

# PLASMA WAKE EXCITATION BY LASERS OR PARTICLE BEAMS\*

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## Abstract

Plasma accelerators may be driven by the ponderomotive force of an intense laser or the space-charge force of a charged particle beam. Plasma wake excitation driven by lasers or particle beams is examined, and the implications of the different physical excitation mechanisms for accelerator design are discussed.

## INTRODUCTION

Plasma-based accelerators [1] have attracted considerable attention owing to the ultrahigh field gradients sustainable in a plasma wave, enabling compact accelerators. These relativistic plasma waves are excited by displacing electrons in a neutral plasma. Two basic mechanisms for excitation of plasma waves are actively being researched: (i) excitation by the nonlinear ponderomotive force (radiation pressure) of an intense laser or (ii) excitation by the space-charge force of a dense charged particle beam.

There has been significant recent experimental success using lasers and particle beam drivers for plasma acceleration. In particular, for laser-plasma accelerators (LPAs), the demonstration at LBNL in 2006 of high-quality, 1 GeV electron beams produced in approximately 3 cm plasma using a 40 TW laser [2]. In 2007, for beam-driven plasma accelerators, or plasma-wakefield accelerators (PWFAs), the energy doubling over a meter to 42 GeV of a fraction of beam electrons on the tail of an electron beam by the plasma wave excited by the head was demonstrated at SLAC [3]. These experimental successes have resulted in further interest in the development of plasma-based acceleration as a basis for a linear collider, and preliminary collider designs using laser drivers [4] and beam drivers [5] are being developed.

The different physical mechanisms of plasma wave excitation, as well as the typical characteristics of the drivers, have implications for accelerator design. In the following, we identify the similarities and differences between wave excitation by lasers and particle beams. The field structure of the plasma wave driven by lasers or particle beams is discussed, as well as the regimes of operation (linear and nonlinear wave). Limitations owing to driver emittance are also discussed.

## PLASMA WAKE EXCITATION

Although large amplitude, relativistic plasma waves (or wakefields) can be driven either by particle beams or laser

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pulses, the physical forces that drive the wave are different. Consider the electron plasma density perturbation excited by a laser or beam driver. Combining the plasma fluid momentum equation, plasma continuity equation, and Gauss's law, in the linear regime, the electron plasma density perturbation in an initially uniform plasma takes the form of a driven harmonic oscillation [1]

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2\right) \frac{n}{n_0} = -\omega_p^2 \frac{n_b}{n_0} + c^2 \nabla^2 \frac{a^2}{2}, \quad (1)$$

where  $n$  is the plasma electron density,  $n_0$  the ambient density,  $\omega_p = (4\pi n_0 e^2/m)^{1/2}$  is the plasma frequency,  $m$  the electron mass,  $-e$  the electron charge,  $n_b$  is the beam density, and  $a = eA/mc^2$  is the normalized vector potential of the laser. The drive term [on the right-hand side of Eq. (1)] can either be a particle beam ( $n_b$ ) or a laser pulse ( $a^2$ ). As seen from Eq. (1) there are some common features of beam-driven and laser-driven excitation. For example, the accelerating bucket size is given by the plasma wavelength  $\lambda_p = 2\pi c/\omega_p$ . The wave excitation is most efficient for driver durations less than, or on the order of, the plasma period. The phase velocity of the wave is determined by the driver velocity. And the characteristic accelerating field for large density perturbations ( $n \sim n_0$ ) is on the order of the cold nonrelativistic wavebreaking field  $E_0 = mc\omega_p/e$ . For example, a plasma density of  $10^{18} \text{ cm}^{-3}$ , yields  $\lambda_p \simeq 33 \mu\text{m}$  and  $E_0 \simeq 96 \text{ GV/m}$ ; this field is approximately three orders of magnitude greater than that obtained in conventional linacs.

Although, from Eq. (1), excitation of the plasma density perturbation from either beam or laser drivers appears equivalent, the field structure is different. Consider a beam driver ( $a = 0$ ) in the linear regime; the longitudinal and transverse fields are, assuming cylindrical symmetry and a highly-relativistic drive beam, [6]

$$E_z/E_0 = -k_p^3 \int d\zeta' \int r' dr' \cos[k_p(\zeta - \zeta')] \times I_0(k_p r_<) K_0(k_p r_>) n_b(r', \zeta')/n_0, \quad (2)$$

$$(E_r - B_\theta)/E_0 = -k_p^2 \int d\zeta' \int r' dr' \sin[k_p(\zeta - \zeta')] \times I_1(k_p r_<) K_1(k_p r_>) \partial_{r'} n_b(r', \zeta')/n_0, \quad (3)$$

where  $\zeta = z - ct$  is the co-moving variable,  $r_<$  ( $r_>$ ) are the smaller (larger) of  $r$  and  $r'$ , and  $I_m$  and  $K_m$  are modified Bessel functions of the  $m^{\text{th}}$  kind. Equations (2) and (3) indicate that the radial extent of the beam-driven wakefields is given by the larger of the plasma skin depth  $k_p^{-1}$  and the

beam radius. For narrow bunches ( $k_p r_b \ll 1$ , where  $r_b$  is the beam radius) the fields extend a skin depth independent of the beam size.

For a laser driver ( $n_b = 0$ ) in the linear regime, the fields are given by [1]

$$\vec{E}/E_0 = - \int dt' \sin[\omega_p(t-t')] \vec{\nabla} a^2(t')/2. \quad (4)$$

The radial extent of the fields driven by a laser is on the order of the transverse laser intensity profile, i.e., the laser spot size. Transversely, the laser ponderomotive force is determined by the local gradient in laser intensity.

It is desirable to have independent control over the accelerating and focusing forces in an accelerator, i.e., one would like to independently tune the focusing forces for matched beam propagation. For a given normalized emittance  $\epsilon_n$  and beam energy  $\gamma_b$ , the matched spot size of the beam is  $r_b = (\epsilon_n/k_\beta \gamma_b)^{1/2}$ , where  $k_\beta$  is determined by the focusing force  $F_r/(\gamma m c^2) = -k_\beta^2 r$ . For a laser driver, the transverse focusing force is determined, from Eq. (4), by the local transverse gradient of the laser intensity  $F_r \propto \partial_r a^2$ . Hence, by shaping the transverse laser intensity profile, the amplitude of the focusing force can be controlled. In practice this may be done by combining higher-order laser modes [7], all of which can be guided in a parabolic plasma channel.

Since the self-fields of the beam extend a plasma skin depth, to shape the transverse fields in a PWFA requires using a broad beam such that the beam radius is many plasma skin depths  $k_p r_b \gg 1$ . In this situation, the return current passes through the drive beam, and, as a consequence, the beam is subject to the filamentation instability [6].

Most present LPA or PWFA experiments operate in a highly-nonlinear regime. This nonlinear regime is characterized by expulsion of plasma electrons from behind the driver and formation of a co-moving ionic cavity. This nonlinear regime has several attractive features for electron acceleration. In particular, in the cavity, the focusing forces are linear (determined by the ion density)  $(E_r - B_\theta)/E_0 = k_p r/2$ , and the accelerating forces are transversely uniform  $E_z/E_0 = k_p \zeta/2$ . The nonlinear PWFA regime, referred to as the blow-out regime [8], requires the beam density be greater than the plasma density  $n_b > n_0$ , and the beam dimensions be less than a skin depth  $k_p r_b < 1$  and  $k_p L < 1$ .

This cavitated regime can also be accessed with a laser-driver [9, 10], and for laser drivers is referred to as the bubble regime. The condition to enter this regime using a laser driver is given by the nonlinear ponderomotive force balancing the space-charge force of the bare ions  $k_p^{-2} \nabla_\perp^2 (1 + a^2)^{1/2} \sim n/n_0 - 1$ , or, for a Gaussian pulse profile,  $a^2/(1 + a^2)^{1/2} \sim k_p^2 r_0^2/4$ . Therefore, for laser-drivers, by increasing the laser intensity, the nonlinear bubble regime can be accessed. Note that one can also enter this regime by using a sufficiently tight laser focus to produce a large transverse ponderomotive force. As the laser intensity increases, the regions of focusing and defocusing of electrons become highly asymmetric [1]. This

asymmetry in the wake may be an issue if acceleration of positrons is desired (e.g., for high-energy physics applications). Positrons can be accelerated and focused on the electron density spike at the back of the cavity, where the attractive properties of the nonlinear bubble regime are lost [11]. As the plasma wave becomes more nonlinear, the phase region where positron acceleration and focusing is possible becomes narrower.

By reducing the laser intensity, the LPA enters the quasi-linear regime. In the quasi-linear regime the fields are nearly symmetric for electrons and positron acceleration and focusing. In addition there is no self-trapping, stable laser propagation can be achieved in a plasma channel, and the transverse focusing forces can be controlled via the transverse laser intensity profile as discussed above.

Accessing the linear regime of PWFA (to facilitate positron acceleration) requires  $E_z/E_0 \lesssim 1$ . Assuming a bi-Gaussian electron beam with  $k_p r_b \ll 1$ , the solution to Eq. (2) is  $E_z/E_0 \approx \sqrt{2\pi} (n_b/n_0) (k_p L) \exp(-k_p^2 L^2/2) (k_p r_b)^2 \ln(1/k_p r_b) \propto N_b n^{1/2}$ . Hence operating in the linear regime requires low plasma density or low beam charge. For fixed bunch charge (i.e., fixed driver energy to be transferred to a witness bunch), operating in the linear regime requires low plasma densities. Lower plasma densities result in smaller accelerating gradients  $E_z = 2E_0 k_p r_e N_b \ln(1/k_p r_b) \propto N_b/L^2 \propto N_b n \propto 1/N_b$ .

In the nonlinear blow-out regime of PWFA, particle-in-cell simulations have shown [12] that the linear beam length scaling for the accelerating gradient holds in the nonlinear regime, namely  $E_z \propto N_b/L^2 \propto N_b n$ , assuming the resonant condition  $k_p L \approx \sqrt{2}$  (i.e., optimizing the beam length). The operational density in the nonlinear blow-out regime is determined simply by the availability of short drive bunches, and the size of the accelerating field is proportional to the plasma density. For example, given a 30  $\mu\text{m}$  beam length, indicates one should operate at  $\sim 10^{17} \text{ cm}^{-3}$  to maximize the accelerating gradient.

### Wake phase velocity

The phase velocity of the plasma wave is approximately equal to the driver propagation velocity. The velocity of the beam driver is typically ultra-relativistic, e.g.,  $\gamma_b = \gamma_p \sim 10^4$ . These large phase velocities have several advantages: no trapping of background plasma electrons (dark current free), negligible slippage between the drive and a witness bunch, and reduction of beam-plasma instabilities.

For LPAs the phase velocity can be, comparatively, low. The wake phase velocity is approximately the laser driver propagation velocity (linear laser group velocity). Although in the nonlinear regime the wake phase velocity is significantly reduced due to laser evolution [14]. For example, using  $\lambda_0 = 1 \mu\text{m}$  laser wavelength in typical plasma densities  $n \sim 10^{17} - 10^{19} \text{ cm}^{-3}$ , the Lorentz factor of the linear laser group velocity is  $\gamma_g \simeq \lambda_p/\lambda_0 \approx \gamma_p \sim 10 - 100$ . This relatively low plasma wave phase velocity can allow

trapping of background plasma electrons [13]. The low phase velocity also results in slippage between the plasma wave and the beam. The distance over which the beam slips from an accelerating to a decelerating region of the plasma wave, or the dephasing length, is  $L_{dph} \sim \lambda_p \gamma_p^2$ . This slippage may limit the energy gain  $\Delta\gamma \propto \gamma_p^2$ . One solution to slippage is to taper the plasma density longitudinally [15], i.e., on the scale of the dephasing length, slowly increase the plasma density, thereby decreasing the plasma wavelength  $\lambda_p \propto n^{-1/2}$  and maintaining the phase of the beam in the plasma wave.

### Driver emittance

Plasma-based acceleration can be limited by the laser-plasma or beam-plasma interaction length. This interaction length may be set by either the characteristic propagation distance of the driver, or driver-plasma instabilities. For a beam-driver, the characteristic scale length for beam evolution is the beta function  $\beta = \gamma r_b^2 / \epsilon_n$ , over which the beam diverges. In the nonlinear blow-out regime, the body of the beam may be self-guided in the cavity, but the head of the beam (outside the cavity) will continue to diverge, leading to beam head erosion. The rate of head erosion will be proportional to the beam emittance. A straightforward solution to extending the beam-plasma interaction length is to use a low emittance beam. For example, using a beam with a geometric emittance of  $\epsilon_n / \gamma_b = 10^{-10}$  m-rad and a  $10 \mu\text{m}$  beam radius, yields  $\beta = 1$  m.

The length over which a tightly focused laser diffracts is the Rayleigh range  $Z_R = \pi r_0^2 / \lambda_0$ , where  $r_0$  is the spot size and  $\lambda_0$  is the wavelength. In the nonlinear bubble regime, the body of the laser may be guided in the cavity, but the head of the laser will be outside the cavity and will continue to diffract, leading to erosion of the head of the laser. The Rayleigh range is typically the shortest length scale for laser evolution. For example,  $Z_R = 2$  mm for  $r_0 = 25 \mu\text{m}$  and  $\lambda_0 = 1 \mu\text{m}$ . The geometric emittance of the photon beam is fixed by the laser wavelength, and therefore some form of external guiding must be employed. Preformed plasma density channels (i.e., tailoring the transverse plasma density profile such that there is a density minimum on axis) have been successfully demonstrated as an effective mechanism for guiding a relativistically intense laser [16]. Hydrogen capillary discharge waveguides have been used to generate long (few cm), low density (few  $10^{18} \text{ cm}^{-3}$ ) plasma channels suitable for a high-energy LPA [2]. By tailoring the plasma both transversely (for laser guiding) and longitudinally (for beam-wake phase-locking, as discussed above) both diffraction and dephasing may be overcome in an LPA. With taper, the single stage energy gain in an LPA is in principle limited by laser energy depletion.

## SUMMARY AND CONCLUSIONS

In this paper we have discussed some of the similarities and differences between plasma acceleration using

laser drivers or particle beam drivers. The different physical mechanisms, as well as the typical characteristics of the drivers, have important implications for the design of plasma-based accelerators.

The field structure of the plasma wave can be strongly dependent on the driver. For example, in the linear regime, the fields of a tightly focused electron beam extend a plasma skin depth, independent of the transverse bunch structure, and therefore shaping the transverse fields by shaping the drive bunch is problematic. In contrast, the transverse fields of the LPA are determined by the local transverse gradient in laser intensity, and therefore the transverse fields (and the focusing forces) can be controlled by controlling the transverse laser intensity profile [7].

The nonlinear cavitated regime can be accessed by either a beam or laser driver. In this regime the phase region where positron acceleration is possible is greatly reduced. For beam-drivers of fixed charge, operating in the linear regime (to facilitate positron acceleration) requires using low plasma density (and consequently lower gradient).

In practice the phase velocity of the beam driven plasma wave is typically much larger than the phase velocity of the laser-driven wave  $\gamma_b \gg \gamma_g$ . One consequence is the potential for self-trapping of background plasma electrons in LPAs. Another consequence is slippage between a relativistic witness beam and a laser-driven plasma wave. This slippage can limit the energy gain in a uniform plasma, but may be overcome using plasma tapering.

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