Title

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When Less Isn't More: A Real-World Fraction Intervention Study

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Abstract

Although an understanding of fractions is a critical precursor for other mathematical concepts, including proportional reasoning, algebra, and success in STEM fields, surveys of mathematics education in the United States indicate that school-age children lack age-appropriate math skills and proficiency. Thus, understanding the critical precursors of fraction knowledge is important for the development of instructional materials. The aim of the present study was to examine whether instructional format affected children's learning and transfer of fraction concepts, and whether individual variables such as executive function and math knowledge moderated these effects. Six- to 8-year-old children participated in a longitudinal, pre/post test design, in which they received a fraction-training intervention. Critically, we manipulated the extent to which real-world instruction was grounded in visual vs. symbolic representations. We find that 1st and 2nd graders were able to learn fraction concepts following this intervention, despite having no formal fraction education. The extent to which the instructional stimuli were grounded in visual vs. symbolic representations affected children's proportional reasoning knowledge in a transfer task, and condition effects were moderated my children's working memory and prior math knowledge. This work has implications for instructional design and curriculum development in the classroom.

Keywords: Numerical cognition, fractions, proportional reasoning, education, learning.

Introduction

An understanding of fractions is a critical precursor for other mathematical concepts, including probability, proportional reasoning, algebra, and much of the STEM fields (Bailey, Hoard, Nugent, & Geary, 2012; Department of Education, 1997). In fact, early fraction knowledge predicts the acquisition of algebraic knowledge well into middle and high school. However, surveys of mathematics education in the United States indicate that school-age children lack age-appropriate math skills and proficiency (NAEP, 2009; NCES, 2010; Siegler et al., 2012; also see Hurst & Cordes, 2016). Thus, improving students' math knowledge and reasoning ability about proportions early in a child's education is important. Furthermore, understanding what instructional format may best lead to both the learning and transfer of difficult math concepts (i.e., proportions) should be a fundamental component of instruction and curriculum development.

The aim of the present study was to examine whether the instructional format in which fraction concepts are taught would affect the learning and transfer of novel fraction concepts (Core Curriculum; New Common Core Mathematics Standards, 2000), as well as whether individual variables such as executive function and prior math knowledge would moderate any observed effects. On the one hand, one approach to teaching mathematics to young children involves the use of concrete instantiations, such as vibrant, perceptually-rich visual displays or real-world contextualized examples (e.g., Van de Walle, 2007), presumably because these high-contrast items are attention-grabbing, motivating, and often found in a child's natural environment (NCTM, 2000). Perceptually rich education materials are abundantly available and often populate children's classrooms in an effort to keep children interested in the materials being taught (Peterson & McNeil, 2012). Even teachers prefer perceptually rich materials (Peterson & McNeil, 2012), as they presumably increase children's engagement in the task at hand.

On the other hand, much work suggests a "less is more" approach to teaching children about difficult math concepts. This work suggests that perceptually rich or concrete materials may hinder mathematics concept learning (and perhaps problem solving and computation) because extraneous perceptual information gets integrated into representation of the target concept (e.g., Kaminski & Sloutsky, 2009, 2013; Kaminski, Sloutsky, & Heckler, 2009; McNeil & Fyfe, 2012; Mix, 1999, 2008; Peterson & McNeil, 2012; Posid & Cordes, 2014). For example, Posid and Cordes (2014) asked young children (3-6 years) to decide which of two arrays contained a target number of items, where half of the arrays were homogenous in make-up (e.g., all of the same kind of animal) and the other half of the arrays were heterogeneous in make-up (all different animals). They found that children were less accurate when arrays were heterogeneous in make-up, particularly when the task was more difficult (when children were asked to find a larger target numerosity), and that this homogeneity advantage remained present across development (Posid & Cordes, 2014; also see: Mix, 1999, 2008). Similarly, Kaminski & Sloutsky (2013) taught young children (kindergarten through second grade) to read bar graphs, while manipulating whether the graphs contained colorful and irrelevant information or monochromatic bars. They found that the children trained on graphs with irrelevant colorful features often tried to use that extraneous perceptual information incorrectly, decreasing their overall accuracy compared to their peers who were trained on monochromatic bar graphs (Kaminski & Sloutsky, 2013). Together, these studies suggest that these more perceptually rich concrete instantiations over-communicate information to the learner, compared to their generic counterparts, thereby hindering learning from the relevant mathematical structure or relations at hand.

The impact of perceptual information when learning about *proportions* or *fractions* is even less known. In one study, Kaminski and Sloutsky (2009) investigated kindergarteners' ability to identity proportions across two sets of stimuli, which varied in their degree of concreteness of the instantiations. Children in the concrete condition failed to compare novel proportions while children in the generic condition successfully compared novel proportions, following a sparse training and no instruction. These findings suggest that simple proportional relations can be learned following generic instantiations, while concrete instantiations do not promote this same type of learning.

In this vein, young children's learning of fractions may also benefit from more generic instantiations. Specifically, fractions are traditionally introduced as symbols (e.g., 1/2), where visual representations may add extraneous information that could be interpreted ambiguously or incorrectly. For example, additional visual information could (a) add a layer of *perceptual* richness by conveying concept-irrelevant information and/or distracting information to the learner from the specific math concept to-be-learned, (b) add extraneous *conceptual* information, such as sharing, when real-world instantiations such as the use of a pizza pie are used, or (c) add a combination of the two. Thus, the present study addresses whether symbolic or visual instantiations in particular provide pre-fraction learners with a better ability to learn about novel fraction concepts. Because little work has addressed whether perceptual or conceptual visual instantiations may be detrimental to the young learner, the present study utilizes minimalistic, black-and-white stimuli (perceptually but not conceptually rich) to begin to address this important research question.

Overview of the Current Study

The aim of the present study was to examine whether instructional format affected children's learning and transfer of fraction concepts, and whether individual variables would moderate any observed visual vs. symbolic instantiation effects. Critically, we manipulated the extent to which real-world instruction was grounded in visual vs. symbolic representations, while incorporating actual educational practices into the training paradigm (material, context, multi-session lessons). First and second graders participated in a pre/post-test design in which they received fraction instruction over several intervention sessions (Ordinal Comparisons, Addition and Subtraction, Decomposition), followed by a test of transfer (Fraction and Proportional Reasoning).

Method

Participants

Seventy-three 1st and 2nd grade children (M_{Age}=6.9 years) participated in this study. Children were tested in their own elementary school during regular school hours. All children were tested in a quiet room with a single female experimenter.

Materials

Pre- and Post-Test: Fraction Battery.

The fraction pre- and post-test batteries were identical and consisted of three fraction-knowledge tasks, which asked participants to make judgments about either symbolic or visual fraction information (also see Hurst & Cordes, 2016; Polinsky, Posid, & Sloutsky, 2017; Posid & Sloutsky, 2015, 2017). The first task was an Ordinal Task, in which participants were asked to judge which of two sets was numerically larger and included visual fraction comparison (e.g., 2/3 vs. 1/3, represented as black-and-white circles divided into three equal parts, with two parts and one part shaded, respectively) and a symbolic fraction comparison (e.g., 2/3 vs. 1/3). Children next completed a Matching Task, in which they matched either a symbolic fraction (e.g., 1/3) to a visual fraction (e.g., a black-and-white circle divided into three parts with one part shaded) or vice-versa. Children then completed an Addition and Subtraction Task, in which they were asked to add visual fractions (e.g., black-and-white circles) or symbolic fractions (e.g., 1/4 + 2/4). Due to the difficulty level of the Matching Task and Addition/Subtraction Task, four answer choices were offered in a multiple-choice format. For all fraction tasks in the pre-/post-test battery, only fractions <1 were used (e.g., 2/3 but not 4/3).

Fraction Training:

The fraction training intervention consisted of three sessions, which were identical in content and instruction across conditions. Each training session consisted of two parts: one-on-one guided instruction between the child and experimenter followed by a block of practice problems used to measure immediate learning. Importantly, although the content and instruction was consistent across conditions, children participated in one of three training conditions: (1) Visual Only (black-and-white circles only; *n*=22), (2) Symbols Only (symbolic fractions only; *n*=25), or (3) Visual + Symbols (both black-and-white and symbolic fractions shown side-by-side; *n*=23). The instruction and practice block were exclusively run in the child's randomly assigned condition (see Figure 1).

The first training task was an Ordinal Comparison task, in which children were instructed on how to compare two fractions with either the same denominator (e.g., 1/4 vs. 2/4) or same numerator (e.g., 1/4 or 1/8; Figure 1). Critically, the fraction instruction throughout all three training tasks was meant to address two concepts prevalent in the fraction-learning literature to date. First, children were taught to use counting to identify and manipulate the numerators and denominators presented, as children notoriously demonstrate a "whole number bias" when learning about fractions (e.g., DeWolf & Vosniadou, 2014; Hurst & Cordes, 2016; Ni & Zhou, 2005; Obersteiner, van Dooren, Van Hoof, & Verschaffel, 2013 Obersteiner, Van Hoof, Verschaffel, & Van Dooren, 2016; Polinsky et al., 2017). Second, children were taught about fraction magnitude knowledge, specifically as it relates to the part-whole concept (e.g., Siegler, Thompson, & Schneider, 2011; Stafylidou & Vosniadou, 2004). This instruction and feedback was followed by a block of practice trials in which children continued to compare fractions, but without scaffolding or feedback from the experimenter.

The second training task was an Addition and Subtraction task, in which children were taught how to systematically add or subtract two fractions. Children were shown a correct strategy for solving this type of fraction problem. This was followed by a practice block, in which children continued to add and subtract fractions, but without the input and feedback from the experimenter.

Finally, children completed a Fraction Decomposition task, in which they were asked to add or decompose a series of fractions (e.g., 1/6 + 2/6 + 1/6; Figure 2). The instruction was similar to that of the addition and subtraction task, with step-by-step instructions on how to identify the fractions, recognize that the denominator (or total number of pieces) was the same using counting, and then add the numerators (or shaded number of pieces) to find the answer. Again, this was followed by a block of practice trials in which children continued to decompose fractions, but without the instruction and feedback of the experimenter.

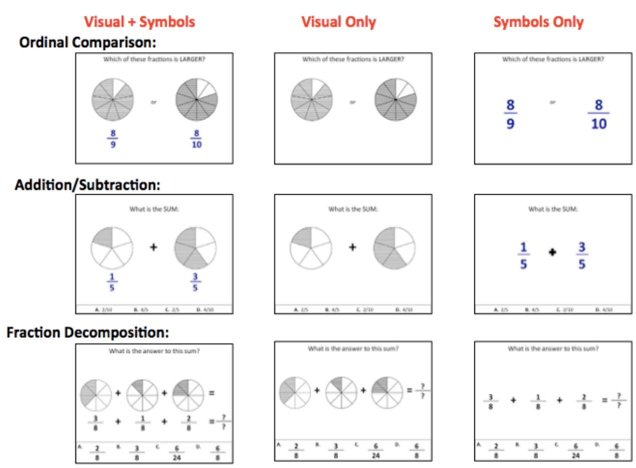

Figure 1. Examples of the Stimuli used in the Test portion of the Training Sessions.

Transfer Task:

The transfer task was used to measure children's fraction and proportion knowledge, and consisted of a series of visual questions. Included questions asked children to make judgments related to either probability (e.g., "If you were to roll a dice, what is the probability that that it would land on a 2?" or "If you were to reach into this box of candy without looking, what is the probability that you would randomly pick out a cherry piece?) or fractions (e.g., "Can you express the triangles as a fraction of the entire set?" or "If you were to reach in to one of two fish tanks, are hoping to pick a fish of a certain color, which fish tank should you reach into?").

All tasks were administered on a Macintosh laptop. These programs were created using RealBasic software, which also recorded participants' reaction time and answers during the tasks.

Procedure

Pre/Post-Test Fraction Battery:

The procedure consisted of three phases: The Ordinal Task, the Matching Task, and the Addition/Subtraction Task. The Ordinal Task consisted of three blocks: natural number comparisons (warm up), visual fraction comparisons, and symbolic fraction comparisons. All fractions were <1 so as to match the visual and symbolic fractions featured in the second and third blocks. Each block in the Ordinal Task consisted of 32 trials, for a total of 96 trials. The Matching Task consisted of three blocks of 12 trials each, for a total of 36 trials. The Addition/Subtraction task consisted of four blocks of 12 trials each, for a total of 48 trials.

Fraction Training:

The procedure consisted of three training sessions: Ordinal Comparisons, Addition and Subtraction, and Decomposition. The Ordinal Comparisons training session consisted of two phases: Same Denominator comparison training and Same Numerator comparison training. Each instruction phase consisted of two examples, followed by a block of 10 test questions (20 total test trials). The Addition and Subtraction training session consisted of two phases: Addition and Subtraction. Each instruction phase contained two examples, followed by a block of 20 test trials (40 total test trials). The Decomposition training session consisted of two phases of Decomposition instruction (fractions whose sum was <1 and fractions whose sum was >1), each followed by one blocks of test trials, plus a final block of "intermixed" test trials, for a total of 30 test trials.

Transfer Task:

The transfer task consisted of seven blocks of proportional reasoning and fraction reasoning picture problems and included: (1) spinner proportions, (2) dice rolling, (3) determining the proportions of shapes in a set, (4) determining the proportion of candy in a bowl, (5) interpreting a pie graph, (6) representing a set of shapes as a fraction, and (7) determining the proportion of fish in a fish tank (for a total of 36 Transfer Task questions). The transfer task was administered approximately 2 weeks after post-test. It was also given in multiple-choice format due to the difficulty of the task itself and for consistency of testing format across sessions.

Results

The present study examined three outcome variables of interest: (1) Learning at Training: Did children perform above chance on the test trials following each intervention session? That is, did they learn the information they had just practiced with the experimenter? (2) Pre-Post Test Gains: Did children improve on our Fraction Battery from Pre-Test to Post-Test? (3) Transfer Task: Did children perform above chance on our Transfer Task? Critically, we examined the impact of Training Condition (Visual+Symbol, Visual Only, Symbols Only) on all three of these variables of interest. Finally, we examined the role of individual variables as moderators of Condition effects on our dependent variables.

Learning at Training

Results indicate that young children were able to learn fraction concepts despite having no formal instruction in the classroom, as indicated by their above-chance performance during each phase of training (vs. chance: Ordinal: $p<.001$, *Cohen's d*=2.8; Addition/Subtraction: $p<.001$, *Cohen's d*=2.4; Decomposition: $p<.001$, *Cohen's d*=4.4; Figure 2). A moderate – but non-significant (*p's*<.2) -- trend demonstrated the impact of children's experimental condition across these tasks. That is, children's performance in the Symbols-Only and Visual+Symbols conditions were similar across the three day-of training tasks; however, children's accuracy in the visual-only condition was lower.

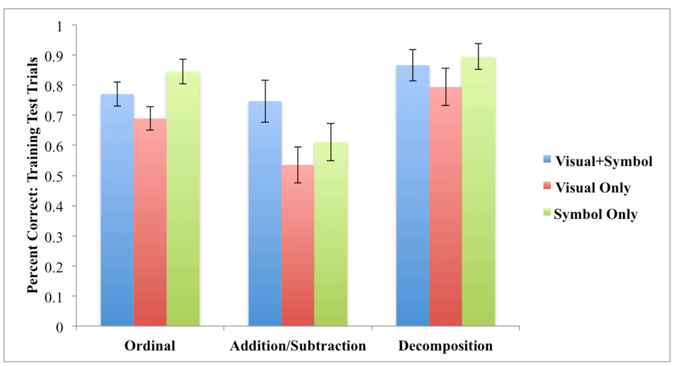

Figure 2. Accuracy on the test trials of each Fraction Training Session by Condition. Error bars reflect Standard Error of the Mean.

Pre- to Post-Test Gains

We also assessed whether children made substantial gains from pre- to post-test within our Fraction Battery. For the purposes of this analysis, difference scores were created for each participant (post – pre) for each portion of the Fraction Battery (Ordinal, Matching, and Addition/Subtraction; pre-test: no effect of Condition: $p>.1$), such that each participant had a single difference score for each task representative of any gains made. Then, an average of these difference scores was created to identify children's total gains across the training tasks. A significantly positive difference score (versus zero) would indicate significant gains made by that participant.

Children demonstrated significant gains between pre- and post-test ($t(69)=10.6$, $p<.001$, *Cohen's d*=2.6). Notably, condition differences were not observed in any of our three tasks within the Fraction Battery (*p's*>.1), suggesting that, at least for 1st and 2nd graders, any instructional format can promote learning of these difficult math concepts.

Transfer Task

Performance accuracy on the Transfer Task was calculated for questions pertaining to Proportional Reasoning and Fraction Reasoning. Overall, children performed above-chance on both types of questions in the Transfer Task (Proportional Reasoning: $p<.001$, *Cohen's d*>.1; Fraction Reasoning: $p<.001$, *Cohen's d*>2.5; Figure 3). Condition differences were observed for the Proportional Reasoning portion of the Transfer Task ($F(2, 63)=6.5$, $p=.003$), such that children in the Visual+Symbol condition out-performed their peers in both the Visual Only and Symbols Only conditions. Of note, children in the Visual Only and Symbols Only conditions performed significantly above-chance, and out-performed a secondary sample of untrained controls whose accuracy did not exceed chance-level ($t(6)=1.7$, $p=.15$, *Cohen's d*=1.4). In contrast, no Condition effects were observed for the Fraction Reasoning questions ($p>.7$), mirroring the lack of Condition effects observed in the pre- to post-test gains.

Individual Variability

A series of regression analyses were run in order to investigate whether individual variables predicted children's performance across our dependent variables. Each regression model tested the following independent variables: pre-fraction knowledge (as assessed through our pre-test Fraction Battery), prior Math Knowledge (assessed through a portion of the Woodcock Johnson and a 3-minute speeded arithmetic test administered at pre-test), Inhibitory Control (assessed through a numerical stroop task administered at pre-test), Working Memory (assessed through a serial ordering task administered at pre-test), and the child's grade in school.

Children's accuracy during the day-of training tasks (composite score) was significantly predicted by their grade in school (*Beta*=.224, $p=.06$), prior math knowledge (*Beta*=.573, $p<.001$), and their pre-test fraction knowledge (*Beta*=.253, $p=.04$; Model: $R^2=.416$, $p<.001$). Children's performance at post-test was significantly predicted by children's prior math knowledge (*Beta*=.566, $p<.001$; Model: $R^2=.401$, $p<.001$), while children's pre- to post-test gains were significantly predicted by their grade in school (*Beta*=.382, $p=.009$), working memory (*Beta*=.306, $p=.012$), and pre-test fraction knowledge (*Beta*=.485, $p=.001$; Model: $R^2=.220$, $p=.014$). Additional SEM modeling was conducted to examine whether our significant predictors specifically *moderated* any effects of Condition on our dependent variables. We find that prior math knowledge does moderate the effects of Condition on children's day-of training accuracy ($p=.02$) and post-test fraction performance ($p=.002$), while working memory moderated children's gains from pre- to post-test ($p<.001$).

Finally, regression and SEM modeling were conducted for children's performance on the Transfer Task. Specifically, children's accuracy on the proportional reasoning portion of the Transfer Task was significantly predicted by their grade in school (*Beta*=.227, *p*=.059), Condition (Beta=.222, p=.064), inhibitory control (*Beta*=.250, *p*=.038), and pre-test fraction knowledge (*Beta*=.257, *p*=.086). Again, secondary SEM models indicated that both math knowledge and working memory individually and significantly moderated performance on the Transfer Task. Specifically, children with High working memory did not show Condition differences, while those with low working memory did (Low WM: $F(2, 28)=9.0$, $p=.001$; High WM: $F(2, 27)=1.7$, $p=.206$). These children benefited most from Visual+Symbol. Like working memory, children with high math knowledge did not show Condition differences on the Transfer Task, whereas children with low math knowledge did. These children benefited most from Visual+Symbol (Low Math: $F(2, 26)=4.7$, $p=.019$; High Math: $F(2, 29)=1.7$, $p=.1$).

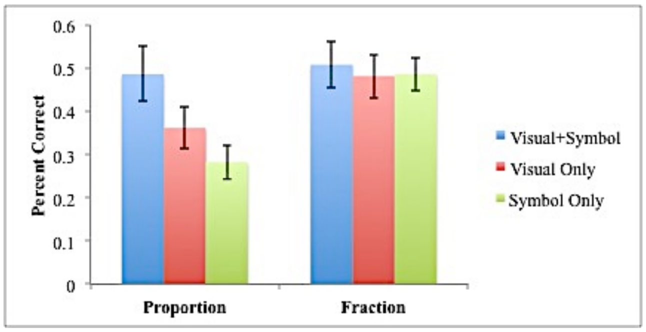

Figure 3. Accuracy on the Transfer Task by Condition. Error bars reflect Standard Error of the Mean.

Discussion

The aim of the present study was to examine whether instructional format affected children's learning and transfer of fraction concepts, as well as to investigate whether individual variables such as executive function and math knowledge moderated any effects of visual vs. symbolic instantiations. Results indicate two important patterns of performance. First, using real-world instructional stimuli from the current Core Curriculum (Core Curriculum; New Common Core Mathematics Standards, 2000), children as young as 1st and 2nd grade successfully learned new fraction concepts, as indicated by their above-chance performance on day-of training, in their gains observed from pre- to post-test on our Fraction Battery, and their above-chance performance on the Transfer Task. Because an understanding of fractions is an important precursor for other mathematical concepts, including probability, proportional reasoning, algebra, and much of the STEM fields (Bailey, Hoard, Nugent, & Geary, 2012; Department of Education, 1997), it is critical that elementary school children are involved in a curriculum that employs these critical foundations in fraction education. Although previous surveys of mathematics education in the United States suggest that children lack age-appropriate math skills (NAEP, 2009; NCES, 2010; Siegler et al., 2012; also see Hurst & Cordes, 2016), the present study suggests that current curriculum is utilizing content that may help close this gap in years to come.

Second, and more importantly, the present study finds that the instructional format in which the to-be-learned concepts are presented to children is important. Specifically, those children in the Visual+Symbol condition faired best both during immediate learning within our intervention sessions and in our test of transfer two weeks following post-test. Of note, children in the Visual only condition never out-performed their peers in either of the other two conditions, suggesting that less is *not* necessarily more when teaching children about new and conceptually challenging fraction concepts. Moreover, children who were low in math knowledge and low in working memory at pre-test benefited most from the Visual+Symbol condition, suggesting that the redundant perceptual information was particularly helpful.

The findings from the present study suggesting that "less" is not "more" when teaching children about new fraction concepts is interesting given much work suggesting that extraneous perceptual information may interfere with children's ability to learn mathematical concepts or make mathematical reasoning judgments (e.g., Kaminski & Sloutsky, 2009, 2013; Kaminski et al., 2009; McNeil & Fyfe, 2012; Mix, 1999, 2008; Peterson & McNeil, 2012; Posid & Cordes, 2014). This pattern of findings could be accounted for by two explanations. First, perhaps either fractions themselves *or* novel fraction concepts are a stand-alone category. That is, perhaps "less is more" when children are learning about whole numbers or non-fraction numerical concepts. However, this is unlikely given ample research to suggest that both children and adults demonstrate a whole number bias, especially when learning about or solving fraction problems that are novel or difficult (e.g., see DeWolf & Vosniadou, 2014; Hurst & Cordes, 2016; Ni & Zhou, 2005; Obersteiner et al., 2013; Obersteiner et al., 2016; Polinsky et al., 2017). That is, when solving fraction or proportion problems, children often apply their intuitions about whole numbers to fraction concepts (for example, they might say that $1/4 + 1/4 = 2/8$, as they incorrectly assume that you should add the numerators and the denominators, as you would if you were adding whole numbers).

Another explanation for the seemingly divergent pattern of findings observed in the present study comes from the nature of the stimuli used in the study itself. Although condition differences emerged, the stimuli were more perceptually impoverished than those used in previous work reporting "less is more" during mathematical learning and reasoning (Kaminski & Sloutsky, 2009, 2013; Kaminski et al., 2009; McNeil & Fyfe, 2012; Mix, 1999, 2008; Peterson & McNeil, 2012; Posid & Cordes, 2014). Specifically, the conditions in this study varied by whether the instantiations contained visual vs. symbolic vs. visual + symbolic information. However, the visual stimuli were always black-and-white circles, while the symbolic stimuli were a single monotone color (e.g., black). In contrast, real-world mathematics education includes much more diverse and vibrant displays, the use of 2D and 3D objects, the use of "interesting" pictures and colors (e.g., a pizza pie to represent a pie graph or visual fraction; Peterson & McNeil, 2012), and so on. Therefore, perhaps the "perceptually impoverished" framework employed in the present study muted the real-world effects of varying concrete vs. generic instantiations when teaching children about fractions and proportions. Currently, work from our laboratory is exploring this variation to visual vs. symbolic instantiations, through the use of a perceptually rich training paradigm. We are currently exploring whether similar Condition effects and individual moderators will emerge when perceptually rich (e.g., pizza pies rather than black-and-white circles, Sesame Street-like numbers with colors and eyes, etc.) stimuli are used in a similar training intervention.

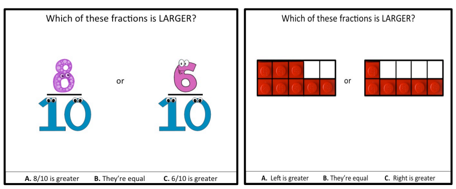

Figure 4. Perceptually rich instantiations of the stimuli used in the present study.

In conclusion, the present study utilized a real-world fraction training intervention and finds that children can learn fraction information prior to formal education, using instructional material from the current Core Curriculum. Importantly, although all children demonstrated gains following training, those who received redundant perceptual information tended to out-perform their peers following immediate learning and in a transfer test of proportional reasoning. Additionally, children's prior math knowledge and working memory moderated our observed effects, indicating these should be taken into consideration when children are taught novel or difficult fraction concepts. This work has implications for instructional design and curriculum development in the classroom.

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