 except those in actinium where state assignments are uncertain, we have no K-forbiddenness, the K-change never exceeding one.

In a spherical nucleus, the retardation could be explained in terms of simple arguments about $\Delta\ell$ or Δj hindrance but when marked nuclear deformation is present j and ℓ are no longer good quantum numbers. For example, let us consider the nuclear states of Np^{237} connected by the 59.6-kev E1 transition seen in the alpha decay of Am^{241}. The Nilsson eigenfunction $[N, n_z, \Lambda] = [642]$ which we assign to the ground state (5/2+) of Np^{237} is composed at deformation $\delta \approx 0.3$ of about 84% $i_{13/2}$, 14% $g_{9/2}$, 1.3% $i_{11/2}$, 0.4% $d_{5/2}$ and 0.3% $g_{7/2}$ character. The eigenfunction [523] assigned to the 59.6-kev state is at the same deformation about 79% $h_{9/2}$, 7.6% $f_{7/2}$, 7.4% $f_{5/2}$ and 6.1% $h_{11/2}$. There seems to be ample admixtures of j-values other than the principal value to allow E1 transitions to occur. However, STROMINGER and RASMUSSEN[144] found that if the Nilsson wave functions were used to make exact calculations of transition probabilities the various contributions from the small parts of the wave function tended generally to cancel one another. The calculations of STROMINGER and RASMUSSEN[144] using equations given in NILSSON'S paper[149] are summarized in Fig. 3.42. The transition probabilities for E1 transitions for various pairs of eigenfunctions which could be connected by low energy E1 transitions in the 82-126 proton shell were calculated for the three values of prolate

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148. S. G. Nilsson and J. O. Rasmussen, Nuclear Phys. 5, 617 (1958).

148a. G. Kramer and S. G. Nilsson, Nuclear Phys. 35, 273 (1962).

149. S. G. Nilsson, Dan. Mat.-fys. Medd. 29, No. 16 (1955).

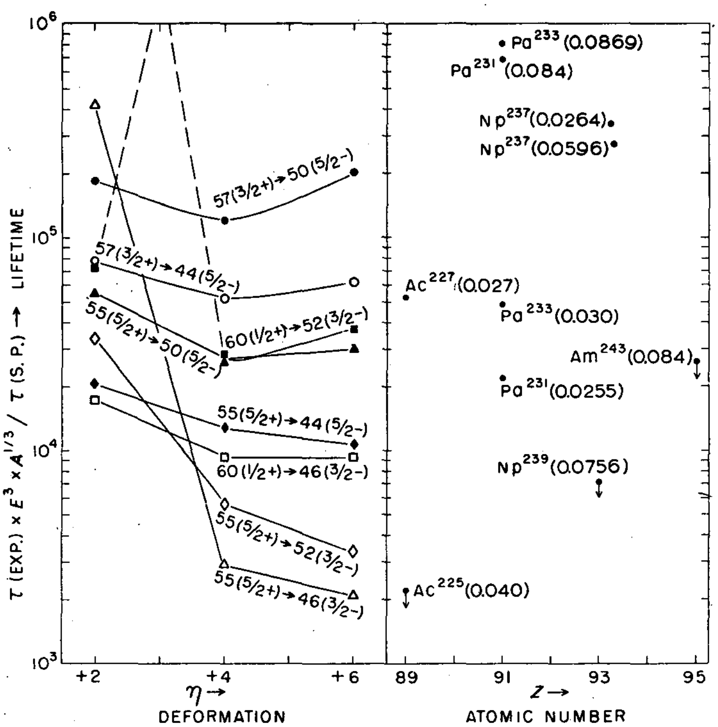

Fig. 3.42 Delayed E1 transitions. (Left side of diagram) Calculated lifetimes as a function of deformation. Note that for purposes of comparison the calculated values are multiplied by ($E^3 \times A^{1/3}$) and divided by the value obtained from the single proton formula. The deformation parameter η is approximately twenty times as large as the deformation parameter δ defined by Eq. 3.30. The number labels refer to the numbered orbitals in Nilsson's paper (Dan. Matt. Fys. Medd. 29, No. 16 (1955)). (Right side of diagram) Experimental lifetimes as a function of atomic number for odd-proton nuclei in the actinide element region. Note that a transition which proceeds as rapidly as predicted by the single proton formula would have a value of unity on the ordinate scale of the figure. The numbers in parentheses are the energies of the transitions in Mev.

deformation for which Nilsson gives eigenfunctions. CHASE and WILETS[150] have made similar calculations for E1 transitions in Lu^{175} where they noted a theoretical retardation of about 10^4.

It is satisfying to note that the calculations always yield reduced transition probabilities lying within the same region as the experimental data and the conclusion seems valid that the model provides a satisfactory qualitative explanation of the E1 transition half lives.

Detailed agreement in individual cases is not obtained and is probably not to be expected. Quite generally the transitions proceed by small parts of the wave function; hence the transition probability will usually be quite sensitive to small details of the wave function and will strongly reflect small K-impurities in the model wave functions. We see for example that the calculation of the transition (5/2+[642] → 5/2-[523]) fails by a factor of 25 to agree with experiments in Np^{237}. In view of the fact that the experimental half life is 2.8×10^5 times slower than the single proton estimate, the remaining factor of 25 is not large.

In Table 3.5 which appears in Sec. 3.4.7 the selection rules in the asymptotic quantum number for electromagnetic transitions in deformed nuclei are summarized. A consideration of these rules as applied to E1 transitions reveals that the E1 transitions under discussion violate these selection rules in the limit of large nuclear deformation. The only allowed E1 transitions are between orbitals which must be separated in energy by several Mev.

In summary, one can state that in the limit of strong deformation the E1 transitions in the heavy element region should be strictly forbidden by the selection rules in N, n_z, and Λ. Also in the limit of a spherical nucleus they should be strictly forbidden by the selection rules in ℓ and j. At intermediate nuclear deformations some transition probability is expected because of small impurities in the wave functions. However, the small parts of the wave function always give contributions to the E1 transition probability which tend to cancel each other as shown by detailed calculations. The retardation is generally several orders of magnitude from the value calculated by use of the single-proton formula. This retardation is more pronounced than the analogous retardation in beta transitions caused by the operation of selection rules in the asymptotic quantum numbers; in the case of beta decay the observed retardation is about a factor of 10.

150. D. M. Chase and L. Wilets, Phys. Rev. 101, 1038 (1956).

E1 transitions in odd-neutron nuclei. As a supplement to Table 3.21 we present Table 3.22 containing lifetime data on E1 transitions in odd-neutron actinide element isotopes. In this case, the transition lifetimes are compared with a "single-neutron transition formula." Some additional weak E1 transitions (316.1 kev and 334.5 kev) have also been observed[151] branching from the 0.19-μsec state in Pu^{239}, and these transitions are more highly retarded than the other E1 transitions. The additional hindrance may be associated with the K-forbidden nature of these transitions ($\Delta K = -2$).

3.5.9 Coulombic Excitation of Levels in Heavy Element Isotopes. It is possible to produce nuclear excitations by the long range electric interactions of a target nucleus with bombarding particles. When the incident energy is so low that the Coulomb repulsion prevents the bombarding particles from entering the nucleus such nuclear excitations can be studied without interference from the nuclear interactions. The term "Coulombic excitation" is applied to this process. Coulombic excitation occurs most strongly when low-lying collective excitations of the nucleus can be induced by the electric quadrupole field of the bombarding charged particles. Coulombic excitation is extremely valuable in exploring the low-lying rotational and vibrational spectra of nuclei.

A comprehensive review of theoretical and experimental aspects of the Coulombic excitation process including a complete summary of experimental data collected by 1956 is given by ALDER, BOHR, HUUS, MOTTELSON and WINTHER.[152] Another excellent but shorter review is provided by HEYDENBURG and TEMMER.[153] The special features of the Coulombic excitation process when ions heavier than helium are used are treated by NEWTON.[154]

Excitation by this process is limited in practice mostly to those levels which can be reached by electric quadrupole (E2) excitation because the matrix elements for the transitions of most multipole orders are small whereas the electric quadrupole moments of many nuclei are quite sizeable as a result of the collective motions. In the regions of spheroidal nuclei one can excite

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151. J. M. Hollander, W. G. Smith, and J. W. Mihelich, Phys. Rev. 102, 740 (1956).

152. K. Alder, A. Bohr, T. Huus, B. R. Mottelson and A. Winther, Rev. Modern Phys. 28, 432 (1956).

153. N. P. Heydenburg and G. M. Temmer, Ann. Revs. of Nuclear Science 6, 77 (1956).

154. J. O. Newton, University of California Radiation Laboratory Report UCRL-3797, (1957).

Table 3.22 Experimental E1 Transition Lifetimes for Odd-Neutron Nuclei in the Actinide Element Region

Daughter nucleus	Energy of photon (kev)	Half-life of parent level (seconds)	Partial half-life of photon (seconds)	Retardation factor from single-particle formula	Reference
${}^{145}_{94}Pu^{239}$	106	1.9×10^{-7}	3.4×10^{-7}	5.2×10^{6}	a
${}^{145}_{94}Pu^{239}$	61.4	1.9×10^{-7}	1.9×10^{-6}	5.7×10^{6}	a
${}^{141}_{90}Th^{231}$	185	8×10^{-10}	8×10^{-10}	6×10^{4}	b, c, e
${}^{141}_{90}Th^{231}$	143	8×10^{-10}	4×10^{-9}	1.4×10^{5}	b, c, e
${}^{135}_{88}Ra^{223}$	50	6×10^{-10}	1×10^{-9}	1.3×10^{3}	c, d, e

a. D. W. Engelkemeir and L. B. Magnusson, Phys. Rev. 99, 135 (1955).

b. F. S. Stephens, Jr., University of California Radiation Laboratory Report UCRL-2970 (Thesis)(June 1955).

c. D. Strominger, University of California Radiation Laboratory Report UCRL-3374 (Thesis)(June 1956).

d. E. K. Hyde, Phys. Rev. 94,1221 (1954).

e. H. Vartapetian, Compt. rend. 246, 1109 (1958).

most readily the 2+ level of the ground state rotational band in even-even nuclei and observe the decay of this level back to ground. With heavy ions of the proper energy it is possible by multiple excitation to raise some nuclei to 4+, 6+ and higher states. One then observes a cascade of E2 gamma rays. In odd-A nuclei one may expect to excite the first two excited levels of the ground state rotational band in a one-stage excitation process and to observe a pattern of three gamma rays in their de-excitation.

Coulombic excitation is important in connection with the concerns of this chapter for many reasons. In the first place the observed cross sections are much larger than one would predict from considerations of the independent particle model and confirm the importance of collective effects. The observed cross sections lead directly to the evaluation of a reduced transition probability, B(E2) (ground state → excited state) which, aside from simple statistical factors, is identical with the reduced transition probability for the inverse process defined in Eqs. 3.38 and 3.41, from which, in turn, one may evaluate the intrinsic quadrupole moment, Q_o. Furthermore one may sometimes excite rotational levels which are not observed in the decay of radioactive nuclides, and the discovery of these levels may help materially in the description of the ground state properties. In the case of Np^{237} NEWTON[155] was able to effect excitation from the 5/2 ground state to the 7/2 and rotational 9/2 levels, the second of which had not been observed in the decay of Am^{241}, U^{237} or Pu^{237} to this nucleus. NEWTON[155-157] has contributed several important papers to the study of the properties of heavy element nuclei and their interpretation within the framework of the Bohr-Mottelson unified model. Figure 3.43 summarizes his studies of odd-A nuclei.

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155. J. O. Newton, Nuclear Phys. 5, 218 (1958).

156. J. O. Newton, Nuclear Phys. 3, 345 (1957).

157. J. O. Newton, Physica 22, 1129 (1956).

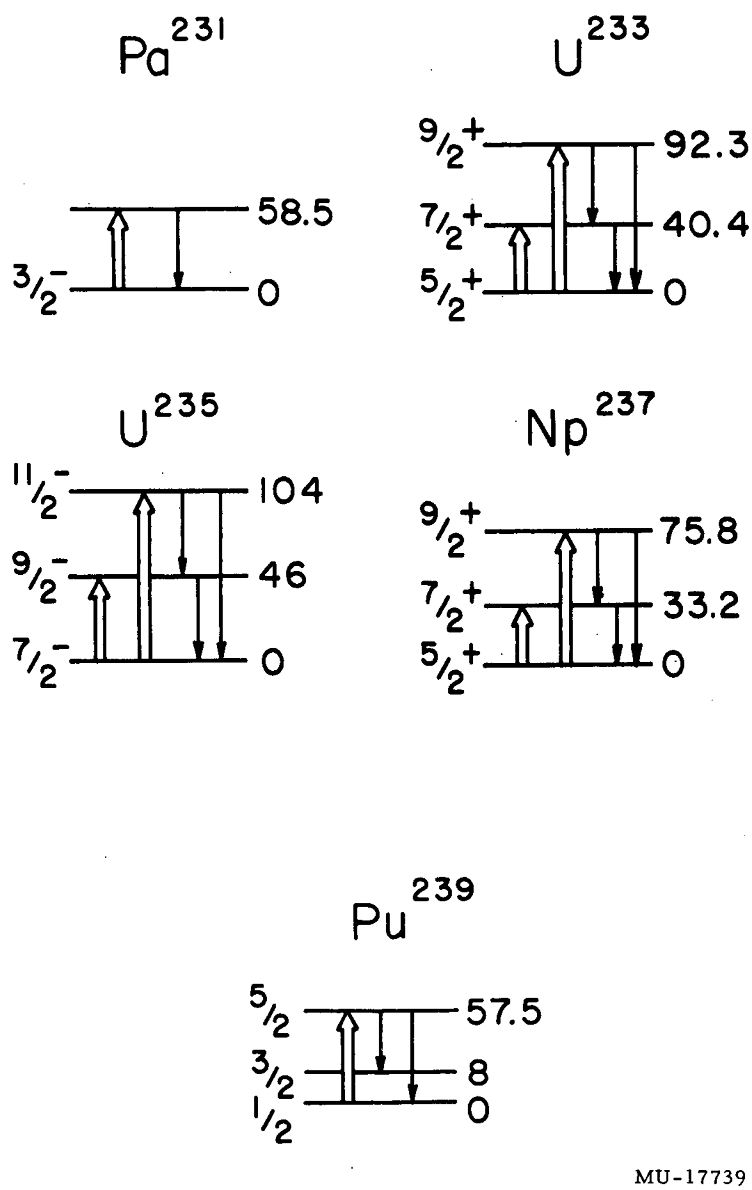

Fig. 3.43 Summary of levels observed in heavy element nuclei by the Coulombic excitation process. Odd-A nuclei.

The even nuclei of thorium and uranium, particularly Th^{232} and U^{238} have been studied by many investigators.[158-165] Some of the chief results are summarized in Figs. 3.44 and 3.45. One of the most impressive results of the Coulombic excitation process is the excitation of many members of the ground state rotational band up to the 12+ member.[162] This was done with the aid of argon ions accelerated in the Berkeley Heavy Ion Linear Accelerator. This is a remarkable confirmation of the correctness of the rotational description of the low-lying states. We have considered these results previously in Table 3.11. Another interesting result is the excitation of β and γ vibrational states in Th^{232} and U^{238}, particularly the former. If one accepts the description of these states as deformations of an ellipsoid, as given in Sec. 3.4 above, one can use the data to calculate quantitatively the shape oscillations and potential energy changes involved in these vibrations. STELSON[165] prepared an interesting figure based on the Th^{232} data which we reproduce here as Fig. 3.46. The work of DURHAM, RESTER and CLASS[164] with the conversion electrons emitted in the de-excitation process has been quite significant for the identification of these β and γ vibrations and for the study of the competition between electric monopole and electric quadrupole transitions in the decay of the states.

Still another interesting feature is the discovery by ELBEK, STEPHENS, DIAMOND and PERLMAN[163] of the excitation of collective states in Th^{232} and U^{238} by an electric octupole interaction. These findings are included in Figs. 3.44 and 3.45.

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158. G. M. Temmer and N. P. Heydenburg, Phys. Rev. 93, 351 (1954); Phys. Rev. 93, 906 (1954); Phys. Rev. 94, 1252 (1954).

159. R. H. Davis et al., Phys. Rev. 103, 1801 (1956).

160. P. H. Stelson and F. K. McGowan, Phys. Rev. 99, 112 (1955); Bull. Am. Phys. Soc. II 2, 207 (1957); Proceedings of International Conference on Nuclear Physics, Paris (1958); Phys. Rev. 120, 1803 (1960).

161. M. S. Moore et al., Bull. Am. Phys. Soc. II 1, 88 (1956).

162. F. Stephens, Jr., R. Diamond and I. Perlman, Phys. Rev. Letters 3, 435 (1959).

163. B. Elbek, F. S. Stephens, R. Diamond and I. Perlman, unpublished results (1960).

164. F. E. Durham, D. H. Rester and C. M. Class, Bull. Am. Phys. Soc. 5, 110 (1960); Phys. Rev. Letters 5, 202 (1960); Proceedings of the International Conference on Nuclear Structure at Kingston, Ontario (1960); Nuclear Phys. 22, 104 (1961).

165. P. H. Stelson, Proceedings of International Conference on Nuclear Structure at Kingston, Ontario, August 29 to September 3, 1960, edited by D. A. Bromley and E. W. Vogt, University of Toronto Press (1960).

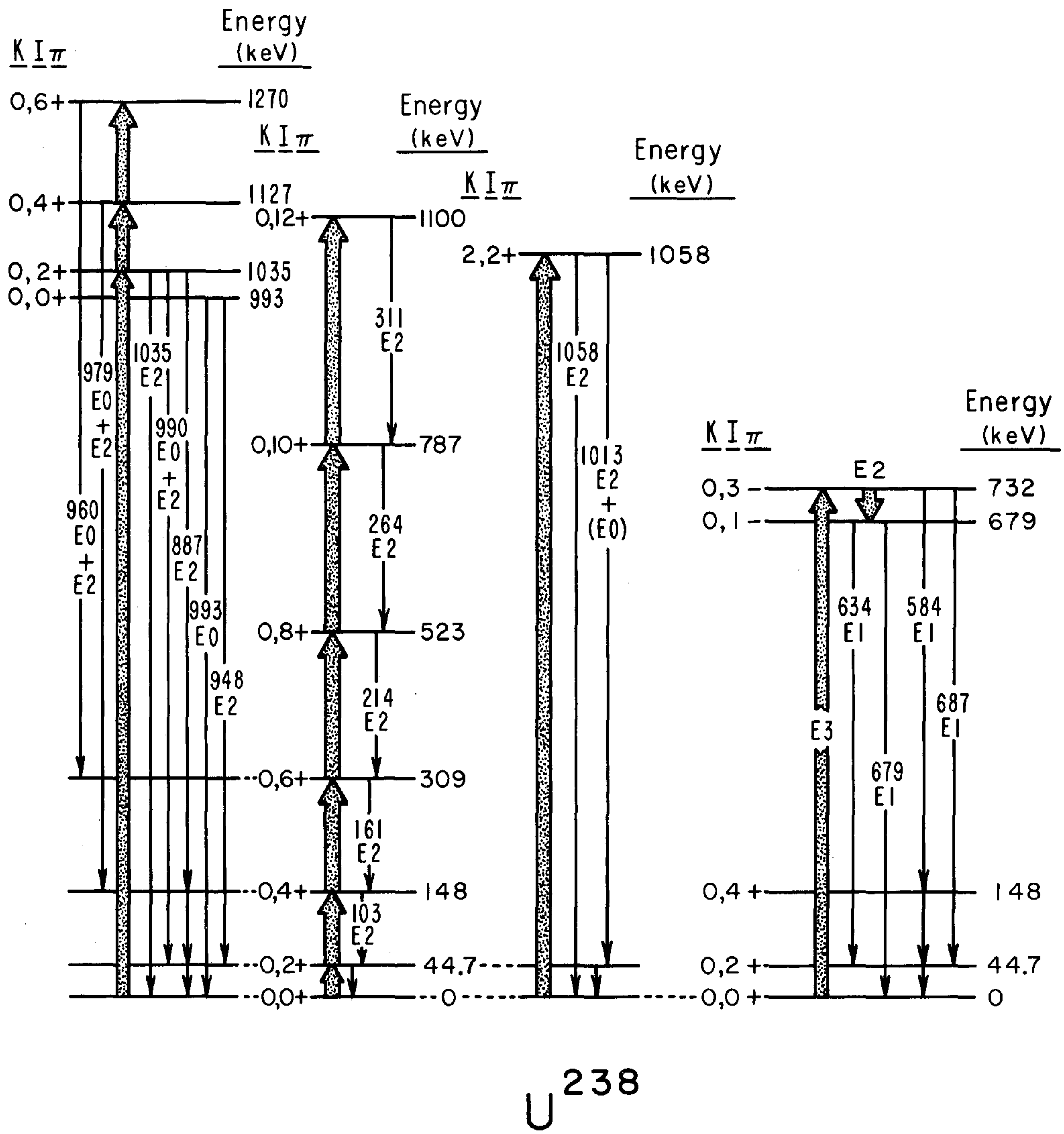

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Fig. 3.44 Summary of data from Coulombic excitation of levels in U^{238} as given by F. Stephens and R. Diamond early in 1963. Stippled arrows indicate steps in the excitation process, but not all the possible paths of excitation for the higher-lying levels are included.

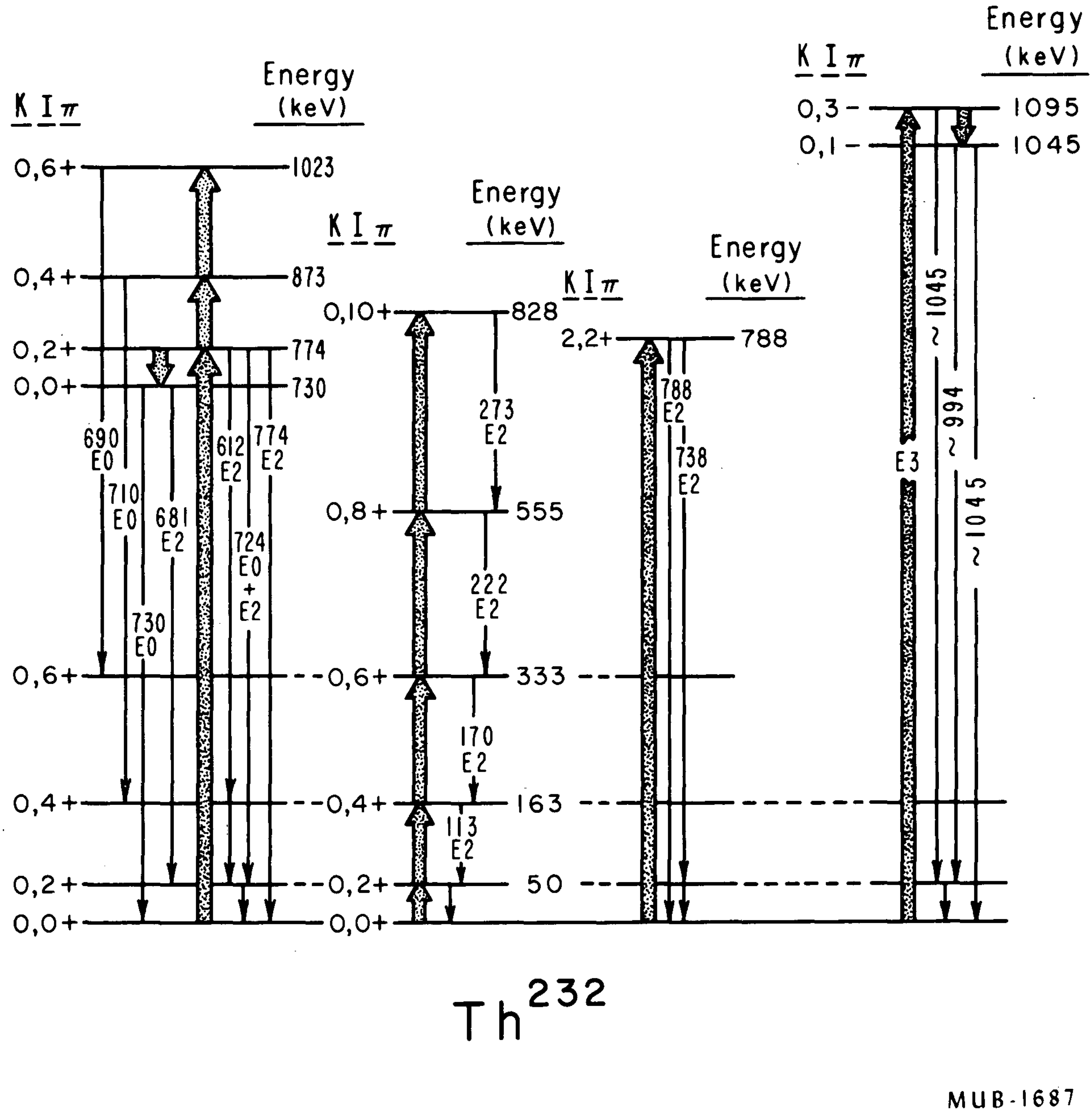

Fig. 3.45 Summary of data from Coulombic excitation of levels in Th^{232} as given by F. Stephens and R. Diamond early in 1963. Stippled arrows indicate steps in the excitation process, but not all the possible paths of excitation for the higher-lying levels are included.

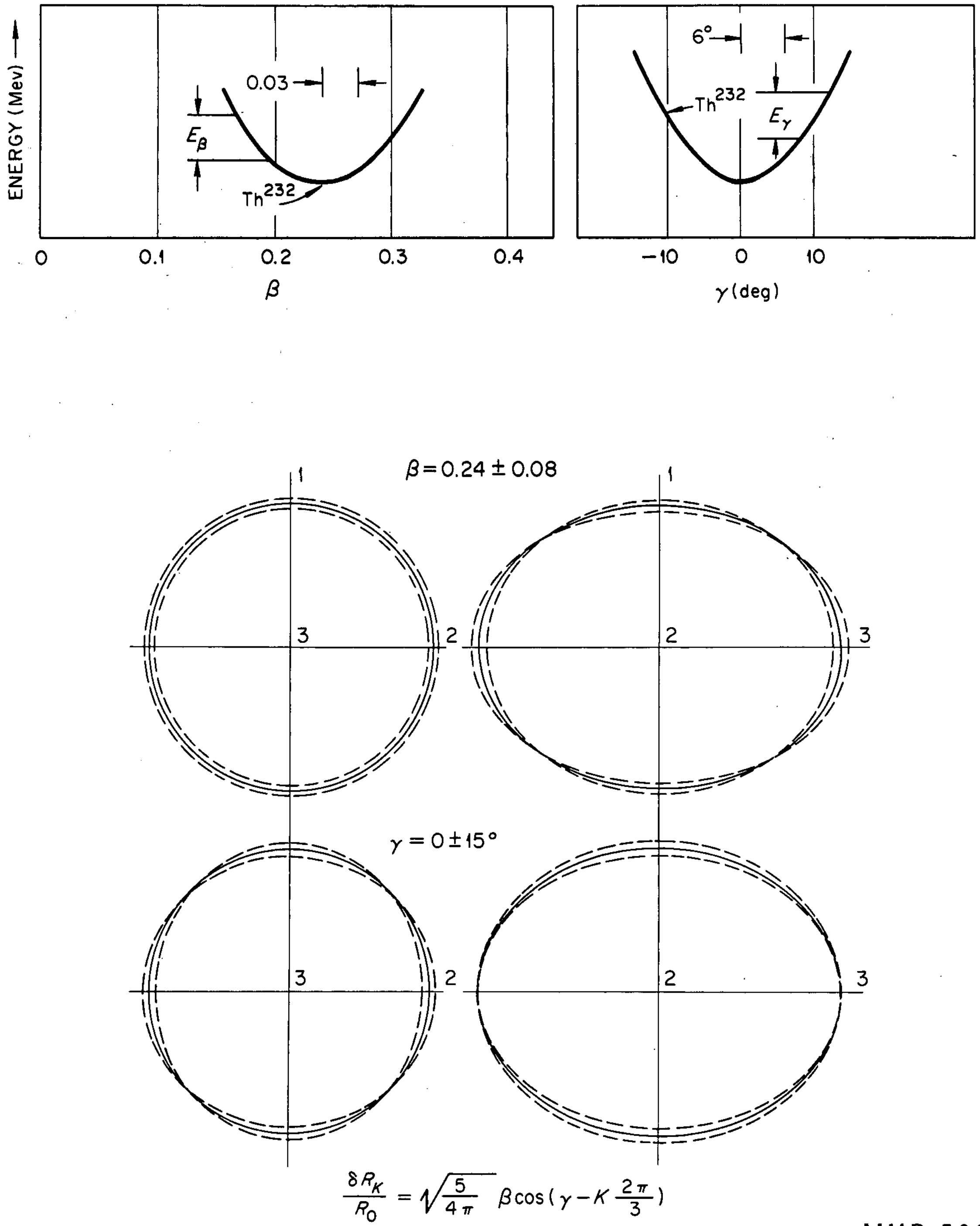

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Fig. 3.46 Top of figure: Variation of the potential energy with β and γ for Th^{232} in the region near equilibrium deformation. The zero point amplitudes are shown. Bottom of figure: Illustration of the fluctuations in shape of Th^{232} in the ground state. The zero point amplitudes have been increased by about a factor of three to show more clearly the changes in shape. The 3 axis is the symmetry axis. Vibrations in β are shown in the upper figures and vibrations in γ in the lower figures. Prepared by Stelson.

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13. J. O. Newton, Nuclear Properties of the Very Heavy Elements, Progress in Nuclear Physics 4, 234 (1955). A review article which summarizes extensively the applications of the unified model to the heavy element nuclei.

14. S. A. Moszkowski, General Survey of Nuclear Models, Contribution to Vol. 39, Handbuch der Physik, edited by S. Flügge (Springer-Verlag, Berlin, 1957).

15. B. B. Kinsey, Nuclear Reactions, Levels and Spectra of Heavy Nuclei, Contribution to Vol. 40, Handbuch der Physik, edited by S. Flügge (Springer-Verlag, Berlin, 1957); see especially pp. 334-356.

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