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Institute of Urban & Regional Development  
University of California, Berkeley

Reprint No.80

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## SOCIAL INDICATORS AND THE LIFE CYCLE

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### INTRODUCTION

SOCIAL indicators are measures that are useful for social action toward achieving societal goals. One of these *social* goals that is often articulated is the full development of *individuals*; another is the equality of individual development among the members of our society. I want to propose a set of individually oriented measures based on the life cycle view of development that may be useful for social policy formulation.

A life cycle is a way of viewing human development as a sequence of stages. The individual passes from one stage to another as he matures. The most eloquent statement of this process is to be found in the work of Erikson; I follow his ideas in much of what follows [1, 2].

The developmental process described by Erikson is called *epigenetic* since an individual must have passed through all the previous stages to progress to a new stage. All is not determined by the very first set of experiences (in contrast to Freud's *genetic* principle). Each stage determines how a person reacts to the next, yet much of what he experiences in any single transition is determined by the environment at that time. Erikson suggests that the transition between stages is accompanied by a crisis in which the individual works out his new self. His proposed sequence is shown in Table 1.

To connect these considerations with social policy, I assume that the social system should and can provide the means by which a person progresses along the life cycle path. Even the most intimate aspects of development are socially related. In the Kibbutz, the process of weaning is an overtly social decision. In American society, the process is less overt but still social (witness Dr. Spock).

As a basis for these indicators we shall want to measure the transition rates (the probabilities for making the transitions between stages), the population in each stage and the societal costs of aiding people through the life cycle sequence. Some will pursue deviant paths, and we shall also want to look at the costs of drop-outs. Lastly, we might try to get a handle on the benefits to the society of those who succeed in reaching a certain developmental level. These measures may need to be combined with others to develop usable indicators.

### PREVIOUS WORK ON LIFE CYCLE MEASURES

As presented here the life cycle is bound up with psychoanalytic theory. However, variants of this concept are quite useful for other social studies and can be divorced, in part, from depth psychology. Economists look at the savings and accumulation of capital as processes occurring within the life cycle of individuals and families. The human capital

approach often uses a concept of a life cycle in its formulation. Moss, for example, discusses income, consumption and wealth patterns as a function of life cycle development [3-5].

The approach offered by those who are concerned with human resources seems more in the line with problems explored here [6, 7]. Cohort analysis of a working life gets at data needed to determine the transition rates through the life cycle stages.

TABLE 1\*

| Stage<br>conflicts-product                       |   | Population<br>of this stage                 | Measures of<br>failure                                                    | Policy problems                           |
|--------------------------------------------------|---|---------------------------------------------|---------------------------------------------------------------------------|-------------------------------------------|
| Infancy<br>Basic-trust<br>Mistrust               | } | Births                                      | Autism<br>motor skills                                                    | Pre-natal<br>care of<br>mothers           |
| Early childhood<br>Autonomy<br>Shame and doubt   |   | 1 yr olds                                   | Creeping<br>Abandoned children                                            | Day care                                  |
| Play age<br>Initiative<br>Guilt                  | } | 3 yr olds<br>Nursery school<br>population   | Inability to speak                                                        | Nursery schools                           |
| School age<br>Industry<br>Inferiority            |   | Elementary school<br>population             | Imagination failure<br>Learning failures                                  | Schools                                   |
| Adolescence<br>Identity<br>Identity<br>confusion | } | High school seniors<br>Military inductees   | Staticness of a<br>society                                                | Manipulable<br>Environments<br>Open space |
| Young adulthood<br>Intimacy<br>Isolation         |   | Marriages<br>Under 25 drivers               | Early divorce rate                                                        | Home formation<br>housing                 |
| Maturity<br>Generativity<br>Stagnation           | } | Parents                                     | Illegitimacy, quality<br>of career, sexual<br>impotence,<br>abortion rate | Work structure                            |
| Old Age<br>Integrity<br>Despair                  |   | Medicare and<br>social security<br>retirees | Hospital admissions<br>Isolated old<br>Handicapped                        | Leisure<br>Health care                    |

\* The above set of stages is taken from Erikson [1, 2]. An interesting discussion of the manifestations of these stages as well as other formulations is to be found in a series of articles in [20]. See also, Graves [16].

Otis Duncan has proposed that we use a socio-economic life cycle to get a handle on measuring discrimination against Negroes [8]. The life cycle he proposes is concerned with family background, schooling, jobs, income and expenditures. His work is complementary to what is presented here in that he gives an extensive discussion of problems related to the statistical resources needed to produce such indicators. I have tried to introduce psychological variables more explicitly and give one formulation for how the various data about life-cycle transitions could be put together.

#### SOME EFFECTS OF A LIFE CYCLE APPROACH TO POLICY

A life cycle indicator orients the evaluation of social policy and the social quality of life towards the individual and his development. Conventional indicators are programmatic in orientation. They look at housing *or* education *or* health services. A life cycle indicator cuts across these programmatic classifications. It can show the integrated effects of programs on individuals, suggest tradeoffs among programs and point up failings in the collective impact of a set of social programs.

An orientation toward a life cycle forces new judgments of value. We are led to ask questions such as: How do we want a social program to increase life cycle longevity? Should social programs produce as many people as possible who mature and not worry too much about those who do not? Or, do we want people to get through adolescence, at least, by means of our social programs?

This orientation may lead to a reformulation of social policy questions. For example, the emphasis on the crises associated with life cycle transitions suggests that helpers operating during those crises would be of use. Some of these helpers could be professionals, but it is more likely that amateurs will be needed to fulfill the demand. Friends function in this role and we might want to consider a societal policy to foster friendship.

#### ALTERNATIVE LIFE CYCLES

There exist a multiplicity of developmental life cycle models. The set of stages that should be used is not simply decidable. Ignoring for the moment the "scientific" basis for a set of stages (from developmental psychology or psychiatry) and the possibility that a single set of stages will turn out to be best, we, at this time, must choose a set on different grounds. I think that appropriate criteria may lie in how well a set points up problems that we may be aware of intuitively, how well a set can measure the changes that current social programs induce and how well a set points up desired values of a society and the social planner. This will mean that any set will be culturally biased. This seems necessary in that life cycle indicators are concerned with a culture.

As social pathways change, a life cycle indicator set must be responsive if it is to measure modal or normal development. If life span changes or the number of careers in a life increases, as both are, then the life cycle model needs to reflect these [10].\*

For a social system, individual functioning cannot be the only value. The conflict lies in the fact that social systemic values of maintenance, efficiency and distributive equity are not well reflected in individual measures. This need not be a problem. It only means that life cycle measures are one of a host of indicators.

#### A LIFE CYCLE INDICATOR SET

Let us look more closely at the life cycle as discussed by Erikson as a preliminary to developing some indicators. In Table 1 the eight developmental stages are listed (with the conflicts involved in each stage and the resulting gain in "self" in resolving these conflicts), possible measures of the population of each stage, possible measures of the failure rate in getting through each stage and policies that might be more related to one stage than another.

Erikson's developmental process does not include enough stages for adulthood. Maslow provides a set of more differentiated stages for indicating the quality of adult development in his treatment of *self-actualization* and these could be used in a more complex model [11]. Another more articulated set of developmental stages for adults is related in [12].

The following procedure is a simple one for using life cycle data in indicators.

Given a population in a steady-state condition, let us look at life histories. For the moment, *assume* that all people live to old age and then die; this assumption can be relaxed. Now say that there are eight stages in the life cycle process, and the probability of going successfully from stage  $i$  to stage  $i + 1$  is called  $p_{i+1}$ . Those who do not make the transition

\* A life cycle orientation may be useful in future studies. It may serve as a predictor by centering the attention of the forecaster on changes in how people spend their time. It will also be useful as an auditing device to check that the institutional changes that are predicted make sense when played out on a human life.

successfully are said to have "dropped out". A person "remains" at the stage in which he dropped out or he may evolve on a "deviant" path. The fraction of people who will successfully get to a certain level  $k$  is  $p_1 \times p_2 \times \dots \times p_k$ ; the rest will have dropped out at various points.\*

Now let me suggest some indicators based on these statistics:

1. *Whole life indicator* equals  $p_1 \times p_2 \times p_3 \times \dots \times p_8$ , which is the fraction of people who have "full" lives.
2. The plot of  $P_i = p_1 \times p_2 \times \dots \times p_i$

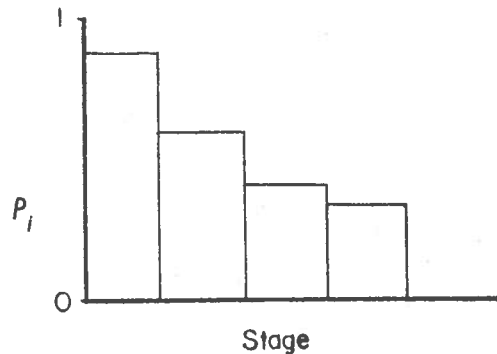


FIG. 1.

is a measure of *mortality along the way*.

3. Let  $C_i$  be the annual *cost of the society* if someone drops out at stage  $i$ . Then

$$D = \sum_i C_i [P_{i-1} - P_i] N_0$$

(size of the population =  $N_0$  and  $P_0 = 1$ ) is the total annual cost of the society of all the dropouts. This is program dependent. The number of dropouts is:

$$N = \sum_i [P_{i-1} - P_i] N_0 = [P_0 - P_8] N_0 = (1 - P_8) N_0$$

Let us apply this model to currently available data about blacks and whites in the United States.

For each stage, I have made an estimate of the transition probabilities,  $p_i$  (Table 2). Note that, except for infant mortality, all the transition rates assume that the population has zero morbidity and zero mortality. It may be that those who survive physiologically are psychodevelopmentally different from those who do not—a burden that this rough calculation will have to bear.

\* If we view (1) the series of steps in the life cycle process as independent ones and (2) the transition from each step to the next step as a fairly random process and (3) if we include the possibility of making a transition from failure to success in the next step — then the life cycle transition network becomes a complex Markov process (see Spiegelman [13]). However, I think such an "unpruned tree" formulation of the life cycle process is unnecessarily complex.

I have assumed, for simplicity in a first model, that the probability of making a transition from outside the system (from being a drop out) to success in the system is small. There is, therefore, a unique positive development process. This does not mean that each individual develops in identical ways, but that there are universal characteristics of the developmental process.

TABLE 2.

| Stage             | Indicator                                                                                | $p_i^*$ | $p_i$              | $P_i$ | $P_i$              |
|-------------------|------------------------------------------------------------------------------------------|---------|--------------------|-------|--------------------|
|                   |                                                                                          | White   | Black or non-white | White | Black or non-white |
| Birth             | Number of babies surviving first year†                                                   | 0.97    | 0.94               | 0.97  | 0.94               |
| 1 yr†             | ?                                                                                        | 1.0     | 1.0                | 0.97  | 0.94               |
| 3 yr†             | ?                                                                                        | 1.0     | 1.0                | 0.97  | 0.94               |
| Elementary school | School retention rates till grade eight                                                  | 0.97    | 0.97               | 0.94  | 0.91               |
| High school       | Probability of graduating high school if entered§                                        | 0.75    | 0.58               | 0.70  | 0.53               |
| Marriage          | Fraction of women ever married**                                                         | 0.94    | 0.93               | 0.66  | 0.49               |
| Parents           | Number of women ever having children††                                                   | 0.86    | 0.86               | 0.57  | 0.42               |
| Work              | Fraction of workers with discretionary jobs—defined as professional, managerial, craft‡‡ | 0.39    | 0.19               | 0.22  | 0.08               |

\* Note that if numbers for white and non-white are same, the data did not distinguish race.

† For lack of data, or a good indicator, I have set these equal to 1.0

‡ Table 70, *Statistical Abstracts*. Sum of infant and fetal deaths.

|| Table 178, *Statistical Abstracts*.

¶ Another useful figure is that white draft acceptees are 61 per cent, while black acceptees are 50 per cent of those who report for pre-induction examination. (*Social and Economic Status of Negroes in the U.S.*, 1969; BLS No. 375; CPR, P-23, No.29, GPO, p. 84).

§ Table 9, *Digest of Educational Statistics*, 1969. Number of people aged 25-29 yr with four or more years of high school.

\*\* Table S-3, *HEW Trends*, 1966-7. Average over ages 35-49, women ever married.

†† Table 63, *Statistical Abstracts*.

‡‡ Table 323, *Statistical Abstracts*.

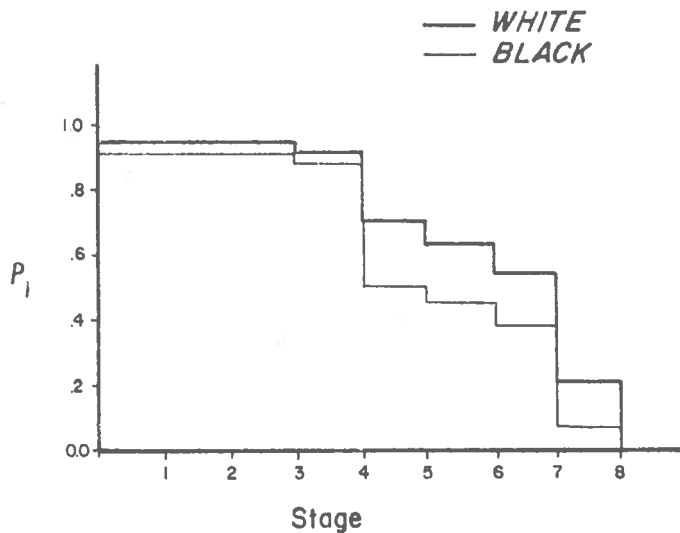


FIG. 2.

In the above graph of the  $P_i$ 's note how the difference between whites and non-whites grows as the stages of development go on.  $P_8$  for whites is 0.22; for blacks it is 0.08. From the  $p_i$ 's in Table 2 it would seem that extra effort should be put into programs that would

decrease infant mortality, would increase the number of non-whites successfully getting through adolescence and increase the quality of the work-life available to all.\*

#### PROGRAM EVALUATION

A life cycle orientation is useful for program evaluation and selection. To implement this approach we shall need a decision rule, and the following one seems reasonable:

We want to be sure that if a program marginally decreases the number of dropouts at one stage, another program will make sure that this group of extra survivals will not drop out in the next stage.

We now apply this rule to decide on the relative allocation of funds among two programs each benefiting adjacent stages. Say that it costs  $\$n_j$  to increase the number of survivors at stage  $j$  by 1, that  $T$  is the total number of dollars available,  $E_j$  is the increase number of survivors at stage  $j$  and  $D_j$  is the amount of money spent in stage  $j$ .

Then:

$$T = D_j + D_{j+1}$$

and

$$E_j = E_{j+1}$$

will determine how the monies should be divided where  $E_j = D_j/n_j$ .

We may formulate these considerations more generally.

Say that we have fixed funds for social programs and that each dollar will be equally effective in changing any one of the  $p_i$ 's. Let us say that the cost of changing the  $p_i$ 's is independent of its value.

Say

$$\frac{d \text{ cost}}{d p_i} = c_i \quad (1)$$

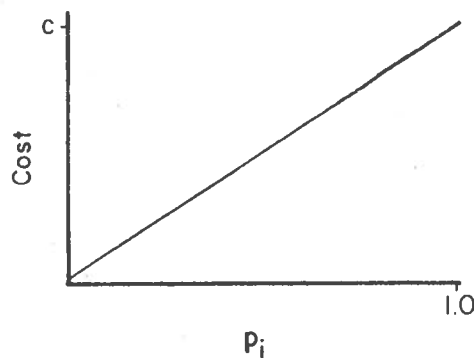


FIG. 3.

\* Note that the improvement in indicative conditions should not be confused with certain improvement in the state represented by an indicator. No indicator is perfect. For example, it may be better to give adolescents some guaranteed income rather than force them to stay in high school more. "Better", when used here, is meant in the total developmental sense for a person. We need to combine many proxies for measures of development to get a variety of estimates of the  $p_i$ .

The effects of programs on the  $p_i$ 's must be carefully evaluated. If one groups the population into one class, then dramatic changes in a subsector of the population will be hidden. This is because the  $p_i$ 's can only have values between zero and one. For example, if the 10 per cent of the population that is black had a deterioration in a transition rate from 0.8 to 0.4, the effect on the total population  $p_i$  would be 0.04 and might not be noticed. Also, the  $p_i$ 's will suffer from substantial uncertainties due to uncertainties in the  $p_i$ 's. A 10 per cent error in each of the eight  $p_i$ 's result in a 30 per cent error in  $P_8$ .

Now

$$P_8 = \prod_{i=1}^8 (p_i + d_i) \quad (2)$$

where  $p_i$  is the current value of the  $p_i$ 's.  $d_i$ 's are the current changes due to expenditure of monies,  $c_i d_i$ , which total

$$T = \sum_i c_i d_i \quad (3)$$

We want to maximize  $P_8$  subject to a constraint on the total monies,  $T$ .<sup>\*</sup> It is straightforward to show that if the cost of changing a  $p_i$  is independent of the stage ( $c_i$  are all equal), then  $P_8$  is largest when all the new transition rates are equal ( $p_i + d_i$  are all equal). This will mean that some of the resources from some programs will be transferred to others (some of the  $d_i$  are positive and others are negative), as well as some of the additional resources will be spread among the various stages (due to  $T$ ). Finally, it turns out that if the amount of resources for addition are small ( $T/c \ll 1$ ), then most of the change in the value of  $P_8$  is due to redistribution. *It may be more effective in total results (though politically and bureaucratically impossible) to redistribute resources, rather than add small amounts to the total resource mix pool.*

Say, however, that one cannot redistribute resources and can only use new resources to aid programs and affect the  $p_i$ . Then it is most effective to put resources in the situations which have the smallest  $p_i$ . This is not difficult to see. Say all the  $p_i$  except one were equal to 0.5 and the eighth one equaled 0.1 ( $P_8 = 7 \times 10^{-4}$ ). Say that enough funds were available to change any  $p_i$  by 0.1. If we put our resources in  $p_8$  the change would be a doubling in  $P_8$ , while if it were invested in any single one of the others the change would be only 20 per cent. *Resources should be distributed in such a way that comparable results are available in various life cycle stages. Also, it does little good (in a global sense) to improve a program for one stage if other stages are in much worse circumstances.*

These observations about expenditures may be applied to policy concerning medical expenditures (and presumably, but not surely, health). Illustrated below is the annual medical expenditure per person for various age groups in the United States (Table S-421 *HEW Trends*) [18], Fig. 4.

We note that medical expenditures rise quite rapidly with age. It is also true that there are fewer older people in the society than younger ones. Say we took a small amount of money away from older people and invested it in young children. Because there are more young children, this amount of money would have to be spread around a bit and the amount of extra money each child received would be smaller than the amount of money per adult we took away. Still, the percentage change in increase in expenditures on young children could be larger than the percentage change in the decrease in expenditures on older people. Let us see how this would work out.

There are, in the U.S., 18 million children between age 0 and 5, and 12 million adults of age 50–55. They spend \$60/yr and \$175/yr, respectively for medical care. If the older people spent 1 per cent less for care, \$1.75/person per year, then 12 millions  $\times$  \$1.75 would be available for 18 million children, which comes to \$1.15/child per year. This is a 2 per cent

\* See Appendix I for demonstrations of the following assertions.

improvement in child care expenditures. *It would be entirely cost-effective in a life cycle sense if a 2 per cent increase in child expenditures resulted in a 1 per cent decrease in need for medical care at the 50-55 yr stage in life.* Of course, those who are 50-55 now might not want to give up their health (actually medical expenditures) for future generations.\*

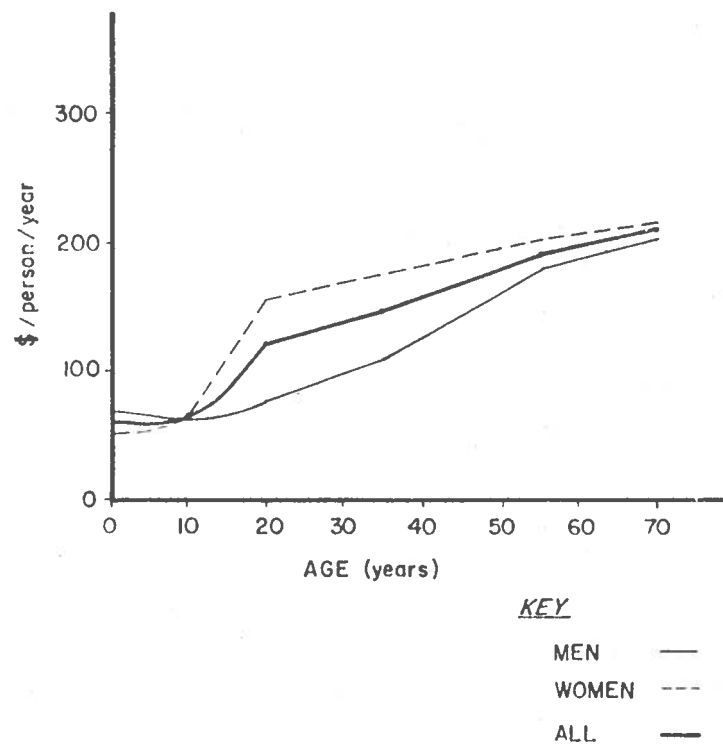


FIG. 4.

Returning to our model, we note some exceptions:

1. We know that the shape of the cost function is not so simple. There may be economies of scale. Also, it is unlikely that any reasonable expenditure will get any  $p_i=1.0$ .

\* More formally, let  $N_i$  be the number of persons at stage  $i$  and  $z_i$  be their dollar expenditures. If the ratio:

$$\frac{N_i z_i}{N_j z_j} \quad [i > j; \text{improving your health when you are older does not help your youth (in a social sense this is not true—e.g. improved health of mothers probably improves infant health)}]$$

is greater than the inverse ratio of the percentage changes in the costs, then an investment in a younger generation is life cycle cost effective in the long run. I have not discussed appropriate discounting questions here.

Thus the cost function would look more like:

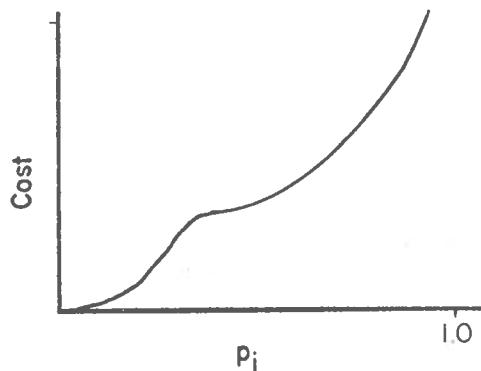


FIG. 5.

The effect of this would be to make the optimal  $p_i$ 's unequal since the slope of the cost function is not constant.

2. In a poorer society that needed some very highly trained people, a developing country for example, one might want to have the maximization criterion on  $P_8$  supplemented by a criterion of a minimum number of people being sufficiently trained. Somewhat different policy recommendations would result.
3. See Appendix 2 for discussion of a more comprehensive approach.

#### CONCLUSION

The life cycle provides a natural orientation for the examination of the total impact of public programs on individuals. As I have suggested, it also forces comparisons of benefit among different clients of public services and suggests how consequent choices might be made. Much of social statistics now gathered could be reorganized in life cycle terms rather than the programmatic form currently used. This should encourage cross-program analysis and thinking.

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## APPENDIX 1

We want to maximize (2) subject to the constraint (3)

$$\frac{d \text{ cost}}{d p_i} = c_i \quad (1)$$

$$P_n = \prod_{i=1}^n (p_i + d_i) \quad (2)$$

$$T = \sum_i c_i d_i \quad (3)$$

We may solve for  $d_n$  in equation (3), substitute in (2) and differentiate (2). It is neater to use the technique of Lagrange multipliers.

$$\Delta P_n + \lambda \Delta T = 0$$

Therefore we get  $\prod_{\substack{i=1 \\ i \neq j}}^n (p_i + d_i) + \lambda c_j = 0 \quad j = 1, \dots, n$

and the total cost constraints.

If we set all the  $c_j$  to be equal, say to  $c$ , then the equations are symmetric in the  $p_i + d_i$ . This implies that all the  $p_i + d_i$  are equal.

The problem is also equivalent to maximizing the volume of an  $n$ -dimensional rectangle, given a fixed sum for the edges. It is well known that the solution is an  $n$ -dimensional cube.

We may now calculate the change in  $p_n$  as a result of maximization. Since all of the  $p_i + d_i$  are equal, their mean is equal to any one of them.

$$\begin{aligned} p' &= \text{the mean value of the } p_i + d_i = \frac{\sum_{i=1}^n p_i + d_i}{n} \\ &= \frac{\sum p_i}{n} + \frac{\sum d_i}{n} = \frac{\sum p_i}{n} + \frac{T}{cn} \end{aligned}$$

Let  $\frac{T}{c} = \varepsilon$ , and  $P'_n$  be the maximum value of  $P_n$ , then

$$\begin{aligned}
 P'_n &= \prod_{i=1}^n (p_i + d_i) = p'^n = \left( \frac{\sum p_i + \varepsilon}{n} \right)^n \\
 &= \frac{1}{n^n} (\sum p_i + \varepsilon)^n \\
 &= \frac{1}{n^n} \left[ (\sum p_i)^n + \varepsilon \cdot \frac{\partial}{\partial \varepsilon} \Big|_0 (\sum p_i + \varepsilon)^n + 0(\varepsilon^2) \right],
 \end{aligned}$$

expanding in a Taylor series in  $\varepsilon$ .

$$\begin{aligned}
 &\cong \frac{1}{n^n} \left[ (\sum p_i)^n + n\varepsilon (\sum p_i)^{n-1} \right] \\
 &= \frac{1 + \varepsilon n (\sum p_i)}{n^n} (\sum p_i)^n
 \end{aligned}$$

The maximized value of the product is the sum of two parts, one of which is due to a redistribution of resources,  $\frac{1}{n^n} (\sum p_i)$  and the other due to the additional resources and redistribution,  $\frac{\varepsilon n}{n^n} (\sum p_i)^{n-1}$ . If  $\frac{Tn}{c} \ll 1$  ( $\varepsilon \ll 1$ ), then improvements in  $P'_n$  will be mostly a result of redistribution.

Redistribution is often difficult, while new unallocated resources can sometimes be spent with greater discretion. Still, there are some situations where no redistribution, but discretionary spending can make a substantial difference. One of these situations is the following:

Say that most of the transition rates are about the same except for one,  $p_n$ , which is somewhat smaller than the rest.

Let  $p_1 \approx \dots \approx p_{n-1} \approx p$  and  $p_n = \delta p$ , where  $\delta \ll 1$ .

$$P \approx p_1 \times \dots \times p_n = p^{n-1} p_n = p^n \delta$$

$$\begin{aligned}
 P' &= \frac{1 + \frac{\varepsilon n}{\sum p_i}}{n^n} (\sum p_i)^n \\
 &= \frac{1}{n^n} \left[ 1 + \frac{\varepsilon n}{(n-1+\delta)p} \right] [(n-1+\delta)p]^n \\
 &= \frac{p^n}{n^n} \left[ 1 + \frac{\varepsilon n}{(n-1+\delta)p} \right] (n-1+\delta)^n \\
 &\cong \frac{p^n}{n^n} \left[ 1 + \frac{\varepsilon n}{p(n-1)} - \frac{\varepsilon \delta n}{p(n-1)^2} \right] [(n-1)^n + \delta n(n-1)^{n-1}]
 \end{aligned}$$

expanding in Taylor series in  $\delta$ .

$$= p^n \left[ 1 + \frac{\varepsilon n}{p(n-1)} - \frac{\varepsilon \delta n}{p(n-1)^2} \right] \left[ \left( \frac{n-1}{n} \right)^n + \delta \left( \frac{n-1}{n} \right)^{n-1} \right]$$

Note that  $\lim_{n \rightarrow \infty} \left( \frac{n-1}{n} \right)^n = \frac{1}{e} = \frac{1}{2.71828\dots}$

$$\begin{aligned} &\cong p^n \left[ 1 + \frac{\varepsilon n}{p(n-1)} - \frac{\varepsilon \delta n}{p(n-1)^2} \right] \left[ \frac{1}{e} + \frac{\delta}{e} \frac{n}{n-1} \right] \\ &\cong \frac{p^n}{e} \left\{ 1 + \frac{\varepsilon n}{p(n-1)} + \delta \left[ \frac{n}{n-1} + \frac{\varepsilon}{p} \left( \frac{n}{n-1} \right)^2 - \frac{\varepsilon}{p} \frac{n}{(n-1)^2} \right] \right\} \end{aligned}$$

ignoring terms in  $\delta^2$ .

Let us say that  $\frac{n}{n-1} \approx 1$  and correspondingly  $n \gg 1$ . Then:

$$\begin{aligned} &\cong \frac{p^n}{e} \left\{ 1 + \frac{\varepsilon}{p} + \delta \left[ 1 + \frac{\varepsilon}{p} \right] \right\} \\ &= \frac{p^n}{e} \left( 1 + \frac{\varepsilon}{p} \right) (1 + \delta) \\ P' &\cong \frac{P}{\delta e} \left[ 1 + \frac{\varepsilon}{p} \right] (1 + \delta) \\ \frac{P'}{P} &\cong \frac{1}{\delta e} (1 + \delta) \left( 1 + \frac{\varepsilon}{p} \right) \end{aligned}$$

The  $1/\delta e$  factor represents the effects of redistribution, while the term in  $\varepsilon$  represents having more monies to spend.

If we do not redistribute, but put all of our resources into the poorest case, then:

$$P_{n0} = p^{n-1}(p_n + \varepsilon) = p^{n-1}(\delta p + \varepsilon) = P \left( 1 + \frac{\varepsilon}{\delta p} \right)$$

The ratio of the best situation ( $P'$ ) to the simple no redistribution case is:

$$\frac{P'}{P_{n0}} = \frac{(1 + \delta) (1 + \varepsilon/p)}{\delta e (1 + \varepsilon/\delta p)}$$

To get a feeling for this ratio, let us try a representative example: set  $p = 0.5$ ,  $\delta = 0.1$ ,  $\varepsilon = 0.2$  and  $n = 8$ , then:

$$\frac{P'}{P_{n0}} = 1.1 \quad \frac{P_{n0}}{P} = 5$$

We do not do too badly if we do not redistribute.

## APPENDIX 2

How would we deal with both systemic (programmatic) problems and individual (life-cycle) concerns in a coordinated fashion? The matrix below suggests how we might do so:

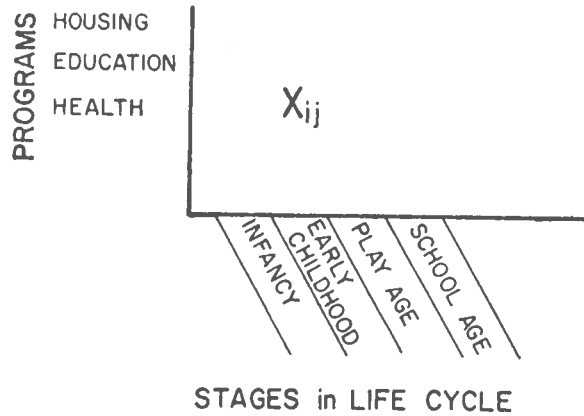


FIG. 6.

We can look at how a program distributes its resources (dollars, people, facilities) among the life cycle stages. We could also try to assess how much each program contributes to the possibility for an individual getting through a certain stage (i.e. the change in a  $p_i$ ).\*

A formal apparatus is easily set-up to deal with this schema.

For the first, let  $X_{ij}$  be contribution of the  $i$ th program to the  $j$ th life cycle step. Say we evaluate programs in terms of how much they contribute to the probability of success in the life cycle process, then  $X_{ij}$  is the contribution to  $p_j$ .

We could maximize the  $p_j = \sum_i X_{ij}$  where the  $X_{ij}$ 's are program dependent and where the cost of a program could be  $D_i = S_i \sum_j X_{ij}$  (linear marginal cost—a bad assumption).  $S_i$  is a scaling factor. We minimize the cost  $T = \sum_i D_i$ .†

\* We might want to try to include as mediating variables some emotional values such as the degree of self-actualization, diversity, surprise and creativity. Appleyard [14] in discussing open space, suggests that skills be classified by their physical, manipulative, sensory, cognitive and social nature.

† See Bergmann [15] for another version of the impact study.

