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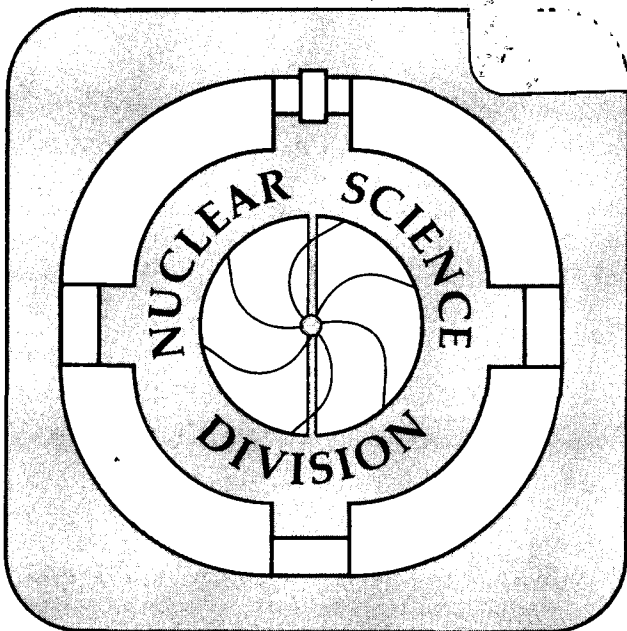April 13-14, 1986; and to be published
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PION INTERFEROMETRY OF ULTRA-RELATIVISTIC HADRONIC COLLISIONS

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Pion interferometry of ultra-relativistic hadronic collisions is described in the context of the inside-outside cascade model using a current ensemble method capable of describing an arbitrary distribution of pion sources with an arbitrary velocity distribution. The results are quite distinct from the usual Gaussian and Kopylov parameterizations. Extraction of the temperature parameter, effective source lifetime, and transverse size requires a full three-dimensional analysis of the correlation function in terms of the momentum difference.

1. INTRODUCTION

Correlations in momentum space between pairs of identical particles have long been used to provide information about the space-time structure of the source of the particles[1,2]. The correlation function is defined as

$$C(k_1, k_2) = \frac{P_2(k_1, k_2)}{P_1(k_1) P_1(k_2)} \tag{1}$$

where $P_1(k)$ and $P_2(k_1, k_2)$ are the single particle and two-particle inclusive distributions, respectively. If one takes the two particle wave function to be a Bose symmetrized (or Fermi antisymmetrized) plane wave state, one finds that $C(k_1, k_2)$ is related to the Fourier transform of the space-time density distribution of particle sources. Experimental results for two pion correlations in hadronic collisions are usually fit by expressions which assume that this density of sources is given by a gaussian in space and time or a uniformly radiating disc with exponential time dependence[3,4]. Neither of these parameterizations, however, contains the correlations between the dynamics and geometry appropriate for high energy hadronic collisions.

Such collisions are expected to be described by the inside-outside cascade. Due to Lorentz time dilation[5], the production of a particle with longitudinal rapidity y cannot be localized to within $\Delta z \sim \tau_0 \sinh(y)$, where $\tau_0 \sim 1$ fm/c. This means that the momentum and the production point are strongly correlated, in contrast to the situation at lower energy. It also results in Lorentz invariance under boosts along the beam (z) axis and a central plateau in rapidity distributions. In ref. 6, it was shown that such correlations could significantly alter the interference patterns. However, the Wigner function approximations used in that study lead to Lorentz noncovariant expressions. Therefore, we employ the general current ensemble formalism developed in ref. 4 to investigate this question. We extend this formalism to allow each source to be moving with an arbitrary velocity in an arbitrary direction. Details of this extension will be published elsewhere[7]. In particular our goal is to find out exactly how $\pi^- \pi^-$ correlation measurements can be used to verify the basic phenomenon of space-momentum correlations.

2. METHOD

We start by considering an ensemble of currents with Fourier transform

$$j(k) = \sum_{i=1}^{N} e^{i\phi_i} e^{ikx_i} j_0(y - y_i, k_T) \qquad , \qquad (2)$$

where y_i is the longitudinal rapidity of pion source i in a frame where $k^\mu = (m_T \cosh y, \vec{k}_T, m_T \sinh y)$, i.e., the laboratory frame. The longitudinal rapidity of the pion in this frame is y, and its transverse mass $m_T^2 = m^2 + k_T^2$ is invariant under boosts along the z axis. Source i, as characterized by a current $j_0(x)$, is centered initially at space-time point x_i. The Fourier transformed current $j_0(k)$ is interpreted as the amplitude for the source to emit a pion with momentum k in the lab frame. We further assume that the phases ϕ_i of the different sources are random[4]. The ensemble is specified by the probability $D(x_0, y_0)$ for finding a source with rapidity y_0 at x_0. The single pion inclusive distribution is then

$$P_1(k) = <|j(k)|^2> = <N> \int dy_0 D(q = 0, y_0) |j_0(y - y_0, k_T)|^2 \qquad , \qquad (3)$$

where $D(q, y_0)$ denotes the Fourier transform of $D(x, y_0)$ in x. Similarly, the two pion inclusive distribution is

$$P_2(k_1, k_2) = <|j(k_1)|^2 |j(k_2)|^2> \qquad . \qquad (4)$$

The correlation function is given by

$$C(k_1,k_2) = 1 + \frac{|G(k_1,k_2)|^2}{G(k_1,k_1)G(k_2,k_2)} \quad , \tag{5}$$

where

$$G(k_1,k_2) = \int dy_0 D(k_2 - k_1, y_0) J_0^*(y_1 - y_0, k_{1T}) J_0(y_2 - y_0, k_{2T}) \quad . \tag{6}$$

We characterize the inside-outside cascade model by

$$D(x,y_0) = D(\tau, \eta, y_0) = \frac{1}{\tau_0} \delta(\tau - \tau_0)\delta(\eta - y_0)\exp\left(-\frac{r^2}{r_T^2}\right) \quad , \tag{7}$$

where $\tau = (t^2 - z^2)^{1/2}$ is the proper time, $\eta = \frac{1}{2} \ln ((t+z)/(t-z))$ is the rapidity-like variable, y_0 is the source rapidity, and r_T is the radius of the production region in the transverse direction. The parameter τ_0 characterizes the proper time at which interactions between the particles cease. From Eq. (6) it is clear that G depends not only on D but also on the dynamical details described by J_0.

We consider a simple pseudothermal source function given by

$$J_0(y - y_0, k_T) = \exp\left(-\frac{m_T}{2T} \cosh(y - y_0)\right) = \exp\left(-\frac{E}{2T}\right) \quad , \tag{8}$$

where E is the pion energy in the rest frame of the pion emitter. This parameterizes the sources in terms of an effective temperature T. We also consider another model

$$J_0(y - y_0, k_T) = E^{1/2}\exp\left(-\frac{E}{2T'}\right) \quad , \tag{9}$$

which one expects to be characteristic of a source in thermal equilibrium. This thermal model results in nonanalytic expressions for C, and hence we present results here only for the pseudothermal model. We find, however, that the results for the two models agree closely if T' and T are related by a complex expression which we empirically can approximate by

$$T \approx 1.42T' - 12.7 \text{ MeV} \quad . \tag{10}$$

Results for the thermal source are presented in more detail in ref. 7.

3. RESULTS AND CONCLUSIONS

The invariant single inclusive pion distribution with the pseudothermal source is

$$P_1(k) = \frac{d^3n_\pi}{dyd^2k_T} = aK_0(m_T/T) \quad , \tag{11}$$

where a is a normalization constant. Thus T characterizes the average transverse momenta of the pions, which are uniformly distributed in rapidity. The function G is found to be

$$G(k_1,k_2) = aK_0(u)\exp\left(-\frac{q_T^2 r_T^2}{4}\right) \tag{12}$$

where

$$u = \left\{2m_{1T}m_{2T}\left(\frac{1}{4T^2} + \tau_0^2\right)\cosh \Delta y \right.$$

$$\left. + (m_{1T}^2 + m_{2T}^2)\left(\frac{1}{4T^2} - \tau_0^2\right) + \left(\frac{i\tau_0}{T}\right)(m_{1T}^2 - m_{2T}^2)\right\}^{1/2} \quad , \tag{13}$$

and $\vec{q}_T = \vec{k}_{2T} - \vec{k}_{1T}$. The rapidity difference is $\Delta y = y_2 - y_1$, and $K_0(z) \propto \int dt \exp(-z \cosh t)$ is the conventional modified Bessel function of a complex argument.

In this model the correlation function depends on three parameters, τ_0, T, and r_T, which we would like to extract. It is a function of four variables, the two transverse masses m_{1T}, m_{2T}, the rapidity difference Δy, and the angle ϕ between $\vec{k}_{1T}$ and $\vec{k}_{2T}$ (which enters through q_T). The structure of G is obviously rather different than conventional Gaussian or Kopylov parameterizations[3,4]. This means that a full three dimensional analysis of pion correlations is neccessary to disentangle T, τ_0, and r_T.

This procedure involves three steps. First, T is determined by fitting the single inclusive distribution P_1 as a function of m_T by Eq. (11), as shown in Figure 1. Next, we examine the correlation function for the case where $\vec{q}_T = 0$, thus eliminating the dependence on r_T. Since T is known from the first step, τ_0 can now be determined. In practice, this is done by examining either C as a function of Δy for fixed m_T ($m_{1T} = m_{2T}$ for this case) or C as a function of m_T for fixed Δy. These two cases are shown in Figures 2 and 3, respectively. Finally, r_T is determined by fitting the correlation function by Eqs. (12) and (13) for the general case where $\vec{q}_T$ is not zero. C can be examined as a function of any of the four variables, m_{1T}, m_{2T}, Δy, or ϕ, while the other three are held fixed. An example of C vs. ϕ for fixed m_{1T},
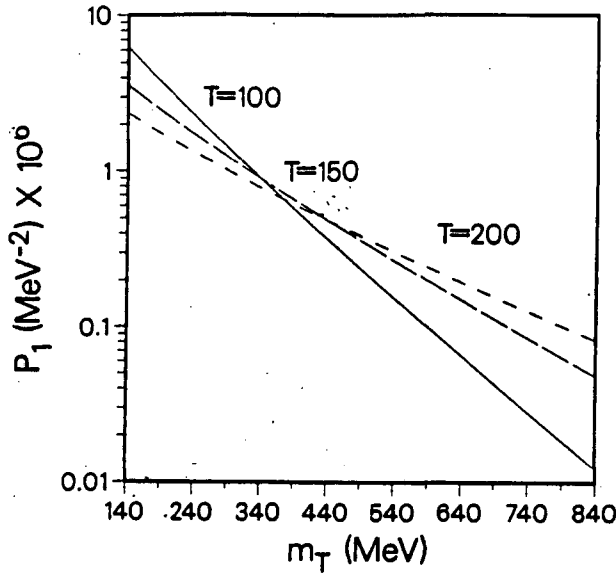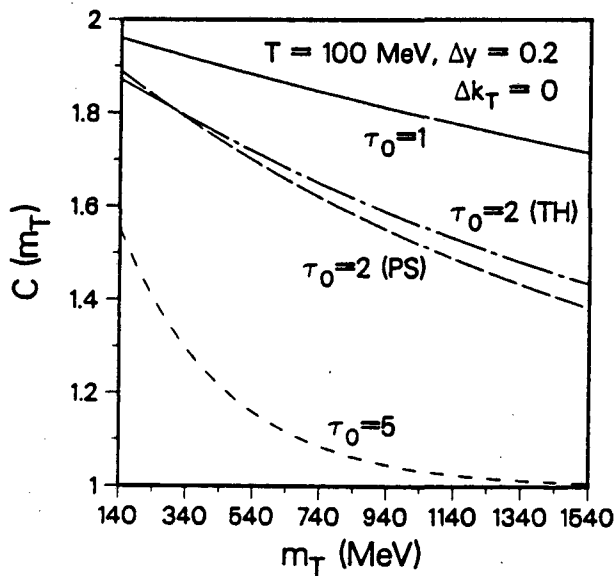
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FIGURE 1
Invariant single particle distribu-
tion P_1 vs. m_T for several values
of the temperature parameter T (in
MeV) in the pseudothermal model.
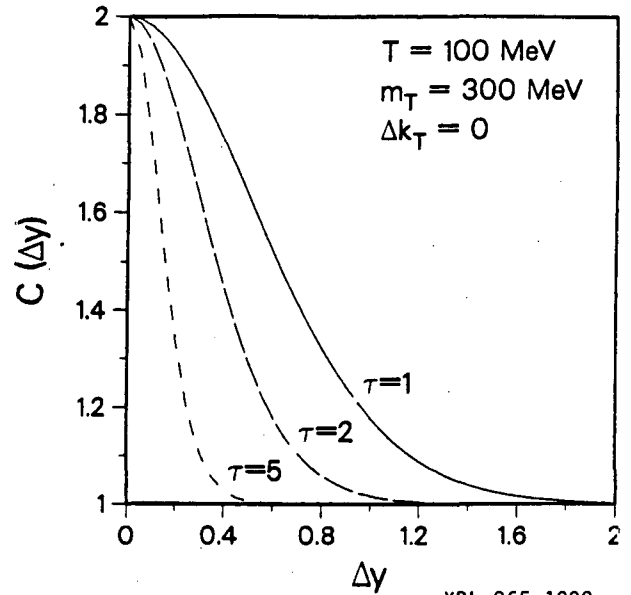
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FIGURE 2
Correlation function C vs. Δy for $q_T = 0$
and fixed m_T. Labels are values of the
lifetime τ_0 in fm. Pseudothermal and
thermal results are identical to within
the accuracy of the figure.

XBL 865-1989

FIGURE 3
C vs. m_T for $q_T = 0$ and fixed Δy.
Several values of τ_0 (in fm) are
shown. For $\tau_0 = 2$ fm, pseudo-
thermal and thermal results are
shown separately. Otherwise,
pseudothermal and thermal results
are identical to within the accuracy
of the figure.
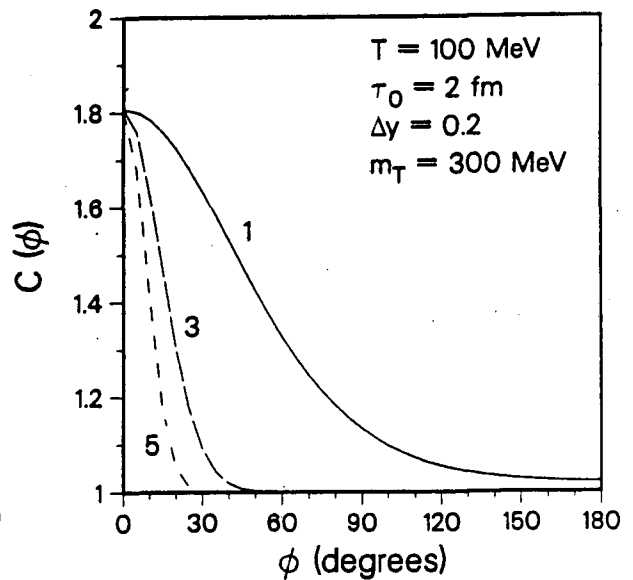
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FIGURE 4 ·
C vs. ϕ for fixed m_{1T}, m_{2T}, and Δy.
Labels indicate values of the transverse
radius r_T in fm. Pseudothermal and
thermal results are identical to within
the accuracy of the figure.

m_{2T}, and Δy is shown in Figure 4.

Since in the general case, the effects of T, τ_0, and r_T are all mixed together, this three step approach is necessary to extract meaningful parameters from correlation data. The usual procedure is to plot the correlation function as a function of one variable, for example $|\vec{q}|$, integrating over all other variables. We have verified that the source radius thus obtained is a nonlinear function of the three parameters in our model. The more complex analysis outlined above is required to see the inside-outside cascade nature of the dynamics.

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