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ARE RICH PEOPLE SMARTER ?

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ARE RICH PEOPLE SMARTER ?

Abstract

We investigate general models of wealth accumulation induced by financial investment. These models assume only that returns are stochastic and that a minimal level of wealth is required in order to participate in financial investments. When homogeneous investment talents are assumed the generated distribution of wealth converges to the Pareto (power-law) distribution of wealth which is empirically observed at the high-wealth range. However, when a small degree of diversity of investment capabilities is introduced, the resulting distribution of wealth becomes inconsistent with the empirical distribution. We conclude that the empirical Pareto wealth distribution suggests that chance, rather than talent, is the dominant factor in the process of wealth accumulation by financial investment. Our findings conform with market efficiency and may have implications regarding the origins, the economic significance, and the desirability of social inequality.

1. Introduction

In ancient Greece only the wealthy land owners had voting rights (Caldwell and Merrill, 1950). The logic behind this law was that voting privileges should be given only to the wise citizens, and wealthy people have proven their wisdom by becoming (and managing to remain) rich. The idea that the rich are rich because they are smarter has been with us ever since, and has become a central part of modern western ideology and culture.

This idea is manifested in financial markets in many ways. For example, many fund managers are compensated according to the performance of their funds, because it is believed that the fund's performance is directly linked to the manager's talents. On the basis of stock price prediction market "Gurus" are created, and followed. Modern western mythology is filled with success stories, in which the poor but ambitious and talented hero becomes a millionaire. Indeed, the notion that in the financial survival-of-the-fittest the smartest investors surface to the top, seems very natural and plausible.

On the other hand, the theory of market efficiency tells us that being smart does not make one a financial winner. The most that a smart investor can (and indeed must) do is diversify away diversifyable risk. There are numerous studies showing that public information can not be exploited to obtain abnormal returns (see for example, Fama 1970, Fama and French 1992). Samuelson heads his 1989 paper which deals with the possibility of reaping abnormal profits with the quotation:

"Forsake search for needles that are so very small in haystacks that are so very large".

He latter writes :

"Those lucky money managers who happen in any period to beat the comprehensive averages in total return seem primarily to have been merely lucky. Being on

the honor roll in 1974 does not make you appreciably more likely to be on the 1975 honor roll." (Samuelson 1989).

Thus, there are two competing explanations for the uneven distribution of wealth in society: i)talent or ii)luck. Both explanations seem very reasonable, and one would probably expect the true explanation to be some combination of the two. Finding out which of these two factors is more dominant has deep philosophical and social implications. If the uneven distribution of wealth is mainly due to diverse talents, then it would seem that inequality is a natural and positive driving force of economic evolution. If, however, it turns out that chance is the dominant factor responsible for the distribution of wealth, this may raise some doubts regarding the economic role and the desired level of inequality.

Three remarks should be made about the title of this paper. The first regards the definition of "smartness". Throughout this paper we define smartness as investment talent - the ability to obtain on average abnormal returns. Thus, we use a narrow definition of "smart" which is relevant to financial markets, and does not necessarily relate to other manifestations of talent such as poetry, philosophy, etc. The second remark is about the kind of answer that we expect for the question asked in the title. There are probably examples of very rich people who are not very bright, and beggars who used to be financial wizards. We will not be concerned with specific examples but, rather, we will look for a connection between wealth and talent in a statistical sense. The third remark is that our analysis is restricted to the high-wealth range (the right tail of the wealth distribution). Thus, the precise question that we ask is whether the distribution of wealth among rich people reflects their relative talents.

A direct measurement of the relation between investment talent and wealth could in principal be undertaken by interviewing investors, asking them about their wealth and keeping track of their investments from that time onwards. One could then rate the performance of the investments by some objective performance measure (Sharpe index or Jensen index, for example), and check for correlation

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between wealth and investment performance. To the best of our knowledge such a measurement was not carried out, probably due to the difficulty of obtaining information about the wealth of individuals. A somewhat similar direct approach has been employed in order to establish whether there is a relation between **income** and returns. The results, however, where not conclusive (Blume, Crockett and Friend 1974, Yitzhaki 1987).

In this paper we use a different approach, one which is indirect. We look for clues about the connection between talent and wealth by analyzing the empirical distribution of wealth. We suggest a very general stochastic process as a model for the process of wealth accumulation by investment. We investigate two cases: the first is the homogeneous talent case, in which differences in wealth arise only from chance; the second is the diverse talent case in which inequality is the outcome of both chance and different investment capabilities. We test these two competing versions by studying the exact form of the wealth distributions which they generate, and comparing them with the empirical distribution.

When examining the wealth distribution in society one typically finds two distinct regions. At the lower-wealth range the distribution of wealth can be approximated by the log-normal distribution. At the high-wealth range the distribution is described by the Pareto distribution (for example, see Stiendl 1965). While the lower range accounts for the vast majority of the population (usually about 95%), the top range is extremely important as it accounts for most of the wealth[1]. The main factors influencing the wealth of a person at the lower range are usually salaries and consumption. In contrast, the wealth of individuals in the high-wealth range typically changes mainly due to financial investments. In this paper we are interested in the process of wealth accumulation via investments and we therefore concentrate on the form of the distribution at the high-wealth range.

A century ago Pareto (1897) discovered that at the high range, wealth (and also income) are distributed according to a power-law distribution, which became

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[1] In the U.S. the top 1% of the population holds over than 40% of the total wealth (Wolff 1995).

known as the Pareto distribution. The Pareto distribution is given by the following probability density function:

$$P(W) = CW^{-(1+\alpha)} \qquad for \quad W \geq W_0 \qquad (1)$$

where W stands for wealth, W_0 is the lower end of the high wealth range, and C and α are positive constants.

Pareto's finding, also known as the Pareto Law, has been verified in numerous studies for various countries (see, for example, Atkinson and Harrison 1978). Three different examples of the Pareto wealth distribution are illustrated in Figures 1a-1c. Figure 1a shows a measurement of the wealth distribution, which was performed in Great Britain. The wealth range (horizontal axis) is divided into wealth classes, and the number of persons in each class is specified by the vertical axis. Notice that this is a double-logarithmic plot. The solid line represents the empirical data. A Pareto distribution should appear linear on a double-logarithmic scale, with slope $-(1+\alpha)$. The dashed line is a Pareto fit to the data. One can see that the empirical distribution is fitted rather well by the Pareto distribution (correlation coefficient -0.975), however, since the number of wealth classes is small this measurement is not very definitive. (Source: National Income and Expenditure, Great Britain, 1970). An alternative way to examine the empirical distribution of wealth, is to measure the percentage of the population with wealth exceeding different wealth levels. A Pareto distribution of wealth would yield a straight line on a double-logarithmic scale, with slope $-\alpha$. Figure 1b shows such a measurement, which was done in Sweden. The empirical data are represented by dots, the solid line is the Pareto fit. The empirical distribution is in excellent agreement with the Paretian fit (correlation coefficient -0.999). The value of the slope is -1.66 (Source: Steindl ,1965).

Recent evidence supporting the Pareto distribution of wealth in the U.S. is provided by the 1996 Forbes 400 list (Fig. 1c). A Pareto distribution of wealth

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implies a relation between the rank of a person in the wealth hierarchy and his wealth. For a Pareto distribution with exponent α the expected relation is: $W = An^{-1/\alpha}$, where W is the wealth, A is a constant n is the rank (i.e. for the person ranked 200 in the wealthiest people list $n = 200$). (For derivation of this relation see, for example, Takayasu 1990). Figure 1c shows the wealth of the richest people in the U.S. as a function of their rank. The solid line is a Paretian fit with slope -0.729, which corresponds to $\alpha = 1.35$ (correlation coefficient -0.998). (Source: Levy and Solomon, 1997).

The first to suggest an explanation for the Pareto distribution of wealth was Pareto himself (Pareto 1906). Pareto suggested that the distribution of wealth corresponds to an underlying distribution of human abilities. However, Pareto has not offered a mathematical model that would explain the distribution of abilities and it's relation to the Pareto law. Pareto's explanation was advanced by Davis who introduced the "law of the distribution of special abilities" which asserts that that the probability of an additional unit of ability was independent of the level of ability, (Davis 1941). This model, however, leads to a normal distribution of ability and therefore presumably to a normal, rather than Pareto, distribution of wealth. A different model for the distribution of ability was formulated by Boissevain (1939) who considered the distribution of abilities that could be represented as a product of several factors each of which follows a binomial distribution. Boissevain's model explains the positive skewness in the distributions of wealth and income, but leads to a log-normal distribution, not the empirically observed Pareto distribution.

The main models that offer an explanation for the precise form of the Pareto wealth distribution are the Markov chain model of Champernowne (1953), the stream model of Simon (1955) and the birth-and-death model of Wold and Whittle (1957)[2] . Although these models are quite different from each other in their details

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[2] For a review of models generating Pareto distributions see Steindl (1965), Arnold (1983), and Slottje (1989).

they do have some common features:

- Stochastic **multiplicative** dynamics

- Lower bound

- Homogeneous talent

Levy and Solomon (1996) have shown that the above three elements are indeed the only essentials needed in order to insure that a process will generate a Pareto distribution. In all of the processes which are based on these elements (including those of Champernowne, Simon, and Wold and Whittle) the **only** reason for the inequality in the generated distribution is the stochastic process - **chance**. Differences in talent are not assumed and therefore play no role in the process of wealth differentiation.

In this study we investigate whether the homogeneous talent assumption is essential in order to obtain the Pareto wealth distribution. More specifically, we ask if the Pareto distribution can be generated by reasonable wealth accumulation processes which combine both chance and talent.

We find that the introduction of even a small degree of diversity with respect to investment talent leads to a wealth distribution which is significantly different from the Pareto distribution. Thus, diverse talents are found to be inconsistent with the empirical wealth distribution at the high wealth range. This leads us to conclude that at the high wealth range chance, rather than talent, is the dominant factor in the process of wealth accumulation.

The rest of this paper is organized as follows. Section 2 lays the general framework of bounded stochastic multiplicative processes. Section 3 demonstrates that the homogeneous talent assumption leads to the Pareto wealth distribution. In section 4 we examine the effects of the introduction of a small degree of talent heterogeneity. In section 5 we summarize our results and discuss their implications.

2. Stochastic Multiplicative Processes with a Lower Bound

A stochastic multiplicative process is a process in which the value of each element is **multiplied** by a **random** variable with each time step. Many economic processes, and specifically the accumulation of wealth via investment of capital, are stochastic and multiplicative by nature. For example, if a person invests his money in a portfolio which yields 10% with probability 1/2 and −5% with probability 1/2 each year, his wealth will follow a stochastic multiplicative process. The main difference between multiplicative and additive processes is that in additive processes (such as random walks) the changes in value are independent of the value, whereas in multiplicative processes the changes are proportional to the value.

Formally, the stochastic multiplicative wealth accumulation process is given by:

$$W_i(t+1) = W_i(t)\tilde{\lambda} \tag{2}$$

where $W_i(t)$ is the wealth of investor i at time t and $\tilde{\lambda}$ represents the return, which is a random variable drawn from a distribution $f_i(\tilde{\lambda})$. Generally, each investor may have a different distribution of returns on his investment, hence the sub-index i in $f_i(\tilde{\lambda})$.

For people at the high-wealth range, changes in wealth are mainly due to financial investment, and are therefore multiplicative. For people at the lower wealth range, changes in wealth are mainly due to salaries and consumption, which are basically additive rather than multiplicative. In reality there is no sharp boundary between the lower and the upper wealth regions. As the stochastic multiplicative process (equation(2)) describes the dynamics only at the higher wealth range, we introduce a threshold wealth level, W_0, above which the dynamics is multiplicative. We assume that only those people with wealth exceeding W_0 participate in the stochastic multiplicative investment process. Formally, we require that:

$$W_i(t) \geq W_0 \quad \forall i \; \forall t. \tag{3}$$

In the case that there is an overall drift towards lower wealth values (as in Champernowne 1953) one can define the lower bound W_0 in absolute terms. In order for the lower bound to be meaningful in the general case, where there is a general drift towards higher wealth values (as when there is inflation), the value of the lower bound should be defined in terms of the average wealth, i.e. $W_0 = \omega \frac{1}{N} \sum_i W_i(t)$, where N is the number of investors and ω is given in absolute terms.

When people's wealth changes they may cross the boundary between the upper and lower wealth regions. As we do not model the dynamics at the lower wealth range, and for the sake of simplicity, we assume that the market has reached an equilibrium in which the flow of people across the boundary is equal in both directions. This means that the number of people participating at the the stochastic multiplicative investment process remains constant. The above assumption simplifies the analysis, but the results presented here are robust to the relaxation of this assumption.

3. Homogeneous Talent

In the homogeneous talent model we assume that all investors face the same return distribution [3] i.e. :

$$f_i(\tilde{\lambda}) = f(\tilde{\lambda}) \quad \forall i \tag{4}$$

Note that although all investors face the same return distribution $f(\tilde{\lambda})$, $\tilde{\lambda}$ is drawn separately for each investor. A "lucky" investor is one for which many high values of λ are drawn. Such a lucky investor will become richer than others. As investors face the same distribution of returns, the differentiation in wealth in this case is

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[3] Even if all investors have similar investment talent one would still generally expect them to have different distributions of returns, due to different attitudes towards risk. This point is discussed in the last section of the paper.

due entirely to chance. The stochastic multiplicative process with a lower bound and homogeneous talent is given by:

$$W_i(t+1) = W_i(t)\tilde{\lambda}$$

$$f_i(\tilde{\lambda}) = f(\tilde{\lambda}) \quad \forall i \qquad (5)$$

$$W_i(t) \geq W_0 \quad \forall i \ \forall t$$

It can be shown that this process leads to a steady state Pareto wealth distribution for any non-degenerate initial wealth distribution (see Appendix A). This result is very general and is robust to various generalizations of the dynamics. The steady state Pareto distribution is independent of the choice of the return distribution function $f(\tilde{\lambda})$. $f(\tilde{\lambda})$ can even be time dependent. The value of the lower bound, (W_0), can effect the degree of inequality (value of α in the Pareto distribution), but not the fact that the distribution of wealth is Paretian. The number of investors does not change the resulting wealth distribution. In fact, the assumption of a constant number of investors can be replaced by the assumption of a stochastic inflow of investors into the market. The Pareto distribution of wealth also remains intact if one introduces the assumption that investors consume a certain fixed (or stochastic) proportion of their wealth in each time period.

Monte Carlo simulations of the stochastic multiplicative dynamics with a lower bound and homogeneous talent confirm the above results, and allow an estimate of the time it takes the wealth distribution to converge to the Paretian distribution. We have conducted simulations in which the number of investors is 1000, and they all start out with $100 Million . W_0 is set to 20% of the average wealth. $f(\tilde{\lambda})$ is given by:

λ	probability
1.10	1/2
0.95	1/2

We have recorded the distribution of wealth at different times. The results are shown in Figure 2, which is a two-way logarithmic plot of the probability density as a function of wealth (in units of the average wealth). The dashed vertical line at 0.2 represents the minimal wealth threshold W_0. The distribution after 10 investment periods (Figure 2a) is still pretty symmetric, and centered around the average wealth (1.0 on the horizontal axis). However, after 100 time periods the wealth distribution is very close to the Paretian distribution (Figure 2b). The distribution remains Paretian from then on. Figure 2c shows the wealth distribution after 10,000 time periods. Monte Carlo simulations of many variations of the basic homogeneous talent model confirm the robustness of the Pareto-law result.

We have shown that stochastic multiplicative processes with a lower bound and homogeneous talent lead to the convergence of the wealth distribution to the Pareto distribution. This result has proven robust in many variations of the basic process. In all of these variations, however, we assume homogeneous talent and therefore the differentiation of wealth is due entirely to chance. Does this mean that talent is homogeneous in the investment world? Or, is it possible to formulate a model which will explain the Pareto wealth distribution as a consequence of both chance and talent? In the next section we describe bounded stochastic multiplicative processes with diverse talents, and we investigate the distributions of wealth generated by these processes.

4. Diverse Talents

Investment talent is the ability to obtain superior return distributions on investments. In a market composed of investors with diverse talents, investors with different skills face different return distributions. Thus, we return to the general case where the return distribution, $f_i(\tilde{\lambda})$, is investor specific.

As a first step toward the analysis of the general heterogeneous talent model, we examine a simplified two-population example. Consider a market in which

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most of the population faces the following "normal" return distribution:

	λ	probability
$f_{normal}(\tilde{\lambda})$:	1.10	1/2
	0.95	1/2

while a minority of "smart" investors receive the superior distribution:

	λ	probability
$f_{smart}(\tilde{\lambda})$:	1.11	1/2
	0.96	1/2

As more and more investment periods pass the "smart" investors become on average richer than the "normal" investors. As the "normal" investors become relatively poorer, more and more of them will cross the lower wealth threshold, W_0, and will exit the market. One might suspect that in the long run the "normal" population will be completely wiped out. However, recall that there is an inflow of investors into the market. This is an inflow of investors from below the threshold who have acquired enough wealth in order to participate in the investment process. (We do not model the process of wealth accumulation below the threshold, but assume that the market is in a steady state in which the inflow of new investors balances the outflow of investors leaving the market. This assumption simplifies the analysis but is not essential to our results.) Some of the new investors entering the market are of the "normal" type[4]. As the number of "normal" investors declines, so does their proportion in the outflow from the market. Eventually, a balance is reached when the outflow of players of each type matches the inflow of that type, and the size of each subgroup converges to a certain (mean) value.

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4 One can think of different ways in which to compose the population of new investors: a) for each investor exiting the market an investor of the same type enters. b) each new investor has a certain probability p for being "smart" and probability (1-p) of being "normal". The choice between the above two options, and the value of the probability p, may change the specific form of the steady state wealth distribution, but not it's essential features.

As the population of each subgroup is homogeneous, the wealth distribution of each subgroup is subject to the dynamics described by equation (5).[5] From the result of the preceding section it follows that the wealth of each subgroup will be divided between the members of that subgroup according to the Pareto distribution. Thus, the wealth distribution among "normal" investors is:

$$P_{normal}(W) = C_{normal} W^{-(1+\alpha_{normal})} \tag{6}$$

and the wealth distribution among "smart" investors is:

$$P_{smart}(W) = C_{smart} W^{-(1+\alpha_{smart})}. \tag{7}$$

Both distributions are Paretian, but with different parameters C and α. As the average wealth of the smart population is greater than the average wealth of the normal population we will have $\alpha_{smart} < \alpha_{normal}$ (see Appendix B). The aggregate distribution of wealth will be:

$$P(W) = C_1 W^{-(1+\alpha_{normal})} + C_2 W^{-(1+\alpha_{smart})} \tag{8}$$

which is **not** a Pareto distribution. (C_1 and C_2 replace C_{normal} and C_{smart} because the normalization constraints have changed, and depend on the relative proportions of the two subgroups, i.e.: $C_1 = \frac{N_{normal}}{N} C_{normal}$; $C_2 = \frac{N_{smart}}{N} C_{smart}$).

Monte Carlo simulations of the above two population model verify our analysis. The wealth distribution is shown in Figure 3. Notice that although the distribution of wealth among the two subpopulations is Paretian, the aggregate distribution is not ($\alpha_{normal} = 1.67, \alpha_{smart} = 0.63$).

In the general heterogeneous talent case there are many different subgroups.

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[5] The interaction between the different subgroups is only through the lower bound W_0, which depends on the average wealth of all investors.

One can view this as a simple generalization of the above two-population example. The distribution of wealth within each subgroup is the Pareto distribution with a certain α, but the aggregate distribution is not Paretian. Instead, it is concave when plotted on a double-logarithmic scale. See Figure 4 for a sketch of the general heterogeneous talent case.

Simulations of heterogeneous talent models do in fact yield wealth distributions which are significantly different than the Pareto distribution and are concave when plotted on a double-logarithmic scale. Figure 5 depicts the steady-state wealth distribution in a market in which each investor faces a different return distribution. For all investors the return distribution $f_i(\tilde{\lambda})$ is taken as a normal distribution with a standard deviation of 20%. However, the mean of the distribution $f_i(\tilde{\lambda})$, μ_i is different for each investor. We assume that μ is distributed normally in the population with a mean value of 10% and a standard deviation of 2%. Even though the distribution of talent is rather narrow (for 85% of the investors μ_i is in the range 8% − 12%), the resulting distribution of wealth is very different than the Pareto distribution (Figure 5). The Kolmogorov-Smirnov goodness-of-fit test confirms that one can safely reject the hypothesis that the generated distribution is Paretian. Comparing the cumulative distributions of the sample distribution with the best fit Pareto distribution we obtain a D value of 0.310, which is much larger than the critical D value of 0.103 ($= 1.63/\sqrt{250}$) needed in order to reject the hypothesis that the distribution is Paretian at a 99% confidence level. (For comparison, the D value obtained for the distribution in Figure 2c is 0.010, which allows one to safely accept the hypothesis that the distribution is Paretian).

Different forms of the return distributions (discrete, truncated-normal, log-normal), different distributions of talent, and variations on many other details of the model yield the same result. Even if one assumes that investors have finite life spans, and that they inherit their wealth to siblings who may have different talents, the concave wealth distribution on double-logarithmic scale persists, and

the Pareto distribution is not obtained[6]. It seems that any stochastic multiplicative wealth accumulation model which assumes even a mild degree of diverse talents leads to a distribution of wealth which is inconsistent with the empirical Pareto distribution.

5. Discussion

We have presented a generic stochastic multiplicative process with a lower bound, as a model for the evolution of speculative wealth. When homogeneous return distributions are assumed, the distribution of wealth converges to the Pareto distribution which is empirically observed at the high wealth range. When heterogeneous return distributions are assumed, the resulting distribution of wealth is inconsistent with the empirical distribution. These results are very robust, and remain intact when many different generalizations of the basic model are explored.

In principal, one should investigate all possible models incorporating heterogeneous return distributions in order to decisively conclude that heterogeneous return distributions are inconsistent with the Pareto wealth distribution. It may be possible to come up with a model where the effect of heterogeneous return distributions is precisely offset by some other heterogeneity effect. However, it seems that such a model could not be based on reasonable economic foundations and could not be general or robust.

Thus, the evidence in this paper suggests that investors in the stock market, with wealth exceeding a certain level, hold portfolios yielding similar return distributions (although the portfolios themselves may be different). One implication is that investors can not differ to much in their investment talents. If some investors would have been more talented than the others we would have expected them to

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[6] Reasonable mortality rates must be assumed. One can think of the death and inheritance process as the same investor who continues to invest, but with different talent. In the case where investors live for an extremely short time, there is strong time averaging of talent, we return to the homogeneous (average) talent framework, and the Pareto distribution is obtained.

achieve superior return distributions. However, this would lead to a non-Paretian wealth distribution, in disagreement with the empirical data.

The following problem arises: homogeneous talent seems to be a necessary condition for the Pareto wealth distribution, but it is not a sufficient condition. The requirement that investors face similar return distributions, which is necessary to insure the Pareto wealth distribution, is stronger than the assumption of homogeneous talent. One could rightfully claim that even if investors have similar investment talents, they may choose different return distributions because of different attitudes towards risk. One possible solution to this problem may be offered if we assume that investors have long investment horizons, i.e. they do not plan to spend a significant proportion of their wealth in the near future. As we are dealing with investors at the high-wealth range, this assumption seems reasonable. In this case it can be shown that under mild assumptions regarding the form of the utility functions, long-horizon investment implies that all investors should choose the one-period return distribution with the maximal geometric-mean, regardless of their preferences.[7]

If all investors aim at maximizing the geometric mean, and if they are all equally talented, it seems plausible that their portfolios will yield roughly the same

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[7] Latané (1959) shows that the probability that terminal payoff of an investment with a certain geometric-mean will be greater than the payoff of any other investment with a lower geometric-mean approaches 1 as the investment horizon becomes infinite. Samuelson (1971) argues that for power utility functions, which imply myopic behavior, the optimal investment is not necessarily that with the highest geometric mean, regardless of the investment horizon. Markowitz (1976) suggests a reconciliation between the maximum geometric-mean criterion and the maximum expected utility framework. Kroll, Levy and Rappoport (1988) find empirical evidence suggesting that investors attempt to maximize geometric mean in a multi-period investment experiment. Leshno and Levy (1997) show that for almost any utility function and long horizon investments the maximum expected utility criterion implies choosing the portfolio with maximal geometric mean (this is true also for myopic utility functions as long as investors are allowed to keep some safety-money uninvested).

return distributions. This does not imply that investors hold the same portfolios. Due to different expectations investors may very well hold different portfolios, and thus realize different returns in each period (different λ's). However, the distribution of returns ($f(\tilde{\lambda})$) is likely to be similar for equally talented investors. This is precisely the condition necessary to ensure the convergence of the wealth distribution to the Pareto distribution. (See Appendix C for one possible framework that may lead to the above scenario).

There are many people who became rich because of their talents, initiative, and hard work. Some people became rich by creating value via investment in real assets, some by closing one-shot "jack-pot" deals, and some by winning lotteries. These different routes of getting rich are certainly not described by similar return distributions. There are many ways of getting into the top wealth range, some of which probably involve more talent than luck. However, once above a certain level of wealth, the main forces responsible for the redistribution of wealth seem to be financial investments. The evidence in this paper suggests that the vast majority of investors are equally talented at choosing their investments and that the main factor responsible for the uneven distribution of wealth at the high wealth range is pure chance.

This result is very much in the spirit of the Efficient Market Hypothesis. In an inefficient market in which there are investment "bargains" we would expect sophisticated investors to take advantage of these "bargains", and as a result to obtain superior return distributions. The empirical wealth distribution suggests that investors face similar return distributions, which leads to the conclusion that either "bargains" do not exist, or that the investors taking advantage of "bargains" alternate (such that every investor has an equal probability of getting a "bargain").

The result of this paper does not mean that only luck matters, and that any investment strategy is as good as any other. On the contrary, it means that one must apply his investment skills just in order to have a fair chance in the competition with other investors. Our findings suggest that because investors in

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the high wealth range seem to have similar investment talents, at the margin it is only luck that differentiates between them.

An implication for investors is that they should recognize that there is a limit to what they can (and should) do in terms of portfolio optimization. Namely, there seems to be a "benchmark" return distribution (the $f(\tilde{\lambda})$ which is common to all investors). Since the evidence suggests that all reasonable investment strategies lead to the same benchmark return distribution, it seems that sophisticated investment strategies (based on market timing, undervalue stock picking, etc.) have no value added relative to "standard wisdom" (optimal diversification) strategies.

Finally, our conclusion regarding the origins of inequality at the high wealth range may have implications regarding the significance of the role that inequality plays as a driving force of economic development. Many people believe that the distribution of wealth among investors represents a system of rewarding investment talent, or efficient resource allocation, and therefor it contributes to economic development. However, the empirical distribution of wealth suggests that investors in the high-wealth range are equally talented at allocating resources. Those who are most successful owe it primarily to their luck, and not to abnormal investment abilities. Thus, the differentiation of wealth in the top wealth range does not reflect a system which rewards skill, and therefor it does not constitute a force driving optimal resource allocation.

Appendix A

The bounded stochastic multiplicative process with homogeneous talent is given by:

$$W_i(t+1) = W_i(t)\tilde{\lambda}$$

$$f_i(\tilde{\lambda}) = f(\tilde{\lambda}) \quad \forall i$$

$$W_i(t) \geq W_0 \quad \forall i \quad \forall t.$$

In order to show that this process leads to a convergence of the wealth distribution to the Pareto distribution, let us denote the probability density of having wealth W at time t by P(W(t)). From probability conservation we have:

$$(9)$$

$$P(W(t+1)) = P(W(t)) + \int_{-\infty}^{+\infty} P(W(t)/\lambda)f(\lambda)d\lambda - \int_{-\infty}^{+\infty} P(W(t))f(\lambda)d\lambda$$

the second term on the right hand side is the inflow of probability to state W and the third term represents the outflow from state W. Since $f(\lambda)$ is a probability distribution, $\int_{-\infty}^{+\infty} f(\lambda)d\lambda = 1$, and the first and third term on the right side cancel out. Starting from an arbitrary (non-degenerate) probability density, the probability density changes according to (9) until it eventually converges to the steady state density distribution, which is time independent. As we are interested in the steady state stable distribution, we are looking for a time independent distribution $P(W)$ which solves equation (9). Since we require that the distribution does not vary with time we must have $P(W(t+1)) = P(W(t))$. Substituting in equation (9), we obtain:

$$P(W(t)) = \int_{-\infty}^{+\infty} P(W(t)/\lambda)f(\lambda)d\lambda. \qquad (10)$$

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It can be easily verified by substitution that the Pareto probability distribution $P(W) = CW^{-(1+\alpha)}$, satisfies equation (10), where α is the solution to:

$$\int_{-\infty}^{+\infty} \lambda^{(1+\alpha)} f(\lambda) d\lambda = 1 \qquad (11).$$

The uniqueness of the Pareto solution is based on the fact that the only positive g-harmonic functions on $\Re$ are exponentials (Choquet 1960, Furstenberg 1965). We would like to emphasize that the analysis is quite general and does not rely on any specific form of the return distribution $f(\lambda)$. Note that without the lower bound there would be no steady state. Instead, the distribution $P(W(t))$ would converge to the lognormal distribution with a variance growing to infinity. For further details see Levy and Solomon (1996).

Appendix B

The lower the value of the exponent α in the Pareto distribution, the higher the average wealth. $-\alpha$ is the slope of the distribution function on the double-logarithmic scale. It is therefore intuitively clear that the more moderate the slope (smaller α) the more weight is given to higher wealth states. Formally:

$$E(W) = \int_{W_0}^{+\infty} P(W)W\,dW = C\int_{W_0}^{+\infty} W^{-(1+\alpha)}W\,dW = \frac{C}{(\alpha-1)}W_0^{1-\alpha}.$$

(We assume $\alpha > 1$, otherwise $E(W)$ is infinite. Empirical values of α are typically $1.2 - 1.6$) . From the normalization condition $\int_{W_0}^{+\infty} P(W)dW = 1$ we obtain $C = \alpha W_0^\alpha$. Substituting in the above equation we obtain :

$$E(W) = W_0\frac{\alpha}{(\alpha-1)},$$

which is a monotonically decreasing function of α.

Appendix C

One possible framework that may lead to the scenario in which investors realize different returns that are drawn from the same distribution is the following:

Assume:

- One period returns on assets are normally distributed.

- There is a riskless asset yielding a return r_f.

- Investors have long investment horizons and general risk aversion preferences ($u \in U_2^*$ in the terminology of Leshno and Levy, 1997) and therefore maximize the geometric mean of their portfolio.

- Investors have different expectations and they revise their expectations with time.

The geometric mean of a normal distribution with mean μ and standard deviation σ can be approximated by $GM = exp(\mu - \frac{\sigma^2}{2})$ (this can be shown by a Taylor expansion of $\log GM = E \log(\tilde{x})$, and holds only as long as μ and σ are not very large). If the means, variances, and covariances of asset returns are known, there is a unique portfolio which maximizes the geometric mean. This portfolio is the point in the Mean-Variance plane in which one of the constant-geometric-mean lines is tangent to the Capital Market Line (portfolio G^* in Figure 6).

If investors where to know the means and the covariance matrix they would all hold portfolio $G*$. However, if investors are not given the means and the covariance matrix but form subjective expectations of them they would disagree on the tangency portfolio T, and therefor also on the maximum-geometric-mean portfolio G^*. As a result each investor will hold a different portfolio G_i, and as a consequence investors will realize different returns. As investors revise their expectations over time, their portfolios will also change with time, i.e.: $G_i = G_i(t)$. If investors are equally talented in the estimation of the means and the covariance matrix, their portfolios will "hover" randomly around portfolio G. Formally, this

means that

$$P(G_i(t) = G) = P(G_j(t) = G) \quad \forall i, j, t, G$$

where the P's are probability density functions and G is any portfolio in the mean-variance plane. The probability density $f(\tilde{\lambda})$ of obtaining a return $\tilde{\lambda}$ is therefore the same for all investors:

$$f_i(\tilde{\lambda}) = \int_{all\ G} P(G_i(t) = G) P(\tilde{\lambda}|G) dG = f_j(\tilde{\lambda}) \quad \forall i, j.$$

Hence, the result of the random averaging over portfolios is that investors face the same distribution of returns, while at each time they realize different returns.

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FIGURE 1a
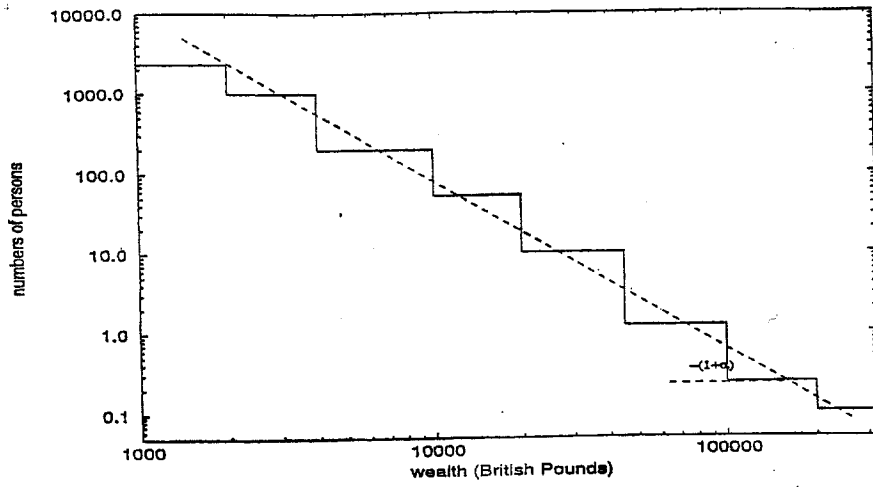
FIGURE 1b

FIGURE 1c

FIGURE 2a

t=10

probability density

wealth (in units of the average wealth)

FIGURE 2b

t=100

probability density

wealth (in units of the average wealth)

FIGURE 2c

t=10,000

probability density

wealth (in units of the average wealth)

FIGURE 3

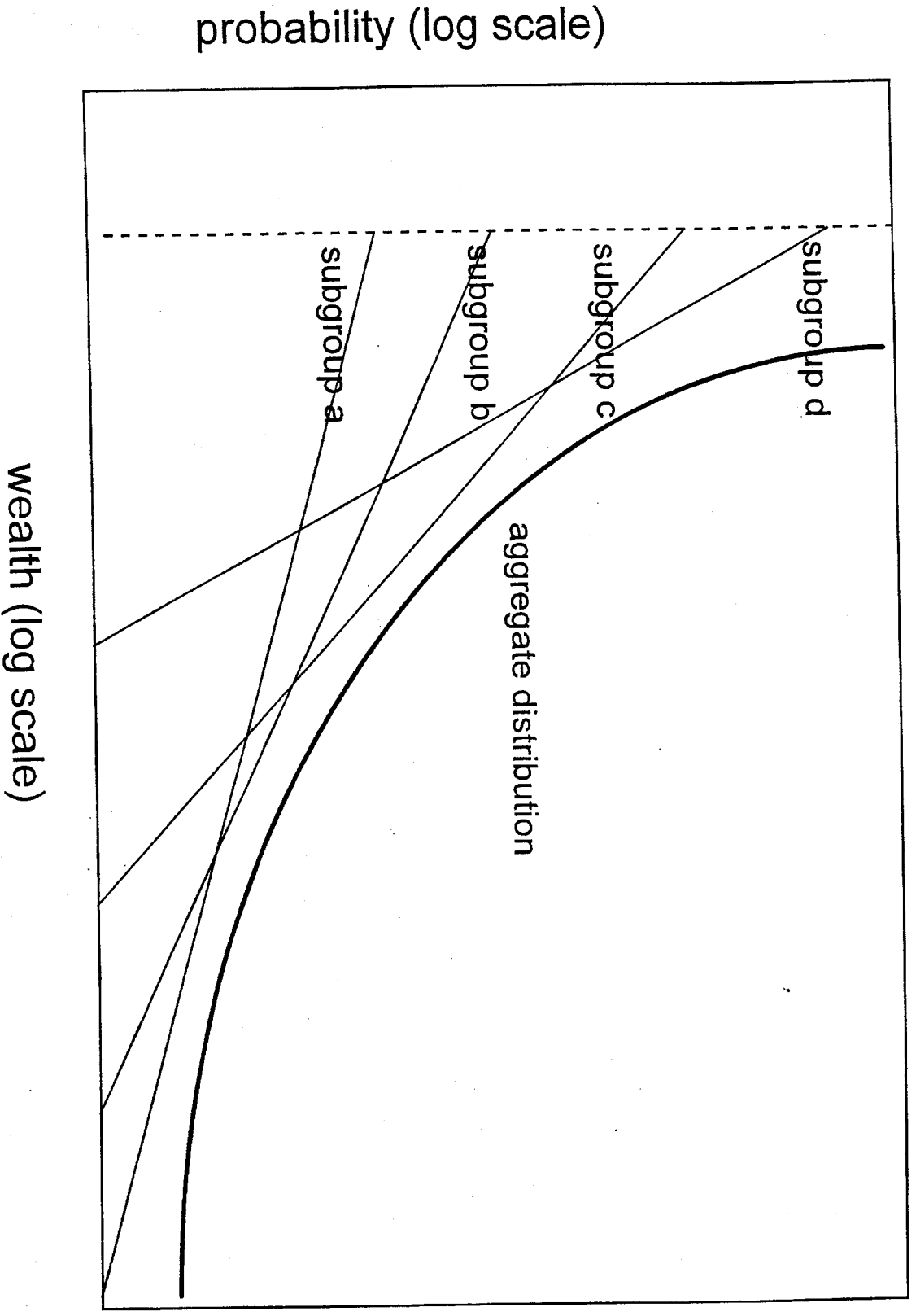probability (log scale)

wealth (log scale)

subgroup a

subgroup b

subgroup c

subgroup d

aggregate distribution
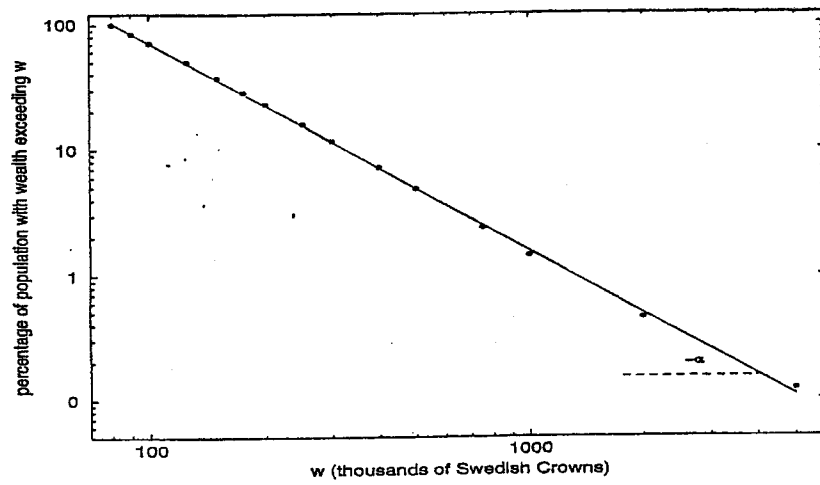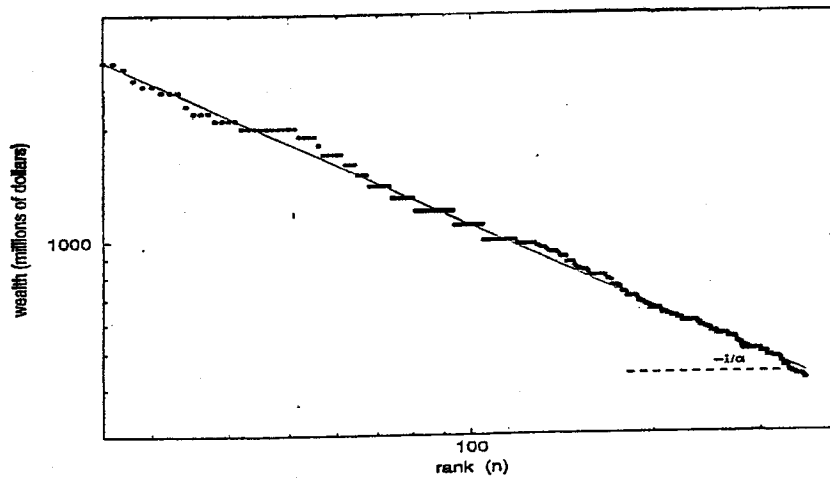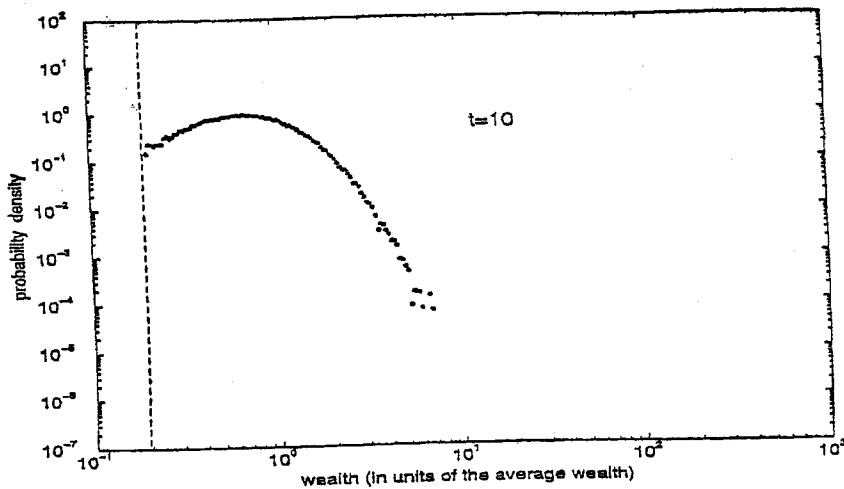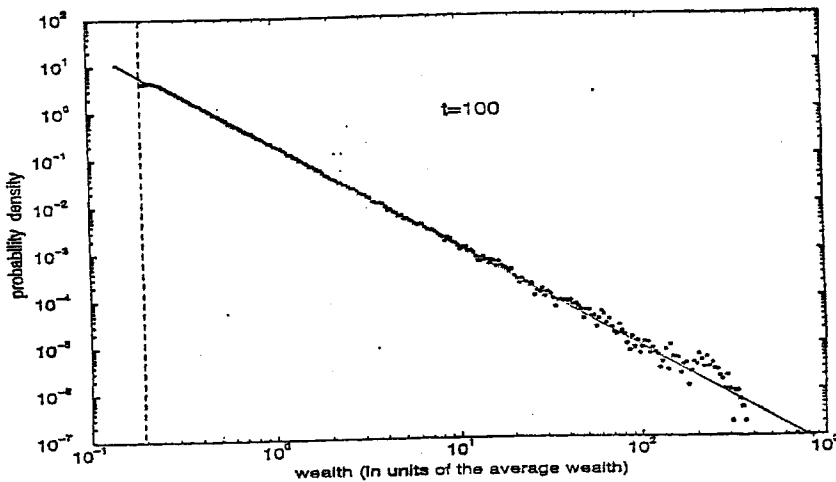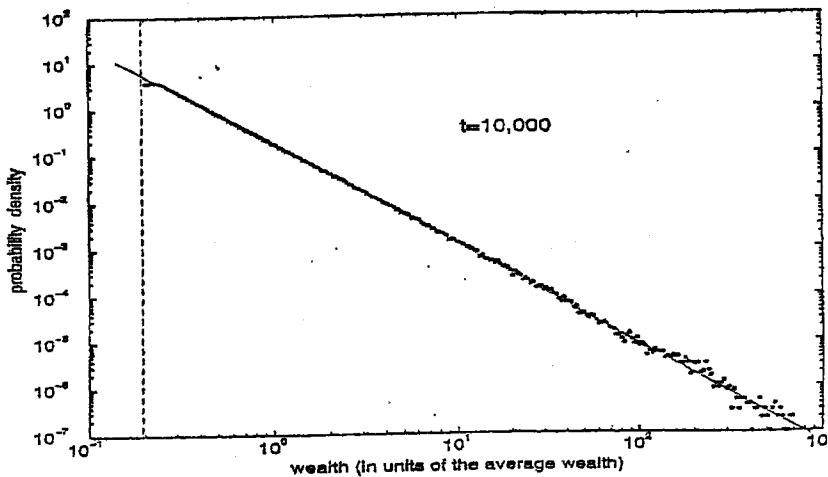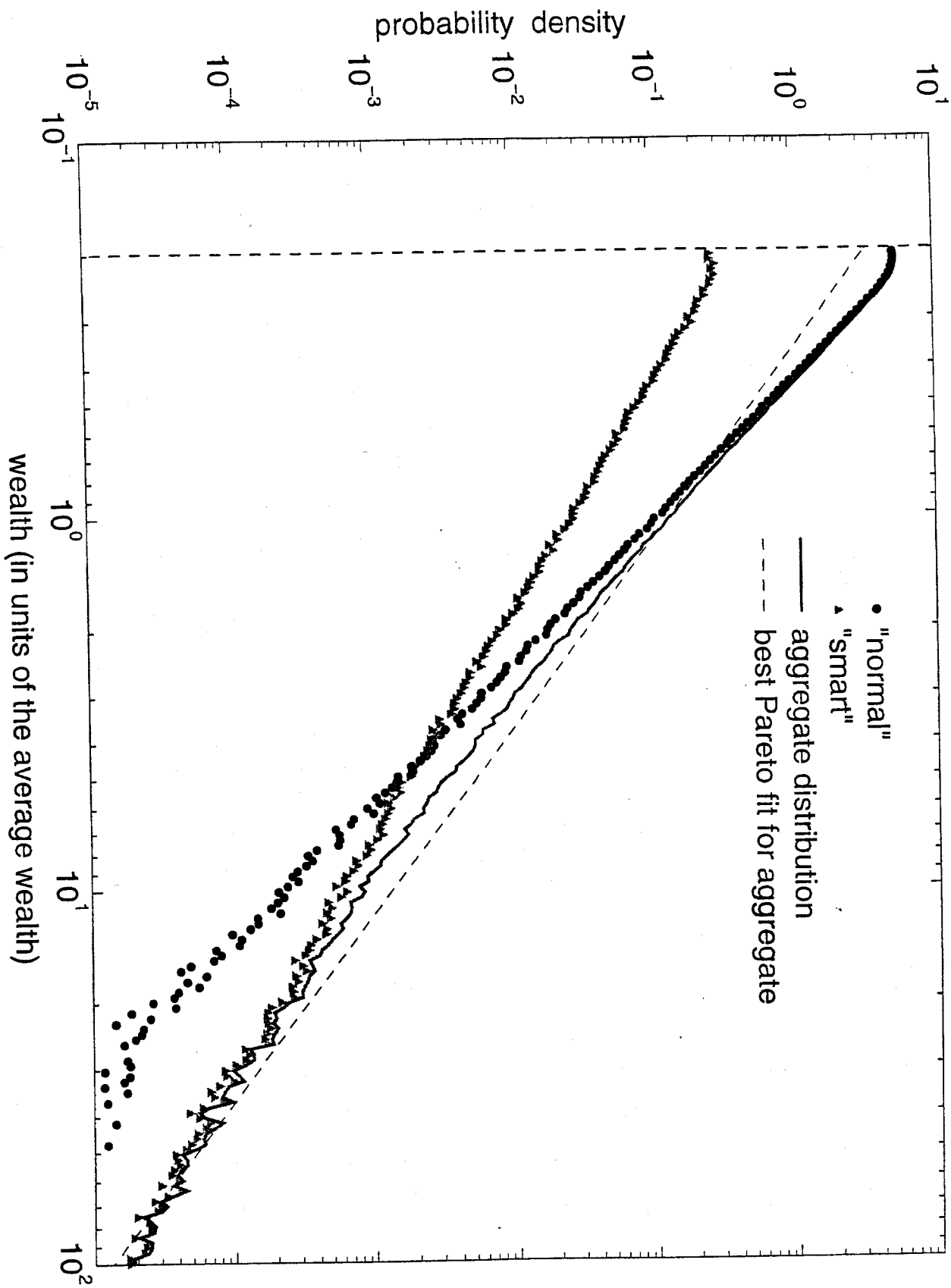
FIGURE 4

FIGURE 5

FIGURE 6