Title
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Journal
San Francisco Estuary and Watershed Science, 6(3)

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Publication Date
2008

DOI
https://doi.org/10.15447/sfews.2008v6iss3art3

OCTOBER 2008

SAN FRANCISCO ESTUARY & WATERSHED SCIENCE

Published by the California Bay-Delta Authority Science Program and the John Muir Institute of the Environment

Sample Design-based Methodology for Estimating Delta Smelt Abundance

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ABSTRACT

A sample design-based procedure for estimating pre-adult and adult delta smelt abundance is described. Using data from midwater trawl surveys taken during the months of September, October, November, and December for the years 1990 through 2006 and estimates of size selectivity of the gear from a covered cod-end experiment, stratified random sample ratio estimates of delta smelt abundance were made per month. The estimation procedure is arguably an improvement over the dimensionless delta smelt indices that have been used historically in that (1) the volume sampled is used in a manner that leads to directly interpretable numbers and (2) standard errors are easily calculated. The estimates are quite imprecise, i.e., coefficients of variation in the range of 100% occurred. The point estimates are highly correlated with the monthly indices, and conclusions on abundance declines are quite similar. However, both the estimates and indices may suffer from selection biases if the trawl samples are not representative of the true densities. Future work is needed in at least three areas: (1) gathering additional information to determine the validity of assumptions made, in particular determining the possible degree of selection bias; (2) developing procedures that utilize survey data gathered from earlier life history stages, such as larval surveys; (3) embedding a life-history model into the population estimation procedure.

KEYWORDS

Gear selectivity, Horvitz-Thompson, *Hypomesus transpacificus*, ratio estimators, stratified random sampling

SUGGESTED CITATION

Newman KB. 2008. Sample design-based methodology for estimating delta smelt abundance. San Francisco Estuary and Watershed Science 6(3): article 3. Available from: http://repositories.cdlib.org/jmie/sfews/vol6/iss3/art3

INTRODUCTION

Delta smelt (*Hypomesus transpacificus*) is a fish endemic to upper (or northern) San Francisco Estuary

(Bennett 2005). It is a small (adult FL < 80 mm typically), short-lived (one to two years) fish. It was listed in 1993 as a threatened species under the Federal and California State Endangered Species Acts (USFWS 1993) and is of considerable public interest for both environmental and economic reasons.

A key survey that was used as supporting evidence for the threatened species listing is the fall midwater trawl (FMWT) survey, which is conducted during the months of September, October, November, and December in the Estuary. The survey, which samples for pre-adult (age 0) and adult (age 1) delta smelt as well as other fish species, began in 1967. Tows are taken once a month at around 100 locations or stations. The catches from these tows are used to construct an annual FMWT index for delta smelt abundance (http://www.delta.dfg.ca.gov/data/mwt/charts.asp). Declines in the annual FMWT index beginning in the 1980s (Sweetnam and Stevens 1993; USFWS 1993) led to the threatened species listing of delta smelt. Surveys at larval and juvenile life history stages have also indicated precipitous declines in abundance over the last twenty-plus years (Greiner and others 2007; Sommer and others 2007).

The annual FMWT index is the sum of four monthly indices. To calculate a monthly index, the sampling region is partitioned into fourteen areas or strata. Figure 1 shows the current configuration of sampling locations (stations) and areas. Within each area, the average number of fish caught per trawl is calculated. Letting $\bar{f}_{m,a}$ denote the average in month m and area a:

$$\bar{f}_{m,a} = \frac{1}{n_a} \sum_{s=1}^{n_a} f_{m,a,s}$$

where n_a is the number of stations in area a (generally constant between months) and $f_{m,a,s}$ is the number of fish caught during month m in area a at station s. The monthly index is a weighted sum of the $\bar{f}_{m,a}$, h=1,...,14, where the weights are estimates of water volume in each area (presumably the volume occupied by delta smelt) in ten thousands of acre feet. Letting w_a denote the weight for area a, the monthly index, denoted I_m, is

$$I_m = \sum_{a=1}^{14} w_a \bar{f}_{m,a}, \; m = Sep, Oct, Nov, Dec. \qquad (1)$$

The annual index is then

$$I = \sum_{m=Sep}^{Dec} I_m. \qquad (2)$$

The indices, both monthly and annual, may be somewhat difficult to interpret and are to some degree technically deficient, e.g., lacking measures of uncertainty, and these criticisms are discussed in the next section. A primary purpose of this article is to present a first step in the development of estimates of delta smelt abundance that are simpler to interpret, and are more statistically rigorous in the sense of clearly stated assumptions, use of standard survey sampling methodology, and inclusion of standard errors.

Before proceeding with criticism of the indices and presentation of the alternative estimation procedure, however, it should be emphasized that the ostensibly more rigorous statistical estimates of delta smelt presented herein do not differ in substantial ways from the FMWT indices, however technically flawed they might be. Relatedly, biases present in the new estimates are largely ones that the indices would share, particularly selection bias. From a management perspective, what is important is that both the indices and the new abundance estimates indicate a steady, consistent decline in the abundance of delta smelt (Sommer and others 2007).

I also emphasize that additional steps are needed, and are in process, to further develop estimation procedures, ones which incorporate life history processes and utilize data from surveys of other life history stages. Areas of future research and data analysis which could yield more statistically defensible and practically useful estimates of delta smelt abundance are presented at the end of the article.

CRITICISM OF THE INDICES

The first criticism is two-fold: (a) the units of the (monthly) indices are the sum of the product of water volumes and fish counts, rather than fish counts alone; (b) the area weights, w_a in equation (1), which are measures of water volume, are constant within

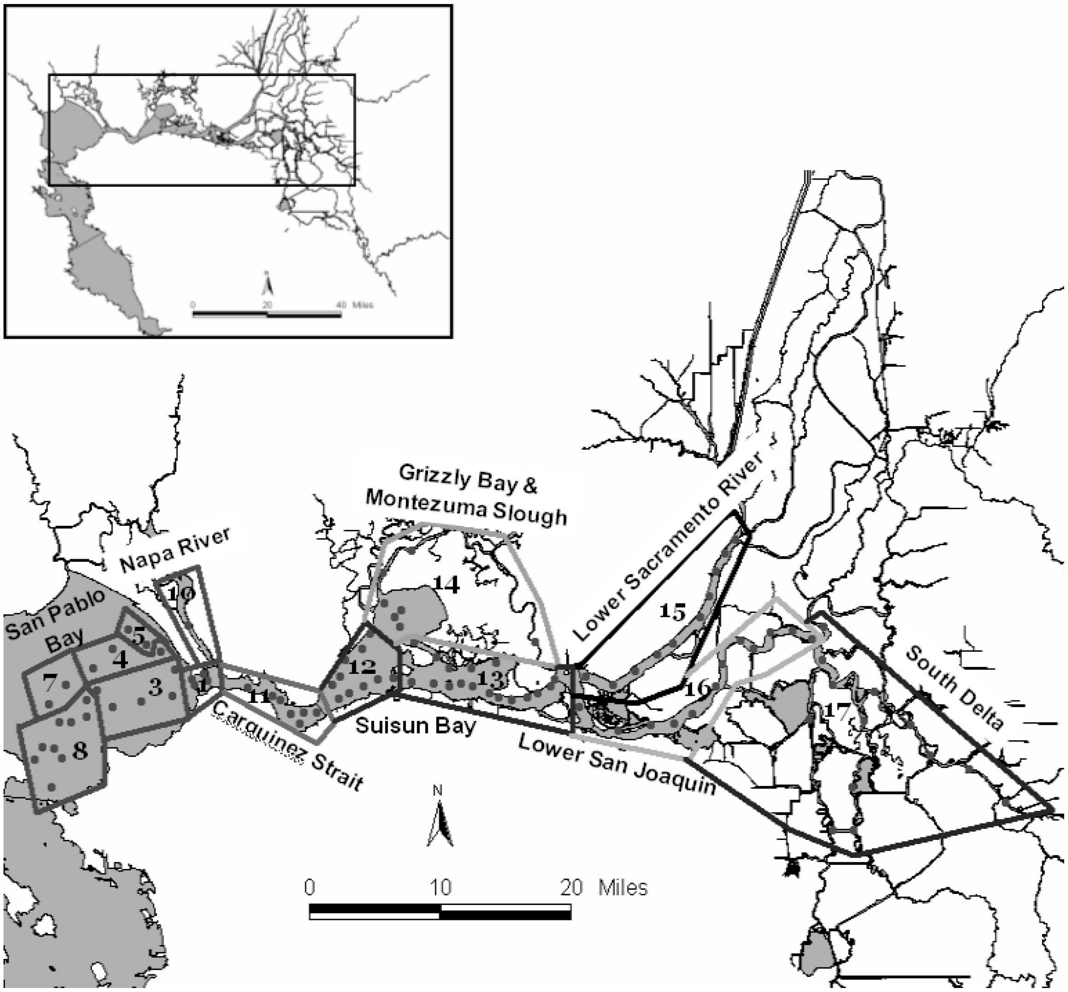

Figure 1. Sampling station locations for fall midwater trawl and areal stratification, separated by straight lines and numbered. Stations in strata 2, 6, and 9 have not been sampled since 1973 and have been removed from the index calculation.

each area, even though the volume sampled by the trawl has varied considerably between tows. Figure 2 shows the extent of variation in tow volumes between stations by month and year. Area weights should change if the volume filtered changes. For example, suppose the true abundance was the same in a given area during September for two years in a row but the volume filtered in each tow during the second year was double the volume filtered in the first year. With constant weights the September index for the second year will be approximately twice that of the first year even though the abundances did not change. In fairness to the indices, however, changes in the abundance of delta smelt have been sizeable enough to dwarf inaccuracies due to variation in volume sampled.

A second criticism of the index is that size-selectivity of the midwater trawl gear is not accounted for. The probability of a delta smelt being caught, given that it is present in the volume swept by the trawl, varies among fish of different size. Thus the number of fish caught at a given station in a given month will depend not only on the abundance of fish present but also the size distribution. The fact that the fork lengths of delta smelt have declined since 1967 (Sweetnam 1999; Bennett 2005) confounds interpretation of the index. As an extreme and artificial case, suppose the fish stayed at the same station during two consecutive months, there was no mortality nor immigration, the fish were all the same length, say 40 mm, in the first month, and then they all grew to the same length, 50 mm, in the second month. Further assume that a constant volume of water was sampled in each tow. Because of gear selectivity, the expected number of fish caught in the second month would be greater than the number for the first month. Thus the month-

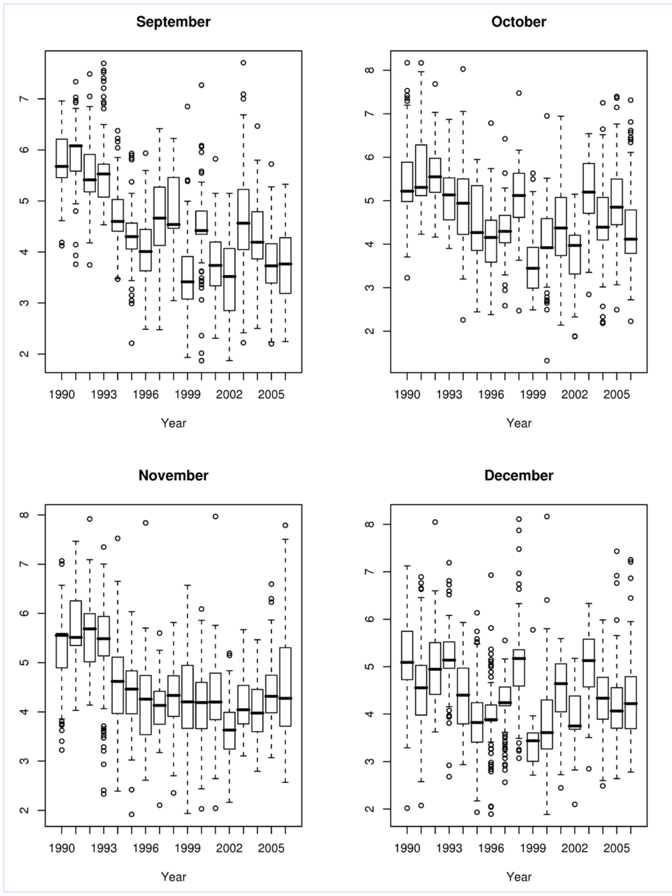

Figure 2. Tow volumes (acre-feet) by month and year

ly index will increase for the second month, but the number of fish has not changed.

The third criticism questions the utility of an annual index [equation (2)]. Interpretation of the annual index is potentially clouded by between year variation in monthly survival. To make the effect of variation in survival more apparent, suppose that the annual index for year *y* was based on catches from a single station, i.e.,

$$I_y = f_{S,y} + f_{O,y} + f_{N,y} + f_{D,y},$$

where the *f*'s are the catches at the station by month (with *S*, *O*, *N*, *D* denoting the months September through December). Further suppose that the probability of catching a fish, given that it is present in the volume swept by the trawl, is constant, denoted *p*, both within and between years (thus eliminating the gear selectivity issue). Let $F_{m,y}$ be the total abundance in the area during month *m* in year *y* and assume the fish are distributed at random throughout the area around the station. The expected catch in month *m* can be written as $E[f_{m,y}] = p(v/V)F_{m,y}$, where *v* is the volume swept, and *V* is the volume of water in the area; i.e., *v* and *V* are constant between months. Assume that there is no emigration, immigration, or births during the fall months, but that there is natural mortality. The probability of surviving from month *m* to month *m*+1 is denoted $\phi_{m,y}$. The expected value of the index, in a given year, is then

$$E[I_y] = N_{S,y} p \frac{v}{V}(1 + \phi_{S,y} + \phi_{S,y}\phi_{O,y} + \phi_{S,y}\phi_{O,y}\phi_{N,y})$$

If the survival probabilities remain constant between years, then $E[I_y] = F_{S,y}k$, where *k* is a constant, and the variation between annual indices would, on average, be a reflection of changes in the abundance in September. However, between year differences in survival probabilities do exist and interpretation of differences in annual indices is problematic. For example, suppose that F_S for two consecutive years is 500,000 but for the first year ϕ_S, ϕ_O, ϕ_N = (0.7,0.8,0.9) and for the second year ϕ_S, ϕ_O, ϕ_N = (0.5,0.6,0.7). The expected abundances by month for the first year are 500,000; 350,000; 280,000; and 252,000; while for the second year they are 500,000; 250,000; 150,000; and 105,000. Letting pv/V=0.001, the expected index value for the first year is 1382, while for the second year it is 1005. If primary concern was over the abundance prior to spawning, i.e., the December abundance, then indices are not reflecting the fact that the abundance for December in the first year is more than twice the abundance the second year.

The procedure described next yields estimates of fish, as opposed to a relative index, it addresses the issues of variation in volume swept and gear selectivity, and produces standard errors for the estimates. The complication of between year variation in survival and the annual index is avoided as only monthly estimates are made. Future work will address variation in monthly survival probabilities.

DESIGN-BASED ESTIMATION PROCEDURE

The estimation procedure is a slight variation of a stratified random sample ratio estimator (Cochran 1977; Thompson 2002), where the auxiliary variable is the volume of water sampled during a tow. The variation is due to the use of a gear selectivity-based expansion of the caught fish, which complicates the variance calculations in particular.

While the FMWT survey dates back to 1967, complete length information for the catches, which is needed for the gear selectivity expansion, was only available from 1990. Estimates of delta smelt abundances were calculated on a per month basis for the months September through December for the years 1990 through 2006.

Appendix A provides technical details on the gear selectivity model used for the expansion of observed catches to the total number in the sample volume. The gear selectivity model was fit using data collected during a covered cod-end experiment (Sweetnam and Stevens 1993), where a cover was attached to the cod-end of a midwater trawl which trapped fish that slipped through the cod-end.

Point Estimation

Informal description. The trawl data are stratified by year, by month, and by area, where the areas are the same 14 non-overlapping regions of the estuary (Figure 1) used in the current delta smelt index calculation. Given 17 years, 4 months, and 14 areas there are 952 (17*4*14) strata. Within each stratum, at each sample station, the number of fish caught in the tow is expanded to yield an estimate of the total number of fish in the tow volume, caught and uncaught. The expansion is made using a model for gear selectivity based on length of fish (Appendix A). Using data from all the stations within a stratum, a stratum-specific ratio of the expanded abundance to volume filtered is calculated. This ratio is then multiplied by the total volume (in acre-feet) of the stratum to yield an estimate of the total abundance within the stratum.

Formal description. Let $f_{y,m,a,s}$ denote the number of delta smelt in the volume of water swept by the trawl net at station s in area a during month m and year y. Likewise let $v_{y,m,a,s}$ denote the volume of water swept by the net (at that place and time). Let $F_{y,m,a}$ and $V_{y,m,a}$ be the total number of fish and total water volume in year y, month m, and area a. Total water volume per area will be assumed constant over time, thus V_a suffices. The number of stations (equivalently tows) in a given year, month, area stratum is denoted $n_{y,m,a}$. The number of fish actually caught in a particular tow (at station s) is $z_{y,m,a,s}$, and $L_{y,m,a,s,i}$, i=1,..., $z_{y,m,a,s}$, is the length of the ith fish caught in that tow.

The estimate of total abundance (in year y and month m) is:

$$\hat{F}_{y,m} = \sum_{a=1}^{14} \hat{F}_{y,m,a} = \sum_{a=1}^{14} V_a \hat{R}_{y,m,a} = \sum_{a=1}^{14} V_a \frac{\sum_{s=1}^{n_{y,m,a}} \hat{f}_{y,m,a,s}}{\sum_{s=1}^{n_{y,m,a}} v_{y,m,s,a}} \quad (3)$$

with

$$\hat{f}_{y,m,a,s} = \sum_{i=1}^{z_{y,m,a,s}} \frac{1}{\widehat{\Pr}(L_{y,m,a,s,i})}, \quad (4)$$

where $\widehat{\Pr}\, L_{y,m,a,s}$ is the estimated probability that a fish of length L is caught. The estimate comes from the gear selectivity model (equation (9) in Appendix A). Equation (4) is an example of a Horvitz-Thompson (Horvitz and Thompson 1952) estimator of a population total, in this case the population is all fish in the volume of water the net is towed through.

Variance Calculation

The variance of the estimated total for a given month is a modification of the formula for a stratified random sample ratio estimate of the total that uses separate ratios per stratum (Cochran 1977; Thompson 2002). The modification is due to the additional variation caused by the expansions of fish present in the tow volume, leading to a two-stage variance formula:

$$\widehat{Var}(\hat{F}_a) = \frac{V_a^2}{\bar{v}_{y,m,a}^2} \left[\frac{1}{n_{y,m,a}^2} \sum_{s=1}^{n_{y.m.s}} \sum_{i=1}^{z_{y,m,a,s}} \left(\frac{1-\hat{\Pr}(L_{y,m,a,s,i})}{\hat{\Pr}(L_{y,m,a,s,i})^2} \right) + \frac{S_{\hat{R}_{y,m,a}}^2}{n_{y,m,a}} \right] \quad (5)$$

where $$S_{\hat{R}_{y,m,a}}^2 = \frac{\sum_{s=1}^{n_{y,m,a}} \left(\hat{f}_{y,m,a,s} - \hat{R}_{y,m,a} v_{y,m,a,s} \right)^2}{(n_{y,m,a} - 1)}.$$

Mathematical details of the derivation are provided in Appendix B. A demonstration of the calculation of a point estimate and variance for a single stratum is shown in Appendix C.

Implicit to the variance formula is independence between sampling units. If sampling locations are chosen by a simple random sample (and are non-overlapping in space), then independence is assured. If data are combined from two or more months and are based on samples taken at the same location, then some degree of dependence is introduced, perhaps some temporal correlation, and the variance formula would need to be modified.

There is another layer of uncertainty, sampling error in the gear selectivity parameters, which has been ignored in equation (5) and the estimated variances may be underestimates to some degree. The bootstrapping procedure described next accounts for this uncertainty.

Bootstrapping Confidence Intervals

The normal distribution-based approach to calculating confidence intervals, e.g., $\hat{\theta} \pm 2 * se(\hat{\theta})$, while simple to carry out, can be quite inaccurate when the sampling distribution of the point estimate is not close to normal. Additionally, as will be the case for some of the monthly delta smelt point estimates, when the coefficient of variation exceeds 50%, such normal distribution-based 95% confidence intervals would include negative values.

An alternative is bootstrapping (Davison and Hinkley 1997). There are several ways to carry out bootstrapping, but the general idea is to view the sample as if it were the population and then to resample from the sample and carry out the same estimation procedures applied to the original sample. Although the emphasis here is on confidence intervals, the bootstrapping procedure can be used to calculate standard errors as well. For the particular problem at hand, sampling error in the gear efficiency estimates, which was ignored in the previous theoretical calculations, can be included.

With the stratified random sampling framework, independent bootstrap sampling is done within each year-month-area stratum. Within each stratum, two levels of sampling occur: the resampling of stations and the resampling of fish in the volume trawled. To exactly mimic the actual sampling process, the sampling of stations should be done without replacement. However, the volume of water sampled within a stratum is so small relative to the entire volume of a stratum, treating the sampling as with replacement is sufficiently accurate. A third level of sampling is added which reflects the uncertainty in the gear efficiency calculations. The number of stations within a stratum are sometimes relatively small, and the bootstrap performance can be relatively poor with such small samples. For example, area 4 has only three stations, and there is a relatively high chance that a resample will consist of three repeats of the same station, and the variance would be zero for that sample.

The steps in the bootstrapping algorithm are the following. For a single iteration of the bootstrap resampling:

1. The covered cod-end experiment data is resampled parametrically by randomly sampling the 812 caught fish (Table 1), where each fish was caught with probability $\widehat{\Pr}(L)$ and the estimated probability value is the maximum likelihood estimate from the logistic model. The logistic gear efficiency model is then re-fit to the resampled fish to yield a fitted gear selectivity model,

$$\Pr(L)^* = \frac{\exp(\hat{\beta}_0^* + \hat{\beta}_1^* L)}{1 + \exp(\hat{\beta}_0^* + \hat{\beta}_1^* L)} \quad (6)$$

2. For area a, the n_a stations in the stratum are sampled with replacement.

3. For station s in area a, a sample of observed fish, $z^*_{a,s}$, is generated using the following binomial distribution,

$$z^*_{a,s} \sim \text{Binomial}\left(\left[f\right]^*_{a,s}, p^*_{a,s}\right),$$

where $\left[f\right]^*_{a,s}$ is the *rounded* bootstrap-generated number of actual fish at the station, calculated

Table 1. Approximate catches by length (mm) of delta smelt in the August 1991 covered cod-end experiment. $\hat{r}_I(L)$ is the observed fraction of fish of length L caught by the inside net.

Group	Length	Outside	Inside	$\hat{r}_I(L)$
1	21.25	1	0	0.00
2	23.75	1	0	0.00
3	26.25	0	0	NA
4	28.75	2	0	0.00
5	31.25	2	0	0.00
6	33.75	1	1	0.50
7	36.25	7	2	0.22
8	38.75	8	6	0.43
9	41.25	20	6	0.23
10	43.75	33	27	0.45
11	46.25	91	29	0.24
12	48.75	77	50	0.39
13	51.25	153	31	0.17
14	53.75	77	27	0.26
15	56.25	62	9	0.13
16	58.75	19	5	0.21
17	61.25	10	2	0.17
18	63.75	2	2	0.50
19	66.25	2	1	0.33
20	68.75	1	2	0.67
21	71.25	0	2	1.00
22	73.75	0	4	1.00
23	76.25	0	8	1.00
24	78.75	0	9	1.00
25	81.25	0	11	1.00
26	83.75	0	6	1.00
27	86.25	0	1	1.00
28	88.75	0	2	1.00
Total		569	243	

using equations (4) and (6), $p^*_{a,s} = z_{a,s} / \hat{f}^*_{a,s}$.

4. For station s in area a, given $z^*_{a,s}$, an estimate of the number of actual fish, $\hat{f}^*_{a,s}$, is calculated by $z^*_{a,s} / p^*_{a,s}$

5. Given the $\hat{f}^*_{a,s}$, the stratified sample ratio formula (equation (3)) is used to calculate a bootstrapped total abundance estimate.

The above steps essentially mimic the sampling and estimation procedure carried out with the real data. The generation of observed fish using the binomial distribution is based on the result that the overall probability of capturing fish of varying sizes can be found by integrating the joint probability of capture and fish size over size, i.e., $p = \int \Pr(L) g(L) dL$, where g(L) is the probability distribution for size. The probability distribution for size classes can be estimated by $\hat{f}(L)/\hat{f}(\cdot)$, where $\hat{f}(\cdot)$ is the estimated total number of fish and $\hat{f}(L)$ is the estimated total number of size L fish. For a given trawl with z total fish captured with lengths $L_1,\ldots,L_Z$, the overall capture probability can then be approximated as follows:

$$p = \int \Pr(L) g(L) dL \approx \frac{\sum_{i=1}^{z} \Pr(L_i) \frac{1}{\Pr(L_i)}}{\sum_{i=1}^{z} \frac{1}{\Pr L_i}} = \frac{z}{\hat{f}(\cdot)}.$$

In other words p is estimated by the actual total number of caught fish divided by the estimated number present in the trawled volume.

RESULTS

The observed number of delta smelt caught each month for the years 1990 through 2006 are shown in Table 2. Delta smelt caught by the midwater trawl during the fall months are predominantly age 0 fish, although some age 1 fish are caught. However, exactly which fish are age 0 and are age 1 is not routinely determined and estimates were based on the total number of fish caught by the midwater trawl.

Data on volumes swept were missing for some of the stations where there were no delta smelt recover-

Table 2. Numbers of delta smelt caught by month for 1990-2006, summed over all 14 sampling areas

Year	September	October	November	December
1990	88	42	157	15
1991	104	213	237	30
1992	61	2	48	22
1993	334	414	85	131
1994	56	7	6	17
1995	96	322	346	73
1996	16	21	11	82
1997	9	93	62	123
1998	185	87	14	68
1999	192	374	131	130
2000	415	107	54	125
2001	68	409	17	25
2002	14	42	27	44
2003	13	118	15	36
2004	9	17	19	6
2005	2	8	7	7
2006	30	4	4	1

ies. This occurred 16% of the time, for 155 of the 952 year-month-area samples. A value of 6,351 m^3, which was based upon the size of the net mouth opening, net length, and typical length of time of towing (Dave Contreras, California Department of Fish and Game, personal communication) was substituted for the missing values.

The monthly point estimates and standard errors for delta smelt abundances are shown in Table 3. The standard errors are based on equation (5), thus exclude error in the gear efficiency estimates. The bootstrap standard errors, however, were quite close to these theoretical estimates (Pearson correlation coefficient = 0.999, median difference of theoretical – bootstrap = 1.8) suggesting that the variance due to error in the gear efficiency model had relatively little impact on the total standard error. The standard error from area 10 cannot be estimated and has been set equal to zero because there is only one sampling station in that area; in practice, delta smelt are almost never recovered in area 10 and the standard error would be zero anyway. The coefficients of variation (not shown) range from 22% to 130%, with a median value of 41%. The bootstrap confidence intervals (95% level), based on 1000 bootstrap samples, for the monthly estimates are shown in Table 4 and indicate the relatively high degree of uncertainty in the point estimates. That uncertainty is also apparent in Figure 3, which contains side-by-side boxplots of the bootstrap sample point estimates by month and year. Note that the zero valued lower bounds are not technically correct since at least one fish was caught in any given year-month, but with the bootstrap resampling there was a relatively high probability of getting zero recoveries in some cases, e.g., December 2006 when only one fish was caught (Table 2).

Table 3. Monthly estimates, in thousands of fish, and theoretical standard errors (in subscripts) of ages 0 and age 1 delta smelt abundances for 1990-2006, summed over all 14 sampling areas

Year	September	October	November	December
1990	553_{277}	286_{89}	887_{333}	72_{32}
1991	613_{217}	1114_{355}	1182_{324}	186_{65}
1992	464_{148}	21_{21}	310_{108}	136_{67}
1993	2703_{990}	3029_{1138}	605_{178}	866_{208}
1994	442_{334}	75_{44}	48_{28}	91_{46}
1995	983_{252}	2760_{712}	2761_{761}	554_{178}
1996	124_{50}	134_{46}	66_{37}	618_{282}
1997	64_{32}	924_{422}	577_{208}	691_{167}
1998	1882_{527}	616_{149}	77_{49}	366_{100}
1999	1760_{500}	2876_{930}	762_{163}	1405_{621}
2000	4433_{1333}	830_{221}	394_{132}	1087_{421}
2001	735_{285}	3659_{1114}	102_{46}	144_{69}
2002	125_{51}	336_{142}	230_{91}	277_{100}
2003	96_{42}	964_{488}	137_{78}	242_{97}
2004	77_{44}	98_{46}	146_{59}	37_{22}
2005	13_{11}	53_{31}	56_{30}	45_{23}
2006	309_{123}	26_{17}	35_{25}	4_{5}

Table 4. Bootstrap confidence intervals (95% level) summed over all 14 sampling areas for age 0 and age 1 delta smelt abundances (in thousands of fish) for 1990-2006

Year	September		October		November		December	
1990	141	1109	129	459	361	1549	21	137
1991	260	1048	488	1817	599	1894	79	319
1992	234	767	0	72	125	525	28	265
1993	1016	4841	1198	5647	288	959	510	1319
1994	26	1237	10	176	6	115	16	183
1995	530	1507	1561	4542	1481	4303	247	968
1996	46	228	54	224	11	143	179	1196
1997	12	133	292	1877	218	988	399	1043
1998	1054	2927	348	921	13	182	204	571
1999	920	2800	1307	4684	466	1107	447	2740
2000	2216	7121	425	1279	170	670	461	1972
2001	255	1378	1719	5805	29	203	41	310
2002	42	225	117	656	82	441	103	517
2003	21	182	311	1988	24	309	76	441
2004	10	182	25	196	43	271	0	83
2005	0	36	6	126	9	122	9	93
2006	112	579	0	67	0	90	0	16

The monthly point estimates (in thousands of fish) for the delta smelt abundances, summed over strata, are plotted against year in Figure 4. The FMWT monthly indices, (multiplied by 10 to make comparison easier), are also plotted in Figure 4. The point estimates and the monthly indices are highly correlated (r = 0.97, 0.98, 0.95, and 0.98 for September through December, respectively). The deviations between the point estimates and the indices are largely a reflection of the effect of accounting for gear selectivity and accounting for variations in volume filtered. General results, however, about the status of the delta smelt population levels are the same for both measures: precipitous declines are apparent.

BIAS, PRECISION, AND FUTURE DIRECTIONS

The quality of estimates of abundance can be measured by the amount of bias and variance. Bias is a systematic departure from the underlying true values, i.e., either consistent under- or over-estimation, and is largely due to assumptions of the estimation procedure not being met. Some of the important assumptions of the estimation process are discussed below along with concerns about violations of these assumptions. Variance, on the other hand, is a measure of non-systematic, random deviations from the underlying true values, i.e., the degree of precision, and factors affecting variance are also discussed. Given the inherent variability in fish densities throughout the Estuary over time, however, and the fact that delta smelt are a dynamic population,

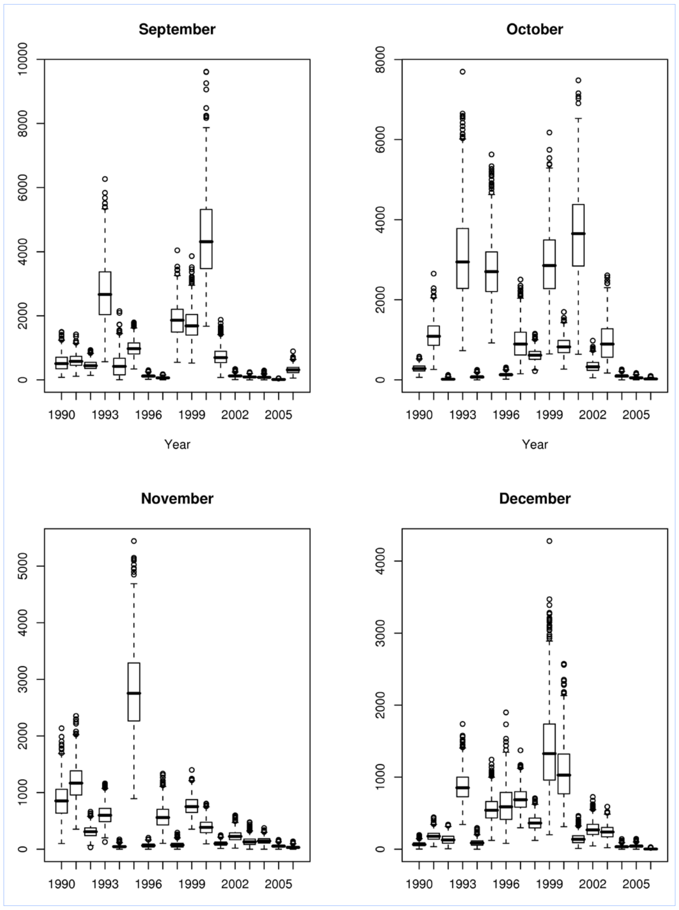

Figure 3. Bootstrap sample estimates of abundance by month and year

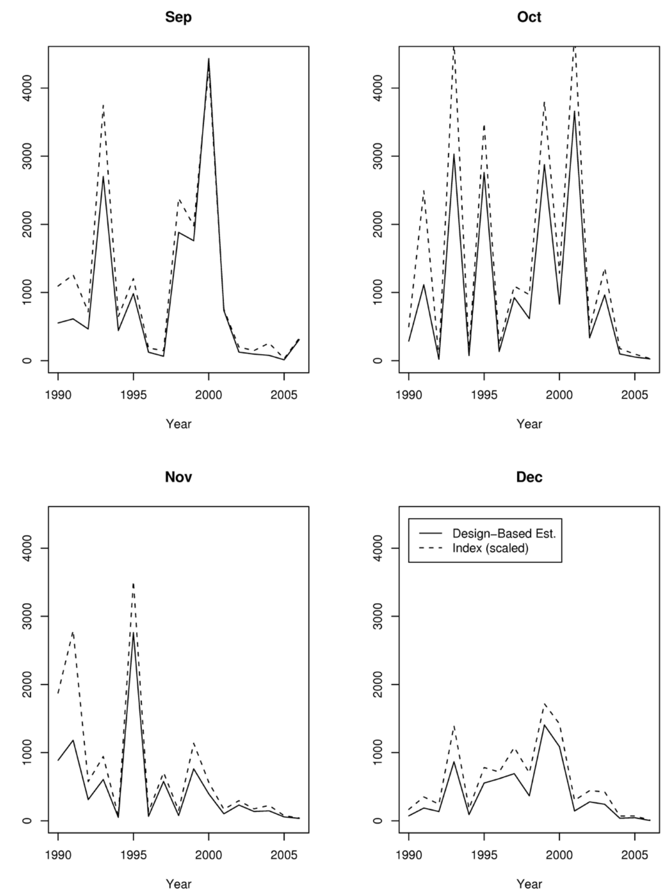

Figure 4. Stratified ratio estimates (solid lines, thousands of fish) of monthly delta smelt abundance (1990–2006) and monthly fall midwater trawl indices (dashed lines, multiplied by 10)

the best way to improve the precision of abundance estimates may be to develop alternative estimation procedures that explicitly recognize underlying population dynamics and spatial-temporal factors, and initial thoughts about such an alternative are given.

Bias

Within a stratum, the estimate of total abundance is a function of three components (see equation (3)),

$$\text{Stratum Volume x } \frac{\text{Estimated Sample Abundance}}{\text{Sample Volume}}$$

where "Estimated Sample Abundance" is the sum over the sample stations of expanded estimates of the number of smelt in the volume swept by the midwater trawl and "Sample Volume" is the sum of those swept volumes. Bias in any one of these terms can lead to bias in the stratum abundance estimate.

Stratum volume, V_a, is not constant over time, e.g., tidal variation affects water volume, but the assumption is that V_a is on average unbiased. A more critical concern, perhaps, is whether the total volume, $\sum_{a=1}^{14} V_a = 1{,}706{,}000$ acre-feet, is an unbiased estimate of the volume of water occupied by delta smelt during the fall months.

Regarding the volume of water sampled by the trawl ("Sample Volume"), measurements of individual tow volumes were calculated from flowmeters pulled alongside the vessel during net retrieval and from estimated average net mouth area during the tow.

However, actual tow volumes could on occasion be less than the estimated values in shallow areas where planing doors, which held the net mouth open, periodically contacted the bottom, and tension from water pressure on net meshes caused the net mouth to partially collapse (Randy Baxter, California Dept. of Fish and Game, personal communication). In those cases, the tow volume measurements would be overestimates and abundance estimates would be biased low. As an aside, the monthly estimates are relatively robust to variation in reported tow volumes. Substitution of the median tow volume, from all tows, for individual tow volumes led to monthly estimates very similar (Pearson correlation coefficient = 0.978) to those shown in Table 3.

Bias in the estimated sample abundance is potentially the most serious bias and there are two possible sources of bias. Bias could arise in the expansion of actual catch to estimated fish present in the volume swept. If fish below some length L_{min}, say, had zero probability of capture, then the abundance estimate would clearly be an underestimate. However, the fact that average fork length during September is 40 to 50 mm (Bennett 2005) and that the midwater trawl caught fish as small as 28 mm supports the assumption that the vast majority of fish present during the fall months did have a positive probability of being caught. The expansion could still be biased if the capture probability estimate was biased. The calculation of $\widehat{\Pr}(L)$, based on the covered cod-end data of Sweetnam and Stevens (1993), assumed that: (a) the cover outside the cod-end was 100% effective; (b) there was no gear avoidance in the volume swept by the trawl; (c) the probability of capture by the cod-end was a logistic function of length; (d) probability of capture was independent of towing distance, towing speed, and fish abundance within the volume swept. Avoidance of gear due to avoidance of the survey vessel itself, trawl doors, or the trawl is always a concern (Gunderson 1993), and would lead to negative biases in abundance estimates. While the fraction of the catch of length L fish retained in the cod-end, relative to fish caught in the cover, tended to increase with increasing length, it was not a very smooth increase (Appendix A), which does call into question the appropriateness of the logistic model.

Even assuming that the expansion of catch in the tow volume to actual numbers present was unbiased, say estimated sample abundance ≈ abundance in the volume towed, bias could arise if the water the trawl sampled was not representative of the water volume in the stratum, i.e., selection bias. This would not be a problem if the fish were uniformly distributed throughout the volume of water in a region, any sampling of the water by the trawl would be representative. However, if there were systematic spatial inhomogeneities in the fish density, such as fish tended to cluster near the surface and away from shoreline, and if the trawl systematically under- or over-sampled higher density volumes, then bias would result.

Concerns over selection bias are triggered by large differences in estimates of abundance presented here and recent estimates by Kimmerer (2008), who used the spring Kodiak trawl (SKT) survey data. The Kodiak trawl survey began in 2002, samples during the months of January through May, and overlaps to a large degree the area sampled by the FMWT, except for the San Pablo Bay areas (areas 1, 3, 5, 7, and 8; Figure 1) which has had relatively few recoveries. As an example of the wide discrepancy in values, the December 2004 abundance estimate based on the FMWT is 37,000 fish (Table 3) while the January 2005 abundance estimate based on the SKT is over 800,000 fish. Kimmerer (2008) also used a ratio estimator, sample abundance to water volume sampled, but did not stratify. Much of the difference can be attributed to considerably greater number of delta smelt caught by the Kodiak trawl: the December 2004 FMWT survey caught six delta smelt (Table 2) at a total of 112 sampling locations and sampled approximately 632,000 m^3 of water, while the January 2005 SKT survey caught 220 delta smelt at a total of 38 sampling locations and sampled approximately 900,000 m^3 of water (Dave Contreras, California Dept. of Fish and Game, personal communication). The Kodiak trawl tends to sample the upper portions of the water column while the midwater trawl takes an oblique tow from the lower to upper portions of the water column. It is unclear, however, whether either trawl is taking a representative sample of the water volume, i.e., selection bias could be present in both surveys. Careful investigation of the abundance of delta smelt by position in the water column combined with estimation of the water volume sampled by depth, by gear type, is clearly necessary.

Increasing Precision

Even if bias in the abundance estimates was minimal, imprecision is large, e.g., coefficients of variation exceed 50% for over 30% of the year-month estimates. For the stratified random sample ratio estimator, the imprecision is largely a sample size issue. Variability in fish numbers between tows, potentially a function of fish aggregation and relative rareness of the fish, is considerable enough that even in highly favorable conditions delta smelt will not be caught in every tow.

Precision can be increased by increasing the number of stations and sample size determination is possible using the variance formula (equation 10 in Appendix B). Given the observed large coefficients of variation for 100 stations, even a doubling of the number of stations may not yield satisfactory levels of precision for management actions. This, however, will likely be prohibitively expensive and practically impossible. An alternative is to use a combination of design- and model-based inference.

Alternative Estimation Procedures

The stratified random sample ratio estimator is largely a design-based estimator. A design-based estimator just uses the fact that probability samples are taken, where the probability of including a particular sampling unit is known, to calculate point estimates and standard errors. For example, the sample average, $\bar{y}$, from a simple random sample of size n from a population of size N is unbiased by design, each sampling unit has probability n/N of being selected and the average value of $\bar{y}$ over the $\binom{N}{n}$ samples is the population average.

In contrast, with model-based inference underlying structure is assumed about the population of interest. In a trivial sense, the use of water volume as the auxiliary variable in the ratio estimator is an example of model-based inference: as the water volume increases, the number of fish in the sample is assumed to increase in a linear manner. Less trivially, other covariates could be included in the estimation procedure so long as covariate measurements are available for both the sampled and unsampled volumes. For example, salinity or turbidity could be used as covariates so long as these measurements were available for the unsampled portions of an area. The sample data would be used to fit a regression model such as

$$f_{y,m,s} = \beta_0 + \beta_1 v_{y,m,s} + \beta_2 salinity_{y,m,s} + \varepsilon_{y,m,s}$$

where v is volume sampled. Then for unsampled volumes, the number of fish would be estimated by plugging in the corresponding covariate values.

A limitation of the estimation approach presented in this paper is that the abundance estimates were calculated independently on a per year, per month, and per area basis, with no connection in time or space between estimates. This meant that estimates of total abundance for one month could exceed the estimated total for the previous month even though no births have occurred and even if the system were closed in the sense that immigration into the fourteen areas was unlikely. For example, the estimated abundance in December 1999 is nearly double that for November 1999. This deviation is partly a function of sampling variation but it is also a reflection of the lack of spatial-temporal connectivity in the estimation procedure.

An alternative that could be much more statistically efficient is to develop a spatial-temporal model for abundances such that estimates for a given month and area are a function of data from the given month and area as well as data from adjacent months and areas. Additional data from other surveys besides the FMWT, such as the 20 mm surveys (samples larvae) and the summer townet surveys (samples juveniles), could inform the estimates, too. Such an estimation procedure would be underpinned by a life history model (Newman and Lindley 2006), and a small step in that direction is described in a companion paper. Such a model-based approach, which recognizes both the underlying continuity in the spatial and temporal distribution of delta smelt as well as the population dynamics of the species, could potentially serve as a tool for understanding reasons for the decline in delta smelt abundances. Life history parameters, such as survival probabilities or fecundity rates, for example, could be modeled as functions of biological and environmental covariates thought to influence population abundance.

ACKNOWLEDGEMENTS

Randy Baxter provided useful background information on the fall midwater trawl survey and, along with Gonzalo Castillo and Ted Sommer, made several helpful suggestions. Russell Millar graciously examined the covered cod-end data, and noted the inconsistencies in the inside and outside catch lengths. Dave Contreras and Kelly Souza provided the fall midwater trawl data, and Dave produced the map of the estuary. Wim Kimmerer and two referees provided many helpful suggestions and criticisms.

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APPENDIX A: GEAR SELECTIVITY ESTIMATION

To estimate the number of fish present in the volume swept by the midwater trawl, the selectivity of the gear is needed. Exactly what is meant by gear selectivity needs defining.

To begin, suppose at time t and location x a given fish of size or shape would be present in the absence of fishing gear passing through or by this location. The probability that such a fish would be caught by the gear is defined to be the long run relative frequency of times the gear would capture such a fish if the fishing process could be carried out repeatedly under (nearly) identical conditions. For delta smelt it will be assumed that length, L, is the only factor affecting this probability and it will be denoted $p(L)$. Note that $1-p(L)$ is the probability of either *evading* the gear or *escaping* the gear. A fish evades if it is stimulated by the gear to leave or avoid location x prior to time t. A fish escapes gear if it remains present at location x at time t as the gear passes through, or occupies, that location but the fish is able to escape from the gear; e.g., it slips through the mesh of a net. A gear is said to be *selective* if the $p(L)<1$ for at least some L, and *nonselective* if $p(L)=1$ for all L. Absolute gear selectivity will be defined to be the same as $p(L)$, a probability of capture that varies with fish size. In contrast, relative gear selectivity is defined, with reference to two or more gear types, as the ratio of capture probabilities for fish of size L; i.e., $p_i(L)/p_j(L)$ is the ratio of capture probabilities for gear type i to gear type j.

Millar (1992) developed a general procedure for estimating the gear selectivity of various fishing gear, including trawl gear, given data from gear efficiency studies. One type of the trawl gear efficiency study he considers is a covered cod-end study where a relatively fine mesh net is attached outside the cod-end, which presumably catches all fish that pass through the cod-end. The procedure is next explained and then applied to data from a covered cod-end experiment discussed by Sweetnam and Stevens (1993).

Millar's General Approach

Millar formulated a probability distribution for the catch of length L fish by one gear type *given* the total catch of length L fish by two or more gear types fishing the same region. The essence of his idea as it pertains to trawl studies can be stated as follows, where for simplicity only two gear types are considered. First define p_1 to be the probability that a fish comes into contact (is exposed to, say) with gear type 1 given that it contacted by either gear type; then $p_2 = 1 - p_1$ is the probability for gear type 2. Assume that the number of length L fish coming in contact with any gear, n_L, is a Poisson random variable with rate λ_L, $n_L \sim \text{Poisson}(\lambda_L)$. The number of fish of length L coming into contact with gear type 1 is then $\text{Poisson}(p_1\lambda_L)$, and for gear type 2 it is Poisson $((1-p_1)\lambda_L)$. Let $r_i(L)$ be the probability that a fish of length L is caught by gear type i conditional on it contacting that gear type. Then the number of fish of length L caught by gear type i, say $y_i(L)$, is $\text{Poisson}(r_i(L)\,p_i\lambda_L)$. It can then be shown that the probability distribution for $y_1(L)$ conditional on the total number caught, $y.(L) = y_1(L) + y_2(L)$, is binomial:

$$y_1(L)|y.(L) \sim Binomial\left(y.(L), \frac{r_1(L)p_1}{r_1(L)p_1 + r_2(L)(1-p_1)}\right).$$

The key advantage, thus, of conditioning on the total catch is that the parameters specifying the density, and implicitly the size distribution, of fish of length L, namely the λ_L's, have been eliminated from the distribution of catches.

The practical question for applications then reduces to the particular formulation of $r_1(L)$ and $r_2(L)$ and whether or not all the parameters are estimable given the observed catches, $y_1(L)$ and $y_2(L)$, say.

An illustrative example given by Millar is the case of alternate hauls with two different size mesh trawl nets, where the net with the finer mesh size is assumed to be non-selective. Denote the selective net gear 1 and the non-selective net gear 2; $r_2(L)$ then equals 1 for any L. A logistic model is assumed for

$r_1(L)$, i.e.,

$$r_1(L) = \frac{\exp(\beta_0 + \beta_1 L)}{1 + \exp(\beta_0 + \beta_1 L)} \quad , \tag{7}$$

and

$$y_1(L) \mid y.(L) \sim Binomial\left(y.(L), \frac{r_1(L)p_1}{r_1(L)p_1 + r_2(L)(1-p_1)} = \frac{p_1 \exp(\beta_0 + \beta_1 L)}{\exp(\beta_0 + \beta_1 L) + (1-p_1)}\right)$$

Millar then applies this model to an alternate haul study of haddock, yielding estimates of p_1, β_0, and β_1.

Millar (1992, page 967) makes brief mention of covered cod-end studies where he implicitly assumes that the outer mesh is non-selective. While he does not describe his reasoning, he states that the probability distribution for the number of fish caught by the inner cod-end net, $y_I(L)$, conditional on the total number of fish caught, is binomial with probability $r_I(L)$. His reasoning is not necessarily based on his general model due to the uniqueness of the covered cod-end trawl, because contact by the inner net in a sense implies contact by the outer cover net. A conclusion similar to his, however, can be arrived at by the following argument. Let $n(L)$ be the number of fish of length L present in the region to be fished, and let $y_O(L)$ be the number of fish caught in the outer cover. Then the joint distribution of $y_I(L)$ and $y_O(L)$ is trinomial

$$\left(y_I(L), y_O(L)\right) \sim \text{Trinomial}\left(n(L), pr_I(L), p\left(1 - r_I(L)\right) r_O(L)\right),$$

where p is the probability of coming into contact with the combined gear. Assuming the outer mesh is non-selective, $r_O(L) = 1$. The conditional probability for $y_I(L)$ given $y.(L)$is then

$$y_I(L) \mid y.(L) \sim Binomial\left(y.(L), \frac{pr_I(L)}{pr_I(L) + p(1 - r_I(L))r_O(L)} = r_I(L)\right) \tag{8}$$

where a reasonable formulation for $r_I(L)$ is the logistic model in equation (7).

Application to a Delta Smelt Gear Efficiency Study

During August 28-29, 1991, a covered cod-end study was carried out by the California Department of Fish and Game (Sweetnam and Stevens 1993) using a standard midwater trawl net, where the cod-end had a 1/2 inch mesh size and the cover was 1/8 inch mesh size. A total of 243 delta smelt were caught in the inner (cod-end) net and 569 delta smelt were caught in the outer cover. The original data giving the exact lengths of fish are no longer available but the approximate catches by lengths can be calculated using the frequency histogram shown in the 1993 report (page 32).

Table 1 contains the constructed data. The column labeled $\hat{r}_I(L)$ is the number caught by the inner net divided by the total number caught, i.e., an empirical estimate of $r_I(L)$. The data are at odds with the model in equation (8) in that one would expect $\hat{r}_I(L)$ to increase monotonically as length increases, but it varies in a non-systematic way between lengths 36.25 and 66.25 mm. Once a fish reaches 71.25 mm in length, however, it was estimated to be caught with certainty.

With the above concern in mind, the cod-end model in equation (8), with the logistic formulation, was fit and yielded the following capture probability:

$$r_I(L) \equiv \widehat{\Pr}(L) = \frac{\exp\left(\hat{\beta}_0 + \hat{\beta}_1 L\right)}{1 + \exp\left(\hat{\beta}_0 + \hat{\beta}_1 L\right)} = \frac{\exp(-3.89 + 0.0585 * L)}{1 + \exp(-3.89 + 0.0585 * L)}. \tag{9}$$

For example, if a fish is of length 55 mm, its probability of capture is 34%. Figure 5 plots the fitted values for $r_I(L)$ against length (the line) and includes the observed fractions of the catch in the inner mesh, relative to total catch, for each length class (the points). The fitted line is smoothing the observed relative fractions.

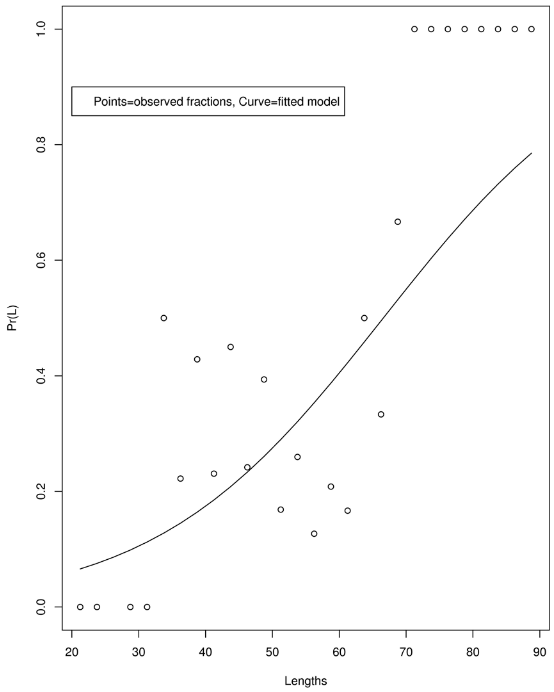

Figure 5. Fitted values for Pr(L) for the Millar model for the August 1991 covered cod-end experiment. The conditional and unconditional Pr (L) (r_J) values are identical for the Millar model. Plotted points are the observed fractions of catch (by length class) from the inside net.

APPENDIX B: MATHEMATICAL DETAILS OF VARIANCE ESTIMATION

Assuming independence between strata, the variance of the total is the sum of variances for the individual strata:

$$Var\left(\widehat{F}_{y,m}\right) = \sum_{a=1}^{14} Var\left(\widehat{F}_{y,m,a}\right). \qquad (10)$$

The variance of the estimated total within a stratum is (see Cochran 1977, eq'n 6.13; or Thompson 2002, pp 68-69):

$$Var(\widehat{F}_{y,m,a}) = (V_a)^2 Var(\widehat{R}_{y,m,a}) = (V_a)^2 Var\left(\frac{\sum_{s=1}^{n_{y,m,a}} \widehat{f}_{y,m,a,s}}{\sum_{s=1}^{n_{y,m,a}} v_{y,m,s,a}}\right) \qquad (11)$$

Given the tiny volume of water sampled relative to the total water volume within an area, the finite population correction factor can safely be assumed negligible.

Accounting for the uncertainty in the estimate, $\widehat{f}_{y,m,a,s}$, involves using the two-stage variance formula, which for two random variables, X and Y, is $Var(Y) = E_X[Var(Y|X)] + Var_X[E(Y|X)]$. In this particular setting, Y is $\frac{\sum_{s=1}^{n_{y,m,a}} \widehat{f}_{y,m,a,s}}{\sum_{s=1}^{n_{y,m,a}} v_{y,m,a,s}}$, and the $f_{y,m,a,s}$ and $v_{y,m,a,s}$ terms make up X. To reduce notation X will be retained in the following:

$$Var(\widehat{f}_{y,m,a,s}) = E_X\left[Var\left(\frac{\sum_{s=1}^{n_{y,m,a}} \widehat{f}_{y,m,a,s}}{\sum_{s=1}^{n_{y,m,a}} v_{y,m,s,a}} \mid X\right)\right] + Var_X\left[E\left(\frac{\sum_{s=1}^{n_{y,m,a}} \widehat{f}_{y,m,a,s}}{\sum_{s=1}^{n_{y,m,a}} v_{y,m,s,a}} \mid X\right)\right] (12)$$

First Component

Regarding the first component of equation (12):

$$E_X\left[Var\left(\frac{\sum_{s=1}^{n_{y,m,a}} \widehat{f}_{y,m,a,s}}{\sum_{s=1}^{n_{y,m,a}} v_{y,m,s,a}} \mid X\right)\right] = E_X\left[\frac{\sum_{s=1}^{n_{y,m,a}} Var(\widehat{f}_{y,m,a,s} \mid X)}{(\sum_{s=1}^{n_{y,m,a}} v_{y,m,s,a})^2}\right]$$

$$= E_X\left[\frac{1}{(\sum_{s=1}^{n_{y,m,a}} v_{y,m,a,s})^2} \sum_{s=1}^{n_{y,m,a}} Var\left(\sum_{i=1}^{z_{y,m,a,s}} \frac{1}{\widehat{\Pr}(L_{y,m,a,s,i})}\right)\right]$$

$$\approx \frac{1}{(\sum_{s=1}^{n_{y,m,a}} v_{y,m,s,a})^2} \sum_{s=1}^{n_{y,m,s}} \sum_{i=1}^{z_{y,m,a,s}} \frac{1-\widehat{\Pr}(L_{y,m,a,s,i})}{\widehat{\Pr}(L_{y,m,a,s,i})^2}. \qquad (13)$$

The last step in the above derivation is based on an estimate of the variance of a Horvitz-Thompson (Horvitz and Thompson 1952) estimate of a population total. Given a population with N individuals, where the probability that individual i is selected is π_i, $i = 1,\ldots,N$, and n individuals are selected, the Horvitz-Thompson estimate of N is $\widehat{N} = \sum_{i=1}^{n} \frac{1}{\pi_i}$. Defining an indicator variable I(j) to equal 1 when fish j is caught and 0 when it is not, the variance of $\widehat{N}$ is as follows:

$$Var(\widehat{N}) = \sum_{j=1}^{N} Var\left(I(j)\frac{1}{\pi_j}\right) = \sum_{j=1}^{N} \frac{1}{\pi_j^2} Var(I(j))$$

$$\sum_{j=1}^{N}\frac{\pi_j(1-\pi_j)}{\pi_j^2}=\sum_{j=1}^{N}\frac{1-\pi_j}{\pi_j},$$

assuming that the capture of one animal is independent of the capture of any other animal. In practice, however, N is unknown as are the π_j's for the unobserved animals. An unbiased estimate of the variance in practice (see equation (6) on page 54 of Thompson 2002) is

$$\widehat{Var}\left(\widehat{N}\right)=\sum_{j=1}^{n}\frac{1-\pi_j}{\pi_j^2}.$$

The capture probability, Pr(L), is estimated, but that uncertainty has been ignored here, thus the variance will be somewhat underestimated. To properly account for this uncertainty requires a "triple" variance formula.

Second Component

Looking at the second component of equation (12):

$$Var_X\left[E\left(\frac{\sum_{s=1}^{n_{y,m,a}}\hat{f}_{y,m,a,s}}{\sum_{s=1}^{n_{y,m,a}}v_{y,m,s,a}}\mid X\right)\right]=Var_X\left[\frac{\sum_{s=1}^{n_{y,m,a}}\hat{f}_{y,m,a,s}}{\sum_{s=1}^{n_{y,m,a}}v_{y,m,s,a}}\right]$$

$$\approx\frac{1}{\bar{v}_{y,m,a}^2}\frac{\sum_{s=1}^{n_{y,m,a}}(\hat{f}_{y,m,a,s}-\hat{R}_{y,m,a}v_{y,m,a,s})^2}{(n_{y,m,a}-1)n_{y,m,a}} \quad (14)$$

where $\bar{v}_{y,m,a}$ is the average volume of samples taken within the area.

Total Variance

Given equations (13) and (14), the variance estimator within a stratum is:

$$\widehat{Var}(\hat{F}_a)=\frac{V_a^2}{\bar{v}_{y,m,a}^2}\left[\frac{1}{n_{y,m,a}^2}\sum_{s=1}^{n_{y,m,s}}\sum_{i=1}^{z_{y,m,a,s}}\left(\frac{1-\widehat{\Pr}(L_{y,m,a,s,i})}{\widehat{\Pr}(L_{y,m,a,s,i})^2}\right)+\frac{S_{\hat{R}_{y,m,a}}^2}{n_{y,m,a}}\right] \quad (15)$$

where $S_{\hat{R}_{y,m,a}}^2=\dfrac{\sum_{s=1}^{n_{y,m,a}}\left(\hat{f}_{y,m,a,s}-\hat{R}_{y,m,a}v_{y,m,a,s}\right)^2}{\left(n_{y,m,a}-1\right)}$.

APPENDIX C: DEMONSTRATION OF ESTIMATION PROCEDURE

The estimation procedure is demonstrated numerically for area 1, which includes 4 stations, 336, 337, 338, and 339, during September 1995. There were two fish caught, both with length 49 mm, one at station 336 and one at station 338. The data relevant to the calculation are shown below.

Station	Fish Length (mm)	Estimated # of fish	Volume Swept (m^3)	Volume Swept (acre-feet)
336	49	3.783	5866	4.7556
337	N/A	0	6351	5.1488
338	49	3.783	4761	3.8597
339	N/A	0	2728	2.2117
Total		7.566	19706	15.9759

There are three steps to calculate an estimate:

1. *Expand the number of observed length L fish to total number of length L fish.* The expanded number of fish represented by a 49 mm fish is 3.783. This is estimated by inverting the probability of catching a 49 mm fish, $1/\widehat{\Pr}$ (length 49 mm fish is caught), where Pr(L) is based on the fitted gear selectivity model (see equation (9) in Appendix A). The probability that a length 49 mm fish is caught is exp(–3.89+0.0585*49)/[1+exp(–3.89+0.0585*49)] = 0.2643462. Thus, the estimated number of 49 mm fish in the volume trawled is 1/0.2643462= 3.783.

2. *Calculate the ratio of total fish to total volume sampled.* The estimated total number fish in the four tows is

   $$\hat{f}_{1995,Sept,1,\cdot}=3.783+0+3.783+0=7.566.$$

   The estimated ratio of fish to volume swept (in acre-feet):

$$\hat{R}_{1995,Sept,1} = \frac{\hat{f}_{1995,Sept,1,\cdot}}{v_{1995,Sept,1,\cdot}} = \frac{7.566}{15.9759} = 0.4735782,$$

where $v_{1995,Sept,1,\cdot}$ is the total sample volume swept.

3. *Estimate total fish in stratum, F_h, by multiplying the ratio by total volume.* The total volume for area 1 is 81,000 acre feet. The estimated number of fish in 1995 in September in area 1 is

$$\hat{F}_{1995,Sept,1} = 81000 * 0.4735782 = 38{,}360.$$

The expansion from the observed number to the estimated number is considerable, and this is due to the sampled volume being about 0.02% of the total volume. If the fish density was relatively constant throughout each area, this would not necessarily be worrying; however, as will be evidenced by the standard errors, density is quite variable.

The variance for the estimated total is calculated using equation (5). Some of the values needed in the formula include the average volume swept at the four stations (3.994 acre-feet), the probability of catching a 49 mm fish (0.264), and the estimated ratio, $\hat{R}$ (0.4736). The estimate of the variance for the total is

$$\widehat{Var}(\hat{F}_{1995,Sept,1}) = \frac{81{,}000^2}{3.993974^2}\left[\frac{1}{4^2}\left(\frac{1-0.264^2}{0.264^2} + 0 + \frac{1-0.264^2}{0.264^2} + 0\right)\right]$$

$$+\frac{81{,}000^2}{3.993974^2}\left[\frac{(3.783-\hat{R}*4.7556)^2 + (0-\hat{R}*5.1488)^2 + (3.783-\hat{R}*3.8597)^2 + (0-\hat{R}*2.2117)^2}{4(4-1)}\right]$$

$$= 541{,}248{,}696 + 452{,}710{,}045 = 993{,}958{,}741$$

The first component in the previous sum reflects the uncertainty in the gear effectiveness expansions while the second component reflects the between sample variation of the ratio estimates. The coefficient of $\sqrt{\text{variance}}\,/\,\text{point estimate}$, is 82%.