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In this note we use Regge's continuation to complex angular momentum[1,2] in order to define and describe particles or resonances that do not correspond to a usual Breit-Wigner type pole of the partial-wave amplitudes. From the point of view of the analytic structure of the S matrix regarded as a simultaneous function of angular momentum and energy, however, there is only a quantitative difference between these and ordinary particles. There are at least two definite examples of such resonances, namely the I = 0, π-π, S-wave "virtual state"[3] and the well-known singlet n-p, S-wave "virtual state,"[4] both near the threshold. The general situation is described here, which is valid for any value of angular momentum and energy, and it is suggested that some of the higher spin resonances recently observed in strong interactions might belong to this category. Futhermore, we determine the connection between range and scattering parameters and the trajectory in the complex angular-momentum plane of poles of the S matrix that correspond to virtual particles. This trajectory in turn is related to high-energy cross sections for processes in which these particles are exchanged in the cross channel.[2,5]

Our considerations are based on the fact that the total two-body elastic-scattering amplitude can be written, using a Watson-Sommerfeld transformation in the complex ℓ-plane,[1] as a sum of--in general few--pole terms plus a regular remainder. The pole terms control the asymptotic behavior of the amplitude, and also the bound states and resonances in the partial-wave amplitudes. One can thus separate explicitly the singular parts of the amplitudes. This procedure is a substitute for the use of subtractions in the dispersion relations. For a given set of quantum numbers, the poles are separated by more than one unit of angular momentum. Therefore, in the vicinity of a bound state or resonance, the total amplitude may be approximated by a Regge-pole term of the form.[6]

$$A(q, \cos\theta) = \beta(q)\, P_{\alpha(q)}(-\cos\theta) / \sin \pi\alpha(q), \qquad (1)$$

where $\alpha(q)$ is the position of the pole of the S matrix in the complex ℓ plane as a function of the momentum q. The partial-wave projections of Eq. (1) are

$$A(q, \ell) = \frac{1}{\pi}\, \beta(q) \Big/ \left\{[\alpha(q) - \ell]\, [\alpha(q) + \ell + 1]\right\}, \qquad (2)$$

which clearly shows the pole in the ℓ plane at $\ell = \alpha(q)$. For $q^2 < 0$, $\alpha(q)$ is a real and increasing function of q; for $q^2 > 0$, $\alpha(q)$ has a positive imaginary part.

We first discuss the threshold behavior of $\alpha(q)$. It will be shown that the behavior of phase-shifts near threshold is consistent with a square-root singularity of $\alpha(q^2)$ at $q^2 = 0$. Therefore near $q^2 = 0$, $\alpha(q^2)$ can be written in the form

$$\alpha(q) = \alpha_R(0) + (-q^2)^{1/2}(d\alpha_I/dq)_0 + 1/2\, (d^2\alpha_R/dq^2)_0\, q^2. \quad (3)$$

Thus, for small $q^2 > 0$, the imaginary and real parts of α are given respectively by

$$\alpha_I = q(d\alpha_I/dq)_0$$

and

$$\alpha_R = \alpha_R(0) + 1/2\ (d^2\alpha_R/dq^2)_0\ q^2. \qquad (4a)$$

These expressions have also been verified numerically and for the triplet n-p scattering and give exactly the deuteron binding energy in terms of the scattering parameters.[7] Once the three parameters in Eq. (3) are determined, the behavior of <u>all</u> partial-wave amplitudes due to the pole term (1) and near $q^2 = 0$ are obtained by inserting Eqs. (4) into Eq. (2). We do not write the general expression here, but if $\alpha_R(0)$ is very close to the integer $\ell = 0$ and we have $q^2(d^2\alpha_R/dq^2)_0 << 1$, we obtain

$$A(q,\ell) \cong (\beta/\pi)\ /[\alpha_R(0) + cq^2 + iq(d\alpha_I/dq)_0], \qquad (5)$$

where $c = (1/2)\ d^2\alpha_R/dq^2)_0 - (d\alpha_I/dq)_0^2$. Equation (5) is precisely the amplitude corresponding to the effective-range approximation $q \cot\delta = a^{-1} + rq^2/2$, and we obtain by comparison

$$[\alpha_R(0)/(d\alpha_I/dq)_0] = -1/a,$$

$$\beta(0)/\alpha_R(0) = a\pi,$$

and

$$(d^2\alpha_R/dq^2)_0/(d\alpha_I/dq)_0 - 2(d\alpha_I/dq)_0 = -r.$$

If we neglect the curvature $(d^2\alpha_R/dq^2)_0$ for the time being, we find

$$(d\alpha_I/dq)_0 = \frac{r}{2};\ \alpha_R(0) = -\frac{1}{2}\ \frac{r}{a};\ \beta(0) = -\frac{\pi}{2}\, r, \qquad (6)$$

where a is the scattering length, and r is the effective range.

We give now a quite independent calculation of the parameters of

Eq. (6), using the range of the forces involved. If we take as a model a short-range potential, a square well of range r_0 and strength $V_0 = (\pi/2 - \epsilon)^2/r_0^2$ not quite strong enough to make an S-wave bound state, the position of the pole in the ℓ plane is given by $\alpha_R(0) \cong -\pi\epsilon/4$. The scattering length a is related to ϵ by $a = 2r_0/\pi\epsilon$. Hence we have

$$\alpha_R(0) = -\frac{1}{4}\frac{r_0}{a};\ (d\alpha_I/dq)_0 = \frac{1}{4}r_0;\ \beta(0) = -\frac{\pi}{2}\ r_0. \qquad (7)$$

The two estimates agree roughly. Their difference gives us the curvature of the trajectory

$$(d^2\alpha_R/dq^2)_0 = \frac{1}{4}\ r_0\,(r_0/2 - r). \qquad (8)$$

Both for singlet and triplet n-p Regge poles as well as for the I = 0, $\pi\pi$ pole, the curvature as determined from the effective range formula is negative, i.e. the trajectories turn at the threshold. This behavior expresses the fact that there are no S-wave resonances without an S-wave bound state. For trajectories near $\ell = 1$ (or higher), the situation is different. Here we can have a P-wave resonance without a P-wave bound state and in this case we expect the curvature to be positive. The real part of α as a function of s or q^2 is shown in Fig. 1, and the parameters are given in Table I. It is important to note that the parameters of the cusp depend only on the range of the forces and not on scattering length. Therefore one would expect approximately the same cusp at all thresholds. Below the threshold, q^2 is less than zero, and $\alpha(q)$ is real and is given by[7]

$$\alpha \equiv \alpha_R(q) = \alpha_R(0) - (-q^2)^{1/2}\,(d\alpha_I/dq)_0 + \frac{1}{2}\,(d^2\alpha_R/dq^2)_0\,q^2. \quad (9)$$

This expression allows us to extrapolate α to the point s = 0 or $q^2 = m_\pi^2$ in the case of the π-π, I = 0 pole (or the so-called ABC pole).

The quantity $\alpha_{ABC}(s = 0)$ is also shown in Table I. The exchange of the I = 0, π-π system in the crossed channel results in a total cross section in the forward direction which varies as $E^{-(1-\alpha(0))}$, where E is the laboratory energy.[2,5]

Using the above method, we can also discuss the threshold behavior of a Regge trajectory very close to an integer ℓ. Again, if we assume that the single pole dominates the ℓth partial wave in question, we can compare the amplitude (2) with that corresponding to the effective-range formula $q^{2\ell+1}\cos\delta = a^{-1} + rq^2/2$. Then near $q^2 = 0$ we find that $\alpha_R = \alpha_R(0) + Aq^2$, $\alpha_I = Bq^{2\ell+1}$ or $\alpha = \alpha_R(0) + B(-q^2)^{(2\ell+1)/2} + Aq^2$. This discontinuity is superimposed upon a generally smooth trajectory at the threshold. Note that $\beta(q)$ is real near the threshold.

We now discuss the general situation where a resonance is observed without the trajectory of the pole crossing an integer value of ℓ or J. First, the threshold can occur, in principle, close to an integer $\ell \neq 0$. This case may be expected to be qualitatively the same as the case $\ell = 0$ discussed above. More interesting is the following situation. In the case of resonances, the function $\alpha_R(E)$ is an increasing function of E even above the threshold, except possibly for a small cusp at the threshold. When $\alpha_R(E)$ becomes equal to an integer, Eq. (2) gives a Breit-Wigner resonance if $\alpha_R(E)$ is assumed to vary linearly with E locally near E_r.[5] We consider the case where the $\alpha_R(E)$ curve turns very close to a physical integer.[8] In this case the expansion of $\alpha_R(E)$ is of the form

$$\alpha_R(E) = \alpha_R(E_r) + \frac{1}{2}(E-E_r)^2 (d^2\alpha_R/dE^2) + \ldots$$

the partial-wave amplitude is given by

$$A(E,\ell) \simeq \frac{(2/\pi)\,\beta(E)/[(2\ell+1)(d^2\alpha_R/dE^2)]}{(E-E_r)^2 + i(\Gamma/2) + C}, \qquad (11)$$

where $\Gamma/2 \cong 2\alpha_I/(d^2\alpha_R/dE^2)_0$, and C is a very small constant. This amplitude corresponds to two energy poles $E = E_r \mp (\sqrt{\Gamma}/2)(1 - i)$. Therefore, if in Eq. (11) a single-pole term is taken literally as the total amplitude, one obtains a resonance cross section that is approximately given by $1/[(E-E_r)^4 + (\Gamma/2)^2]$. If we take one of the energy poles only, the resonance shape becomes $1/[(E-E_r - \sqrt{\Gamma}/2)^2 + \Gamma/4]$. Such a resonance can be of importance only if the curvature at the turning point is small. No examples of resonances of this type are known at present. At any rate, the virtual particles, although somewhat different in character, are special manifestations of the poles of the S-matrix in the complex angular-momentum plane.

I should like to thank Professor Geoffrey F. Chew for many discussions, suggestions, and encouragement.

Table I. Parameters of the Regge poles $\alpha = \alpha(q^2)$ near threshold $q^2 = 0$. Above threshold the imaginary part of α behaves as $\alpha_I = (d\alpha_I/dq)_0\, q$, and the real part as $\alpha_R = \alpha_R(0) + \frac{1}{2}(d^2\alpha_R/dq^2)_0 q^2$. The residue of the pole is essentially given by $\beta(0)$, and $\alpha(s = 0)$ will determine the power of the total cross section in the crossed channels.

Pole	$\alpha_R(q = 0)$	$(d\alpha_I/dq)_0$ (m_π^{-1})	$(d^2\alpha_R/dq^2)_0$ (m_π^{-2})	$\beta(0)$ (m_π^{-1})	$\alpha(s = 0)$
π-π, $I = 0$ [a]	-1/16	1/8	-1/32	$-\pi/4$	-3/16
n-p, $^1S_0$ [b]	-2.5×10^{-2}	0.42	-0.39	-2.57	--
n-p, $^3S_1$ [b]	6×10^{-2}	0.36	-0.18	-2.27	--

a. Evaluated from Eq. (7) on the basis of a scattering length $a \approx 2m_\pi^{-1}$ (see reference 2) and an assumed range $r_0 \approx r \approx 1/2\ m_\pi^{-1}$. Here $\alpha(s = 0)$ is evaluated from Eq. (10), $s = 4(q^2 + 1)$.

b. Evaluated from range and scattering parameters fitted by a square-well potential (see reference 3). For the triplet state, the approximate expression for the position of the trajectory at the threshold is $\alpha_R(q = 0) \approx (1/3)\, E_B^{1/2} r_0$, where E_B is the binding energy and r_0 the range of the forces (see reference 7). The calculations for this case are nonrelativistic.

FOOTNOTES AND REFERENCES

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† On leave from Syracuse University, Syracuse, New York.

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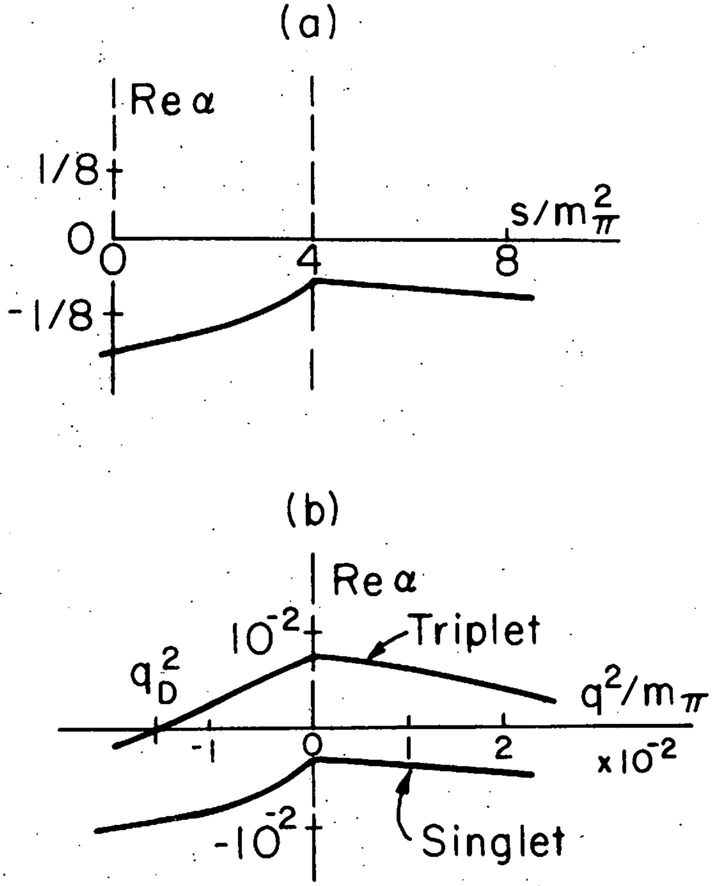
7. The deuteron case shows that, in the analytic continuation of Eq. (3) below threshold, we have to take the negative sign of the square root.
8. If exchange potential is assumed, only alternate values of ℓ give physical resonances.

FIGURE LEGEND

Fig. 1. Trajectory of the Regge poles near threshold. The real part of α is plotted against (a) $s = 4(q^2 + 1)$ in the case of $I = 0$, for a $\pi+\pi$ virtual particle, and (b) q^2 in the case of n-p singlet and triplet poles.


(a)
Re α
1/8
0
0
4
8
s/m$_{\pi}^{2}$
-1/8
(b)
Re α
10^{-2}
Triplet
q_D^2
q^2/m_π
-1
0
1
2
x10^{-2}
-10^{-2}
Singlet

100

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