Title
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Publication Date
1979-08-01

LBL-9371 c.2
EEB-W-79-09

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UNIVERSITY OF CALIFORNIA

ENERGY & ENVIRONMENT DIVISION

Presented at the Izmir International Symposium - II On Solar Energy Fundamentals and Applications, Izmir, Turkey, August 6-8, 1979

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Prepared for the U.S. Department of Energy under Contract W-7405-ENG-48

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The work described in this report was funded by the Office of Buildings and Community Systems, Assistant Secretary for Conservation and Solar Applications of the U.S. Department of Energy under contract No. W-7405-ENG-48. Additional support for the presentation was provided by the United Nations under the TOKTEN program.

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ABSTRACT

The behavior of room temperature in a passive solar building without backup heat is of great interest to the building designer. This paper presents programs for card-reading programmable hand-calculators which compute room temperature over the course of a design day. Instructions for calculating the input parameters, and for running the programs are given, and a brief review of the theory is provided. The program can presently be used only for single-zone unmanaged, direct-gain buildings.

INTRODUCTION

Floating, or non-thermostat-controlled, room temperature in a passive solar building is an important measure of the building's performance. Optimally designed buildings will provide temperature which floats within the comfort range of the occupants without the use of heaters or air conditioners. Even if some supplementary heating or cooling is used, the floating performance of the building is of interest to the designer in assuring that full use is made of the solar energy collected by a building, and that the optimum window area is chosen.

Solar energy collected by a passive solar building is useful only to the extent that it either balances heating loads during sunny periods or can be stored for use at night. If solar heat gain increases room temperature beyond the comfort range, then the excess heat is either lost through ventilation or else it results in discomfort. In either case, a design whose maximum temperature is lower than the designer's upper limit, (e.g., 80°F (26.7°C)) is preferable to one which heats up beyond this limit.

The lowest floating temperature is also of interest to the designer. The magnitude of the minimum, and the time of day at which it occurs, will determine, for a given occupant's thermal preference characteristics, whether supplementary heat is needed.

Tradeoffs can be made in building design which affect different aspects of floating temperature behavior. Increasing the window area increases the maximum room temperature, but may also decrease the minimum temperature. Adding to the building's thermal mass decreases the magnitude of daily fluctuations in temperature without changing the daily average. Thermal mass can also delay the times of temperature extrema; for a Trombe wall, these delays can exceed 12 hours.

This paper presents a hand-calculator program which can be used to predict the floating temperature of a building, given a few simple building parameters and weather data. The calculations describe the building's response to a design day -- that is, a day with idealized (sinusoidal) weather. Two versions of the program are given; one for a Hewlett-Packard HP-67 calculator and the other for a Texas Instruments TI-59. Listings of the program are given in Appendix A.

The programs described here can be run in less than one-half hour; in some cases (e.g., those in which a few parameters are varied from an initial design), the run-time is considerably less. The methodology used in the programs can be generalized beyond the level of detail available in the programs. Some of these extensions can be done as hand calculations using intermediate outputs of the programs.

Use of this program will allow the building designer to easily predict the floating performance of a proposed single-zone, unmanaged passive solar building. The effect on floating temperature of varying parameters such as properties of the thermal mass and area of windows can be seen and, thus, optimal values can be chosen for such parameters.

The theoretical basis of the programs is described in Appendix B, and detailed derivations can be found in Ref. 1. Some familiarity with the theory will be helpful to the user of the programs, as it will assist in evaluating the input parameters to the program. As can be seen, the theory parallels that used in many public-domain building energy use analysis computer programs (e.g., NBSLD[2], DOE-2[3,4], TWOZONE[5], BLAST[6]), except that Fourier transformations are used instead of Laplace transformations, and some additional approximations are made.

The program has been validated by comparing its predictions with the measurements of floating temperature performed in two passive solar test cells at Los Alamos Scientific Laboratories.[7] Predictions of the temperature elevation (above average ambient temperature) were accurate to within ±10% of measurements at all hours of the day for two tests days (see figure, page 27).[8]

THEORY: RESPONSE FUNCTIONS

This section describes the use of response functions in the programs. Their derivation is discussed Appendix B. The response functions describe the response of temperatures in the building to driving forces such as sunlight and outside air temperature. They are functions of frequency (denoted by ω); that is, they give the effect of regular variations in the driving forces at a frequency ω on the variation of building temperatures at the same frequency. The most interesting frequency is generally one cycle per day.

The surface temperatures on the inside surface of building elements (e.g., walls and floors) are described by the materials response functions R_1 and R_2. These are complex-valued functions which are evaluated by the sub-programs for each building element. R_1 describes the response to sunlight inside the room, and R_2 gives the response to outside temperature.

The overall building performance is described by three building response functions A, B, and C, which are computed using the R_1 and R_2 results for each building element. A and C have the form of design heat loss rates. The A function relates the floating of room temperature to heating or cooling loads, while the C function describes the building response to ambient temperature variations. Response to sunlight is given by the B function.

Room temperature as a function of time is calculated using the building response functions A, B, and C, and weather data for a design day.

PROGRAM INPUT PARAMETERS

The passive solar programs require some simple weather inputs, and some data on building parameters. The building parameters are in a form which differs somewhat from other building models, since a simple format which accounts for most of the passive solar effects is desired.

Before any building parameters can be evaluated, the building must be divided into a small number of different types of construction sections. Each construction section is associated with a surface which faces the inside of the building. A typical house, for example, might have envelope wall sections, ceiling, floor, partition walls, and windows.

If a section is of thermally light construction (e.g., a window or a wall or floor with an insulating material on the inside) the 'U-value' (overall heat transfer coefficient) should be calculated, and no further data entries are needed. If a section is heavy (e.g., masonry construction; solid wood, gypsum board), then more detail is needed.

A heavy wall is divided conceptually into two layers: a thermally massive, inside-facing layer (e.g., concrete walls or floor) and an outside insulating layer (e.g., foam insulation plus sheathing or dry soil). For the inside thermally massive layer, data are needed on conductivity, heat capacity per unit volume, and thickness of the massive layer. For the insulating layer, only U-value of the insulation is needed. If there is no exterior insulation, the outside film coefficient is used.

For thermally massive elements connected to the room by an insulating layer, such as face brick on the outside of an insulated wall, the thermal mass of the outer layer is ignored. For walls with two layers of similar but not exactly equal thermal properties, such as drywall or plaster on solid wood, averaged parameters are used as an approximation. For example, a 4-inch wood wall covered on the outside by sheathing and on the inside by 1/2-inch plaster may be considered as a 4-1/2-inch wall with an insulating layer outside whose resistance is equal to that of the sheathing plus the outside air film. The properties of the thermal mass are the weighted average properties of the wood and plaster.*

Walls which cannot be approximated by this two-layer method are beyond the scope of the present programs because different formulas are needed to evaluate their response functions R_1 and R_2. Hand calculation methods which can compute these response functions are discussed in Ref. 1, appendix 2.4. An example of such a section might be a 1-inch-thick wood panel covering the inside of an 8-inch concrete wall or floor.

The program is fastest to run when the number of different surfaces or sections is minimized. The TI-59 program can only treat three different sections. To reduce the number of sections, different walls or floors with similar thermal properties can be combined into a single section; and sections which are much lighter than the rest of the house can be considered light.

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*To make the whole wall's U-value come out correct, one should average the inverse of conductivity rather than conductivity itself.

One can, for example, combine 4-inch frame envelope walls, 8-inch frame ceilings and floor, and 4-inch frame partition walls into a single section (call it a 6-inch frame wall) if the outside layer's U-value (U_r) is adjusted such that the heat transfer coefficient is the same as it would have been if each section were treated separately. (See Appendix E for an illustration of this process.)

In detailed building energy use analysis programs, such as DOE-2, NBSLD, BLAST and TWOZONE, frame walls are considered as two different sections; one with the properties of the solid wood (stud) fraction, and the other with the properties of the insulated section. This approach will also work in the hand-calculator programs, at the expense of adding an extra surface.

Once the number of surfaces has been established, the user must estimate how much of the solar radiation entering the house is absorbed on each surface. The fraction of solar gain absorbed on a surface is designated by α. The sum of α's for all heavy surfaces is less than one, because some fraction of the sunlight is absorbed by light surfaces or furniture. This fraction is released immediately to the room, and is called α_R.
There are at present no simple methods for determining the values of the α's from theory, even for simple room geometries. For complex geometries, even the detailed computerized methods break down. Empirical evaluation is a possibility. Fortunately, building performance does not appear to depend crucially on the exact evaluation of these parameters.

As a rough guideline, for a dark floor and light walls and ceiling, $\alpha_{floor} \cong .45$, $\alpha_{envelope\ walls} \cong .10$, $\alpha_{partition\ walls} \cong .20$, $\alpha_{ceiling} \cong .10$, $\alpha_R \cong .15$. These estimates are based on computer runs using the "Lumen

II" program[9] to calculate radiation balance for a prototype passive solar room in winter.

If the walls are dark, this changes to: $\alpha_{floor} \cong .30$, $\alpha_{envelope\ walls} \cong .20$, $\alpha_{partition\ walls} \cong .35$, $\alpha_{ceiling} \cong .05$, $\alpha_R \cong .10$.

Following is a list of input parameters needed.

<u>For each massive surface:</u>

- K- The thermal conductivity of the inside massive layer (in Btu/^{o}F-ft-hr). (Note that many handbooks express K in other units, such as Btu-in/^{o}F-ft^2-hr).
- ρc - The heat capacity per unit volume (Btu/^{o}F-ft^3). This is usaly obtained by finding the density ρ and specific heat c from handbook tables, and multiplying these values. $\rho c \simeq 9$ for wood and varies from about 15 to 30 for concrete.
- d - The thickness (in feet) of the massive part of the section. For the partition walls use half the wall thickness.
- h - The inside film heat transfer coefficient coupling the surface to the room air (in Btu/^{o}F-hr-ft^2). A typical value is 1.5, although sparsely furnished buildings with few partition walls may have lower values ($\sim$1).

- U_r - The U-value of the resistance between the outside of the massive part of the section and the ambient air (in Btu/^{o}F-ft^2-hr). Typically, $U_r \cong 5$ for a bare thermal mass or about 0.1 for an insulated mass. Concrete slab floors on grade with perimeter insulation have $U_r \sim 0.01$. For partition walls, $U_r = 0$.

- α - The fraction of solar energy entering the house which is absorbed on the surface (including multiple reflections). α is dimensionless.
- A - The total area of the surface facing the room (ft^2).

For the whole building:

- $\hat{U}_q$ - The design heat loss of the building per degree temperature difference through all quick heat transfer mechanisms, including window heat loss, infiltration loss, and conduction through quick construction sections (in Btu/^oF-hr).
- α_R - The fraction of solar energy entering the house which is absorbed on light surfaces, furniture, carpet, etc. (α_R is dimensionless.)
- H - The daily average heater output plus internal loads (Btu/hr). Typically, internal loads are about 2000 Btu/hr for a residential unit in the United States.

Weather parameters:

- ω_o - Daily frequency: 2π radians/24 hr.
- t_d - The length of the day (in hours) from sunrise (or time of first solar gain through the window) to sunset (or time of last solar gain).
- $|S_1|$ - Amplitude of daily solar gain through the windows (Btu/hr). In practice this is obtained by requiring that daily total solar gain is correct; that is, $\int |S_1| \sin \omega_1 t$ equals the daily solar gain. Thus

$$S_1 = \frac{\pi}{2t_d} \times \text{daily solar gain} \quad .$$

Daily solar gain can be obtained for a sunny day from ASHRAE solar heat gain factors,[10] or it can be approximated by multiplying window transmissivity by measured solar heat flux on a surface oriented in the same direction as the window. Typical winter transmissivities for south-facing windows are $\sim$0.85 for single-pane, 0.75 for double-pane, and 0.65 for triple-pane glass. To model cloudy days, the value of S_1 must be reduced. Solar gain through all windows is considered in computing S_1. Errors can result if east or west window area is large compared to south window area.

- $\bar{T}_A$ - Average ambient temperature in ^oF.
- $|\Delta T_A|$ - The amplitude of diurnal temperature fluctuations (^oF) or one-half the difference between maximum and minimum temperature.
- t_ϕ - The number of hours between sunrise and maximum ambient temperature. Note that minimum temperature is modeled as occurring 12 hours from maximum, so choose t_ϕ for best overall fit of sinusoidal temperature,

  $T_A = \bar{T}_A + \Delta T_A \cos(\omega_o(t-t_\phi))$

  to real temperature.
- $\bar{S}$ - For a weather cycle in which $|S_1|$ varies sinusoidally from day to day, the average value of $|S_1|$, (Btu/hr).
- ΔS_w - For a weather cycle, the amplitude of variation of $|S_1|$ over the cycle, (Btu/hr).

- ω_w - Frequency of weather variations (in radians/hr). ω_w is smaller than ω_o; typically $\omega_w \cong .1\omega_o$.
- ΔT_{A_w} - Amplitude of weather-cycle variation in ambient temperature (^oF).
- t_a - The time in the weather cycle at which the ambient temperature is maximized (hrs).

- t_s - The time in the weather cycle at which solar gain is maximized (hrs).

Results of weather cycle variations are not presently computed in the HP-67 program; however, the response functions can be obtained from the program and the weather cycle response may be computed by hand, as discussed in the HP program description.

PROGRAM OPERATION

This section describes the operation of the program from the point of view of the user. It assumes that the user has already evaluated all the building and weather parameters. A more detailed description of the HP-67 and TI-59 programs is given in Appendix C. A listing of the programs will be found in Appendix A. The HP-67 program and the TI-59 program are different in structure, so they are described separately below. The instructions must be followed exactly to assure correct output.

To check the performance of the programs and the selection of input parameters, a sample problem is set up and solved in Appendix E.

HP-67 Program

This program consists of three sub-programs. The first, sub-program 'R_{12}', calculates the R_1 and R_2 functions for a surface, given the building parameters for its construction section. The functions as evaluated at five frequencies (0, ω_w, ω_o, $2\omega_o$, and $3\omega_o$) and the results are read out on a data card. This program is run once for each surface.

The results on the data card produced by this program are valid for any material surface with the same construction as the one computed, so that if many runs are to be made on buildings of similar construction, the user may wish to build up a library of R_{12} data cards for commonly used construction sections.

For construction sections beyond the scope of the R_{12} sub-program, response functions can be computed manually and written onto data cards to be used in the rest of the calculation.

The second sub-program, 'ABC', computes the Building Response Functions from the data stored on R_{12} data cards. Each data card is read into the calculator once, some additional data are entered, and the calculator computes the effects of the new surface on A, B, and C. Any number of surfaces may be used. When the ABC sub-program is completed, the results are written on a data card; the card contains A and B (evaluated at all five frequencies) and $C(\omega_W$ and $\omega_O)$. Note that $C(0) \equiv A(0)$.

The final sub-program, 'T_R', takes the data from the 'ABC' sub-program and computes coefficients of $e^{i\omega t}$ from Eq. (B11) in Appendix B. The program next evaluates the temperature for each hour of the design day using (B11). It displays t, the time (relative to sunrise), for one second; then displays $T_R(t)$ for 5 seconds (or prints it); then displays t for the next hour and the new T_R. The first time and temperature displayed correspond to (solar) midnight, the second to 1a.m., etc. The coefficients of Eq. (B11) used to evaluate T_R are retained in memory.

At present, the effects of internal load and heater output must be added manually by reading A(0) from the ABC data card, decoding the entry using subroutine 'd' of the ABC or T_R sub-programs, and adding the temperature difference H/A(0) to the results for T_R. Response to weather cycles longer than one day must also be computed manually, as described at the end of the T_R sub-program.

<u>R_{12} sub-program</u>: Run this once for each material. Angle mode must be set to 'radians'.

1) Enter Input: ω_o in STO A

$\frac{\omega_o}{\omega_w}$ in STO B* $\left(\frac{\omega_o}{\omega_w}\right.$ is the period of the weather cycle in days$\left.\right)$

K in STO 1

ρc in STO 2

d in STO 3

h in STO 4

U_r in STO 5

0 (zero) in STO I.

2) Press E.

Wait ∿2 minutes.

Program will stop and read 'Crd' in display.

3) Feed in blank data card (both sides).

Note that the data card applies to this particular material.

Output:

The output R_1 and R_2 are encoded; can be interpreted with D routine of ABC program. The 'D' routine places the magnitude of R_1 or R_2 in the 'x' register and the phase angle in 'y'

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*Program will not run if this storage register contains 'zero'.

$R_1(0)$ in STO 0

$R_2(0)$ in STO 1

$R_1(\omega_w)$ in STO 2

$R_2(\omega_w)$ in STO 3

$R_1(\omega_o)$ in STO 4

$R_2(\omega_o)$ in STO 5

$R_1(2\omega_o)$ in STO 6

$R_2(2\omega_o)$ in STO 7

R_1 $(3\omega_o)$ in STO 8

R_2 $(3\omega_o)$ in STO 9.

ABC sub-program: Run this once for the whole building after all R_{12} data cards have been obtained. Angle mode must be set to 'radians'.

1) First: press E.

   Calculator will display '8'.

2) Press 'g' MERGE (or 'f' MERGE on HP 97).

3) Read in R_{12} data card for 1st material.

4) Input h in STO A

   A in STO B — for the 1st material.

   α in STO C

5) Press R/S; wait ~ 2 minutes.

   Calculator will stop and display '8'.

6) Repeat steps (2) - (5) for each surface.

7) After inputting all R_{12} data cards,

enter $\hat{U}_q$ in STO A

α_R in STO B.

Set Flag 0 (press 'h' 'SF' '0').

8) Press R/S; wait ~2 minutes.

Display will read 'Crd'.

9) Feed in blank data card for ABC data.

Note that output is encoded. Output for A and C can be decoded using by pressing 'd'. Output for B can be decoded by pressing 'D' The magnitude of the response function appears in 'x' register and the phase angle in the 'y' register.

Output:	A(0)	in STO 0
	B(0)	in STO 1
	$A(\omega_w)$	in STO 2
	$B(\omega_w)$	in STO 3
	$A(\omega_o)$	in STO 4
	$B(\omega_o)$	in STO 5
	$A(2\omega_o)$	in STO 6
	$B(2\omega_o)$	in STO 7
	$A(3\omega_o)$	in STO 8
	$B(3\omega_o)$	in STO 9
	$C(\omega_w)$	in STO 10
	$C(\omega_o)$	in STO 11.

<u>$T_R(t)$ sub-program:</u>

1) Read ABC data card. (This may be unnecessary if ABC has just been calculated.)

2) Press P $\leftrightarrow$ S.

3) Enter weather data

0 (zero) in STO I

$|\Delta T_A|$ in STO 5

t_ϕ in STO 6

t_d* in STO 7

S_1 in STO 8

$\overline{T_A}$ in STO 9

ω_o in STO C.

4) Press 'E'.

Output: After $\sim$1 minute calculator will flash for 1 sec the time+ at midnight, then for 5 sec, the temperature at that time; then flash for 1 sec the time 1 hour later and flash for 5 sec the temperature at 1 a.m.; etc. Note that these temperatures do not include the effects of internal loads or supplementary heat.

On HP 97, hour will flash, T_R will be printed.

T_R also provides the following coefficients from Eq. (B11)

$|S_1|\ d_o\ \frac{B(0)}{A(0)}$ in STO 0

$|S_1|\ d_o\ \frac{B(\omega_w)}{A(\omega_w)}$ in STO 2 - 3‡

*Program may fail for t_d = 6 or 12; if there is a problem, try 6 or 12±.001.

+Relative to sunrise.

‡Magnitude of complex number is in the first register; argument is in the second.

$|S_1|\ d_1\ \frac{B(\omega_o)}{A(\omega_o)}$ in STO 4 - 5*

$|S_1|\ d_2\ \frac{B(2\omega_o)}{A(2\omega_o)}$ in STO 6 - 7*

$|S_1|\ d_3\ \frac{B(3\omega_o)}{A(3\omega_o)}$ in STO 8 - 9*

$\frac{C(\omega_w)}{A(\omega_w)}$ in STO 10 - 11*

$\frac{C(\omega_o)}{A(\omega_o)}$ in STO 12 - 13*

Subroutine D of T_R decodes representations of R_1, R_2, and B into polar format.

Subroutine d of T_R decodes representations of A and C.

<u>Calculating response to weather cycles:</u>

As discussed in Appendix D, only three terms need be added to the daily response calculation to compute response on any given day of a weather cycle. These terms are given in Eq. (D4) of Appendix D, "Long-term Weather Response."

To perform the computation; set $|S_1|$ equal to the amplitude of solar gain on the day in question:

$$|S_1| = \bar{S} + \Delta S_w \cos \omega_w (t-t_s)$$

where t is evaluated at noon of the day of interest.

*Magnitude of complex number is in the first register; argument is in the second.

$\bar{T}_A$ is set equal to the average temperature for the whole cycle. The program is then run and results recorded for T_R at each hour. To these results are added the three terms from Eq. (D4).

The first term, $(\bar{S} - |S_1|)\, B(0)/A(0)\, d_o$, is computed by pressing RCL 0 and multiplying the result by $(\bar{S} - |S_1|)/|S_1|$. This temperature change is added to T_R at all hours.

The second term, $\Delta S_w\, B(\omega_w)/A(\omega_w)\, d_o\, e^{i\omega_w(t-t_s)}$, is obtained as follows: Compute $\omega_w(t-t_s)$ and then enter '1'. Press RCL 3, then RCL 2. Press 'B' for complex multiplication. Then multiply the result by $\Delta S_w/|S_1|$ to get the complex number represented by this term. The temperature change is the real part of this number; Press →R and read the result from the x register. In principle, this result is different each hour of the day, but the variation is usually so slow that only two or three hours need be calculated; the rest can be obtained by linear interpolation.

The third term, $\Delta T_{A_w}\, C(\omega_w)/A(\omega_w)\, e^{i\omega_w(t-t_a)}$, is obtained similarly to the second. $\omega_w(t-t_a)$ is computed, then '1' is entered. The programmer presses P⟷S, then RCL 1 and RCL 0, then (subroutine) B. The result is multiplied by ΔT_{A_w} and the real part taken by pressing →R.

Addition of these three terms completes the calculation of response to long-term weather.

TI - 59 Program

This program consists of two sub-programs, PSA-1 and 2, which take input data on the building and weather and compute building response. The response is calculated at each frequency, and the results added cumulatively.

When the next term changes the results by a sufficiently small amount, usually when the frequency is greater than 3 cycles per day ($\omega > 3\omega_o$), the user terminates the calculation.

The first sub-program, PSA-1, accepts all the building parameter data and most of the weather data in its first steps. It calculates and displays the steady-state room temperature response of the building, then Fourier analyzes the solar gain function S (t) into its amplitudes d_n. These results are displayed (and printed) by the calculator, and must be retained by the user for manual entry in PSA-2.

The second sub-program, PSA-2 uses the data which were stored in the memory registers by PSA-1, and the d's generated from PSA-1. The program calculates the room temperature component at a given frequency, then adds these results to the sum of those previously computed at other frequencies. After four to five passes through this program, the results for $T_R(t)$ converge to the final answer. To get the room temperature, the user must input the steady-state term computed in PSA-1.

This program expresses the time delays somewhat differently than they are defined in the parameter section of this paper. It uses the variable ϕ to represent the phase delay (in hours) of a given weather term.

- For the ΔT_A terms, ϕ = - (the number of hours from midnight to the time of the temperature maximum).
- For the ΔT_{A_W} term, ϕ = - (the number of hours from the beginning of the temperature cycle to the time of maximum temperature).
- For the ΔS_W term ϕ = - (the number of hours from the beginning to the weather cycle to the maximum of solar amplitude S_1).

- For the $|S_1|$ solar terms, ϕ = - (the number of hours from midnight to sunrise).

  Note the negative signs in the definitions of all ϕ's.

PSA-1 sub-program instructions: (using Master Library module)

1) Read in program by entering 1, then pressing INV 2nd WRITE and feeding in the first side of the card. Next, enter 2, press the same keys, and read in side 2.

2) Choose the surface (i=0,3) for which you will enter the parameters. Surface 0 refers to general parameters such as $|S_1|$ and $\hat{U}_q$. Up to three surfaces are allowed.

2a) Enter surface number (i) and press E. Calculator will display '20'.

3) Enter parameter in order for surface i

(0)	Enter d_i	or	$\lvert S_1 \rvert$	Press A
(1)	Enter $(\frac{1}{U_R})$ i	or	$\hat{U}_q$	Press A
(2)	Enter h_i	or	$\bar{T}_A$	Press A
(3)	Enter K_i	or	π/t_d	Press A
(4)	Enter $(\rho c)_i$	or	H	Press A
(5)	Enter α_i	or	α_R	Press A
(6)	Enter A_i	or	ΔT_A	Press A

Then return to Step 2a.

3b) To correct a parameter, return to step (2), and enter the number of the surface; then press E and enter the substep number shown in step 3 (e.g., '4' for ρc) and press B, then enter the correct parameter value and press 'R/S'.

4) Enter the number of surfaces used; press E. Calculator will display '20'.

5) Press C; calculator computes H/A(0) and prints result.

6) Press 'R/S'; calculator computes and adds steady-state ambient temperature and prints $\overline{T}_A$.

7) Enter t_d*; press R/S. Calculator prints $d_0 |S_1|$ B(0)/A(0).

8) Press 'R/S'. Calculator prints steady-state component of T_R. Record this value for future use.

9) Compute Fourier components of solar gain d_n for each frequency of interest 'n'.

   a) Enter n (you will need n=1, 2, and 3 in most cases).

   b) Press D*.

   c) Calculator displays Im(d_n) (imaginary part of d_n). Record this value for later entry.

   d) Press x⟷t.

   e) Calculator displays Re(d_n) (real part of d_n): Record this value for later keyboard entry.

   f) Go back to step 9a) and enter another value of n.

10) Press 2nd D; calculator computes and displays d_0.

11) Keep calculator on and run sub-program PSA-2.

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*For t_d = 6, 12, or few other values, calculator will attempt to divide by zero, resulting in an error. If this happens, re-enter t_d as .001 larger or smaller and try again.

PSA-2 sub-program instructions:

1) Read in PSA-2 (after having run PSA-1 to fill the calculator memories with data). Enter '1', press INV 2nd WRITE and feed in the first side of the card. Then enter 2, press the same keys, and read in side 2.

2) Run through steps 3 - 10 once for each frequency of interest (except $\omega=0$). Choose a frequency ω.

3) Computes $R_1(\omega)$ $R_2(\omega)$, and $A(\omega)$ $B(\omega)$ and $C(\omega)$.
Enter the period length in days (ω_0/ω); press A.
Calculator will display $Im(R_1(\omega))$ for the first surface.
To see Re $(R_1(\omega))$, press '$x \longleftrightarrow t$'.
To see Re $(R_2(\omega))$, press RCL 10.
To see Im $(R_2(\omega))$, press RCL 11.

4) Press R/S. (If R_1 or R_2 has been examined, press R/S twice.)

5) For two or more surfaces press 'R/S' to compute R_1 and R_2 for each surface. Display is as shown in Step 3.

6) Press R/S

7) Compute ambient temperature term at frequency ω. If there is no such term, go to step 9.
Enter ΔT_A or $|\Delta T_{A_W}|$.
Press B.
Calculator displays Imaginary Part of weather term. To see real part, press '$x \longleftrightarrow t$'. To see the term hour by hour, press 'R/S'. Calculator flashes time for 1 sec and temperature from this term for 4 sec.

8) Enter ϕ for the weather term, press E; calculator will display '2'.

9) To compute solar term at frequency ω, enter $Re(d_n)$ and press C. (If $\omega = \omega_w$, enter d_o). Enter $Im(d_n)$ and press 'R/S'. Enter $|S_1|$ or $|\Delta S_w|$ and press 'R/S'.

Calculator will display imaginary part of solar term at frequency ω. Press 'x⟷t' to see real part.

To see the term hour-by-hour, press R/S; calculator flashes hour for 1 sec and solar term for 4 sec.

10) Enter ϕ for solar term, press E. Calculator will display '2'.

11) Go back to Step (2) and run through the program for another frequency.

12) If all frequencies of interest have been run, enter 0 and press STO 23 and STO 4. Then enter the steady-state part of T_R obtained from Step 8 of PSA-1 and press STO 3. Press E, then press D and calculator will print $T_R(t)$ for every other hour of the day beginning with midnight. If no printer is available, these 12 temperatures are found successively in RCL 48-59.

ACKNOWLEDGMENT

This passive solar building model was developed as part of a project on analytic building calculations at Lawrence Berkeley Laboratory, initiated by Sam Berman of LBL and Robert Richardson of New York University.

We wish to thank Ray Kinoshita for her assistance in performing the calculations in support of the frame wall model used herein. Her comments on the program operating instructions and evaluation of the parameters were also helpful in improving the usability of the program.

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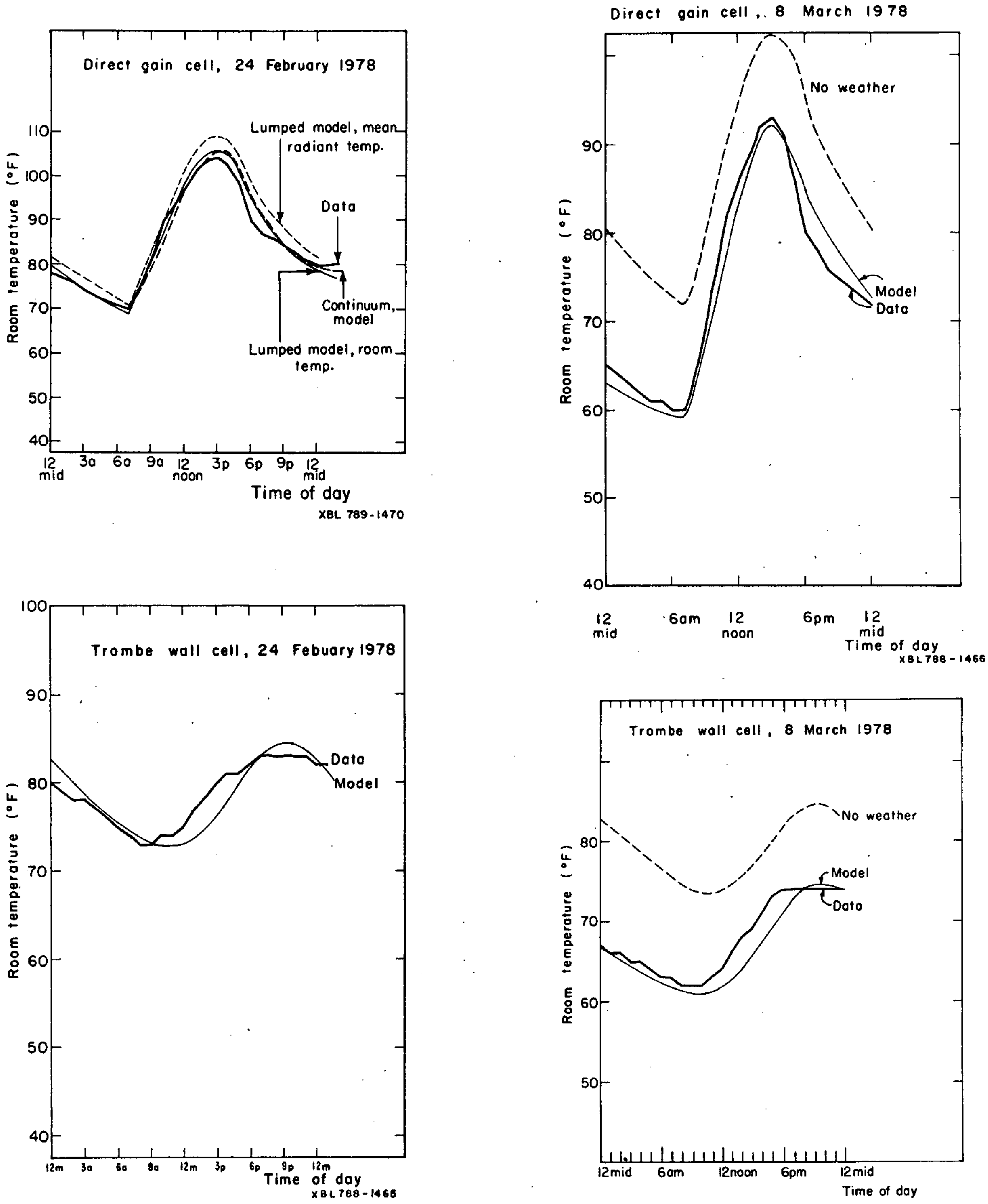
10. ASHRAE, Handbook of Fundamentals. 1977. Chapter 26, Tables 17-25



Predicted room temperature and observed data as a function of time of day for 24 February 1978 (left), and 8 March 1978 (right). Top: LASL direct gain cell; bottom: LASL Trombe wall cell. For 8 March, the curve labeled "no weather" assumes that all previous days had the same weather; the curve labeled "model" accounts for the previous two weeks' weather.

APPENDIX A: Listing of the R_{12}, ABC, $T_R(t)$, PSA-1, and PSA-2 Sub-Programs

R_{12}

001	*LBLA	21 11
002	→R	44
003	STOD	35 14
004	R↓	-31
005	STOE	35 15
006	R↓	-31
007	→R	44
008	RCLD	36 14
009	+	-55
010	X⇄Y	-41
011	RCLE	36 15
012	+	-55
013	X⇄Y	-41
014	→P	34
015	RTN	24
016	*LBLB	21 12
017	STOD	35 14
018	R↓	-31
019	STOE	35 15
020	R↓	-31
021	RCLD	36 14
022	×	-35
023	X⇄Y	-41
024	RCLE	36 15
025	+	-55
026	X⇄Y	-41
027	RTN	24
028	*LBL0	21 00
029	STOC	35 13
030	ENT↑	-21
031	e^x	33
032	RCLC	36 13
033	CHS	-22
034	ENT↑	-21
035	e^x	33
036	GSBA	23 11
037	2	02
038	÷	-24
039	STO6	35 06
040	X⇄Y	-41
041	STO7	35 07
042	X⇄Y	-41
043	RCLC	36 13
044	CHS	-22
045	ENT↑	-21
046	e^x	33
047	CHS	-22
048	GSBA	23 11
049	STO8	35 08
050	X⇄Y	-41
051	STO9	35 09
052	RTN	24
053	*LBLC	21 13
054	EEX	-23
055	4	04
056	×	-35
057	INT	16 34
058	1	01
059	0	00
060	×	-35
061	X⇄Y	-41
062	*LBL2	21 02
063	X<0?	16-45
064	GTO1	22 01
065	+	-55
066	RTN	24
067	*LBL1	21 01
068	2	02
069	Pi	16-24
070	×	-35
071	+	-55
072	GTO2	22 02
073	*LBLE	21 15
074	RCLI	36 46
075	X=0?	16-43
076	GTO9	22 09
077	2	02
078	÷	-24
079	1	01
080	-	-45
081	3	03
082	X=Y?	16-33
083	SF2	16 21 02
084	R↓	-31
085	*LBLd	21 16 14
086	RCLA	36 11
087	×	-35
088	RCL2	36 02
089	×	-35
090	RCL1	36 01
091	÷	-24
092	2	02
093	÷	-24
094	√X	54
095	STO0	35 00
096	RCL3	36 03
097	×	-35
098	GSB0	23 00
099	RCL1	36 01
100	RCL0	36 00
101	×	-35
102	ENT↑	-21
103	→P	34
104	STO0	35 00
105	X⇄Y	-41
106	STOC	35 13
107	CHS	-22
108	X⇄Y	-41
109	1/X	52
110	RCL4	36 04
111	×	-35
112	RCL5	36 05
113	×	-35
114	RCLC	36 13
115	RCL0	36 00
116	GSBA	23 11
117	RCL9	36 09
118	RCL8	36 08
119	GSBB	23 12
120	P⇄S	16-51
121	STO8	35 08
122	X⇄Y	-41
123	STO9	35 09
124	P⇄S	16-51
125	0	00
126	RCL4	36 04
127	RCL5	36 05
128	+	-55
129	RCL7	36 07
130	RCL6	36 06
131	GSBB	23 12
132	P⇄S	16-51
133	RCL9	36 09
134	RCL8	36 08
135	P⇄S	16-51
136	GSBA	23 11
137	1/X	52
138	X⇄Y	-41
139	CHS	-22
140	P⇄S	16-51
141	STO9	35 09
142	X⇄Y	-41
143	STO8	35 08
144	P⇄S	16-51
145	RCLC	36 13
146	CHS	-22
147	RCL0	36 00
148	1/X	52
149	RCL5	36 05
150	×	-35

```
151   RCL9      36 09
152   RCL8      36 08
153   GSBB      23 12
154   RCL7      36 07
155   RCL6      36 06
156   GSBA      23 11
157   P⇄S       16-51
158   RCL9      36 09
159   RCL8      36 08
160   GSBB      23 12
161   GSBC      23 13
162   RCL9      36 09
163   RCL8      36 08
164   0            00
165   R↑        16-31
166   STOi      35 45
167   ISZI   16 26 46
168   P⇄S       16-51
169   CLX         -51
170   RCL5      36 05
171   GSBB      23 12
172   GSBC      23 13
173   P⇄S       16-51
174   STOi      35 45
175   ISZI   16 26 46
176   F2?    16 23 02
177   GTOc   22 16 13
178   P⇄S       16-51
179   GTOE      22 15
180   *LBL9     21 09
181   RCL1      36 01
182   RCL3      36 03
183   ÷           -24
184   STO8      35 08
185   1/X          52
186   RCL4      36 04
187   x           -35
188   RCL5      36 05
189   x           -35
190   RCL4      36 04
191   +           -55
192   RCL5      36 05
193   +           -55
194   STOC      35 13
195   1/X          52
196   RCL5      36 05
197   RCL8      36 08
198   ÷           -24
199   1            01
200   +           -55
201   x           -35
202   RCL5      36 05
203   P⇄S       16-51
204   RCLC      36 13
205   ÷           -24
206   X⇄Y         -41
207   0            00
208   X⇄Y         -41
209   GSBC      23 13
210   STOi      35 45
211   ISZI   16 26 46
212   X⇄Y         -41
213   0            00
214   X⇄Y         -41
215   GSBC      23 13
216   STOi      35 45
217   ISZI   16 26 46
218   P⇄S       16-51
219   RCLB      36 12
220   1/X          52
221   GTOd   22 16 14
222   *LBLc  21 16 13
223   WDTA      16-61
224   P⇄S       16-51
```

ABC

```
001   *LBLA     21 11
002   →R           44
003   STOD      35 14
004   R↓          -31
005   STOE      35 15
006   R↓          -31
007   →R           44
008   RCLD      36 14
009   +           -55
010   X⇄Y         -41
011   RCLE      36 15
012   +           -55
013   X⇄Y         -41
014   →P           34
015   RTN          24
016   *LBLC     21 13
017   EEX         -23
018   4            04
019   x           -35
020   *LBL3     21 03
021   INT       16 34
022   1            01
023   0            00
024   x           -35
025   X⇄Y         -41
026   *LBL2     21 02
027   X<0?      16-45
028   GTO1      22 01
029   +           -55
030   RTN          24
031   *LBL1     21 01
032   2            02
033   Pi        16-24
034   x           -35
035   +           -55
036   GTO2      22 02
037   *LBLD     21 14
038   CF2    16 22 02
039   *LBL6     21 06
040   1            01
041   0            00
042   ÷           -24
043   STOE      35 15
044   FRC       16 44
045   1            01
046   0            00
047   x           -35
048   STOD      35 14
049   CLX         -51
050   RCLE      36 15
051   INT       16 34
052   F2?    16 23 02
053   GTO4      22 04
054   EEX         -23
055   CHS         -22
056   4            04
057   x           -35
058   *LBL5     21 05
059   RCLD      36 14
060   X⇄Y         -41
061   RTN          24
062   *LBLc  21 16 13
063   EEX         -23
064   2            02
065   x           -35
066   GTO3      22 03
067   *LBLd  21 16 14
068   SF2    16 21 02
069   GTO6      22 06
070   *LBL4     21 04
071   EEX         -23
072   CHS         -22
073   2            02
074   x           -35
075   GTO5      22 05
076   *LBLE     21 15
077   CF0    16 22 00
078   CLRG      16-53
079   P⇄S       16-51
080   CLRG      16-53
081   P⇄S       16-51
082   *LBLe  21 16 15
083   8            08
084   STOI      35 46
085   R/S          51
086   F0?    16 23 00
087   GTO0      22 00
088   *LBL7     21 07
089   0            00
090   STOI      35 46
091   RCLE      36 15
092   STO7      35 07
```

```
093  *LBLa   21 16 11
094  RCLi       36 45
095  GSBD       23 14
096  RCLA       36 11
097  CHS          -22
098  x            -35
099  0             00
100  ENT↑         -21
101  1             01
102  GSBA       23 11
103  RCLA       36 11
104  x            -35
105  RCLB       36 12
106  x            -35
107  P⇄S        16-51
108  RCLi       36 45
109  GSBd    23 16 14
110  GSBA       23 11
111  GSBc    23 16 13
112  STOi       35 45
113  P⇄S        16-51
114  RCLi       36 45
115  GSBD       23 14
116  RCLC       36 13
117  x            -35
118  RCLA       36 11
119  x            -35
120  ISZI    16 26 46
121  P⇄S        16-51
122  RCLi       36 45
123  GSBD       23 14
124  GSBA       23 11
125  GSBC       23 13
126  STOi       35 45
127  P⇄S        16-51
128  ISZI    16 26 46
129  RCLI       36 46
130  9             09
131  X≤Y?       16-35
132  GTOb    22 16 12
133  GTOa    22 16 11
134  *LBLb   21 16 12
135  RCL3       36 03
136  GSBD       23 14
137  RCLB       36 12
138  x            -35
139  RCLA       36 11
140  x            -35
141  RCL9       36 09
142  GSBd    23 16 14
143  GSBA       23 11
144  GSBc    23 16 13
145  STO9       35 09
146  RCL5       36 05
147  GSBD       23 14
148  RCLB       36 12
149  x            -35
```

```
150  RCLA       36 11
151  x            -35
152  RCL7       36 07
153  GSBd    23 16 14
154  GSBA       23 11
155  GSBc    23 16 13
156  STO7       35 07
157  F0?     16 23 00
158  GTOB       22 12
159  STOE       35 15
160  GTOe    22 16 15
161  *LBL0      21 00
162  RCLE       36 15
163  STO7       35 07
164  *LBL9      21 09
165  RCLI       36 46
166  2             02
167  +            -55
168  STOI       35 46
169  0             00
170  RCLA       36 11
171  RCLi       36 45
172  GSBd    23 16 14
173  GSBA       23 11
174  GSBc    23 16 13
175  STOi       35 45
176  ISZI    16 26 46
177  0             00
178  RCLB       36 12
179  RCLi       36 45
180  GSBD       23 14
181  GSBA       23 11
182  GSBC       23 13
183  STOi       35 45
184  RCLI       36 46
185  1             01
186  9             09
187  X=Y?       16-33
188  GTOB       22 12
189  DSZI    16 25 46
190  GTO9       22 09
191  *LBLB      21 12
192  RCL9       36 09
193  GSBd    23 16 14
194  0             00
195  RCLA       36 11
196  GSBA       23 11
197  GSBc    23 16 13
198  STO0       35 00
199  RCL7       36 07
200  GSBd    23 16 14
201  0             00
202  RCLA       36 11
203  GSBA       23 11
204  GSBc    23 16 13
205  STO1       35 01
206  P⇄S        16-51
```

```
207  WDTA       16-61
208  RTN           24
209  R/S           51
```

$T_R(t)$

```
001  *LBLB      21 12
002  STOD       35 14
003  R↓           -31
004  STOE       35 15
005  R↓           -31
006  RCLD       36 14
007  x            -35
008  X⇄Y          -41
009  RCLE       36 15
010  +            -55
011  X⇄Y          -41
012  RTN           24
013  *LBLD      21 14
014  CF2     16 22 02
015  *LBL6      21 06
016  1             01
017  0             00
018  ÷            -24
019  STOE       35 15
020  FRC        16 44
021  1             01
022  0             00
023  x            -35
024  STOD       35 14
025  CLX          -51
026  RCLE       36 15
027  INT        16 34
028  F2?     16 23 02
029  GSB4       23 04
030  EEX          -23
031  CHS          -22
032  4             04
033  x            -35
034  RCLD       36 14
035  X⇄Y          -41
036  RTN           24
037  *LBLd   21 16 14
038  SF2     16 21 02
039  GTO6       22 06
040  *LBL4      21 04
041  EEX          -23
042  2             02
043  x            -35
044  RTN           24
045  *LBLE      21 15
046  P⇄S        16-51
047  *LBLa   21 16 11
048  RCLi       36 45
049  GSBd    23 16 14
```

```
050    1/X        52
051    STOA       35 11
052    X⇄Y        -41
053    CHS        -22
054    STOB       35 12
055    ISZI       16 26 46
056    GSB0       23 00
057    RCLi       36 45
058    GSBD       23 14
059    GSBB       23 12
060    P⇄S        16-51
061    RCL8       36 08
062    P⇄S        16-51
063    x          -35
064    RCLB       36 12
065    RCLA       36 11
066    GSBB       23 12
067    DSZI       16 25 46
068    SPC        16-11
069    STOi       35 45
070    ISZI       16 26 46
071    X⇄Y        -41
072    STOi       35 45
073    9          09
074    RCLI       36 46
075    X=Y?       16-33
076    GTO9       22 09
077    3          03
078    X=Y?       16-33
079    GSB8       23 08
080    RCLI       36 46
081    5          05
082    X=Y?       16-33
083    GSB8       23 08
084    ISZI       16 26 46
085    GTOa       22 16 11
086    *LBL8      21 08
087    7          07
088    +          -55
089    STOI       35 46
090    RCLi       36 45
091    GSBd       23 16 14
092    RCLB       36 12
093    RCLA       36 11
094    GSBB       23 12
095    STOi       35 45
096    X⇄Y        -41
097    ISZI       16 26 46
098    RCLi       36 45
099    X⇄Y        -41
100    STOi       35 45
101    ISZI       16 26 46
102    X⇄Y        -41
103    STOi       35 45
104    RCLI       36 46
105    9          09
106    -          -45
107    STOI       35 46
108    RTN        24
109    *LBL0      21 00
110    P⇄S        16-51
111    RCLI       36 46
112    3          03
113    -          -45
114    2          02
115    X>Y?       16-34
116    GTO7       22 07
117    ÷          -24
118    RCLC       36 13
119    x          -35
120    STOD       35 14
121    CHS        -22
122    RCL7       36 07
123    x          -35
124    1          01
125    →R         44
126    1          01
127    +          -55
128    →P         34
129    RCLC       36 13
130    x          -35
131    RCL7       36 07
132    ÷          -24
133    Pi         16-24
134    RCL7       36 07
135    ÷          -24
136    X²         53
137    RCLD       36 14
138    X²         53
139    -          -45
140    ÷          -24
141    P⇄S        16-51
142    RTN        24
143    *LBL7      21 07
144    0          00
145    RCLC       36 13
146    RCL7       36 07
147    x          -35
148    Pi         16-24
149    X²         53
150    ÷          -24
151    P⇄S        16-51
152    RTN        24
153    *LBL9      21 09
154    P⇄S        16-51
155    RCL7       36 07
156    2          02
157    ÷          -24
158    1          01
159    2          02
160    -          -45
161    STOI       35 46
162    P⇄S        16-51
163    *LBLA      21 11
164    P⇄S        16-51
165    RCLI       36 46
166    RCL6       36 06
167    -          -45
168    RCLC       36 13
169    x          -35
170    RCL3       36 03
171    +          -55
172    RCL2       36 02
173    RCL5       36 05
174    x          -35
175    P⇄S        16-51
176    →R         44
177    STO1       35 01
178    RCLI       36 46
179    3          03
180    x          -35
181    RCLC       36 13
182    x          -35
183    RCL9       36 09
184    +          -55
185    RCL8       36 08
186    →R         44
187    ST+1       35-55 01
188    RCLI       36 46
189    2          02
190    x          -35
191    RCLC       36 13
192    x          -35
193    RCL7       36 07
194    +          -55
195    RCL6       36 06
196    →R         44
197    ST+1       35-55 01
198    RCLI       36 46
199    RCLC       36 13
200    x          -35
201    RCL5       36 05
202    +          -55
203    RCL4       36 04
204    →R         44
205    ST+1       35-55 01
206    RCL0       36 00
207    ST+1       35-55 01
208    P⇄S        16-51
209    RCL9       36 09
210    P⇄S        16-51
211    ST+1       35-55 01
212    RCLI       36 46
213    PSE        16 51
214    RCL1       36 01
215    PRTX       -14
216    ISZI       16 26 46
217    SPC        16-11
218    RCLI       36 46
219    2          02
220    4          04
221    X≤Y?       16-35
222    RTN        24
223    GTOA       22 11
224    R/S        51
```

PSA-1

```
000  76  LBL
001  43  RCL
002  22  INV
003  97  DSZ
004  09   09
005  57  ENG
006  03   3
007  03   3
008  42  STO
009  08   08
010  85   +
011  53   (
012  07   7
013  42  STO
014  07   07
015  65   ×
016  43  RCL
017  09   09
018  54   )
019  95   =
020  42  STO
021  06   06
022  76  LBL
023  32  X:T
024  73  RC*
025  08   08
026  42  STO
027  05   05
028  73  RC*
029  06   06
030  72  ST*
031  08   08
032  43  RCL
033  05   05
034  72  ST*
035  06   06
036  69  OP
037  38   38
038  69  OP
039  36   36
040  97  DSZ
041  07   07
042  32  X:T
043  76  LBL
044  57  ENG
045  92  RTN
046  76  LBL
047  15   E
048  42  STO
049  00   00
050  65   ×
051  07   7
052  95   =
053  42  STO
054  01   01
055  02   2
056  00   0
057  44  SUM
058  01   01
059  91  R/S
060  76  LBL
061  11   A
062  72  ST*
063  01   01
064  69  OP
065  21   21
066  91  R/S
067  76  LBL
068  12   B
069  44  SUM
070  01   01
071  91  R/S
072  72  ST*
073  01   01
074  69  OP
075  21   21
076  91  R/S
077  76  LBL
078  13   C
079  43  RCL
080  00   00
081  42  STO
082  09   09
083  43  RCL
084  21   21
085  42  STO
086  10   10
087  43  RCL
088  25   25
089  42  STO
090  11   11
091  76  LBL
092  59  INT
093  71  SBR
094  43  RCL
095  43  RCL
096  28   28
097  85   +
098  43  RCL
099  29   29
100  35  1/X
101  85   +
102  53   (
103  43  RCL
104  27   27
105  55   ÷
106  43  RCL
107  30   30
108  54   )
109  95   =
110  35  1/X
111  42  STO
112  08   08
113  65   ×
114  43  RCL
115  33   33
116  95   =
117  44  SUM
118  10   10
119  01   1
120  75   -
121  43  RCL
122  08   08
123  55   ÷
124  43  RCL
125  29   29
126  95   =
127  65   ×
128  43  RCL
129  32   32
130  95   =
131  44  SUM
132  11   11
133  69  OP
134  29   29
135  71  SBR
136  43  RCL
137  69  OP
138  29   29
139  97  DSZ
140  09   09
141  59  INT
142  03   3
143  07   7
144  02   2
145  03   3
146  69  OP
147  04   04
148  43  RCL
149  24   24
150  55   ÷
151  43  RCL
152  10   10
153  95   =
154  42  STO
155  12   12
156  42  STO
157  14   14
158  69  OP
159  06   06
160  91  R/S
161  03   3
162  07   7
163  01   1
164  03   3
```

```
165  69  OP
166  04   04
167  43  RCL
168  22   22
169  44  SUM
170  14   14
171  69  OP
172  06   06
173  91  R/S
174  42  STO
175  08   08
176  02   2
177  03   3
178  69  OP
179  04   04
180  43  RCL
181  08   08
182  69  OP
183  06   06
184  35  1/X
185  65   ×
186  89   π
187  95   =
188  42  STO
189  23   23
190  65   ×
191  01   1
192  02   2
193  95   =
194  35  1/X
195  65   ×
196  43  RCL
197  20   20
198  65   ×
199  43  RCL
200  11   11
201  55   ÷
202  43  RCL
203  10   10
204  95   =
205  42  STO
206  13   13
207  44  SUM
208  14   14
209  03   3
210  07   7
211  03   3
212  06   6
213  69  OP
214  04   04
215  43  RCL
216  13   13
217  69  OP
218  06   06
219  91  R/S
220  03   3
221  07   7
222  03   3
223  05   5
224  00   0
225  01   1
226  69  OP
227  04   04
228  43  RCL
229  14   14
230  69  OP
231  06   06
232  98  ADV
233  91  R/S
234  76  LBL
235  14   D
236  42  STO
237  05   05
238  65   ×
239  89   π
240  55   ÷
241  01   1
242  02   2
243  95   =
244  42  STO
245  02   02
246  33  X²
247  94  +/-
248  85   +
249  43  RCL
250  23   23
251  33  X²
252  95   =
253  65   ×
254  01   1
255  02   2
256  55   ÷
257  43  RCL
258  23   23
259  95   =
260  35  1/X
261  42  STO
262  03   03
263  01   1
264  06   6
265  03   3
266  01   1
267  69  OP
268  04   04
269  43  RCL
270  05   05
271  69  OP
272  06   06
273  43  RCL
274  08   08
275  94  +/-
276  49  PRD
277  02   02
278  00   0
279  42  STO
280  01   01
281  42  STO
282  04   04
283  36  PGM
284  05   05
285  17  B'
286  69  OP
287  21   21
288  36  PGM
289  04   04
290  13   C
291  03   3
292  05   5
293  69  OP
294  04   04
295  43  RCL
296  01   01
297  69  OP
298  06   06
299  32  X⇌T
300  02   2
301  04   4
302  69  OP
303  04   04
304  43  RCL
305  02   02
306  69  OP
307  06   06
308  22  INV
309  37  P/R
310  99  PRT
311  32  X⇌T
312  99  PRT
313  98  ADV
314  32  X⇌T
315  37  P/R
316  91  R/S
317  76  LBL
318  19  D'
319  01   1
320  06   6
321  00   0
322  01   1
323  69  OP
324  04   04
325  43  RCL
326  23   23
327  65   ×
328  01   1
329  02   2
```

```
330  95   =
331  35  1/X
332  69  OP
333  06   06
334  98  ADV
335  91  R/S
336  35  1/X
337  99  PRT
338  98  ADV
339  91  R/S
```

PSA-2

```
000  76  LBL
001  43  RCL
002  22  INV
003  97  DSZ
004  09   09
005  57  ENG
006  03   3
007  03   3
008  42  STO
009  08   08
010  85   +
011  53   (
012  07   7
013  42  STO
014  07   07
015  65   X
016  43  RCL
017  09   09
018  54   )
019  95   =
020  42  STO
021  06   06
022  76  LBL
023  32  X:T
024  73  RC*
025  08   08
026  42  STO
027  05   05
028  73  RC*
029  06   06
030  72  ST*
031  08   08
032  43  RCL
033  05   05
034  72  ST*
035  06   06
036  69  OP
037  38   38
038  69  OP
039  36   36
040  97  DSZ
041  07   07
042  32  X:T
043  76  LBL
044  57  ENG
045  92  RTN
046  76  LBL
047  65   X
048  00   0
049  42  STO
050  01   01
051  36  PGM
052  05   05
053  17   B'
054  36  PGM
055  04   04
056  13   C
057  92  RTN
058  76  LBL
059  39  COS
060  42  STO
061  02   02
062  94  +/-
063  42  STO
064  01   01
065  36  PGM
066  06   06
067  13   C
068  42  STO
069  10   10
070  42  STO
071  12   12
072  32  X:T
073  42  STO
074  11   11
075  42  STO
076  13   13
077  43  RCL
078  06   06
079  42  STO
080  02   02
081  94  +/-
082  42  STO
083  01   01
084  36  PGM
085  06   06
086  12   B
087  94  +/-
088  42  STO
089  02   02
090  32  X:T
091  42  STO
092  01   01
093  92  RTN
094  76  LBL
095  11   A
096  35  1/X
097  65   X
098  89   π
099  55   ÷
100  01   1
101  02   2
102  95   =
103  42  STO
104  23   23
105  43  RCL
106  21   21
107  42  STO
108  14   14
109  42  STO
110  18   18
111  43  RCL
112  25   25
113  42  STO
114  16   16
115  00   0
116  42  STO
117  15   15
118  42  STO
119  17   17
120  42  STO
121  19   19
122  43  RCL
123  00   00
124  42  STO
125  09   09
126  76  LBL
127  23  LNX
128  71  SBR
129  43  RCL
130  43  RCL
131  31   31
132  65   X
133  43  RCL
134  23   23
135  55   ÷
136  43  RCL
137  30   30
138  55   ÷
139  02   2
140  95   =
141  34  √X
142  42  STO
143  08   08
144  65   X
145  43  RCL
146  28   28
147  65   X
148  43  RCL
149  30   30
150  65   X
```

151	02	2
152	95	=
153	35	1/X
154	42	STO
155	07	07
156	43	RCL
157	08	08
158	65	×
159	43	RCL
160	27	27
161	95	=
162	42	STO
163	06	06
164	71	SBR
165	39	COS
166	43	RCL
167	07	07
168	42	STO
169	03	03
170	94	+/-
171	42	STO
172	04	04
173	36	PGM
174	04	04
175	13	C
176	44	SUM
177	12	12
178	32	X⇌T
179	44	SUM
180	13	13
181	43	RCL
182	28	28
183	65	×
184	43	RCL
185	30	30
186	33	X^2
187	65	×
188	43	RCL
189	08	08
190	33	X^2
191	65	×
192	02	2
193	95	=
194	42	STO
195	04	04
196	43	RCL
197	29	29
198	42	STO
199	03	03
200	36	PGM
201	04	04
202	13	C
203	43	RCL
204	28	28
205	35	1/X
206	44	SUM
207	03	03
208	43	RCL
209	03	03
210	42	STO
211	04	04
212	43	RCL
213	10	10
214	49	PRD
215	03	03
216	43	RCL
217	11	11
218	49	PRD
219	04	04
220	36	PGM
221	04	04
222	12	B
223	36	PGM
224	05	05
225	15	E
226	42	STO
227	10	10
228	32	X⇌T
229	42	STO
230	11	11
231	43	RCL
232	28	28
233	35	1/X
234	49	PRD
235	10	10
236	49	PRD
237	11	11
238	43	RCL
239	12	12
240	42	STO
241	03	03
242	43	RCL
243	13	13
244	42	STO
245	04	04
246	36	PGM
247	04	04
248	13	C
249	42	STO
250	12	12
251	32	X⇌T
252	42	STO
253	13	13
254	91	R/S
255	43	RCL
256	29	29
257	65	×
258	43	RCL
259	32	32
260	95	=
261	49	PRD
262	12	12
263	49	PRD
264	13	13
265	43	RCL
266	12	12
267	44	SUM
268	16	16
269	43	RCL
270	13	13
271	44	SUM
272	17	17
273	43	RCL
274	29	29
275	94	+/-
276	49	PRD
277	01	01
278	49	PRD
279	02	02
280	69	OP
281	21	21
282	65	×
283	43	RCL
284	33	33
285	94	+/-
286	95	=
287	49	PRD
288	01	01
289	49	PRD
290	02	02
291	49	PRD
292	10	10
293	49	PRD
294	11	11
295	43	RCL
296	01	01
297	44	SUM
298	14	14
299	43	RCL
300	02	02
301	44	SUM
302	15	15
303	43	RCL
304	10	10
305	44	SUM
306	18	18
307	43	RCL
308	11	11
309	44	SUM
310	19	19
311	69	OP
312	29	29
313	71	SBR
314	43	RCL
315	69	OP

316	29	29
317	97	DSZ
318	09	09
319	23	LNX
320	91	R/S
321	76	LBL
322	55	÷
323	43	RCL
324	14	14
325	42	STO
326	03	03
327	43	RCL
328	15	15
329	42	STO
330	04	04
331	36	PGM
332	04	04
333	18	C'
334	42	STO
335	03	03
336	32	X:T
337	42	STO
338	04	04
339	92	RTN
340	76	LBL
341	15	E
342	42	STO
343	05	05
344	01	1
345	02	2
346	42	STO
347	09	09
348	04	4
349	08	8
350	42	STO
351	08	08
352	76	LBL
353	38	SIN
354	43	RCL
355	05	05
356	42	STO
357	02	02
358	43	RCL
359	23	23
360	49	PRD
361	02	02
362	71	SBR
363	65	×
364	74	SM*
365	08	08
366	02	2
367	44	SUM
368	05	05
369	69	OP
370	28	28
371	97	DSZ
372	09	09
373	38	SIN
374	91	R/S
375	76	LBL
376	99	PRT
377	43	RCL
378	23	23
379	55	÷
380	02	2
381	55	÷
382	89	π
383	95	=
384	35	1/X
385	42	STO
386	07	07
387	76	LBL
388	90	LST
389	43	RCL
390	23	23
391	65	×
392	43	RCL
393	07	07
394	66	PAU
395	99	PRT
396	95	=
397	42	STO
398	02	02
399	71	SBR
400	65	×
401	66	PAU
402	66	PAU
403	66	PAU
404	66	PAU
405	99	PRT
406	97	DSZ
407	07	07
408	90	LST
409	92	RTN
410	76	LBL
411	12	B
412	42	STO
413	01	01
414	42	STO
415	02	02
416	43	RCL
417	18	18
418	49	PRD
419	01	01
420	43	RCL
421	19	19
422	49	PRD
423	02	02
424	71	SBR
425	55	÷
426	91	R/S
427	71	SBR
428	99	PRT
429	91	R/S
430	76	LBL
431	13	C
432	42	STO
433	01	01
434	91	R/S
435	42	STO
436	02	02
437	91	R/S
438	49	PRD
439	01	01
440	49	PRD
441	02	02
442	43	RCL
443	16	16
444	42	STO
445	03	03
446	43	RCL
447	17	17
448	42	STO
449	04	04
450	36	PGM
451	04	04
452	13	C
453	71	SBR
454	55	÷
455	91	R/S
456	71	SBR
457	99	PRT
458	91	R/S
459	76	LBL
460	14	D
461	98	ADV
462	04	4
463	08	8
464	22	INV
465	90	LST
466	91	R/S

APPENDIX B: Theory

This section describes the mathematical model which is the basis for the programs. The model looks at the response of interior (that is, room-side) surfaces of building elements, such as envelope and partition walls, on which solar heat may be absorbed. An equation is developed to describe the response of material surface temperatures to weather variables in terms of Fourier materials response functions R_1 and R_2, which are evaluated in the programs.

Next, the room temperature response is computed from the surface temperature results. Room temperature can be given as a function of weather variables in terms of three Building Response Functions, called A, B, and C. These functions are also evaluated in the programs.

The final results for room temperature are derived from the weather description and the Building Response Functions, and hourly temperature is displayed or printed by the programs.

We now proceed to the theory.

Solar energy entering a direct-gain building passes through the room air until it reaches a surface (e.g., a floor). When it strikes the surface, some fraction is absorbed and the rest is reflected. The reflected component eventually is absorbed on some surface. The fraction of sunlight which is absorbed a given surface depends on the sun angles and room geometry in a very complex way. We assume that the internal solar radiation balance is already known, either from direct measurement or through simulation; and that the amount of sunlight absorbed on a surface "j" is given by

α_j S, where S is the total amount of sunlight entering the building (in watts or Btu hr^{-1}).

For each surface "j", the surface temperature, T_{sj}, can be determined by a surface heat balance, which is expressed as:

$$h_j A_j (T_{sj} - T_R) - A_j K_j \left. \frac{\partial T_j(x,t)}{\partial x} \right|_{x=o} = \alpha_j S \qquad \text{(B1)}$$

where

h_j is the combined radiation/convection film heat transfer coefficient for the jth surface (watts m^{-2} $^oC^{-1}$ or Btu hr^{-1} ft^{-2} $^oF^{-1}$),

A_j is the area of the surface (m^2 or ft^2),

T_R is the room temperature, (°C or °F),

$T_j(x,t)$ is the temperature distribution within the j^{th} material, (^oC or ^oF),

K_j is the conductivity of the j^{th} material (watts m^{-1} $^oC^{-1}$ or Btu hr^{-1} $^oF^{-1}$ ft^{-1} ,

α_j is the fraction of sunlight absorbed on the j^{th} surface,

x is the distance into the material (m or ft), and

S is the solar gain transmitted through all the windows (watts or Btu/hr).

This equation sets heat losses from the surface (left-hand side of (B1)) equal to heat gains (right-hand side). It assumes that the surface transfers heat directly to the room air, rather than being in radiative contact with other surfaces, which results in a substantial simplification of the computation effort.[B2]

Heat-flows within a material satisfy the diffusion equation:

$$K_j \frac{\partial^2 T_j(x,t)}{\partial x^2} = (\rho c)_j \frac{\partial T_j(x,t)}{\partial t} \tag{B2}$$

where $(\rho c)_j$ is the heat capacity per unit volume of the j^{th} material (watt-hr $^oC^{-1}m^{-3}$ or Btu $^oF^{-1}$ ft^{-3}).[B3]

Equations (B1) and (B2) describe heat-flows at the inside surface of a material and in its interior; at the outside surface we assume that the material is coupled to the ambient air (at temperature T_A) by a pure resistance which can be described by a heat transfer coefficient U_r. (For an uninsulated material this coefficient is just equal to an exterior surface film coefficient).

This description allows the solution for surface temperatures in terms of the driving forces of solar gain and ambient temperature. This solution can be written in simple form if we look at the amplitudes of temperature (and solar) fluctuations at a steady harmonic frequency. The result can be expressed as:

$$T_{sj} = (h_j T_R + \alpha_j S/A_j) R_{1j} + T_A R_{2j} \tag{B3}$$

where R_{1j} and R_{2j} are frequency-dependent response functions whose forms are given in Table B1. These response functions give all the information needed to describe the thermal behavior of the material and its surface. They are evaluated and can be displayed in the programs.

Response functions are computed for all surfaces of materials with significant thermal mass. They would be needed, for example, for surfaces of masonry materials, and also for surfaces of wood or drywall if the house is relatively lightweight. For surfaces of materials with little thermal mass, such as upholstery furniture, thin wood, carpet, insulation, etc., no computation of response functions is necessary. Solar absorption on these surfaces is accounted for by the parameter α_R, which appears below in Eq. (B4), and represents the portion of solar energy absorbed on light surfaces. If the light weight surface is an envelope wall, the U-value of the wall is computed by conventional methods. The sum of U-value times areas for all thermally light elements, including windows, is added to the heat loss rate due to infiltration, given by the heat capacity of air (0.018 Btu/^{o}F-ft^3 at sea level) times the volume of the building (in ft^3) times the air change rate (in air changes per hour), to produce a single term, called $\hat{U}_q$, which describes all quick heat transfer. Heat losses through massive elements are already taken into account in the R_2 functions.

In most cases, only a few sets of response functions need be evaluated. For example, if both walls and floor are made of masonry with similar thermal properties, they can be combined into a single surface. No distinction need be made between directly solar-illuminated materials and those in the shade -- all that is required is that the <u>total</u> solar absorption on the surface be correctly specified (as a fraction of total solar heat gain S).

We combine the surface temperature results into an expression for room temperature using a heat balance for the room air. This is given by:

$$\sum_{j=1}^{N} \hat{h}_j (T_R - T_{sj}) + \hat{U}_q (T_R - T_A) = H + \alpha_R S \tag{B4}$$

where

$\hat{h}_j = h_j A_j$,

H is the heater output,

α_R is the fraction of sunlight absorbed directly into the room air or on the surfaces of light-weight objects (e.g., upholstery),

$\hat{U}_q$ is the quick heat transfer coefficient, the sum of U values time areas for all pure conductances (e.g., windows) plus the loss rate due to infiltration.

This heat balance says that heat losses from the room air to material surfaces plus losses directly to the outside air are equal to heat gains from the heater or from solar absorption on light material surfaces (which conduct immediately into the room air).

We can use Equations (B4) and (B3) to derive the room temperature; at any frequency, the amplitude of room temperature is given by:

$$T_R \cdot A(\omega) = S \cdot B(\omega) + T_A \cdot C(\omega) + H \tag{B5}$$

where

$$A(\omega) = \sum_{j=1}^{N} \hat{h}_j \, (1-h_j \, R_{1j}) + \hat{U}_q \quad , \tag{B6a}$$

$$B(\omega) = \sum_{j=1}^{N} \alpha_j \, h_j \, R_{1j} + \alpha_R \ , \quad \text{and} \tag{B6b}$$

$$C(\omega) = \sum_{j=1}^{N} \hat{h}_j \, R_{2j} + \hat{U}_q \ . \tag{B6c}$$

For the case of Trombe wall buildings, an additional term is included in each of the three building response functions; this term is not presently computed in the programs.

The building response functions are combined with weather data to give an expression for room temperature at each hour. Weather data are expressed in idealized (approximate) form, as simple sine waves. Ambient temperature is given by

$$T_A(t) = \bar{T}_A + |\Delta T_A| \cos\left(\omega_o(t - t_\phi)\right) = \bar{T}_A + \underset{\sim}{\Delta T_A} \, e^{i\omega_o t} \tag{B7}$$

where

$\bar{T}_A$ is the average temperature for the day,

$|\Delta T_A|$ is the amplitude of diurnal temperature variation,

t_ϕ is the number of hours from sunrise until the hour at which temperature reaches its maximum,

$\underset{\sim}{\Delta T_A}$ is the complex number given by $|\Delta T_A| \, e^{-i\omega_o t_\phi}$, and

$\omega_o = 2\pi/24$ hrs.

Solar heat gain is given by a half-sine wave corresponding to a period of t_d hours of sunlight per day.

$$S(t) = \begin{cases} |S_1| \sin \omega_1 t & \text{daytime} \\ 0 & \text{night} \end{cases} \tag{B8}$$

where

$$\omega_1 = \pi/t_d \quad .$$

To perform the calculations, S(t) is Fourier analyzed into components at frequencies of zero, one, two, and three cycles per day. Further terms are unnecessary for two reasons: their size is smaller than the first few terms, and the building response functions reduce their effect on room temperature. This Fourier analysis allows us to express S(t) as:

$$S(t) = |S_1| \sum_{n=0}^{3} d_n e^{in\omega_0 t} \tag{B9}$$

where $\omega_0 = 2\pi/24$ hours
and t is measured beginning at sunrise.

This expression requires that

$$d_n = \frac{\omega_0 t_d}{\pi^2} \qquad n=0$$

$$\frac{\omega_0}{t_d} \frac{1 + e^{-in\omega_0 t_d}}{\left(\frac{\pi}{t_d}\right)^2 - (n\omega_0)^2} \qquad n \neq 0 \quad . \tag{B10}$$

The programs calculate d_n and evaluate T_R using the equation

$$T_R(t) = |S_1| \sum_{n=0}^{3} \frac{B(n\omega_o)}{A(n\omega_o)} d_n e^{in\omega_o t} + \bar{T}_A + \Delta \underset{\sim}{T}_A \frac{C(\omega_o)}{A(\omega_o)} e^{i\omega_o t} . \qquad (B11)$$

If a heater is present, then T_R is increased by the constant H/A(0) where H is the heater output and A(0) turns out to equal the conventional design heat loss of the building per degree.

Notes and References

B1. Simulating radiant energy interchange in buildings in very complex. One computer program which performs this calculation is G.P. Mitalas and D. G. Stephenson, "Fortran IV Programs to Calculate Radiant Energy Interchange Factors." Computer Program No. 25 of the Division of Building Research, National Research Council of Canada, Ottawa, Canada, 1966.

B2. D. B. Goldstein. Some Analytic Models of Passive Solar Building Performance. Lawrence Berkeley Laboratory, LBL-7811, November, 1978, and Garland Press, New York City, 1979. See Section 2.2.5 and Appendix 2.4.

B3. If the material consists of several layers, a separate diffusion equation is needed for each layer; but in practice, all layers beyond the first (inside) layer can usually be modeled as pure resistances, as is done in the programs.

Table B1. Equations for response functions.

Materials Response Functions R_1 and R_2:

$$R_1(\omega) = \frac{\cosh kd + \frac{1}{RKk} \sinh kd}{(h+\frac{1}{R}) \cosh kd + (Kk + \frac{h}{RKk}) \sinh kd}$$

$$R_2(\omega) = \frac{\frac{1}{R}}{(h+\frac{1}{R}) \cosh kd + (Kk + \frac{h}{RKk}) \sinh kd}$$

where

K is the thermal conductivity of the material,

d is the thickness of the material,

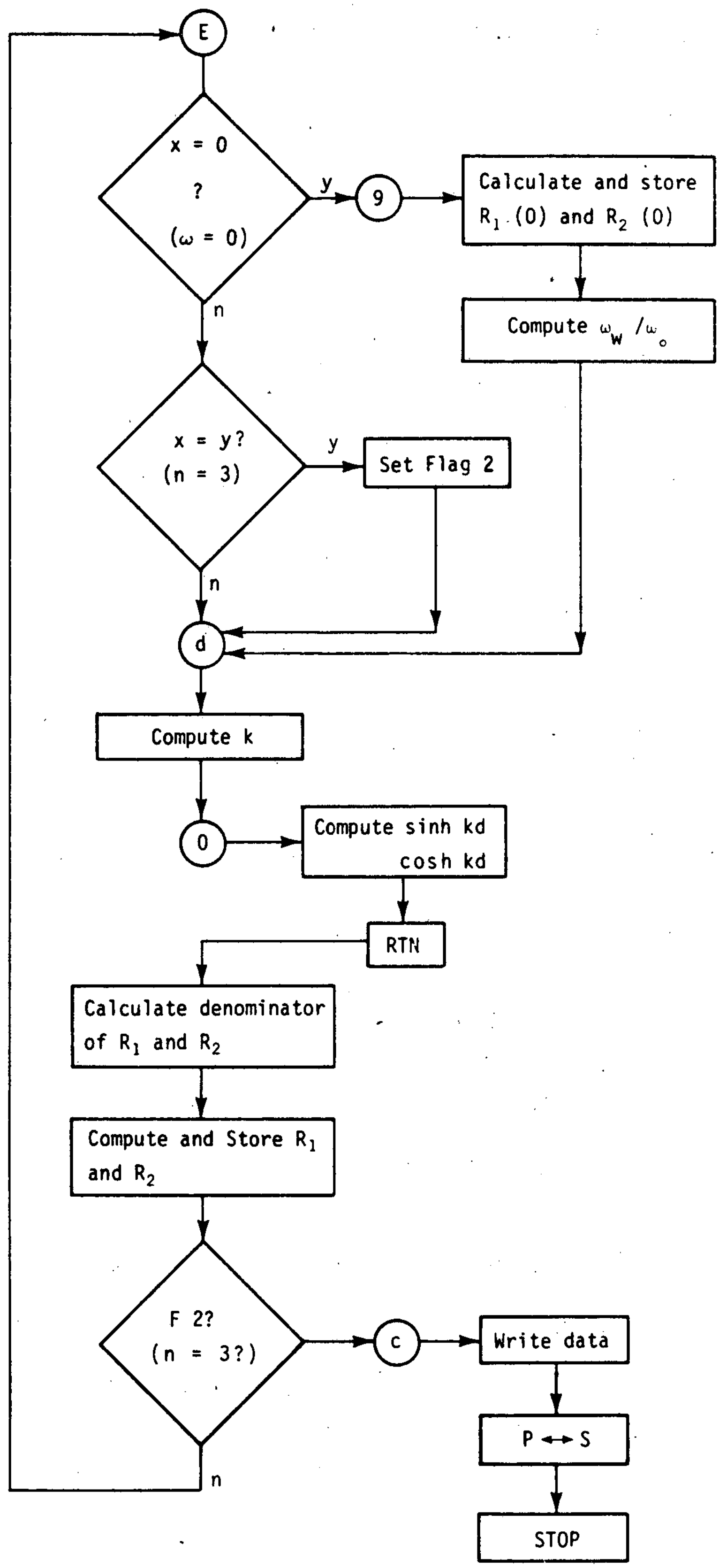
R is the thermal resistance of the exterior insulating layer, $(R = U_r^{-1})$,

$k = \sqrt{i\omega\rho c/K}$ with ρc = the volumetric heat capacity of the material, and

h is the film heat transfer coefficient for the surface.

APPENDIX C: Detailed Description of the Programs

R_{12} Program



XBL 7910-4517

R_{12} Sub-Program

Calculates $R_1(\omega)$ and $R_2(\omega)$ for a surface, at $\omega = 0,\ \omega_w,\ \omega_o,\ 2\omega_o,\ 3\omega_o$

Inputs:	STO	1	K
	STO	2	ρc
	STO	3	d
	STO	4	h
	STO	5	U_r
	STO	A	ω_o
	STO	B	ω_o/ω_w (or 1; can't be zero)
	STO	I	0

Outputs: (encoded on data card)

STO	0	$R_1(0)$
STO	1	$R_2(0)$
STO	2	$R_1(\omega_w)$
STO	3	$R_2(\omega_w)$
STO	4	$R_1(\omega_o)$
STO	5	$R_2(\omega_o)$
STO	6	$R_1(2\omega_o)$
STO	7	$R_2(2\omega_o)$
STO	8	$R_1(3\omega_o)$
STO	9	$R_2(3\omega_o)$

Outputs are decoded with subroutine D of ABC or T_R programs. Note that outputs are calculated in registers 10-19 and switched to the primary registers for readout. Note also that $R_2(2\omega_o)$ and $R_2(3\omega_o)$ will not be used in subsequent calculations.

R_{12} Sub-Program: Subroutines

A- Complex addition: $R_1e^{i\theta_1} + R_2e^{i\theta_2}$

Requires STO D and STO E

Input: x R_1

y θ_1

z R_2

t θ_2

Output: complex sum (polar) R in x, θ in y.

B- Complex multiplication: $R_1e^{i\theta_1} \times R_2e^{i\theta_2}$

input, output, and storage requirements like A.

0- Cosh and sinh: Calculates cosh r (1+i) and sinh r (1+i) for r real. Requires STO C, D, E; calls subroutine A

Input: x r

Output: STO 6-7 cosh r (1+i) (polar)

STO 8-9 sinh r (1+i) (polar).

C- Complex encode: Concatenates two components (R and θ) of a complex number into one number for storage. Decoding is accurate to 5-place accuracy; can be done with subroutine D of ABC and T_R programs. Valid for R $\tilde{<}$ 10 and $\theta <$ 10 radians.

Loses one significant figure for R $\gtrless 10^{\pm 1}$; two for R $\gtrless 10^{\pm 2}$, etc.

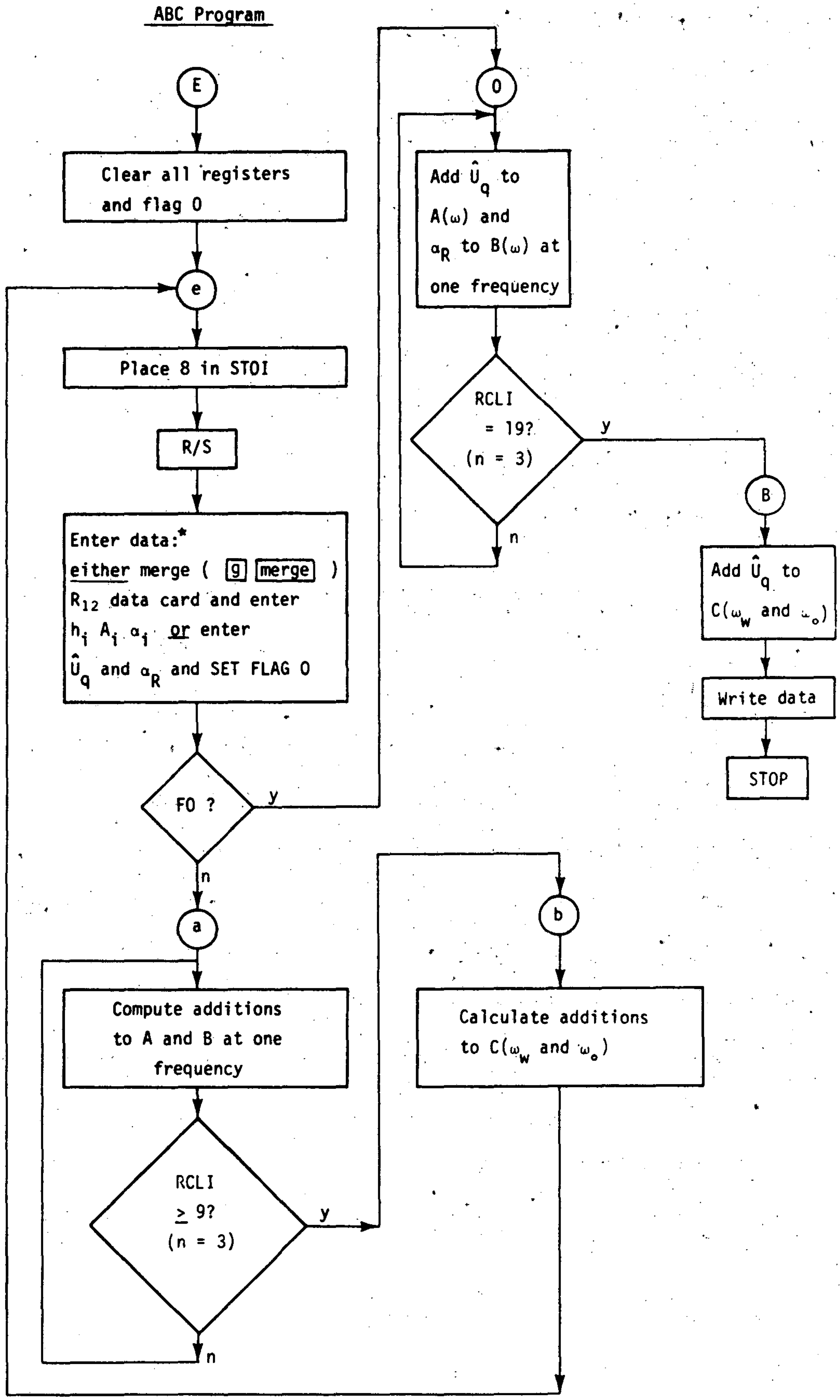
Accuracy is lost from θ for large R and from R for small R.

requires LBL 1 and LBL 2

Input: x - R

y - θ

Output: x - encoded number.



* - performed by user

XBL 7910-4515

ABC Sub-Program

Calculates $A(0,\omega_w,\omega_o,2\omega_o,3\omega_o)$, $B(0,\omega_w,\omega_o,2\omega_o, 3\omega_o)$, and $C(\omega_w,\omega_o)$ given R_1 and R_2.

Inputs: R_{12} data card and h_i A_i α_i for each surface

Data card fills STO 0 - STO 8

STO A - h_i

STO B - A_i

STO C - α_i

Outputs: (encoded on data card)

STO	0	A(0)
STO	1	B(0)
STO	2	$A(\omega_w)$
STO	3	$B(\omega_w)$
STO	4	$A(\omega_o)$
STO	5	$B(\omega_o)$
STO	6	$A(2\omega_o)$
STO	7	$B(2\omega_o)$
STO	8	$A(3\omega_o)$
STO	9	$B(3\omega_o)$
STO	10	$C(\omega_w)$
STO	11	$C(\omega_o)$

A's and C's are decoded with 'd' subroutine; B's are decoded with 'D'.

Note that program reverses $p \longleftrightarrow s$ before output is finished, so $A(0) \rightarrow B(3\omega_o)$ are accumulated in STO 10-19.

$C(\omega_w)$ is accumulated in STO 9.

$C(\omega_0)$ is accumulated in STO 7, but transferred to STO E during data or card entry.

All storage registers are used.

ABC Sub- Program: Subroutines

A- Complex addition: $R_1 e^{i\theta_1} + R_2 e^{i\theta_2}$

requires STO D and STO E

Input: x R_1

y θ_1

z R_2

t θ_2

Output: complex sum (polar form in x-y)

C- Complex encode:

Concatenates two components (R and θ) of a complex number into one number to 5-digit accuracy.

Good for $R \lesssim 10$ and $\theta < 10$ radians.

Loses one significant figure to $R \gtrless 10^{\pm 1}$, two for $R \gtrless 10^{\pm 2}$, etc.

Accuracy is lost from θ for large R and from R for small R.

Requires LBL 1, LBL 2, and LBL 3.

Input: x - R

y - θ

Output: encoded number - x

C- complex encode

like C only for $R \lesssim 1000$ and $\theta < 10$.

D- Complex decode - inverse of C.

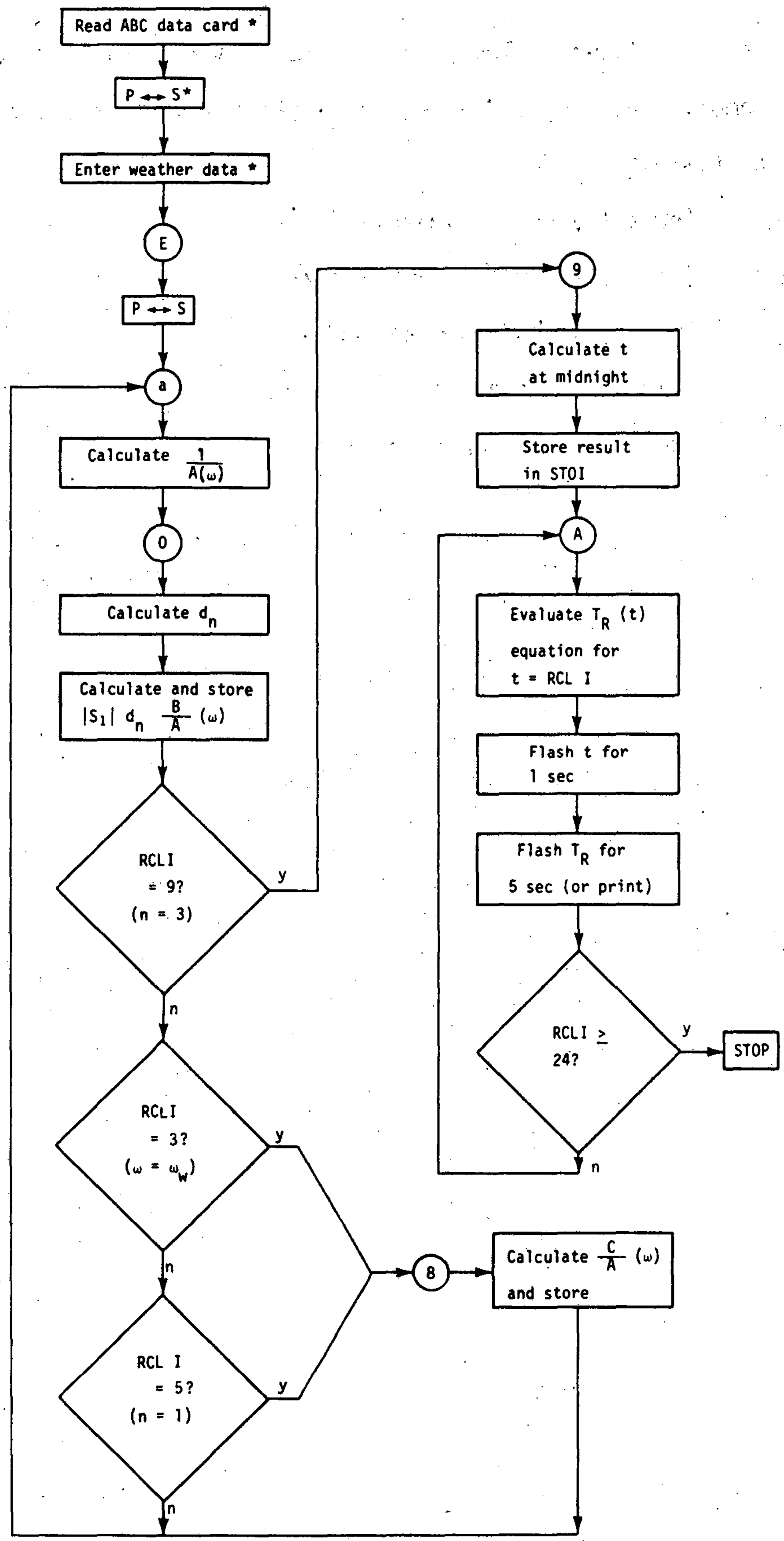
d- inverse of c.

D and d both require flag 2, LBL 4, LBL 5, LBL 6, STO D, and STO E.

D and d are structured so as to preserve the contents of the y and z stack registers. Thus, two complex numbers can be recalled from memory, decoded, and added or multiplied using these routines, as follows:

```
RCL 1   (recall encoded number)
 D      (decode and place in x and y)
RCL 2   (recall second encoded number)
 D      (decode and place in x-y; number from RCL 1 now in z-t)
 A      (add the two complex numbers)
```

T_R (t) Program



* Performed by user

XBL 7910-4516

$T_R(t)$ Sub-Program

Evaluates coefficients of $e^{in\omega_o t}$ in the following equation:

$$T_R(t) = |S_1| \sum_{n=0}^{3} \frac{B(n\omega_o)}{A(n\omega_o)} d_n e^{in\omega_o t} + \Delta T_A \frac{C(\omega_o)}{A(\omega_o)} e^{i\omega_o t} + \overline{T}_A$$

$$+\left(|S_1| \frac{B(\omega_w)}{A(\omega_w)} d_o e^{i\omega_w t} + \frac{C(\omega_w)}{A(\omega_w)} e^{i\omega_w t} \right) \quad \text{(C1)}$$

(Terms in parenthesis are not correct, but are evaluated in the form given.)

Evaluates $T_R(t)$ using Eq. (C1) (but not the part in parentheses) for each hour of the day, beginning at solar midnight.

Inputs: ABC data card fills STO 0-11

STO	I	0
STO	15	$\|\Delta T_A\|$
STO	16	t_ϕ
STO	17	t_d
STO	18	$\|S_1\|$
STO	19	$\overline{T}_A$
STO	C	ω_o

Outputs: Time (t) at midnight flashed for 1 sec, then room temperature

$T_R(t)$ flashed for 5 sec (or printed)

Time (t) at 1 a.m....

Iterates for $\tilde{>}$ 24 hours.

Outputs of $T_R(t)$:

STO	0	$\lvert S_1 \rvert\ d_o \cdot \frac{B}{A}\ (0)$
STO	2-3	$\lvert S_1 \rvert\ d_o\ \frac{B}{A}\ (\omega_w)$
STO	4-5	$\lvert S_1 \rvert\ d_1\ \frac{B}{A}\ (\omega_o)$
STO	6-7	$\lvert S_1 \rvert\ d_2\ \frac{B}{A}\ (2\omega_o)$
STO	8-9	$\lvert S_1 \rvert\ d_3\ \frac{B}{A}\ (3\omega_o)$
STO	10-11	$\frac{C}{A}\ (\omega_w)$
STO	12-13	$\frac{C}{A}\ (\omega_o)$

Subroutines:

D- decodes complex numbers $\sim 10^0$ (e.g., $B(\omega)$)

requires flag 2, LBL 4, LBL 5, LBL 6, STO D and STO E

Input: encoded number in x

Output: complex number (polar) in x-y

d- like D; decodes complex numbers $\sim 10^2$ (e.g., $A(\omega)$, $C(\omega)$)

B- complex multiply $R_1 e^{i\theta_1} \times R_2 e^{i\theta_2}$

requires STO D, E

Input; x R_1

y θ_1

z R_2

t θ_2

Output: complex product (polar) in x-y

D- calculates d_n using contents of STO I for n,

requires STO D

Input: STO I; STO I = 1,3; calculates d_o

	5	d_1
	7	d_2
	9	d_3

STO C - ω_o

STO 17 - t_d

Output: d_n in x-y

A- calculates $T_R(t)$ given coefficients of Eq. (B11) in STO 10 - STO 19 and STO 0 - STO 3, as described in "$T_R(t)$ Program Output" above, with primary and secondary storage registers reversed, and

$|\Delta T_A|$ in STO 5

t_ϕ in STO 6

$\bar{T}_A$ in STO 9

ω_o in STO C

t in STO I

Iterates until t > 24 hr.

Sub-Program PSA-1

PSA-1 stores weather and building data in memories 20-46. It then computes and prints the steady-state temperature terms that arise from the heater, ambient temperature, and solar gain. The sum is printed last. Finally the Fourier components for the solar gain component can be calculated, but are not stored.

Procedure:

LBL E (46-59) sets the counter at 20 + (i x 7) where i is the index of the surface (equal to 0 for the weather terms).

LBL A (60-66) stores an input parameter at the location specified by the counter and adds one to the counter.

If the user wishes to change a parameter the surface number is re-entered (LBL E) and then the parameter number and its new value and entered via LBL B (67-76). LBL B also adds one to the counter so further changes can be made through LBL A. After all parameters are entered the number of surfaces used is stored via LBL E.

LBL C (77-233) computes, stores, and prints the steady-state components from the input parameters entered in LBL E and LBL A. Steps 77-90 are used for initialization. A loop from steps 91-140 computes $A_j(0)$, $B_j(0)$ and U_j for each surface j and keeps a running sum. A subroutine call (93-94) to SBR RCL (00-45) is used to swap the locations of the jth surface parameter with those of the first surface so that direct address arithmetic can be used in the loop. A second call at 135-136 swaps the locations back again. Steps 142-233 print an identifier for each steady-state term and compute and print their values. The hours of sunshine must be entered at step 174-175 and is echoed by print-out at steps 180-183.

The formulas used in this section are:

$$A_i(0) \equiv (\text{Area})_i \left(\left(\frac{1}{U_r}\right)_i + \frac{1}{h_i} + \frac{d_i}{K_i} \right)^{-1} \qquad (95\text{-}118)$$

$$A(0) = \sum_{i=1}^{i_{max}} A_i(0) + \hat{U}_q$$

(A(0) was initialized $\hat{U}_q$ is steps 83-86).

$$B_i(0)\ (1 - U_i/h_i)\ \alpha_i$$

(119-132, with U_i computed in steps 110-112. U_i is the U-value of the i^{th} construction section.)

$$B(0) = \sum_{i=1}^{i_{max}} B_i(0) + \alpha_R$$

(B(0) was initialized to α_R in steps 87-90)

$$d_o = t_d/12\pi$$

(174-194 with input and echo to print of t_d)

$$T_H = H(0)/A(0)$$

(142-160 with print commands)

$T_S = |S_1|\, d_o B(0)/A(0)$ (195-219 with print command)

The definition of the variables can be found in the input list. In addition to the above calculations LBL C also recalls and display T_A (161-173) and prints $T_R = T_H + T_A + T_S$ (220-233)

LBL D computes the Fourier components for sunshine from the following formula

$$d_n = \frac{\omega_1}{12(\omega_1^2 - (n\pi/12)^2)} \qquad (1 + e^{-in\omega_o t_d})$$

where $\omega_o = \pi/12$ and $\omega_1 = \pi/t_d$

n is entered in steps 236-237 and the first factor accumulated in steps 236-262. The master-library routines for complex exponentiation and multiplication are then used in steps 273-290 to compute d_n. Steps 291-316 generate the label and calculate d_n in both polar and rectangular form. Only the latter is automatically printed.

LBL D' (317-339) comutes and displays the zeroth Fourier component $d_o = \omega_o/\pi\omega_1 = t_d/12\pi$

Storage registers:

10 - ΣA_j (0)	27 d_1
11 - ΣB_j (0)	28 $(1/U_r)_1$
12 - T_H (0)	⋮
13 - T_S (0)	34 d_2
14 - T_R (0)	⋮
20 - S	41 d_3
21 - $\hat{U}_q$	⋮

Sub-Program PSA-2

PSA-2 calculates the frequency dependence of the response functions. From these functions the time dependent temperature terms can be calculated and summed with specified phase lags between them. The results for every other hour of a sample day are summed into memories 48-59.

Procedure:

LBL A (94-320): Computes the material response functions, and the building response functions for a specified ω. ω is calculated in steps 94-104 from the period entered by the user. After counters and initializations are performed (95-125) the imaginary and real components of the R_1, R_2 A, B and C are calculated in the loop from 126-320. The subroutine calls 128-129 and 313-374 to SBR RCL(00-45) perform the identical purpose as in LBL C of PSA-1. The formulas for R_1 and R_2 are taken from Table B1. R_1 is calculated in steps 130-219 and 238-254.

The complex hyperbolic functions cosh and sinh are evaluated by a subroutine call to SBR cos (58-93) which uses the master library's complex trigonometric functions and the relationships cosh z = cos iz and sinh z = - i sin iz. The numerator is calculated in steps 130-180 and the denominator is calculated in re-arranged form in steps 181-219. R_1 (ω) is then calculated with a complex divide in steps 238-254.

$R_2(\omega)$ is calculated in steps (130-237). Since $R_2(\omega)$ is so similar to $R_1(\omega)$ the initial sums are carried out in the same location. The final complex divide is calculated in steps 220-237.

A 'R/S' is present at step 254 so that the $R'(\omega)$ can be examined if it is so desired. $A(\omega)$, $B(\omega)$, and $C(\omega)$ are computed as:

$$A(\omega) = \sum_{j=1}^{j_{max}} A_j h_j (1-h_j R_{1j}) + \hat{U}_q \qquad (273\text{-}302)$$

$$B(\omega) = \sum_{j=1}^{j_{max}} (\alpha_j h_j R_{1j}) + \alpha_R \qquad (255\text{-}272)$$

$$C(\omega) = \sum_{j=1}^{j_{max}} (A_j h_j R_{2j}) + \hat{U}_q \qquad (273\text{-}294 \text{ and } 303\text{-}310)$$

where the A_j are the $_j$th sunface areas. Steps 311-320 merely control the loop.

LBL B (410-429) computes a frequency dependent weather term;

$$T_R(\omega) = \Delta T_A \, C(\omega)/A(\omega).$$

The complex divide $(C(\omega)/A(\omega))$ is computed by a call (424-425) to SBR '÷' (321-339) which divides a complex number in registers 01 and 02 by $A(\omega)$ via the complex divide routine of the master library.

LBL C (430-458) computes a frequency dependent solar term:

$$\Delta T(\omega) = |S_1| \frac{d_n B(n\omega_o)}{A(n\omega_o)} \qquad (438\text{-}455)$$

The real and imaginary components of d_n and $|S_1|$ are entered in steps 430-438. If a weather frequency solar term is examined, substitute the magnitude of this term for $|S_1|$ and use d_o in place of d_n. The complex divide by $A(n\omega_0)$ is performed by a call to SBR '÷'.

Both LBL B and LBL C can call SBR PRT (375-404). SBR PRT prints hourly temperature increments over the period of the term.

LBL E (340-374): This subroutine calculates bihourly temperature increments for a given frequency dependent solar or weather term. The results are summed into registers 48-59. The phase of the term (ϕ) is entered at the beginning. The subroutine calculates the temperatures in a loop from the formula:

$$T(t) = \text{Real part}\left(\Delta T e^{i\omega(t + \phi)}\right) \quad (352\text{-}373)$$

The ΔT from LBL C or LBL D are complex numbers. LBL E calls SBR X (46-57) to perform the complex multiply.

LBL D (459-466): LBL D advances the paper and prints the contents of registers 48-59.

- A note on speeding up the program. If the program is not intended to be modified it can be noticeably sped up by replacing labels by absolute addresses. In particular, the loops to LBL SIN and LBL LIST in LBL B and LBL PRT respectively, are very slow. Delete LBL SIN (352-353) and insert one pause. Insert one step at 370 and rewrite the decrement statements as DSZ, 9, 352 (steps 370-373). Do the same with LBL LIST. Since these labels are called many times, this replacement is very noticeable in terms of program execution time.

APPENDIX D: Long Term Weather Responses

To evaluate the effect of multi-day cycles of temperature and sunlight, we assume that the cycle can be described by sinusoidal terms. Ambient temperature is taken to be of the form.

$$T_A(t) = \bar{T}_A + |\Delta T_{A_W}| \, e^{i\omega_W(t-t_a)} + \Delta \underset{\sim}{T}_A \, e^{i\omega_o t} \tag{D1}$$

where ω_W is the frequency at which weather variations take place (typically 2π/1 week),

$|\Delta T_{A_W}|$ is the amplitude of weather-variation of temperature,

t_a is the time at which the ambient temperature is at its maximum,

and $\bar{T}_A$ is now the average temperature over the whole cycle.

Solar gain is still taken as a half sine-wave for each day, but the amplitude is assumed to be sinusoidally modulated, as shown:

$$S(t) = \begin{cases} \left(\bar{S} + \Delta S_W \cos\omega_W \, (t-t_s)\right) \sin \omega_1(t-t_{sr}) & \text{day} \\ 0 & \text{night} \end{cases} \tag{D2}$$

where

$\bar{S}$ is the average amplitude of solar gain (average $|S_1|$),

ΔS_W is the amplitude of modulation of $|S_1|$ over the cycle,

t_s is the time at which the solar energy in the cycle is at its maximum, and

T_{sr} is the time of the most recent sunrise.

It can be shown [D1] that room temperature is then given by:

$$T_R(t) = (S + \Delta S_w \cos\omega_w (t-t_s)) \sum_{n=1}^{3} \frac{B(n\omega_o)}{A(n\omega_o)} d_n e^{in\omega_o t} + \bar{S} \frac{B(0)}{A(0)} d_o$$

$$+ \Delta S_w \frac{B(\omega_w)}{A(\omega_w)} d_o e^{i\omega_w (t-t_s)} + \bar{T}_A + \Delta T_{A_w} \frac{C(\omega_w)}{A(\omega_w)} e^{i\omega_w (t-t_a)}$$

$$+ \underset{\sim}{\Delta T_A} \frac{C(\omega_o)}{A(\omega_o)} e^{i\omega_o t} + \frac{H}{A(0)} \quad . \qquad \text{(D3)}$$

Comparing Eq. (B11) with Eq. (D1), it can be seen that if we set $S_1 = \bar{S} + \Delta S_w \cos \omega_w (t-t_s)$, Eq. (D1) says

$$T_R(t) = (\text{old terms}) + (\bar{S} - |S_1|) \frac{B(0)}{A(0)} d_o$$

$$+ \Delta S_w \frac{B(\omega_w)}{A(\omega_w)} d_o e^{i\omega_w (t-t_s)} + \Delta T_{A_w} \frac{C(\omega_w)}{A(\omega_w)} e^{i\omega_w (t-t_a)} \quad . \qquad \text{(D4)}$$

Thus only three new terms and only two new combinations of Building Response functions are needed to account for multi-day weather cycles.

Notes and References

D1. D. B. Goldstein, Some Analytic Models of Passive Solar Building Performance. Lawrence Berkeley Laboratory, LBL-7811, November, 1978, and Garland Press, New York City, 1979. See Section 2.4 and Appendix 2.4.

APPENDIX E: Example Problem Using the Program

This section sets up a simple problem of modelling a passive solar house on a sunny winter design day, and computes the input parameters to the programs. Intermediate and final results are given. The user can employ this example to check the answers he computes using the programs.

The example house is a single-story wood-frame residential structure of conventional American construction. Its only passive solar features are its bare (or tile-covered) slab-on-grade floor and its large south-facing collector window area. As the results in Table E.4 show, it should not be considered "optimized."

The use of a wood-frame house for the example illustrates some of the simplifications that can be made in using the model to describe a house. In principle, seven material surfaces would be needed for a model of this building: the stud portion of envelope walls, of partition walls, and of the ceiling; the cavity portion of envelope walls, partition walls, and ceiling; and the floor. But to increase computational speed and convenience, in this problem we use only three surfaces. This procedure can be shown to lead to less than 1°F errors.

Consider a 30 X 50 foot single-story house with 8-foot ceiling. The walls have R-11 insulation between 2 X 4 studs; the ceiling uses R-30 insulation and 2 X 6 joists; while the floor is a bare concrete slab on grade. There are 250 ft^2 of south-facing double-glazing and 30 ft^2 of glazing on each of the other elevations. We assume 1.5 ft^2 of partition wall per ft^2 of envelope wall.

We consider three surfaces:

1) Wall and ceiling studs (25% of envelope wall area,
+ 15% of partition wall area
+ 10% of ceiling area)

2) Wall and ceiling cavities

3) Floor

For the first two, we consider a wall section composed of the following layers (from inside to outside)

1) Inside film resistance
$R = .68$ hr - ft^2- ^{o}F/Btu (walls)
.61 (ceiling)

2) Gypsum wall board
$K = .075$ Btu/^{o}F-ft-hr $\rho c = 13$ Btu/ft^3 - ^{o}F
$d = 1/24$ ft

3) Insulation
R - 11 wall
R - 30 ceiling

or

studs: $K = .068$ Btu/^{o}F - ft - hr $\rho c = 9$ Btu/ft^3 - F
d = .292 envelope wall
= .146 partition wall
= .458 ceiling

4) (Ceiling studs only) Insulation over studs R - 11

5) Stucco plus exterior film coefficient R = .41

or attic (for the ceiling) $R \cong 3$

Areas are:

1) Studs: $.25 \times (\underbrace{8 \times (30 + 50) \times 2}_{\text{gross wall area}} - \underbrace{(250 + 3 \times 30)}_{\text{window area}})$

$+ .15 \times 1.5 \times 8 \times (30 + 50) \times 2$

$+ .10 \times 1500$

$= 235 + 288 + 150$

$= 673 \text{ ft}^2$

2) Cavities $.75 \times (8 \times (30 + 50) \times 2 - (250 + 3 \times 30) + .85 \times 1.5$

$\times 8 \times (30 + 50) \times 2 + .90 \times 1500$

$= 705 + 1632 + 1350$

$= 3687 \text{ ft}^2$

3) Floor: 1500 ft^2

For two-layer wall approximation; we average the parameters as follows:

1) Studs

The massive layer has the average properties of the gypsum board and the studs.

For the envelope walls, we take the weighted average of 1/K and ρc for 1/2" of gypsum and 3 1/2" of wood

$$K_e = ((1/2" \times \frac{1}{.075} + 3\ 1/2" + \frac{1}{.068}) \div 4")^{-1} = .0688$$

$$(\rho c)_e = (1/2" \times 13 + 3\ 1/2" \times 9) \div 4" = 9.5$$

$$d_e = 1/3 \text{ ft}$$

For the ceiling, we take the weighted average of 1/K and ρc for 1/2" gypsum and 5 1/2" wood:

$$K_c = (1/2" \times \frac{1}{.075} + 5\ 1/2" \times \frac{1}{.068}) \div 6")^{-1} = .06853$$

$$(\rho c)_c = (1/2" \times 13 + 5\ 1/2" \times 9) \div 6" = 9.333$$

The thickness d is 1/2 foot

For the partition walls, we do the same average for 1 3/4" of wood (half the stud thickness)

$$K_p = (1/2" + \frac{1}{.075} \times 1\ 3/4" \times \frac{1}{.068}) \div 2\ 1/4")^{-1} = .06944$$

$$(\rho c)_p = (1/2" \times 13 + 1\ 3/4" \times 9) \div 2\ 1/4" = 9.889$$

The thickness d is 2 1/4 inches or .1875 ft

We take the area-weighted average of K, ρc, d, and h to derive the properties of the first material (the studs).

$$K_1 = (235\ ft^2 \times .0688 + 288\ ft^2 \times .06944 + 150\ ft^2 \times .06853) \div 673\ ft^2$$

$$= .06901\ Btu/^oF\text{-}hr\text{-}ft$$

$$(\rho c)_1 = (235\ ft^2 \times 9.5 + 288\ ft^2 \times 9.889 + 150\ ft^2 \times 9.333) \div 673\ ft^2$$

$$= 9.629\ Btu/^oF\text{-}ft^3$$

$$d_1 = (235\ ft^2 \times 1/3 + 288\ ft^2 \times .1875 + 150\ ft^2 \times .5) \div 673\ ft^2$$

$$= .3081\ ft$$

$$h_1 = (235\ ft^2 \times \frac{1}{.68} + 288\ ft^2 \times \frac{1}{.68} + 150\ ft^2 \times \frac{1}{.61}) \div 673\ ft^2$$

$$= 1.508\ Btu/ft^2\text{-}hr\text{-}^oF$$

$$A_1 = 673\ ft^2$$

To compute U_r, we require that the steady-state heat loss through the approximate version of material 1 is the same as for the exact case.

For the exact case, the heat loss rate is

$$U_eA_e + U_cA_c$$

or

$$\frac{1}{(\frac{.075}{1/24})^{-1} + (\frac{.068}{.292})^{-1} + .41} \times 235\ ft^2 + \frac{1}{(\frac{.075}{1/24})^{-1} + (\frac{.068}{.458})^{-1} + 11 + 3} \times 150\ ft^2$$

$= 51.22$ Btu/^oF-hr

For the model, the heat loss rate is

$$\left(\left(\frac{K_1}{d_1}\right)^{-1} + U_{r1}^{-1} \right)^{-1} A_1$$

so $$\left(\frac{A_1}{51.22} - \frac{d_1}{K_1} \right)^{-1} = U_{r1}$$

or $U_{r1} = .117$ Btu/^oF-hr-ft^2

Note that we have ignored the inside film resistance in computing U_r for both cases.

2) Cavities

The same process is followed. The massive layer is the gypsum board and the outside layer contains everything else. Since the massive layer has only one component, the averaging process is unnecessary and K_2, $(\rho c)_2$, and d_2 are just given by the parameters of gypsum board. A_2 is 3687 ft^2. We calculate the average h for this surface as the area-weighted average:

$$h_2 = (705\ ft^2 \times \frac{1}{.68} + 1632\ ft^2 \times \frac{1}{.68} + 1350\ ft^2 \times \frac{1}{.61}) \div 3687\ ft^2$$

$= 1.532$ Btu/ft^2-hr-^oF

We still have to calculate U_{r2}

For the exact case, the heat loss rate is

$$U_eA_e + U_cA_c$$

or

$$\frac{1}{\left(\frac{.075}{1/24}\right)^{-1} + 11 + .41} \times 705\ \text{ft}^2 + \frac{1}{\left(\frac{.075}{1/24}\right)^{-1} + 30 + 3} \times 1350\ \text{ft}^2$$

$$= \quad 99.15\ \text{Btu}/^{\circ}\text{F-hr}$$

For the model, the heat loss rate is

$$\left(\left(\frac{K_2}{d_2}\right)^{-1} + U_r^{-1}\right)^{-1} A_2$$

so

$$\left(\frac{A_2}{99.15} - \frac{d_2}{K_2}\right)^{-1} = U_{r2}$$

or

$$U_{r2} = .0273\ \text{Btu}/^{\circ}\text{F-ft}^2\text{-hr}$$

3) Floor

For the floor parameters, we assume that the slab floor is coupled to the ground, and that the ground-water migration is not important in the area of the house. The mean length of a path of heat flow from the floor through the ground to the outside air is on the order of 20 feet, so, we assume a 20-foot floor thickness. (The results are not sensitive to exact floor thickness.)

We use typical materials properties for concrete (and soil) of

$K = .8\ Btu/^oF\text{-}ft\text{-}hr$, $\rho c = 20\ Btu/^oF\text{-}ft^3$; $h = \frac{1}{.61}$

$= 1.639\ Btu/^oF\text{-}ft^2\text{-}hr$

For U_r, we take the outside film coefficient at the soil, $h = 6\ Btu/ft^2\text{-}^oF\text{-}hr$, though the results would not change if h were changed from this value.

To compute the α's, we use the "typical" values given in the text for a dark floor and light walls. Thus $\alpha_3 = .45$, $\alpha_R = .15$.

To compute α_1, and α_2, we average the solar gain on the envelope walls, partition walls, and ceiling over surfaces 1 and 2 on an area-weighted basis. The total solar gain absorbed on these surfaces is

$$.10 + .20 + .10 = .40, \text{ so } \alpha_1 = .40 \times \frac{673\ ft^2}{673\ ft^2 + 3687\ ft^2}$$

and

$$\alpha_2 = .40 \times \frac{3687\ ft^2}{673\ ft^2 + 3687\ ft^2} \text{ or } \alpha_1 = .062 \text{ and } \alpha_2 = .338$$

We next compute the final remaining parameter for the house, $\hat{U}_q$. This term is composed of infiltration losses (assumed to be at 0.6 air changes per hour) and window losses.

Infiltration loss rate is $(8 \times 30 \times 50)\ ft^3 \times 0.018\ Btu/^oF\text{-}ft^3 \times 0.6$ air changes/hr = 129.6 $Btu/^oF\text{-}hr$. Window losses through double glazing with $U = 0.49\ Btu/^oF\text{-}ft^2\text{-}hr$ (1/2" air space) are given by UA or $0.49 \times (250\ ft^2 + 3 \times 30\ ft^2) = 166.6\ Btu/^oF\text{-}hr$. So $\hat{U}_q = 296.2\ Btu/^oF\text{-}hr$.

Weather Parameters

The example we take describes a northern California climate on a relatively cool but clear winter design day in January. We set $\overline{T}_A = 45^0$ and $\Delta T_A = 10^0F$, so the daily high temperature is 55 and the low is 35. We set the length of day at nine hours ($t_d = 9$ hr). Maximum ambient temperature is at 3 pm or 7 1/2 hours after sunrise ($t_\phi = 7.5$ hr).

Solar gain is taken from Ashrae solar heat gain tables for 40°N. latitude for January and summed over all four elevations:

S:	$250\ ft^2$ X 1630 Btu/day	= 407,500 Btu/day
E & W:	2 X $30\ ft^2$ X 508 Btu/day	= 30,480 Btu/day
N:	$30\ ft^2$ X 118 Btu/day	= 3,540 Btu/day
	Total	441,520 Btu/day

This total is modified by a shading coefficient of about 0.85 for double-pane glass and multiplied by 0.9 to account for opaque window frame area, so total daily solar gain entering the house is 337,760 Btu/day. S_1 is given by: (daily solar heat gain) X $\frac{\pi}{2t_d}$ so S_1 = 58,950 Btu/hr.

We set $\omega_w = \frac{1}{10}\omega_o$ to calculate response functions; however, this will not be used in the calculation. To run this problem on the TI-59, the phase terms are: $\phi = -7.5$ for the solar terms and $\phi = -15$ for the temperature term.

This completes the derivation of the building and weather parameters. They are summarized in Table E.1.

Results of the Calculation

We next display intermediate results to check the operation of each of the programs. The parameters for each surface are used to compute R_1 and R_2. Table E.2 gives the values of the response functions for each of the three surfaces, while Table E.3 gives the building response functions. The room temperature results are given in Table E.4 under the assumption of no heater output. To include the effect of 2000 Btu/hr of internal loads, add $4^{o}F$ $\left(= 2000 \text{ Btu/hr}/A(0)\right)$ to each entry in Table E.4. (This effect is calculated directly in the TI-59 program).

As seen in the results from Table E.4, the response of the example building is not optimal. Afternoon temperatures are too warm and morning temperatures too cool. As an alternate to this design, one could consider the effects of more insulation and smaller collector area, or more thermal mass.

Table E.1

House Parameters for Example Problem

	Surface 1	Surface 2	Surface 3
K	.06901	.075	0.8
ρc	9.629	13	20
d	.3081	.04167	20
h	1.508	1.532	1.639
$U_r{}^*$	.117	.0273	6
A	673	3687	1500
α	.062	.338	.45

$\hat{U}_q = 296.2$

$\alpha_R = .15$

$\Delta T_A = 10$

$t_\phi = 7.5$

$t_d = 9$

$S_1 = 58950$

$\bar{T}_A = 45$

$^*U_r^{-1}$ is used for input in the TI-59 program.

Table E.2

Response Functions for the Example House*

R_1 (hr-^{0}F-ft^2/Btu)

ω	Surface 1	Surface 2	Surface 3
0	.6309	.6414	.5956
ω_0	$.5473e^{-i0.1851}$	$.6374e^{-i0.0891}$	$.2933e^{-i0.4385}$
$2\omega_0$	$.5022e^{-i0.2163}$	$.6258e^{-i0.1752}$	$.2372e^{-i0.5069}$
$3\omega_0$	$.4780e^{-i0.2443}$	$.608e^{-i0.2558}$	$.2064e^{-i0.54383}$

R_2

ω	Surface 1	Surface 2	Surface 3
0	.0484	.0172	.0236
ω_0	$.0291e^{-i1.2979}$	$.0171e^{-i0.1281}$	0
$2\omega_0$	$.0161e^{-i1.9614}$	$.0167e^{-i0.2531}$	0
$3\omega_0$	$.0102e^{-i2.4303}$	$.0162e^{-i0.3725}$	0

*Note that $e^x = e^{x-2\pi}$ for any x.

Table E.3

Building Response Functions for the Example House

ω	A(Btu/°F-hr)	B	C(Btu/°F-hr)
0	502.2	.9802	502.2*
ω_o	$2332.3e^{i0.514}$	$.7364e^{-i0.1783}$	$402.0e^{-i0.1016}$
$2\ \omega_o$	$2953.0e^{i0.562}$	$.6847e^{-i0.2231}$	‡
$3\ \omega_o$	$3493.4e^{i0.588}$	$.6504e^{-i0.2631}$	‡

*Set equal to A(0)

‡Not computed

Table E.4

Room Temperature as a Function of Time

t	Solar Time	T_R (no heater or internal loads)
-7.5	12 m	68.0
-6.5	1 am	67.1
-5.5	2	66.4
-4.5	3	65.9
-3.5	4	65.3
-2.5	5	64.6
-1.5	6	64.1
-.5	7	64.2
.5	8	65.5
1.5	9	68.4
2.5	10	72.7
3.5	11	77.7
4.5	12 n	82.1
5.5	1 pm	85.0
6.5	2	85.7
7.5	3	84.3
8.5	4	81.5
9.5	5	78.4
10.5	6	75.6
11.5	7	73.6
12.5	8	72.3
13.5	9	71.3
14.5	10	70.3
15.5	11	69.2
16.5	12 m	68.0

U.S.GPO:1980-698-101 F-101

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