Title
THE Eo MASS

Author
Stevenson, M. Lynn.

Publication Date
1958-03-14

UNIVERSITY OF CALIFORNIA

Radiation Laboratory

BERKELEY, CALIFORNIA

UCRL-8199

UNIVERSITY OF CALIFORNIA
Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

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March 14, 1958

Printed for the U.S. Atomic Energy Commission

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ABSTRACT

The mass of the Σ^0 hyperon is obtained from a single associated production event in which both strange particles are observed to decay. The event is unusual, first because the Σ^0 hyperon decays directly into an electron-positron pair plus a Λ, and secondly because all the tracks are long and of higher-than-average quality. The value of Q_Σ obtained from this event is 76.4 ± 3.3 Mev, where $Q_\Sigma \equiv M_{\Sigma^0} - M_\Lambda$. For M_Λ = 1115.2 ± 0.14 Mev the mass of the Σ^0 is 1191.6 ± 3.3 Mev. Upon averaging this value with the existing measurements of Alvarez et al and Eisler et al, one obtains $Q_\Sigma = 75.3^{+0.9}_{-1.4}$ Mev and hence $M_{\Sigma^0} = 1190.5^{+0.9}_{-1.4}$ Mev.

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INTRODUCTION

During an investigation of the production and decay characteristics of hyperons and K mesons in a liquid-hydrogen bubble chamber, a single event of particular interest was found that enabled us to determine the Q value of the Σ^0 as 76.4 ± 3.3 Mev. The Q value is defined as $Q_{\Sigma^0} \equiv M_{\Sigma^0} - M_{\Lambda}$. Using M_{Λ} = 1115.2 ± 0.14 Mev,[1] we obtain M_{Σ^0} = 1191.6 ± 3.3 Mev.

Figure 1 shows a photograph of an associated production reaction produced by an incident pion (track No. 1) of 1.085 ± 0.015 Bev/c momentum. The primary reaction that occurs at the point A is $\pi^- + P \rightarrow \Sigma^0 + K^0$. The Σ^0 decays directly into an electron-positron pair, (tracks 8 and 9 respectively) and a Λ (neutral track No. 2). The Λ subsequently decays at point "Λ" into a π^- and a proton, (tracks 6 and 7 respectively).† Finally, the associated K^0 (neutral track No. 3) decays at the point "K^0" into a π^- and a π^+ (tracks 4 and 5 respectively).

Feinberg calculates that for equal relative parities of the Σ^0 and Λ particles, the Σ^0 should decay by internal pair creation, $\Sigma^0 \rightarrow \Lambda + e^+ + e^-$, in 1/184 of the time (and 1/165 for opposite parities).[2] We have observed

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*This work has been performed under the auspices of the U. S. Atomic Energy Commission.

†This event is similar to an event reported by Eisler et al[4] which gave a Q value of 80 ± 5 Mev.

[1]M. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nuclear Sci. 7, 407 (1957).

[2]G. Feinberg, Phys. Rev. 109, 1019 (1958).

over 100 events in which the Σ^0 decays by its normal mode $\Sigma^0 \rightarrow \Lambda + \gamma$. The one event that we have seen is therefore consistent with the predicted number.

Strictly speaking, the Q of this reaction would be $M_{\Sigma^0} - 2M_e - \Lambda$. However, we have in this paper defined Q as just $Q \equiv M_{\Sigma^0} - M_\Lambda$ to conform with the definition of Q for the normal decay mode.

MEASUREMENTS

As far as over-all quality of the tracks is concerned this event is above average. All tracks (including the neutral ones) are long and enable the angles as well as the momenta to be measured with higher than normal accuracy. However, no matter how high the quality of the tracks, the measured values will not perfectly conserve momentum and energy. It is therefore, necessary to vary the measured values until one obtains an internally consistent set of values.

The fitting is done piecemeal first by forcing tracks 3, 4, and 5 to fit the decay of a K^0, subject to the condition that

$$\chi^2 \equiv \sum_i \sigma_{\alpha_i} (\alpha_i - \beta_i)^2$$

should be a minimum.

Here β_i is the adjusted "least-squares" value of α_i and $\sigma_{\alpha_i}^{-1}$ is the mean square measurement error for the quantity α_i. For the charged tracks, namely 4, and 5, α_i represents both momentum and angle measurements, whereas for the neutral track 3, α_i represents only angle measurements. The momentum of the K^0 determined in this way will be referred to as $\vec{P}_K^{(L.S.)}$.

A similar "least-squares" fit is made on tracks 2, 6, and 7 to fit the decay of a Λ of mass 1115.2 Mev into a proton (mass = 938.21 Mev) and a π^- (mass = 139.69 Mev).[1] The Λ momentum determined by this least-squares method will be referred to as $\vec{P}_\Lambda^{(L.S.)}$.

The next step is to add the measured electron and positron momenta vectorially. This momentum will be called $\vec{P}_{pr}^{(0)}$. Now, from the vector addition of $\vec{P}_{pr}^{(0)}$ and $\vec{P}_\Lambda^{(L.S.)}$, we obtain the Σ^0 momentum, $\vec{P}_\Sigma^{(L.S.)}$.

The measurement of the incident π^- momentum from the curvature of its track (No. 1) is completely disregarded because of the very large error associated with the curvature measurement of such high-momentum tracks. However, the direction of the incident π^-, designated by the unit vector $\hat{P}_{inc}^{(0)}$, is used.

The three vectors $\hat{P}_{inc}^{(0)}$, $\vec{P}_{K}^{(L.S.)}$, and $\vec{P}_{\Sigma}^{(L.S.)}$ in general will not be coplanar and therefore will satisfy neither momentum nor energy conservation. In general these three quantities should be varied in such a way that a χ^2 function analagous to that one used in fitting the Λ and K^0 decays be a minimum. However, by a fortuitous circumstance, momentum and energy conservation can be achieved by (a) varying $\hat{P}_{inc}^{(0)}$ by only 0.02° in dip (1/50th of a standard deviation) and 0.12° in azimuth (1/3 of a standard deviation) and (b) changing $\vec{P}_{\Lambda}^{(L.S.)}$ by 3.4 Mev/c (1/20th of a standard deviation) in magnitude and 0.1° in dip angle (~1/10th of standard deviation), with no change in azimuth.

In order to summarize the values of the measured quantities as well as to display how these values were altered , the momenta as well as the azimuth and dip angles are tabulated in Table I. Table II summarizes the included angles between the pertinent momenta before and after the final adjustment. Here ψ_i is the azimuthal angle measured in a plane normal to the magnetic field and λ_i is the angle by which the track dips out of this plane. A track with $\psi_i = 0^{\circ}$ and $\lambda_i > 0$ would move from left to right and upwards out of the paper.

TABLE I

Measured and adjusted quantities for particles.*

Particle	Quantity	Measured values	Least squares values	Final adjusted values
K^0 (track No. 3)	P_K		384.3 ± 9 Mev/c	Same as least-squares values
	ψ_3	115.3 ± 0.3°	115.3°	
	λ_3	21.7 ± 1.2°	21.3°	
π^- (track No. 4)	P_4	387.9 ± 77.9 Mev/c	418.8 Mev/c	
	ψ_4	101.1 ± 0.3°	101.1°	
	λ_4	9.4 ± 2.0°	12.1°	
π^+ (track No. 5)	P_5	117.0 ± 6.5 Mev/c	119.3 Mev/c	
	ψ_5	226.5 ± 0.3°	226.5°	
	λ_5	25.3 ± 1.2°	25.5°	
Λ (track No. 2)	P_Λ		651.4 ± 30 Mev/c	654.8 ± 20 Mev/c
	ψ_2	102.7 ± 0.3°	102.7°	102.7°
	λ_2	6.5 ± 1.2°	7.5°	7.4°
π^- (track No. 6)	P_6	206.9 ± 15.2 Mev/c	202.2 Mev/c	
	ψ_6	98.4 ± 0.3°	98.4°	
	λ_6	24.3 ± 1.2°	24.0°	
π^+ (track No. 7)	P_7	477.5 ± 63.9 Mev/c	461.9 Mev/c	
	ψ_7	104.5 ± 0.3°	104.4°	
	λ_7	0.8 ± 1.2°	0.3°	

Table I (cont'd)

Particle	Quantity	Measured values	Least squares values	Final adjusted values
e^- (track No. 8)	P_{e^-}	23.2 ± 1.1† Mev/c	↑	↑
	ψ_8	73.7 ± 0.3°		
	λ_8	4.0 ± 1.2°		
e^+ (track No. 9)	P_{e^+}	90.7 ± 3.9† Mev/c	Same as measured values	$e^+ + e^-$ same as measured values
	ψ_9	65.3 ± 0.3°		
	λ_9	23.4 ± 1.2°		
"pr" = $e^+ + e^-$	P_{pr}	112.1 ± 4.0 Mev/c	↓	↓
	ψ_{pr}	67.0 ± 0.3°		
	λ_{pr}	19.7 ± 0.9°		
Σ^0	P_Σ		745.0 ± 30§	748.3 ± 20 Mev/c
	ψ_Σ		97.9°	97.9°
	λ_Σ		9.5°	9.4°
Incident π^- (track No. 1)	P_{inc}		1082.0 ± 15 Mev/c§	1085.4 ± 15 Mev/c
	ψ_{inc}	103.7 ± 0.3°	103.6°	103.6°
	λ_{inc}	-0.9 ± 1.0°	-0.9°	-0.9°

*All momenta have been corrected for ionization in the liquid hydrogen. The quantity ψ_i is the azimuthal angle measured in the plane normal to the magnetic field, with $\psi=0°$ corresponding to a track going from left to right in Fig. 1; λ_i is the angle at which the track dips out of this plane.

†Corrected for radiation loss as well as ionization loss.

§Obtained from momentum conservation alone.

TABLE II

$\theta_{\mu\nu}$	Space angles $\theta_{\mu\nu} = \cos^{-1} \hat{P}_\mu \cdot \hat{P}_\nu$ (degrees) Measured Values	Least Square Values	Final Adjusted Values
θ_{iK}	23.7 ± 1.4	23.4 ± 1.2	23.4
$\theta_{i\Lambda}$	7.5 ± 1.4	8.4 ± 1.2	8.4
$\theta_{e^+e^-}$	21.0 ± 1.5	21.0 ± 1.5	21.0
θ_{pre^-}	16.8 ± 1.5	16.8 ± 1.5	16.8
θ_{pre^+}	4.2 ± 0.4	4.2 ± 0.4	4.2
$\theta_{pr\Lambda}$	36.9 ± 0.4	36.5 ± 0.4	36.6
$\theta_{pr\Sigma}$		31.4 ± 0.7	31.4
$\theta_{\Lambda\Sigma}$		5.2 ± 0.7	5.1
$\theta_{i\Sigma}$		11.8 ± 1.5	11.7

MASS DETERMINATION

The rest mass in Mev of the Σ^0 is simply $M_{\Sigma^0} = \sqrt{E_\Sigma^2 - c^2 P_\Sigma^2}$ where $E_\Sigma = E_{pr} + E_\Lambda$. From the final adjusted values given in Table I and using the value for the Λ mass of 1115.2 Mev,[1] we obtain $E_\Sigma = 1407.1$ Mev and $P_\Sigma = 748.3$ Mev/c.

From these values one obtains for the value of the Σ^0 mass, $M_{\Sigma^0} = 1191.6 \pm 3.3$ Mev* and $Q_{\Sigma^0} \equiv M_{\Sigma^0} - M_\Lambda = 76.4 \pm 3.3$ Mev.*

This value of Q_{Σ^0} when averaged with the following existing determinations of the Q value, $Q_{\Sigma^0} = 71.7^{+3.8}_{-7.1}$ (Alvarez et al.[3]) and

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*For the purpose of propagating errors, it is much more informative to express the Q value directly in terms of the measured quantities P_{pr}, P_Λ and $\cos\theta_{pr\Lambda}$. Here $\theta_{pr\Lambda}$ is the angle between $\vec{P}_{pr}$ and $\vec{P}_\Lambda$. After suitable approximations (for this event only), we have

$$Q_\Sigma \approx \frac{P_{pr}}{M_\Lambda}\left[\sqrt{P_\Lambda^2 + M_\Lambda^2} - P_\Lambda \cos\theta_{pr\Lambda}\right],$$

from which we obtain

$$\left(\frac{\delta Q}{Q}\right)_{rms} \approx \left\{\left(\frac{\delta P_{pr}}{P_{pr}}\right)^2 + \left(\frac{\frac{P_\Lambda}{E_\Lambda} - \cos\theta_{pr\Lambda}}{\frac{E_\Lambda}{P_\Lambda} - \cos\theta_{pr\Lambda}}\right)^2 \left(\frac{\delta P_\Lambda}{P_\Lambda}\right)^2\right\}^{\frac{1}{2}},$$

and

$$\delta Q_{rms} \approx Q\left\{\left(\frac{\delta P_{pr}}{P_{pr}}\right)^2 + 6.43 \times 10^{-2}\left(\frac{\delta P_\Lambda}{P_\Lambda}\right)^2\right\}^{\frac{1}{2}}.$$

From the last expression, one can see that the error in Q comes almost exclusively from the measurement of the electron pair momenta. The latter's error in turn comes primarily from multiple coulomb scattering and, secondarily, from turbulence.

[3]Alvarez, Bradner, Falk-Vairant, Gow, Rosenfeld, Schmitz, and Tripp, Interactions of K^- Mesons in Hydrogen, UCRL-3775 (July 1957). This Q value was obtained by the use of the $\Sigma - \Sigma^-$ mass difference reported by these authors, the Σ^- mass of 1196.5 ± 0.5 Mev and the Λ mass of 1115.2 ± 0.14 Mev.[1]

$Q_{\Sigma^0} = 77.4 \pm 3.5$ (Eisler et al.[4]) gives as the value of $\overline{Q}_{\Sigma^0}$,

$$\overline{Q}_{\Sigma^0} = 75.3^{+0.9}_{-1.4} \text{ Mev},$$

with a mass of

$$\overline{M}_{\Sigma^0} = 1190.5^{+0.9}_{-1.4} \text{ Mev}.$$

To obtain this average Q value, two values with gaussian likelihood, functions must be averaged with a value that has a skew likelihood function. The likelihood function for the average Q is just the product of the three individual likelihood functions.

For ease in forming this product, we averaged the Eisler et al value[4] with ours to give a composite value of $Q' = 76.9 \pm 2.4$, which of course is still gaussian. This function together with the Alvarez et al function[3] (neither of which is normalized) is shown in Fig. 2. Also shown in Fig. 2 is the final likelihood function. Each shaded area of this function represents 0.159 of the total area under the likelihood function. This is just the fraction of the area that lies beyond one rms displacement on a normal gaussian distribution. The boundaries of the shaded regions then form the upper and lower error bounds. The average Q value, which is indicated by the vertical arrow, is then just that value that splits the unshaded area into equal parts.

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[4] Eilser, Plano, Samios, Steinberger, and Schwartz, Associated Production of Σ^0 and θ_2^0, Mass of the Σ^0, Nevis Report 60, R-198, December 1957.

ACKNOWLEDGMENTS

This mass measurement is the by-product of an associated production experiment done in collaboration with Frank S. Crawford, Jr., Marcello Cresti, Myron L. Good, Klaus Gottstein, Ernest M. Lyman, Frank T. Solmitz, and Harold K. Ticho, all of whom have contributed in some way to this Σ^0 mass determination. We wish to thank Professor Luis W. Alvarez for his stimulation and guidance throughout this associated production experiment. Our thanks also go to the many people involved in the operation of the bubble chamber, notably, James Donald Gow, Robert D. Watt, and Glenn J. Eckman, and to the scanners under the direction of Dr. Hugh Bradner. And finally a special word of appreciation goes to Hugo Bayona who originally found this event.

FIGURE LEGENDS

Fig. 1. Associated production reaction $\pi^- + p \rightarrow \Sigma^0 + K^0$. The incident pion (track No. 1) produces a Σ^0 and a K^0 at point "A". The Σ^0 decays immediately into an electron-positron pair (tracks 8 and 9 respectively) and a Λ (neutral track labeled No. 2). The Λ subsequently decays at point "Λ" into a π^- (track No. 6) and a proton (track No. 7). The K_0 (neutral track No. 3) decays at point "K^0" into a π^- (track No. 4) and a π^+ (track No. 5).

Fig. 2. Σ^0 mass likelihood functions. Our gaussian Q value and that of Eisler et al[4] are averaged together to yield the gaussian distribution represented by the dotted curve. This composite gaussian likelihood function is then combined with the likelihood function of Alvarez et al[3] which is represented by the dotted skew curve to give the final combined likelihood function indicated by the solid line. From the final likelihood function this average Q value is determined as $\overline{Q}_{\Sigma^0} = 75.3^{+0.9}_{-1.4}$ Mev.

Information Division
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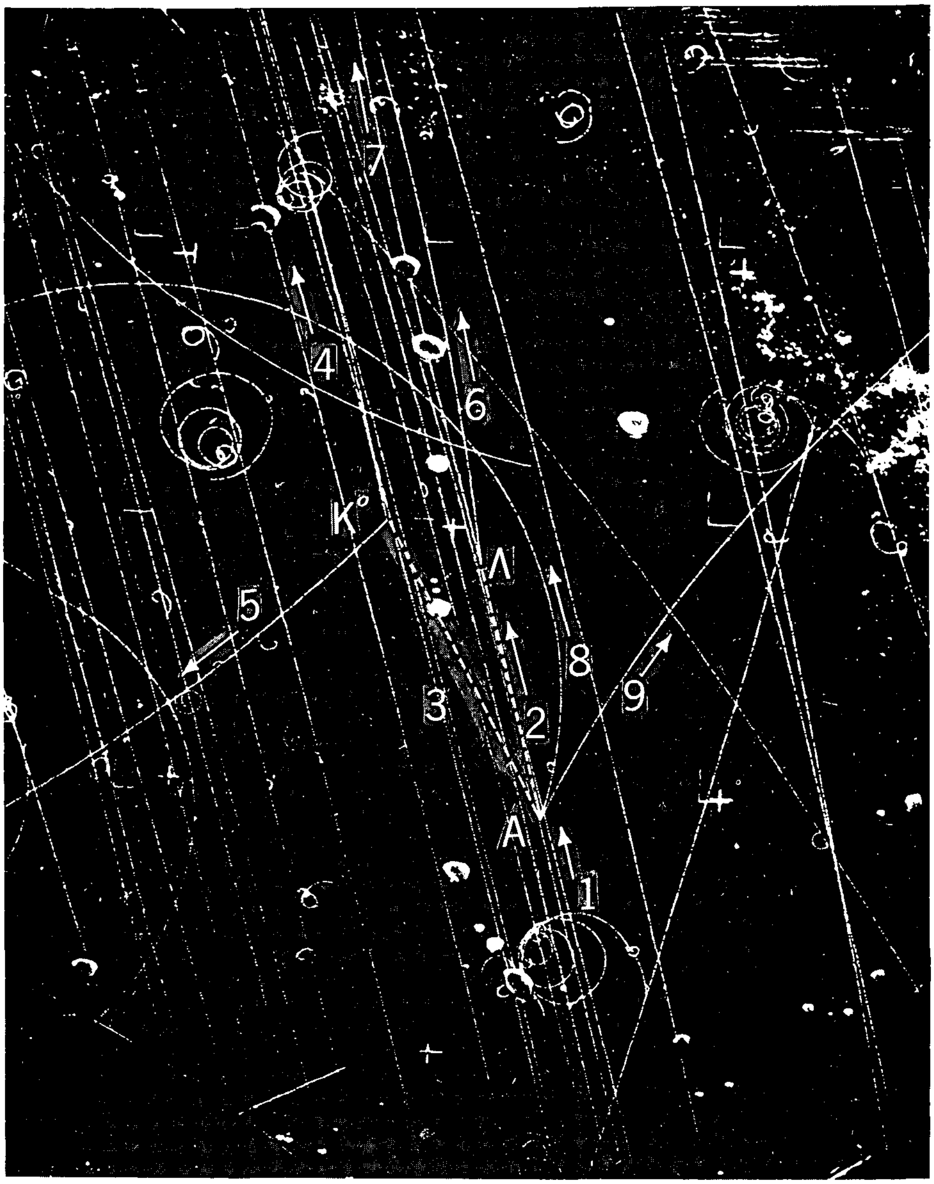

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Fig. 1.

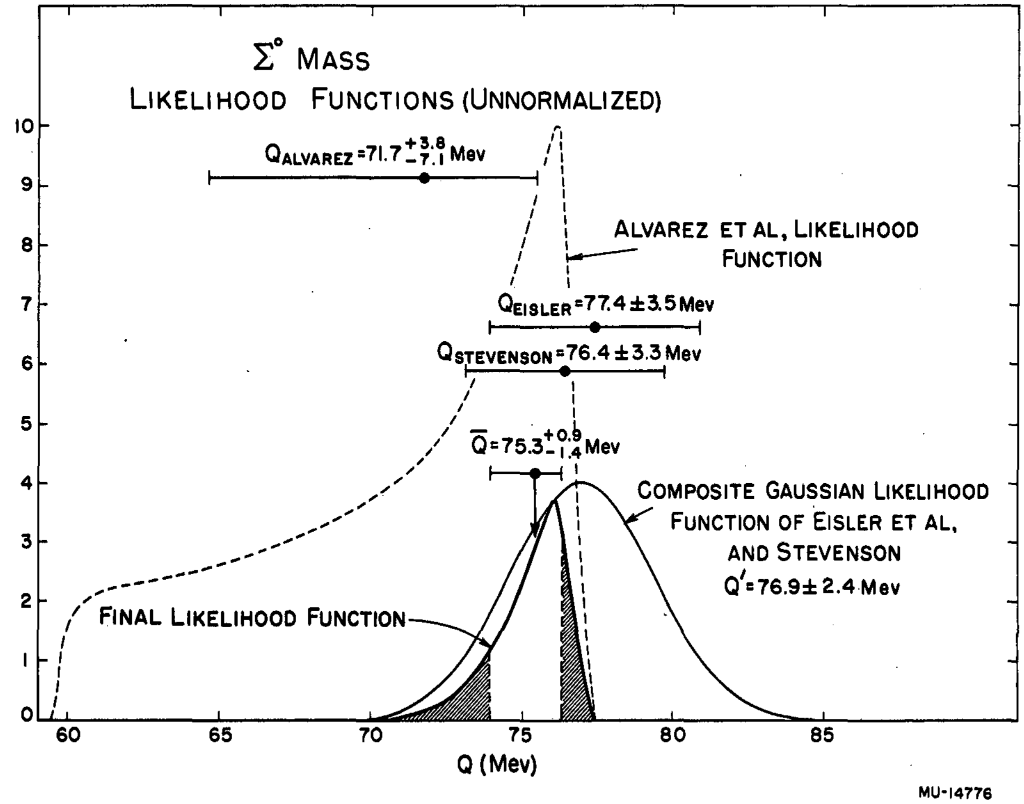
Σ° MASS
LIKELIHOOD FUNCTIONS (UNNORMALIZED)
Q_ALVAREZ = 71.7 +3.8 −7.1 Mev
ALVAREZ ET AL, LIKELIHOOD FUNCTION
Q_EISLER = 77.4 ± 3.5 Mev
Q_STEVENSON = 76.4 ± 3.3 Mev
Q̄ = 75.3 +0.9 −1.4 Mev
COMPOSITE GAUSSIAN LIKELIHOOD FUNCTION OF EISLER ET AL, AND STEVENSON
Q' = 76.9 ± 2.4 Mev
FINAL LIKELIHOOD FUNCTION
0
1
2
3
4
5
6
7
8
9
10
60
65
70
75
80
85
Q (Mev)
MU-14776

Fig. 2.