Lawrence Berkeley National Laboratory
Recent Work

Title

EFFECT OF VIBRATION-ROTATION INTERACTION ON E2 TRANSITION PROBABILITIES IN SPHEROIDAL NUCLEI

Permalink

https://escholarship.org/uc/item/99t2z1xd

Authors

Person, Lucy Wu.
Rasmussen, John O.

Publication Date

1960-08-01

UNIVERSITY OF CALIFORNIA

Ernest O. Lawrence

Radiation Laboratory

BERKELEY, CALIFORNIA

DISCLAIMER

UCRL-9230

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory

Berkeley, California

Contract No. W-7405-eng-48

EFFECT OF VIBRATION-ROTATION INTERACTION
ON E2 TRANSITION PROBABILITIES IN SPHEROIDAL NUCLEI

Lucy Wu Person and John O. Rasmussen

August, 1960

EFFECT OF VIBRATION-ROTATION INTERACTION
ON E2 TRANSITION PROBABILITIES IN SPHEROIDAL NUCLEI

Lucy Wu Person and John O. Rasmussen

Lawrence Radiation Laboratory and Department of Chemistry

University of California
Berkeley, California

August, 1960

INTRODUCTION

Rotational spectra have been well established as an important feature of the excited states in deformed nuclei. The deviation from the simple rotational spectrum has been found for the most part to be small and often to exhibit the character of rotation-vibration corrections, especially in even-even nuclei. An examination of rotational spectra in Sm^{152}, Er^{166}, Th^{232}, U^{234}, and Pu^{238} reveal a deviation from a simple structure which can be partially accounted for by the rotation-vibration correction.[1] This rotation-vibration correction in molecules is well known.

In the following sections, the sequence of the rotation-vibration perturbation are developed using a simplified model in the beginning in order to keep the essential points foremost. A rather detailed application is then made to the accurately measured energies of Sm^{152},[2,3] Er^{166},[3] Th^{232},[4] U^{234},[5] and Pu^{238}.[6] It is also shown that the experimental values agree reasonably well with the theoretical results.

CALCULATION OF ROTATION-VIBRATION INTERACTION

In order to make clear the origin and generality of the particular rotational perturbation in which we are most interested, it is useful to consider a simple theoretical model. Subsequently, we shall generalize this model in order to make it applicable to actual nuclei.

Therefore, we first restrict ourselves to nuclear systems strictly analogous to diatomic molecules.[7] The Hamiltonian for the system is simply

$$\mathcal{H} = T\ rot + V\ vib + T\ vib, \tag{1a}$$

$$\mathcal{H} = \hbar^2/2\mathcal{Q}\,(I_x^2 + I_y^2) + 1/2\ C\,(\alpha-\alpha_o)^2 + 1/2\ B\left[(d/dt)(\alpha-\alpha_o)\right]^2, \tag{1b}$$

where

$\mathscr{Q}$ is the principal moment of inertia,

I_x is the x component of the total angular momentum,

I_y is the y component of the total angular momentum,

α is the distance between two atoms along the nuclear symmetry axis, and

$$\alpha = \beta R_0/106^{10} \tag{2}$$

where we use $R_0 = 1.2 \times 10^{-13} A^{1/3}$ cm, and

α_0 is the equilibrium distance between two atoms along the nuclear axis

B the mass parameter

C the force constant parameter or the surface tension

And $I = f\alpha^2 B$ in cgs units $\tag{3a}$

$\quad\; I = f\beta^2 B$ in Mev. $\tag{3b}$

(Where (3a) and (3b) are corrections to the irrotational flow moment-of-inertia expression; the constant f has the value of 3 in the irrotational flow model but here it will be taken as a parameter.)

$$\alpha = \alpha_0 + \delta \tag{4}$$

δ is the distance from the equilibrium internuclear distance.

If we put (3a) and (4) into the first term of (1b), we get

$$\hbar^2 \vec{I}^2/2f B\alpha_0^2 (1 + \frac{\delta}{\alpha_0})^2 \tag{5a}$$

where $\vec{I}$ is the total angular momentum of nuclei $\vec{I}^2 = I_x^2 + I_y^2$.

Since $\delta \ll \alpha_0$ we can expand $(1 + \delta/\alpha_0)^{-2}$ in binomial form, the (5a) turns out to be $(\hbar^2 \vec{I}^2/2f B\alpha_0^2)(1 - 2\delta/\alpha_0 + \dots)$ $\tag{5b}$

The second term defined in (5b) is an effect of rotation-vibration interaction, and it constitutes the main part of the interact, in Hamiltonian

$$\mathscr{H}_{int} = (\hbar^2/2f B\alpha_0^2)(-2\, \delta/\alpha_0)\, \vec{I}^2 \tag{6}$$

This term experses the interaction between vibration bands with $\triangle n_\nu = 1$ where n_ν is the vibrational quantum number.

Whether or not the $\mathscr{H}_{int}$ will be important depends essentially on the spectrum of energies for the vibrational band. If the spacing between the bands is large compared with the rotational energies i.e., if the rotation is adiabatic with respect to the particle motion, then the $\mathscr{H}_{int}$ is a small perturbation and we can use second-order perturbation theory.

Since it is not certain just what value of f in eq. (3b) is appropriate for nuclei, we shall estimate an upper limit on the interaction of the ground rotational band and first excited beta vibrational band by attributing all the deviation from $I(I+1)$ spacing in the ground band to this one interaction.

The rotational spectra corresponding to the close lying configurations will be mixed by the rotation-vibration interaction, and the resulting energy spectrum will be given by

$$E' = \frac{h\nu - \sqrt{h\nu - 4\,H_{int}^2}}{2} \tag{7a}$$

or

$$E = \frac{\hbar^2}{2\mathcal{I}} I(I+1)\epsilon_{II'}^2 (E_I' - E_I) + h\nu \tag{7b}$$

$$\epsilon_{II}' = \sqrt{\frac{-H_{int}}{h\nu - E}} \tag{8a}$$

or

$$\epsilon_{II}' = \sqrt{F\,I^2\,(I+1)^2/(E_I' - E_I)} \tag{8b}$$

where ν is the vibration frequency corresponding to B, (the mass parameter) and C, (the force constant parameter) ϵ_{II}' is the mixing amplitude.

We are interested in the electric-quadrupole reduced transition probabilities between mixed states.[8] The results are:

with mixing $B(E2\ 0'+ \longrightarrow 2+)$
$$= Z^2\,e^2\,5/4\pi\,[2\,\alpha_0 \sqrt{\frac{\hbar}{2\sqrt{BC}}} + \epsilon\,\alpha_0^2]^2 \tag{9a}$$

without mixing $B(E2\ 0'+ \longrightarrow 2+)$
$$= Z^2\,e^2\,5/4\pi\,[2\,\alpha \sqrt{\frac{\hbar}{2\sqrt{BC}}}]^2 \tag{9b}$$

and $B(E^2\ 2'+ \longrightarrow 0+)$
$$= Z^2\,e^2\,1/4\pi\,[2\,\alpha_0 \sqrt{\frac{\hbar}{2\sqrt{BC}}} - \epsilon\,\alpha_0^2]^2\ . \tag{10}$$

We can estimate the lower limiting values of B and C from the correction formulae given by Birbran et al.[9]

$$E = h\nu + (\hbar^2/2\mathcal{J}) \, I(I+1) - F\,I^2(I+1)^2 \tag{11}$$

By assuming $\Delta E\gamma = -\left(\dfrac{\hbar^2}{\mathcal{J}}\right)^2 \dfrac{1}{6\beta^2 C\gamma}$ (12)

as mentioned in Bohr et al.,[1] and the correction due to the interaction with the β oscillation can be determined by taking into account that the moment of inertia is a function of β as in (3), we then get

$$\Delta E_\beta = -1/2C_\beta \, (d/d_\beta \hbar^2/2\mathcal{J})^2 \, I^2(I+1)^2 \tag{13a}$$

$$= -1/2C_\beta B^2 \, (\hbar^2/\mathcal{J})^2 \, I^2(I+1)^2 \tag{13b}$$

$$\Delta E_o = \Delta E_\beta + \Delta E_\gamma = -\left(\frac{\hbar^2}{\mathcal{J}_o}\right)^2 \left[\frac{1}{6\beta^2 C\gamma} + \frac{1}{2\beta^2 C_\beta}\right] I^2(I+1)^2 \tag{14}$$

$$= -F\,I^2(I+1)^2 .$$

Applying these calculations to Sm^{152}, Er^{166}, Th^{232}, U^{234}, and Pu^{238} we find that since $C_\gamma \leq C_\beta$, we can get lower limiting values of C'_β and B'_β by the following formula:

$$B'_\beta \geq 2/3 \, (\hbar^2/\mathcal{J}_o)^2 \, 1/F\beta^2(\hbar\omega_\beta)^2 \tag{14a}$$

$$C'_\beta \geq 2/3 \, (\hbar/\mathcal{J}_o)^2 \, 1/F\beta^2 . \tag{14b}$$

Here we assume C'_β and B'_β are the same for the different vibrational-rotational bands in a nucleus. The experimental energies of the first and second excited states of the first rotational bands (E_2+ and E_4+) are substituted into equation (11) to give values for $\hbar/2\mathcal{J}_o$ and F. Substituting F and the experimental energies of the first and second excited states of the β vibrational band (0'+, 2'+) and E_2+ into equation (8b) we get ϵ_{22}', the mixing amplitude. The values of ϵ_{22}' are given in Table I, column (a). Substituting into

equation (14a) $(\hbar^2/2\vartheta_o)$'F' E_o+' and the experimental values of the β deformation parameters[10] we obtain the lower limits of B'_β which are given in Table I, column (b) in both Mev and cgs units. Similarly, we calculate the lower limits of C'_β from equation (14b). These values are tabulated in column (c) of Table I.

TABLE I

Theoretical values of mass parameters and force constant parameters.

Nucleus	(a) ϵ_{22}' (mixing amplitude) in spin 2 states	(b) B'_β		(c) C'_β	
		Mev^{-1}	gm	Mev	gm/sec^2
Sm^{152}	0.087	228.4	4.36×10^{-22}	107.0	4.69×10^{20}
Er^{166}	0.018	153.2	2.71×10^{-22}	332.4	113.75×10^{20}
Th^{232}	0.023	533.0	7.65×10^{-22}	278.8	9.21×10^{20}
U^{234}	0.017	458.6	6.53×10^{-22}	302.4	9.87×10^{20}
Pu^{238}	0.012	536.0	9.47×10^{-22}	464.5	15.14×10^{20}

The lower limiting values of C'_β, B'_β, ϵ_{22}' and the experimental value[10] of α are substituted into Eqs. (9a), (9b), and (10) to give the upper limiting value of B (E2 0 + ⟶ 2'+) with mixing and without mixing and the values of B (E2 0 + ⟶ 2'+) with mixing. These values are tabulated in columns (a), (b), and (c) of Table II both in cgs units and Bsp units.

TABLE II

Calculated values of electric quadrupole reduced transition probabilities.

Nucleus	(a) B(E2 0"+ ⟶ 2+) (with mixing)		(b) B(E2 0'+ ⟶ 2+)* (without mixing)		(c) B(E2 0+ ⟶ 2'+) (with mixing)	
	e^2 x 10^{-48} cm^4	Bsp	e^2 x 10^{-48} cm^4	Bsp	e^2 x 10^{-48} cm^4	Bsp
Sm^{152}	0.30	12.5	0.21	8.5	0.13	5.3
Er^{166}	0.29	10.9	0.26	9.7	0.23	8.4
Th^{232}	0.28	6.7	0.24	5.8	0.21	4.9
U^{234}	0.35	8.2	0.31	7.3	0.27	6.5
Pu^{238}	0.30	6.8	0.27	6.1	0.24	5.5

*B(E2 0'+ ⟶ 2+) = B(E2 0 + ⟶ 2'+) (without mixing).

DISCUSSION

From the calculated B (E2) values it is indicated that the B (E2 0+→2'+) value is depressed and B (E2 0'+ —→ 2+) is enhanced. Since we have estimated the interaction by perturbation of rotational spacings, the depression and enhancement are very sensitive to the values of energies of the rotational bands; for example, a 5% error in the experimental energies will result in approximately a 20% error in the calculated B (E2) values.

There are few experimental data for comparison. The only available coulombic excitation datum is B (E2 0+ —→ 2'+) = 3 ± 1 in Bsp[11] for Sm[152]. There is qualitative agreement between the calculated upper limiting values for Sm[152] (5.28 in Bsp) with the experimental values. Generally, the enhancement of the de-excitation transition caused by mixing is increased as the nucleus approaches to the borderline of the region of strongly deformed nuclei. At the same time the depression of the Coulombic excitation transition 0+ —→ 2'+ becomes more pronounced.

On the other hand, from equations (8a), (8b), (9a), (9b), (10), and (11) we get the expression for the branching ratio of the B(E2) transition from the 2+ state of the β vibrational band and first rotational band

$$\frac{B(E22'+ \longrightarrow 2+)}{B(E22'+ \longrightarrow 0+)} = \frac{\left(C_{000}^{222}\right)^2 B(E2)_{vib} \left(1 - \epsilon_{22'}^2\right)^2}{\left(C_{000}^{022}\right)^2 \left(\sqrt{B(E2)_{vib}} - \epsilon_{22'} \sqrt{B(E2)_{rot}}\right)^2}$$

(A-10)

$$\frac{B(E22'+ \longrightarrow 4+)}{B(E22'+ \longrightarrow 0+)} = \frac{\left(C_{000}^{422}\right)^2 \left[\sqrt{B(E2)_{vib}} (1 - \epsilon_{22'}\epsilon_{44'}) + (\epsilon_{44'} - \epsilon_{22'})\sqrt{B(E2)_{rot}}\right]^2}{\left(C_{000}^{022}\right)^2 \left[\sqrt{B(E2)_{vib}} - \epsilon \sqrt{B(E2)_{rot}}\right]^2}$$

(A-1b)

where $B(E2)_{rot}$ is the electric quadrupole transition between the 2+ state and ground state of first rotational band without considering the effect of mixing— these values can be easily calculated from experimental values[10] of B(E2 2+ —→0+).

$B_{vib}(E2)$ is the electric quadrupole transition between the 2'+ state of the β vibration band and ground state without considering the effect of mixing — these values are assumed to be 5 times the values of B_{sp} respectively, except $Sm^{152},^{12} Th^{232*}$ which we could estimate easily from experimental data.

$\epsilon_{22'}, \epsilon_{44'}$ is the mixing amplitude which can be evaluated the same way as we mentioned before. The calculated results are tabulated in Table III.

TABLE III

The theoretical values of branching ratio of $B(E2\ 2'+ \longrightarrow 0+)$: $B(E2\ 2'+ \longrightarrow 2+)$: $B(E2\ 2'+ \longrightarrow 4+)$.

Nucleus	$B(E2\ 2'+ \longrightarrow 0+)$	$B(E2\ 2'+ \longrightarrow 2+)$	$B(E2\ 2'+ \longrightarrow 4+)$
Sm^{152**}	1	3.52	9.37
Er^{166}	1	1.79	5.06
Th^{232}	1	1.88	6.17
U^{234**}	1	1.84	5.44
Pu^{238}	1	1.66	4.10
without mixing	1	1.43	2.57

*In the Th^{232} case the β and γ vibrational bands are so close together that the experimentally measured values[12] of $B(E2)$ will be a mixture of two bands and then the estimated value is not accurate.

**Sm^{152} Th^{232} cases the third excited states of first rotational band are not known so we use $\epsilon_{44'} = \frac{20}{6} \epsilon_{22'}$ as an approximation.

From the Table III we see that the $B(E2\ 2'+ \longrightarrow 4+)$ transitions are enhanced by mixing more than the $B(E2\ 2'+ \longrightarrow 2+)$ transition, and the enhancement in general increases toward the edge of the spheroidal nuclei region. This is the same conclusion as we get from the $B(E2)$ absolute values.

REFERENCES

1. A.Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat-fys. Medd $\underline{27}$, No. 16, p. 89 (1953).

2. O. Nathan, Nuclear Phys. $\underline{2}$, p. 564 (1957).

3. I. Marklund et. al. Nuclear Phys. $\underline{15}$, p541-546 (1960).

4. Stephens, Diamond, and Perlman, UCRL-8902, Sept. 1959 (unpublished results).

5. C. Gallagher, Jr. and T. D. Thomas, UCRL-8804, Aug. 1959.

6. I. Perlman and J. O. Rasmussen, Handbuck der Physik $\underline{17}$, p. 193 (1957).

7. G. Herzberg, Spectra of Diatomic Molecules, D. Van Norstrand Company Inc., p. 103 (1959).

8. S. Nilsson, Kgl.Danske Videnskab Selskab, Mat-fys Medd. $\underline{29}$, No. 16, p. 29 (1955).

9. B. L. Birbrair et al., Soviet Phys.- JETP $\underline{9}$, p. 568 (1959).

10. K. Alder et al., Revs. Modern Phys. $\underline{28}$, p.530 (1956); Bent Elbek (unpublished data), 1960.

11. R. K. Sheline, Revs. Modern Phys. $\underline{32}$, p.21, 1960.

12. F. K. McGowan and P. Stelson, Bull. Am. Phys. Soc. $\underline{2}$, p. 207 (1957).