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COSMOLOGICAL CONSEQUENCES OF THE QUARK-HADRON PHASE TRANSITION

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ABSTRACT

A first-order phase transition is likely to have occurred in the early universe at the end of the quark era and the beginning of the hadron era. This phase transition would have created large amplitude isothermal fluctuations in the distribution of baryon number in the universe. These fluctuations have significant impact on the subsequent synthesis of isotopes of hydrogen, helium and lithium out of protons and neutrons. It turns out to be possible that all of the mass in the universe today is baryonic.

1. INTRODUCTION

It is clear that at sufficiently early times ($t \ll 10\mu\text{s}$), when the universe was at sufficiently high temperature ($T \gg 100 \text{ MeV}$), the strongly interacting particles in the universe could not have been ordinary, confined mesons and baryons. The mean number density of strongly interacting particles exceeded 1 fm^{-3} . It is certainly reasonable to suppose that at these early times the fundamental degrees of freedom were quarks and gluons in some kind of deconfined phase.

The nature of the transition from this high temperature phase to the gas of confined hadrons is not understood. One plausible mechanism is the first order phase transition. The purpose of this review is to summarize recent work which has shown that a first order phase transition of this kind has very interesting consequences for cosmology.

Early discussions of the cosmic quark-hadron phase transition (e.g., Suhonen¹; Kajantie and Kurki-Suonio²) addressed the issues of phase dynamics, but did not yield evidence of a clear signal for astronomers of the physics of the phase transition. Witten³ drew attention to the likelihood that a first order phase transition would have created inhomogeneities in the distribution of baryons, providing the first suggestions of present day legacies of the phase transition. For technical reasons, the signals Witten suggested might presently exist, nuggets of strange matter and long wavelength gravitational waves, are not likely to have been made⁴.

Applegate and Hogan^{5,6} drew attention to the possibility of interesting consequences for nucleosynthesis of inhomogeneous distributions of baryons, and discovered the important effect, described below, of neutron-proton segregation. Alcock, Fuller and Mathews^{7,8} showed that inhomogeneous distributions of

baryons are very likely to occur during a cosmic quark-hadron phase transition, as a result of a well known chemical physics effect that is everyday knowledge among chemical engineers and materials scientists.

We review here what is known about the putative cosmic quark-hadron phase transition, and summarize the important unknown physics.

2. COSMOLOGY

It will be useful to summarize a few results from modern cosmology. The universe is expanding and cooling. At the very early times (very high temperatures) that are discussed below, the age of the universe is given by:

$$t \approx 0.1 M_p / T^2 \approx 10\mu s (100 \text{ MeV} / T)^2 \quad (1)$$

where $M_p = 10^{22}$ MeV is the Planck mass, and is the "large number" in cosmology which ensures that the universe expands slowly.

A full description of the dynamics of the universe requires some understanding of the geometry of curved space-time. The study of a small region of linear size $l \ll t$, which will be called a local "patch", may be accomplished using a local Cartesian system. Physical or "proper" distances will be denoted by 3- vectors $\mathbf{x}$ with a local origin which is chosen arbitrarily. Since the universe expands, the separation $\mathbf{x}$ between two elements of the universe increases with time. This is described in terms of an expansion parameter $a(t)$, where $a(t_0) = 1$ at some fiducial time t_0 . Let $\mathbf{u}$ be the separation of two elements at $t = t_0$. Then the separation at later time t is $\mathbf{x} = a(t) \mathbf{u}$. Henceforth $\mathbf{u}$ will be referred to as the "comoving coordinate" because it is unaffected by the expansion.

The Hubble recession law gives the velocity of a point with respect to the origin,

$$\mathbf{v} = \frac{d\mathbf{x}}{dt} = H\mathbf{x} , \quad (2)$$

where $H = (\dot{a}/a)$ is the Hubble coefficient.

One important number in cosmology is the fraction Ω_B of the mean density of the present universe that is baryonic. Traditional studies⁹ of the synthesis of ^2H , ^3He , ^4He and ^7Li have clearly indicated that $\Omega_B \leq 0.1$. This result has motivated a lot of speculative suggestions for the remaining 0.9 of the mass. We will revisit the nucleosynthesis argument below.

3. THERMODYNAMICS

The assumption at the heart of this work is that there is a first-order phase transition in QCD between a high temperature phase (which we will call the quark-gluon plasma) and a low temperature phase (which is clearly the gas of mesons and baryons). This phase transition occurs at some temperature $T_C \sim 100$ MeV, and has a latent heat per unit volume $L \sim (100 \text{ MeV})^4$. The high temperature phase and the low temperature phase can coexist in equilibrium at T_C . This leads to the all important phenomenon of "cosmic phase separation": there was an epoch in the early universe when the temperature was very close to T_C and when part of the volume of the universe was occupied by quark-gluon plasma and the rest was occupied by hadron gas. These two volumes were separated by a phase boundary which had a free energy per unit area (surface tension) $\sigma \sim (100 \text{ MeV})^3$. During this epoch the universe expanded without cooling, and the fraction of the universe which was occupied by hadron gas increased from zero to unity. The duration of this phase was $t \sim 0.03 M_P L^{-1/2} \approx 30 \mu\text{s}$.

The thermodynamic description of this phase transition is very similar to the description of quark-gluon plasma formation and decay in a heavy ion collision. For a number of reasons, however, the cosmic phase transition is easier to characterize. First, the duration of the phase transition ($\sim 10 \mu\text{s}$) is so great that weak interactions come into equilibrium. This means that flavor equilibrium among the quarks is maintained and that there is only one independent chemical potential. We will use the chemical potential for baryon number (μ). Second, the density of baryon number is extremely low ($\mu \ll T$). This means that for the purposes of studying the overall dynamics of the system, the net baryon number of the universe may be ignored. The baryon number is a perturbation on the system.

The phase diagram which summarizes these issues is shown in Figure 1. Note the very large range in chemical potential shown. The trajectory of the universe in this phase plane is indicated; note that $\mu \approx 1 \text{ eV} \approx 10^{-8} T$ at the point of the phase transition.

The important consequences for cosmology of this phase transition arise because baryon number is more soluble in the quark-gluon plasma than in the hadron gas. This phenomenon may be understood as follows. The baryon number density n_B is related to the thermodynamic potential $\Omega(T, \mu, v)$ by

$$n_B = \frac{1}{V} \frac{\partial \Omega}{\partial \mu} = (\mu/v) (\partial^2 \Omega / \partial \mu^2)_{\mu=0} , \quad (3)$$

where the right hand expression is valid for $\mu \ll T$. When chemical equilibrium exists between the two phases, the chemical potential μ is the same in the two

phases. The ratio of baryon number densities in the two phases is then where suffixes Q and H refer to the quark and hadron phases, respectively, and the expression is evaluated at $T = T_c$. This ratio will not, in general, be near unity; in solid-liquid phase transitions the equivalent ratio is usually very different from unity.

FIGURE 1
Phase diagram for QCD

$$R = \frac{n_{B,Q}}{n_{B,H}} = \frac{\partial^2 \Omega_Q / \partial \mu^2}{\partial^2 \Omega_H / \partial \mu^2}, \quad (4)$$

We will argue below that $R \sim 100$. This means that the baryon number of the universe preferentially remained in the quark phase during the period of phase separation. As the fraction of the universe occupied by the quark phase shrunk, the majority of the baryon number of the universe was concentrated into fractionally small volumes, with most of the volume of the universe significantly depleted in baryon number.

In the absence of complete thermodynamic prescriptions for this system, we have performed model calculations in which the quark-gluon plasma was treated as an ideal gas of quarks and gluons, with a "bag" term, and in which the hadron gas was treated as an ideal gas of mesons and baryons. The resulting expressions, keeping only the lowest order non-vanishing terms in μ/T , are:

$$\Omega_Q = - \frac{7\pi^2}{180} N_c N_f V T^4 \left[1 + \frac{10}{21\pi^2} \left(\frac{\mu}{T} \right)^2 \right] - \frac{\pi^2}{45} N_g V T^4 + B V, \quad (5)$$

$$\Omega_H = - \frac{1}{2\pi^2} V T^2 \sum_{\text{baryon}} g_b m_b^2 \sum_{n=1}^{\infty} (-1)^{n+1} n^{-2} \left(1 + \frac{n^2 \mu^2}{T^2} \right) K_2 \left(\frac{n m_b}{T} \right) - \frac{1}{2\pi^2} V T^2 \sum_{\text{meson}} g_m m_m^2 \sum_{n=1}^{\infty} n^{-2} K_2 \left(\frac{n m_m}{T} \right), \quad (6)$$

where N_c is the number of color degrees of freedom, N_f the number of massless quarks, N_g the number of gluons, g_b and g_m are baryon and meson statistical weights, m_b and m_m are baryon and meson masses, and K_2 is a modified Bessel function. The summations in equation (6) are over the independent meson and baryon degrees of freedom. T_c is found by letting $\Omega_Q = \Omega_H$, and the ratio R may then be estimated. For example, if $B = (145 \text{ MeV})^4$, $N_f = 2$, we find $T_c = 106 \text{ MeV}$ and $R = 100$.

The reason for the large value of R is the following. A large penalty in free energy is paid when a unit of baryon number is placed in the hadron phase, because of the large mass of the baryons. However, in the quark phase there is no mass penalty, and in addition the three quarks carry more entropy. In consequence it is always thermodynamically advantageous to place most of the baryon number into the quark phase.

It is important to stress now that the ideal gas model is only a guide to what might really happen in this system. Lattice calculations^{10,11} have indicated that the ideal gas model gives a reasonable estimate of $\partial^2 \Omega_Q / \partial \mu^2$; the signal for $\partial^2 \Omega_H / \partial \mu^2$ is lost in the noise. For the purpose of cosmology it is best to regard R as an unknown parameter with magnitude in the vicinity of $R \sim 100$.

4. DYNAMICS OF COSMIC PHASE SEPARATION

A full description of the cosmic quark-hadron phase transition is complicated by a variety of non-equilibrium effects. These include initial supercooling followed by reheating to $T = T_c$, and the development of eddy currents in the quark phase.

The initial supercooling arose because, as the universe cooled to temperatures just below T_C , bubbles of the hadron phase did not appear immediately. The positive free energy of the surface of the bubbles (i.e., the surface tension σ) inhibited bubble formation until sufficient supercooling occurred that the penalty in free energy of the surface of the bubbles was overwhelmed by the bulk advantage of creating regions of hadron phase. At this point, small bubbles of the new phase spontaneously appeared, and grew rapidly, releasing latent heat into the surrounding medium. This release of latent heat reheated the universe back to T_C , and no further nucleation of new phase occurred.

Quantitative analysis of this process², based on classical nucleation theory, strongly suggests that very little supercooling occurred, $T_C - T \sim 0.01T_C$. This is because the universe was expanding very slowly compared to the natural QCD timescales, permitting the rare large thermal fluctuations which lead to stable growth of new phase. This same analysis yields a quantity that is of great importance later during nucleosynthesis: this is the mean separation between nucleation sites,

$$\lambda \approx 0.3 \frac{\sigma^{3/2} t}{T_C^{1/2} L} . \quad (7)$$

The dependence of λ on uncertain quantities, especially the surface tension σ , makes a prior determination of λ impossible. We will treat λ as a parameter in subsequent discussions of the consequences of the phase transition.

During the ensuing period of near phase equilibrium, a very interesting and complicated velocity field develops. The overall expansion of the universe was manifested in a peculiar velocity field during the phase transition. Examining conditions in the "small patch" described above, there must be a velocity field $\mathbf{v}(\mathbf{x})$ which, on a large scale, exhibits Hubble expansion. However, within each phase the energy density is constant, since it is fixed by the thermodynamics. The velocity field must satisfy the equation of continuity,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 , \quad (8)$$

whence we find $\nabla \cdot \mathbf{v} = 0$. This appears to contradict the assertion that there is large scale expansion. The apparent paradox is reconciled by noting that $\nabla \cdot \mathbf{v}$ is singular at the phase boundary. All of the expansion of the universe occurs at the phase boundary. This is clearly very different from the normal Hubble law $\mathbf{v} = H \mathbf{x}$, with $\nabla \cdot \mathbf{v} = 3H$.

The system is further complicated by gravitational instability. There is a substantial difference in energy density between the two phases, $\Delta\rho = \rho_Q - \rho_H \sim L$. Gravitational forces between different regions of the different phases are not in equilibrium. These forces give rise to accelerations and additional peculiar velocities. A crude estimate yields $\Delta v \sim H\lambda$, which is of order the mean recession velocity between neighboring distinct regions of like phase.

This response of the system to the gravitational instability creates a very complicated velocity field, as large bubbles of one phase "plow through" the other phase. The new peculiar velocity has non-zero vorticity, and perhaps is best described as a system of "eddy currents", or even slow convection. The Reynolds number for the flow is ~ 100 , too small for fully developed turbulence to arise. The significance of these eddy currents will be made clear in the next section.

5. BARYON TRANSPORT DURING THE PHASE TRANSITION

The eddy currents described above lead to the mixing of baryon number within each phase. We describe this mixing in terms of an effective diffusion coefficient $D \sim \Delta v \lambda \sim H\lambda^2$. Working again in the "small patch" we can construct a diffusion equation for the baryon number density n_B . In this patch the mean velocity of the baryons at any point $\mathbf{x}$ is:

$$\mathbf{v}_B = H \mathbf{x} - n_B^{-1} D \nabla n_B, \quad (9)$$

where the first term is the Hubble expansion and the second is drift due to diffusion. Combining with the equation of continuity yields:

$$\frac{\partial n_B}{\partial t} = -3Hn_B - H(\mathbf{x} \cdot \nabla)n_B + \nabla \cdot D \nabla n_B, \quad (10)$$

where the terms on the right hand side represent expansion, Hubble drift and diffusion. This equation describes the evolution of n_B within a particular phase. It must be solved in each phase separately, with the two solutions satisfying the ratio R at the phase boundary. Note that this is a boundary condition on a moving surface.

Equation (10) assumes a much simpler form if rewritten in comoving coordinates, since the expansion is removed. The equation becomes:

$$\frac{\partial n_B^C}{\partial t^C} = \nabla^C \cdot D^C \nabla^C n_B^C, \quad (11)$$

where superscript c denotes the use of comoving coordinates. This, of course, is the standard form of the diffusion equation. The "comoving diffusion coefficient" is related to the physical, or proper diffusion coefficient, by $D^c = a^{-2} D$.

A complete three dimensional treatment of this mixing is impracticable at present, and unwarranted until clearer understanding of the eddy currents emerge. An "equivalent sphere" model is shown in Figure 2. The sphere is centered at a point where a bubble of quark-gluon plasma will disappear. The

FIGURE 2

Distribution of baryon number density versus comoving radius

full volume of the sphere represents a patch of volume $\sim l^3$. The phase boundary is always a spherical shell, which starts at the edge of the sphere and moves toward the center. The proper diffusion coefficient in the quark phase is, in this example, $D = 4 \times 10^9 \text{ cm}^2 \text{ s}^{-1}$. The quantity $f_v^{1/3}$ is a comoving radius coordinate.

The inhomogeneity develops as the phase boundary moves inward. Chemical equilibrium sweeps baryon number ahead of the boundary. Diffusion sweeps the excess baryon number deep into the quark-gluon plasma. As the boundary moves inwards, the amplitude of the inhomogeneity grows rapidly. The high central

peak in this figure probably would not occur in a more realistic calculation, because in the inner core the phase boundary moves faster than the speed of sound, which means that quasi-equilibrium cannot be maintained.

This model shows that the solubility ratio at the phase boundary, together with mixing, leads to a very substantial inhomogeneity in the distribution of the baryon number.

6. LIGHT ELEMENT SYNTHESIS

The inhomogeneities described above remain frozen in comoving coordinates until $t \sim 1s$, $T \sim 1MeV$. At this point there is a very interesting development. Neutrons and protons are now the only baryons in the system, and for $T > 1MeV$ the neutrons and protons are strongly coupled by weak interactions with the thermal neutrinos. The ratio of number densities of neutrons and protons is thermal. Below $T \sim 1MeV$, the weak interactions become too slow to affect the number densities, and free decay of neutrons becomes the only reaction of importance.

The neutron has a long scattering mean free path in the ambient plasma of e^+ , e^- and γ , because it is uncharged. Accordingly, the neutron has a much larger diffusion coefficient than the proton. For this reason, neutrons begin to diffuse rapidly out of the clumps of baryons. This process can be so efficient that, by the onset of nucleosynthesis, the neutrons are largely separated from the protons.

The first stage of nucleosynthesis is the formation of deuterons. Clearly this important reaction is greatly affected by the separation of most of the neutrons from most of the protons. A further complication is that, as most of the neutrons are consumed in the proton rich regions, neutrons begin to diffuse back into the these cores and sustain further nucleosynthesis.

A full description of these processes involves solving the coupled, non-linear differential equations that describe the nuclear reaction rates of importance¹² and include spatial diffusion of neutrons. We have carried out this program by differencing the equations on an eight-zone representation of the initial baryon number distribution. The evolution of neutron number density in these eight zones is shown in Figure 3, for the case $\ell = 40m$. Weak reaction decoupling occurs after about 1 sec when the temperature is $\approx 1 MeV$. As one can see from this Figure there is some neutron diffusion up to this time. After $t \approx 1 sec$, when the neutrons are no longer rapidly converted into protons, the outer zones are able to approach equilibrium. At $t \sim 200 sec$, the onset of nucleosynthesis causes a factor of 2 drop in the neutron density as the available protons are consumed. Afterwards, the neutron density decreases more gradually, and is given by the timescale for neutron decay into the

protons which are required for further nucleosynthesis.

The innermost zone appears to evolve almost independently from the outer zones. The density in this zone remains high until $t \sim 10$ sec at which point the nucleosynthesis begins and the neutron density is quickly diminished as baryons are converted into ${}^4\text{He}$. The reason that this zone does not communicate efficiently with the outer zones is its high baryon density. The resulting high neutron-proton scattering rate drastically shortens the baryon diffusion length.

Given our calculated fluctuation shapes and diffusion constants, the amount

FIGURE 3

Comoving neutron density versus time for each of the 8 zones

of neutron flow is determined by the average separation between nucleation sites. Figure 4 shows an example of our calculated nucleosynthetic yields as a function of the separation between fluctuations, for a model universe with $\Omega_B = 1$. The optimum separation distance for producing "acceptable" yields of ${}^2\text{H}$, ${}^3\text{He}$ and ${}^4\text{He}$ appears to be $\sim 30\text{--}100\text{m}$ at the time of the phase transition, which is comparable to our earlier estimates based upon classical nucleation theory. For separations which are too close, $l < 10\text{m}$, ${}^4\text{He}$ and ${}^7\text{Li}$ are overproduced while ${}^2\text{H}$ and ${}^3\text{He}$ are underproduced. This

is due to the flow of neutrons back into the high density zone after the onset of nucleosynthesis. For separations $\ell > 100$ m, there is not enough neutron diffusion to produce significant ^2H in the low density zones. For the optimal conditions, $\ell \sim 30$ m, ^7Li is "overproduced" by some factor which is presently very controversial.

The problem with determining by observations an estimate of the primordial ^7Li abundance is that it cannot be measured in the interstellar gas (for

FIGURE 4

Computed yields of four light isotopes for a model with $\Omega = 1$, $\ell = 40$ m. Boxes show regions where yields agree with observational determinations. Dotted lines indicate "best choices" for ℓ

spectroscopic reasons: the strong absorption lines are in the far ultra-violet). Weak absorption due to ${}^7\text{Li}$ can be found in old stars. The interpretation of this absorption is extremely unclear, however, because of convincing empirical evidence that stars slowly destroy their ${}^7\text{Li}$ with time¹³. The primordial ${}^7\text{Li}$ abundance is not known with any confidence, and should not be used to distinguish between cosmological models.

The success of the models described above at producing appropriate amounts of ${}^2\text{H}$, ${}^2\text{He}$ and ${}^4\text{He}$ is very suggestive. These models contain no exotic dark matter. It may turn out that the material of our universe is the familiar mix of baryons and leptons which is studied in the laboratory!

7. CONCLUSIONS

This study of the cosmic quark-hadron phase transition is at an early stage, but already some very interesting conclusions may be reached. The ratio R of solubilities is very likely to be large, and this drives the formation of substantial inhomogeneities in the distribution of the baryons. Baryon transport due to eddy currents plays an important role in this process.

The inhomogeneities are subsequently modified by neutron-proton segregation, which in turn has a profound effect on light element synthesis. In particular, these models produce acceptable light element yields when $\Omega_B = 1$.

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