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SYNCHROTRON RADIATION FROM A WEAKLY
RELATIVISTIC ELECTRON PLASMA*

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ABSTRACT

Recent measurements (Lichtenberg et al., 1971) of synchrotron radiation from a mirror confinement device have differed markedly from the radiation predicted by single particle emission theory. In this paper the emission is recalculated with the inclusion of plasma effects. We use a slab model, with the magnetic field in the plane of the slab, and assume only electron motion perpendicular to the field. Integrating the single particle emission spectrum over the electron distribution we obtain the total emissivity. After calculating the complex index of refraction from the electron distribution function, we find the net radiation emitted perpendicularly to the slab. Our results show qualitative agreement with the experimental results.

I. INTRODUCTION

Lichtenberg, Schwartz, and Lieberman (1971) have measured the synchrotron radiation emitted in the midplane perpendicular to the magnetic field of a mirror machine, the Berkeley Mirror Experiment I. The plasma parameters were: $T_e \approx 60$ KeV; $n_e \approx 5 \times 10^{12} \text{ cm}^{-3}$; diameter $L \approx 0.5$ cm; the experimental spectrum is shown in Fig. 1.

The experimental data were compared to the single particle emission formula integrated over loss-cone electron distributions of the form $F(p_\perp) \propto p_\perp^s \exp(-p_\perp^2/2m_0KT_e)$, where s is a small integer, m_0 is the electron rest mass, and $p_\perp$ is the momentum of the electron in the plane perpendicular to the magnetic field. The theoretical spectra obtained were in agreement with the position of the large experimental peak, but gave a much broader half-width for the large peak and did not explain the existence of the small peak at lower frequencies.

We have attempted to obtain a better theoretical spectrum by taking into account the emission, absorption, and transmission properties of the plasma. We use an infinite slab model, with the magnetic field B parallel to the surface of the slab, and we calculate the radiation emitted perpendicular to the slab surface; see Fig. 2. As in Lichtenberg et al. (1971) we assume a loss cone electron distribution function for the perpendicular momenta and neglect completely the parallel momenta. We also neglect ion motion. Under these conditions, the emitted synchrotron radiation is entirely in the extraordinary mode. Using the dispersion relation for the extraordinary mode in a Vlasov plasma, we calculate the complex index of refraction, the imaginary part of which is the absorption coefficient for the plasma.

The synchrotron emission spectrum is then calculated by considering the transfer of radiation in a dispersive medium.

II. THE SLAB MODEL

In our calculations, we employ the slab model of Bekefi (1966), which we briefly summarize; see Fig. 2 which is essentially Bekefi's Fig. 1.13. Consider a medium characterized by an emission coefficient j_ω (energy emitted per unit time per unit frequency interval per unit volume) and an absorption coefficient α_ω (fractional energy absorbed per unit length). The radiation emitted perpendicular to the slab of Fig. 2 will be given by

$$I_\omega = \frac{1}{N^2} \frac{j_\omega}{\alpha_\omega} (1 - \Gamma) \left(\frac{1 - e^{-\alpha_\omega L}}{1 - \Gamma e^{-\alpha_\omega L}} \right) \quad (1)$$

where a subscript ω denotes a function of ω ; Γ is the reflection coefficient, defined as $\Gamma = |(n_R - 1)/(n_R + 1)|^2$, where n_R is the real part of the index of refraction; and N is the ray refractive index. Denoting the angle between the magnetic field $\underline{B}$ and the wave number $\underline{k}$ by θ , N can be written as

$$N = n_R \left(\frac{1}{1 - \frac{1}{n_R} \frac{\partial^2 n_R}{\partial \theta^2}} \right)^{\frac{1}{2}} \quad (2)$$

for $\theta = \pi/2$, where by symmetry we have set $\partial n_R / \partial \theta = 0$. There is, however, no reason to expect that $\partial^2 n_R / \partial \theta^2$ vanishes here; nevertheless, we will be dealing with fairly tenuous media only, so that n_R

is always close to 1.0, and $\partial^2 n_R / \partial \theta^2$ must be quite small. Therefore, we set $\partial^2 n_R / \partial \theta^2$ equal to zero, and we have $N = n_R$. At the densities we are considering, we find that there is little difference between using the correct Γ , or setting Γ equal to zero. Thus, we set Γ equal to zero and equation (1) becomes

$$I_\omega = \frac{1}{n_R^2} \frac{j_\omega}{\alpha_\omega} \left(1 - e^{-\alpha_\omega L} \right) \quad (3)$$

For $\theta = \pi/2$, and for the two dimensional distribution functions considered, the synchrotron radiation emitted by a single electron is given by (Bekefi, 1966, p. 180)

$$\eta_\omega = \frac{e^2 \omega^2}{8\pi^2 \epsilon_0 c} \left[\sum_{m=1}^{\infty} \beta_\perp^2 J_m'^2(x) \delta(m\omega_0 - \omega) \right] \quad (4)$$

where $\omega_0 = (eB/m_0)(1 - \beta_\perp^2)^{\frac{1}{2}}$ is the relativistic gyrofrequency; $\beta_\perp \equiv v_\perp/c \equiv (1 + m_0 c^2 p_\perp^{-2})^{-\frac{1}{2}}$; $x \equiv (\omega/\omega_B)(p_\perp/m_0 c)$; $\omega_B \equiv (eB/m_0)$; J_m is the Bessel function of order m and a prime denotes the derivative with respect to its argument; e is the absolute value of the electronic charge; B is the magnitude of the magnetic field. The emission is entirely in the extraordinary mode. The total emissivity j_ω is then given by

$$j_\omega = \int_0^\infty \eta_\omega F(p_\perp) 2\pi p_\perp dp_\perp \quad (5)$$

The remaining quantities needed in Eq. 3 come directly from the complex index of refraction, to be derived in the next section.

III. THE COMPLEX INDEX OF REFRACTION

The dispersion relation for the extraordinary mode perpendicular to the field is (Bekefi, 1966, pgs. 9,229)

$$n^2 = K_{yy} + K_{xy}^2/K_{xx} \quad (6)$$

where $n = n_R + in_I$ is the complex index of refraction, and the K_{ij} are elements of the dielectric tensor, given by (Bekefi, 1966, p. 229)

$$K_{ij} = \delta_{ij} + \frac{\omega_p^2}{\omega^2} \int_0^\infty 2\pi p_\perp dp_\perp \sum_{m=-\infty}^\infty \frac{p_\perp}{\left(1 + \frac{p_\perp^2}{m_0^2 c^2}\right)^{\frac{1}{2}}} \frac{1}{\left(1 - \frac{m\omega}{\omega_0}\right)} \frac{\partial F(p_\perp)}{\partial p_\perp} H_{ij} \quad (7)$$

where δ_{ij} is the Kronecker delta function, and the relevant H_{ij} are given by

$$\begin{aligned} H_{xx} &= \left(\frac{m J_m(x)}{x} \right)^2 \\ H_{yy} &= [J'_m(x)]^2 \\ H_{xy} &= \frac{im J_m(x) J'_m(x)}{x} \end{aligned} \quad (8)$$

where $x = k_\perp v_{\perp 0}/\omega_0$, and all quantities vary as $\exp[i(\omega t - k \cdot x)]$

where ω is assumed to have a small negative imaginary part. Since we do not yet know the index of refraction n_R we do not know the

correct value of the wave number $k_\perp$ to go with each value of ω . However, if we use the free space value of $k_\perp = \omega/c$; we find that for the parameter range of interest, $0.9 < n_R < 1.1$, so that the free space wave number is within 10% of the correct value. In this approximation, the argument of the Bessel functions becomes formally identical to the argument of the Bessel functions in Eq. (4)

$$x = \frac{\omega}{\omega_B} \frac{p_\perp}{m_0 c} \quad (9)$$

We then use the formula $\lim_{y \rightarrow 0} \frac{1}{x \pm iy} = P\left(\frac{1}{x}\right) \mp i\pi\delta(x)$ so that the integral splits into a real and an imaginary part. A numerical integration is done for each value of m which makes a significant contribution; for the imaginary part only positive m 's contribute. After n is calculated from Eq. (6), we find α_ω from

$$\alpha_\omega = -2k_I = -2 \frac{\omega}{c} n_I \quad (10)$$

IV. THE RESULTS

For a typical set of parameters, the absorption coefficient α_ω versus frequency is shown in Fig. 3. Negative α_ω implies wave growth and positive α_ω implies wave damping. There is a very simple physical explanation for the sign of α_ω (Bekefi, 1966, p. 227). A wave of frequency ω will resonate with those particles whose relativistic mass shift has given them a gyrofrequency $\omega = \omega_B (1 - \beta_\perp^2)^{\frac{1}{2}}$. Inverting this equality gives us a momentum of $p_\perp = m_0 c [(\omega_B/\omega)^2 - 1]^{\frac{1}{2}}$. If our loss-cone distribution function has positive slope at this

momentum, we have wave growth; if negative slope, we have the usual cyclotron damping. Indeed, the simple low density theory of Bekefi, Hirshfield, and Brown (1961), gives an absorption coefficient proportional to $[\partial F(p_{\perp})/\partial p_{\perp}]$; our results at very low densities were found to be in exact agreement with this theory. Our results are also in agreement with the work of Jayson (1965) for a Maxwellian distribution in the limit $m_0 c^2/T_e \gg 1$.

Due to the exponential factor in Eq. (3), the existence of negative α_{ω} can lead to very pronounced shifts in the intensity spectrum. For the same parameters as in Fig. 3, we have shown in Fig. 4 the variation of intensity I_{ω} with slab thickness L . The curve for $L = 0$ is the normalized emissivity of Eq. (5); this is the spectrum presented in Ref. 1. An important result of this work is the shift towards higher ω/ω_B of the large peak with increasing L ; this shift would continue with increasing L until the peak corresponded to that frequency with the most negative α_{ω} , or until the wave amplitude becomes so large that linear theory is no longer valid. In Figs. 5, 6, and 7 we show the variation of intensity with T_e , s , and n_e .

In Fig. 8, we show the best fit obtained for the experimental curve of Fig. 1; this was found for $L = 0.5$ cm, $s = 4$, and $T_e = 70$ KeV. The synchrotron radiation spectrum was observed using detectors of wide bandwidth ($\sim 10\%$). As such, the experimental spectrum depends heavily on the manner in which the data was unconvolved. Considering the uncertainties thus contained in the experimental spectrum, the agreement between theory and experiment shown in Fig. 8 is quite satisfactory.

V. CONCLUSIONS

The synchrotron radiation emitted perpendicular to the magnetic field in the presence of a loss-cone distribution function can suffer substantial shifts in its spectrum due to the presence of conditions favoring stimulated emission ($\alpha_{\omega} < 0$). Plasma effects on synchrotron emission have previously been shown to be negligible provided $(\omega_p^2/\omega^2) \ll 1$ (Drummond and Rosenbluth, 1960, 1961, 1963; Trubnikov, 1958). In our work $(\omega_p^2/\omega^2) \sim 10^{-2}$ and plasma effects have been demonstrated to be of importance in a detailed calculation of the emission. In the vicinity of the higher harmonics the inequality is strengthened and we expect plasma effects to be in fact negligible. However, our work suggests that some care should be exercised in calculating synchrotron emission near $\omega \lesssim \omega_B$, particularly in the case where the velocity distribution is non-Maxwellian.

We conclude that this effect can be quite important in the interpretation of radiation data to yield details of magnetic field, density, temperature, distribution function, and size of the plasma. These effects may be important in other loss-cone situations, such as the earth's magnetosphere (Hasegawa, 1971) and pulsar magnetospheres (ter Haar, 1972).

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FOOTNOTES AND REFERENCES

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FIGURE CAPTIONS

- Figure 1: Experimental spectrum of synchrotron radiation emitted perpendicular to the magnetic field, where $\omega_B \equiv eB/m_0$.
- Figure 2: The slab model.
- Figure 3: Absorption coefficient versus frequency.
- Figure 4: Variation of intensity spectrum with slab thickness.
- Figure 5: Variation of intensity spectrum with temperature.
- Figure 6: Variation of intensity spectrum with s , the index of the distribution function.
- Figure 7: Variation of intensity spectrum with density.
- Figure 8: Best fit to the experimental curve of Fig. 1.

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Fig. 1

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Fig. 2

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Fig. 3

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Fig. 4

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Fig. 5

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Fig. 6

XBL731-2083

Fig. 7

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Fig. 8

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