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DECAY OF THE  $\pi$ -MESON

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October 24, 1951

Berkeley, California

DECAY OF THE  $\pi$ -MESON

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October 24, 1951

The decay of the charged  $\pi$ -meson through virtual nucleon pairs and the annihilation of a pair with the creation of a neutrino and an electron or  $\mu$ -meson is permitted by the accepted couplings between these particles<sup>1</sup>. Conservation of angular momentum and parity together with Furry's theorem<sup>2</sup> forbid the decay for most choices of the meson field and the  $\beta$ -decay coupling. The forbiddenness is independent of perturbation theory<sup>3</sup>. Assuming the  $\pi$  to be a pseudoscalar meson at rest, in order that the final spinor pair have the same transformation properties under reflection and rotation as the  $\pi$ , the  $\beta$ -coupling must be  $\alpha_1 \alpha_2 \alpha_3$  or  $\alpha_1 \alpha_2 \alpha_3 \alpha_4$ . Therefore only the axial vector or pseudoscalar  $\beta$ -decay theories can lead to the pseudoscalar  $\pi \rightarrow e, \nu$  decay<sup>4</sup>.

If electrons and  $\pi$ -mesons are similarly coupled to nucleons, a pseudoscalar matrix element in  $\beta$ -decay leads to an electron decay five times as often as a  $\mu$  decay. For axial vector it has been shown<sup>1</sup> that the matrix element involves

$$1 - \frac{(p\nu)^2}{E_\nu E_{e,\mu}}$$

$\pi \rightarrow \mu, \nu$  is then  $10^4$  times as probable as  $\pi \rightarrow e, \nu$ . Any mixture of scalar, vector, axial vector, and tensor is not in contradiction with experiment<sup>5</sup>.

\*

Work initiated at the California Institute of Technology.

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The decay of the pseudoscalar  $\pi$  into a photon and electron (or a  $\mu$ -meson) and a neutrino is not limited by such rigid selection rules. For axial vector  $\beta$ -decay this mode is more probable than  $\pi \rightarrow e, \nu$  because of the latter's singularly small matrix element. The emission of a photon by the  $\pi^+$  or the electron in Figs. (1b) and (1d) does not alter any of the selection rules forbidding the non-radiative decay. Only the graph of Fig. (1c) can contribute. For vector  $\beta$ -decay and direct meson coupling the matrix element for  $\pi \rightarrow e + \nu + \gamma$  a photon of momentum  $\underline{k}$  and polarization  $\underline{e}$  is

$$-i \sum_{\mu} \int \frac{d^4 p}{(2\pi)^4} \frac{4\pi e g_{\beta}}{(4m_{\pi} E_{\gamma})^{\frac{1}{2}}} \text{Spur} \left( \gamma_5 \frac{1}{\underline{p} - M} \underline{e} \frac{1}{\underline{p} - \underline{k} - M} \gamma_{\mu} \frac{1}{\underline{p} - \underline{q} - M} \right) \cdot \langle \Psi_e^{\dagger} \gamma_{\mu} \Psi_{\nu} \rangle \quad (1)$$

$\underline{q}$  is the meson 4-vector. The decay rate is

$$\begin{aligned} \gamma^{-1} &= \left( \frac{e^2}{4\pi} \right) \left( \frac{g^2}{4\pi} \right) \left( \frac{m_{\pi}}{M} \right)^2 \left( g_{\beta}^2 \frac{m_{\pi}^5}{\pi} \right) \left( \frac{2}{15\pi^3} \right) \\ &= 4 \times 10^2 g^2 \text{ sec.}^{-1} \end{aligned} \quad (2)$$

for  $g_{\beta} = 10^{-49} \text{ erg cm}^3$ .

The lifetime is between  $6 \times 10^{-4}$  and  $6 \times 10^{-5}$  sec. corresponding

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to a  $g^2$  of 4 or 40. Gradient coupling gives a divergent matrix element for (1) spoiling the equivalence theorem. Tensor  $\beta$ -decay diverges for both couplings. Conservation of angular momentum and parity still forbid a decay through scalar  $\beta$ -decay.

For axial vector  $\beta$ -coupling the emission of a photon by the electron as in Fig. (1b) is expected to be more probable than emission by the heavy nucleon or  $\pi$ . (Emission from the  $\pi$  does not alter the very small matrix element associated with the  $\beta$ -decay.) If the  $\mu$  and electron are assumed to have the same axial vector coupling with nucleons the ratio  $(\pi \rightarrow e + \nu + \gamma) : (\pi \rightarrow \mu + \nu)$  is again independent of the perturbation treatment of the meson field and the divergent integrals arising in the separate calculation of each lifetime. The ratio of the probability for radiative  $\pi - e$  decay to non-radiative  $\pi - \mu$  decay is

$$\left(\frac{e^2}{4\pi}\right) (3.1) = 5.7 \times 10^{-3} \quad (3)$$

A symmetrical coupling scheme with axial vector coupling predicts one non-radiative electron decay and over 10 radiative electron decays for every  $10^4$   $\pi - \mu$  decays. The ambiguities arising from the divergences and the pseudoscalar direct coupling constant preclude conclusions for vector and tensor  $\beta$ -coupling.

I wish to thank Professor R. P. Feynman for helpful discussion. This work was performed under the auspices of the Atomic Energy Commission.

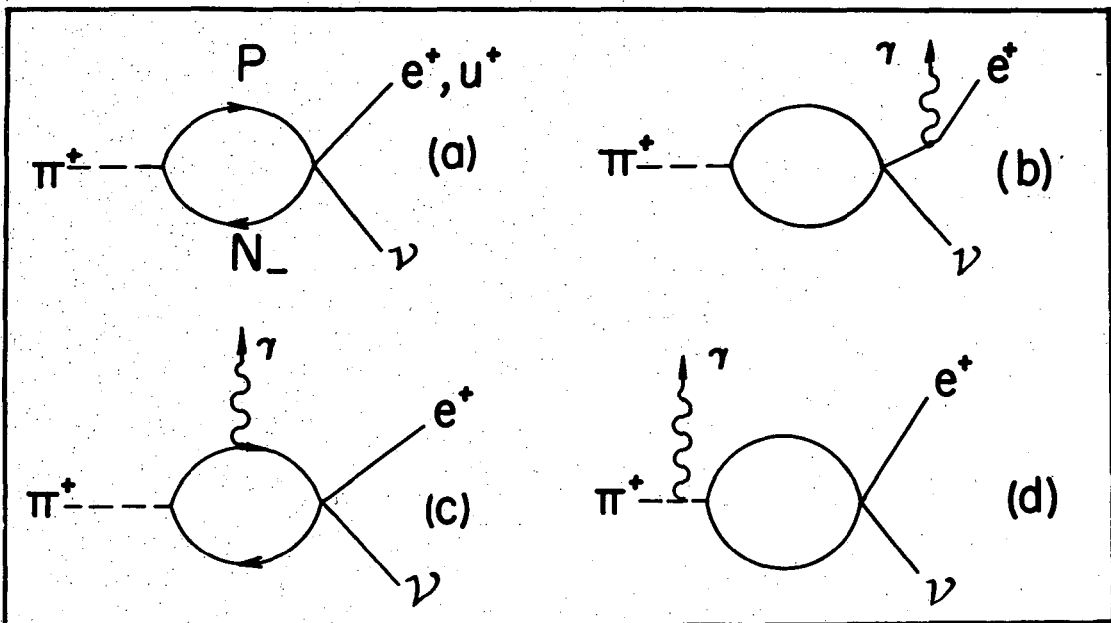
## REFERENCES

1. M. Ruderman and R. Finkelstein, Phys. Rev. 76, 1458 (1949).
2. W. H. Furry, Phys. Rev. 51, 125 (1937).
3. If Furry's theorem forbids the Feynman graph of Fig. (1a), it will also forbid all more complex graphs. Except for the original meson line, for every meson emitted from a closed loop, one is also absorbed so that the "evenness" or "oddness" of the matrix element is unchanged. For exact cancellation of graphs with an odd number of "even" operators it is necessary to assume that the square of the coupling constant between neutral mesons and protons is equal to that between neutrals and neutrons. All of the forbidden cases of Ref. (1) except scalar mesons with gradient coupling and vector  $\beta$ -decay vanish to all orders.
4. If one adopts the spinor reflection rule of Yang and Tiomno, Phys. Rev. 79, 496 (1950), their interactions (12) and (13) are the appropriate ones. Referring the  $\beta$ -decay labeling to the proton-neutron coupling our results are independent of the  $\mu$ ,  $e$ , and  $\nu$  inversion property.
5. H. L. Friedman and J. Rainwater, Phys. Rev. 82, 334 (1951).



## FIGURE CAPTION

Figure 1: Feynman's diagrams contributing to charged  $\pi$  decay.



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Fig. 1