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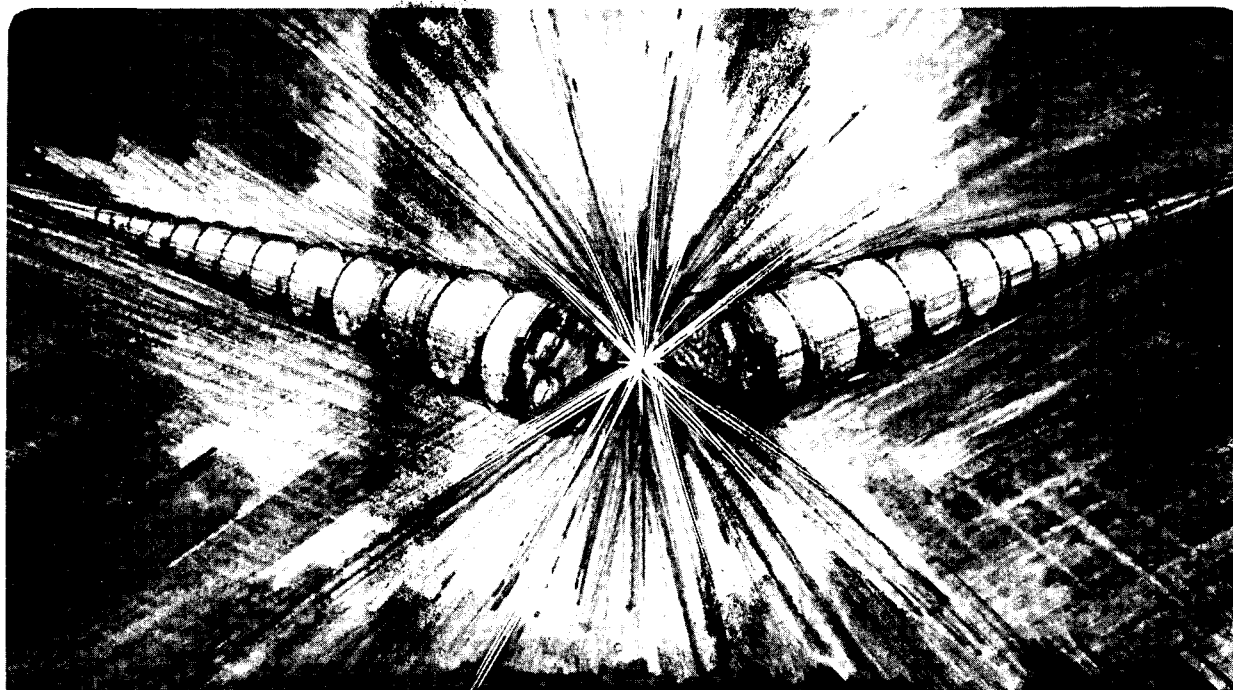
A RELATIVISTIC KLYSTRON

R. Marks

September 1985

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A Relativistic Klystron\*

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Abstract

Theoretical analysis is presented of a relativistic klystron; i.e. a highly-relativistic bunched electron beam which is sent through a succession of tuned cavities and has its energy replenished by periodic induction accelerator units. Parameters are given for a full-size device and for an experimental device using the FEL at the ETA; namely the ELF Facility.

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## I. Introduction

The TBA is a long beam into which DC energy is periodically injected and from which radiation is periodically extracted. The original concept of Sessler<sup>1</sup> envisions extraction by FEL interaction. A modification suggested by Panofsky<sup>2</sup> would replace the FEL by Klystron interaction. Many of the principles of the original TBA carry over to the Klystron TBA. This report is a discussion of the theory of such a device and issues involved in design.

The basic idea is to transport a highly-relativistic bunched beam through a succession of tuned cavities, each of which extracts a small proportion of the total beam energy. Periodically, an induction accelerator replenishes the energy. We are thinking here of beam parameters similar to those suggested in the traditional TBA - 1 kA at 20 MeV, propagation for several kilometers. We would like to generate radiation in the wavelength range 1-3 cm.

The first necessity is a modulated beam. This is somewhat of a problem at relativistic energies, especially since emittance is critical. This is an issue which shall not be treated in this report, except for the mention of several possibilities. One is a laser-modulated photocathode, as used in the lasertron. Another is a racetrack-type buncher. A third is an FEL operating solely as a buncher; this may not be a desirable means of modulation in an actual TBA but could prove useful as an experimental source.

The next relevant topic is the cavity structure. Klystrons have well-known deficiencies at short wavelengths. The bunch size must be less than half a wavelength. The holes in the cavity through which the beam travels (we ignore grid possibilities due to emittance growth) must be less than  $\sim 3/4$  of a wavelength in diameter to ensure cutoff. And the cavity ("gap") length must be much less than a wavelength, assuming a dominant cavity mode.

The cavity size is fairly critical. To extract even a small percentage of the power of the beam under consideration in a distance much less than a wavelength requires intense electric fields which exceed practical breakdown limits. Additionally, the resonant frequency of the dominant cavity mode is a function of the cavity radius only, and a high-order radial mode must be chosen if the cavity radius is to be reasonably large.

Fortunately, an alternative exists. Since the electrons move at essentially  $c$ , one need only utilize a traveling-wave structure of the same phase velocity. This is most easily achieved by employing a cylindrical cylinder with diameter several times the wavelength. Of course, such a structure stores most of its energy in radial fields, and if one were interested in large single-pass extraction efficiency, a coupled-cavity slow-wave structure would probably be superior. However, since only small fractions of the beam energy are to be removed in a given cavity, we entertain the hope that the simplest cavity configuration may prove effective.

Another important issue is debunching, caused by differential energy loss and by space-charge effects. We shall demonstrate below that, although the latter are important, they can be counteracted by the former.

These and other significant subjects are discussed below. At the end, we present a set of parameters apropos to two distinct physical devices. The first is a long, 20 MeV TBA. The second is a proposed experimental device operating at 1 cm using as a current source the 3 MeV bunched beam emerging from the ELF free-electron laser. Such an experiment can verify models of beam-cavity interaction, demonstrate a difficult beam transport line, and test the ability of the cavity electric field to counteract space-charge effects.

## II. Beam-Cavity Interaction

We are interested in computing the interaction between an axially-symmetric, longitudinally-bunched axial current and an azimuthally-symmetric cavity (Figure 1a) at a steady-state angular frequency  $\omega$ . The cavity is not closed but connected to infinitely-long cylindrical beam pipes. We assume the pipe radius  $a$  is small enough to insure cutoff of propagation, at the given frequency, of all modes except the dominant  $TE_{11}$ ; since this mode lacks azimuthal symmetry, we assume that it is not excited. The next mode is the  $TM_{01}$ , which is certainly excited and has a cutoff wavelength  $\lambda_c = a/.383$ . The penetration length of the evanescent mode is  $\frac{1}{2\pi[\lambda/\lambda_c]^2 - 1}^{-1/2}$  times the wavelength  $\lambda$ . We assume that  $a \leq .35 \lambda$ ; this insures a penetration length less than  $0.36 \lambda$ .

Suppressing a time dependence of the form  $e^{-i\omega t}$ , the electric field obeys the wave equation

$$\nabla^2 \underline{E} + \frac{\omega^2}{c^2} \underline{E} = \frac{-4\pi i \omega}{c^2} \underline{j} + 4\pi \nabla \rho \quad (1)$$

subject to the condition that  $\hat{n} \times \underline{E} = 0$  on the walls.

Define modes of the empty cavity as solutions to the eigenvalue problem

$$\nabla^2 \underline{e}_j + \frac{\omega_j^2}{c^2} \underline{e}_j = 0 \quad (2)$$

$$\nabla \cdot \underline{e}_j = 0$$

$$\hat{n} \times \underline{e}_j = 0 \text{ on the walls.}$$

If  $S$  formed a closed surface, we could readily establish that an infinite set of modes satisfy these relations; furthermore that the  $\omega_j$  are real and that the  $\underline{e}_j$  are orthogonal in the sense

$$(\underline{e}_i, \underline{e}_j) = \int d^3v \underline{e}_i^* \cdot \underline{e}_j = 0 \text{ for } \omega_i \neq \omega_j \quad (3)$$

More significantly the  $\underline{e}_j$  form a complete set. To satisfy the requirement [of a closed surface], we assume that the pipes are shorted by a conducting plate at some large distance from the cavity. We assume that the existence and location of the plates does not materially affect the problem as long as the plates are far enough from the cavity to insure that modes of interest have vanishingly weak fields there. Taking the modes to be orthonormal, we expand  $\underline{E} = a_j \underline{e}_j$  and solve for  $a_j$ , with the result:

$$a_j = \frac{4\pi}{\omega^2 - \omega_j^2} \left\{ -i\omega(\underline{e}_j, \underline{j}) + c^2(\underline{e}_j, \nabla\rho) \right\} \quad (4)$$

One can readily establish that the axial component of the modal field has the Fourier integral expansion:

$$e_{jz}(r, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A_j(p) I_0(r\sqrt{p^2 - k_j^2}) e^{ipz} dz \quad (r < a) \quad (5)$$

with the inverse

$$A_j(p) I_0(r\sqrt{p^2 - k_j^2}) = \int_{-\infty}^{\infty} e_{jz}(r, z) e^{-ipz} dz \quad (6)$$

Using (5) and the fact that  $\nabla \cdot \underline{e}_j = 0$ , one determines a form for the radial modal field. One can then find expressions for  $(\underline{e}_j, \nabla\rho)$  and  $(\underline{e}_j, \underline{j})$ . Assuming the charge moves only axially, not radially, one determines that  $(\underline{e}_j, \nabla\rho) = 0$ .

Consistent with this assumption, let  $\underline{j}$  be the time-dependent current and let

$$\underline{j} = \hat{z} f(r)g(t-z/v) \quad (7)$$

where  $g(t)$  is a periodic function. Then the component at frequency  $\omega$  is

$$\underline{j} = \hat{z} \cdot 2f(r) g e^{i\omega z/v} \quad (8)$$

where

$$g = \frac{\omega}{2\pi} \int_{-\pi/\omega}^{\pi/\omega} g(t) e^{i\omega t} dt. \quad (9)$$



Using the expression (5) for  $e_{jz}$ , one can evaluate  $(e_j, \underline{j})$  and arrive at the result:

$$a_j = \frac{-i 16\pi \omega f_j^* A_j^* g}{\omega^2 - \omega_j^2} \quad (10)$$

where

$$A_j \equiv A_j\left(\frac{\omega}{v}\right) \text{ and} \quad (11)$$

$$f_j \equiv \int_0^\infty r f(r) I_0\left(r \sqrt{\left(\frac{\omega}{v}\right)^2 - \left(\frac{\omega_j}{c}\right)^2}\right) dr \quad (12)$$

Note that  $f_j$ , while formally a real quantity, becomes complex if  $\omega_j$  becomes complex. This is a convenient and conventional method of allowing for cavity loss; we do not attempt to justify the method mathematically but simply assume that it is valid as long as the imaginary components are small compared to the real.

### III. Specification of Current and Cavity

We assume a time-dependent current  $\tilde{\underline{j}}$ :

$$\tilde{\underline{j}} = \frac{I}{\omega \alpha b^2} \theta(b - r) g_\alpha(t - z/v) \quad (13)$$

where  $g_\alpha(x)$  is a periodic function of period  $\frac{2\pi}{\omega}$  and

$$g_\alpha(x) = \begin{cases} 1 & -\alpha \leq x \leq \alpha \\ 0 & \alpha \leq x \leq \frac{\pi}{\omega}, \quad \frac{-\pi}{\omega} \leq x < -\alpha \end{cases} \quad (14)$$

(see Figure 1b). This is simply a series of charge cylinders traveling axially at velocity  $v$ ;  $I$  is the average beam current. Then

$$f_j = \frac{2I_1(b \sqrt{(\frac{\omega}{v})^2 - (\frac{\omega_j}{c})^2})}{b \sqrt{(\frac{\omega}{v})^2 - (\frac{\omega_j}{c})^2}} \quad (15)$$

$$g = \frac{I}{2\pi} \frac{\sin(\omega\alpha)}{\omega\alpha} \quad (16)$$

For  $\omega \approx \omega_j$  and  $v \approx c$ ,  $f_j \approx 1$ . For bunch length much less than wavelength,  $\frac{\sin(\omega\alpha)}{\omega\alpha} \approx 1$ .

Now consider a specific cavity—namely a cylinder of length  $d$ , radius  $R$ , coaxial with the beam pipe. (see Figure 1b) For analytical purposes, we assume that the field in the region  $r \geq a$  ( $a$  is the pipe radius) is identical to that in a closed cylindrical cavity of length  $d$ . The validity of this assumption depends on the penetration length of the cutoff fields into the beam pipe being much less than the cavity length  $d$ , as well as the pipe radius  $a$  being much less than the scale length for field variation in the radial direction. TM cavity modes have the form:

$$e_{jz}(r,z) = E_0 J_0(X_{0m} \frac{r}{R}) \cos \left( \frac{n\pi z}{d} \right) \begin{cases} 0 < z < d \\ a < r < R \end{cases} \quad (17)$$

(where  $J_0(X_{0m}) = 0$ ), which can be evaluated at  $r=a$  and used to match expression (5), valid for  $r < a$ . There results:

$$A_{j\omega} = A_{mn} = \frac{i2E_0 J_0(X_{0m} \frac{a}{R})}{I_0(a \sqrt{(\frac{\omega}{v})^2 - (\frac{\omega_{mn}}{c})^2})} \frac{\omega/v}{(\frac{n\pi}{d})^2 - (\frac{\omega}{v})^2} e^{-i\frac{\omega d}{2v}} \begin{cases} i \sin \\ \cos \end{cases} \left( \frac{\omega d}{2v} \right) \quad (18)$$

Where the top (bottom) of the bracketed expression is chosen if  $n$  is even (odd).

$E_0$  is determined by normalization. Assuming expression (17) to be valid for all  $r$  (for normalization purposes), then

$$E_0^2 = \frac{\lambda_{mn}^2 X_{om}^2 \epsilon_n}{4\pi^3 d R^4 J_1^2(X_{om})} \quad \begin{cases} \epsilon_0 = 1 \\ \epsilon_n = 2 \quad (n \neq 0) \end{cases} \quad (19)$$

where the resonant wavelength  $\lambda_{mn}$  of the closed cylinder mode satisfies:

$$\left(\frac{2\pi}{\lambda_{mn}}\right)^2 = \left(\frac{X_{om}}{R}\right)^2 + \left(\frac{n\pi}{d}\right)^2 \quad (20)$$

We can clarify the expression for  $A_{mn}$  by defining  $\delta$  such that

$$d = \frac{\pi v}{\omega} (n + \delta); \quad \text{then} \quad (21)$$

$$A_{mn} = \frac{2 E_0 J_0 \left(X_{om} \frac{a}{R}\right)}{I_0 \left(a \sqrt{\left(\frac{\omega}{v}\right)^2 - \left(\frac{\omega_{mn}}{c}\right)^2}\right)} \frac{v/\omega}{1 - \left(\frac{n}{n+\delta}\right)^2} \sin\left(\frac{\pi\delta}{2}\right). \quad (22)$$

Collecting expressions for  $g$  and  $A_{mn}$ , the field amplitude coefficient is

now

$$A_{mn} = \frac{-i 16\pi E_0 v}{\omega^2 - \omega_{mn}^2} J_0 \left(X_{om} \frac{a}{R}\right) \frac{\sin\left(\frac{\pi\delta}{2}\right)}{1 - \left(\frac{n}{n+\delta}\right)^2} M_{mn} I \quad (23)$$

where the factor  $M_{mn}$ , defined by

$$M_{mn} = \frac{\sin(\omega\alpha)}{\omega\alpha} \cdot \frac{2I_1 \left(b \sqrt{\left(\frac{\omega}{v}\right)^2 - \left(\frac{\omega_{mn}}{c}\right)^2}\right)}{b \sqrt{\left(\frac{\omega}{v}\right)^2 - \left(\frac{\omega_{mn}}{c}\right)^2}} \frac{1}{I_0 \left(a \sqrt{\left(\frac{\omega}{v}\right)^2 - \left(\frac{\omega_{mn}}{c}\right)^2}\right)}, \quad (24)$$

is composed of factors typically of order unity.

#### IV. Tuning

The resonant denominator in (23) indicates the need for tuning to the modal resonant frequency. We assume the mode to have a resonant frequency

$$\omega_{mn} = \frac{2\pi c}{\lambda_{mn}} \left(1 - \frac{i}{2Q}\right) \quad (25)$$

where  $\lambda_{mn}$  is given by (20) and  $Q \gg 1$ . Let  $\omega = \frac{2\pi c}{\lambda_{mn}} (1 + \Delta/2)$  where

$\Delta \ll 1$ ;

$$\frac{1}{\omega^2 - \omega_{mn}^2} = \frac{-iQ}{1 - iQ\Delta} \left(\frac{\lambda_{mn}}{2\pi c}\right)^2 \quad (26)$$

Thus

$$\left| \frac{1}{\omega^2 - \omega_{mn}^2} \right| = \frac{Q}{\sqrt{1 + (Q\Delta)^2}} \left(\frac{\lambda_{mn}}{2\pi c}\right)^2 \quad (27)$$

is maximum for  $|Q\Delta| \ll 1$ . It may, however, be preferable to choose a larger value of  $\Delta$ , say  $|\Delta| \approx Q$ , in order to shift the phase of the field in order to stabilize the beam. The phase of the field is actually real for  $Q\Delta \ll 1$  due to the factor  $i$  in (23).

Now insert (19) and (26) into (23):

$$a_{mn} = \left[ \frac{2^7 \epsilon_n \beta \lambda_{mn}}{\pi} \right]^{1/2} \frac{Q}{1 - iQ\Delta} \frac{J_0(X_{om} \frac{a}{R}) M_{mn}}{X_{om} J_1(X_{om}) c} I \frac{\sin(\frac{\pi\delta}{2})}{(n + \delta)^{1/2}} \frac{\beta_p^2 - 1}{\beta_p^2 - \beta^2} \quad (28)$$

where we have introduced  $\beta = v/c < 1$  and

$$\beta_p = \frac{2d}{n\lambda} = \beta \frac{n+\delta}{n} > 1 \quad (29)$$

is the phase velocity (divided by  $c$ ) of the cavity mode (driven at its resonant frequency) viewed as a wave traveling in the  $z$ -direction. In the case  $n=0$ , the quotient involving  $\beta_p$  is replaced by 1.

In conjunction with (28), one needs to keep in mind the relations

$$d = \frac{\lambda\beta}{2} (n + \delta) \quad (30)$$

and

$$R = \frac{X_{om}}{2\pi} \lambda \frac{\beta_p}{\sqrt{\beta_p^2 - 1}} \quad (31)$$

As  $\delta$  approaches its minimum value in  $n[\frac{1}{\beta} - 1]$ ,  $\beta_p$  goes to 1 and R becomes infinite. On the other hand, we would like  $\delta = 1$  to maximize  $\sin(\frac{\pi\delta}{2})$ . It appears that  $\delta=1$  is a reasonable choice, with  $n \gg \delta$  so that  $\beta_p \approx 1$ .

## V. Power

The stored energy in a particular cavity mode is

$$U = \frac{1}{8\pi} |a_{mn}|^2. \quad (32)$$

The Q is defined by

$$Q = \omega \frac{U}{P} \quad (33)$$

where P is the power lost. We assume that essentially all of this power is usefully extracted, with negligible wall loss. Thus the power extracted is

$$P = \frac{W}{8\pi Q} |a_{mn}|^2 = \frac{c}{4Q\lambda} |a_{mn}|^2 \quad (34)$$

which in the present case is

$$P = \frac{2^5 \epsilon_n \beta}{\pi c} \frac{Q}{1 + (Q\Delta)^2} \left[ \frac{J_0(x_{om} \frac{a}{R}) M_{mn}}{x_{om} J_1(x_{om})} I \frac{\beta_p^2 - 1}{\beta_p^2 - \beta^2} \right]^2 \frac{\sin^2 \frac{\pi\delta}{2}}{n + \delta} \quad (35)$$

## VI. Choice of Mode

A standard Klystron operates in the mode  $n = 0$ . However, our expression for  $A_{mn}$  indicates that for  $\beta \approx 1$ , higher order modes may be used. The expression  $\beta_p^2 - \beta^2$  in the denominator is actually a synchronism parameter which can be minimized by making  $\beta$  as large and  $\beta_p$  as small as is possible. Unfortunately, this effect is canceled by the term  $\beta_p^2 - 1$  in the numerator. This term is specific to the cylindrical cavity and comes from the mode normalization; it reflects the fact that, as  $\beta_p \rightarrow 1$ , field energy moves out of longitudinal and into ineffective radial fields.

There remain several persuasive arguments for choosing  $n > 0$ . One is the size of a cavity with  $n = 0$ . For maximum power,  $d/\lambda = 0.37\beta$ . The cavity can of course be lengthened at the expense of power, but the result is to successively accelerate and decelerate electron bunches with an obviously negative effect on beam integrity. Likewise, the cavity radius is  $R = \frac{X_{0m}}{2\pi} \lambda_{mn}$ . For  $m=1$ , this is barely larger than the beam pipe radius and clearly too small. One can address this problem by moving to larger  $m$ , but the term  $J_0(X_{0m} \frac{a}{R})$  continues to keep the field weak in the beam region.<sup>3</sup> Neither length nor width increases affected the total electric field required to extract a given amount of power. This is the fundamental problem with  $m = 0$ . The interaction is required to take place over a short distance, which requires large fields. Consider an  $m = 0$  cavity designed to extract 1% of the energy of a 20 MeV beam. Regardless of the cavity dimensions, the relevant interaction takes place over only one half of a wavelength. For 1 cm radiation, this implies a peak field strength of at least 63 MV/m. The absence of perfect beam overlap and the fact that the strongest fields exist away from the beam can easily result in a maximum field strength of several times this size. These fields suggest a major breakdown problem.

On the other hand, the  $n > 0$ , the interaction takes place more gently, as the beam continually loses its energy while it remains in synchronism with the field;<sup>4</sup> additional length is not just wasted space. Also, as  $\frac{n+\delta}{n}$  goes to its minimum value at a given wavelength,  $R$  increases without limit while we remain in the  $m = 1$  mode. Thus small radial dimensions are avoidable.<sup>5</sup>

One final and perhaps vitally important advantage of  $n > 0$  modes is the fact that the phase of the field can be arranged to provide a net longitudinal bunching force to counter the effect of space-charge debunching. This will be elaborated upon below.

It is conceivable that an alternative cavity could combine the advantages of extended interaction and essentially longitudinal fields - namely, a coupled-cavity slow-wave structure. At wavelengths of interest, this would be a delicate and expensive object to construct, but it could be useful when large single-cavity extraction efficiencies are important. This is not the case in the problem at hand.

### VII. Choice of Q

All of the previous analysis concerned the steady-state solution. The fields actually build up in time with the factor  $1 - e^{-t/(t_f)}$ , where the filling time  $t_f = 2Q/\omega$ . For  $Q = 300$  at 30 GHz,  $t_f = 3.2$  ns. Thus a 15 ns pulse brings the fields to 99% of their steady-state level. When one considers that the steady-state power is proportional to  $Q$  and that the transient power goes as  $(1 - e^{-t/t_f})^2$ , the maximum power at the end of a pulse  $t$  seconds long is actually attained for  $t_f \cong 1.25 t$ . However, it may prove advantageous to suffer the somewhat reduced power levels (by a factor of about 2) in order to achieve the phase stability and mode purity of the steady state.

An important issue in choosing the  $Q$  is one's ability to construct a cavity with enough loss to an external circuit. Let us view the cavity as a length of waveguide and open holes in its wall to couple to a second waveguide. What coupling factor in the waveguides corresponds to a particular  $Q$  in the cavity? The power  $P_T$  traveling in one direction along the cavity axis moves at a group velocity of essentially  $c$ . Its energy per unit length is  $U/2d$ , where  $U$  is the total stored energy (the other half belongs to the oppositely - directed traveling wave). Hence  $P_T \approx \frac{cU}{2d}$ . The power extracted is  $P = \omega u/Q$ , so

$$\frac{P}{P_T} \approx \frac{4\pi d}{Q\lambda} \approx \frac{2\pi}{Q} (n + \delta) \approx \frac{2\pi n}{Q} \quad (36)$$

assuming we couple from only one traveling wave. For  $n=20$  and  $Q=300$ ,  $P/P_T = 0.42$  and the coupling factor is  $10 \log(P_T/P) = 3.8$  dB. If we couple from both traveling waves, we need 21% from each (6.8 dB). This may be difficult to arrange. One suggestion is to lay a single-mode rectangular wave guide parallel to the cylindrical cavity. Holes are cut at the points of peak current along the length of the cavity, but only at alternate peaks since the guide wavelength is about twice the cavity wavelength. The cylinder may have a circumference of  $12 \lambda$ , the waveguide a width  $1/2 \lambda$ . Thus many waveguides can be attached, hopefully bringing the  $Q$  down to a useful level. The computation of the  $Q$  of a given coupler design requires further study.

One more issue related to the  $Q$  is the stimulation of adjacent modes. We saw earlier that the tuning bandwidth is of order  $1/Q$ . If  $\omega \approx \omega_{mn}$ , then the power in a mode of resonant frequency  $\omega_{m'n'} = \omega_{mn} (1 + \frac{\epsilon}{2})$  is down from the fundamental mode power by about

$$\left| \frac{\omega^2 - \omega_{mn}^2}{\omega^2 - \omega_{m'n'}^2} \right|^2 \approx \frac{1 + (Q\Delta)^2}{Q\epsilon} \approx \frac{1}{Q\epsilon}$$

When  $\beta_p \approx 1$ , then  $\omega_{mn} \approx \frac{n\pi c}{d}$  so  $\epsilon \approx \frac{2}{n}$ ,  $\frac{1}{Q\epsilon} \approx \frac{n}{2Q}$  which is typically much less than 1. Modes of nearby  $m$  may also be important. Consider as an example  $\lambda_{0,19} = 1$  cm,  $\delta = 1$ ,  $n = 19$ ,  $\beta = 1$ . The  $d = 10$  cm,  $R = 1.23$  cm. We find  $\lambda_{0,20} = 0.95$  cm,  $\lambda_{0,18} = 1.05$  cm,  $\lambda_{1,19} = 0.84$ . However, if  $\delta = 0.2$ , then  $\lambda_{1,19} = 0.96$  cm. In this worst case, the power in the adjacent mode is  $\frac{1}{20}$  the power in the fundamental.



### VIII. Bunch Degradation

There are two primary sources of energy spread: space-charge repulsion and differential energy loss due to finite phase spread of the bunch.

As a simple space-charge model, consider a bunch of charge  $q$  which is uniformly distributed in a sphere of radius  $\ell'$  as measured in the charge's inertial reference frame. It is readily seen that the sphere expands uniformly; the time required for it to double its radius is

$$t_2' = \left[ 2 + \frac{1}{2} \ln(2\sqrt{2} + 3) \right] \sqrt{\frac{m\ell'^3}{2qe}} = 1.62 \sqrt{\frac{m\ell'^3}{qe}} \quad (37)$$

where  $t_2'$  is measured in the beam frame. The time in the lab frame is  $t_2 = \gamma t_2'$ ; in that frame, the charge packet has length  $\ell = \ell'/\gamma$ , radius  $b = \ell'$ . Hence

$$t_2 = 1.62 \sqrt{\frac{m\gamma^5 \ell}{qe}} = 1.15 \gamma^{5/2} \sqrt{\frac{I_{alf}}{I}} \frac{\ell}{c} \quad (38)$$

where  $I_{alf} = \frac{mc^3}{e} \approx 17$  kamps and  $\hat{I} = \frac{qC}{2\ell}$  is the peak current, averaged over the bunch length.

In a beam line, the bunches are spaced  $\lambda$  apart and  $\ell$  is perhaps  $\frac{\lambda}{6}$ , so that the first harmonic of the current is significantly degraded by the time the bunch length doubles. Note also that we have taken the initial radius to be  $\gamma\ell \sim \frac{\ell\lambda}{6}$ , the final radius twice this. In fact, in order to get through a beam pipe of radius  $a < .35\lambda$ , the transverse dimensions must be considerably smaller. In other words, the assumption of spherical (beam frame) bunches of given length strongly overestimates the radial dimensions, so we expect the calculation to also overestimate the debunching time. Since for  $\hat{I} = 1$  kampf, the sphere-model predicts that bunches travel only about  $140\lambda$  ( $\gamma=8$ ) or  $8000\lambda$

( $\gamma=40$ ) before beginning to merge, it is clear that space-charge debunching can be a severe problem.

A potential solution is to locate the bunch at a bunch-stable phase of the wave. Consider the equation for the energy  $U$  of a  $z$ -directed particle of charge  $e$ :

$$\frac{dU}{dt} = e\mathbf{v} \cdot \mathbf{E}(r, \mathbf{v}, t) = ev a_j e_{jz}(r, z) e^{-i\omega t} \quad (39)$$

With the approximation  $z(t) = v(t-t_0)$ , we can integrate:

$$\Delta U = ev a_j \int_{-\infty}^{\infty} e_{jz}(r, v[t-t_0]) e^{-i\omega t} dt \quad (40)$$

where  $e_{jz}(r, z)$  is given by (5). Evaluation of the integral leads to:

$$\Delta U = ev a_j I_0 \left( \frac{r\omega}{\gamma v} \right) A_{j\omega} e^{-i\omega t_0}$$

or alternately

$$\Delta U = -ev I_0 \left( \frac{2\pi r}{\gamma \lambda \beta} \right) \frac{P}{2\pi f_j g} \operatorname{Re}\{(1 - iQ\Delta) e^{-i\omega t_0}\} \quad (42)$$

In pondering this expression notice that the radial dependence is extremely weak for even moderately large  $\gamma$ ; little energy spread develops as a function of radius.

Note also that  $\operatorname{Arg}(1 - iQ\Delta)$  is the phase of the electric field; the phase of the particle in the center of the current bunch is zero. Assume the phases  $\omega t_0$  of the remaining particles are distributed symmetrically about zero. When  $Q\Delta = 0$  (Fig. 2a), the center particle ( $t_0 = 0$ ) loses the maximum energy ( $e$  and  $f_j g$  are both negative) and all particles within  $-\pi/2 < \omega t_0 < \pi/2$  lose energy. When  $Q\Delta \gg 1$  (Fig. 2b), the phase of the field is shifted back by  $\pi/2$ ; now particles with  $\omega t_0 > 0$  gain energy, those with  $\omega t_0 < 0$  lose energy. In a symmetric distribution, the field gains no net energy. The fields in this

case tend to divide the bunch. The choice  $Q\Delta \ll -1$  shifts the field phase forward  $\pi/2$  from the resonant position, providing bunch compression.

To get an idea how much the field can counteract space-charge forces, consider a bunch of length  $\omega\Delta t = \pi/2$ , with the field phase equal to  $\pi/4$  ( $Q\Delta = -1$ ) (Fig. 2c). Consider the electron at  $\omega t_0 = \pi/2$ . It sees a force  $eE_m$  to the left due to the field, and a space-charge force to the right. Consider as an example a 20 MeV beam from which one would like to extract 1% of the power over an interval of 10 cm. The average electron sees a field of  $E_{av} = 2/\pi E_m$ . It must traverse a potential of  $\Delta V = dE_{av} = 0.2$  MV in order to lose 1% of its energy; thus  $E_m = \frac{\pi}{2} E_{av} = \frac{\pi}{2} \frac{0.2 \text{ MV}}{0.1 \text{ m}} \approx 3 \text{ MV/m}$ .

This longitudinal field is independent of reference frame. On the other hand, consider the space-charge field in the particle's reference frame, modeling the charge as a sphere in that frame as was done earlier. By those methods, one obtains the space-charge field  $E_{sc} = \frac{2\hat{I}}{\gamma bc}$ , where again  $\hat{I}$  is the peak current averaged over the bunch length and  $b$  is the charge width (radius),  $b \approx \lambda/3$ . At  $\lambda = 1$  cm and  $\hat{I} = 1$  KA,  $E_{sc} = 0.4$  MV/m. In this case, there is a net decelerating force at the right edge of the beam which is stronger than the (net decelerating) force at the left edge. Thus the bunch must be compressed.

The charge sphere model is not quite appropriate for this example. In the lab frame, bunches may be cylinders of length  $\lambda/3$  and radius also  $\lambda/3$ . In the beam frame, the length is expanded by  $\gamma$ . For  $\gamma=40$ , one should use a long-cylinder model to compute the field. Such a computation results in the expression  $E_{sc} = \frac{\pi\hat{I}}{\gamma bc}$ . The difference is not enough to change the conclusion, which is that space-charge debunching can be counteracted by detuning. Questions remain, however. One concerns the required ratio between the cavity length and the drift space length in order to achieve net containment of

bunches. A second concern is the rate of particle loss from such bunches. A third issue is the start-up delay caused by debunching in the early part of a pulse. Further detailed study, including computer simulation, is indicated in this area.

#### IX. Beam Transport

The most obvious method of beam transport is a series of focusing magnets with a cavity or induction unit located at each focal point. The condition that the beam pass through two holes of radius  $a \sim .35\lambda$  separated by perhaps  $d=10\lambda$  sets a limit on the emittance:  $\epsilon < a^2/d \sim 12\lambda$  mrad. The drawback to this method is that the bunches spend a large proportion of their time out of a cavity. Alternatives require lowering the beam emittance or increasing the wavelength. A transport system designed for an emittance of 10 mrad-cm features a cavity-to-cavity spacing of 0.75 m.<sup>6</sup>

X. Example

Following is a list of parameters for a sample cavity:

$$\lambda = 1 \text{ cm}$$

$$m = 1$$

$$n = 19$$

$$\delta = 1$$

$$d = 10\beta \text{ cm}$$

$$R = 1.22\beta \text{ cm}$$

$$a = 0.35 \text{ cm}$$

$$b = 1/3 \text{ cm}$$

$$Q = 300$$

$$Q\Delta = -1$$

$$\lambda_{mn} = 0.997 \text{ cm}$$

$$\omega\alpha = 1$$

$$M_{mn} = 0.84$$

$$J_0\left(X_{om} \frac{a}{R}\right) = 0.88$$

$$\left\{ \begin{array}{l} \beta = 0.99: \\ \beta = 1: \end{array} \right.$$

$$P = I_a^2 \text{ kW}$$

$$P = 1.6 I_a^2 \text{ kW}$$

$I_a =$  average current in amps

$$|E| < 75 I_a \text{ kV|m}$$

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Andrew Sessler played a key role in the identification of problems and offered many suggestions and useful criticisms. Helpful discussions with Perry Wilson, Simon Yu, Don Hopkins, Jonathan Wurtele, Art Paul, and Philip Morton provided enlightenment.

## References

1. A. M. Sessler, "The Free Electron Laser as a Power Source for a High Gradient Structure," in Laser Acceleration of Particles, P. J. Channell, ed., AIP Proceedings No. 91, New York 1982, pp. 163-189.
2. W.K.H. Panofsky, private communication.
3. Note, however, that if  $I_0(X_{om} \frac{a}{R})$  is small, the assumption of a closed-cavity mode for  $r > a$  may be invalid.
4. Note the similarity to the extended-interaction device described in M. Chodorow and T. Wessel-berg, "A High-frequency Klystron with Distributed Interaction," IRE Trans. Electron Devices, ED-8: 44-55 (1961).
5. In this case, the assumption that  $a$  is small compared to the radial scale length is validated.
6. A. C. Paul, private communication.

Figure 1a: Configuration of Section II

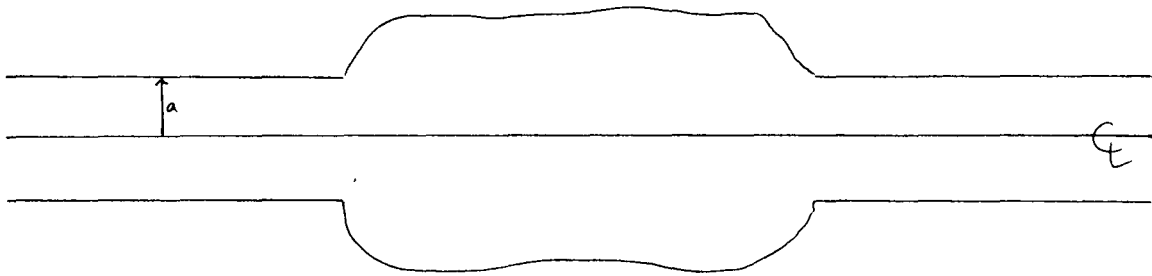
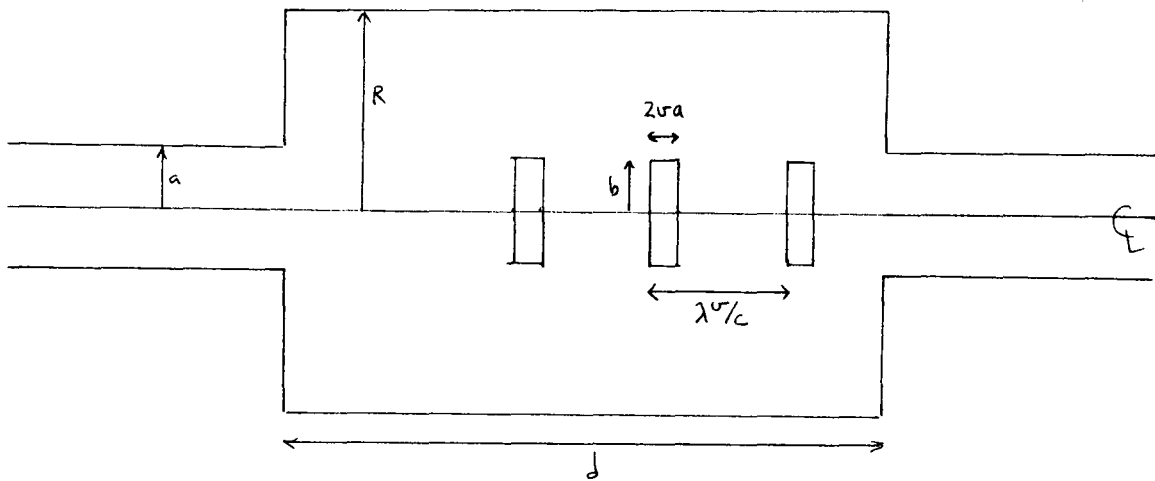


Figure 1b: Specific Cavity of Section III



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Figure 2a:  $Q\Delta = 0$

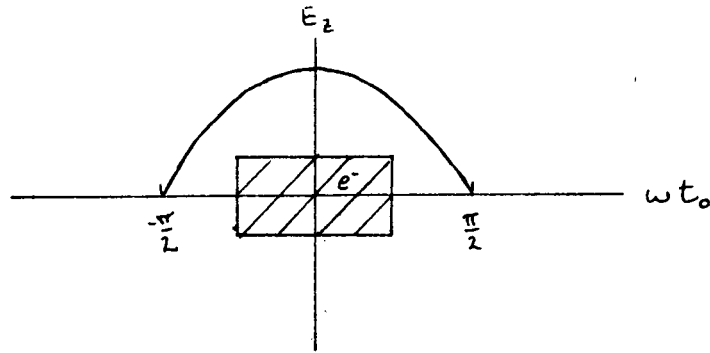


Figure 2b:  $Q\Delta \gg 1$

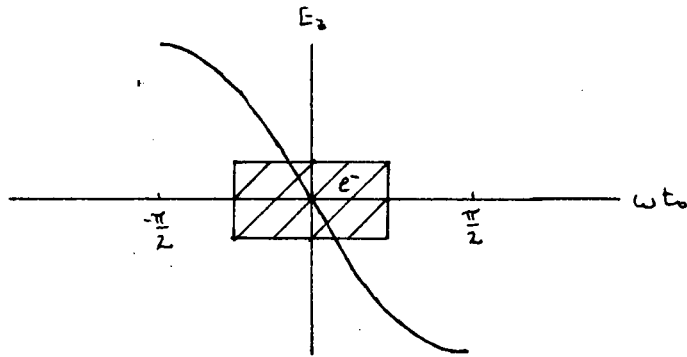
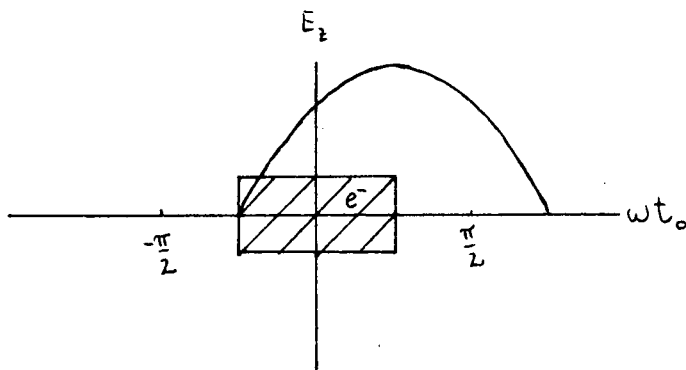


Figure 2c:  $Q\Delta = -1$



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