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# Relativistically Upshifted Higher Harmonic Generation via Relativistic Flying Mirrors

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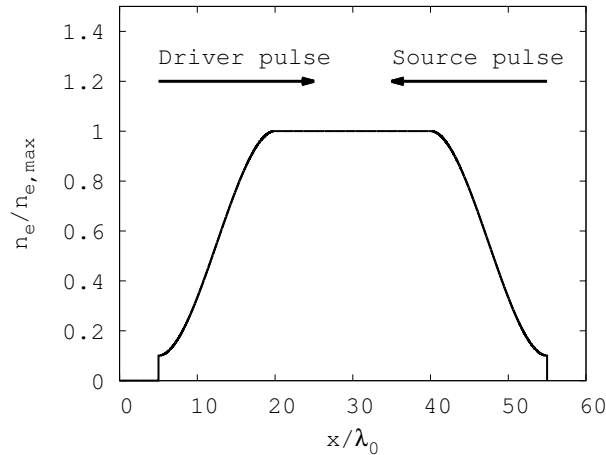
**Abstract.** We have previously shown that laser light can be upshifted to higher frequencies by its reflection off relativistically moving mirrors in plasma. These mirrors were generated with ultrahigh intensity laser pulses. However, the laser light which was reflected off the mirrors had relatively low intensity. We show via simulations that even high intensity light can be reflected off the mirrors and that this can generate relativistically upshifted harmonics.

*Keywords:* higher harmonics, relativistic mirrors, laser, plasma Submitted to: *Plasma*

## 1. Introduction

The rapid progress of ultra-short ultra-high intensity lasers has resulted in a plethora of new topics in the physical sciences requiring detailed study and analysis. One area of need is short wavelength pulses of intense light. Although there are many applications, one appealing aspect is their possible use in studying the fundamental physics of the vacuum. Recently, research into precise measurements in the X-ray regime has been carried out for photon-photon scattering using coherent XFEL light [1], in the  $\gamma$ -ray regime for Delbrück scattering [2] and vacuum breakdown[3]. There are many methods to generate higher harmonics which include such things as Thomson scattering[4, 5, 6], betatron radiation[7, 8], Burst intensification by singularity emitting radiation (BISER) [9] and harmonics from atoms via the recollision process[10, 11] (see also descriptions and references in [12, 13] and references cited in the introduction of [14]). In this paper we examine how to achieve such high intensity short wavelength with relativistic moving mirrors. The idea of the upshift in frequency of electromagnetic waves reflecting off relativistically moving mirrors was first elucidated by Einstein [15]. One way to achieve this upshift is via a laser pulse (source) counter-propagating to breaking plasma waves, which act as relativistically moving mirrors, generated behind an ultrashort high intensity laser (driver) propagating in underdense plasma (see review [16] and references cited therein). The realization of such an upshift with relativistic mirrors has been demonstrated theoretically[17], numerically[18], and experimentally[19, 20, 21, 22, 16]. In addition, relativistic mirrors with large wavelength differences between the driver and source pulses have been investigated with solids using lasers to generate photo-induced plasma fronts and reflect THz radiation both theoretically [23] and experimentally [24, 25]. Since the light reflected off of relativistic mirrors is also substantially shortened, such a method could be relevant for applications at upcoming laser based atto-second international facilities such as ELI-ALPS [26].

Up to now the laser pulse reflecting off the relativistic mirror has been chosen to be of sufficiently low intensity such that the reflected pulse energy is well below the energy stored in the wake wave so as to not perturb the mirror[27]. High intensity pulse reflection has been previously investigated in the case of the interaction of a relativistic mirror formed by a foil with an intense laser pulse [28]. We have shown previously in analytical estimates that sufficiently high intensity relativistically upshifted pulses could be obtained to measure photon-photon scattering with next generation lasers [29]. Here, we show by one dimensional particle-in-cell (PIC) simulations that the laser light can be reflected off the mirror at even relatively high laser source pulse amplitudes and that harmonics of the upshifted light can be obtained. Boosted high order harmonics have been previously seen in a different situation with a thin foil accelerated by radiation[30]. With relativistic mirrors in underdense plasma this may allow the measurement of photon-photon scattering with smaller laser systems. Estimates for relativistic mirror



**Figure 1.** Initial plasma density profile of the simulation. The driver laser enters from the left and the source laser enters from the right.

gamma factor needed to observe Schwinger pair production for the case of a solid density foil have been previously made[31]. This could also be used to estimate the gamma factors necessary for pair creation using relativistic flying mirrors in underdense plasma. We also perform the simulations in the large wavelength difference regime between the driver and source pulses. Large wavelength differences between the driver and source have been previously considered numerically [32].

## 2. Results

In order to examine the behavior of strong laser pulses reflecting off relativistic flying mirrors we performed 1D dimensional particle-in-cell simulations using the code EPOCH [33, 34]. Higher dimensional simulations will be needed to take into account realistic situations such as spatio-temporal couplings, which occur in ultra-high intensity laser pulses [35, 36], in this publication our goal is to consider whether or not ideal high intensity laser pulses can be reflected and generate harmonics with the caveat that the higher dimensional effects could possibly significantly alter the interaction, when taken into account. The simulation size was chosen to be  $60\mu m$  in length. The plasma had the initial profile shown in Fig. 1. The maximum density was  $n_{e,max} = 5 \times 10^{19} \text{ cm}^{-3}$ . The ramps on both sides of the layer were added to mitigate wave-breaking from sharp rising plasma edges, which was investigated for relativistically strong laser pulses propagating in under dense plasmas in the limit of one dimension[37]. The functions of the ramp were  $n_e(x)/n_{e,max} = n_0 + (n_1 - n_0)[3 - 2(x - x_0)/(x_1 - x_0)][(x - x_0)/(x_1 - x_0)]^2$  where  $n_0 = 0.1$ ,  $n_1 = 1$ ,  $(x_0, x_1)$  with  $(5\lambda_0, 20\lambda_0)$ ,  $(40\lambda_0, 55\lambda_0)$  for the left and right side, respectively. 10 macro-particles per cell were used. Although the feasibility of such a short density profile with such a sharp density ramp is difficult given current state of the art with sub-millimeter gas jets [38] and with longer propagation distances changes in the pulse

shape from phenomena such as pulse depletion and downshift of the laser pulses will occur [39, 40, 41, 42, 43, 44], for this study we have concentrated on understanding the basic mechanisms.

The simulation consisted of two counter-propagating laser pulses, driver and source pulses. The driver laser, which was used to generate the breaking plasma waves (relativistic flying mirrors), had a wavelength of  $\lambda_0 = 1\mu m$  and entered from the left of the simulation box propagating in the  $+x$  direction. The source laser had a wavelength of  $\lambda_s = 3\mu m$  and entered from the right propagating in the  $-x$  direction. The driver and source laser pulses were linearly polarized in the  $y$  and  $z$  directions, respectively, so that the reflected pulses could be clearly distinguished. In order to resolve the upshifted laser pulses  $\lambda_0/\Delta x = 9000$  where  $\Delta x$  is the simulation cell size. In the current study we have chosen both the driver and source pulse to be few cycle tightly focused laser pulses corresponding to the non-paraxial regime. This was mainly to keep the simulation regime small and to more clearly see the reflected source pulses. The results of the simulation should be extendable to longer duration and less sharply focused laser pulses with the appropriate adjustment of plasma parameters.

The intensity of the driver pulse was  $2 \times 10^{19}$  W/cm<sup>2</sup> corresponding to a normalized amplitude of  $a_d = 3.8$  where  $a_d \equiv eE_y/mc\omega_0$  having a Gaussian profile. Breaking of the plasma waves generated by the driver laser for  $a_d \gg 1$  is expected when  $a_d \geq (2^{1/4}\omega_0/\omega_{pe})^{2/3}$ , obtained from the equations of motion of a single electron interacting with an one dimensional laser pulse having a constant group velocity based on the laser amplitude and plasma density [45]. In our case the threshold value is 3.14. So the driver pulse has sufficient amplitude to generate wave breaking. The driver laser was given a pulse duration of  $5.7\mu m$  (FWHM), which based on the optimum length,  $l_{opt}$ , for wake field generation for  $a_d^2 \gg 1$  assuming a circularly polarized laser with constant amplitude over  $l_{opt}$  and zero elsewhere propagating in a cold plasma where the ions are at rest and a one dimensional wake wave [46]:

$$l_{opt} = 2|a_d| + \left( \frac{1}{2} + 2 \ln 2 - \ln |a_d| \right) \frac{1}{|a_d|} + O \left[ \frac{1}{|a_d|} \right]^3, \quad (1)$$

where  $l_{opt}$  is normalized by  $c/\omega_{pe}$  with  $\omega_{pe} = \sqrt{4\pi e^2 n_e/m_e}$ . With our current parameters this corresponds to  $l_{opt} \approx 7.74$  or  $5.8\mu m$  ( $c/\omega_{pe} = 0.75\mu m$ ). We consider that the driver pulse is sufficiently long and transversely smooth to produce a clear breaking wave, relativistic mirror. A detailed study of the effects of pulse shortness and transverse modulations is beyond the scope of the current paper.

The intensity of the source pulse was varied between  $1.5 \times 10^{12}$  W/cm<sup>2</sup> to  $1.5 \times 10^{19}$  W/cm<sup>2</sup> with the corresponding amplitudes ranging from  $3 \times 10^{-3}$  to 9.876. The source pulse had a Gaussian profile with a pulse duration of  $4\mu m$  (FWHM) in the propagation direction.

For these parameters the density of the plasma layer is near critical in the linear regime for the source laser pulse,  $n_{e,max}/n_{cr_s} = 0.4$  or  $\omega_s/\omega_{pe} = 1.57$  and very underdense for the driver pulse,  $n_{e,max}/n_{cr_0} = 0.044$  or  $\omega_0/\omega_{pe} = 4.7$ .

The expected upshift can be calculated from the laser group velocity for  $a_d \ll 1$  where the corresponding relativistic gamma factor is  $\gamma_{ph} \approx \omega_0/\omega_{pe}$ . Analytically in one dimension for a nonlinear plasma wave driven by a relativistic laser pulse for  $a_d \gg 1$  the phase velocity driven by a Gaussian laser of duration  $k_p L = 1$  has been shown to approach [47]:

$$\gamma_{ph} \approx 0.45 \frac{1}{\sqrt{N_p}} \frac{\omega_0}{\omega_{pe}} \quad (2)$$

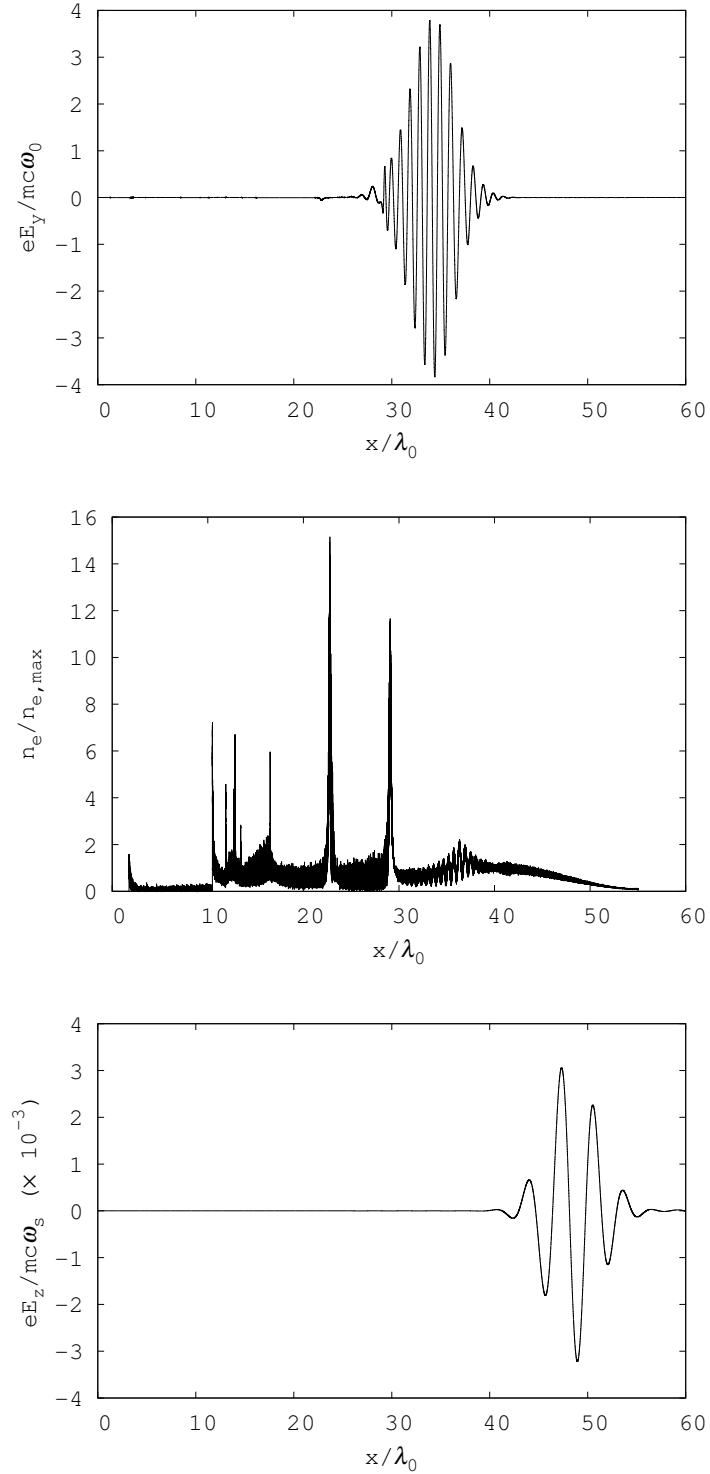
where  $N_p$  is the number of the plasma wave period behind the laser pulse,  $k_0$  is the wave number of the laser pulse, and  $k_p$  is the plasma wave number. The upshift factor expressed in terms of the relativistic gamma factor is  $(\gamma_{ph} + \sqrt{\gamma_{ph}^2 - 1})^2$ . With the current parameters the upshift factor for  $a_d \ll 1$  is 87 and for  $a_d \gg 1$  is 16 and 6.9 for  $N_p = 1$  and 2, respectively, from Eq. 2.

### 2.1. Low Intensity regime ( $a_s \ll 1$ )

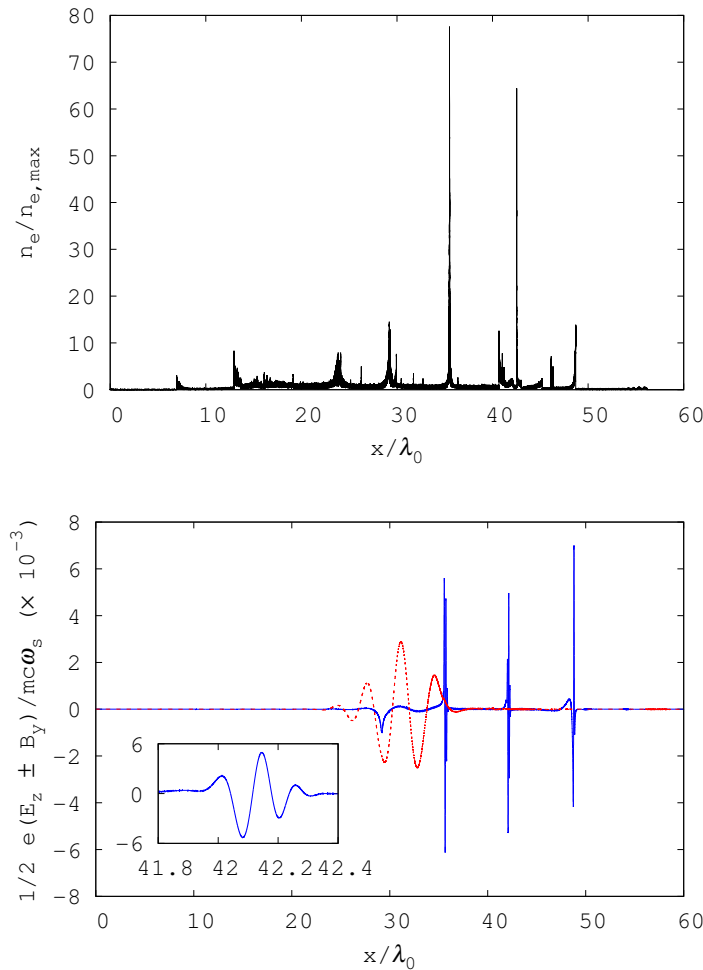
In order to verify previous results we performed simulations in the low source pulse intensity regime where the intensity of the source pulse was  $1.5 \times 10^{12} \text{W/cm}^2$ . Figure 2 shows the driver laser (top figure), which starts from the left edge of the simulation, electron density (middle figure) and source laser (bottom figure), which starts from the right edge of the simulation  $ct/\lambda_0 = 55$  from the start of the simulation. The amplitude of the source laser is in normalized units of  $a_s \equiv eE_z/mc\omega_s$  and has a maximum near 0.003 where  $\omega_s$  is the frequency of the source pulse. Sharp density spikes behind the driver laser pulse can be seen in the range  $20 \leq x/\lambda_0 \leq 30$ . Since the critical density of the source pulse is  $n_{cr_s}/n_{e,max} = 2.48$ , it can be seen that these density spikes can reflect the source pulse in the low source intensity regime.

Figure 3 shows the electron density (top) and source laser (bottom) at  $ct/\lambda_0 = 75$  from the start of the simulation, after the source pulse has collided with the density peaks. We have separated the transmitted and reflected components of the source by calculating the Poynting vector so that  $E_z + B_y$  and  $E_z - B_y$  correspond to the forward (left, red dotted line) and backward (right, blue solid line) propagating electromagnetic waves, respectively. It can be seen that the source pulse has reflected off of three density peaks. In the inset on the lower lefthand side a zoom-up of the middle reflected pulse is shown. We can see that it has much shorter wavelength ( $< 0.2\lambda_0$ ) than the original source pulse ( $3\lambda_0$ ). This corresponds to  $\lambda' \sim \lambda_s/15$  where  $\lambda'$  is the upshifted wavelength. In addition it can be seen that the number of cycles is less than the original source pulse.

In Figure 4 we show the Fourier transform of  $(E_z + B_y)$  (transmitted, red dotted line) and  $(E_z - B_y)$  (reflected, blue solid line) of the source laser pulse at the same time as Fig. 3 where the wavenumber is normalized to that of the source laser pulse  $k_s$ . The thick solid line is the Fourier transform of the original source pulse  $(E_z + B_y)$  after most of it has entered the simulation box near the beginning of the plasma interaction at  $ct/\lambda_0 = 50$ . In the figure a peak for the transmitted spectrum can be seen at  $k_x/k_s \approx 1$



**Figure 2.** Driver laser pulse (top figure), plasma electron density profile (middle figure) and source laser pulse (bottom figure)  $ct/\lambda_0 = 55$  from the start of simulation. The driver laser enters from the left edge of the simulation and the source laser enters from the right edge of the simulation.

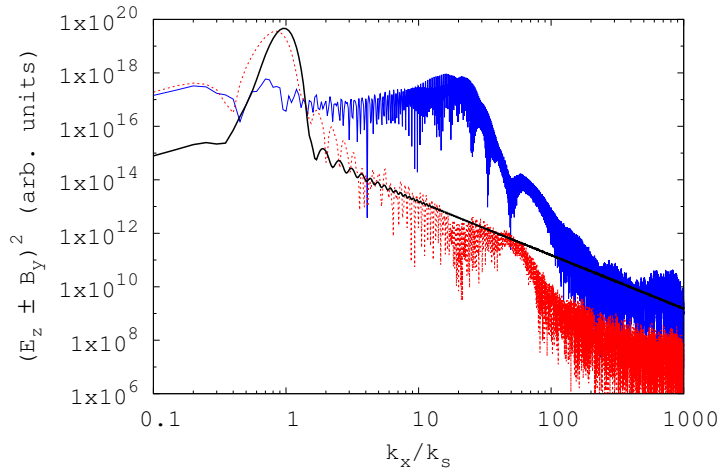


**Figure 3.** Plasma electron density profile (top) and source laser pulse (bottom)  $ct/\lambda_0 = 75$  from the start of simulation. The source laser has been separated into transmitted and reflected components by using the Poynting vector so that  $E_z + B_y$  and  $E_z - B_y$  correspond to the forward (left, red dotted line) and backward (right, blue solid line) propagating electromagnetic waves, respectively. The inset on the lower lefthand side shows a zoom-up of the middle reflected pulse. Note the increase of the reflected pulse amplitude upon the reflection. (color online)

corresponding to the source pulse wave number. In the reflected spectrum there is one broad peak around  $15 \lesssim k_x/k_s \lesssim 24$ . Another peak can be seen around  $k_x/k_s \approx 66$ . The lower values are in the range for the upshift factors for  $a_d \gg 1$  for reflection from the first and second wake wave behind the laser pulse and the upper value is in the range of the upshift factor for  $a_d \ll 1$ .

The reflection coefficient for the partial reflection of an electromagnetic wave from breaking plasma wave has been calculated in previous work for a source pulse which is of sufficiently low intensity so as not to perturb the plasma wave significantly [49, 50]. For simplicity we look at the case when the density spike is approximated by a delta-function





**Figure 4.** Fourier transform of  $(E_z + B_y)$  (transmitted, red dotted line) and  $(E_z - B_y)$  (reflected, blue solid line) of the source laser pulse at the same time as Fig. 3 where the wavenumber is normalized to that of the source laser pulse  $k_s$  with a normalized amplitude of  $a_s = 0.003$ . Fourier transform of the original  $(E_z + B_y)$  after most of it has entered the simulation box near the beginning of the plasma interaction at  $ct/\lambda_0 = 50$  (thick solid line). (color online)

where the reflection coefficient is given by [20]:

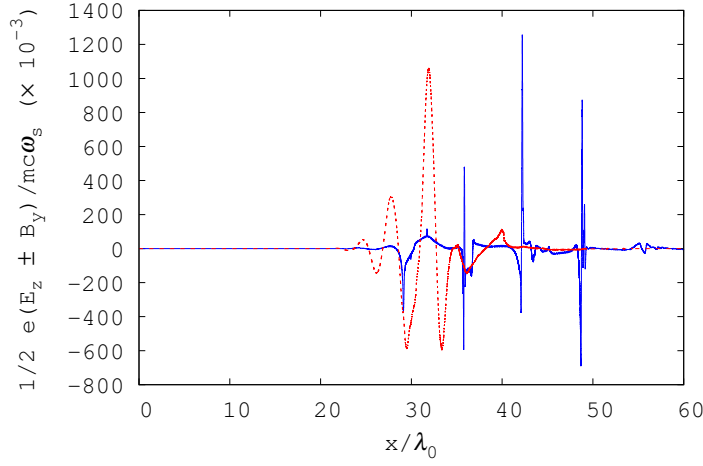
$$R_\delta \approx \left( \frac{\omega_{pe}}{\omega_s \cos^2(\theta/2)} \right)^2 \frac{1}{2\gamma_{ph}} \quad (3)$$

where  $\theta$  is the angle of incidence. Taking  $\theta = 0$  and using Eq. 2 giving  $\gamma_{ph} \approx 16$  for  $N_p = 1$  and  $\omega_s/\omega_{pe} = 1.57$  we get that  $R_\delta \approx 1.27 \times 10^{-2}$ . This can be seen to be in agreement with the ratio of the reflected spectrum broad peak around  $15 \lesssim k_x/k_s \lesssim 24$  (blue solid line) to that of the original pulse  $k_x/k_s \approx 1$  (thick solid line) in Fig. 4.

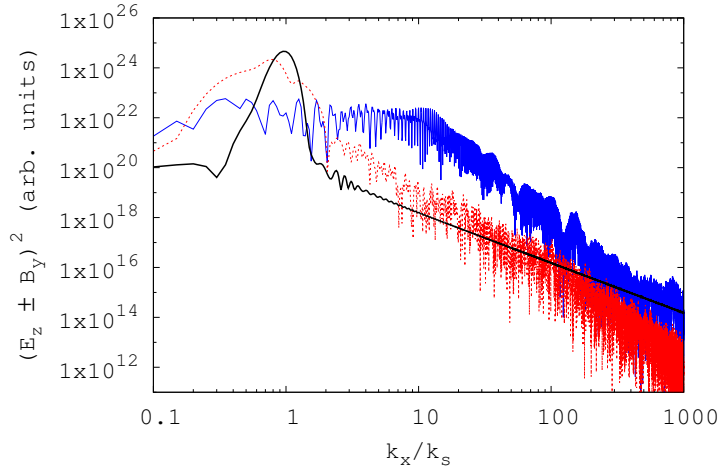
## 2.2. Near-relativistic Intensity regime ( $a_s \approx 1$ )

Figure 5 shows the transmitted,  $E_z + B_y$  (left, red dotted line), and reflected,  $E_z - B_y$  (right, blue solid line), components of the source at the same time as Fig. 3 (bottom)  $ct/\lambda_0 = 75$ . The source pulse intensity is  $I_s = 1.5 \times 10^{17} \text{W/cm}^2$  with a normalized amplitude of  $a_s = 0.99$ , which is in the near-relativistic regime. It can be seen that there are three reflected pulses similar to the low intensity case shown in Fig. 3 (bottom). The reflected pulses are also in approximately the same location as in Fig. 3 (bottom). The differences are mainly in the amplitudes of the reflected pulses compared to the transmitted source pulses. In the low intensity case shown in Fig. 3 (bottom) the reflected pulse amplitudes are nearly a factor of 2 larger than the original source pulse amplitude. For the near-relativistic intensity shown in Fig. 5 the reflected pulse amplitude is only slightly larger than or less than the source pulse amplitude.

Figure 6 shows the corresponding spectra of the transmitted and reflected source pulse at the same time as Fig. 5 where the wavenumber is normalized to that of the



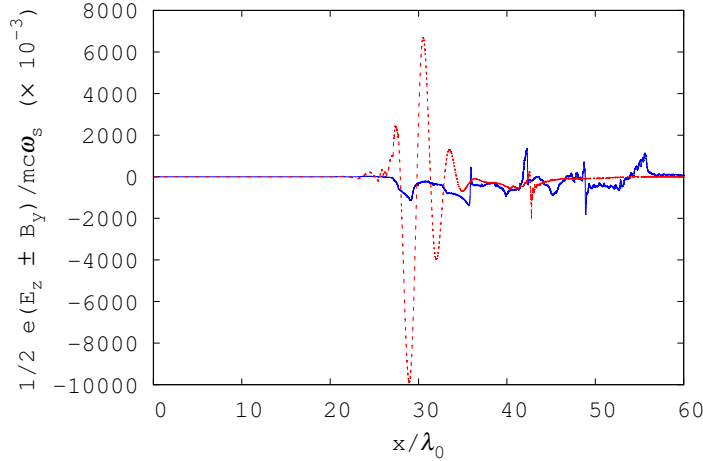
**Figure 5.**  $(E_z + B_y)$  (transmitted, red dotted line) and  $(E_z - B_y)$  (reflected, blue solid line) of the source laser pulse at the same time ( $ct/\lambda_0 = 75$ ) as Fig. 3. The source pulse intensity is  $I_s = 1.5 \times 10^{17} \text{W/cm}^2$  with normalized amplitude of  $a_s = 0.99$ . Note that the reflected pulse amplitude  $\approx$  source amplitude. (color online)



**Figure 6.** Fourier transform of  $(E_z + B_y)$  (transmitted, red dotted line) and  $(E_z - B_y)$  (reflected, blue solid line) of the source laser pulse of the corresponding data shown in Fig. 5 where the wavenumber is normalized to that of the source laser pulse  $k_s$ . The source pulse intensity is  $I_s = 1.5 \times 10^{17} \text{W/cm}^2$  with a normalized amplitude of  $a_s = 0.99$ . Fourier transform of the original  $(E_z + B_y)$  after most of the source pulse has entered the simulation box near the beginning of the plasma interaction at  $ct/\lambda_0 = 50$  (thick solid line). (color online)

source laser pulse  $k_s$ . The thick solid line is after most of the source pulse has entered the simulation box near the beginning of the plasma interaction at  $ct/\lambda_0 = 50$ .

It can be seen that the reflected source pulses (blue solid lines) are still substantially stronger than the transmitted source pulse emissions (red dotted lines) at high wavenumber for both intensities. Compared with  $I_s = 1.5 \times 10^{12} \text{W/cm}^2$  in Fig. 4



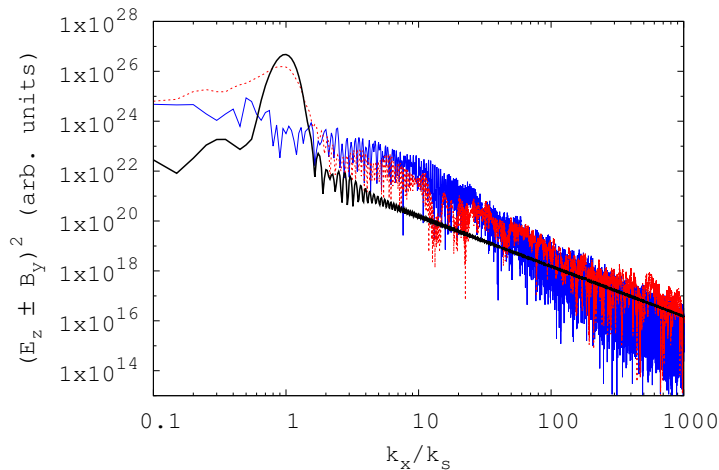
**Figure 7.**  $(E_z + B_y)$  (transmitted, red dotted line) and  $(E_z - B_y)$  (reflected, blue solid line) of the source laser pulse at the same time ( $ct/\lambda_0 = 75$ ) as Fig. 3. The source pulse intensity is  $I_s = 1.5 \times 10^{19} \text{W/cm}^2$  with a normalized amplitude of  $a_s = 9.9$ . Note that the reflected pulses have smaller amplitudes than the original source. (color online)

the reflected spectrum has broadened to higher  $k_s$  values and has multiple peaks. It can be seen that the ratio of the reflected spectrum region around  $k_x/k_s \approx 10$  (blue solid line) to that of the original pulse  $k_x/k_s \approx 1$  (thick solid line) in Fig. 6 is slightly lower than that of the low intensity case.

### 2.3. Relativistic Intensity regime ( $a_s \gg 1$ )

Figure 7 shows the separated the transmitted,  $E_z + B_y$  (left, red dotted line), and reflected,  $E_z - B_y$  (right, blue solid line), components of the source at the same time as Fig. 3 (bottom)  $ct/\lambda_0 = 75$ . The source pulse intensity is  $I_s = 1.5 \times 10^{19} \text{W/cm}^2$  with a normalized amplitude of  $a_s = 9.9$ , which is in the relativistic regime. For the relativistic intensity shown in Fig. 7 the reflected pulse amplitude is less than or much smaller than the respective source pulse amplitudes. Compared to the low intensity case shown in Fig. 3 (bottom) the reflected pulses are less apparent.

Figure 8 shows the Fourier transform of  $(E_z + B_y)$  (transmitted, red dotted line) and  $(E_z - B_y)$  (reflected, blue solid line) of the source laser pulse at the same time as Fig. 7 where the wavenumber is normalized to that of the source laser pulse  $k_s$ . The source pulse intensity is  $I_s = 1.5 \times 10^{19} \text{W/cm}^2$  with a normalized amplitude of  $a_s = 9.876$ , which is in the relativistic regime. Since the threshold for wavebreaking is 1.52 from the threshold formula [45], this source pulse causes wavebreaking. The reflected spectrum substantially differs from the low source amplitude spectrum for  $I_s = 1.5 \times 10^{12} \text{W/cm}^2$  shown in Fig. 4. It can be seen that the ratio of the reflected spectrum region around  $k_x/k_s \approx 10$  (blue solid line) to that of the original pulse  $k_x/k_s \approx 1$  (thick solid line) in Fig. 8 is substantially lower than that of the low intensity case. The spectra are broad, have no clear peaks, and show that the transmitted and reflected spectra are nearly the

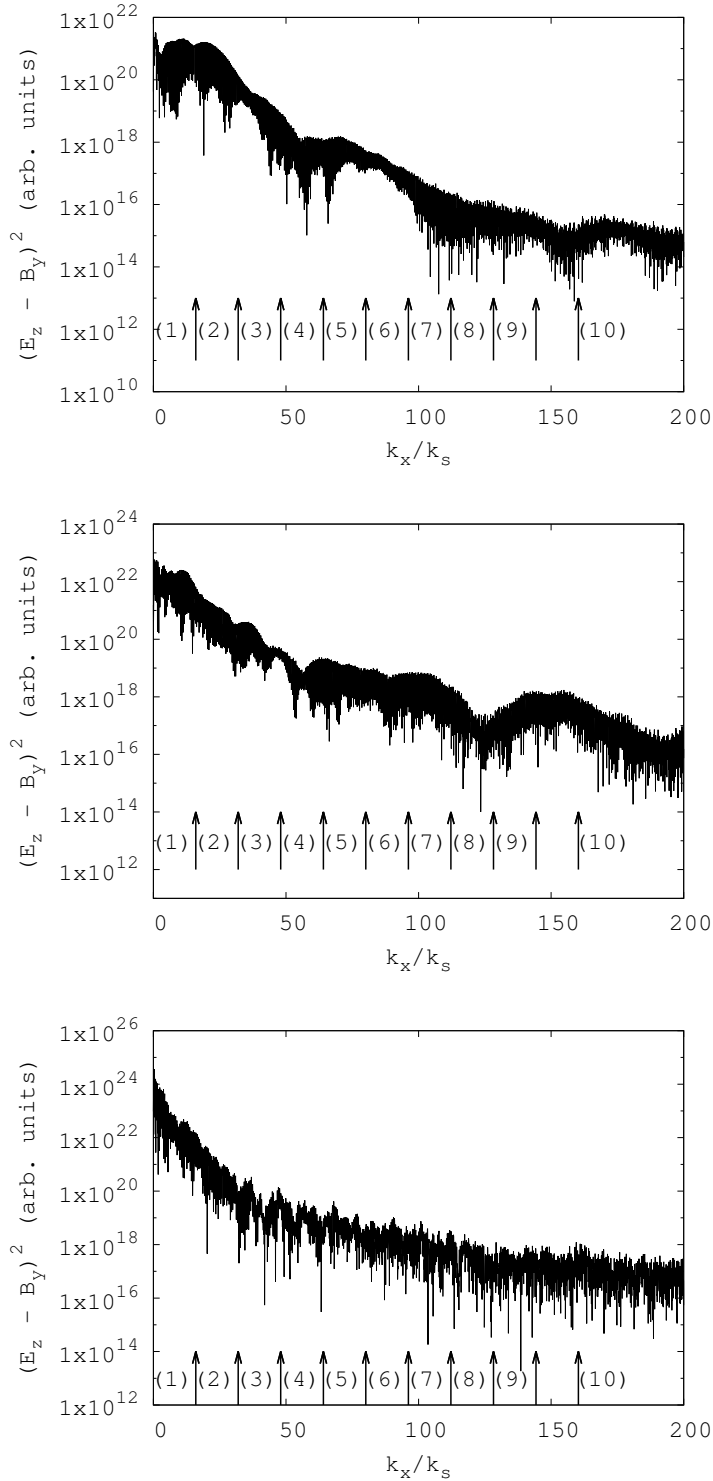


**Figure 8.** Fourier transform of  $(E_z + B_y)$  (transmitted, red dotted line) and  $(E_z - B_y)$  (reflected, blue solid line) of the source laser pulse at the same time as Fig. 7 where the wavenumber is normalized to that of the source laser pulse  $k_s$ . The source pulse intensity is  $I_s = 1.5 \times 10^{19} \text{W/cm}^2$  with a normalized amplitude of  $a_s = 9.9$ , respectively. Fourier transform of the original  $(E_z + B_y)$  after most of the source pulse has entered the simulation box near the beginning of the plasma interaction at  $ct/\lambda_0 = 50$  (thick solid line). (color online)

same amplitude over most of the spectral range. Although further analysis is needed, this could be an indication of a regime similar to the sliding mirror[51, 52]. In the sliding mirror model a laser interacts with a thin over-dense plasma where the electrons move along the surface of the plasma and not perpendicular to it and the transmitted and reflected pulses are predicted to be equal with the only difference being the presence of the incident pulse[51, 52]. The spectrum in Fig. 8 shows this characteristic where the transmitted and reflected spectra are nearly the same except for the peak near  $k_x/k_s = 1$ .

#### 2.4. Harmonic Generation

When the source pulses interacting with the mirror approach relativistic intensity levels ( $a_s \approx 1$ ) the motion of the electrons in the mirror becomes nonlinear. So at high source pulse intensities harmonics of the boosted source pulse light are expected to be observed. In the case of nonlinear Thomson scattering of relativistic electrons these harmonics have been observed [6]. Figure 9 shows the Fourier transforms of the reflected source laser pulse at the same time as Figs. 5 and 8 for source pulse intensities of  $I_s = 1.5 \times 10^{16} \text{W/cm}^2$  (top),  $I_s = 1.5 \times 10^{17} \text{W/cm}^2$  (middle) and  $I_s = 1.5 \times 10^{18} \text{W/cm}^2$  (bottom) with normalized amplitudes of  $a_s = 0.31$ ,  $a_s = 0.99$  and  $a_s = 3.12$ , respectively. These are spectra of the reflected source pulses in the range where the source pulse amplitudes are below and above  $a_s = 0.99$  which is the case where the harmonics are most apparent. The x-axis is linear scale so that the locations of the harmonics can be



**Figure 9.** Fourier transforms of  $(E_z - B_y)$  of the reflected source laser pulse at the same time as Figs. 5 and 8 where the wavenumber is normalized to that of the source laser pulse  $k_s$  and the scale is linear. The source pulse intensities are in the range of harmonic generation  $I_s = 1.5 \times 10^{16} \text{W/cm}^2$  (top),  $I_s = 1.5 \times 10^{17} \text{W/cm}^2$  (middle) and  $I_s = 1.5 \times 10^{18} \text{W/cm}^2$  (bottom) with normalized amplitudes of  $a_s = 0.31$ ,  $a_s = 0.99$  and  $a_s = 3.12$ , respectively. The arrows indicate the position of the harmonics of the relativistic upshift factor calculated from  $\gamma_{ph}$  in Eq. 2 for  $N_p = 1$  where the numbers in the parentheses refer to the harmonic order.

more clearly seen. The arrows in the figures indicate the position of the harmonics of the relativistic upshift factor calculated from  $\gamma_{ph}$  in Eq. 2 where we have chosen  $N_p = 1$ , since the reflected source pulse spectral peak is closest to this value in the low intensity case shown in Fig. 4. The numbers in the parentheses refer to the harmonic order.

For  $a_s = 0.31$  (top) there is a broad peak near the first harmonic (1) ( $10^{21}$  (arb. units)). If we look at amplitudes about five orders of magnitude down from this peak at  $10^{16}$  (arb. units), we can see peaks near the harmonics of (4) and (5). For  $a_s = 0.99$  (middle) there is a small peak-like structure near the first harmonic (1) ( $10^{22}$  (arb. units)). If we look at amplitudes about five orders of magnitude down from this peak at  $10^{17}$  (arb. units), we can see peaks near the harmonics up to around (9) and (10). For  $a_s = 3.12$  (bottom) we can see no clear peak near the first harmonic (1) ( $10^{22}$  (arb. units)) and the spectral power for  $k_x$  below the first harmonic clearly increases with decreasing  $k_x$  and rises to  $10^{24}$  (arb. units). If we look at amplitudes about five orders of magnitude down from either  $10^{24}$  or  $10^{22}$  (arb. units) at  $10^{19}$  or  $10^{17}$  (arb. units), we can see narrow peak-like structures, however, they are not as clear as the lower amplitude cases. The spectrum is also dropping off more quickly with  $k_x$  than in the lower amplitude cases.

The upshift factor for the peak between the labels of (9) to (10) in Fig. 9(middle) corresponds to  $\approx 160$  times the original source pulse wavelength of  $3\mu\text{m}$  resulting in wavelengths in the sub 20 nm range or photon energies  $\approx 60$  eV.

We have not taken into consideration the effect of magnetic fields on the harmonic generation. Since these are 1D simulations, poloidal magnetic fields driven by relativistic electrons generated during the laser-plasma interaction observed in experiments [53] are not produced. In addition, strong axial magnetic fields either produced by a circularly polarized pulse or externally can produce a hole in the breaking wave [54], which would modify the way the source pulse is reflected from the mirror. A detailed study of these magnetic field effects is left for further study.

### 3. Conclusions

In this paper we have studied relativistically upshifting coherent light pulses to higher frequencies via their reflection off of Relativistic flying mirrors (breaking plasma waves). We have shown via one dimensional PIC simulations that even high intensity light near the relativistic regime can be reflected off the mirrors and that this can generate relativistically upshifted harmonics. In the best case out of those considered by us the upshifted harmonics have wavelengths below 20 nm, which is much smaller than the original  $3\mu\text{m}$  source pulse wavelength. With these photon energies or wavelengths one could have pair creation in the multiphoton regime with several focused laser pulses [3]. The intensities which can be achieved by such harmonics requires higher dimensional simulations, which is beyond the scope of the current paper. For high source pulse intensities further investigation into the disruption of the relativistic mirror properties is needed such as has been studied in the interaction of wakefields with regular nonlinear

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