Title

Effect of Stress on Apparent Coefficient of Expansion

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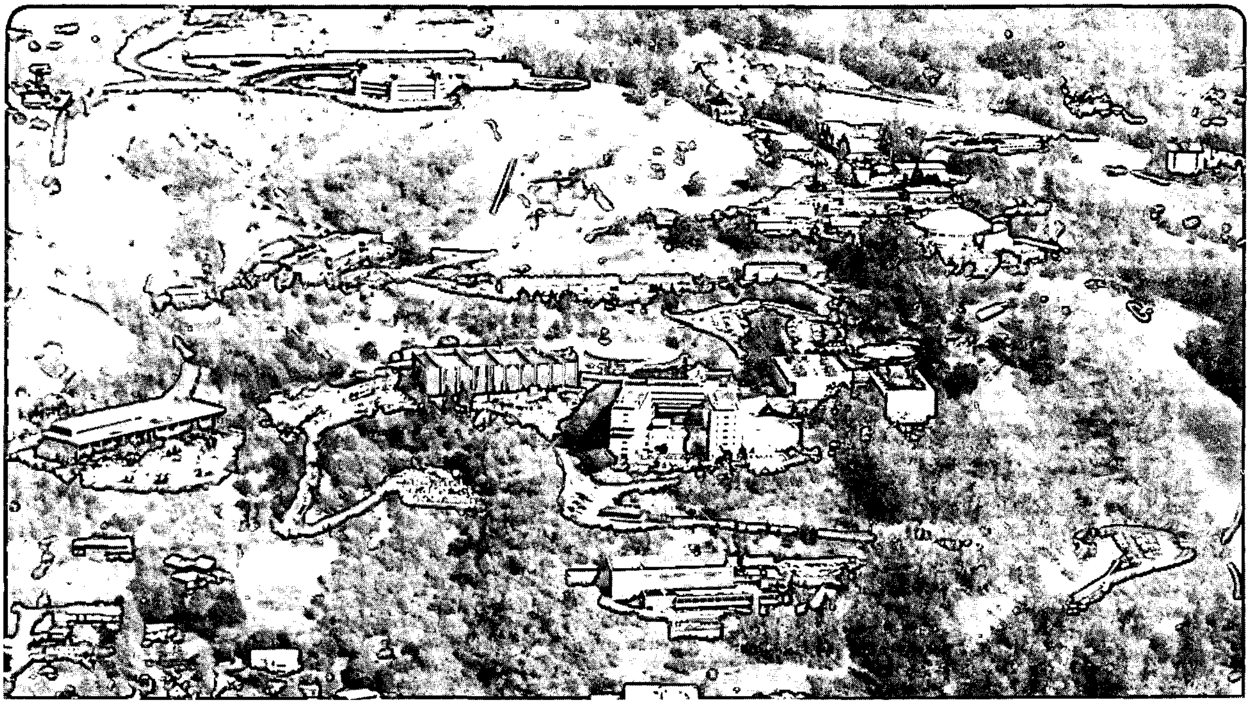
Publication Date

1979-06-01

UC-25
LBID-066 Rev. c.1

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Engineering & Technical Services Division



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AUTHOR R. Meuser	DEPARTMENT Mechanical	LOCATION Berkeley	DATE June 22, 1979

PROGRAM — PROJECT — JOB
High-Field Magnet Development

Analysis

TITLE
Effect of Stress on Apparent Coefficient of Expansion

A revision, 11-3-80

In the literature, the influence of stress upon the apparent coefficient of linear thermal expansion (CLTE) has been viewed as an independent physical property of a material. It has been postulated, for an extreme case where the sign of the apparent CLTE reversed at high stress, that the change probably resulted from a change in the microstructure of the material. No such abstruse explanation is necessary or warranted; a change in apparent CLTE with stress must occur if the elastic modulus changes with temperature. For a well-behaved elastic material, there is a simple unique relationship between the apparent CLTE, the real CLTE, and the hot and cold elastic moduli.

It doesn't seem possible that this is anything new. But it apparently is not well known, and I don't recall having seen it before. But then, I've forgotten a lot.

We consider first a material having linear thermal and elastic properties, and second, non-linear properties and show that the above statement is applicable to either.

Materials Having Linear Properties

We consider a material with the following properties:

At a given stress the length is uniquely determined by the temperature and the behavior is characterized by a constant value of the apparent CLTE. (See "Nomenclature"):

$$\alpha_{ap}(\sigma) \equiv L^{-1}(\partial L/\partial T)$$

At a given temperature the length is uniquely determined by the stress and the behavior is characterized by a constant value of the elastic modulus:

$$E(T) \equiv L \quad (\partial L/\partial \sigma)^{-1}$$

△A

We note that the real CLTE is

$$\alpha = \alpha_{ap}(0)$$

It follows that the change length from one stress-temperature condition to another is independent of the stress-temperature path.

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We consider a path 1, 2, 3, 4, 1 on a temperature-stress graph as shown in Figure 1.

The increase in length, Δ_{ij}, for each leg of the path are:

Leg 1-2 $\quad \Delta_{12} = - \sigma_1 L/E_h$

Leg 2-3 $\quad \Delta_{23} = - \alpha (T_h - T_c)L$

Leg 3-4 $\quad \Delta_{34} = \sigma_1 L/E_c$

Leg 4-1 $\quad \Delta_{41} = \alpha_{ap}(T_h - T_c)L$

Substituting the above into the equation

$$\Delta_{12} + \Delta_{23} + \Delta_{34} + \Delta_{41} = 0$$

yields

$$\alpha_{ap} = \alpha + \frac{\sigma_1}{T_h - T_c} \left(\frac{1}{E_h} - \frac{1}{E_c} \right)$$

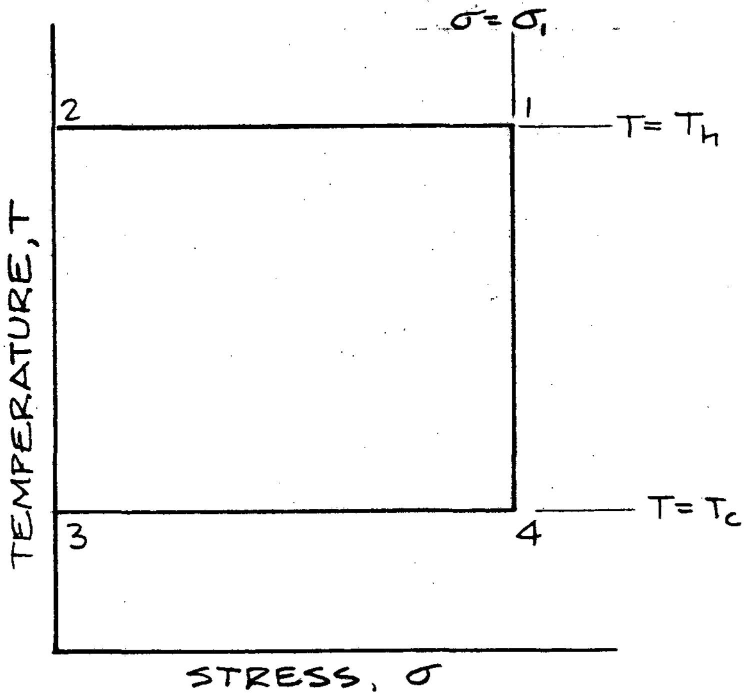

FIG. 1 STRESS-TEMPERATURE PATHS

Material Having Non-Linear Properties

As in the linear case, the thermal and elastic behavior must be reversible. At a given temperature, the change in length in going from stress zero to stress σ_1 at temperature T_h is $\varepsilon_h L$; at temperature T_c it is $\varepsilon_c L$; as illustrated in Fig. 2.

The increase in length, Δ_{ij}, for each leg of the path shown in Fig. 1 is:

Leg 1-2 $\quad \Delta_{12} = -\varepsilon_h L$

Leg 2-3 $\quad \Delta_{23} = -L \int_{T_c}^{T_h} \alpha(T)$

Leg 3-4 $\quad \Delta_{34} = \varepsilon_c L$

Leg 4-1 $\quad \Delta_{41} = \alpha_{ap}(T_h - T_c)L$

Again:

$$\Delta_{12} + \Delta_{23} + \Delta_{34} + \Delta_{41} = 0$$

These equations reduce to:

$$\alpha_{ap} = \bar{\alpha} + \frac{\varepsilon_h(\sigma_1) - \varepsilon_c(\sigma_1)}{T_h - T_c}$$

where

$$\bar{\alpha} \equiv (T_h - T_c)^{-1} \int_{T_c}^{T_h} \alpha \, dt$$

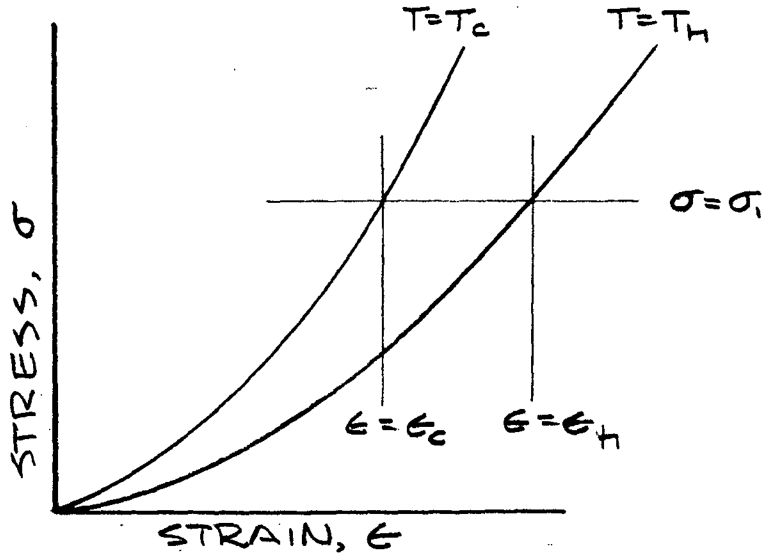

FIG. 2 NON-LINEAR STRESS-STRAIN CURVES.

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Example (Linear):

$E_h = 1.0 \times 10^6$

$E_c = 0.5 \times 10^6$

$\alpha = 5.0 \times 10^{-5}$

$T_h - T_c = 100$

$\sigma = 1.0 \times 10^4$

$\therefore$

$\alpha_{ap} = -5.0 \times 10^{-5} = -\alpha$ △A

Nomenclature:

α	CLTE: $L^{-1}(\partial L/\partial T)$ at $\sigma = 0$
α_{ap}	Apparent CLTE: $L^{-1}(\partial L/\partial T)$ at $\sigma = \sigma_1$
σ	Axial stress (+ = tension)
L	Length of body
E	Elastic modulus: $L^{-1}(\partial L/\partial \sigma)$ at constant T
E_h, E_c	E at $T = T_h$, $T = T_c$
T	Temperature
Δ_{ij}	Increase in length of body for temperature-stress path leg i-j.
ϵ	Axial strain (+ = extension) △A

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