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Los Angeles

Controlling Electric-Field Gradients
for Molecular-Ion Quantum Logic and Wideband Sensing

A dissertation submitted in partial satisfaction
of the requirements for the degree
Doctor of Philosophy in Physics

by

Clayton Ze Chi Ho

ABSTRACT OF THE DISSERTATION

Controlling Electric-Field Gradients for Molecular-Ion Quantum Logic and Wideband Sensing

by

Clayton Ze Chi Ho

Doctor of Philosophy in Physics

University of California, Los Angeles, 2026

Professor Eric R. Hudson, Chair

Trapped atomic ions are among the most capable platforms for quantum computation, yet leading architectures using laser-based gates suffer photon-scattering errors, demanding cooling requirements, and optical delivery challenges that grow more burdensome at scale. Electric-field-gradient gates (EGGS) bypass these obstacles by driving qubit transitions with radio-frequency fields applied directly to easily scalable trap electrodes — the transitions addressed render photon scattering negligible and tolerate thermal motion. The low-lying, electric-dipole transitions necessary for EGGS are found only in molecules, and this dissertation details the pursuit of EGGS-based quantum-logic spectroscopy of the lambda-doublet qubit in H^{35}Cl^+ with a co-trapped $^{40}\text{Ca}^+$ ion in a cryogenic system built to that end.

This pursuit required wideband calibration of the EGGS field gradients across the 10 MHz to 1000 MHz band of the molecular transitions — for which no suitably wideband interaction existed. We resolve this with our discovery of motional Raman transitions, which downconvert an arbitrary-frequency drive into a near-resonant motional displacement, and apply them toward quantum vector signal analysis, a protocol for complete waveform sensing of an electric field, which we demonstrate from 100 kHz to 1 GHz, a range $800\times$ wider than prior work.

The protocol is compatible with quantum amplification, which we use to reach an amplitude sensitivity 3.4(20) dB below the standard quantum limit.

We then extend this family of interactions toward sensing beyond conventional limits. We use subharmonic motional Raman transitions to achieve *nonlinearity*-enhanced frequency sensitivity, surpassing the Fourier-transform limit with classical states alone to reach a frequency uncertainty of 0.56(32) Hz at 80 MHz — to our knowledge, the most precise frequency measurement of a radio-frequency signal on a quantum harmonic oscillator. Separately, we demonstrate broadband frequency super-resolution: a phase-reversal sequence encodes the separation between two conventionally unresolvable tones into a displacement, resolving a 5 Hz splitting near 80 MHz to 1.6 Hz, $200\times$ below the spectral limit. Finally, combining Fock-state enhancement with synchronized readout recovers a quantum advantage that persists to long timescales, improving the *ultimate* frequency precision to $\sigma_\delta = 2\text{ }\mu\text{Hz}$.

The dissertation of Clayton Ze Chi Ho is approved.

Wesley C. Campbell

Paul Hamilton

Isaac L. Chuang

Eric R. Hudson, Committee Chair

University of California, Los Angeles

2026

Dedication

To my family and my friends.

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¹Title inspired by [Wal21]

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²[Hud06], Chapter 4.

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³or “Raw EGGS.”

⁴or “Scrambling EGGS.”

⁵or “Deus EGGS Machina.”

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⁶or “Iacta Alea EGGS.”

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⁷Name by SC.

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List of Acronyms

AA Anti-Aliasing. 176	269
ADC Analog-Digital Converter. 203	DAC Digital-Analog Converter. 49 , 164 , 175 , 176
AM Amplitude Modulation. 47 , 157	DD Dynamical Decoupling. 230 , 233 , 244
AOM Acousto-Optic Modulator. xvii , 110 , 136 , 137 , 139 , 148 , 204 , 211	DDS Direct Digital Synthesis/Synthesizer. 52 , 75 , 117 , 173 , 204 , 261 , 272 , 274
ARTIQ Advanced Real-Time Infrastructure for Quantum Physics. 173 , 178 , 203	DFT Discrete-Fourier Transform. 79 , 81 , 84 , 195
ASF Amplitude Scaling Factor. 174	DMA Direct Memory Access. 177 , 182
AWG Arbitrary Waveform Generator. 52 , 167 , 168 , 170 , 175 , 238 , 260	DPQL Dipole-Phonon Quantum Logic. 144 , 165
BBR Black-Body Radiation. 244	DUC Digital Up-Converter/Conversion. 175 , 176
BCH Baker-Campbell-Hausdorff Lemma. 46	DUT Device Under Test. 272
BSB Blue Sideband. vii , xiv , 38 , 39 , 41 , 43 , 60 , 67–69 , 71 , 72 , 76–78 , 80 , 101 , 116 , 227 , 255 , 275	ECDL External-Cavity Laser Diode. 133 , 142
CPLD Complex Programmable Logic Device. 174	EGGS Electric-Field Gradient Gates. vi , x , xi , xvii , xviii , 1–3 , 5 , 6 , 56 , 166 , 169 , 188 , 189 , 208 , 215 , 216 , 222 , 224 , 225 , 227 , 229–244 , 246
cQED Cavity Quantum Electro-Dynamics. 86 , 333	
CRB Cramér-Rao Bound. 36 , 262 , 265 , 267–	

EIT Electromagnetically-Induced Transparency. 204, 205

EMCCD Electron-Multiplying Charged-Coupled Device. 131, 132, 185, 240

FFT Fast-Fourier Transform. 69

FID Free-Induction Decay. xv, 64, 66, 209, 210, 212

FIFO First-In, First-Out. 181

FIR Finite-Impulse Response. 187

FM Frequency Modulation. 47

FNCS Fiber Noise Cancellation System. 141, 209

FPGA Field-Programmable Gate Array. 172, 175, 176, 178, 181

FSR Free Spectral Range. 142, 146

FTL Fourier Transform Limit. 7, 299

FWHM Full-Width at Half-Maximum. 77, 128, 206, 231, 246, 299, 302

GKP Gottesman-Kitaev-Preskill. 36

GPIB General Purpose Interface Bus. 179, 180

GSL Graduate-Student Locking/Graduate-Student Laser Stabilization. 147

hfgui Homemade Fast Graphical User Interface. 203

HWP Half-Wave Plate. 126, 128, 135, 142

IMD Inter-Modulation Distortion. 170

IP3 3rd-Order Intercept Point. 169, 170

IQ In-Phase/Quadrature. 170, 175–177

JCH Jaynes-Cummings Hamiltonian. xv, 33, 39, 42, 69, 86, 91, 251, 252

LDP Lamb-Dicke Parameter. 3, 4, 117, 169

LDR Lamb-Dicke Regime. 101

LQT Linear Quadrupole Trap. 118, 119, 184

MIMO Multiple Input, Multiple Output. 143

MLE Maximum Likelihood Estimator. 5, 44, 83

MOT Magneto-Optical Trap. 97

MS Mølmer-Sørensen. 42, 48, 230

NCO Numerically-Controlled Oscillator. 175, 176

NDSP Network Device Support Package. 179

NEG Non-Evaporable Getter. 122

NISQ Noisy, Intermediate-Scale Quantum Computers. 216

NIST National Institute of Standards and Technology (Boulder, Colorado (USA)). 178, 203, 216, 238

NMR Nuclear Magnetic Resonance. 336, 337

PBS Polarizing Beam Splitter. 126, 128, 142

PDH Pound-Drever-Hall Stabilization. 142, 145–147, 209

PDL Pulsed Dye Laser. 134
PER Polarization Extinction Ratio. 142
PI Photo-Ionization. 183–185
PLL Phase-Locked Loop. 176
PM Polarization-Maintaining. 135, 146, 202
PM Phase Modulation. 47
PMT Photo-Multiplier Tube. 131, 132, 186, 199
PSD Power Spectral Density. 57
PSK Phase-Shift Keying. xiv, 54
PTB Physikalisch-Technische Bundesanstalt (Braunschweig (Germany)). 61, 216
QA Quantum Amplification. 267
QCCD Quantum Charged Coupled Device. 273
QCRB Quantum Cramér-Rao Bound. 74, 268, 309, 310, 316, 319, 323
QEC Quantum Error Correction. 36
QFI Quantum Fisher information. 35, 36, 74, 268, 309, 318, 326
QFS Quantum Fluctuation Suppression. xii, xix, 309, 312, 315, 316, 321, 323–325, 327, 331
QHO Quantum Harmonic Oscillator. vii, 4–7, 33, 56, 66, 86, 253, 261, 262, 273, 276, 279, 282, 283, 285, 298, 311, 312, 316
QLS Quantum Logic Spectroscopy. x, xviii, 1–4, 6, 10, 16, 25, 30, 56, 91, 216–218, 222, 227, 228, 230, 231, 234, 238, 239, 241, 243, 246
QMC Quadrature-Modulation Compensation. 170
QVSA Quantum Vector Signal Analyzer/Analysis. xi, xviii, xix, 7, 42, 47, 233, 239, 240, 242–247, 252, 253, 255, 257, 258, 260, 261, 263–265, 267, 273, 274, 276, 279, 280, 309, 311, 312
QWP Quarter-Wave Plate. 126
RAP Rapid Adiabatic Passage. xv, 40, 59, 60, 72, 74–79, 116, 117
REMPD Resonance-Enhanced Multi-Photon Dissociation. 232
RF Radio Frequency. 60, 241
RGA Residual Gas Analyzer. 16, 122, 124, 185
RPC Remote Procedure Call. 181
RSB Red Sideband. 39, 60, 66, 67, 72, 74–77, 99, 101, 116, 117, 217, 227, 255, 272
RTIO Real-Time Input/Output. 178
RWA Rotating Wave Approximation. 230, 254, 257
SBC Sideband Cooling. xv, 5, 71, 76, 84, 98, 99, 101, 115–117, 146, 173, 205

SBR Sideband Ratio. [vii](#), [xv](#), [59–61](#), [66–69](#), [285](#), [295](#), [299](#)
[72](#), [116](#)

SCPI Standard Commands for Programmable Instruments. [180](#)

SDF Spin/State-Dependent Force. [44](#), [80](#), [82](#)

SED Scalable Event Dispatcher. [172](#), [173](#)

SLS Stable Laser Systems. [141](#), [147](#)

SMA SubMiniature Version A. [151](#), [152](#)

SMPS Switched-Mode Power Supply. [132](#), [165](#)

SNR Signal-to-Noise Ratio. [4](#), [185](#), [195](#), [199](#), [228](#), [243](#)

SPAM State Preparation and Measurement. [2](#), [68](#)

SQL Standard Quantum Limit. [xii](#), [xix](#), [7](#), [8](#), [35](#), [36](#), [262](#), [265](#), [269–271](#), [281](#), [282](#), [285](#), [295](#), [299](#)

SSB Single-Sideband. [230](#)

TA Tapered Amplifier. [145](#)

TEC Thermo-Electric Cooler. [132](#), [152](#)

TLS Two-Level System. [87](#), [96](#), [99](#)

UHV Ultra-High Vacuum. [123](#), [142](#)

ULE Ultra-Low Expansion. [142](#), [147](#)

UNB Ultra Narrow-Band. [79](#)

VCO Voltage-Controlled Oscillator. [176](#)

VISA Virtual Instrument Software Architecture. [179](#), [180](#)

VNA Vector Network Analyzer. [xviii](#), [153](#), [261](#), [273](#), [274](#)

VSWR Voltage-Standing Wave Ratio. [153](#)

VVA Voltage-Variable Attenuator. [149](#), [154](#)

List of Names

CS Christian Schneider. 118 , 123 , 164	JM Jeremy Metzner. 156
CZCH Clayton Ze Chi Ho. 248	JP Jens Palsberg. 178
DK Dylan Kawashiri. 125 , 164 , 171 , 178 , 282	LM Landon Miller. 178
DTCA David Allcock. 156	MB Matthew Boguslawski. 151
ERH Eric R. Hudson. 222 , 248 , 249	MP Matthew Pulos. 150
FRH Félix-Ribuot Hirsch. 125	PA Pramod Asagodu. 125
GDM Grant D. Mitts. 43 , 118 , 120 , 123–125 , 140 , 145 , 150 , 164 , 178 , 222 , 244 , 248	PSH Paul S. Hamilton. 249
HH Hanchen Huang. 125	PY Peter Yu. 137 , 145 , 148 , 150 , 152 , 165 , 167 , 168
HMK Hannah Marie Knaack. 118	RA Riley Ancona. 118
HW Hao Wu. 118 , 125 , 141 , 160 , 222 , 244 , 248	SC Samuel Crary. xii , 7 , 308
JAR Joshua A. Rabinowitz. 144 , 178 , 248	SCB Shaun C. Burd. 249
	WCC Wesley C. Campbell. 249

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I hope you don't mind if I take rather a while to list out these acknowledgments; if you did, I simply don't care – I was just being polite. There is a very nontrivial chance that this acknowledgments section may balloon to dominate this (already bloated) thesis. In this event, I would sooner be rid of the thesis than the acknowledgments – to scrimp, save, and reserve, in recognizing the support I needed to hobble over the finish line is fundamentally wrong, and I refuse to have any part in that.

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To Josh, I think it is perhaps no surprise that this shift in lab direction and fortune

⁸Whether this is a compliment or an insult, only time may tell.

coincided with having you join EGGS. I take your drive for granted — how many others get into battles over how exactly the lab should be cleaned up, and on back-to-back Saturdays and Sundays as well? I think your boldness and flexibility, your readiness to forge ahead with new directions and tools (old ways be damned), are exactly what EGGS needs in its next generation.

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G. Z. Zhu, G. M. Lao, C. Z. C. Ho, W. C. Campbell, and E. R. Hudson. High-resolution laser-induced fluorescence spectroscopy of $^{28}\text{Si}^{16}\text{O}^+$ and $^{29}\text{Si}^{16}\text{O}^+$ in a cryogenic buffer-gas cell. *J. Mol. Spectrosc.* **384**, 111582 (2022).

“It’s a reason to get up in the morning.
It’s a reason to lose weight, to fit in the red dress.
It’s a reason to smile.
It makes tomorrow all right.”

Dr. Sara Goldfarb, *Requiem For a Dream*

Chapter 1

Introduction

This dissertation details research conducted with the [Electric-Field Gradient Gates \(EGGS\)](#) experiment in the Hudson group at UCLA, exploiting electric-field gradients for quantum computation and for sensing beyond conventional limits. We describe the construction and characterization of a cryogenic $^{40}\text{Ca}^+$ trapped-ion experiment built in pursuit of [EGGS](#), together with initial progress toward [Quantum Logic Spectroscopy \(QLS\)](#) of the lambda-doublet qubit transitions in H^{35}Cl^+ . Calibrating the titular field gradients across the 10 MHz to 1000 MHz band of these transitions was challenged by the absence of a suitably wideband interaction; to address this, we discovered and developed the theory of motional Raman transitions, and leveraged it for vector analysis of in-situ field gradients with sub-micron spatial resolution. We then extended this theory to a broader family of interactions — subharmonic motional Raman — that offer metrological gain through nonlinearity. Building on these techniques, we further demonstrated frequency super-resolution below the conventional spectral limit and, by combining quantum enhancement with synchronized readout, recovered a quantum advantage that persists to long timescales to enable improvements in *ultimate* sensitivity.

This thesis can be organized into three parts: Part I (chapters [2](#) to [4](#)) outlines relevant theory; Part II (chapters [5](#) to [7](#)) presents the experimental implementation and efforts towards

Quantum Logic Spectroscopy with EGGS; Part III (chapters 8 to 11) develops the theory of motional Raman transitions and their application towards sensing.

1.1 Electric-Field Gradient Gates for Quantum Computation

A quantum computer promises efficient solution of problems intractable to classical computation, among them the factoring of large integers and the simulation of quantum chemistry and materials. Realizing such a device requires a physical system that satisfies the DiVincenzo criteria: a scalable register of well-characterized qubits, high-fidelity State Preparation and Measurement (SPAM), and a universal set of coherent operations. Among candidate platforms, trapped atomic ions are the most capable, offering unmatched gate fidelities and coherence times, all-to-all connectivity, and naturally identical qubits.

These strengths notwithstanding, leading trapped-ion architectures face obstacles at scale from a reliance on laser fields to drive gates. First, delivering laser light to the $\sim 10^3$ ions required for gates remains difficult: bulk, free-space optics do not scale [Ran+25], and integrated photonics remain challenged to supply the low-noise, high-power ultraviolet light required to address atomic transitions [Meh+16]. Second, laser-driven gates incur fundamental errors from photon scattering, and improving fidelity with higher laser powers further aggravates the delivery problem [Oze+07]. Third, suppressing the anharmonicity of the laser field generally requires cooling the ion into the Lamb-Dicke regime, further complicating optical delivery.

EGGS addresses these obstacles by substituting a laser for electric-field gradients applied directly to the trap electrodes. Because the EGGS drive leverages only trapping infrastructure, it is inherently scalable; because the relevant transitions lie in the radio-frequency band, photon-scattering errors are negligible (via Fermi’s golden rule); and because the field gradient from the electrodes is approximately uniform over the ion, gates can be operated for ions

in thermal motion [HC21]. However, use of low-lying, electric-dipole-allowed transitions requires molecules: for H^{35}Cl^+ , opposite-parity lambda-doublet transitions range from ≈ 80 MHz to 500 MHz.

1.2 *Controlling* Electric-Field Gradients

In combining into a single experiment the frontier areas of **Quantum Logic Spectroscopy (QLS)**, **gradient-field quantum gates**, and **molecular control**, **EGGS** adopts also their challenges¹.

In **QLS**, the internal state of an optically inaccessible spectroscopy ion (here, H^{35}Cl^+) is mapped through a shared motional mode onto a co-trapped logic ion (here, $^{40}\text{Ca}^+$) for high-fidelity detection. As this requires the coupling of spin to motion between multiple ions, it can be regarded as a two-qubit gate — typically the hardest ingredient to implement in a quantum computer. Molecular state readout therefore required establishment of a quantum-information capability novel to UCLA: efficient sympathetic ground-state cooling and coherent control over motion and the $^{40}\text{Ca}^+$ optical qubit.

Control of the electric field gradients themselves proved deceptively difficult. The **Lamb-Dicke Parameter (LDP)** that quantifies the coupling of spin to motion is set by the ratio of the ion’s zero-point extent to the field radius, which for gradient gates is set by the ion-electrode distance and is small ($\eta \sim 10^{-5}$ in our experiment) relative to the $\eta \sim 10^{-2}$ typical for laser-driven gates; this resulted in a correspondingly weak **EGGS** signal. Precise control of electric fields in the unexplored 0.01 GHz to 1 GHz band is uncommon in trapped ions, where high-order harmonics of motional resonances are readily excited and create a spurious background that can obscure the **EGGS** signal; in this regime we further discovered motional Raman transitions, a source of *resonant* motional background — these sources of background are unique to **EGGS**, as magnetic fields do not couple to charge. A second difficulty was

¹To quote Hartmut Häffner, on a similar endeavor: “Sometimes, the whole is less than the sum of its parts.”

introduced by the small [Lamb-Dicke Parameter](#): as carrier excitation scales as η^{-1} relative to that on the sidebands, the carrier is strongly driven, and accurate characterization of the field gradients is required to suppress large stark shifts and off-resonant transfer outside the search manifold. Though this is routine with magnetic-field gradients, H^{35}Cl^+ possesses transitions spanning 60 MHz to 550 MHz, necessitating wideband calibration — for which a protocol did not previously exist.

Finally, the internal structure that renders molecular ions attractive for quantum logic also makes them considerably more difficult to control than atomic ions. As their rovibrational states are typically within the THz band, the internal state is continually redistributed by ambient blackbody radiation, severely limiting the length of data acquisition and thus the achievable [SNR](#). Likewise, state-selective loading is challenging, as few ionization pathways are sufficiently selective to yield H^{35}Cl^+ , and undesired species are readily loaded.

1.3 Dissertation Outline

Chapter 2: [Paul Traps](#)

The classical theory of ion trapping is reviewed — Mathieu dynamics are solved both analytically and approximately, trap stability conditions are outlined, and micromotion effects are considered. Collective motion of ion chains are analyzed in the normal mode picture, and the challenge of mode decoupling in mixed-species chains is discussed with relevance to [Quantum Logic Spectroscopy](#).

Chapter 3: [Quantum Motion](#)

The ion motion of Chapter 2 is quantized and described in the Fock and phase-space formalisms of the [Quantum Harmonic Oscillator \(QHO\)](#). Properties, applications, and experimentally-relevant features of key [QHO](#) states are summarized together with protocols for their generation and detection. This control over nonclassical states enables quantum

enhancement and amplification of small signals, used in Chapters 8 and 11, with work underway to extend benefits to EGGS. Techniques are presented for engineering QHO phase-space trajectories to achieve a desired response (typically frequency) to an input drive, as used in Chapters 7 and 10. As motional decoherence is the major limitation for coherent motional operations in this experiment, its basic theory is reviewed, and techniques for its characterization are discussed (to excess).

Chapter 4: Light & Structure

The interaction of radiation with matter is reviewed, with a dominant focus on motional effects: spin-motion interactions are described in the resolved sideband regime, and dynamics under Doppler cooling and Sideband Cooling are derived. State detection via optical fluorescence is described as a Poisson process, and statistics for time-resolved photon counting for adaptive state detection via the Maximum Likelihood Estimator (MLE) are derived. Finally, these processes are specialized to the $^{40}\text{Ca}^+$ ion used in this thesis.

Chapter 5: Experimental Implementation

This chapter documents the experimental hardware on which all measurements in this thesis are made, with overly verbose and needlessly thorough fashion, nominally for posterity. First, the main apparatus is presented: the trap, closed-cycle 4 K cryogenic system, vacuum chamber, ion loading apparatus. Next, the optics and light delivery are described, covering laser hardware and stabilization, beam paths, magnetic-field apparatus, and imaging systems. Trapping electronics are covered at length due to their contribution to experiment characteristics (e.g. motional dephasing), including the helical resonator, RF amplitude stabilization, parametric modulation, trap RF cancellation, and the EGGS signal path. Finally, the control software is described, spanning the LabRAD server–client platform, the ARTIQ control system, and the software extensions developed in-house. Where useful, calibration techniques and operator challenges are recorded.

Chapter 6: Experimental Techniques

Standard experimental techniques for system operation and characterization are presented. Ion loading and chain management procedures for both $^{40}\text{Ca}^+$ and H^{35}Cl^+ are provided. Ion-based beam optimization is discussed, together with spectroscopic characterization of $^{40}\text{Ca}^+$ transitions and system dephasing. Micromotion compensation receives central focus for its relevance to [EGGS](#), and as a case study of field nulling: three standard techniques (visual compensation, Doppler spectral modulation, sideband comparison) are examined, while the more recent parametric modulation/count correlation technique is presented in depth, with demonstration of a fast, robust procedure for automated compensation.

Chapter 7: Electric-Field Gradient Gates

This chapter presents the [QLS](#) search with [EGGS](#) for the lambda-doublet qubit transition in H^{35}Cl^+ . A cursory review of diatomic molecular structure is provided to aid discussion of the molecular structure of H^{35}Cl^+ in the ground rovibrational state. Implementation of [EGGS](#) gate primitives are presented, and are extended for [QLS](#) using a bichromatic interaction. Important challenges to the search, e.g. dipole field-induced stark shifts and motional Raman background, are discussed, as well as mitigation strategies, e.g. dynamical decoupling. To suppress deleterious dipole field components, the [EGGS](#) field gradient is characterized with sub-micron spatial resolution; field nulls are shown to be in disagreement with the trap RF null, and are frequency-dependent. Finally, preliminary search results are presented, showing excitation at 83.512 MHz; the validity of these peaks are discussed.

Chapter 8: Motional Raman

Field sensing on [QHO](#) platforms has heretofore been limited to narrowband measurements about a static secular frequency. To overcome this, we present our discovery of motional Raman excitation, which uses parametric modulation for quantum heterodyne downconversion

of an arbitrary-frequency drive, generating a near-resonant displacement in the [QHO](#) baseband. We develop the [Quantum Vector Signal Analyzer/Analysis \(QVSA\)](#) protocol, which interferes two motional Raman displacements to enable tunable, complete waveform sensing of an electric field, and demonstrate operation from 100 kHz to 1 GHz, $800\times$ wider than prior works. Compatibility of the [QVSA](#) with quantum amplification is demonstrated, using squeezing to reach an amplitude sensitivity of $51(10) \mu\text{V}/\sqrt{\text{Hz}}$, 3.4(20) dB below the [Standard Quantum Limit \(SQL\)](#). Potential of the [QVSA](#) for challenging in-situ measurements is showcased through wideband network analysis of qubit control lines.

Chapter 9: *Subharmonic Motional Raman*

Frequency sensing on [QHOs](#) is bounded by the Fourier Transform Limit (FTL), which ties achievable resolution to coherence time; this constraint is exacerbated when using nonclassical states due to their accelerated decoherence. We instead present a protocol for arbitrary-frequency electric-field sensing that surpasses the FTL with nonclassical states alone, via *nonlinearity*-enhancement: we use subharmonic motional Raman to drive a K -quanta process, narrowing the linewidth by $2/K$ and thereby enhancing frequency sensitivity by $K/2$. We achieve a frequency sensitivity 12.3(9) dB beyond the FTL, reaching 0.56(32) Hz for an 80 MHz field — to our knowledge, the most precise frequency measurement of a radio-frequency field on a [QHO](#).

Chapter 10: *Super-Duper Resolution*²

Frequency resolution is conventionally bounded by the [FTL](#) linewidth, requiring a time T to resolve a separation $\Delta\omega$ between two tones that diverges as $\Delta\omega \rightarrow 0$. We show that resolution is instead limited by quantum fluctuations independent of $\Delta\omega$, and demonstrate the first broadband protocol for super-resolution spectroscopy in the radio-frequency domain, using a phase-reversal sequence to encode the frequency separation $\Delta\omega$ between two tones into a

²Name by [SC](#).

displacement. We resolve two 80 MHz tones with separation $\Delta\omega = 5$ Hz to an uncertainty $\sigma_{\Delta\omega} = 1.6$ Hz using a 1 ms probe, representing an improvement $200\times$ beyond the conventional limit and accelerating data acquisition by over 10^5 .

Chapter 11: Quantum-Enhanced Correlation Sampling

Use of nonclassical states to suppress quantum noise below the [Standard Quantum Limit \(SQL\)](#) often precludes improvements in *ultimate* sensitivity due to their accelerated decoherence. We restore persistent quantum advantage by combining Fock-state quantum enhancement with a quantum heterodyne (Qdyne) protocol, its first realization on any platform beyond NV centers, to decouple sensitivity from the decoherence-limited interrogation time τ . Measurements can be acquired at short τ for optimal quantum-enhanced gain, while precision accumulates with the total measurement time T . We achieve a quantum-enhanced precision that reaches $\sigma_\delta = 2$ μ Hz, for a fractional uncertainty of $\sim 10^{-7}$.

“Two years ago, I was afraid of [trapping] anything. I figured [trapping] would lead to trying and trying would lead to failure. But now I find I can’t stop [trapping] . . .”

Tyra Collette, *Friday Night Lights*

Chapter 2

Paul Traps

2.1 The Paul Trap

To meaningfully investigate physics on a test model beyond a few significant figures, we have to first isolate the model in question from the environment, to eliminate confounding degrees of freedom and sources of noise. Straightforwardly, one would expect that a suitable arrangement of static electric fields would be able to confine a charged particle (such as an ion), à la Millikan [Mil11]. However, Earnshaw’s theorem forbids this: the motion of a point charge is unstable in a static configuration of electric fields, i.e.

$$\nabla \cdot \vec{F}_{\text{Electric}} = \nabla \cdot (q\vec{E}) = q \left(\frac{\rho}{\epsilon_0} \right) = 0, \quad (2.1)$$

where we consider an infinitesimal volume around the equilibrium point, which we assume contains no charge. A lawyer’s approach, channeling Feynman’s aphorism that “anything not expressly forbidden must be allowed” [Gri08], suggests two remedies: use of a time-varying, rather than static, electric field, or addition of an external field (e.g. a magnetic field). The former approach is the principle that underlies Paul traps (shown in Figure 2.1), and the latter, Penning traps. Compared to Penning traps, Paul traps are cost-effective, easy to implement [BG86,Gho95], and localize the ions with ease [McM+24], making them well-suited

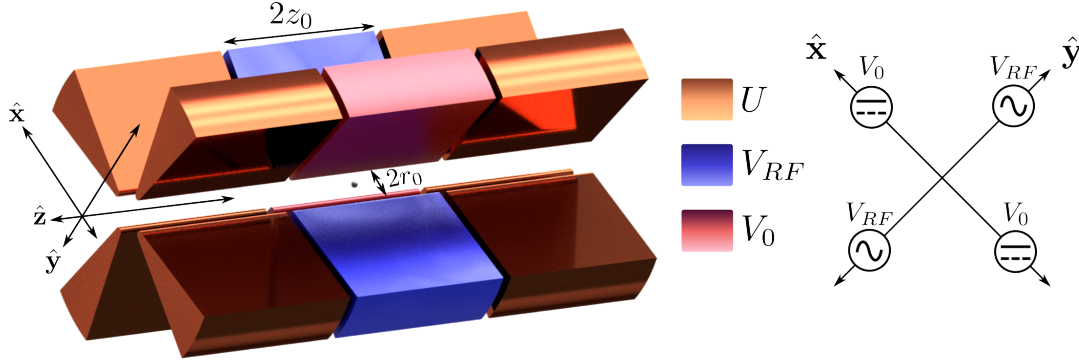


Figure 2.1: **Diagram of an RF Paul trap.**

DC voltages U are applied on the outermost electrodes to generate an axially confining potential (i.e. along $\hat{z}$), while an oscillating voltage V_{RF} is applied to two diametric center electrodes to generate a radially confining ponderomotive potential. Bias voltages V_0 applied on the other pair of diametric center electrodes break the symmetry between the two radial axes $\hat{x}$, $\hat{y}$. The radial and axial ion-electrode distances r_0 , z_0 are defined as half the distance between opposing electrodes.

to tabletop Quantum Information Processing (QIP) and [Quantum Logic Spectroscopy \(QLS\)](#) experiments.

2.1.1 Field Potentials

The simplest potential to analyze is a parabolic potential of the form

$$\phi = \alpha x^2 + \beta y^2 + \gamma z^2, \quad (2.2)$$

so favored because it results in simple harmonic motion, and is relatively straightforwardly produced by a set of equilateral hyperbolic electrodes (Figure 2.1). As different geometries and implementations of the Paul trap are possible, each with slightly different potentials and motion, analysis is restricted to the Linear Quadrupole Trap (LQT), which operates most similarly to the blade-style trap used in this thesis¹. In an LQT, two effective parabolic potentials are applied to confine the ion: radial confinement is produced by a voltage V_{RF}

¹The taxonomy of ion traps is fast-evolving and often arcane, as novel trap architectures are developed in the poor illumination of academia – see [Mal22] for an authoritative and comprehensive treatise on the subject.

oscillating at Ω_{RF} (with an optional DC offset V_0) and carried on a quadrupolar set of electrodes at the center of the trap, while axial confinement is produced by a static voltage U carried on a set of electrodes at opposite ends of the trap. Subject to the Laplacian condition $\nabla^2\phi = 0$ implicit in eq. (2.1), these potentials can be expressed

$$\phi_{RF} = \frac{x^2 - y^2}{2r_0^2} (V_{RF} \cos(\Omega_{RF}t) + V_0), \quad (2.3)$$

$$\phi_{DC} = \frac{2z^2 - (x^2 + y^2)}{2z_0^2} U, \quad (2.4)$$

where r_0 and z_0 are the radial and axial ion-electrode distances, respectively (Figure 2.1). Typically, the radial electrodes have significant axial extent to create an approximate axial symmetry, reducing the radial potential to two dimensions and allowing the implicit axial component in eq. (2.3) to be neglected [Pau90].

2.1.2 Ion Dynamics

Using Newton's second law, the motion of a single ion under eqs. (2.3) to (2.4) potentials is then

$$\ddot{x} + \frac{q_e}{m} \left(\frac{V_{RF} \cos(\Omega t) + V_0}{r_0^2} - \frac{U}{z_0^2} \right) x = 0 \quad (2.5)$$

$$\ddot{y} + \frac{q_e}{m} \left(-\frac{V_{RF} \cos(\Omega t) + V_0}{r_0^2} - \frac{U}{z_0^2} \right) y = 0 \quad (2.6)$$

$$\ddot{z} + \frac{q_e}{m} \left(\frac{2U}{z_0^2} \right) z = 0, \quad (2.7)$$

where q_e is the ion charge, and m is the ion mass. These dynamics are described by a Mathieu equation, i.e.

$$\ddot{x} + (a + 2q \cos(2\tau)) x = 0. \quad (2.8)$$

Using the substitution $\tau = \frac{\Omega}{2}t$, (eq. (2.5) - eq. (2.7)) can be recast as Mathieu equations with parameters

$$\begin{aligned} q_x &= \frac{2q_e V_{RF}}{m r_0^2 \Omega^2} & a_x &= \frac{4q_e}{m \Omega^2} \left(\frac{V_0}{r_0^2} - \frac{U}{z_0^2} \right) \\ q_y &= -q_x & a_y &= -\frac{4q_e}{m \Omega^2} \left(\frac{V_0}{r_0^2} + \frac{U}{z_0^2} \right) \\ q_z &= 0 & a_z &= \frac{8q_e U}{m \Omega^2 z_0^2}. \end{aligned} \tag{2.9}$$

As there is no pseudopotential in the axial case (i.e. $q_z = 0$), the axial dynamics are completely described by simple harmonic motion with frequency $\omega_z = \sqrt{\frac{2eU}{mz_0^2}}$. Analysis of the radial dynamics (i.e. $x(t)$ and $y(t)$) described by (eq. (2.5) - eq. (2.6)), however, is less straightforward, due to the presence of the time-dependent driving. In the following sections, we explore two methods for obtaining analytical solutions to (eq. (2.5) - eq. (2.6)): an approximate, intuitive approach using perturbation theory, and a more rigorous analysis with Floquet theory. Trajectories for the exact and first-order (pseudopotential) approximation are compared in Figure 2.2.

2.1.3 Approximate Dynamics – Two-Timing

An approximate solution can be found by noting that there are two timescales for the dynamics: fast motion, corresponding to driving at Ω , and slow (or secular) motion, which yields the dynamics of interest. As dynamics on different timescales are typically independent, the motion can thus be separated into corresponding fast and slow components, which considerably simplifies the analysis. This technique is known to ion trappers as the “pseudopotential approximation” and is an application of the “two-timing” (or more generally, multi-scale analysis) technique from the field of nonlinear dynamics² [Str94].

²Two-timing reduces to the standard Poincaré-Lindstedt perturbation method in the case of weakly anharmonic oscillators [LT88].

Formally, we assume that the motion along an arbitrary axis is described by

$$\begin{aligned}\ddot{x} + (\epsilon^2 a + 2\epsilon q \cos(2t)) x &= 0 \\ x(t) &= x_0(t, t_1) + \epsilon x_1(t, t_1) \quad (t_1 = \epsilon t, \epsilon \ll 1)\end{aligned}\tag{2.10}$$

where t, t_1 are the fast and secular timescales, respectively. The time derivatives $\dot{x} = \frac{d}{dt}x$ are transformed as $\frac{d}{dt} = \partial_t + \epsilon \partial_{t_1}$, which inserted into eq. (2.10) and expanded gives the following terms:

$$\begin{aligned}O(\epsilon^0) : \quad & \partial_t^2 x_0 = 0 \\ O(\epsilon^1) : \quad & \partial_t^2 x_1 + 2\partial_{tt_1}^2 x_0 + 2q \cos(2t)x_0 = 0 \\ O(\epsilon^2) : \quad & \partial_{t_1}^2 x_0 + 2\partial_{tt_1}^2 x_1 + ax_0 + 2q \cos(2t)x_1 = 0.\end{aligned}\tag{2.11}$$

As we are interested in stable, steady-state solutions, we ignore solution terms that diverge on the fast timescale. Solution of the $O(\epsilon^0)$ equation then gives $x_0(t, t_1) = f(t_1) = x_0(t_1)$, indicating that the secular motion is static on the fast timescale. This is used to solve the $O(\epsilon^1)$ equation to relate the fast motion to the secular motion, i.e. $x_1(t, t_1) = \frac{q}{2} \cos(2t)x_0(t_1)$. As the term of interest in the $O(\epsilon^2)$ equation is $\partial_{t_1}^2 x_0$, the equation can be simplified by averaging over a period of the fast motion to become $f''(t_1) + \left(a + \frac{q^2}{2}\right) f(t_1) = 0$, which is a simple harmonic motion with frequency $\omega = \sqrt{a + \frac{q^2}{2}}$.

Together, these results give the motion as

$$x(t, t_1) = x_0(t, t_1) + x_1(t, t_1) = A \cos(\omega t + \phi) \left(1 + \frac{q}{2} \cos(2t)\right),\tag{2.12}$$

where A and ϕ are the amplitude and phase set by initial conditions. This consists of a slow, secular oscillation at ω (typically known as the “secular” frequency), on which a fast oscillation (known as the “micromotion”) is superimposed. In our solution of the $O(\epsilon^2)$ equation in eq. (2.11), we saw that the secular motion arises from the net, time-averaged

dynamics of the fast motion. Intuitively, this can be understood by an emotional numerical integration of the motion — as the ion is displaced outwards during a micromotion cycle, it experiences a restorative force greater than that experienced during the inwards portion of the micromotion cycle. In this sense, the micromotion can be said to *enable* the secular motion.

Further insights can be teased out of eq. (2.12). Firstly, the harmonicity of the motion, as well as the validity of our derivation, depends on the Mathieu q-parameter q (see difference in approximation accuracy for different q in Figure 2.2). For large $q \gtrsim 0.4$, the trajectory becomes distorted as the micromotion begins to dominate, and applied fields become Doppler-shifted in the reference frame of the ion. Secondly, the presence of the micromotion creates additional resonances at $\Omega \pm \omega$, a consequence of the parametric nature of the Mathieu equation. Further discussion of the motional dynamics is explored in sections 2.3 and 6.2.

2.2 Analytical Dynamics

2.2.1 Floquet's Theorem

Though the previous analysis provided an intuitive understanding of the motion, an exact (or arbitrarily close) description of the dynamics is required to understand the limitations of our trapping and illuminate interesting features of the motion, as well as to improve the accuracy of our model for precision measurements. The standard approach to solving the Mathieu equation is to use Floquet's theorem [Lei+03,Mih24], which essentially assumes that a stable solution exists and is periodic, such that it can be described in general as

$$x(t) = Ae^{i\beta t} \sum_{n=-\infty}^{\infty} C_{2n} e^{2int} + Be^{-i\beta t} \sum_{n=-\infty}^{\infty} C_{2n} e^{-2int}, \quad (2.13)$$

where A , B , C_{2n} are arbitrary coefficients determined by initial conditions, and β is the secular frequency, which is the parameter of interest. In general, both C_{2n} and β depend on

the Mathieu a and q parameters. Insertion of eq. (2.13) into the Mathieu equation eq. (2.8) and use of the orthogonality of the complex exponentials yields a recurrence relation for the C_{2n} coefficients:

$$C_{2n+2} + C_{2n-2} = D_{2n}C_{2n}, \quad D_{2n} \equiv \frac{(2n + \beta)^2 - a}{q}. \quad (2.14)$$

This can be rearranged to give a continued fraction expression for D_{2n}

$$\begin{aligned} D_{2n} &= \frac{C_{2n+2}}{C_{2n}} + \frac{C_{2n-2}}{C_{2n}} \\ &= \frac{1}{D_{2n+2} - \frac{C_{2n+4}}{C_{2n+2}}} + \frac{1}{D_{2n-2} - \frac{C_{2n-4}}{C_{2n-2}}} \\ &= \frac{1}{D_{2n+2} - \frac{1}{D_{2n+4} - \dots}} + \frac{1}{D_{2n-2} - \frac{1}{D_{2n-4} - \dots}} \end{aligned} \quad (2.15)$$

which gives the exact form of β as

$$\beta^2 = a + q \left(\frac{1}{D_2 - \frac{1}{D_4 - \dots}} + \frac{1}{D_{-2} - \frac{1}{D_{-4} - \dots}} \right), \quad (2.16)$$

where we have set $n = 0$ as we are interested in the lowest-order trajectory. In practice, the k th-order approximation is obtained by truncating the continued fraction at $2k \pm 2$. From eq. (2.16), the lowest-order solution is $\beta^{(1)} = \sqrt{a + \frac{q^2}{2}}$, in agreement with the earlier result eq. (2.12) obtained using the pseudopotential approximation. As solution of eq. (2.16) entails polynomial equations in β , closed form expressions with arbitrary a and q exist only up to the $k = 4$ order due to the Abel-Ruffini theorem. This result is:

$$\beta^{(4)} = \frac{1}{8} \sqrt{\frac{2048a + 16q^2(4 + a)(16 + a^2) + q^4(16 + 9a)}{32 - 2q^2(6 + 5a)}}, \quad (2.17)$$

with a similar form in [Brk+06].

Using typical trap parameters $q = 0.2248$, $a = 0.0016$ for our experiment, we plot the relative error of $\beta^{(k)}$ as a function of the order k (Figure 2.3).

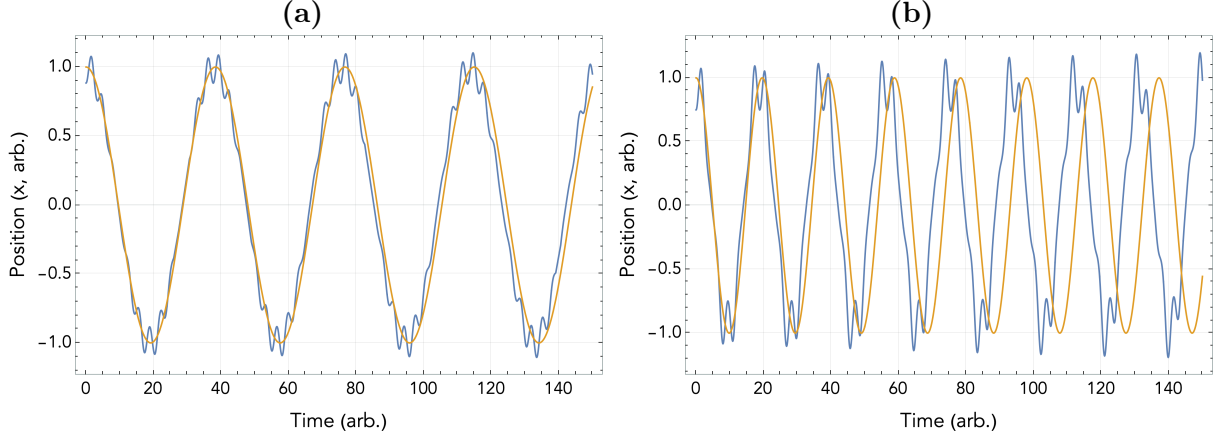


Figure 2.2: **Sample Mathieu trajectories.**

Mathieu trajectories for (a) $q = 0.2248$ and (b) $q = 0.45$, with $a = 0.0016$ for both, show the faster micromotion sitting additively atop the slower secular motion. The pseudopotential approximation (blue) exhibits good agreement with the exact solution (orange) at low q (a), but visibly departs at higher q (b) due to increased frequency error (eq. (2.16)) and distortion from the micromotion terms.

2.2.2 Mathieu Stability

Our result from the pseudopotential approximation in section 2.1.3 suggests that the ion trajectory is not stable for all values of a and q . Given that these parameters are mass-dependent ($a, q \propto \frac{1}{m}$), a single set of trap voltages may not be able to trap all possible mass species simultaneously³. Moreover, practical limitations (e.g. insufficient voltage) prevent the realization of arbitrary a and q , potentially precluding trapping of even the target species. As our experiment requires trapping a mixed-species ion chain for [Quantum Logic Spectroscopy \(QLS\)](#), a more complete understanding of trap stability is therefore of interest.

Following the Hill determinant approach in [\[KRS18, But18, Mih24\]](#), we begin by formulat-

³This is the operating principle of quadrupole mass spectrometers, such as in a typical [Residual Gas Analyzer \(RGA\)](#).

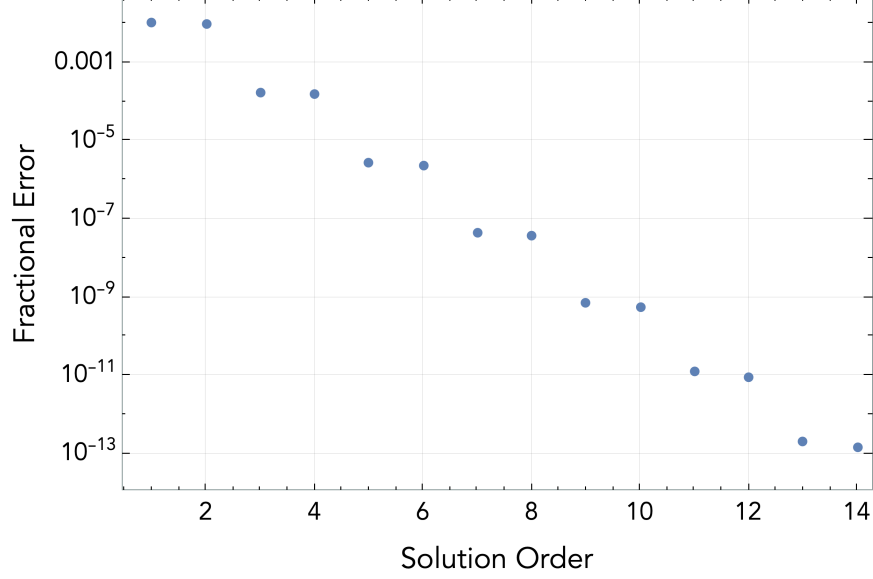


Figure 2.3: **Fractional error of higher-order solutions for the secular frequency β .** Solutions are obtained from eq. (2.16) for typical trap parameters $q = 0.2248$, $a = 0.0016$. Each solution of order n is generated by truncating the continued fraction for β at D_{2n} , expanding to order $n + 1$ in a , q , then numerically solving the resulting polynomial via root-finding. The fractional error is taken relative to the maximum-order solution shown. The error decreases in pairs of adjacent orders due to the pairing $D_{2n} + D_{-2n}$ in eq. (2.16), which, since $D_{2n} = [(2n + \beta)^2 - a]/q$, leaves only the even-order terms — i.e. the perturbation expansion for β contains only terms q^{2n} . Further, since $a \ll q$, the expansion is governed primarily by q .

ing eq. (2.14) as a matrix equation:

$$\begin{pmatrix} -D_{-2n} & 1 & & & \\ & 1 & -D_{-2n+2} & 1 & \\ & & & \ddots & \\ & & & & 1 & -D_{2n-2} & 1 \\ & & & & & 1 & -D_{2n} \end{pmatrix} \begin{pmatrix} C_{-2n} \\ C_{-2n+2} \\ \vdots \\ C_{2n-2} \\ C_{2n} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}. \quad (2.18)$$

Since a trajectory transitions from stable to unstable when the oscillation frequency β becomes imaginary, we set $\beta^2 = 0$, giving $D_{2n} = \frac{a-(2n)^2}{q}$ (via eq. (2.14)). Nontrivial solutions exist only when the determinant vanishes, i.e. $\det(M) = 0$, which yields an implicit function $f(a, q) = 0$ that can be numerically plotted to show the desired stability regions. Historically,

in the absence of computers, solutions to eq. (2.18) were instead obtained by perturbatively expanding a as a power series in q .

The Mathieu stability diagram (also known as an Ince-Strutt diagram) calculated from eq. (2.18) is plotted in fig. 2.4. Regions of instability form “tongues” which emanate from particular integer values $a = n^2$ — these are parametric resonances, which form when the oscillator’s resonance is near a certain multiple of the drive frequency. The presence of a linear damping term in eq. (2.8), such as that from laser cooling, results in a “shrinkage” of these instability regions — this is useful to ensure that the ion remains stably trapped while exciting a parametric resonance, such as for compensation of micromotion [Nad+21].

As we are interested in overall stability, that is, stability along all 3 spatial axes, we can combine the stability regions along the respective axes to form a composite *total* stability diagram. Such a diagram is straightforward for the linear quadrupole trap: the axial motion is unconditionally stable due to the lack of an axial pseudopotential and can thus be ignored, while the two radial modes differ only in the sign of their a -parameter (ignoring the radial contribution of the DC potential, which is negligible), so the composite diagram can be approximately formed by the simple intersection of the region in fig. 2.4 with its horizontal reflection. Shown in fig. 2.4 is the first combined stability region with $0 < q \lesssim 0.908$ and $|a| \lesssim 0.23$. Though stability in principle extends to relatively large $q \lesssim 0.908$, experimental considerations (e.g. micromotion) favor smaller q -parameters, with $q < 0.4$ being a heuristic for general “happiness” of the ion [Gho95]. For this reason, higher stability regions are not used in quantum-facing applications, though they have been leveraged for faster scanning in mass spectrometers [GCS97].

Though the discussion of stability has been presented in terms of trapped ion dynamics, with particular relevance to mass filtering⁴, the applications are of course more varied — these results are general to any system well-described by a Mathieu equation (eq. (2.8)).

⁴And, by extension, most of industrial mass spectrometry.

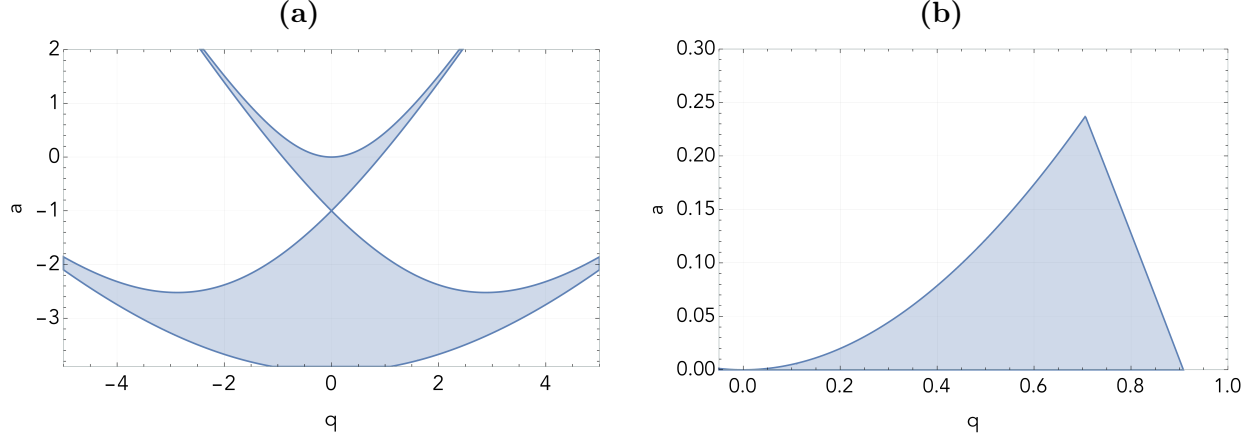


Figure 2.4: **Mathieu stability regions relevant to ion trapping.**

Regions of stability are shaded in blue. **(a)** General Mathieu stability diagram (also known as an Ince-Strutt diagram). Shaded regions denote (a, q) for which β is real and the motion is bounded. “Tongues” of instability emanate from parametric resonances at $(q, a) = (0, n^2)$. **(b)** First stability region in a linear quadrupole trap with ponderomotive confinement along both radial axes (x, y), which differ only in the sign of a ; the anti-confining contribution from the axial (z) DC confinement a_z is small relative to the radial a component and is ignored for simplicity. This region is formed by the intersection of the shaded regions in (a) with its horizontal reflection $a \rightarrow -a$.

2.2.3 Mass Filtering

In trapped-ion quantum computing (and its adjacent endeavors), mass filtering is largely a tertiary concern: as experiments (and, in fact, entire groups) typically use only one or two ion species, they are able to use enriched sources and highly-selective ionization schemes to mitigate the loading of unwanted (or, “parasitic”) ions. However, we are self-destructively interested in the co-trapping of molecular ions, and so, this ancient evil returns. The challenge of mass filtering is brought to the fore for molecular ions, the loading of which entails impure sources with non-selective ionization mechanisms.

The conventional approach to mass filtering entails the configuration of trap voltages to destabilize the mass-dependent Mathieu a and q parameters for parasitic species while preserving them for target species. Relative Mathieu stability regions for different ions are plotted in Figure 2.5. As the ratio of the Mathieu parameters a/q is approximately mass-independent, a “scan line” [Gho95] is formed on which all species fall, depicted in Figure 2.5.

The task of mass filtering is thus reduced to tailoring the slope of this line by configuring trap voltages. Ideally, all parasitic species can be filtered using two settings of the trap voltages: smaller masses with larger (a, q) can be ejected by increasing the voltages to “push” species along the operating line towards the right boundary, and larger masses with smaller (a, q) can be ejected by the reverse procedure. As always, this task is difficult in practice, as not all a and q parameters are arbitrarily achievable: for example, a two-fold increase in q requires a 6 dB increase in trap RF power.

Another common approach to ejection from mass spectrometry is the application of resonant excitations [Gho95]. This can be done with either a dipolar [Goe+92] (known as a tickle) or quadrupolar AC field (i.e. a parametric resonance) [Sch+20,Sud+00a], each with different characteristics. Use of a tickle corresponds to the addition of a driving term at the secular frequency ω_0 to the Mathieu equation eq. (2.8), resulting in a linear growth of the oscillation amplitude with time and a typical Breit-Wigner lineshape. Use of parametric excitation corresponds to the addition of a further quadrupolar drive term at $2\omega_0$, which results in an exponential growth of the oscillation amplitude with time, enabling greater mass selectivity. Though the growth of the motion under parametric excitation is initially slow and depends on the ion position, these drawbacks can be mitigated by addition of an initial tickle to amplify the motion [Sch+20]. Higher order parametric resonances can somewhat improve mass selectivity due to their nonlinear amplitude dependence, though these are prohibitively challenging to excite in practice [Sud+00a,00a].

A key challenge in mass filtering with ion chains arises from the strong Coulomb coupling between the ions. In the ion chain (as opposed to ion cloud) regime, the ions move collectively, with their motion instead described by normal modes of oscillation. As the motion of individual ions are no longer independent, attempts to eject the parasitic ion via excitation in turn excite the target ion, such that the entire chain can be lost.

To improve the likelihood of successful mass filtering, the trap strength can be reduced to separate the ions and effectively “decouple” their motion. Though this further constrains

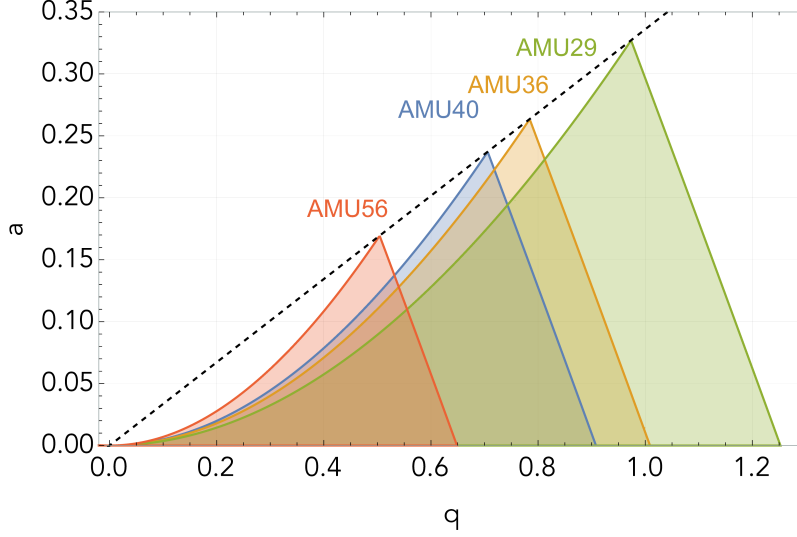


Figure 2.5: **Applying Mathieu stability regions for mass filtering.**

Axes are normalized to the a, q -parameters of an AMU40 ion (i.e. $^{40}\text{Ca}^+$); species outside the blue shaded AMU40 region are unstable and ejected from the trap. As both $a, q \propto 1/m$, all mass species fall on a parametric “operating line” (dashed, black) a/q with slope determined by the trap voltages; the operating line depicted is configured to selectively confine AMU40. Lighter species (larger a, q) can be ejected by pushing the operating line toward the right boundary; heavier species (smaller a, q) by the reverse.

filtering based on Mathieu stability, it enables resonant excitation ejection strategies, since the heating rate of an ion is proportional to the square of its mode participation [Wüb+12]. Parametric excitation is favored for this task, as the difference in the mode participation of each ion becomes amplified by the exponential growth of the motion [Sch+20].

2.3 Micromotion

We consider two experimentally relevant imperfections to the motion. Firstly, stray electric fields produce a constant force that offsets the ion’s equilibrium position, adding to eq. (2.8) an inhomogeneous constant term $\frac{eE}{m}$. Secondly, imbalances in the oscillating potential between the radial electrodes contribute an effective driving term. To see this, we consider a relative

phase delay $\delta\phi$ between the potential on the two electrodes, which we express as a vector:

$$\begin{aligned}\vec{V}_{RF} &= \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} V \cos(\Omega t + \frac{\delta\phi}{2}) \\ V \cos(\Omega t - \frac{\delta\phi}{2}) \end{pmatrix} \\ &\approx V \cos(\Omega t) \begin{pmatrix} 1 \\ 1 \end{pmatrix} + V \frac{\delta\phi}{2} \sin(\Omega t) \begin{pmatrix} -1 \\ 1 \end{pmatrix},\end{aligned}\tag{2.19}$$

where the in-phase and out-of-phase components represent quadrupolar (x^2) and dipolar (x) potentials, respectively. The same approach can be used to examine the case of amplitude imbalance (i.e. $|V_1| \neq |V_2|$). Under these effects, the Mathieu equation eq. (2.8) becomes

$$\ddot{x} + \frac{\Omega^2}{4} (a + 2q \cos(\Omega t)) x = \frac{eE}{m} + \frac{\delta\phi}{2} \frac{qr_0\Omega^2}{2} \sin(\Omega t).\tag{2.20}$$

The lowest order solution [Ber+98] is:

$$x(t) = x_0(t) + \underbrace{\frac{eE}{m\omega^2} \left(1 + \frac{q}{2} \cos(\Omega t)\right)}_{\text{offset}} + \underbrace{\frac{\delta\phi}{2} \frac{qr_0}{2} \sin(\Omega t)}_{\text{phase delay}},\tag{2.21}$$

where $x_0(t)$ is the homogeneous solution from eq. (2.12).

As micromotion and secular motion are typically decoupled, excess micromotion does not fundamentally affect the secular motion. The key problem comes when incident light becomes frequency modulated by the ion's micromotion in its rest frame [Ber+98]:

$$\begin{aligned}\mathbf{E}(t) &= \mathbf{E}_0 \exp \left[i \left(\vec{k} \cdot \vec{x} - \omega t \right) \right] \\ \Rightarrow \mathbf{E}(t) &= \mathbf{E}_0 \exp \left[i \vec{k} \cdot \vec{x}_0 \right] \exp \left[-i \left(\omega t - \vec{k} \cdot \vec{x}'(t) \right) \right] \\ &= \mathbf{E}_0 \exp \left[i \vec{k} \cdot \vec{x}_0 \right] \exp \left[-i \left(\omega t - \beta \cos(\Omega t + \delta) \right) \right]\end{aligned}\tag{2.22}$$

where $x'(t)$ are the excess micromotion terms in eq. (2.21), and we have used $\vec{k} \cdot \vec{x}'(t) = \beta \cos(\Omega t + \delta)$. As eq. (2.22) takes the form of frequency modulation, micromotion sidebands

are produced at $\pm n\Omega$ in the ion’s spectra, with more power being diverted to these sidebands as the excess micromotion (and thus the modulation index β) increases. For high $\beta \approx 1$, cooling becomes threatened as peak broadening limits the Doppler temperature, while spectroscopy on narrow transitions becomes difficult as little power remains in the fundamental peak.

When multiple ions are trapped in a chain, excess micromotion can threaten the chain stability. Collisions with other ions [Che+13] or neutral atoms (from e.g. background gasses) [CSH14] couple the excess micromotion to the secular motion, and can often lead to decrystallization of the chain.

2.4 Ion Chains

So far, we have considered only the motion of a single ion. However, much like human society, and echoing the orthodoxy of after-school television, their great strength lies in their ability to share and communicate with another. To invoke the archetypal example, the singular advantage of a quantum computer is entanglement, which necessarily requires some means of “sharing” between the qubits — in trapped ion quantum computers, the collective motion of the ions serves as this means. This principle similarly underpins QLS, which transfers motional quanta from an ion of interest to a co-trapped helper ion, enabling interrogation of systems otherwise inaccessible.

2.4.1 System Lagrangian

Though the Coulomb coupling introduced by additional ions compounds the challenge of exact analytical description, an approximate description using normal mode analysis remains tractable and largely sufficient for the cases of interest. A historical overview of the more peculiar nonlinear dynamics of multiple ions in a Paul trap can be found in the introduction of [Lan+12].

We begin by first describing the kinetic and potential energy of a chain of N harmonically

trapped ions, each with charge e . For simplicity, the ion chain is assumed to be axially confined, for which a necessary⁵ condition is that the radial confinement exceeds the axial confinement, i.e. $\omega_{x,y} > \omega_z$,

$$T = \sum_{i=1}^N \sum_{\alpha=x,y,z} \frac{m_i \dot{x}_{i\alpha}^2}{2} \quad (2.23)$$

$$U = \sum_{i=1}^N \sum_{\alpha=x,y,z} \frac{m_i \omega_{i\alpha}^2 x_{i\alpha}^2}{2} + k e^2 \sum_{i=1, j>i}^N \left(\sum_{\alpha=x,y,z} (x_{i\alpha} - x_{j\alpha})^2 \right)^{-\frac{1}{2}} \quad (2.24)$$

where $x_{i\alpha}$ and $\omega_{i\alpha}$ are the position and single-ion secular frequency, respectively, of the i th ion along the α direction ($\alpha \in \{x, y, z\}$), and $k = \frac{1}{4\pi\epsilon_0}$, where ϵ_0 is the permittivity of free space.

2.4.2 Equilibrium Positions

For an axially confined chain, the equilibrium points will lie along the z -axis where there is zero net force on each ion, a condition expressed by

$$\begin{aligned} F_{iz} &= - \left. \frac{\partial U}{\partial x_{iz}} \right|_0 = 0 \\ &= - m_i \omega_{iz}^2 x_{iz} + \sum_{j \neq i}^N \frac{k e^2 (x_{iz} - x_{jz})}{\left(\sum_{\alpha=x,y,z} (x_{i\alpha} - x_{j\alpha})^2 \right)^{\frac{3}{2}}} \bigg|_{x_{ix}, x_{iy}=0} \\ \Rightarrow 0 &= - m_i \omega_{iz}^2 x_{iz} + k e^2 \sum_{j \neq i}^N \frac{\text{sgn}(x_{iz} - x_{jz})}{(x_{iz} - x_{jz})^2} \end{aligned} \quad (2.25)$$

Rearranging eq. (2.25) gives a set of equations:

$$x_{iz} = \frac{k e^2}{m \omega_{iz}^2} \sum_{j \neq i}^N \frac{\text{sgn}(x_{iz} - x_{jz})}{(x_{iz} - x_{jz})^2}. \quad (2.26)$$

⁵But not sufficient! See section 2.4.2.

From eq. (2.9), the axial secular frequencies are $\omega_{iz} = \sqrt{\frac{2eU}{m_i z_0^2}}$, which makes the term $m_i \omega_{iz}^2 = \frac{2eU}{z_0^2}$ mass-independent. The surprising result is that the equilibrium positions of the chain are independent of both the ion masses as well as their ordering within the chain, a fact more readily seen by nondimensionalizing eq. (2.26):

$$u_i = \sum_{j \neq i}^N \frac{\text{sgn}(u_i - u_j)}{(u_i - u_j)^2}, \quad (2.27)$$

where we have made the substitution $u_i = l_0^{-1} x_{iz}$ and defined

$$l_0 \equiv \left(\frac{ke^2}{m\omega_{iz}^2} \right)^{\frac{1}{3}} \quad (2.28)$$

as the characteristic length scale of the system. Solution of eq. (2.27) is best approached with numerical computation in all but the simplest cases of $N = 2, 3$, where it is tractable using symmetry arguments [Jam98]. An approximate analytic solution to eq. (2.27) was derived in [MJ98], building on an observation by [Gar96]:

$$u_i \approx \alpha(N) \sin \left(\frac{\sin^{-1}(\beta(N) [i - (N+1)/2])}{3} \right), \quad (2.29)$$

where $\alpha(N) \approx 3.94N^{0.387}$ and $\beta(N) \approx 1.75N^{-0.982}$ are empirically determined coefficients. Ion spacings are smallest at the center of the chain, where the Coulomb repulsion is greatest, and monotonically increase towards the exterior of the chain, where the Coulomb repulsion is weakest. The minimum spacing (i.e. at the center of the chain) is $u_{\min} \approx 2.018N^{-0.559}$ [Jam98], and is obtained by empirical fits to the numerical solutions.

These results have assumed that each ion is singly-charged, which is of course untrue for chains with highly-charged ions (HCIs), such as for QLS tests of fundamental physics. For an HCI with charge $q = Ze$ at normalized position u_i in the chain, eq. (2.27) is modified with the substitution $u_i \rightarrow Z^{\frac{1}{3}} u_i$. The additional Coulomb repulsion exerted by the HCI results in asymmetry of the inter-ion spacings about the center, and provides a simple and robust

method for establishing the presence of HCIs [Koz+18].

Zigzagging

A core assumption when describing the system in section 2.4.1 was that the chain was axially confined, i.e. the ion equilibrium positions extended only along the z -axis, where we gave the condition $\omega_{x,y} > \omega_z$ as a necessary (but not sufficient) condition. As it turns out, the dynamics of the chain impose additional conditions on stability.

Intuitively, if too many ions are confined within a chain, the Coulomb repulsion between the ions must eventually exceed the radial trapping force, destroying the regular axial structure of the chain [Sch93]. At this phase transition, the ion equilibrium positions at the center of the chain become radially displaced, forming a “kink,” or a “zig-zag.” Zig-zagging can be recovered from the equations of motion in eq. (2.36): as more ions are added to the chain, the frequencies of higher-index radial modes decrease and eventually become imaginary, which results in an unstable trajectory. The center-first pattern of zigzagging consequently arises because the center ions participate most in these higher-index radial modes.

Though zig-zagging can be determined by solution of eq. (2.36), a less computationally-intensive description of zig-zagging is helpful for informing trap parameters. For example, [Ste97] gives the zigzag condition $\frac{\omega_r}{\omega_z} > 0.73N^{0.86}$. This topic received much focus in the mid-90s due to its implications for the then-burgeoning field of trapped ion quantum computing [Win+98,Dub93,Sch93,MJ98]. Particularly, the constraint on ω_r and ω_z sets a limit on the number of ions that can reasonably be trapped while also meeting Mathieu stability criteria. A solution innovated by GD Lin at the Monroe group, and now popular with IonQ, is to use anharmonic potentials, which approximately equalizes the inter-ion spacings to mitigate decrystallization from the Coulomb repulsion [Lin+09].

2.4.3 Normal Modes

If the oscillations of the ions are small relative to their spacings, we can Taylor expand the potential energy term in eq. (2.23) about the equilibrium positions of the ions. To second order, this gives

$$\begin{aligned}
U(x_{i\alpha}) &\approx U(\bar{x}_{i\alpha}) \\
&+ \sum_{i=1}^N \sum_{\alpha \in \{x,y,z\}} \left. \frac{\partial U}{\partial x_{i\alpha}} \right|_{\bar{x}} (x_{i\alpha} - \bar{x}_{i\alpha}) \\
&+ \sum_{i,j=1}^N \sum_{\alpha, \beta \in \{x,y,z\}} \frac{1}{2} \left. \frac{\partial^2 U}{\partial x_{i\alpha} \partial x_{j\beta}} \right|_{\bar{x}} (x_{i\alpha} - \bar{x}_{i\alpha}) (x_{j\beta} - \bar{x}_{j\beta}), \tag{2.30}
\end{aligned}$$

where the $\bar{x}_{i\alpha}$ are the equilibrium positions. The constant term $U(\bar{x}_{i\alpha})$ can be ignored, while the first-order term $\left. \frac{\partial U}{\partial x_{i\alpha}} \right|_{\bar{x}}$ disappears at the equilibrium positions, leaving only the second-order term, which is

$$\begin{aligned}
\mathbf{M}_{ij}^{(\alpha)} &\equiv \left. \frac{\partial^2 U}{\partial x_{i\alpha} \partial x_{j\beta}} \right|_{\bar{x}} \\
&= m_i \omega_{i\alpha}^2 \delta_{ij} \delta_{\alpha\beta} - k e^2 \sum_{i' \neq i} \frac{(\delta_{ij} - \delta_{i'j})}{|x_{iz} - x_{i'z}|^3} (\delta_{\alpha\beta} - 3\delta_{\alpha z} \delta_{\beta z}) \\
&= m_i \omega_{iz}^2 \left[\left(\frac{\omega_{i\alpha}}{\omega_{iz}} \right)^2 \delta_{ij} \delta_{\alpha\beta} - \sum_{i' \neq i} \frac{(\delta_{ij} - \delta_{i'j})}{|u_i - u_{i'}|^3} (\delta_{\alpha\beta} - 3\delta_{\alpha z} \delta_{\beta z}) \right] \tag{2.31}
\end{aligned}$$

where $\delta_{\alpha\beta}$ is the Kronecker delta, and we have defined the tensor $\mathbf{M}^{(\alpha)}$ for convenience. The effect of $\delta_{\alpha\beta}$ and $\delta_{\alpha z} \delta_{\beta z}$ in eq. (2.31) is to eliminate any cross terms in $\{x, y, z\}$, conveniently leaving the motion along all three dimensions independent⁶.

⁶to second order, at least.

Evaluated along each dimension, the tensor $\mathbf{M}_{ij}^{(\alpha)}$ is

$$\mathbf{M}_{ij}^{(x,y)} = m_i \omega_{iz}^2 \begin{cases} \left(\frac{\omega_{i\alpha}}{\omega_{iz}} \right)^2 - \sum_{i' \neq i} \frac{1}{|u_i - u_{i'}|^3} & i = j \\ \frac{1}{|u_i - u_j|^3} & i \neq j \end{cases} \quad (2.32)$$

$$\mathbf{M}_{ij}^{(z)} = m_i \omega_{iz}^2 \begin{cases} 1 + 2 \sum_{i' \neq i} \frac{1}{|u_i - u_{i'}|^3} & i = j \\ -2 \frac{1}{|u_i - u_j|^3} & i \neq j \end{cases} \quad (2.33)$$

Defining $q_{i\alpha} \equiv x_{i\alpha} - \bar{x}_{i\alpha}$ as the small fluctuations of each ion, the Lagrangian eq. (2.23) is simplified to

$$\mathcal{L} = \sum_{i=1}^N \sum_{\alpha \in \{x,y,z\}} \frac{m_i}{2} \dot{q}_{i\alpha}^2 - \sum_{i,j=1}^N \sum_{\alpha \in \{x,y,z\}} \frac{1}{2} \mathbf{M}_{ij}^{(\alpha)} q_{i\alpha} q_{j\alpha}. \quad (2.34)$$

For generality, it is helpful to express eq. (2.34) without explicit dependence on mass, which is achieved by transforming into mass-weighted coordinates with the substitutions $q_{i\alpha} \rightarrow q'_{i\alpha} = \sqrt{m_i} q_{i\alpha}$ and $\mathbf{M}_{ij}^{(\alpha)} \rightarrow \mathbf{M}'_{ij}^{(\alpha)} = \frac{1}{\sqrt{m_i m_j}} \mathbf{M}_{ij}^{(\alpha)}$:

$$\mathcal{L} = \sum_{i=1}^N \sum_{\alpha \in \{x,y,z\}} \frac{\dot{q}'_{i\alpha}}{2} - \sum_{i,j=1}^N \sum_{\alpha \in \{x,y,z\}} \frac{1}{2} \mathbf{M}'_{ij}^{(\alpha)} q'_{i\alpha} q'_{j\alpha}. \quad (2.35)$$

Application of the Euler-Lagrange equation to the Lagrangian eq. (2.35) thus gives

$$\ddot{q}'_{i\alpha} = -\mathbf{M}'_{ij}^{(\alpha)} q'_{j\alpha} \quad (2.36)$$

$$\Rightarrow \omega_{i\alpha}^2 q'_{i\alpha} = \mathbf{M}'_{ij}^{(\alpha)} q'_{j\alpha}, \quad (2.37)$$

where we have assumed oscillatory solutions of the form $q'_{i\alpha}(t) = q'_{i\alpha} e^{i\omega t}$ in eq. (2.37). Equation (2.37) can now be solved as an eigenvalue problem to yield the normal modes $\eta_{n\alpha}$ and the normal mode frequencies $\omega_{n\alpha}$, where n is the mode index.

2.4.4 Mixed-Species Chains

With a full picture of ion chain dynamics, we seek to understand the challenges created by co-trapping a mixed-species chain. Due to the mass-dependence of the secular frequencies, ions of the same mass species couple selectively to one another within a mixed-species chain, leading to a “decoupling” of the motional modes. The result is that the shared motion of the chain becomes prohibitively costly to address — high-fidelity entangling gates on chains with even comparable masses, such as a $^{138}\text{Ba}^+ - ^{171}\text{Yb}^+$ chain, can require laser powers 10^4 times larger than for the single-species case [SCM21]. This problem is particularly existential for QLS (the original goal of this thesis), which fundamentally relies on the sympathetic exchange of motion via the normal modes.

To demonstrate the features of this mode decoupling, the motional modes of a 5-ion $^{138}\text{Ba}^+ - ^{171}\text{Yb}^+$ chain with the configuration Yb–Yb–Yb–Yb–Ba are examined in Figure 2.7, following [SCM21]. Along the axial modes, the mode amplitudes differ only slightly as compared to their single-species counterpart, while the radial modes exhibit almost complete decoupling, with the $^{138}\text{Ba}^+$ ion having near-zero participation in the $^{171}\text{Yb}^+$ -dominated modes. The dire radial decoupling is a consequence of the mass-dependence of the radial pseudopotential, which scales as approximately $\frac{1}{m}$, compared to the $\sqrt{\frac{1}{m}}$ dependence axially. However, since the root of the decoupling stems from the mass-dependence of the secular frequencies, the modes can conversely be re-coupled by equalizing those same secular frequencies. The frequency ratio between the two species along each mode is:

$$\frac{\omega_{ix}}{\omega_{jx}} = \frac{m_j}{m_i} \sqrt{\frac{m_i - \beta}{m_j - \beta}} \qquad \frac{\omega_{iz}}{\omega_{jz}} = \sqrt{\frac{m_j}{m_i}}, \quad (2.38)$$

where the constant $\beta \propto \frac{q^2}{a}$ has been introduced to encapsulate all trap parameters. From eq. (2.38), the radial mode participation can be increased by reducing β , which in the limit of $\beta = 0$ reduces to $\frac{m_j}{m_i}$; this is most easily achieved in experiment by increasing the axial confinement to increase the a -parameter (Figure 2.6(a)). Intuitively, this increase of

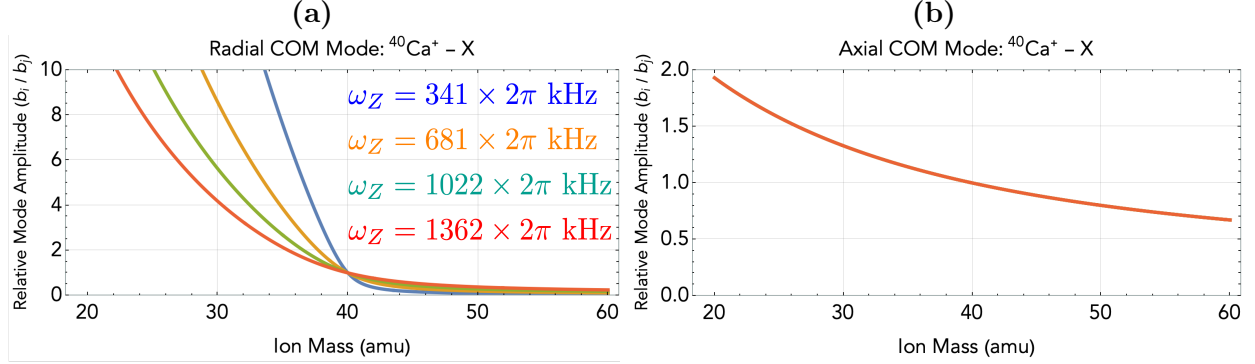


Figure 2.6: **Motional mode participation in a two-ion chain.**

Participation in the center-of-mass mode for a $^{40}\text{Ca}^+ - \text{X}$ ion chain in the (a) radial and (b) axial directions is given as the ratio of the per-ion mode amplitudes $|b_{40}/b_X|$. **(a)** Radial mode participation improves towards the limiting case in (b) as the axial secular frequency (denoted for a single- $^{40}\text{Ca}^+$ ion) is increased. **(b)** The axial secular frequency ratio is dependent only on the mass ratio (eq. (2.38)) and cannot be improved by adjustment of trap parameters.

axial confinement pushes the ions together, thereby maximizing their coupling via Coulomb repulsion. By contrast, the axial secular frequency ratio is dependent only on the mass ratio, precluding any improvement (Figure 2.6(b)). Nevertheless, it should be noted that the radial ratio achieves equality with the axial ratio only in the ideal case of $\beta = 0$, making axial modes fundamentally better suited for mixed-species operations.

Given an ion of interest, therefore, the choice of co-trapped species should be informed primarily by mass, as no straightforward solutions exist to mode decoupling, and axial modes should ideally be used for QLS due to their demonstrably optimal coupling. Some novel, though less well-established, solutions to mode decoupling do exist, though these operate by obviating normal modes as a medium for motional exchange, as opposed to improving the mode mismatch problem: in the context of quantum computation, the *omg* protocol has been proposed to obviate the use of mixed-species chains entirely while retaining those same mixed-species benefits [Viz+24]; applied beamsplitter interaction to directly exchange motional quanta between modes [Hou+24a,Hou+24b]; and the inherent anharmonicity of the Coulomb potential can be used to effect resonant mode-mode transfer and entanglement [MSJ03].

2.4.5 Sympathetic Cooling

Here, we consider sympathetic cooling of a general mixed-species ion chain. In the normal mode picture, the cooling and heating rates of a given mode $n_{i\alpha}$ under Doppler cooling (see section 4.2.1) are given simply by the cooling contribution of each coolant ion scaled by their participations in the mode, which is

$$\frac{dE_{i\alpha}^{\text{cool}}}{dt} = \sum_j (b_{ij}^\alpha)^2 \frac{dE_0^{\text{cool}}}{dt} \quad \frac{dE_{i\alpha}^{\text{heat}}}{dt} = \sum_j (b_{ij}^\alpha)^2 \frac{dE_0^{\text{heat}}}{dt} \quad (2.39)$$

where b_{ij}^α is the mode amplitude of the j th ion in the i th mode along the α direction, and $\frac{dE_0}{dt}$ are the single-ion Doppler cooling and heating rates. The mode participations can be simply factored out to show that the Doppler limit is unchanged in the absence of external sources of heating [Wüb+12]. Though the Doppler limit remains unchanged, the presence of the mode amplitude factors b_{ij}^α in eq. (2.39) does result in diminished cooling efficiency on modes as the mass ratio increases, with up to a $\approx 10\times$ reduction in the radial mode cooling rate for a mass ratio of 0.8.

The sympathetic cooling problem has been shown in simulation to persist into the highly excited regime of $\sim 10^1\text{K}$, as is common during the ion loading stage [Wüb+12]. However, due to the infrequent and stochastic nature of ion loading⁷, testing and confirming analytical predictions remains difficult. Ideal secular frequencies for loading depend strongly on where the ion is created: if the ion is created near the trap center, then higher secular frequencies are needed to improve sympathetic cooling, but if the ion is created off-axis, then lower secular frequencies are needed to reduce their initial energy [Gug+15].

⁷As well as the nontrivial experimental cost due to prolonged loading, which causes significant charging and drastically worsens the vacuum.

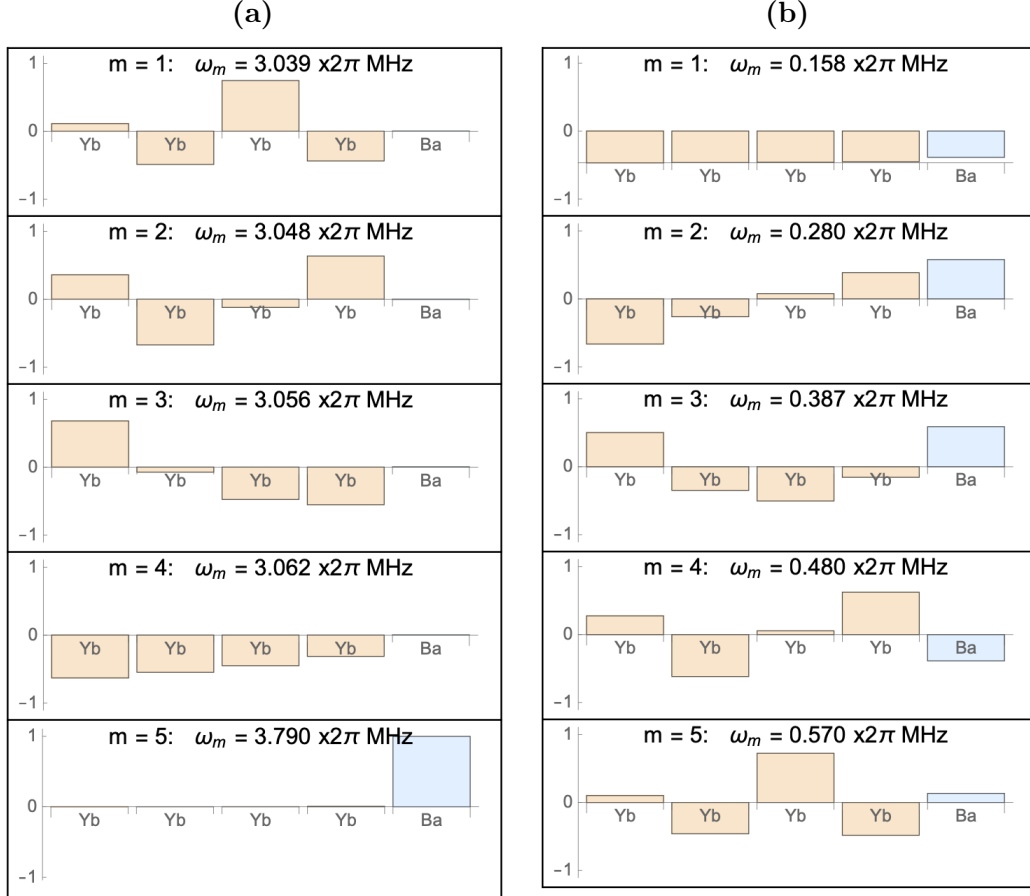


Figure 2.7: **Mode decoupling in a 5-ion Yb–Yb–Yb–Yb–Ba chain.**

Normal-mode amplitudes for the radial (a) and axial (b) modes of a 5-ion $^{138}\text{Ba}^+ - ^{171}\text{Yb}^+$ chain, reproduced following [SCM21], with the $^{171}\text{Yb}^+$ ions in orange and the single $^{138}\text{Ba}^+$ ion in blue. (a) The four $^{171}\text{Yb}^+$ -dominated modes ($m = 1$ –4) near $2\pi \times 3.05$ MHz exhibit negligible $^{138}\text{Ba}^+$ participation, while the lone $^{138}\text{Ba}^+$ ion is isolated into a single mode ($m = 5$, $3.790 \times 2\pi \times \text{MHz}$). (b) By comparison, the axial modes are largely undisturbed, with the $^{138}\text{Ba}^+$ ion retaining significant participation across all modes.

Chapter 3

Quantum Motion

The [Quantum Harmonic Oscillator \(QHO\)](#) provided by the trapped ion motional mode is critical to almost all¹ quantum information with trapped ions. The unique benefit of the trapped ion [QHO](#) is the ability to easily generate spin-motion coupling by exciting the motional sidebands of a spin transition, which can implement both a [Jaynes-Cummings Hamiltonian](#) and an anti-[Jaynes-Cummings Hamiltonian](#). This gives trapped ions the ability to directly control the state in the Fock basis for creation of highly nonclassical states (e.g. [\[San+26\]](#)), a tool not easily found in standard quantum optics experiments. Together with the all-to-all connectivity available in an ion chain and record coherence times [\[Val+25\]](#), these features make the [QHO](#) uniquely suited for quantum computation and electric field sensing.

3.1 The Quantum Harmonic Oscillator

3.1.1 The [Quantum Harmonic Oscillator \(QHO\)](#)

The secular motion of a single ion along each principal axis of a Paul trap is, to excellent approximation, a quantum harmonic oscillator [\[Lei+03\]](#). For a single motional mode with

¹Save for some heralded entanglement approaches [\[Moe+07\]](#).

frequency ω and ion mass m , the Hamiltonian is [SN20]:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 = \hbar\omega \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right), \quad (3.1)$$

where the annihilation and creation operators are defined:

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}} \hat{x} + \frac{i}{\sqrt{2m\hbar\omega}} \hat{p}, \quad \hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}} \hat{x} - \frac{i}{\sqrt{2m\hbar\omega}} \hat{p}. \quad (3.2)$$

For convenience, we define dimensionless quadrature operators [Flü19]:

$$\hat{q} \equiv \sqrt{\frac{m\omega}{2\hbar}} \hat{x} = \frac{(\hat{a} + \hat{a}^\dagger)}{2}, \quad \hat{p} \equiv \sqrt{\frac{1}{2\hbar m\omega}} \hat{p} = \frac{(\hat{a} - \hat{a}^\dagger)}{2i}. \quad (3.3)$$

The eigenstates of eq. (3.1) are the Fock states $|n\rangle$ with energies $E_n = \hbar\omega(n + 1/2)$ spaced equally from $n = 0$ to infinity.

3.1.2 The Wigner Function

While the Fock basis is the natural language for describing spin-motion interactions (see section 4.1), phase-space representations are more convenient for state manipulation and trajectories. The Wigner function provides a quantum analog of a classical phase-space distribution² [GK04] and can be defined in terms of the characteristic function $\chi(\beta) = \text{Tr}[\hat{\rho} \hat{D}(\beta)]$, where $\hat{\rho}$ is the density matrix of the state [WM08] (see also section 3.5.5):

$$W(\gamma) = \frac{1}{\pi^2} \int_{\beta \in \mathbb{C}} \chi(\beta) e^{\gamma\bar{\beta} - \bar{\gamma}\beta} d^2\beta. \quad (3.4)$$

Negativity of the Wigner function is a sufficient but not necessary indicator of nonclassicality (e.g. squeezed states are nonclassical yet Wigner-positive), though pure states with non-

²Though there exists a broader family of quasiprobability distributions (i.e. the Glauber-Sudarshan P and the Husimi Q functions), the Wigner function is generally preferred because it represents nonclassical states efficiently, and, most importantly, is directly measurable [Lei+96, Kie+16, FH20, LD97].

negative Wigner functions are necessarily Gaussian [Hud74]. In phase space, measurements can be understood as overlap between Wigner functions [GK04],

$$\text{Tr}(\hat{\rho}_1 \hat{\rho}_2) = \pi \int_{\gamma \in \mathbb{C}} W_1(\gamma) W_2(\gamma) d^2\gamma, \quad (3.5)$$

which provides useful geometric intuition for understanding and improving measurement sensitivity (see section 3.5.5 and chapter 11).

3.1.3 The Heisenberg Uncertainty Principle

As position and momenta are conjugate variables, their uncertainties Δq , Δp are bound by the Heisenberg uncertainty principle [SN20]:

$$\Delta q \Delta p \geq \frac{1}{2} |\langle [\hat{q}, \hat{p}] \rangle| = \frac{1}{4}. \quad (3.6)$$

The ground motional state $|n = 0\rangle$ saturates this bound with $\Delta q = \Delta p = 1/2$, corresponding to the symmetric Gaussian Wigner function

$$W_0(\gamma) = \frac{2}{\pi} e^{-2|\gamma|^2}. \quad (3.7)$$

The finite phase-space extent of W_0 limits the per-measurement sensitivity to displacements — a limitation expressed by the somewhat grandiose term, the [Standard Quantum Limit \(SQL\)](#)³. The SQL is quantified by the [Quantum Fisher information \(QFI\)](#) of the ground state, which, in dimensionless quadrature units (see eq. (3.3)), is:

$$\mathfrak{F} = 4. \quad (3.8)$$

³As we have learned in our battles to publish [Wu+26], use of the term [SQL](#) is often challenged, with many preferring to reserve it for contexts of entanglement. Our usage, however, is standard in the field of bosonic quantum sensing — see [McC+19a, Wol+19, Aff+20, Bur+19], to name a few. See [Bra+18, Pez+18] for thorough overviews of the term.

The [Quantum Fisher information](#) sets the ultimate achievable precision via the quantum [Cramér-Rao Bound \(CRB\)](#), $\sigma \geq 1/\sqrt{\mathfrak{F}}$, which for the ground state recovers the vacuum uncertainty $\sigma \geq 1/2 = \Delta q$ (eq. (3.6)).

Notably, the uncertainty principle (eq. (3.6)) constrains only the *product* of the quadrature uncertainties, not their distribution. The [SQL](#) can therefore be surpassed by reshaping the motional state to reduce uncertainty along the quadrature of interest — squeezing, the archetypal example, narrows one quadrature at the expense of the other. These strategies are discussed in sections 3.2 and 3.2.2.

3.2 Motional States and Where to Find Them⁴.

Properties of the key motional states relevant to this thesis are summarized in section 3.2; more comprehensive reviews can be found in [[Wal21](#),[FRG25](#)]. The operations described in this section can be composed to create states of greater complexity that leverage different properties. Displaced Fock states $\hat{D}(\alpha)|n\rangle$, for example, break the rotational symmetry of the bare Fock state to enable phase-sensitive displacement sensing [[Wol+19](#),[McC+19b](#)], while the famously termed “quantum Rosetta Stone”⁵ leverages the rapidly oscillating fringes of cat states to generate the periodic grid structure required for bosonic [QEC](#) [[Flü+19](#)].

⁴Title inspired by [[Wal21](#)]

⁵The term “quantum Rosetta Stone” in fact *understates* the importance of the [GKP](#) state — a more appropriately grand term is perhaps “quantum Great Wall.”

State	$\bar{n}$	$\sigma_{\bar{n}}^2$	$P(n)$	$W(\gamma)$
Coherent $ \alpha\rangle$	$ \alpha ^2$	$ \alpha ^2$	$\frac{e^{- \alpha ^2} \alpha ^{2n}}{n!}$	$\frac{2}{\pi} e^{-2 \gamma-\alpha ^2}$
Fock $ n\rangle$	n	0	$\delta_{m,n}$	$\frac{2}{\pi} e^{-2 \gamma ^2} (-1)^n L_n(4 \gamma ^2)$
Squeezed $ \xi\rangle$	$\sinh^2 r$	$\frac{1}{2} \sinh^2(2r)$	$\frac{(2k)! (\tanh r)^{2k}}{4^k (k!)^2 \cosh r} (n = 2k)$	$\frac{2}{\pi} \exp\left(-2 \gamma \cosh r + \bar{\gamma} e^{i\theta} \sinh r ^2\right)$
Cat $ \alpha\rangle \pm \bar{\alpha}\rangle$	$\bar{n}_+ = \alpha ^2 \tanh \alpha ^2$ $\bar{n}_- = \alpha ^2 \coth \alpha ^2$	$\sim \alpha ^2$ $(\alpha \gg 1)$	$\frac{1 \pm \cos(n\Delta\phi)}{1 \pm e^{- \alpha ^2} (e^{\alpha^2} + e^{\bar{\alpha}^2})} \frac{e^{- \alpha ^2} \alpha ^{2n}}{n!}$	$\frac{2\mathcal{N}_{\pm}^2}{\pi} \left[e^{-2 \gamma-\alpha ^2} + e^{-2 \gamma+\alpha ^2} \right]$
Thermal $\hat{\rho}_{\text{th}}$	$\bar{n}$	$\bar{n}(\bar{n} + 1)$	$\frac{\bar{n}^n}{(\bar{n} + 1)^{n+1}}$	$\frac{2}{\pi} \frac{1}{2\bar{n} + 1} \exp[-2 \gamma ^2 / (2\bar{n} + 1)]$

Table 3.1: **A motional state bestiary summarizing properties of motional states relevant to this thesis.** The mean phonon number $\bar{n}$ and its variance $\sigma_{\bar{n}}^2$, Fock-state probability distribution $P(n)$, and Wigner function $W(\gamma)$ are given for state type as a function of the state parameter. The parameters $\{\alpha, \gamma \in \mathbb{C}\}$ denote the 2D position in phase space. Formulae are from [GK04, WM08, Lei+03, Fox06]. Fock state populations for the squeezed state are nonzero only for even n . For the cat state, the angle $\Delta\phi$ denotes the phase space angle between the two wavepackets, i.e. $\Delta\phi = \arg(\alpha - \bar{\alpha})$.

3.2.1 Coherent States

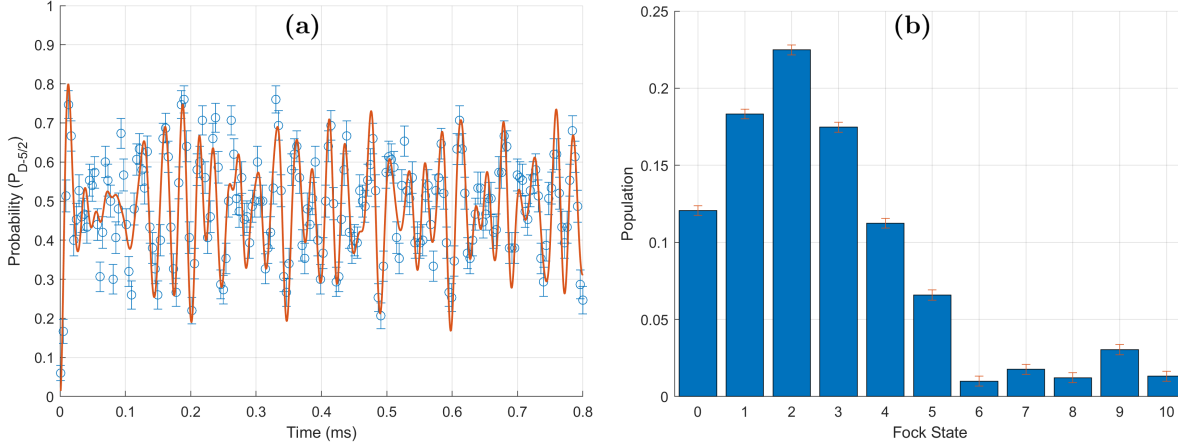


Figure 3.1: **Blue Sideband Rabi oscillations (a) and Fock-state Decomposition (b) of a coherent state with $|\alpha| = 1.54(3)$.** See section 3.5.2 for a protocol overview.

Coherent states are the quantum description of classical oscillator motion [GK04], defined equivalently as eigenstates of $\hat{a}$. Their Wigner function is a Gaussian centered at α with vacuum variance $1/4$ per quadrature, making them minimum-uncertainty states. Coherent states are generated by the driven-oscillator Hamiltonian

$$\hat{H} = \frac{\hbar\Omega}{2} (\hat{a}e^{-i\phi} + \hat{a}^\dagger e^{i\phi}), \quad (3.9)$$

which, when applied for time t , produces the displacement operator

$$\hat{D}(\alpha) = \exp [\alpha \hat{a}^\dagger - \bar{\alpha} \hat{a}] \quad (3.10)$$

with $\alpha = \Omega e^{i\phi} t/2$. In practice, displacements are implemented by applying a voltage oscillating at the secular frequency to a trap electrode (a “tickle”) or the shim electrodes (see sections 5.1.1 and 5.3.6).

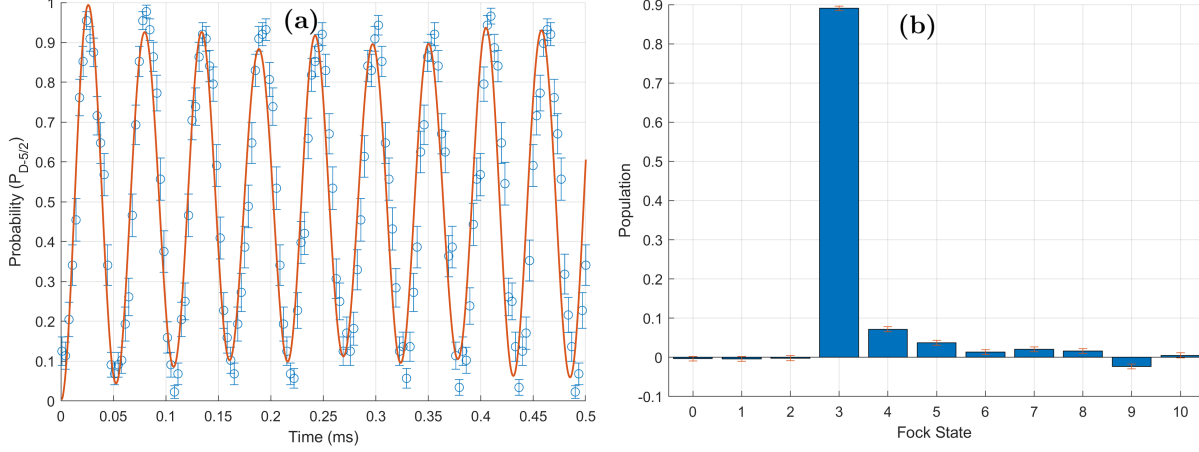


Figure 3.2: **Blue Sideband Rabi oscillations (a) and Fock-state Decomposition (b) of a Fock state $|n = 3\rangle$.** See section 3.5.2 for a protocol overview.

3.2.2 Fock States

The Wigner functions of the Fock states exhibit oscillatory structure governed by the n^{th} Laguerre polynomial, superimposed on the Gaussian envelope of the ground-state Wigner function $W_{n=0}$. This function has full rotational symmetry in phase space (depending only on $|\gamma|^2$) and exhibits concentric rings of negativity for all $n \geq 1$. This rotational symmetry makes the Fock states well suited to sensing displacements of unknown phase, and further renders them robust to dephasing (though not to motional heating, whose effect worsens with n). Beyond symmetry arguments, this robustness can be understood from the fact that Fock states are eigenstates of the dephasing Lindbladian (with jump operator $\hat{L} = \hat{a}^\dagger \hat{a}$), i.e. they constitute Zurek’s pointer basis for this noise channel [Zur81]. Though the Fock states are not minimum-uncertainty states (since $\Delta q^2 = \Delta p^2 = (2n + 1)/4$, eq. (3.6)), they nonetheless provide metrological advantage by virtue of their Wigner negativity (see sections 3.1.2 and 3.5.4).

Generation Fock states are generated by sequential anti-Jaynes-Cummings Hamiltonian (i.e. BSB) and Jaynes-Cummings Hamiltonian (i.e. RSB) interactions [Mee+96] (see section 4.1). Starting from $|\downarrow, 0\rangle$, BSB and RSB π -pulses are alternated to add phonons: $|\downarrow, 0\rangle \rightarrow |\uparrow, 1\rangle \rightarrow |\downarrow, 1\rangle \rightarrow |\uparrow, 2\rangle \rightarrow \dots$. Since the sideband Rabi frequency $\Omega_{n,n+1} = \Omega_0 \eta \sqrt{n+1}$ varies with

n , the π -pulse duration must be adjusted for each step. In experiment, we found that these pulse times required manual calibration rather than simply following the expected $\sqrt{n+1}$ scaling. We have created Fock states up to $n = 3$ with $> 90\%$ fidelity; higher Fock states were (sadly) not required and therefore not pursued. To reduce the lengthy calibrations and drift-induced infidelities associated with the π -pulse sequence, we later instead implemented Fock state generation using [Rapid Adiabatic Passage \(RAP\)](#), following the motional overlap technique of [\[Wol+19\]](#) (see section 3.5.4 for an overview).

The sideband pulse sequence can also be adapted to create a Fock-state superposition:

$$|\phi_n\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |n\rangle). \quad (3.11)$$

Specifically, a $\pi/2$ -pulse creates a spin superposition $|\psi\rangle = (|\downarrow\rangle + |\uparrow\rangle)/\sqrt{2}$, after which the $|\downarrow\rangle$ component is shelved to remove it from the dynamics while preserving its motional state $|n=0\rangle$. The sideband pulse sequence is then applied to the $|\uparrow\rangle$ component to generate the target Fock state. This superposition saturates the Margolus–Levitin bound on the speed of quantum evolution [\[ML98\]](#), and is thus optimal for quantum-enhanced phase sensing [\[McC+19a\]](#). Additionally, as a superposition state, the fringe contrast is directly proportional to the off-diagonal coherences, making it a standard probe of motional dephasing⁶ [\[Tur+00a,00b\]](#). While motional dephasing studies with the simplest Fock interferometer $|\phi_1\rangle$ were tested in our experiment, this approach was not adopted due to the extensive calibrations involved.

3.2.3 Squeezed States

Squeezed states are the archetypal resource for quantum-enhanced sensing: the wavepacket is “squeezed” to redistribute vacuum fluctuations between quadratures, reducing noise along one axis at the expense of the other while preserving the minimum-uncertainty product (eq. (3.6)).

⁶The excess decoherence of $|\phi_n\rangle$ under dephasing (the contrast decays exponentially with rate proportional to n^2) makes it a particularly sensitive probe of decoherence [\[00b\]](#).

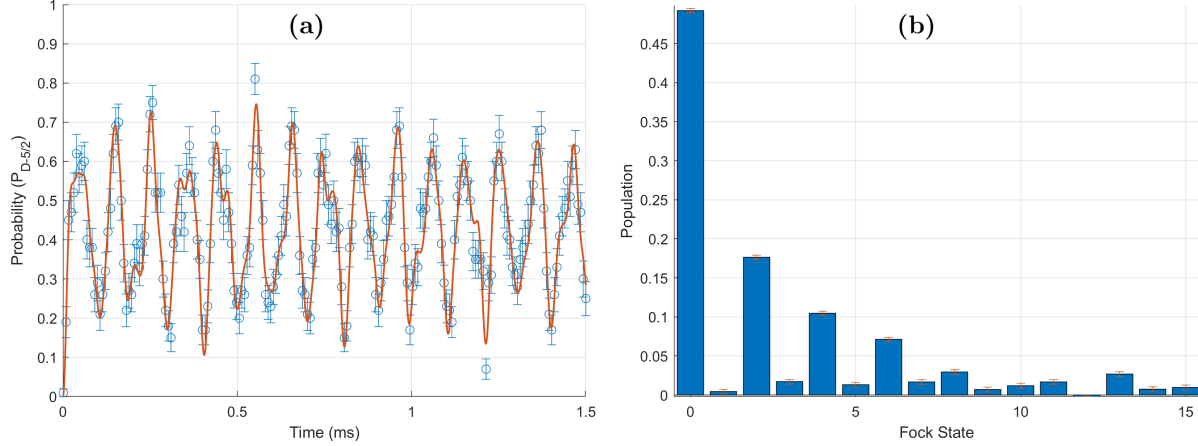


Figure 3.3: **Blue Sideband Rabi oscillations (a) and Fock-state Decomposition (b) of a squeezed state $r = 1.32(3)$.**

See section 3.5.2 for a protocol overview.

Indeed, whenever quantum enhancement is discussed, the conversation invariably turns to LIGO’s use of squeezed light [Col11,Abb+16,Col19], which stems from Caves’ foundational proposal [Cav81] — to this author’s knowledge, LIGO remains the only *real* and *demonstrated* instance of useful quantum enhancement.

Discrimination between squeezed and thermal states is difficult with only a *qualitative* evaluation of the BSB Rabi oscillations as their Fock state probability distributions both decay exponentially with n . Further, the monotonic, super-Poissonian Fock state distribution makes numerical simulation challenging as the simulated Fock basis cannot be easily truncated.

Generation Parametric modulation of the trap potential at 2ω generates the squeezing Hamiltonian [Bur+19,Bur20]:

$$\hat{H}_{\text{sq}} = \frac{i\hbar g}{2} (\hat{a}^2 e^{i\theta} - \hat{a}^{\dagger 2} e^{-i\theta}), \quad (3.12)$$

which, when applied for time t , produces the squeeze operator

$$\hat{S}(\xi) = \exp \left[\frac{1}{2} (\bar{\xi} \hat{a}^2 - \xi \hat{a}^{\dagger 2}) \right] \quad (3.13)$$

where $\xi = r e^{i\theta} = gt e^{i\theta}$ is the complex squeeze parameter.

Alternatively, squeezing can be implemented by driving the second-order motional sidebands with two laser tones, which produces the same Hamiltonian as eq. (3.12) [Mee+96]. A robust variant of this laser-based approach is reservoir engineering, which uses engineered spin-motion couplings to dissipatively pump the ion into a target motional state [PCZ96,Kie+15]; the protocol robustness is further enhanced by the fact that the squeezing is determined by only the *relative* tone parameters (i.e. $\Omega_{\text{BSB}}/\Omega_{\text{RSB}} = e^{i\phi} \tanh(r)$).

Applications Perhaps the most important property of squeezing is that the squeeze operator does not commute with the displacement operator (eq. (3.10)), which enables quantum amplification of displacements [Bur+19]. In our experiment, this was used to amplify the Quantum Vector Signal Analyzer/Analysis (QVSA) interaction (see chapter 8 and section 8.5.2), implemented using the apparatus for parametric micromotion compensation (see sections 5.3.3 and 6.2.4) rather than the resonant coupling circuit of [Bur20]. This repurposing of the parametric apparatus required large drive powers, leading to severe difficulties in achieving high-fidelity squeezing (see section 5.3.3). With maximum power, we achieved a Rabi frequency $\Omega_{\text{sq}} \approx 5.5(2) \times 2\pi$ kHz, and were able to maintain metrological gain up to a squeezing parameter $r = 0.95$.

Other notable applications of squeezing include acceleration of phonon-mediated interactions such as the Mølmer-Sørensen (MS) gate and the Jaynes-Cummings Hamiltonian (JCH) [Bur+19,Bur+21,Bur+24], code-state generation for bosonic quantum computing [Flü+19], and more efficient motional state tomography [Kie+16].

3.2.4 Cat States

Cat states are superpositions of coherent states [Mon+96] and can be understood as atom interferometers: the two coherent-state components trace distinct paths in phase space, and their recombination encodes information about the path difference via a geometric

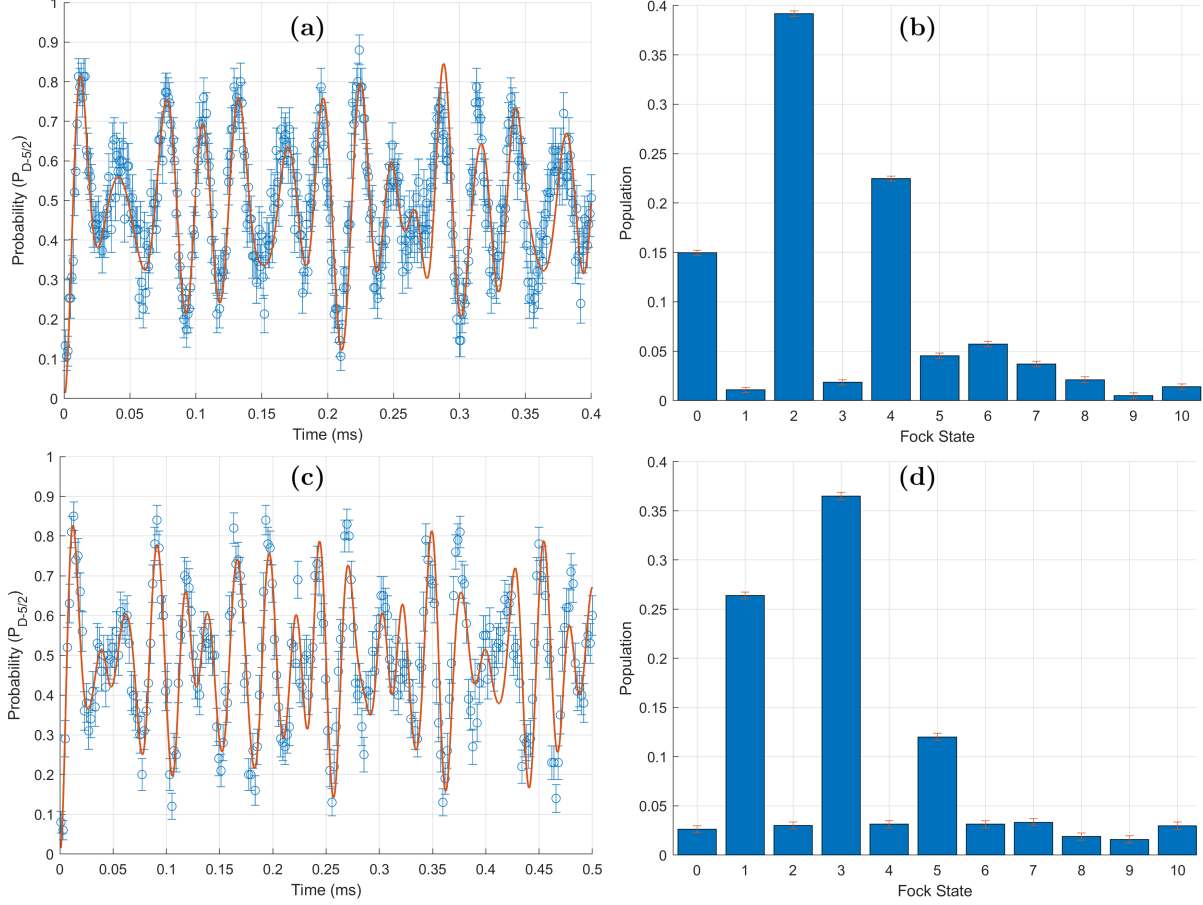


Figure 3.4: **Blue Sideband** Rabi oscillations (a, c) and **Fock-state Decomposition** (b, d) of an even (odd) cat state $|\alpha_{\text{even}}| = 1.713(3)$ ($|\alpha_{\text{odd}}| = 1.778(3)$). See section 3.5.2 for a protocol overview.

phase [Lei+03]. The Wigner function consists of two Gaussian wavepackets centered at $\pm\alpha$, connected by oscillatory interference fringes with strong Wigner negativity near the origin. These fringes define the coherence of the superposition [WM08,DS07] — decoherence between the wavepackets destroys the fringes and reduces the cat to a statistical mixture of the two coherent states [Kie+16]. Cat states enable quantum-enhanced displacement sensing with a sensitivity that scales with $|\alpha|$ [Hem+13], serve as building blocks for bosonic quantum error correction, and provide tests of quantum mechanics at mesoscopic scales [Tur+00a].

Cat states are used in our experiment for quantum-enhanced mass spectroscopy, the subject of GDM's forthcoming thesis.

Generation In our experiment, cat states are created by a [Spin/State-Dependent Force \(SDF\)](#) implemented with bichromatic laser tones resonantly addressing the first motional sidebands. The [SDF](#) Hamiltonian and resulting propagator are:

$$\hat{H}_{\text{SDF}} = \frac{\hbar\Omega\eta}{2} \hat{\sigma}_X (\hat{a} + \hat{a}^\dagger), \quad \hat{U}(t) = \hat{D}(\alpha \hat{\sigma}_X), \quad (3.14)$$

where $\alpha = \frac{\Omega\eta t}{2}$. Starting from $|\downarrow, 0\rangle$, the [SDF](#) displaces the two spin components in opposite directions, creating the entangled state $|+, +\alpha\rangle + |-, -\alpha\rangle$. A projective measurement of the spin then collapses the motion onto either $|\text{cat}_+\rangle = \mathcal{N}_+(|\alpha\rangle + |-\alpha\rangle)$ or $|\text{cat}_-\rangle = \mathcal{N}_-(|\alpha\rangle - |-\alpha\rangle)$, depending on the outcome; either parity can be selected by preceding the measurement with a carrier π -pulse. Since photon scattering during readout decoheres the motional state, only the projection onto the dark spin state is usable.

This constraint was the original motivation for our implementation of the adaptive [Maximum Likelihood Estimator \(MLE\)](#) readout technique (see section 4.3.3). Our standard readout time of ~ 1 ms was comparable to the radial dephasing timescale, leaving the cat substantially decohered by the time it was projected. Adaptive [MLE](#) reduced the readout time to ~ 100 μs , largely resolving this issue.

The projective measurement requirement can also be bypassed entirely by preparing each coherent-state component separately. A $\pi/2$ -pulse creates a spin superposition, one component of which is shelved to an auxiliary state. The [SDF](#) is then applied to displace the remaining component, which is subsequently shelved to a second auxiliary state. The procedure is repeated for the other spin component with the opposite displacement, and the two components are finally returned to the qubit manifold [[Mon+96](#)]. This approach requires more pulses but eliminates the need for measurement-conditioned state selection.

3.3 Phase Space Trajectories

From eq. (3.5), measurement outcomes are determined by the relative positions of phase-space wavepackets. Controlling the motion of these wavepackets is therefore central to all motional sensing and control protocols in this thesis. Displacements are the fundamental primitive for phase-space control, and can be composed into sequences to shape their response.

3.3.1 Displacements

Application of a sinusoidal uniform driving force is described by the interaction-picture Hamiltonian [Wol+19, Lee+05, Val+22]:

$$\hat{H} = i\hbar (f(t)\hat{a}^\dagger - \bar{f}(t)\hat{a}), \quad (3.15)$$

where $f(t) = i\Omega(t)e^{i(\delta(t)+\phi(t))}$, $\Omega(t)$ is the effective Rabi frequency, $\delta(t)$ is the detuning from the secular frequency ω_0 , and $\phi(t)$ is the drive phase. The evolution is described by the propagator

$$\hat{U}(t_0, t) = \hat{D}(\alpha(t_0, t)) e^{i\Phi(t_0, t)}, \quad (3.16)$$

where t_0 is the initial time of the interaction, and $D(\alpha(t_0, t)) = \exp [\alpha(t_0, t)\hat{a}^\dagger - \bar{\alpha}(t_0, t)\hat{a}]$ is the displacement operator. The accumulated displacement $\alpha(t_0, t)$ and geometric phase $\Phi(t_0, t)$ are

$$\alpha(t_0, t) = \int_{t_0}^{t_0+t} f(t') dt' = i \int_{t_0}^{t_0+t} \Omega(t') e^{i(\delta(t')+\phi(t'))} dt' \quad (3.17)$$

$$\Phi(t_0, t) = \text{Im} \left[\int_{t_0}^{t_0+t} dt' \int_{t_0}^{t'} dt'' f(t') \bar{f}(t'') \right]. \quad (3.18)$$

Interpretation The time-dependent drive function $f(t)$ can be understood as the ion's instantaneous velocity phasor in phase-space, which is integrated to yield the ion's total

displacement α . The displacement α can be decomposed into phase space quadratures by taking $(q(t), p(t)) = (\text{Re}[\alpha], \text{Im}[\alpha])$. Nonzero detuning $\delta(t)$ results in curvature of the phase-space trajectory, while $\Omega(t)$ and $\phi(t)$ set the speed and direction of travel, respectively. As $\Omega(t)$ and $\phi(t)$ can be well-controlled, it is convenient to combine them into a complex drive parameter $\mathbf{\Omega}(t) = \Omega(t)e^{i\phi(t)}$.

The geometric phase Φ arises from the non-commutativity of displacements: two successive displacements compose as $\hat{D}(\alpha)\hat{D}(\beta) = e^{i\text{Im}(\alpha\bar{\beta})}\hat{D}(\alpha + \beta)$ (via the [Baker-Campbell-Hausdorff Lemma \(BCH\)](#)), such that chaining of the infinitesimal displacements $f(t') dt'$ from the trajectory yields the double integral expression in eq. (3.17) [Vut+18]. For a closed trajectory, Φ corresponds to the total enclosed area; this is the key principle underlying geometric phase gates [MS99, SM00, Mil99].

Typical Cases A time-independent resonant drive ($\Omega(t) = \Omega_0$, $\phi(t) = \phi_0$, $\delta(t) = 0$) results in a linear trajectory $\alpha(t_0, t) = i\Omega_0 t$ that grows with time and phase-space direction set by ϕ_0 . A similar time-independent resonant drive with nonzero constant detuning (i.e. $\Omega(t) = \Omega_0$, $\phi(t) = \phi_0$, $\delta(t) = \delta_0 \neq 0$) produces a trajectory

$$\alpha(t) = \frac{\Omega_0 e^{i\phi_0}}{\delta_0} (e^{i\delta_0 t} - 1), \quad (3.19)$$

which traces a circle of radius $R = |\Omega_0/\delta_0|$ in phase space traversed with period $T = 2\pi/\delta_0$. To more easily visualize the trajectory, eq. (3.19) can be expressed as a parametric function of t with phase space components $(q(t) = \Omega_0 (\cos(\delta_0 t) - 1)/\delta_0, p(t) = \Omega_0 \sin(\delta_0 t)/\delta_0)$.

Multiple Displacements As eq. (3.15) is linear in $\hat{a}$, $\hat{a}^\dagger$, multiple simultaneous drives $f_k(t)$ can be combined to yield a net driving term $f(t)$. Additionally, since the displacement is linear in $f(t)$ (eq. (3.17)), the net displacement can be found by simply adding the displacement

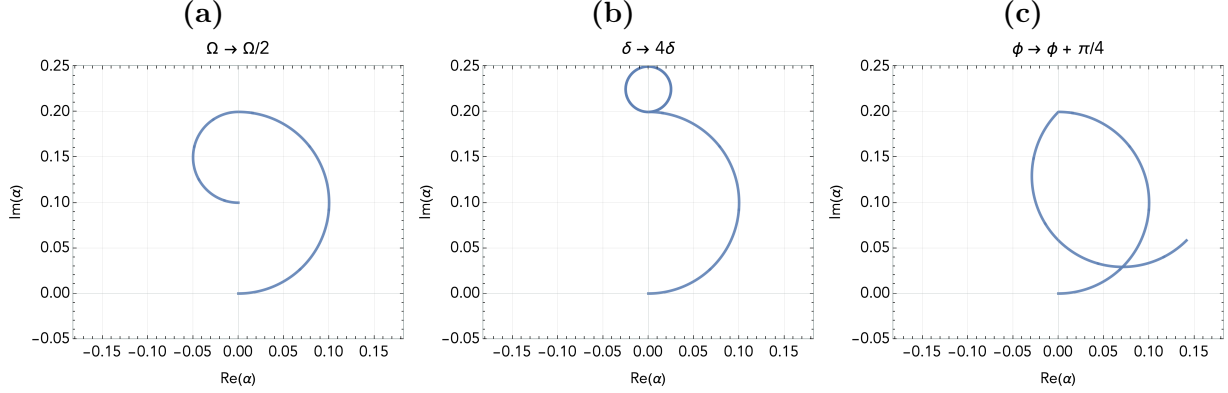


Figure 3.5: **Phase-space trajectories under modulation.**

A phase space trajectory that begins at the origin and has detuning δ (eq. (3.19)) is modulated at time $t = 1/\delta$. **(a) Amplitude Modulation (AM)** ($\Omega \rightarrow \Omega/2$) reduces the orbital radius $R = |\Omega_0/\delta_0|$ but leaves the angular velocity unchanged. **(b) Frequency Modulation (FM)** ($\delta \rightarrow 4\delta$) reduces the orbital radius and increases the angular velocity, winding the trajectory into a tighter loop. **(c) Phase Modulation (PM)** ($\phi \rightarrow \phi + \pi/4$) rotates the center of the orbit; both orbital radius and angular velocity are unaffected.

from each drive α_k :

$$\alpha_{\text{tot}}(t) = \sum_k \alpha_k(t). \quad (3.20)$$

By contrast, the geometric phase Φ is nonlinear in $f(t)$ and must be evaluated directly from the double integral in eq. (3.17).

3.3.2 Pulse Shaping

Amplitude pulse-shaping suppresses unintended excitation from fast off-resonant terms. This is crucial when exciting motional Raman transitions, as these operate in the RF band (1 MHz to 1000 MHz)⁷ where spectra are dense with near-resonant transitions, such as higher-order motional sidebands and trap Mathieu harmonics at $p\Omega_{\text{RF}} \pm k\omega_i$, ($p, k \in \mathbb{Z}$). These spectator transitions can carry Rabi frequencies comparable to the target motional Raman transition, which, as a second-order process, is suppressed by a factor $1/\omega_s^2$, where ω_s is the Raman carrier frequency. Motional Raman transitions further entail strong, off-resonant excitation

⁷Colloquially, this author separates the RF band into the following bands on relevance to the ion trapper: the noise band (DC $\sim 10^2$ kHz), the secular band (10^2 kHz $\sim 10^1$ MHz), the Mathieu band (10^1 MHz $\sim 2 \times 10^2$ MHz), and finally, the **QVSA** band (2×10^2 MHz $\sim 1 \times 10^3$ MHz).

of the baseband secular frequency with strength proportional to that of the motional Raman transition itself (see chapter 8), presenting a background that, without pulse-shaping, is otherwise irreducible and can dominate the motional Raman transition. As we are here concerned with the magnitude of the displacement, we ignore the drive phase ϕ and assume constant detuning $\delta(t) = \delta$ such that only the envelope $\Omega(t) = \Omega_0 f(t)$ varies; discussion of geometric phase is neglected entirely.

Rectangular Pulses

For comparison, we first consider the unshaped case by examining a pulse with rectangular envelope of duration t_1 . The displacement (eq. (3.17)) evaluates to

$$\alpha(t_1) = i \int_0^{t_1} \Omega_0 e^{i\delta t} dt = \frac{\Omega_0}{\delta} (e^{i\delta t_1} - 1), \quad |\alpha| \leq \frac{2\Omega_0}{\delta}. \quad (3.21)$$

The off-resonant excitation oscillates with an amplitude bounded by $2\Omega_0/\delta$, which is non-negligible when $\delta \sim \Omega_0$; with laser-based fields, this is often the case when driving MS gates. Intuitively, the problem is that truncation of the pulse at arbitrary time strands the ion on this oscillatory circle, yielding nonzero residual displacement. While this off-resonant excitation could in principle be nulled by judicious selection of $t_1 = 2n\pi/\delta$, such fine choice of t_1 is experimentally challenging for Mathieu-band drives: large detunings imply fast oscillations ($\lesssim 10$ ns), and pulse jitter⁸ and finite system bandwidths at this timescale result in an averaging of the excitation to half the peak value.

⁸Pulse jitter arises more from hardware behavior rather than jitter in the ARTIQ control clock, which is (specified to be) accurate to 10 ps.

General Pulse Shapes

For an arbitrary smooth envelope $f(t)$ on $[0, t_1]$, repeated integration by parts yields the asymptotic series

$$\begin{aligned}\alpha(t_1) &= i\Omega_0 \int_0^{t_1} f(t) e^{i\delta t} dt \\ &= i\Omega_0 \sum_{n=0}^{\infty} \frac{(-1)^n}{(i\delta)^{n+1}} (f^{(n)}(t_1) e^{i\delta t_1} - f^{(n)}(0)).\end{aligned}\tag{3.22}$$

Principles for effective pulse-shape design then follow from eq. (3.22):

- **Vanishing endpoints:** Elimination of the leading $n = 0$ term requires $f(0) = f(t_1) = 0$. In principle, this precludes envelopes such as Gaussians that are nonzero at the endpoints, though this does not in practice pose a problem due to finite [DAC](#) resolution. These problems can be mitigated by applying an additional envelope with the properties of interest [[Smi03](#), [Mil+21](#)].
- **Smoothness:** Smooth ramping at the endpoints (i.e. $f'(0) = f'(t_1) = 0$) eliminates the $n = 1$ term. Higher-order terms can be eliminated by vanishing higher-order derivatives at the endpoints; this is cumbersome to implement in practice and offers marginal improvement, as higher-order terms are already suppressed by a factor δ^n .
- **Symmetry:** As the magnitude of the general expression $|ae^{i\delta t_1} - b|$ (where a, b are arbitrary coefficients) is minimized for $|a| = |b|$, surviving terms are minimized by a symmetric envelope with $f^{(n)}(0) = (-1)^n f^{(n)}(t_1)$.

Off-resonant excitation can thus be made identically zero by a C^∞ envelope with all derivatives vanishing at the endpoints — such envelopes exist (e.g. bump functions [[BL20](#)]), though lose their benefits in practice due to the finite resolution of digital hardware.

The simplest illustrative case is a triangular envelope of half-width $\tau = t_1/2$, which is symmetric with vanishing endpoints but a discontinuous first derivative at the corners⁹.

⁹For convenience, we assume that the envelope continues outside the region of interest $t \in [0, t_1]$ such

While the $n = 1$ term is nonvanishing, it is suppressed by a factor $\tau\delta$, and can therefore be made sufficiently small by choosing $\tau \gg 1/\delta$. More generally, the time-constant of the edges (as in e.g. a Tukey/cosine-taper window) sets the order parameter $\tau\delta$ and thus determines the minimum detuning for which off-resonant excitation is suppressed.

Intuitively, the effect of pulse shaping can be understood as smoothly “growing” the oscillation radius as the pulse is ramped on, and smoothly “shrinking” the radius as the pulse is ramped off — this ensures that the ion deterministically returns to the origin. During roll-on, the instantaneous orbit radius grows from zero to result in a trajectory that “spirals” outward and is centered on the origin, with a magnitude fixed at half the maximum value of the unshaped case. Due to this nonzero residual displacement, off-resonant excitation can still seed dephasing even when using shaped pulses. During roll-off, the radius shrinks back toward zero, compressing the spiral inward — this is the key step which enables suppression of residual displacement. In principle, pulse roll-on is non-essential to achieve reasonable suppression, as long as pulse roll-off is present and sufficiently slow.

Experimental Implementation

In practice, a $\sin^2$ envelope (equivalently, the Hann window) is typically used whenever generic pulse-shaping is required:

$$f_{\text{Hann}}(t) = \sin^2\left(\frac{\pi t}{t_1}\right) \quad (0 \leq t \leq t_1), \quad (3.23)$$

where t_1 is the pulse length (Figure 3.6). This envelope is favored as it is symmetric, has vanishing zeroth and first derivatives at both endpoints, and is straightforward to implement in hardware. As suppression is determined primarily by the behavior at the endpoints (see eq. (3.22)), we often extend this envelope as a Tukey/cosine-taper to minimize cost to that higher-order derivatives vanish at the endpoints.

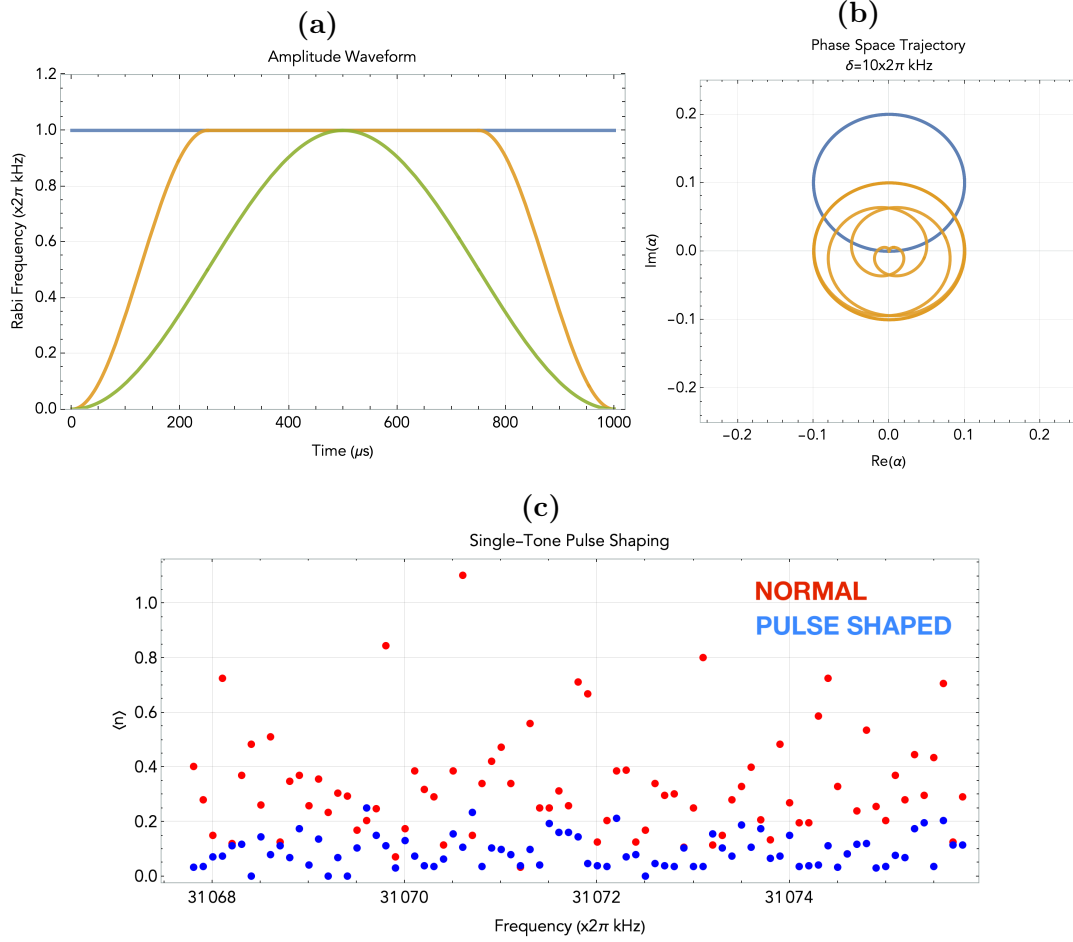


Figure 3.6: **Effect of pulse-shaping on phase-space trajectories.**

(a) A rectangular (blue), Tukey/cosine-taper (orange), and a $\sin^2$ /Hann (green) envelope. (b) Phase-space trajectories for fixed detuning $\delta = 10 \times 2\pi$ kHz with a rectangular (blue) and Tukey/cosine-taper (orange) amplitude envelope; for clarity, only the trajectory under the first half of the pulse is shown. Following the rectangular pulse, the ion is stranded along the circular orbit with nonzero displacement (eq. (3.21)); with the shaped pulse, the orbit spirals smoothly outward and back, suppressing off-resonant excitation. (c) Experimentally measured residual excitation $\langle n \rangle$ versus drive frequency for a rectangular (red) and Tukey-shaped (blue) pulse.

pulse area:

$$f_{\text{Tukey}}(t) = \begin{cases} \sin^2\left(\frac{\pi t}{2\tau}\right) & 0 \leq t \leq \tau \\ 1 & \tau \leq t \leq T + \tau \\ \sin^2\left(\frac{\pi(t-T)}{2\tau}\right) & T + \tau \leq t \leq T + 2\tau \end{cases} \quad (3.24)$$

where τ is the roll-on time of the edges, and T is the length of the rectangular portion of

the pulse, for a total pulse time of $T + 2\tau$ (Figure 3.6). Compilation of shaped composite pulse sequences is implemented for the Phaser AWG (see section 5.4.1) by a homebuilt pulse compiler, which we call “SpinEchoWizard” to evoke the printer setup wizards of the mid-2000s. On the AD9910 DDSs, pulse shaping is implemented via RAM mode; RAM mode further supports options to mirror and taper the pulse, such that only the rising edge need be specified (via e.g. the bidirectional ramp feature on the AD9910 section 5.4.1), enabling efficient use of the limited AD9910 RAM depth (1024 words).

A natural and noteworthy consequence of using shaped pulses (with the same total pulse time) is a broadened frequency response near resonance (i.e. $\delta \sim 1/\tau$), making it unsuitable for minimization of near-resonant excitation. This follows from the time-bandwidth product: as a rectangular pulse shape is maximally wide in the time-domain, all other pulse shapes must have wider frequency-domain width.

3.3.3 Composite Sequences

Single displacements are rarely applied in isolation: composing a sequence of displacements enables properties beyond that achievable by a primitive block, with significant application in sensing [Ge+19a,Bur+21,Bur+24] [Wu+25a,Wol+19,Mil+21,Kel+21] [McC+19b,Sim20] (see chapter 10 for our application to super-resolution) and robust entangling gates [Val+22].

We treat a sequence as a set of N displacement pulses, each of fixed length τ . The n -th block spans $[t_n, t_n + \tau]$, within which the complex drive parameter $\mathbf{\Omega} = \Omega(t)e^{i\phi(t)} = \Omega_n e^{i\phi_n} \equiv \mathbf{\Omega}_n$ is held constant. For simplicity we further assume the detuning $\delta(t) = \delta$ is fixed across the entire sequence.

This is restrictive — formalisms such as Monroe’s AM/FM gates handle time-varying δ — but is well-suited to our purposes here. This thesis is primarily concerned with sensing, where δ is typically the quantity being measured and thus cannot be controlled by design. Though noise sensing formalisms [McC+19b,Kel+21] allow continuously varying $\delta(t)$, the detuning can be linearized within a block by choosing τ to be much shorter than the timescale on

which $\delta(t)$ varies.

From eq. (3.17), the displacement contributed by the n -th block is

$$\begin{aligned}\alpha_n &= i\Omega_n \int_{t_n}^{t_n+\tau} e^{i\delta t} dt = \frac{\Omega_n}{\delta} (e^{i\delta(t_n+\tau)} - e^{i\delta t_n}) \\ &= \underbrace{i\tau \text{sinc}\left(\frac{\delta\tau}{2}\right) e^{i\delta\tau/2}}_A \underbrace{\Omega_n e^{i\delta t_n}}_{\beta_n}.\end{aligned}\tag{3.25}$$

The prefactor A represents the change in the displacement when off-resonant, while β_n encodes the waveform and thus the characteristics of the sequence. Within β_n , the term $e^{i\delta t_n}$ tracks the time-evolved phase that is accumulated between blocks.

Phase Modulation For phase modulation, the amplitude is held constant, $\Omega_n = \Omega_0 e^{i\phi_n}$, such that the displacement contributed by the n -th block is:

$$\alpha_n = \Omega_0 A e^{i\delta t_n} e^{i\phi_n},\tag{3.26}$$

with total displacement:

$$\alpha_{\text{tot}} = \sum_{n=0}^{N-1} \alpha_n = \Omega_0 A \sum_{n=0}^{N-1} e^{i(\delta t_n + \phi_n)}.\tag{3.27}$$

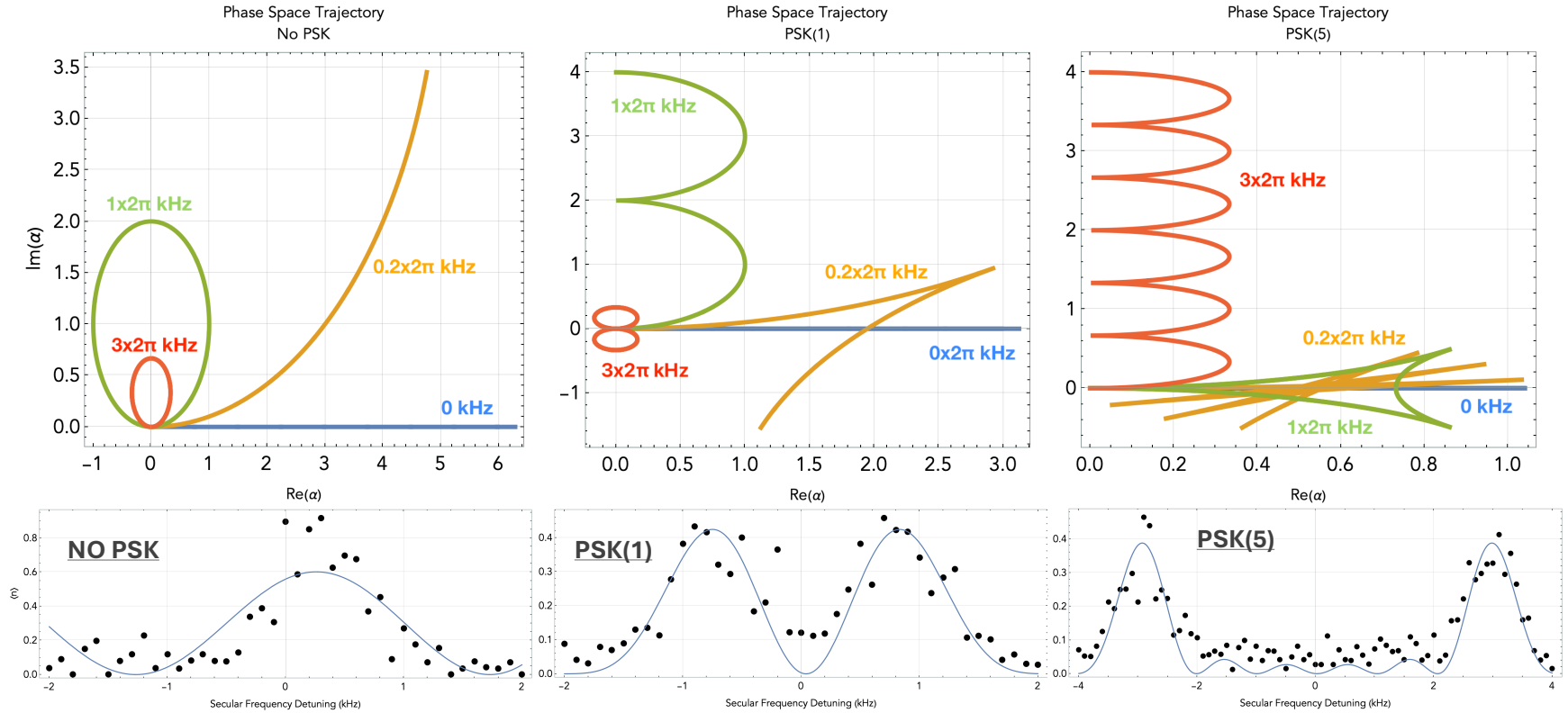


Figure 3.7: **Phase-space trajectories under phase modulation (Phase-Shift Keying (PSK)).**

Columns show the No PSK, PSK(1), and PSK(5) cases, with phase waveforms (0), (0, π), and (0, π , 0, π , 0) applied over the total pulse time, respectively. **(top row)** Phase-space trajectories for several detunings. Each phase flip reverses the instantaneous direction of travel and suppresses the net displacement off resonance, but at $\delta = \pi/\tau$ the flips resonate with the trajectory and the displacement instead accumulates. **(bottom row)** Experimentally measured motional excitation $\langle n \rangle = |\alpha|^2$ versus secular-frequency detuning, for a total pulse time $T = 1$ ms; data are fitted to the analytic profile of eq. (3.27). Phase-shift-keying splits the central peak of the “No PSK” case into side-lobes at $\delta = \pm\pi/\tau$, separating further with the number of flips and suppressing residual excitation near resonance.

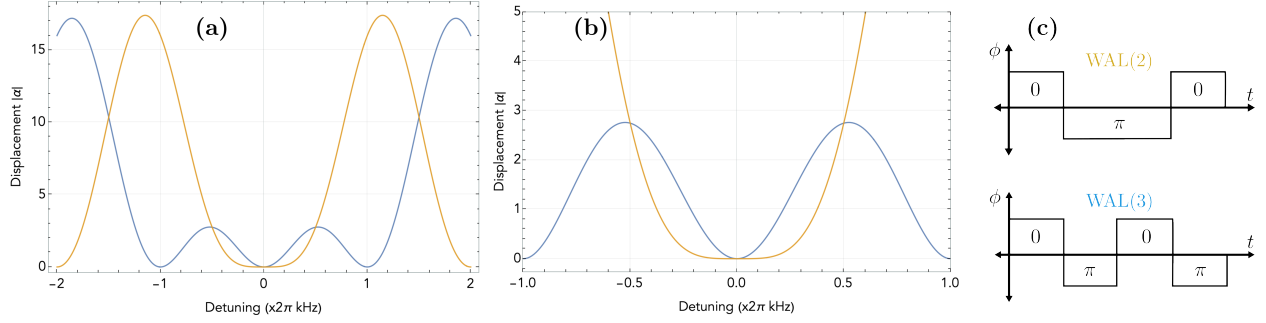


Figure 3.8: **Motional excitation spectrum under spin-echo type sequences.**

(a) Calculated displacement $|\alpha|$ versus detuning for Walsh(2) (orange) and Walsh(3) (blue) phase-modulated sequences (eq. (3.27)). While both sequences completely reverse the displacement at $\delta = 0$, the flat response of the Walsh(2) sequence provides improved robustness to detuning errors. **(b)** A zoomed-in view of (a), highlighting the different responses of the two sequences as $\delta \rightarrow 0$. **(c)** The Walsh-modulated waveforms.

Spin-Echo Sequences Spin-echo-type sequences interleave free-evolution delays between displacement blocks to enable frequency sensing [Wol+19, McC+19a, McC19, Kel+21, Mil+21]. Prior to quantum heterodyne protocols [Wan+22, Wu+25b], spin-echo sequences carried primary importance for sensing of AC signals [DRC17, Kot+11]. For simplicity, all delays are assumed to be of the same fixed duration T . Intuitively, spin-echo sequences operate in phase space by mapping a detuning into the magnitude of the displacement: an initial displacement makes the wavepacket sensitive to detuning, while delays rotate the wavepacket in phase space at rate δ , and subsequent displacements interfere via a δ -dependent relative phase. Equivalently, spin-echo sequences can be understood as shaping the ion’s sensitivity function to the detuning [DRC17].

Motional Ramsey The simplest spin-echo sequence is the motional Ramsey [Wol+19, Sim20]: a displacement pulse is applied, followed by a free-evolution delay of duration T , and a second displacement with phase π relative to the first. Substitution of $\phi_0 = 0$, $\phi_1 = \pi$, $t_0 = 0$,

$t_1 = \tau + T$ into eq. (3.27) yields

$$\begin{aligned}\alpha_{\text{tot}} &= \Omega_0 A (1 - e^{i\delta(\tau+T)}) \\ \Rightarrow |\alpha_{\text{tot}}| &= 2\Omega_0 \left| A \sin\left(\frac{\delta(\tau+T)}{2}\right) \right|.\end{aligned}\tag{3.28}$$

In the small-detuning limit $\delta(\tau + T) \ll 1$, the displacement is linear in δ ,

$$|\alpha_{\text{tot}}| \approx \Omega_0 |A\delta| (\tau + T),\tag{3.29}$$

enabling its use as a frequency probe. Note that the signal grows linearly with the total interrogation time T , following the standard Ramsey scaling.

3.4 Motional Coherence

Motional coherence sets the maximum time for which the [QHO](#) can be coherently manipulated, and is the limiting factor for almost all operations in our experiment. For sensing on motional modes, long coherence times enable longer interrogation times and thus improved sensitivities — this was critical to convincingly demonstrate the state-of-the-art sensitivities possible with our motional Raman protocol (see chapters 8 and 10). For [Quantum Logic Spectroscopy \(QLS\)](#) with [Electric-Field Gradient Gates \(EGGS\)](#) (see chapter 7), low motional dephasing is essential and remains an area of active improvement: as the [EGGS](#) signal is small ($\Omega_{\text{EGGS}} \sim 10^2 \times 2\pi$ kHz), longer interrogation times yield a larger, more robust search signal with a commensurately larger *effective* linewidth.

Given this rate-limiting importance of motional decoherence (motional dephasing, in particular), much effort was dedicated to eliminating sources of noise in our experiment. Like Napoleon, we waged many campaigns against the scourge of decoherence; and, like Napoleon, our initial decisive victories against motional *heating* proved insidious — our success ossified our experimental apparatus of state and precluded the reforms necessary to repeat them

against *dephasing* [Cha66]. This section serves as a chronicle of our experience.

Two processes underlie motional decoherence in trapped ions: **heating** (amplitude damping, characterized by T_1), which adds energy to the oscillator, and **dephasing** (phase damping, characterized by T_2), which randomizes the phase of the motional state without changing its energy. Heating is largely driven by electric field fluctuations near the secular frequency, while dephasing is driven by fluctuations of the secular frequency, typically caused by noise on the trapping RF. Heating and dephasing are related by $1/T_2 = 1/(2T_1) + \Gamma_\phi$, where Γ_ϕ is the pure dephasing rate; when $\Gamma_\phi \rightarrow 0$, dephasing is entirely heating-limited.

In general, decoherence is a non-unitary process, and is described by the Lindblad master equation:

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] + \sum_j \gamma_j \left(\hat{L}_j \hat{\rho} \hat{L}_j^\dagger - \frac{1}{2} \{ \hat{L}_j^\dagger \hat{L}_j, \hat{\rho} \} \right), \quad (3.30)$$

where $\hat{L}_j$ is the Lindblad operator for the j -th decoherence channel, and γ_j its associated rate. The first term reproduces the unitary Schrödinger evolution, while the second captures incoherent dissipation.

3.4.1 Motional Heating

Motional heating is most efficiently described by considering the coupling of the ion to environmental electric-field fluctuations [Tur+00b, Bro+15, Win+98, Lei+03]. Modeling the noise field $E(t)$ as a uniform driving force gives the Hamiltonian

$$\hat{H} = -qE(t)\hat{x}, \quad (3.31)$$

where q is the ion charge. The resulting heating rate from the ground state is [Tur+00b]

$$\dot{\bar{n}} = \frac{q^2}{4m\hbar\omega_0} S_E(\omega_0), \quad (3.32)$$

where $S_E(\omega)$ is the single-sided **Power Spectral Density (PSD)** of the field noise, ω_0 is the

secular frequency, and m is the ion mass.

Models of Electric-Field Noise

The specific form of $S_E(\omega)$ is mechanism-dependent and yields characteristic scalings with the ion-electrode distance d .

Johnson Noise Johnson noise arises from the nonzero resistance $R(\omega)$ of the trap electrodes and external network, and yields a heating rate with d^{-2} dependence [Win+98, Tur+00b]:

$$\dot{n}_R = \frac{q^2}{4m\hbar\omega_0} \frac{4k_B T R(\omega_0)}{d^2}. \quad (3.33)$$

For typical experimental parameters $m = m_{40\text{Ca}^+}$, $\omega_0 \approx 1 \times 2\pi$ MHz, $T \approx 20$ K, $d = 550$ μm , eq. (3.33) predicts $\dot{n}_R \sim 0.5 \text{ s}^{-1}$ for $R(\omega_0) = 1 \Omega$, in reasonable agreement with heating rates (see Figure 3.10) and resistances (see section 5.1.1) measured in *our* trap. This low rate, combined with our cryogenic operation, suggests that our experiment is not limited by anomalous heating.

Patch Potential Noise Patch potential noise (equivalently, anomalous heating) arises from microscopic regions of fluctuating surface potentials (caused by e.g. adsorbed contaminants [Bro+15, Saf+11]). Modeling the trap as a sphere of radius d surrounding the ion, with fraction C of its area covered by equally-distributed patches of characteristic size r_p , gives [Tur+00b]

$$\dot{n}_P = \frac{q^2}{4m\hbar\omega_0} \frac{3C r_p^2 S_V(\omega_0)}{4d^4}, \quad (3.34)$$

where $S_V(\omega)$ is the voltage spectral density per patch. This produces the famous d^{-4} dependence that distinguishes anomalous heating from Johnson noise [Tur+00b].

General Form of $S_E(\omega)$ More generally, $S_E(\omega)$ is parameterized by its dependence on ω_0 , d , and trap temperature T :

$$S_E(\omega) \propto \omega_0^{-\alpha} d^{-\beta} T^\gamma. \quad (3.35)$$

The frequency exponent is naively $\alpha = 1$ for $1/f$ noise; in practice, values from $\alpha \in [-1, 7]$ have been reported across the literature [Bro+15], with well-controlled measurements yielding $\alpha = 0.95(28)$ to $1.57(4)$ [Bro+21].

The distance exponents $\beta = 2$ and 4 correspond to Johnson and anomalous heating, respectively; $\beta \approx 4$ is recovered in well-controlled experiments [Lab+08, Des+06, Sed+18], though values as low as $\beta \approx 2.5$ have been reported [Bro+15, Bro+21].

The temperature exponent γ has been measured to lie in $\gamma \in [0.5, 4]$ with characteristic crossover temperature near $T \sim 70$ K — $\gamma \approx 0.54(4)$ below this temperature, and $\gamma \approx 2.13(5)$ from 70 K to room temperature [CS14]. This behavior is well-modeled by atoms with fluctuating dipole moments adsorbed onto the trap surface [Bro+15, Saf+11]. Cryogenic trap operation (as in our experiment) thus suppresses anomalous heating by $\sim 10^3\times$ relative to that at room temperature [Lab+08, Bro+15].

Experimental Implementation

Heating rates are straightforwardly measured by sideband cooling near the ground motional state, waiting for some variable delay time t , then measuring the mean phonon number $\bar{n}(t)$. $\dot{\bar{n}}$ is then extracted by a linear fit to $\bar{n}(t)$. The Fisher information of most motional readout protocols (e.g. SBR and RAP-readout) is diminished above $\bar{n} \sim 2$; we therefore restrict our measurements to this range.

$\bar{n}$ was initially measured via the Sideband Ratio (SBR) technique (see section 3.5.1), which required measurement of complete red and blue sideband spectra for each delay time to compensate for infidelities and drifts (the effects of which were, in a vicious cycle, exacerbated by the long times required to measure the entire spectrum). With SBR, full heating rate measurements typically took ~ 30 min, complicating diagnostics and troubleshooting. A later

transition to **Rapid Adiabatic Passage (RAP)**-based readout (see section 3.5.4) improved data robustness, enabling a full heating rate characterization in ~ 2 min. However, **RAP** readout exhibited non-monotonic behavior for longer measurement times (~ 200 ms) not expected for a thermal state; we attribute this to either improper calibration of the **RAP** pulse, or to non-thermal heating (from e.g. background-gas collisions or coherent driving).

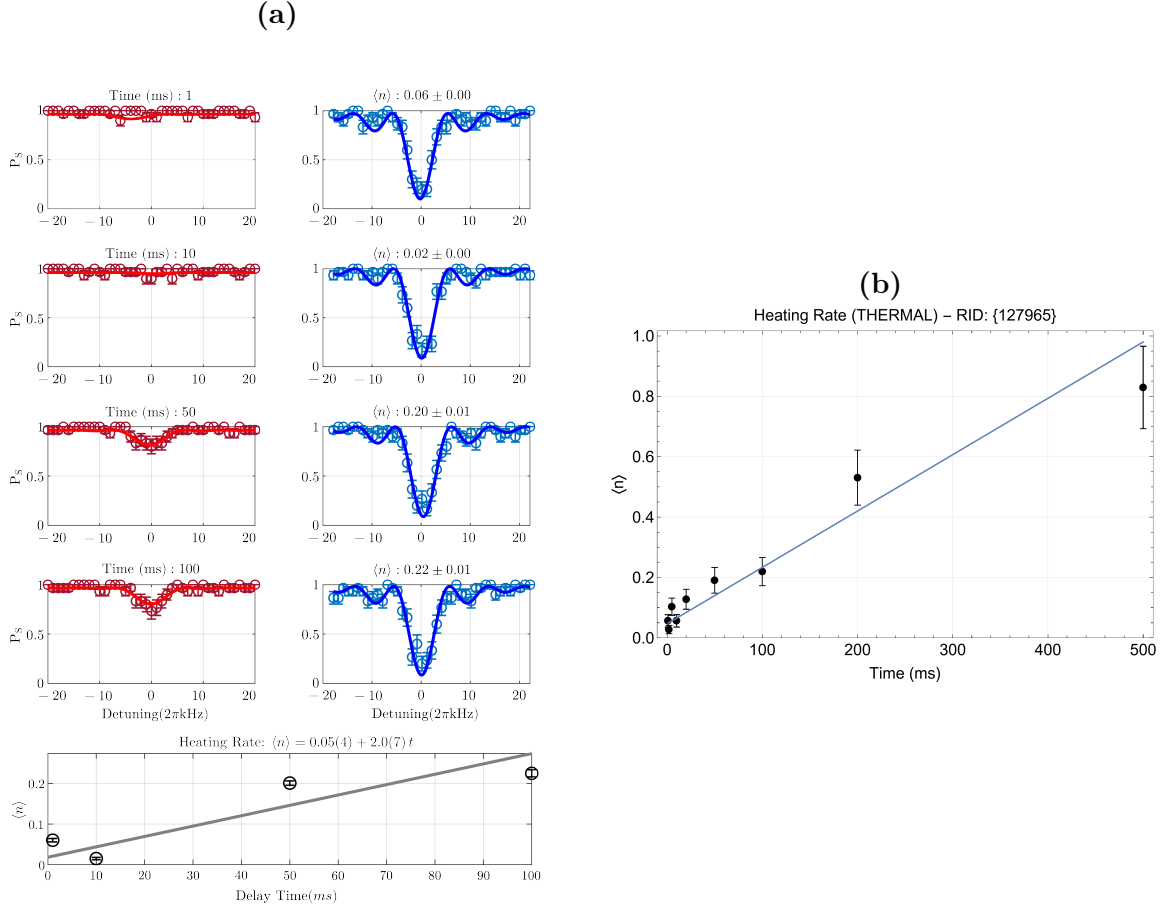


Figure 3.9: **Motional heating rate measurement protocols.**

(a) Heating rate measured with the **Sideband Ratio (SBR)** protocol. **RSB** and **BSB** sideband spectra are acquired at each delay time for robust extraction of the mean phonon number $\bar{n}$ (eq. (3.41)). (b) Heating rate measured with a **Rapid Adiabatic Passage (RAP)** pulse on the **Red Sideband**. As **RAP** is inherently robust to detuning and pulse errors, it allows single-shot measurement at each delay time, improving data acquisition efficiency. Linear fits give heating rates $\dot{n} = 2.0(7)$ quanta/s (a) and $\dot{n} = 1.9(3)$ quanta/s (b), in excellent agreement; both measurements use the radial **RF** mode.

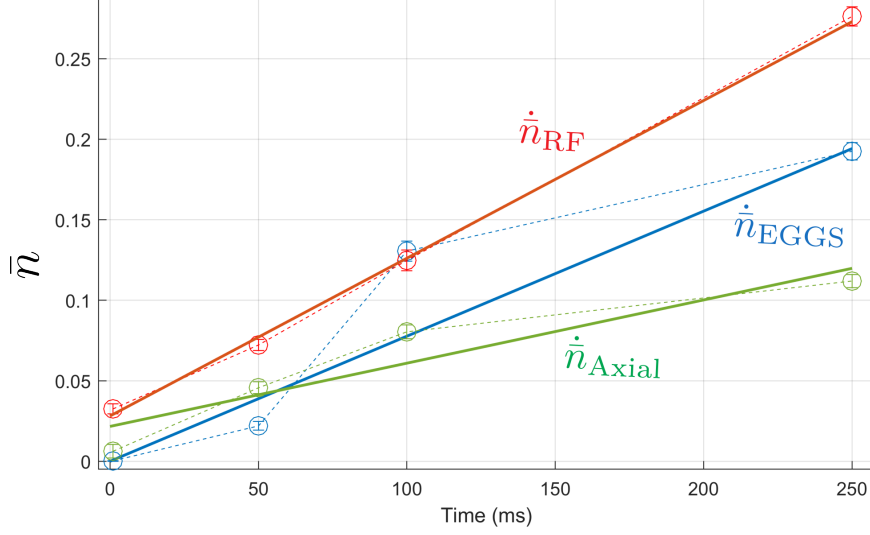


Figure 3.10: **Characterization of motional heating rates on all modes.**

Heating rates $\dot{\bar{n}}_{\text{EGGS}}, \dot{\bar{n}}_{\text{RF}}, \dot{\bar{n}}_{\text{Axial}} = (0.72(9), 0.98(8), 0.39(21)) \text{ s}^{-1}$ (blue, red, green, respectively) measured using an [Sideband Ratio](#) protocol. Solid lines are linear fits to the data; dashed lines are point-to-point guides.

Current Performance Following these improvements, the heating rates on each mode were $\dot{\bar{n}}_x, \dot{\bar{n}}_y, \dot{\bar{n}}_z = (0.72(9), 0.98(8), 0.39(21)) \text{ s}^{-1}$ (Figure 3.10). This is comparable to other traps of similar geometry and size:

- $\dot{\bar{n}}_z = 1.3(2) \text{ s}^{-1}$ for $r_0, z_0 = 555 \mu\text{m}, 4.5 \text{ mm}$ and $\omega_z = 1 \times 2\pi \text{ MHz}$ (room temperature, 2017 Innsbruck blade trap [[Gug17](#)])
- $\dot{\bar{n}}_z = 2.56(13) \text{ s}^{-1}$ for $r_0, z_0 = 800 \mu\text{m}, 7.3 \text{ mm}$ and $\omega_z = 1.2 \times 2\pi \text{ MHz}$ (room temperature, 2008 Innsbruck blade trap [[Ben08](#)]).
- $\dot{\bar{n}}_z = 2.14(16) \text{ s}^{-1}$ for $r_0 \approx 120 \mu\text{m}$ and $\omega_r = 1.1 \times 2\pi \text{ MHz}$ at $T < 20 \text{ K}$ (Innsbruck cryogenic surface trap, trap from Häffner group [[Bra16](#)]),
- $\dot{\bar{n}}_r = 5 \text{ s}^{-1}$ for $r_0 \approx 180 \mu\text{m}$ and $\omega_r = 4.166 \times 2\pi \text{ MHz}$ at $T \approx 4.7 \text{ K}$ ($^{171}\text{Yb}^+$ Monroe Group cryogenic blade trap [[Pag+18](#)]),
- $(\dot{\bar{n}}_x, \dot{\bar{n}}_y, \dot{\bar{n}}_z) = (1.9(3), 0.7(2), 2.3(1)) \text{ s}^{-1}$ for $z_0 = 900 \mu\text{m}$ and $(\omega_x, \omega_y, \omega_z) = (2.2, 2.5, 0.7) \times 2\pi \text{ MHz}$ ($^9\text{Be}^+$ [PTB CryPTEx](#) cryogenic blade trap [[Leo+19](#)]),

3.4.2 Motional Dephasing

Pure dephasing is described by the Lindblad operator $\hat{L}_\phi = \sqrt{\Gamma_\phi} \hat{a}^\dagger \hat{a}$ — i.e. the phonon number operator [Tur+00a,00b]. As $\hat{n} = \hat{a}^\dagger \hat{a}$ is the generator of rotations in phase space, dephasing can intuitively be understood as noisy rotations of the motional state which “smear” the wavepacket rotationally while preserving its energy. Under the Lindbladian (eq. (3.30)) with pure dephasing, the density matrix in the Fock basis evolves as

$$\rho_{nm}(t) = \rho_{nm}(0) e^{-(n-m)^2 \Gamma_\phi t/2}. \quad (3.36)$$

The energy and thus the phonon number distribution are preserved since Fock states are eigenstates of $\hat{n}$, while the off-diagonal coherences ρ_{nm} decay proportionally to $(n - m)^2$. Once fully dephased, the coherences vanish, leaving the density matrix ρ diagonal in the Fock basis and rotationally symmetric in phase space — in this sense, the coherences can be understood to encode the directionality of the motional state. A notable feature of dephasing is that it is a *non-Gaussian* quantum channel [GOP11,AMM20]: a Gaussian input state (e.g. a coherent state) is mapped to a non-Gaussian output state; this makes it difficult to describe dephasing for arbitrary states analytically [AMM20,HLW24] [GRM22,FRG25] and few closed-form results exist beyond those derived in [Tur+00a,00b].

Experimental Implementation

Measurement of motional dephasing is most natural with superposition states [DS07,Tur+00a], where the coherences have a clear interpretation — they encode the ability to interfere the two components — and can be mapped linearly onto spin for readout via the geometric phase [Tur+00a,00b]. For non-superposition states (e.g. coherent states), phase information must instead be extracted via a displacement that maps it onto a motional amplitude, which complicates extraction of the coherences as analytical expressions for the contrast decay do not, in general, exist.

To characterize dephasing, two timescales are of interest: T_2^* , the bare dephasing time in the absence of dynamical decoupling, and T_2 , the white-noise-limited dephasing time. T_2^* is extracted from a motional Ramsey sequence and T_2 from a motional spin-echo sequence (see section 3.3.3 for details). In the spin-echo sequence, the displacement direction is reversed midway through the interrogation to refocus any accumulated phase from constant detunings (i.e. motional dynamical decoupling):

$$\hat{D}(\alpha) \rightarrow \tau_a \rightarrow \hat{D}(-2\alpha) \rightarrow \tau_a \rightarrow \hat{D}(\alpha). \quad (3.37)$$

If the reversals are sufficiently fast, coherently oscillating detunings (e.g. technical noise from the AC line or signal spurs) appear quasi-static and are likewise refocused. Stochastic frequency fluctuations, however, are not reversed, enabling the spin-echo sequence to isolate reversible inhomogeneous dephasing from irreversible pure dephasing. However, reliable T_2/T_2^* measurements must be “deconvolved” from the effects of slow drift and transient glitches, which can reduce signal contrast to masquerade as dephasing (see section 6.3.4 and Figure 6.14 for further details).

As dephasing is itself a diagnostic for other systematics (e.g. trap frequency drift, electrical noise), speedy data acquisition is of cosmic, celestial importance. An initial (v1) protocol measured T_2^* by extracting contrasts with a Ramsey frequency scan for each delay time in separate runs (e.g. frequency scan for $T = 0.2$ ms first, then $T = 0.8$ ms, etc.). An improved implementation (v2) used a Free-induction-decay-style protocol (enabled by improved waveform compilation and experiment sequencing) to characterize T_2/T_2^* by scanning the Ramsey delay time T at fixed phase; both protocols are compared in Figure 3.11. This reduced the overhead of full T_2/T_2^* characterization from two hours (for v1) to two minutes (for v2) and enabled further “laboratory synergy” (e.g. rapid iteration for diagnostics/testing, well-documented baselines for day-to-day variances in T_2/T_2^*).

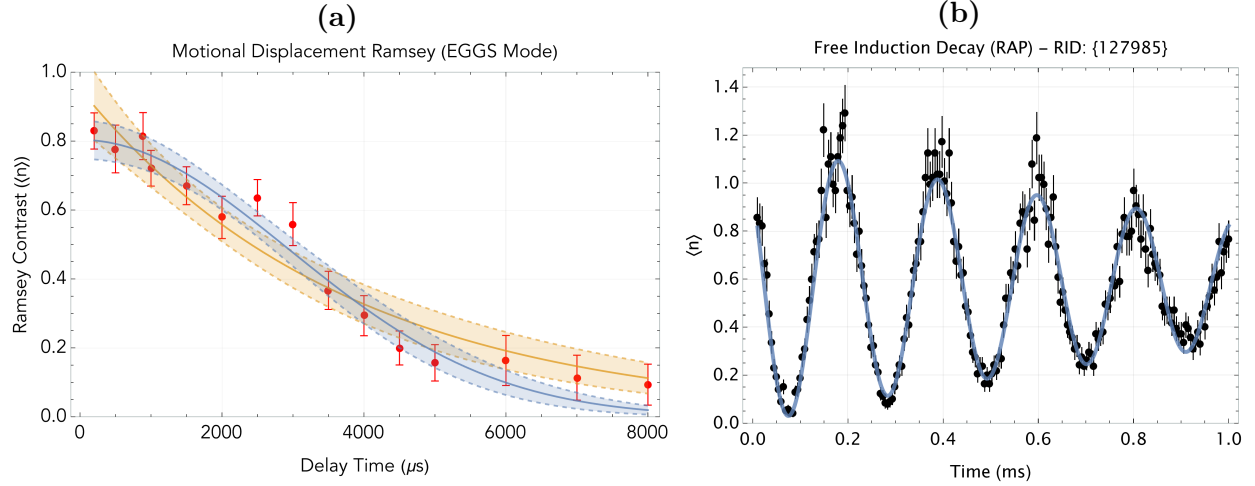


Figure 3.11: **Motional dephasing measurement protocols.**

(a) Original (v1) protocol: the Ramsey contrast is measured by a frequency scan at each delay time, and the resulting contrast decay (red points) are fitted to extract T_2^* . Gaussian (blue) and exponential (orange) profiles are fitted (solid lines) to the data; shaded bands are 95% confidence bands of the fits. (b) **Free-Induction Decay (FID)** (v2) protocol: phase and frequency are fixed as the delay time is scanned to recover the contrast envelope in a single acquisition. Both measurements use the radial EGGS mode.

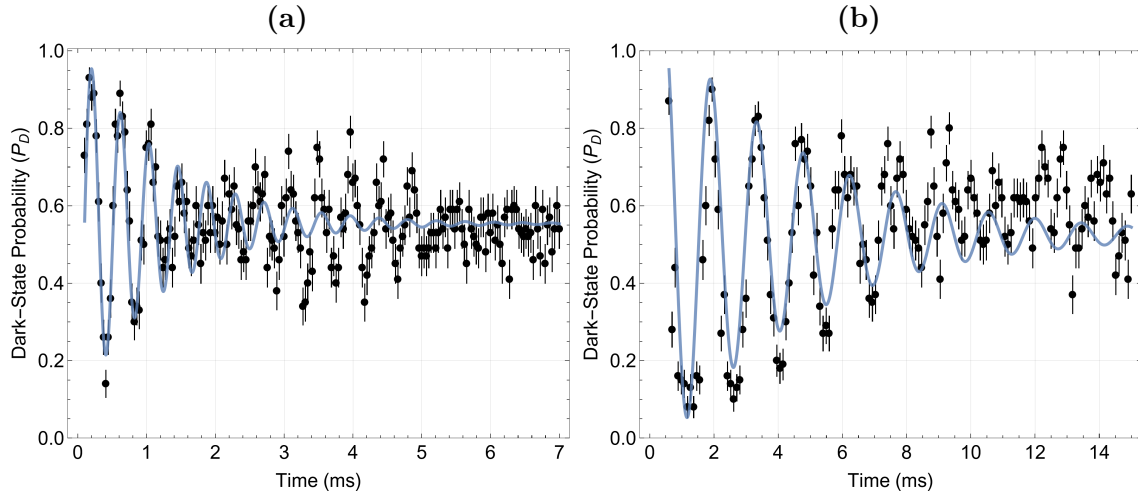


Figure 3.12: **Motional spin-echo removes coherent contributions to dephasing.**

Dephasing is characterized using a **Free-Induction Decay (FID)** protocol with a primitive trajectory (a) and with five motional echoes (b) on the radial RF mode. (a) Without motional spin-echoes, the contrast decay exhibits a beatnote characteristic of coherent, low-frequency noise. (b) Inserting five motional echoes refocuses the slowly varying detuning, removing the beatnote and leaving the residual decay due to irreversible pure dephasing.

Improvement Attempts

Trap RF Drive Chain (see section 5.3.2 for a complete technical overview) Optimization of the trap RF amplitude servo was attempted by tuning servo parameters to improve

the dephasing rate; however, this was not found to have an effect — for this reason, the high-stability servo setpoint (following [Joh+16]) was deprecated. Improvements to the trap RF drive chain were similarly ineffectual. The sole effective change was the collocation of the trap RF drive chain on a single board, which had the further benefit of reducing cable lengths — this improved the radial dephasing rate from $T_2 \sim 1.2 \text{ ms} \rightarrow 3 \text{ ms}$.

Secular Frequency Dependence In a white-noise-limited regime, the dephasing rate increases with trap voltage (which sets the secular frequency) since a given fractional amplitude fluctuation $\delta V_{\text{RF}}/V_{\text{RF}}$ produces larger absolute frequency shift at higher ω_0 [Har13]. Tests revealed no significant dependence of the dephasing rate on the secular frequency, which suggested that our dephasing was not white-noise-limited (i.e. the dephasing was independent of the amplitude noise on the trap RF).

Current Performance A motional free-induction-decay measurement on the radial modes revealed the presence of coherent noise, evident in the beatnote of Figure 3.12(a), which was removed by a motional spin-echo sequence (Figure 3.12(b)). This coherent, low-frequency noise explains the poor progress in improving the T_2 time, as interventions were made to address broadband white noise. A bare dephasing time $T_2^* \approx 2 \text{ ms}$ was measured on the radial modes (Figure 3.13), which improves to $T_2 \approx 4 \text{ ms}$ with five echoes. Surprisingly, a bare $T_2^* = 150(27) \text{ ms}$ (Figure 3.13) is measured on the axial mode, suggesting that the excess radial dephasing is due to a radial-specific noise source.

3.5 Motional Detection

In quantum optics, homodyne and heterodyne detection provide direct access to the quadratures of a bosonic mode. No analogous tool exists for the bosonic mode of a trapped ion: all motional information must instead be extracted indirectly by coupling to spin (see section 4.1), which becomes something of a Faustian bargain — while it enables highly nonclassical control

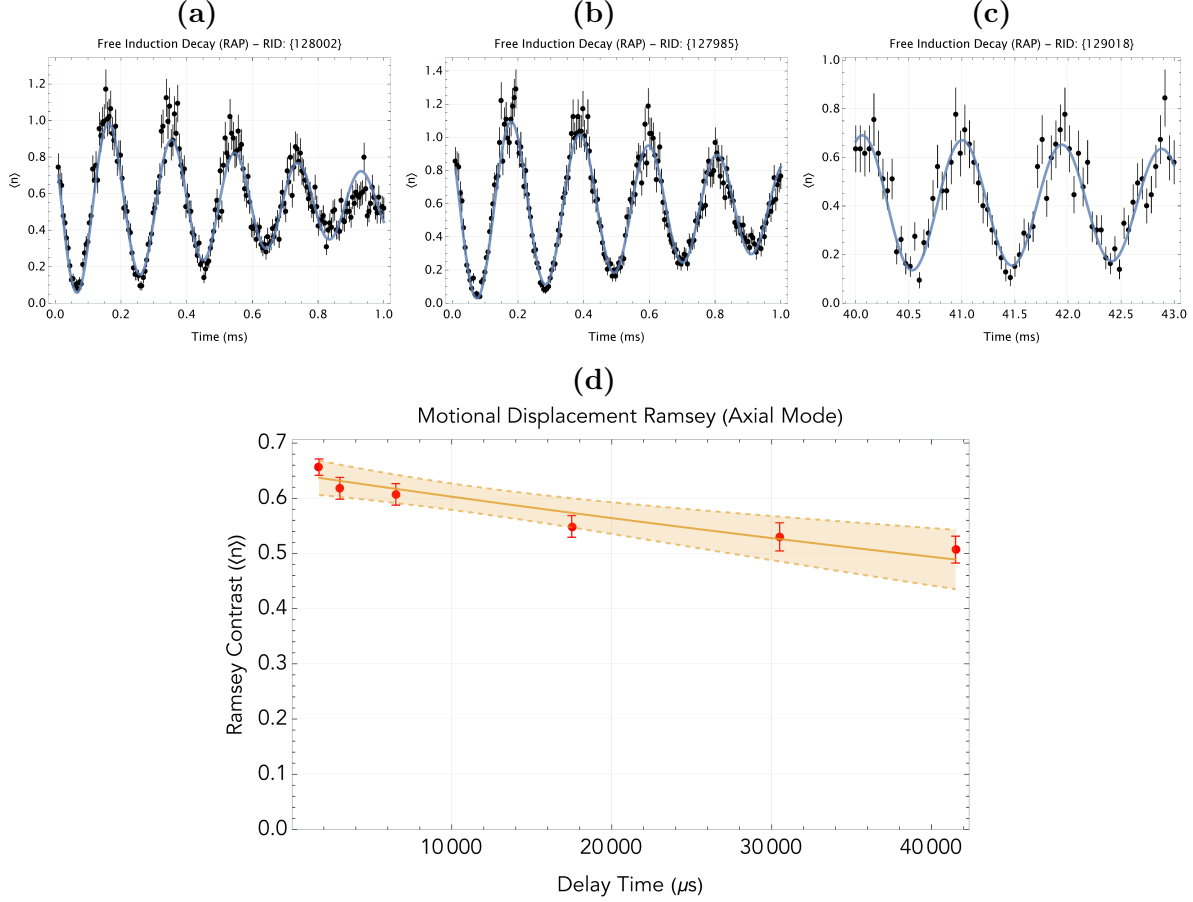


Figure 3.13: **Bare motional dephasing (T_2^*) on all modes with a Free-Induction Decay protocol.**

Bare dephasing times $T_2^* = 717(52) \mu\text{s}$ (RF mode, (a)) and $T_2^* = 1084(78) \mu\text{s}$ (EGGS mode, (b)) are extracted as time constants of damped cosines fitted to the data. (c) Axial mode dephasing is too slow to scan continuously, so the contrast is instead sampled at a series of hold times. (d) An exponential fit to extracted contrasts (e.g. (c)) yields $T_2^* = 150(27) \text{ ms}$ on the axial mode.

of the QHO, it is also the *only* practical means of probing motion, making motional detection considerably more involved than in quantum optics. Here, we survey the techniques involved in this delectably intricate challenge of motional detection.

3.5.1 Sideband Ratio (SBR)

Perhaps the simplest approach to motional state characterization is to directly probe the first Red Sideband (RSB). As a diagnostic, interpretation of the RSB can be simple: if cooled

to the ground motional state, [RSB](#) excitation is impossible, such that any excitation must indicate imperfect preparation of the ground state. However, the [RSB](#) excitation spectrum depends on the interrogation time t and the Rabi frequency Ω_0 , which complicates extraction of $\bar{n}$ without precise calibration and makes the measurement subject to drift.

Instead, the more robust (and canonical) approach for thermometry in the Lamb-Dicke regime is the [SBR](#) technique [[Lei+03](#),[Roo00](#)]. In this technique, identical probe pulses are applied on the red and blue sidebands to extract the excitation probabilities, which are then compared to extract a value for $\bar{n}$. Measurement of the [Blue Sideband \(BSB\)](#) here serves as an effective control measurement to eliminate the common-mode dependence on the interrogation parameters (i.e. interrogation time and Rabi frequency).

Sideband Excitation Given an ion with motional Fock state distribution $\{p_n\}$, the [RSB](#) and [BSB](#) excitation probabilities following a probe pulse are:

$$P_{\text{RSB}}(t) = \sum_{n=1}^{\infty} p_n \sin^2\left(\frac{\Omega_{n,n-1} t}{2}\right), \quad P_{\text{BSB}}(t) = \sum_{n=0}^{\infty} p_n \sin^2\left(\frac{\Omega_{n,n+1} t}{2}\right), \quad (3.38)$$

where $\Omega_{n,n+1} = \eta\sqrt{n+1}\Omega_0$ and $\Omega_{n,n-1} = \eta\sqrt{n}\Omega_0$ are the sideband Rabi frequencies in the Lamb-Dicke regime (to first order) [[Lei+03](#)]. For a thermal state with mean phonon number $\bar{n}$, the Fock state distribution is:

$$p_n = \left(\frac{\bar{n}}{\bar{n}+1}\right)^n \frac{1}{\bar{n}+1} = p_{n-1} \left(\frac{\bar{n}}{\bar{n}+1}\right). \quad (3.39)$$

Using eq. (3.39) and the convenient notation $r \equiv \bar{n}/(\bar{n}+1)$, the [RSB](#) can be expressed in terms of the [BSB](#):

$$\begin{aligned} P_{\text{RSB}}(t) &= \sum_{n=0}^{\infty} p_{n+1} \sin^2\left(\frac{\Omega_{n+1,n} t}{2}\right) = r \sum_{n=0}^{\infty} p_n \sin^2\left(\frac{\Omega_{n,n+1} t}{2}\right) \\ &= r \cdot P_{\text{BSB}}(t). \end{aligned} \quad (3.40)$$

The ratio of the sideband probabilities can therefore be used to extract $\bar{n}$ independently of t and Ω_0 :

$$R \equiv \frac{P_{\text{RSB}}(t)}{P_{\text{BSB}}(t)} = \frac{\bar{n}}{\bar{n} + 1} \Rightarrow \bar{n} = \frac{R}{1 - R}. \quad (3.41)$$

SBR in Practice The uncertainty of eq. (3.41) is:

$$\delta\bar{n} = \frac{1}{(1 - R)^2} \delta R = (\bar{n} + 1)^2 \delta R, \quad (3.42)$$

where δR is the uncertainty of the sideband ratio. As $\bar{n}$ increases, sensitivity is lost due to the vanishing distinguishability between the red and blue sidebands as $R \rightarrow 1$; in practice, sideband ratio thermometry is reliable only for $\bar{n} \lesssim 3$. This reduced sensitivity is evident in the flattening of the $R \rightarrow \bar{n}$ conversion at higher $\bar{n}$ in Figure 3.14(a). Additionally, SPAM infidelities nontrivially modify eq. (3.41). Experimentally, these can be compensated by scanning the probe detuning δ and fitting generic sinc profiles to the resulting spectra, from which the true excitation probabilities are extracted.

The independence of eq. (3.41) from the Rabi frequency and interrogation time arises from the geometric nature of the thermal distribution and does not hold in general: for e.g. coherent and squeezed states, the ratio $P_{\text{RSB}}(t)/P_{\text{BSB}}(t)$ is no longer time-independent nor analytically invertible for $\bar{n}$. However, the sideband ratio can still be converted to $\bar{n}$ for non-thermal states by using numerically calculated $R \rightarrow \bar{n}$ calibration curves (Figure 3.14(a)). As the $R \rightarrow \bar{n}$ curve is no longer unconditionally injective for these states, care must be taken to ensure that R remains within the injective region; for coherent states, this is typically $R \lesssim 0.8$.

3.5.2 Blue Sideband (BSB) Rabi Oscillations

Extraction of the full Fock-state distribution of a motional state is achieved by examining the tea leaves of Blue Sideband (BSB) Rabi oscillations, and is referred to as Fock State

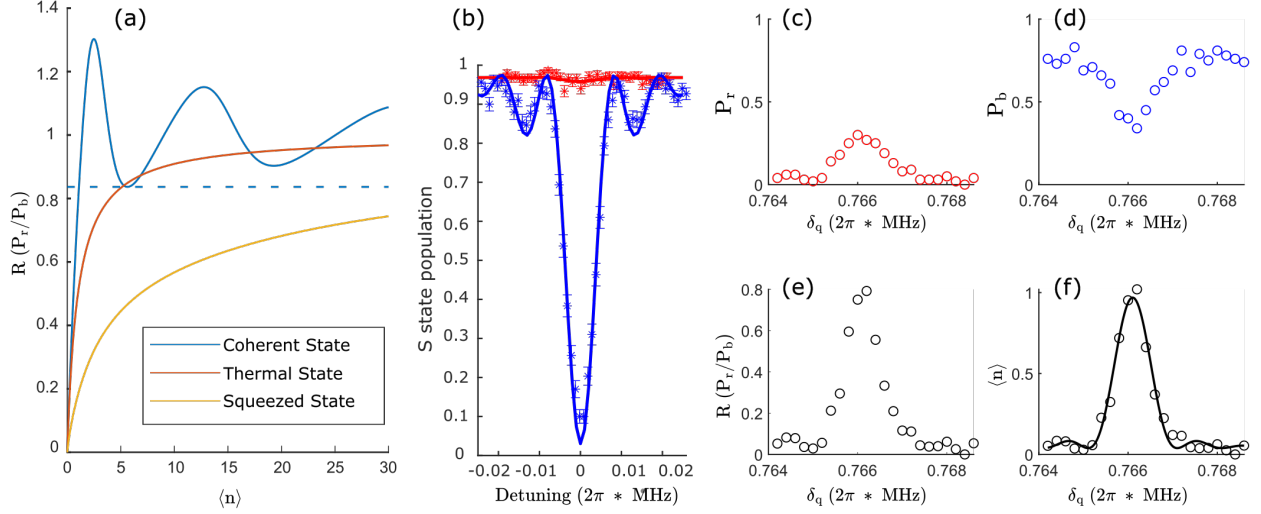


Figure 3.14: **Sideband Ratio (SBR) readout protocol.**

(a) Numerically computed dark-state probability P_D versus coherent displacement $|\alpha|$, used to convert a measured sideband ratio R into a mean phonon number $\bar{n}$ for non-thermal states. The monotonic rise saturates at large $|\alpha|$, illustrating the loss of sensitivity (and eventual loss of injectivity) as $R \rightarrow 1$ that restricts **SBR** thermometry to $\bar{n} \lesssim 3$.

Decomposition. Until fairly recently [FH20], this technique underlay the bulk¹⁰ of experimental implementations of motional state tomography [Lei+96]. In the resolved-sideband regime, each Fock component $|n\rangle$ undergoes **BSB** Rabi oscillations at a characteristic frequency $\Omega_{n,n+1} = \eta\sqrt{n+1}\Omega_0$. As the **Jaynes-Cummings Hamiltonian** does not mix Fock components, the total probability is then a linear combination of the oscillations:

$$P(t) = \sum_{n=0}^{n_{\max}} p_n f_n(t), \quad f_n(t) = \frac{A}{2} \left(1 - e^{-\gamma t \sqrt{n+1}} \cos \left(\Omega_0 \eta \sqrt{n+1} t \right) \right), \quad (3.43)$$

where A is the oscillation amplitude and γ is the decoherence constant.

Experimentally, the parameters A , $\Omega_0\eta$, and γ are first extracted by a nonlinear fit of eq. (3.43) to **BSB** oscillations of an ion cooled near the motional ground state. An **FFT** can be used to extract fast and robust initial guesses for $\Omega_0\eta$. **BSB** oscillations are then measured on the target state to produce the observation vector $(P(t_1), P(t_2), \dots)$. By linearity in $\{p_n\}$,

¹⁰Some other methods exist [LD97], though these are not commonly used in experiment.

these measurements can be cast as a linear problem:

$$\begin{pmatrix} P(t_1) \\ P(t_2) \\ \vdots \\ P(t_N) \end{pmatrix} = \begin{pmatrix} f_0(t_1) & f_1(t_1) & \cdots & f_{n_{\max}}(t_1) \\ f_0(t_2) & f_1(t_2) & \cdots & f_{n_{\max}}(t_2) \\ \vdots & \vdots & & \vdots \\ f_0(t_N) & f_1(t_N) & \cdots & f_{n_{\max}}(t_N) \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \\ \vdots \\ p_{n_{\max}} \end{pmatrix}, \quad (3.44)$$

which is solved via ordinary least squares to extract the populations $\{p_n\}$ ¹¹ [Mee+96].

Populations are left unconstrained and small negative values are permitted, as enforcing positivity biases the estimated p_n [Mee+96,Kie15].

Practicalities At high n , adjacent Fock states oscillate at frequencies separated by $\Delta\Omega_n \approx \eta\Omega_0/2\sqrt{n}$. Resolution is limited by the decoherence-limited interrogation time of the highest Fock state, $T = 1/(\gamma\sqrt{n})$, which sets a maximum resolvable Fock state:

$$n_{\max} \sim \frac{\eta\Omega_0}{4\pi\gamma}. \quad (3.45)$$

For a decoherence-limited system, increasing Ω_0 (e.g. by raising the laser power) further separates the oscillation frequencies and directly raises $n_{\max}$. However, if coherence is instead limited by laser amplitude fluctuations, the number of observable oscillations is effectively fixed and increasing Ω_0 yields no improvement [Roo00]. In addition to the Fourier limit, the sample interval must be sufficiently small to satisfy the Nyquist criterion, i.e. $\Delta t < \pi/(\eta\sqrt{n_{\max}+1}\Omega_0)$.

In our experiment, we observe $1/\gamma \approx 4 \text{ ms}$ ¹² and $\eta\Omega_0/2 \approx 10 \times 2\pi \text{ kHz}$, suggesting $n_{\max} \approx 40$. Spectator modes were found to be a significant source of decoherence for the sideband oscillations, as noted by [Rog+25a]. Sufficient coherence for reasonable decomposition

¹¹While a discrete Fourier transform can in principle extract the Fock-state populations, the fidelity suffers because the Rabi frequencies $\Omega_{n,n+1} = \Omega_0\eta\sqrt{n+1}$ are nonlinear in n and the resulting bases are non-orthogonal due to exponential decay.

¹²For Rabi oscillations.

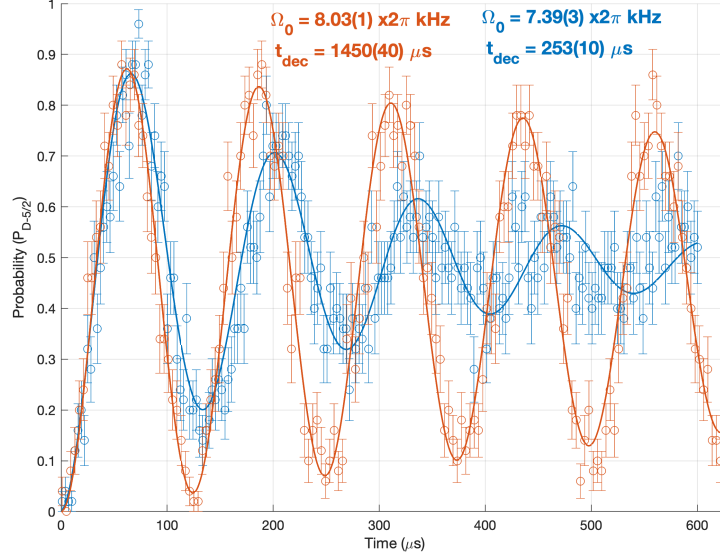


Figure 3.15: **Effect of spectator modes on sideband Rabi oscillations.**

Blue Sideband Rabi oscillations on the radial EGGs mode cooled to $\bar{n} \approx 0.05$ with Sideband Cooling applied to the EGGs mode alone (blue) and to all three motional modes (orange); coherence times are extracted as time constants of fitted damped cosines. Cooling all motional modes extends the coherent interrogation from $t_{\text{dec}} = 253(10) \mu\text{s}$ (single-mode) to $1450(40) \mu\text{s}$ (all modes).

($n_{\text{max}} \sim 10$) required sideband cooling of all motional modes.

3.5.3 Carrier Rabi Oscillations

The unique domain of carrier Rabi flopping is the intermediate regime of $\bar{n} \sim 30$ –100, where its reduced sensitivity to motion — scaling as $\eta^2 \bar{n}$ rather than $\sqrt{\bar{n}}$ — allows operation outside the range of blue-sideband Rabi flopping or sideband thermometry [Roo00, Gli23]. From [Roo00], the excited-state population P_D under carrier Rabi oscillations for a thermal state is given by an ensemble average over Fock-state contributions:

$$P_D(t) = \frac{1}{2} (1 - \text{Re}(Q(t))) \quad Q = \frac{\exp(2i\Omega_0 t)}{1 + \bar{n} (1 - \exp(-i2\Omega_0 t \eta^2))}. \quad (3.46)$$

Equation (3.46) generalizes to k motional modes via the substitution $\eta^2 \bar{n} \rightarrow \sum_k \eta_k^2 \bar{n}_k$. Given the beam geometries outlined in chapter 5 ($\cos \theta_x, \cos \theta_y, \cos \theta_z = (1/2, 1/2, 1/\sqrt{2})$) and secular frequencies $(\omega_x, \omega_y, \omega_z) = (1.3, 1.5, 0.7) \times 2\pi \text{ MHz}$, the combined quantity reduces to

$$\sum_k (\eta_k^2 \bar{n}_k) \approx 1.25 \eta_z^2 \bar{n}_z.$$

3.5.4 Fock-State Overlap

[Rapid Adiabatic Passage \(RAP\)](#) on the motional sidebands enables direct readout of Fock-state populations that is inherently robust to pulse area errors, providing efficient estimation of coherent displacements [\[Wol+19\]](#). The technique was introduced for sideband transitions by [\[Wat+11\]](#), applied to deterministic phonon arithmetic [\[Um+16\]](#) and Dicke-state creation [\[Lec16\]](#), and developed into a systematic Fock-state overlap protocol by [\[Wol+19\]](#), a variant of which is implemented here.

In practice, we use almost exclusively the unenhanced ($n = 0$) overlap measurement. Higher Fock states ($n \geq 1$) offer improved sensitivity through quantum-enhanced metrological gain (section [3.5.4](#)), but each additional Fock state demands extra calibrations and substantially longer pulse sequences, increasing susceptibility to motional decoherence. Despite this restriction, the $n = 0$ overlap is a powerful diagnostic: beyond its robustness to pulse-area errors, it provides near-optimal sensitivity at small displacements (section [3.5.4](#)) and, as a single-shot binary estimator, is more efficient than the [SBR](#) technique, which requires separate interrogation of the [RSB](#) and [BSB](#) and a subsequent ratio calculation. This technique further addresses the non-injectivity of the [Sideband Ratio](#) technique for coherent states (see section [3.5.1](#)): the $n = 0$ overlap probability $P_0(\alpha) = e^{-|\alpha|^2}$ is a monotonic function of $|\alpha|$ and is therefore injective and can be inverted to extract α .

Principle In this technique, an applied displacement $\hat{D}(\alpha)$ is measured by examining the resulting overlap of the displacement applied to a Fock state. This results in an overlap probability:

$$P_n(\alpha) = \left| \langle n | \hat{D}(\alpha) | n \rangle \right|^2 = e^{-|\alpha|^2} \left(L_n^0(|\alpha|^2) \right)^2 \quad (3.47)$$

where L_n^0 is the n^{th} -order Laguerre polynomial [\[Wol+19, Lei+96\]](#). This measurement is best understood in phase space as an overlap integral between the Wigner functions for

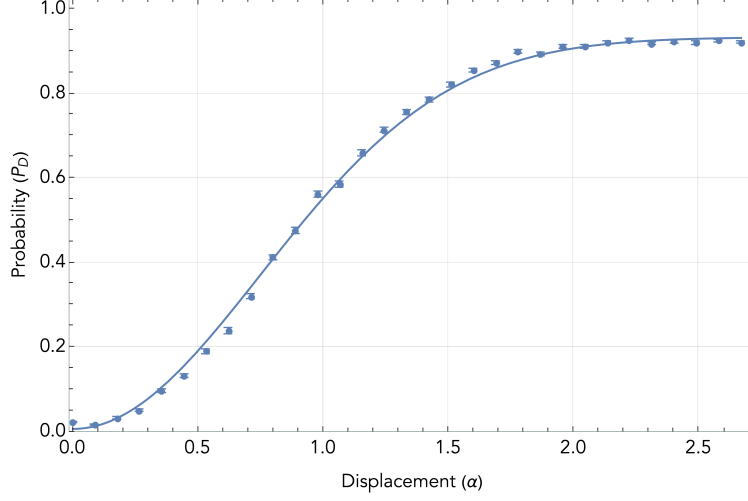


Figure 3.16: $|n = 0\rangle$ **Fock state overlap readout of a coherent state.**

Motional overlap of the $|n = 0\rangle$ Fock state with a coherent state $|\alpha\rangle$; measurements use the RF mode. The analytical form $P_0(\alpha) = C + A \times e^{-|\alpha|^2}$ is fitted to the data (solid line) (see eq. (3.47)) with free parameters A, C to account for protocol infidelities; error bars represent one standard error and are too small to see. The monotonic decay reflects the shrinking overlap of the two Gaussian wavepackets as they separate in phase space, making P_0 injective and a directly invertible estimator of $|\alpha|$.

$\hat{D}(\alpha) |n\rangle$ and $|n\rangle$ [Wol+19]. For the ground $|n = 0\rangle$ state, the Wigner functions are Gaussian wavepackets centered at the origin and at α for $|n = 0\rangle$ and $\hat{D}(\alpha) |n = 0\rangle$, respectively, such that the overlap decays monotonically with α — this underlies the exponential in eq. (3.47) (Figure 3.16). For higher, *nonclassical* Fock states $n \geq 1$, the Wigner function contains negative regions, which can result in net-zero overlap even before the wavepackets fully separate, producing the non-monotonic fringes¹³ (see Figure 11.4). This sharpens the dependence of eq. (3.47) on α and enhances the Fisher information (and thus the sensitivity) beyond that classically achievable (see Figure 11.4).

Additionally, the rotational symmetry of the Fock states (which can be seen by the dependence of eq. (3.47) only on $|\alpha|$) implies an insensitivity to the phase of the displacement, making the Fock states optimal when the phase is unknown (or unknowable) [Wol+19, WS21, FRG25]¹⁴.

¹³Analytically, the fringes arise from the Laguerre polynomials $L_n^0(x)$, which have n zeros.

¹⁴Interestingly, optimality for sensing phase-insensitive displacements is not the unique purview of the Fock state — despite their phase-space asymmetry, squeezed states provide equally optimal gain for phase-insensitive displacements [GRM22].

Sensitivity We consider the estimation efficiency of the overlap measurement. For a binary-outcome experiment, the (classical) Fisher information is

$$F_n(\alpha) = \frac{(\partial_\alpha P_n(\alpha))^2}{\text{var}(P_n)} = \frac{(\partial_\alpha P_n(\alpha))^2}{P_n(1 - P_n)}, \quad (3.48)$$

where the variance uses the Bernoulli distribution $\text{var}(P_n) = P_n(1 - P_n)$. We evaluate this explicitly for $n = 0$:

$$\begin{aligned} F_0(\alpha) &= \frac{4\alpha^2}{e^{|\alpha|^2} - 1} \\ &\approx 4 \quad (\text{for } \alpha \sim 0). \end{aligned} \quad (3.49)$$

This saturates the [Quantum Cramér-Rao Bound \(QCRB\)](#), defined by the [Quantum Fisher information](#) $\mathfrak{F} = 4$ [[Wol+19](#)], and is therefore ideal in the small-displacement limit. For higher Fock states, the [QFI](#) is [[Wol+19,FRG25](#)]:

$$\mathfrak{F}_n = 4(2n + 1). \quad (3.50)$$

The metrological gain thus scales linearly with the Fock number: the $n = 1$ state offers a factor of $\times 3$ improvement in [QFI](#) over the vacuum state, or a ~ 4.8 dB of enhancement (see [Figure 11.4](#) and [table 11.1](#)).

Implementation

RAP Pulses A frequency-chirped [RSB](#) pulse adiabatically transfers $|\downarrow, n\rangle \rightarrow |\uparrow, n - 1\rangle$ for all n above a threshold set by the adiabaticity condition [[Wun+07](#)]:

$$\dot{\Omega}_{n,n-1}(t) \ll \Omega_{n,n-1}^2 \quad (3.51)$$

where $\Omega_{n,n-1}(t) = (\Omega_{n,n-1}(t), \delta(t))$ is the generalized Bloch vector comprising the sideband

Rabi frequency $\Omega_{n,n-1}(t)$ and the instantaneous detuning $\delta(t)$. This adiabaticity condition requires the rate of change of Ω to be small relative to its squared magnitude, ensuring that the ion state is entrained by the pulse throughout the chirp [Wun+07]. When satisfied, the transfer efficiency exceeds 99 % over a broad plateau of chirp parameters, in stark contrast to a resonant π -pulse, which requires precise calibration of both pulse area and frequency.

The frequency chirp is generated by the digital ramp generator on an AD9910 DDS channel¹⁵, while the shaped amplitude envelope is produced by RAM modulation on the same AD9910 channel (see section 5.4.1). A $\sin^2$ (Hann) or Blackman window is typically used for the amplitude envelope, though the specific shape is not critical insofar as it ramps smoothly on and off to suppress off-resonant excitation¹⁶. For the $n = 0$ overlap, a single RAP pulse on the RSB suffices to realize the full protocol: the RAP pulse transfers all population from $|\downarrow, n\rangle \rightarrow |\uparrow, n-1\rangle$ state while isolating $|\downarrow, n=0\rangle$ for subsequent fluorescence detection to yield P_0 .

Higher Fock-State Overlap In general, overlap readout requires isolation of the target population p_n in a state with a means of detection (i.e. the $^2S_{1/2}$ state). The general procedure used in this experiment is adapted from [Wol+19] for our $^{40}\text{Ca}^+$ system — see Figure 3.18 for details. Each higher Fock state requires approximately four extra pulses (including the pulses used to generate the initial Fock state). For our system, three auxiliary states ($m_J = -3/2$, $+1/2$, and $+3/2$ of the $^2D_{5/2}$ manifold)¹⁷ are accessible, enabling a maximum detectable Fock state $|n=3\rangle$. To ensure high fidelity across the many pulses in the sequence, we converted *all* operations (Fock-state creation, shelving, carrier transitions) to RAP pulses, achieving $\sim 98\%$ fidelity per pulse. This reduces the operator overhead otherwise required to calibrate each pulse (transition frequencies, pulse times) and ensures uninterrupted high-fidelity operation

¹⁵Sinara/M-labs Urukul

¹⁶However, not all window functions are created equal, and “ramps smoothly on and off” is often open to interpretation. Gaussian windows, for example, are generally inappropriate: they do not start or finish at zero, resulting in poor suppression of spectral leakage. Window functions with improved spectral characteristics also come at a cost: their slower roll-on generally provides less pulse area per unit time, requiring longer pulses.

¹⁷The $|^2D_{5/2}, m_J = -1/2\rangle$ sublevel is not used due to insufficient Rabi frequency.

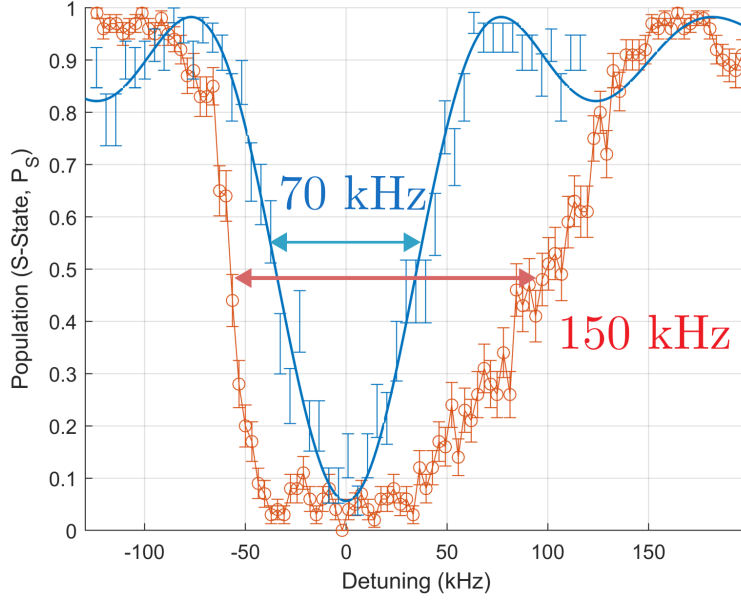


Figure 3.17: **Comparison between sideband excitation and Rapid Adiabatic Passage readout.**

Sideband spectroscopy on the axial **Blue Sideband** following **Sideband Cooling** to $\bar{n} \lesssim 0.1$, comparing direct excitation (blue) and **Rapid Adiabatic Passage (RAP)** (red); scans use a Rabi frequency $\Omega\eta \sim 2\pi \times 21.07(7)$ kHz. **RAP** yields a broad and flat transfer plateau characteristic of adiabatic transfer in contrast to the narrower sinc lineshape with direct excitation. A Stark shift from the nearby (~ 702 kHz) carrier transition results in spectral asymmetry for the **RAP** pulse.

for long-term experiments (see chapter 11).

Calibration Two calibrations are required: first the **RAP** transfer itself, then, for higher Fock states, the individual pulses in the protocol outlined above. **RAP** is calibrated by preparing the ion in the motional ground state via sideband cooling, then applying the **RAP** pulse on the **Blue Sideband (BSB)**¹⁸. The chirp range and pulse duration are adjusted iteratively until the pulse fidelity is sufficiently high ($\gtrsim 97\%$) and both curves produce the robust “plateau” regions characteristic of **Rapid Adiabatic Passage**; as a coarse initial guide, parameters can be estimated from the adiabaticity condition eq. (3.51). Since $\Omega_{n,n-1} = \Omega_0\eta\sqrt{n}$ increases with n , the **RAP** pulse becomes non-adiabatic for sufficiently high n and creates an additional source of infidelity. The pulse time should therefore be maximized to ensure

¹⁸The **BSB** Rabi frequency for $n = 0$ is equal to that on the **Red Sideband (RSB)** for $n = 1$, i.e. $\Omega_{0,1} = \eta\Omega_0 = \Omega_{1,0}$.

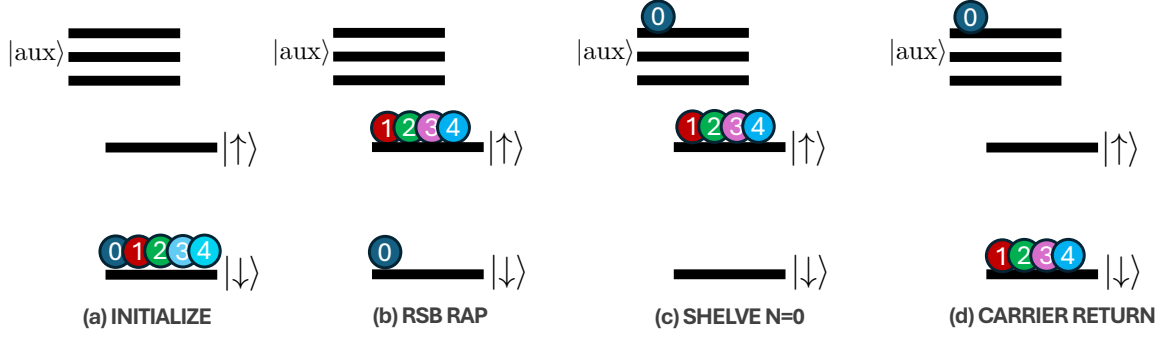


Figure 3.18: **Fock state overlap readout protocol.**

A series of sideband **Rapid Adiabatic Passage** and shelving pulses isolate a single Fock state population in $|\downarrow\rangle$ for state detection; auxiliary states used are the $m_J = -3/2, +1/2, +3/2$ sublevels of the $^2D_{5/2}$ manifold.

(a) The target motional state distribution is prepared.

(b) A **RAP** pulse on the **RSB** transfers $|\downarrow, n\rangle \rightarrow |\uparrow, n-1\rangle$, leaving p_0 in $|\downarrow\rangle$ — the protocol can be terminated here to detect the p_0 population (via e.g. state-selective fluorescence), or continued to detect higher p_n .

(c) The p_0 population is shelved into an auxiliary $^2D_{5/2}$ Zeeman sublevel via carrier π -pulse to remove it from subsequent dynamics.

(d) A carrier π -pulse returns $|\uparrow, n-1\rangle \rightarrow |\downarrow, n-1\rangle$ for the cycle to repeat from (b) until the target p_n is isolated.

sufficient margin to entrain the higher Fock states.

Following calibration, the population transfer fidelity should be measured as the center frequency of the **RAP** pulse is scanned (i.e. a linescan) to verify that the transfer region is broad and confirm the quality of the **RAP** pulse¹⁹ (Figure 3.17). On the axial mode, we typically find a base **BSB** Rabi frequency $\Omega_{0,1} \approx 20 \times 2\pi$ kHz, which enables a pulse time $t_{\text{RAP}} \approx 450 \mu\text{s}$ and a total chirp range $\delta_{\text{max}} - \delta_{\text{min}} \approx 280 \times 2\pi$ kHz. A **RAP** linescan revealed a 90% transfer efficiency range of ~ 102 kHz, and a **FWHM** linewidth of ~ 150 kHz. The asymmetric spectrum is due to the stark shift from the nearby (~ 700 kHz) carrier spin transition.

Practical Limitations The principal limitation of the **RAP** pulse itself is speed²⁰: while adiabaticity requires the sweep to be slow relative to $\Omega_{n,n+1}$, motional dephasing limits

¹⁹A well-calibrated **RAP** pulse produces a wide, flat transfer plateau; a narrow or peaked response indicates marginal adiabaticity.

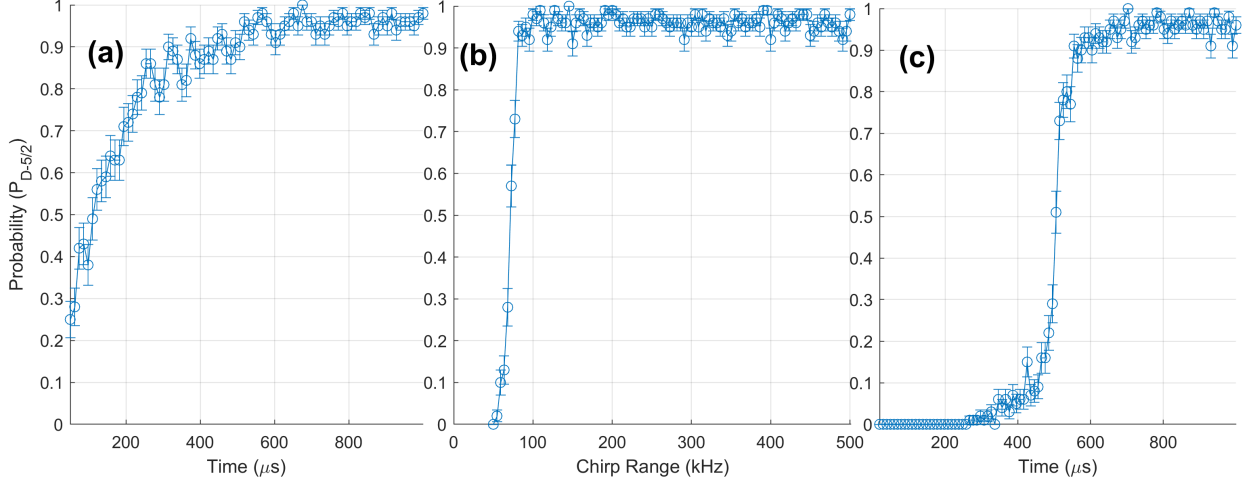


Figure 3.19: **RAP calibrations on the axial sideband.**

A Blackman-windowed **RAP** pulse is applied on the **Blue Sideband** following cooling of the ion to $\bar{n} \approx 0.01$. **(a)** Pulse time is scanned at a chirp range 350 kHz. **(b)** Chirp range is scanned at fixed pulse time 1 ms. **(c)** Evolution as a 1 ms **RAP** pulse with 350 kHz chirp is truncated at different times. Calibration proceeds by obtaining coarse values for chirp range and pulse time from eq. (3.51), then iteratively tuning the parameters following (a, b) to optimize population transfer. Parameter validity is confirmed by scanning the chirp center frequency (Figure 3.17) to observe the characteristic transfer plateau.

entrainment of the Fock state populations, requiring instead a fast pulse relative to the coherence time [Wat+11,IV05].

For higher Fock-state readout, the challenges are compounded. Each higher Fock state adds roughly four pulses, dominated by the **RAP** pulse, which occupies $\sim 400 \mu\text{s}$ and $\sim 800 \mu\text{s}$ on the axial and radial modes, respectively. For the $n = 3$ overlap, the full sequence comprises ~ 13 pulses, yielding a cumulative fidelity of roughly $0.98^{13} \approx 77\%$. As lower-lying Fock components are progressively shelved, the auxiliary states with weaker Rabi frequencies are left, which require longer pulse times and lower fidelities. The cumulative overhead is approximately (for the radial mode) 4 ms for $n = 2$ and 7 ms for $n = 3$, which in addition to exceeding the motional coherence time of the mode ($\tau \sim 1.5$ ms), limits the data acquisition rate and increases susceptibility to drifts, further eroding the already marginal metrological gains. In practice, we achieve a maximum metrological gain of $g_{\text{SQL}} = 3.9(8)$ dB with the

²⁰i.e. optical power — this is a key reason why waste of optical power is disrespectful. See section 6.3.1 for an only marginally related tirade.

$n = 3$ state, representing a significant loss in metrological potential compared to the ideal gain of $g_{\text{SQL}} = 8.5 \text{ dB}$); we attribute this to the cumulative pulse infidelity and motional decoherence.

Several avenues exist for accelerating the protocol. Applying additional laser tones to cancel the non-adiabatic coupling terms — a technique demonstrated in the context of phonon arithmetic [Um+16,Um19] — could substantially reduce the required RAP pulse duration. Alternatively, the Ultra Narrow-Band (UNB) composite-pulse sequences of [Mal+24] offer motional-state-selective population transfer with only a single excitation sequence, eliminating the complex pulse protocol that represents the greatest source of infidelity in our implementation. However, the UNB sequences in their present form incur similar total time costs and suffer reduced state selectivity compared to the RAP-based protocol.

3.5.5 Characteristic Function Tomography

The characteristic function is the expectation value of the displacement operator:

$$\chi(\beta) = \langle \hat{D}(\beta) \rangle \quad (3.52)$$

where $\beta \in \mathbb{C}$. The characteristic function can be understood as measuring the overlap of the state $|\psi\rangle$ with itself after displacement by β , and is further related to the Wigner function $W(\gamma)$ by a Fourier transform (eq. (3.4)) [WM08,FH20]. As the characteristic function can be easily measured by a bichromatic “cat” operation [Lo+15], eq. (3.4) provides a means for efficient Wigner function tomography: the characteristic function can be sampled on a grid, then processed via a Discrete-Fourier Transform (DFT)²¹. This tomographic implementation represents a reduction in data acquisition overhead by a factor of $\approx 100\times$ compared to the standard “displace and measure” protocol of [Lei+96]²², and a reduction of $\approx 20\times$ compared

²¹This is not the textbook discrete Fourier transform, and differs by a phase factor due to eq. (3.4).

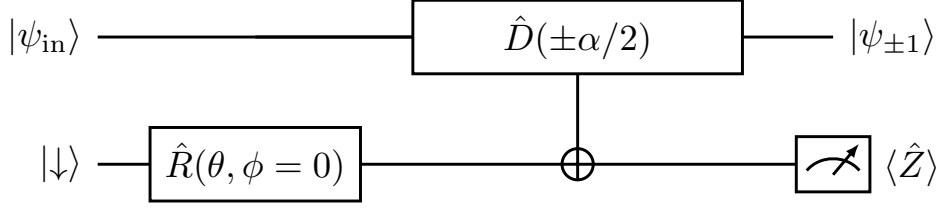


Figure 3.20: **Quantum circuit for direct characteristic function tomography.**

The motional state of interest $|\psi_{\text{in}}\rangle$ is prepared with spin in $|\downarrow\rangle$. A carrier rotation $\hat{R}(\theta, \phi)$ selects the measured quadrature of χ , after which a [Spin/State-Dependent Force \(SDF\)](#) $\hat{D}(\pm\beta/2)$ creates a spin-entangled superposition of the wavepacket (eq. (3.54)); a projective spin measurement $\langle\hat{Z}\rangle$ yields the wavepacket overlap $\text{Re}[\chi(\beta)]$ ($\theta = 0$) or $\text{Im}[\chi(\beta)]$ ($\theta = \pi/2$) (eq. (3.55)).

to the previous state-of-the-art protocol [\[Kie+16\]](#).

Implementation

Measurement of $\chi(\beta)$ is realized by the quantum circuit in fig. 3.20 [\[FH20\]](#).

The ion spin is initialized into $|\downarrow\rangle$ and a motional state of interest $|\psi_{\text{in}}\rangle$ is prepared. A carrier pulse on the optical qubit transition then applies the rotation

$$\hat{R}(\theta, \phi) = \cos\left(\frac{\theta}{2}\right)\hat{I} - i\sin\left(\frac{\theta}{2}\right)\left(\sin(\phi)\hat{X} + \cos(\phi)\hat{Y}\right), \quad (3.53)$$

preparing the spin in a superposition state, where θ is the rotation angle and ϕ is the phase relative to the [Spin/State-Dependent Force \(SDF\)](#). The rotation angle θ controls the relative amplitude of the two wavepackets and thereby selects which quadrature of χ is measured. The relative phase ϕ determines the alignment of the spin state with the [SDF](#); for maximum contrast, we set $\phi = 0$.

A spin-dependent force $\hat{D}(\hat{X}\beta/2)$, generated by a bichromatic laser pulse, then produces

²²The technique in [\[Lei+96\]](#) involved displacement to the target Wigner-space position and measurement of a full [BSB](#) Rabi oscillation at each data point to extract the parity $W(\gamma) = \frac{2}{\pi} \sum_{n=0}^{\infty} (-1)^n P_n(\gamma)$.

the displaced wavepacket superposition:

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left((\cos(\theta/2) - \sin(\theta/2)) \hat{D}(+\beta/2) |\psi_{\text{in}}\rangle \otimes |X_+\rangle + (\cos(\theta/2) + \sin(\theta/2)) \hat{D}(-\beta/2) |\psi_{\text{in}}\rangle \otimes |X_-\rangle \right). \quad (3.54)$$

The phase-space orientation of β is set by the relative phase between the interaction used to generate $|\psi_{\text{in}}\rangle$ and the beat-note of the bichromatic pulse, $\Delta\phi = \phi_{\text{BSB}} - \phi_{\text{RSB}}$. The overlap is obtained by measuring state-dependent fluorescence, equivalent to measurement of $\hat{Z}$:

$$\begin{aligned} \langle \hat{Z} \rangle &= -\frac{1}{2} \langle e^{-i\theta} \hat{D}(+\beta) + e^{i\theta} \hat{D}(-\beta) \rangle \\ &= -\cos(\theta) \text{Re}[\langle \hat{D}(\beta) \rangle] - \sin(\theta) \text{Im}[\langle \hat{D}(\beta) \rangle] \\ &= -\cos(\theta) \text{Re}[\chi(\beta)] - \sin(\theta) \text{Im}[\chi(\beta)] \end{aligned} \quad (3.55)$$

where we have used $\hat{D}^\dagger(\beta) = \hat{D}(-\beta)$. A complete measurement of χ at a given β therefore requires two settings: $\theta = 0$ to extract $\text{Re}[\chi]$, and $\theta = \pi/2$ to extract $\text{Im}[\chi]$. While 2D sampling of $\chi(\beta)$ is most easily implemented by scanning in polar coordinates (r, θ) , this can introduce distortions in the reconstructed Wigner function at larger r [Din+17]; best results are obtained by scanning on a rectangular grid [FH20].

Data Processing & Reconstruction

The measured $\text{Re}[\chi]$ and $\text{Im}[\chi]$ are first fitted to the analytical characteristic function of the expected state (e.g. coherent, squeezed, or thermal) to remove effects of finite readout contrast and state-preparation infidelity. The fitted grid is then zero-padded to increase the resolution of the Fourier-transformed result, and optionally upsampled to extend the spatial range of the reconstructed Wigner function. The Fourier transform itself is a specialized [Discrete-Fourier Transform \(DFT\)](#) that accounts for the non-standard phase conventions of χ , rather than a standard [DFT](#). Following the transform, $\text{Im}[W(\gamma)]$ serves as a diagnostic: since $W(\gamma)$ is

real-valued, a nonzero imaginary part indicates systematic errors in the protocol [Flü19].

As $\chi(\beta)$ and $W(\gamma)$ are Fourier conjugate pairs, features localized in $W(\gamma)$ map to global, oscillatory structure in $\chi(\beta)$. For example, a coherent state $|\alpha\rangle$ has a Gaussian Wigner function displaced to $\gamma = \alpha$; its characteristic function $\chi(\beta) = e^{-|\beta|^2/2} e^{\beta\bar{\alpha} - \bar{\beta}\alpha}$ is therefore a Gaussian envelope at the origin modulated by oscillatory fringes, with frequency proportional to $|\alpha|$ and orientation determined by the displacement direction (Figure 3.21).

Additionally, Hermiticity of $\chi(\beta)$ (i.e. $\chi(\beta) = \chi^*(-\beta)$) implies that $\text{Re}[\chi]$ is even and $\text{Im}[\chi]$ is odd in β . States with rotationally symmetric Wigner functions (e.g. Fock and thermal states) therefore have $\text{Im}[\chi] = 0$ identically, and only the $\text{Re}[\chi]$ (i.e. $\theta = 0$) measurement is required [FH20].

Experimental Considerations

Laser Phase Calibration If the spin is not aligned with the SDF (i.e. $\phi \neq 0$ in the rotation pulse $\hat{R}$), eq. (3.55) becomes:

$$\langle \hat{Z} \rangle = -\cos(\theta) \text{Re}[\chi(\beta)] - \cos(\phi) \sin(\theta) \text{Im}[\chi(\beta)], \quad (3.56)$$

resulting in reduced signal contrast. Calibration of ϕ must generally be performed empirically, as changes in the optical path length introduced by the AOM used to generate the bichromatic beams alter the relative phase. Once calibrated, however, the setting is expected to be robust: any subsequent path-length change affects both the rotation and SDF pulses equally, producing only an inconsequential global phase. Similarly, because their relative alignment is determined only by their relative phase within the sequence, long-term drifts do not introduce error.

Accurate measurement of $\text{Im}[\chi]$ additionally requires precise calibration of the rotation angle $\theta = \pi/2$, which is determined by the pulse area and susceptible to drift. This requirement is obviated for rotationally symmetric states (e.g. Fock states), whose characteristic functions are purely real (see discussion above). Where measurement of $\text{Im}[\chi]$ is required, the rotation

angle can be made robust via composite pulse sequences (e.g. [HKJ13]).

Acquisition Efficiency In keeping with the parsimonious spirit of the technique, several modifications were made to reduce data-acquisition overhead. Adaptive MLE readout (section 4.3.3) was incorporated, achieving a $\sim 20\%$ reduction in total runtime. Hermiticity of the characteristic function $\chi(\beta) = \chi^*(-\beta)$ can be leveraged to sample only half the complex β -plane. Finally, for rapid diagnostics, a one-dimensional slice — scanning only the polar angle of β at fixed $|\beta|$ — is sufficient to determine both the phase and absolute displacement of a coherent state (Figure 3.22); the probability is given by

$$P(\beta, \phi) = \frac{1 + \cos [2\alpha\beta \sin (\phi - \phi_0)]}{2}, \quad (3.57)$$

where α, ϕ_0 are the magnitude and phase of the coherent displacement, respectively.

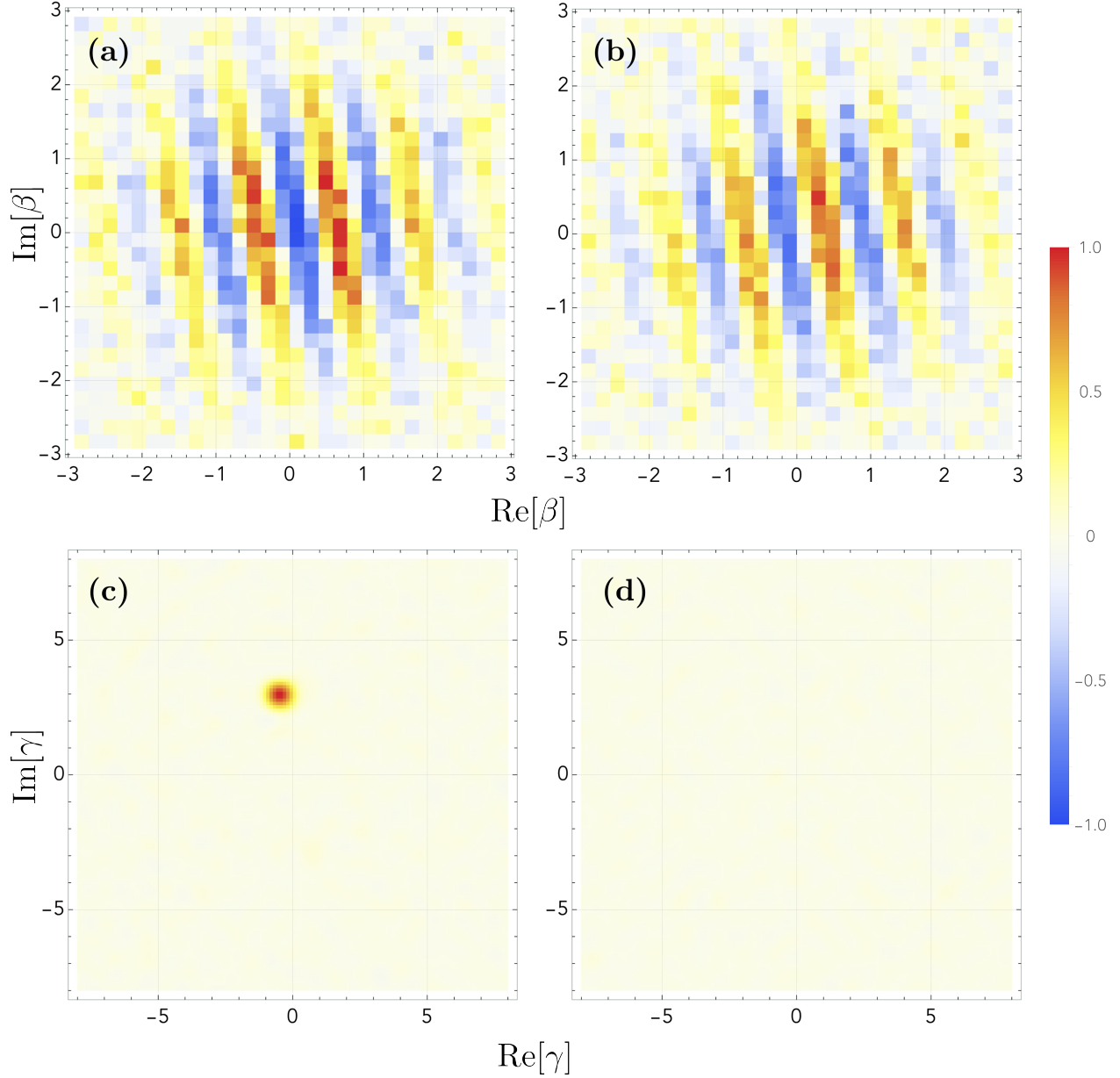


Figure 3.21: **Reconstructed Wigner function of a coherent state via characteristic function tomography.**

A coherent state is generated on the axial mode by [Sideband Cooling](#) to $\bar{n} \approx 0.01$ and applying a displacement $|\alpha| \approx 3$. (a, b) Real and imaginary parts of the characteristic function $\chi(\beta)$ sampled on a rectangular grid in the complex β -plane. The characteristic function of a coherent state consists of a Gaussian envelope modulated by oscillatory fringes, the frequency and direction of which are determined by the displacement. (c, d) Real and imaginary parts of the Wigner function $W(\gamma)$ reconstructed by [Discrete-Fourier Transform \(DFT\)](#) of the characteristic function in (a, b) (eq. (3.4)). $\text{Re}[W]$ shows the expected displaced Gaussian wavepacket at $\alpha = -0.54 + 2.91i$, while $\text{Im}[W]$ vanishes as required for the real-valued Wigner function. Data in (a, b) is zero-padded on all sides by 50 samples prior to the [DFT](#). The orientation of fringes in (a,b) is perpendicular to the displacement axis in (c) due to a complex rotation in eq. (3.4).

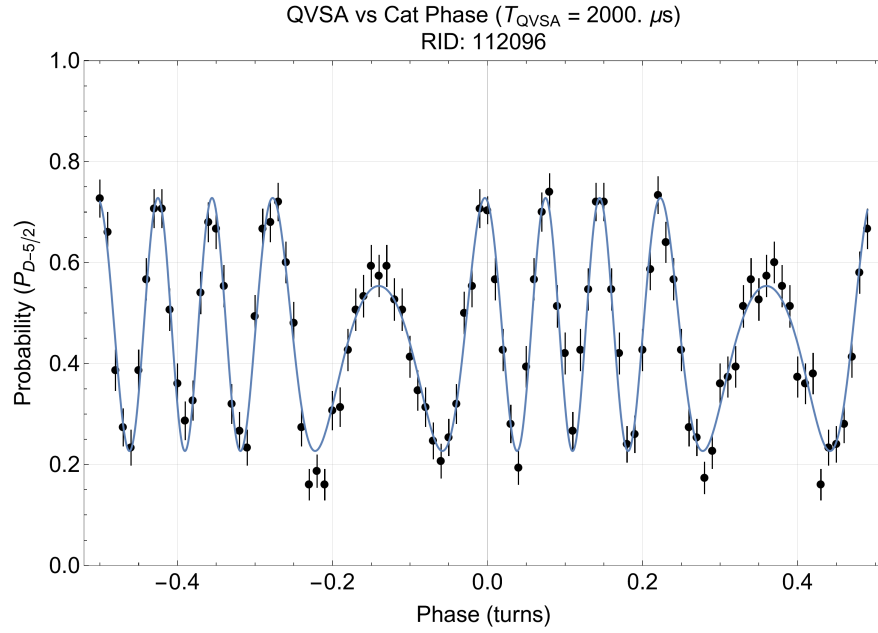


Figure 3.22: **One-dimensional characteristic function tomography.**

The phase of the bichromatic displacement β is scanned at fixed magnitude $|\beta|$ for a coherent state; error bars represent one standard error. Values $\alpha\beta = 7.223(15)$, $\phi_0 = 0.1097(4)$ turns are extracted from a fit to the data using eq. (3.57); these can be used to determine the phase and magnitude of the coherent displacement.

“The [laser] is both clock and calendar.
It’s your school, your home, your
church, your friend ... and your lover.”

Oh Dae-Su, *Oldboy*

Chapter 4

Light & Structure

How does the physicist love the laser? Let us count the ways.

As has been begrudgingly discovered over the course of this thesis, the laser, as a medium of power delivery, is unparalleled — a Watt of laser light in a typical 780HP fiber has a power density of order 10^{10}W/m^2 — this enables otherwise weak interactions to be probed on fast, even ultrafast, timescales. Light can further be shaped to increase its power density and directed with relative ease, transmitted into an isolated chamber with minimal overhead, detected at the single photon level, and has been used to make the most precise measurements in history [Mar+25]. It is thus for good reason that the laser is the standard tool of the atomic physicist. In this chapter, we provide the relevant background for probing matter with light.

4.1 Spin-Motion Coupling

Perhaps the singular advantage of the trapped ion platform is the well-isolated [Quantum Harmonic Oscillator \(QHO\)](#) provided by the ion motion which couples strongly to spin, allowing easy exploration of uniquely quantum phenomena in a continuous variable system. The formalism for describing the interaction between spin and motion is given by the [Jaynes-Cummings Hamiltonian \(JCH\)](#), originally developed to explore [Cavity Quantum](#)

Electro-Dynamics (cQED).

We begin by considering a system with a bosonic degree of freedom, i.e. the motional mode of a trapped ion, coupled to a [Two-Level System \(TLS\)](#), such as the optical qubit manifold in $^{40}\text{Ca}^+$. As we are interested in coherent dynamics, we here ignore dissipative effects such as spontaneous emission. These systems can be described using the following Hamiltonians:

$$\hat{H}_0 = \underbrace{\frac{\hbar\Delta}{2}\hat{\sigma}_Z}_{\hat{H}_{\text{TLS}}} + \underbrace{\hbar\omega_0\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)}_{\hat{H}_{\text{QHO}}} \quad (4.1)$$

where ω_0 is the mode frequency of the bosonic mode, and Δ is the transition frequency of the TLS.

We apply a laser beam to couple states of the system, which we model as a planar electric field:

$$\mathbf{E} = \mathbf{E}_0 \cos(\mathbf{k} \cdot \mathbf{x} - \omega t), \quad (4.2)$$

which in turn produces an interaction Hamiltonian

$$\begin{aligned} \hat{H}_I(t) &= -\mathbf{d} \cdot \mathbf{E} \\ &= -\langle 0 | \mathbf{d} \cdot \mathbf{E}_0 | 1 \rangle \cos(\mathbf{k} \cdot \mathbf{x} - \omega t) \\ &= \hbar\Omega\hat{\sigma}_X \cos(\mathbf{k} \cdot \mathbf{x} - \omega t) \end{aligned} \quad (4.3)$$

where $\mathbf{d}$ is the dipole moment of the TLS, and we have defined the Rabi frequency $\Omega = -\frac{\langle 0 | \mathbf{d} \cdot \mathbf{E}_0 | 1 \rangle}{\hbar}$.

4.1.1 The Lamb-Dicke Parameter (LDP)

The plane-wave term of the laser expressed by $e^{i\mathbf{k}\cdot\mathbf{x}}$ can be expanded

$$\begin{aligned}\mathbf{k} \cdot \mathbf{x} &= k \cos(\theta) \hat{x} = k \cos(\theta) x_0 (\hat{a}^\dagger + \hat{a}) \\ &= \eta (\hat{a}^\dagger + \hat{a})\end{aligned}\tag{4.4}$$

$$\Rightarrow \eta \equiv kx_0 \cos(\theta) = \frac{2\pi}{\lambda} \cos(\theta) \sqrt{\frac{\hbar}{2m\omega_0}}\tag{4.5}$$

where η is the Lamb-Dicke parameter (LDP). Though the Lamb-Dicke parameter η has different equivalent interpretations, that most relevant here relates the spatial wavefunction of the QHO to the wavelength of the radiation. When $\eta^2 \bar{n} \ll 1$, where $\bar{n}$ is the mean phonon number of the QHO, the system is said to be in the Lamb-Dicke regime, where the QHO's wavefunction is small relative to the radiation, and the first-order multipole expansion is thus well-justified; here, the field gradient term is typically expanded only to first order $e^{i\eta(\hat{a}^\dagger + \hat{a})} \approx 1 + i\eta(\hat{a}^\dagger + \hat{a})$. The $\sqrt{\bar{n}}$ term forms part of this Lamb-Dicke criterion since, as the ion's motional energy increases, its trajectory becomes similarly enlarged, such that it again samples the higher-order components of the field gradient. The Lamb-Dicke regime $\eta^2 \bar{n} \ll 1$ therefore establishes an effective criterion for the efficient coupling of spin to motion, such as for resolved sideband cooling or motion-mediated entanglement.

It should be noted that the Lamb-Dicke regime is only an *effective* criterion, and not fundamental — sideband cooling remains possible outside the Lamb-Dicke regime (though more challenging) [Mor+99], while 2-qubit entanglement has been demonstrated near its edge [Kir+09]. In fact, the stronger spin-motion couplings available far outside the Lamb-Dicke regime can be harnessed, such as to create highly nonclassical states of motion [McC+19a], or autonomous bosonic quantum computation [Roj+24].

For Radio-frequency and microwave gradient fields, which have a wavelength much larger than the trap, the wavevector $\mathbf{k}$ is poorly defined. For these fields, the Lamb-Dicke parameter is instead set by an effective wavelength determined by boundary conditions, typically the

trap electrodes used to generate the fields, which give:

$$\eta = \frac{1}{r_0} \sqrt{\frac{\hbar}{2m\omega_0}} \approx \text{O}(10^{-4}), \quad (4.6)$$

where r_0 is the ion-electrode distance of the trap. The gradient-field Lamb-Dicke is significantly smaller as compared to laser-based fields, which typically have $\eta \sim 0.1$. This underscores the great challenge of gradient-gate experiments¹, for which achieving meaningful spin-motion coupling is an existential difficulty.

4.1.2 The Jaynes-Cummings Hamiltonian

Rotating into the interaction frame with respect to $\hat{H}_0$ (eq. (4.1)), eq. (4.3) becomes:

$$\hat{H}_I(t) = \frac{\hbar\Omega}{2} \left(\hat{\sigma}_+ e^{-i\eta(\hat{a}^\dagger(t)+\hat{a}(t))} e^{i(\omega-\Delta)t} + \hat{\sigma}_- e^{i\eta(\hat{a}^\dagger(t)+\hat{a}(t))} e^{-i(\omega-\Delta)t} \right), \quad (4.7)$$

where we have applied the rotating wave approximation $\delta \ll \Delta, \omega$, and $\hat{a}(t) = \hat{a}e^{-i\omega_0 t}$. To simplify the field-gradient term, it can be expressed as a displacement operator $\hat{D}(\alpha) = \exp(\alpha\hat{a}^\dagger - \bar{\alpha}\hat{a})$, such that its matrix elements are

$$\begin{aligned} \langle n | e^{i\eta(\hat{a}^\dagger(t)+\hat{a}(t))} | m \rangle &= \langle n | \hat{D}(i\eta e^{i\omega_0 t}) | m \rangle \\ &= (ie^{i\omega_0 t})^{n-m} \eta^{|n-m|} e^{-\frac{|\eta|^2}{2}} L_n^{|n-m|}(|\eta|^2) \left(\frac{n!}{m!} \right)^{-\frac{\text{sgn}(n-m)}{2}} \\ &= (ie^{i\omega_0 t})^{n-m} D_{n,m} \end{aligned} \quad (4.8)$$

where $L_p^q(x)$ is the generalized Laguerre polynomial, and $n \leq m$ (the $n > m$ case follows from $|D_{n,m}| = |D_{m,n}|$). In the resolved sideband limit $\Omega_{n,m} \ll \omega_0$, if the laser frequency is detuned near a motional resonance, i.e. $\delta = \omega - \Delta - l\omega_0$ with $\delta \ll \Delta, \omega, \omega_0$, then the system can be further approximated as having only two states $|g, n\rangle$, $|e, m\rangle$ coupled to one another, with

¹Such as that in this thesis.

couplings to other states being negligible. The system state is thus

$$|\psi(t)\rangle = c_{g,n} |g, n\rangle + c_{e,m} |e, m\rangle, \quad (4.9)$$

which can be solved using the Schrödinger equation, yielding:

$$\begin{aligned} \dot{c}_{g,n} &= -i^{1+|n-m|} \frac{\Omega_{n,m}}{2} e^{-i\delta t} c_{e,m} \\ \dot{c}_{e,m} &= -i^{1-|n-m|} \frac{\Omega_{n,m}}{2} e^{i\delta t} c_{g,n}, \end{aligned} \quad (4.10)$$

with solution

$$\begin{pmatrix} c_{g,n}(t) \\ c_{e,m}(t) \end{pmatrix} = T(n, m) \begin{pmatrix} c_{g,n}(0) \\ c_{e,m}(0) \end{pmatrix} \quad (4.11)$$

$$\Rightarrow T(n, m) = \begin{pmatrix} e^{-i\frac{\delta t}{2}} \left(\cos\left(\frac{\Omega'_{n,m}t}{2}\right) + i\frac{\delta}{\Omega'_{n,m}} \sin\left(\frac{\Omega'_{n,m}t}{2}\right) \right) & -i^{1+|n-m|} e^{-i\frac{\delta t}{2}} \frac{\Omega_{n,m}}{\Omega'_{n,m}} \sin\left(\frac{\Omega'_{n,m}t}{2}\right) \\ -i^{1-|n-m|} e^{i\frac{\delta t}{2}} \frac{\Omega_{n,m}}{\Omega'_{n,m}} \sin\left(\frac{\Omega'_{n,m}t}{2}\right) & e^{i\frac{\delta t}{2}} \left(\cos\left(\frac{\Omega'_{n,m}t}{2}\right) - i\frac{\delta}{\Omega'_{n,m}} \sin\left(\frac{\Omega'_{n,m}t}{2}\right) \right) \end{pmatrix} \quad (4.12)$$

where $\Omega'_{n,m} = \sqrt{\Omega_{n,m}^2 + \delta^2}$. For $m = 0$, eq. (4.12) gives “carrier” transitions, which leave the motional state of the ion unchanged, while $m \neq 0$ gives “sideband” transitions which change the motional state by a discrete number of quanta.

The model described by eqs. (4.7) and (4.12) is not the typical JCM; this is recovered only for $m = -1$, which corresponds to the excitation of the first-order red sideband. The ability to drive anti-JCH terms at $m = +1$ as well as higher order sidebands at $|m| > 1$ are absent in cQED, affording trapped ions greater control with which to explore more complex and intricate dynamics, such as mesoscopic superposition states [Mon+96] and continuous-variable grid qubits [Flü+19].

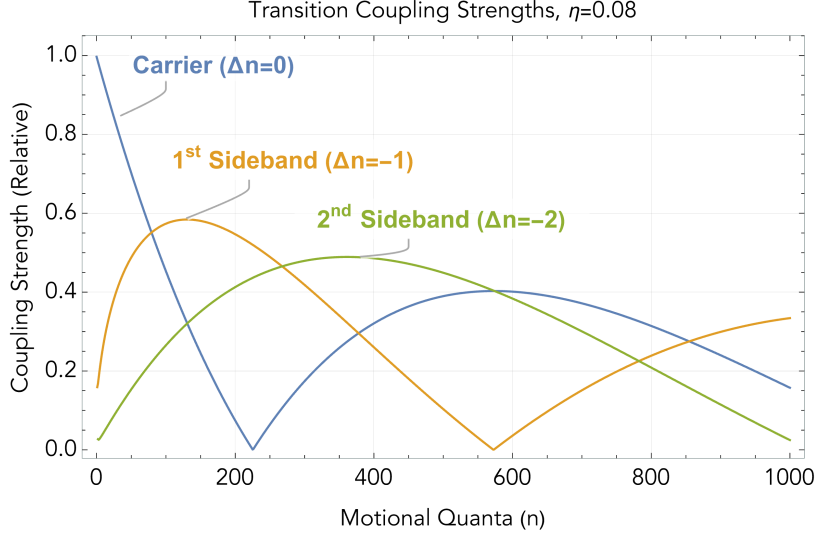


Figure 4.1: **Transition coupling strengths for the Jaynes-Cummings Hamiltonian for $\eta = 0.08$.**

Relative coupling strengths $|\Omega_{n,n+\Delta n}/\Omega_0|$ calculated from eq. (4.8) as a function of motional quanta n for the carrier ($\Delta n = 0$, blue), first red sideband ($\Delta n = -1$, orange), and second red sideband ($\Delta n = -2$, green). Zeroes are determined by the associated Laguerre polynomials, and can result in trapped populations during sideband cooling (see section 4.2.2).

4.2 Laser Cooling

Laser cooling, proposed in 1975 [WD75] and demonstrated in 1978 [WDW78, Neu+78], allows the ion motion to be prepared in the ultracold regime required to probe quantum phenomena. Generally², Doppler cooling is used to initialize the ion into the Lamb-Dicke regime $\eta^2 \bar{n} \ll 1$ and enable coherent laser-based operations on the ion (section 4.2.1). However, due to fundamental limitations on Doppler cooling, the Lamb-Dicke regime is often not comfortably satisfied, which results in a reduction of coherence and measurement contrast due to thermal effects [Hem+13, Roo00]. To enable the coherent interrogation of motion at the level required for Quantum Logic Spectroscopy (QLS), as well as the remainder of the experiments in this thesis, a further stage of cooling, such as Resolved Sideband Cooling (SBC), is then required to bring the ion below the Doppler limit and comfortably into the Lamb-Dicke regime; this is the focus of section 4.2.2.

²It should be noted that the Doppler limit is not intrinsically linked to the Lamb-Dicke regime: traps with high secular frequencies ω_0 can have Doppler limits of $\bar{n} \approx 0.5$ [Mon+95, Xu+25], while traps with very low ω_0 may never reach it in practice.

4.2.1 Doppler Cooling

Intuitively, Doppler cooling operates by leveraging the momentum imparted by a photon onto an atom during absorption. When an ion absorbs or emits a photon to transition between states, conservation of momentum dictates that the ion must carry the difference of the photon's momentum $\Delta \mathbf{p} = -\hbar \mathbf{k}$. To harness this interaction for cooling, we use the principle of Doppler shift: in the rest frame of a moving ion, the frequency of incident radiation is shifted by an amount $\delta\omega = -\mathbf{k} \cdot \mathbf{v}$. Therefore, by shining red-detuned light onto ions, those ions moving towards the light are more likely to absorb it and become slowed by the photon momentum, resulting in a velocity-dependent force that can be used to damp the ion motion — i.e., Doppler cooling.

Photon Absorption

We begin by first considering the effect of photon absorption by an ion with a TLS outside the resolved sideband regime, i.e. $\Gamma \gg \omega_0$, where Γ is the rate of decay from the excited state and ω_0 is the secular frequency of the ion. In this regime $\Gamma \gg \omega_0$, photon absorption/emission events occur at timescales much faster than the secular motion of the ion, allowing the ion to be treated as a free particle with some velocity v .

The change in the ion's momentum due to photon absorption over a unit time is simply the number of absorption events multiplied by the photon momentum per event, which is expressed:

$$F_{\text{scatter}} = \frac{dp}{dt} = \hbar k \Gamma \rho_{ee}, \quad (4.13)$$

where ρ_{ee} is the excited state probability, and k is the wavevector of the photon. The excited state probability is obtained by solution of the Optical Bloch Equations (OBE) in the steady-state, which results in [\[Foo05\]](#)

$$\rho_{ee} = \frac{s/2}{1 + s + \left(\frac{2\delta}{\Gamma}\right)^2}, \quad (4.14)$$

where $s = \frac{2\Omega^2}{\Gamma^2}$ is the saturation parameter of the transition. If the ion is travelling at velocity $\mathbf{v}$, the photon becomes Doppler-shifted in the rest frame of the ion, such that the effective detuning is instead $\delta \rightarrow \delta - \mathbf{k} \cdot \mathbf{v}$. For small velocities $kv \ll \Gamma$, the force can be linearized about $v = 0$:

$$\begin{aligned}
F_{\text{scatter}} &\approx F|_{v=0} + \left. \frac{dF}{dv} \right|_{v=0} v \\
&= F|_{v=0} + \hbar k \Gamma \rho_{ee}|_{v=0} \left(\frac{8\delta k}{\Gamma^2} \frac{1}{1 + s + \left(\frac{2\delta}{\Gamma}\right)^2} \right) v \\
&= F|_{v=0} (1 + \kappa v),
\end{aligned} \tag{4.15}$$

where we have defined $\kappa = \frac{8\delta k}{\Gamma^2} \frac{1}{1 + s + \left(\frac{2\delta}{\Gamma}\right)^2}$ for convenience. The sign of κ , and thus the cooling force, is determined by the detuning δ : for $\delta < 0$ cooling is effected, and vice versa. In the $\delta < 0$ case, the initially red-detuned photon is blue-shifted near resonance by ions moving towards the photon, leading to a damping of the ion's motion. Conversely, for $\delta > 0$, the photon is initially blue-detuned and is red-shifted near resonance by ions moving away from the photon, accelerating the ion's motion.

From eq. (4.15), the time-averaged cooling rate is then:

$$\begin{aligned}
\frac{dE_{\text{cool}}}{dt} &= \langle F_{\text{scatter}} v \rangle = F|_{v=0} \langle v + \kappa v^2 \rangle = F|_{v=0} \kappa \langle v^2 \rangle \\
&= \hbar k \Gamma \rho_{ee}|_{v=0} \kappa \langle v^2 \rangle
\end{aligned} \tag{4.16}$$

where the term in $\langle v \rangle$ averages to zero. The cooling rate (eq. (4.16)) is maximized at

$$\delta_{\text{max}} = -\frac{\Gamma \sqrt{1+s}}{2\sqrt{3}}. \tag{4.17}$$

Heating Processes Two patterns of emission are possible: spontaneous emission, and stimulated emission. Both cases are necessarily preceded by an initial absorption event, which

imparts a momentum $\Delta p = \hbar \mathbf{k}$ to the ion. When followed by spontaneous emission, the emission occurs isotropically and thus averages to zero, such that the overall absorption/emission cycle results in a net momentum change $\Delta p = \hbar \mathbf{k}$. However, when followed by stimulated emission, the emission occurs along the same direction as the incident photon, negating the kick from the initial absorption event for a net momentum change $\Delta p = 0$. As we are interested only in events which result in a net momentum change, we need only consider the effect of spontaneous emission; this approximation can be improved by operating in the low saturation regime $s \ll 1$, where decay of the excited state exceeds the absorption rate.

The discrete and stochastic nature of spontaneous emission results in a random walk of the ion's momentum (i.e. momentum diffusion) since spontaneous emission events are, in the low-saturation regime $s \ll 1$, independent and identically distributed (iid) [Lei+03]. As a random walk, the change in the mean squared momentum over time dt is given by the rate of scattering events $R = \Gamma \rho_{ee}|_{v=0}$:

$$\langle p^2 \rangle = (Rdt) \xi \Delta p^2 = (\Gamma \rho_{ee}|_{v=0} dt) \xi \hbar^2 k^2 \quad (4.18)$$

where we have introduced a geometric factor ξ to extract the averaged contribution along the radiation axis. For exactly isotropic emission, $\xi = \langle \cos^2(\theta) \rangle = \frac{1}{3}$; however, if the radiation pattern of a dipole transition is considered, then $\xi = \frac{2}{5}$ [Ste86, Lei+03]. The spontaneous emission heating rate is finally

$$\frac{dE_{\text{heat}}}{dt} = \frac{d}{dt} \left(\frac{\langle p^2 \rangle}{2m} \right) = \xi \frac{\hbar^2 k^2}{2m} \Gamma \rho_{ee}|_{v=0}. \quad (4.19)$$

A further source of momentum diffusion (and thus heating) arises from the stochastic nature of absorption. This contribution can be modelled identically to eq. (4.19), though without the need for a geometric factor ξ since absorption events change only the momentum

along the radiation axis. Together, the net heating rate becomes:

$$\frac{dE_{\text{heat}}}{dt} = (1 + \xi) \frac{\hbar^2 k^2}{2m} \Gamma \rho_{ee} \quad (4.20)$$

The Doppler Limit In the steady-state, the heating and cooling processes are balanced:

$$\begin{aligned} \frac{dE_{\text{cool}}}{dt} &= \frac{dE_{\text{heat}}}{dt} \\ \Rightarrow \hbar k \Gamma \rho_{ee}|_{v=0} \frac{\frac{8\delta k}{\Gamma^2}}{1 + s + \left(\frac{2\delta}{\Gamma}\right)^2} \langle v^2 \rangle &= (1 + \xi) \frac{\hbar^2 k^2}{2m} \Gamma \rho_{ee}|_{v=0}. \end{aligned} \quad (4.21)$$

Here, the system is at a minimum energy, which can be expressed as a temperature using the equipartition theorem:

$$T_D = \frac{\hbar (1 + \xi) \Gamma^2 (1 + s) + 4\delta^2}{16k_B |\delta|} \quad (4.22)$$

This is known as the Doppler limit T_D , and reaches a minimum at $\delta_D = -\frac{\Gamma}{2} \sqrt{1 + s}$:

$$T_D = \frac{(1 + \xi) \hbar \Gamma \sqrt{1 + s}}{4k_B} \quad \left(\delta_D = -\frac{\Gamma}{2} \sqrt{1 + s} \right) \quad (4.23)$$

Note that the Doppler-optimal detuning δ_D is different from the cooling-optimal detuning δ_{max} in eq. (4.17).

The Recoil Limit The Doppler limit is the key figure of merit for laser-based cooling; techniques that can bring a system below T_D are granted the weighty honorific of “sub-Doppler.” In this sub-Doppler regime, a different figure of merit becomes relevant: the recoil limit. The recoil limit is simply the energy imparted by a photon recoil from a spontaneous emission event:

$$T_R = \frac{p^2}{mk_B} = \frac{\hbar^2 k^2}{mk_B}, \quad (4.24)$$

where k_B is Boltzmann’s constant.

Since photon recoil served as a key heating mechanism for Doppler cooling, one reasonably

assumes that the recoil limit is always below the Doppler limit, and, generally, one is correct. This assumption is further strengthened by the simple expression of the recoil limit, which evokes all the emotion of a fundamental limit. However, one’s suspicions can be aroused by noticing that the Doppler limit lacks an explicit dependence on the photon momentum $\hbar k$, and depends only on the transition linewidth Γ — if a sufficiently narrow transition Γ can be found, then the Doppler limit could in principle be below the recoil limit. These suspicions turn out to be true: sub-recoil Doppler limits at the 10^{-7} K level have in fact long been achieved using ultranarrow $\sim$ kHz-order lines, such as those found in ^{88}Sr [Ste13,Kat+99]. The recoil limit can be similarly beaten by a host of sub-Doppler techniques, such as resolved sideband cooling [Mon+95].

Further Considerations A key assumption used to derive the results in this section is that the dynamics are well-described by a TLS. A more complete description of the cooling dynamics can be achieved by considering relevant sublevels (from e.g. Zeeman splitting) and the decay to (and concomitant repumping from) states outside the cooling manifold; this is covered in great depth by [Jan15]. In general, the inclusion of additional states does not significantly change the cooling dynamics if the relevant component of the transition lineshape remains monotonic and approximately Lorentzian [Lei+03].

More complete treatment of the ion motion is also required in the presence of significant micromotion [Cir+94], where excitation on even red-detuned micromotion sidebands can result in heating, and large ion velocities, such as those encountered during loading [Gug+15,Wüb+12]. Finally, though Doppler cooling typically takes place on dipole transitions to achieve sufficient Rabi frequencies, this is not fundamental — Doppler cooling has been demonstrated on dipole-forbidden transitions [Hen+08].

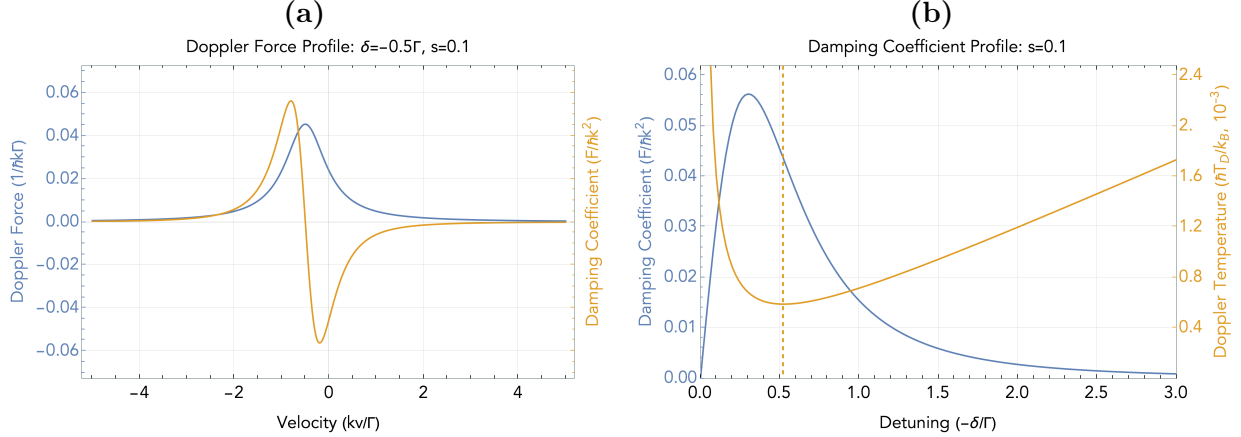


Figure 4.2: **Doppler cooling force profiles.**

(a) Scattering force (blue) and damping coefficient (orange), calculated as $\partial F/\partial v$, for detuning $\delta = -0.5\Gamma$ and saturation parameter $s = 0.1$. As the ion is trapped and the velocity oscillates about zero, only a single beam is required for cooling, unlike for [Magneto-Optical Traps \(MOTs\)](#), which require two beams per axis. For small velocities near $v = 0$, the force is approximately linear in v with negative damping coefficient, resulting in net cooling; vice versa for blue detunings ($\delta > 0$). **(b)** Damping coefficient (blue) evaluated at $v = 0$ and Doppler temperature as a function of detuning for saturation parameter $s = 0.1$. The damping coefficient and Doppler temperature are optimized at different detunings (the Doppler temperature optimum is denoted by the orange dashed line).

4.2.2 Resolved Sideband Cooling

We here outline the theoretical background for resolved sideband cooling (SBC), a sub-Doppler cooling technique that enables an ion's motional mode to be prepared in the ground state. Of all sub-Doppler cooling techniques in use in trapped ion platforms, SBC is the most commonly used, and serves as a de facto minimum requirement for coherent motional operations on the ion. The key requirement of SBC is the use of a transition in the resolved-sideband limit, i.e. $\Gamma \ll \omega_0$, which allows the motional sidebands to be directly addressed to create a *momentum-selective* scattering force, largely eliminating the heating terms from momentum diffusion (eq. (4.20)) present in Doppler cooling.

A further criterion commonly favored for SBC is trap operation in the Lamb-Dicke limit³ $\eta^2 \ll 1$. In the Lamb-Dicke limit, the photon recoil energy is small relative to the harmonic oscillator spacing, such that spontaneous emission events leave the motional state of the ion

effectively unchanged. Under these criteria, SBC can be understood as removing a single quanta with each absorption/emission cycle until it is pumped into the ground motional state. As the ground motional state is a dark state of the laser, further absorption is prevented, eliminating momentum diffusion heating.

Population Dynamics We begin by considering a general Ξ -level system [Mar+94], depicted in Figure 4.3, which most closely resembles the $^2S_{1/2} \rightarrow ^2D_{5/2} \rightarrow ^2P_{3/2}$ manifold in $^{40}\text{Ca}^+$ used for SBC. We assume that the $|2\rangle \rightarrow |0\rangle$ transition is in the resolved-sideband limit, while the $|1\rangle \rightarrow |0\rangle$ and $|1\rangle \rightarrow |2\rangle$ transitions are not, i.e. $\Gamma_{02} \sim 0 \ll \omega_0 \ll \Gamma_{01}, \Gamma_{1,2}$. A laser with Rabi frequency Ω_{02} and detuning δ_{02} is applied to address the red motional sideband of the $|0\rangle \rightarrow |2\rangle$ transition, while a second laser with Rabi frequency Ω_{21} and detuning δ_{21} is applied on the $|2\rangle \rightarrow |1\rangle$ transition. The role of the Ω_{21} laser is to effectively “quench” the population in the long-lived $|2\rangle$ state by coupling it to the fast-decaying $|1\rangle$ state, significantly accelerating the cooling dynamics, which would otherwise proceed glacially due to the long lifetime $\Gamma_{02} \sim 0$ of the $|2\rangle$ state. For these reasons, the $|2\rangle \rightarrow |1\rangle$ transition is typically a dipole-allowed transition, with arbitrary Ω_{12} achievable relative to Ω_{02} .

To proceed, calculation of the photon absorption rates is required. Though the entirety of the 3-level Ξ -system can be considered by solution of the Lindbladian, the system can be simplified significantly by noticing that the $|1\rangle$ state is rarely occupied due to its short lifetime Γ_{10}, Γ_{12} . The $|1\rangle$ state can thus be adiabatically eliminated [BPM07], reducing the system to the 2-level case, with modified detuning and decay rates

$$\tilde{\delta} = \delta_{20} - \delta_{12} \frac{s_{12}}{2} \qquad \tilde{\Gamma} = \Gamma_{10} \frac{s_{12}}{2} \qquad (4.25)$$

$$s_{12} \equiv \frac{2\Omega_{12}^2}{(\Gamma_{10} + \Gamma_{12})^2 + 4\delta_{12}^2}. \qquad (4.26)$$

where s_{12} is the saturation parameter for the $|2\rangle \rightarrow |1\rangle$ transition. The photon absorption

³While the Lamb-Dicke criterion greatly simplifies the cooling dynamics and results in greater cooling efficiency, it is not strictly necessary to achieve sub-Doppler cooling [Mor+99].

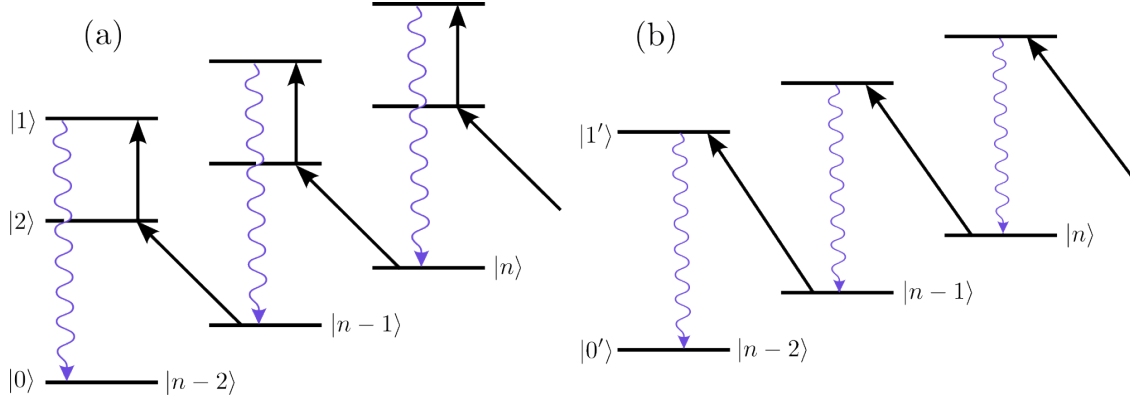


Figure 4.3: **Sideband Cooling with a metastable Ξ -level system.**

The system is composed of a ground $|0\rangle$ state connected to a long-lived metastable $|2\rangle$ state via a resolved-sideband quadrupole transition, and a short-lived $|1\rangle$ state that decays selectively to the $|0\rangle$ state. The $|1\rangle \leftrightarrow |0\rangle$ and $|1\rangle \leftrightarrow |2\rangle$ transitions are assumed to be dipole-allowed. **(a)** A **Red Sideband** pulse on the resolved-sideband $|0\rangle \leftrightarrow |2\rangle$ transition cools the system by one quantum, followed by quenching via coupling to a short-lived $|1\rangle$ state, where it decays back to the $|0\rangle$ state. **(b)** With quenching, the $|1\rangle$ state can be adiabatically eliminated, reducing the system to an effective **Two-Level System** with linewidth $\tilde{\Gamma}$.

rate is now simply the excited state decay rate, which is

$$R_{\text{SBC}} = \tilde{\Gamma} \rho_{ee} = \tilde{\Gamma} \frac{\Omega^2}{4\tilde{\delta}^2 + 2\Omega^2 + \tilde{\Gamma}^2}. \quad (4.27)$$

Heating and Cooling We now consider the interactions within each absorption/emission cycle and their effect on cooling. We first note that the Lamb-Dicke parameters of the two transitions η_{10} , η_{20} must be distinguished as they may be at different frequencies. Though SBC directly addresses the red motional sideband that cools the motion, off-resonant excitation of nearby spectator transitions can heat the ion either directly (e.g. the blue sideband at $\delta = 2\omega_0$), or indirectly via spontaneous emission (e.g. the carrier transition at $\delta = \omega_0$) [Ste86, Lei+03]. For simplicity, we assume operation in the Lamb-Dicke regime $\eta_{10}^2 \bar{n}, \eta_{20}^2 \bar{n} \ll 1$ such that higher-order transitions $|\Delta n| > 1$ can be ignored. Two types of transitions are then possible: first-order sideband transitions $|\downarrow, n\rangle \rightarrow |\uparrow, n \pm 1\rangle$, and carrier transitions $|\downarrow, n\rangle \rightarrow |\uparrow, n\rangle$

(Figure 4.3). From eq. (4.27), the sideband transitions are excited with rates

$$R_{n-1}(\tilde{\delta}) = \tilde{\Gamma} \frac{\Omega_{n,n-1}^2}{4\tilde{\delta}^2 + 2\Omega_{n,n-1}^2 + \tilde{\Gamma}^2} \approx \tilde{\Gamma} \frac{\Omega_0^2}{4\tilde{\delta}^2 + \tilde{\Gamma}^2} \eta_{20}^2 n = R'_\Delta(\tilde{\delta}) \eta_{20}^2 n \quad (4.28)$$

$$R_{n+1}(\tilde{\delta}) = \tilde{\Gamma} \frac{\Omega_{n,n+1}^2}{4\tilde{\delta}^2 + 2\Omega_{n,n+1}^2 + \tilde{\Gamma}^2} \approx \tilde{\Gamma} \frac{\Omega_0^2}{4\tilde{\delta}^2 + \tilde{\Gamma}^2} \eta_{20}^2 (n+1) = R'_\Delta(\tilde{\delta}) \eta_{20}^2 (n+1) \quad (4.29)$$

where, for convenience, we have assumed $\Omega_{n,n\pm 1} \ll \tilde{\Gamma}$, and Ω_0 is the Rabi frequency of the bare carrier transition. Since $\eta^2 \ll 1$, we can assume that, following excitation to $|\uparrow, n \pm 1\rangle$, spontaneous emission returns the motional state unchanged to $|\downarrow, n \pm 1\rangle$.

Though the carrier transitions leave the motional state unchanged, these occur with sufficiently large Rabi frequency $\Omega_{n,n} = \Omega_0 \gg \Omega_{n,n\pm 1}$ that the associated spontaneous emission cannot be ignored: these result in a displacement $e^{i\eta_{10}(\hat{a}^\dagger + \hat{a})} \approx 1 + i\eta_{10}(\hat{a}^\dagger + \hat{a})$, which, depending on the emission direction, can cool or heat the motion. The motional state $|n\rangle$ is therefore changed with probabilities $P(n \rightarrow n+1) \approx \xi \eta_{10}^2 (n+1)$ and $P(n \rightarrow n-1) \approx \xi \eta_{10}^2 n$, where the factor ξ has been added to again account for the emission pattern. The transition rate from carrier excitation is therefore:

$$R_n(\tilde{\delta}) = \tilde{\Gamma} \frac{\Omega_0^2}{4\tilde{\delta}^2 + 2\Omega_0^2 + \tilde{\Gamma}^2} = R'_0(\tilde{\delta}) \xi \eta_{10}^2 \begin{cases} n & n \rightarrow n-1 \\ n+1 & n \rightarrow n+1 \end{cases}. \quad (4.30)$$

Rate Equations By stamp-collecting the heating and cooling terms of eq. (4.28) and eq. (4.30), the SBC dynamics can now be described using a set of rate equations [Ste86, Mar+94]:

$$\dot{P}_n = A_-(n+1)P_{n+1} + A_+nP_{n-1} - A_+(n+1)P_n - A_-nP_n, \quad (4.31)$$

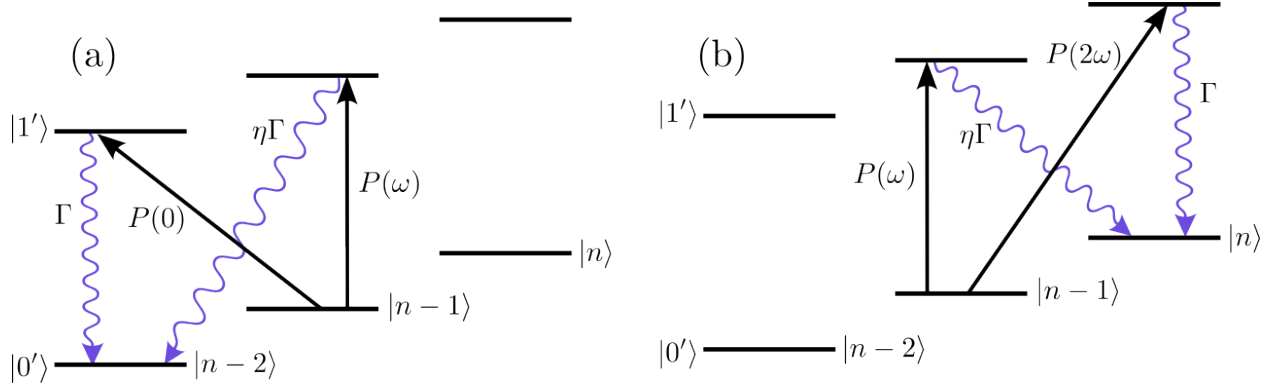


Figure 4.4: **Cooling and heating processes in Sideband Cooling.**

The laser is applied resonantly on the **Red Sideband** and the system is assumed to be in the **Lamb-Dicke Regime**, such that processes are considered only to leading order in η ; processes that leave the motional state unchanged are omitted. **(a)** Cooling processes consist of resonant **Red Sideband** excitation ($P(0)$) followed by a $\Delta n = 0$ decay; and off-resonant carrier excitation $P(\omega)$ followed by a $\Delta n = -1$ decay. **(b)** Heating processes consist of off-resonant excitation on the carrier ($P(\omega)$) followed by a $\Delta n = +1$ decay; and off-resonant **Blue Sideband** excitation $P(2\omega)$ followed by a $\Delta n = 0$ decay. Competition between heating and cooling determines the steady-state phonon number $\bar{n}_s$ (eq. (4.36)); these processes are considered formally in eq. (4.32).

where P_n is the population of the n th motional state, and the rate coefficients are

$$\begin{aligned}
 A_- &= R'_\Delta(\tilde{\delta} = 0)\eta_{20}^2 & + R'_0(\tilde{\delta} = \omega_0)\xi\eta_{10}^2 \\
 A_+ &= R'_\Delta(\tilde{\delta} = 2\omega_0)\eta_{20}^2 & + R'_0(\tilde{\delta} = \omega_0)\xi\eta_{10}^2
 \end{aligned} \tag{4.32}$$

where the detunings have been set to drive the red motional sideband on-resonance (the typical case for SBC), i.e. $\delta_{\text{RSB}} = 0, \delta_{\text{Carrier}} = \omega_0, \delta_{\text{BSB}} = 2\omega_0$. We are interested in the cooling of the system as a whole, as captured by the evolution of the mean phonon number $\dot{\bar{n}}$; this can be derived inductively from eq. (4.32):

$$\dot{\bar{n}} = \sum_{n=0}^{\infty} n \dot{P}_n = -(A_- - A_+)\bar{n} + A_+ \tag{4.33}$$

$$\Rightarrow \bar{n}(t) = (\bar{n}_0 - \bar{n}_s) e^{-(A_- - A_+)t} + \bar{n}_s \tag{4.34}$$

The dynamics from eq. (4.33) are immediately clear: following initialization at some temperature $\bar{n}_0$, the ion is cooled exponentially at a rate $W = A_- - A_+$ until it reaches some steady-state temperature $\bar{n}_s$ set by the competition between our SBC and off-resonant heating. Explicitly, this steady state temperature $\bar{n}_s$ is:

$$\bar{n}_s = \frac{A_+}{A_- - A_+}. \quad (4.35)$$

If we assume that SBC brings $\bar{n} \approx 0$ such that the population is mostly in the $n = 0, 1$ states, then eq. (4.35) can be evaluated explicitly using eqs. (4.28) and (4.30) to give:

$$\bar{n}_s = \left(\frac{1}{4} + \xi \frac{\eta_{10}^2}{\eta_{20}^2} \right) \frac{\tilde{\Gamma}^2 + 2\Omega_0^2 \eta_{20}^2}{4\omega_0^2} \quad (4.36)$$

For large ω_0 , the contribution of blue sideband absorption is negligible, allowing the cooling rate W to be simplified in the small n regime as

$$W \approx \tilde{\Gamma} \frac{\Omega_0^2 \eta_{20}^2}{2\Omega_0^2 \eta_{20}^2 + \tilde{\Gamma}^2}. \quad (4.37)$$

For a fixed value of $\Omega_0 \eta_{20}$, the cooling rate is then maximized:

$$W_{\max} = \frac{\Omega_0 \eta_{20}}{2\sqrt{2}} \quad \text{for} \quad \tilde{\Gamma} = \Omega_0 \eta_{20} \sqrt{2} \quad (4.38)$$

Practical Considerations From eq. (4.36), the minimum temperature can be optimized by either increasing secular frequency ω_0 or by reducing the effective quench parameter $\tilde{\Gamma}$. A higher ω_0 moves the carrier and blue-sideband transitions further away, reducing the probability of off-resonant excitation and thus the strength of the heating process. Similarly, a larger value for $\tilde{\Gamma}$ can worsen the heating by increasing spontaneous emission along $|1\rangle \rightarrow |2\rangle$ and $|1\rangle \rightarrow |0\rangle$. However, optimization of the minimum temperature has to be weighed against cost of cooling rate. Larger secular frequency values reduce the value of η and thus the

sideband coupling strength⁴, per eqs. (4.28) and (4.38), and too low a value for $\tilde{\Gamma}$ results in ineffective quenching, also diminishing the cooling rate (eqs. (4.27) and (4.37)). These tradeoffs are again present with regards to the Rabi frequency Ω_0 : in addition to its direct contribution to the minimum temperature (eq. (4.36)), $\tilde{\Gamma}$ must be increased proportionally to Ω_0 for optimal cooling, per eq. (4.38), further exacerbating the heating problem.

A practical solution to balancing these conjugate criteria is to split the SBC process into two stages: an initial “coarse” stage optimized for cooling rate, and a second “fine” stage optimized for temperature. This is particularly helpful when cooling multiple modes to reduce the influence of spectator modes, where only the target mode requires deep cooling to $\bar{n} \ll 1$.

When applying the lasers, the beam detunings need to be carefully calibrated due to the presence of stark shifts from both the quench and cooling beams, such as in eq. (4.25). If the quench detuning δ_{21} is too far red-detuned, the cooling rate becomes extremely sensitive to the detuning of the cooling laser δ_{20} [Mar+94]. For these reasons, experiments on hyperfine qubits favor pulsed SBC protocols [RQB24], which alternate between sideband pulses and quenching to reduce the effect of stark shifts.

4.3 State Detection

Though the ion does not fluoresce if projected into the dark $|D\rangle$ state, arbitrarily high detection fidelities are not possible. Firstly, despite a lack of ion fluorescence, a background signal is nevertheless measured by the detector due to scattered light (from e.g. the trap blades) and dark counts from the detector itself, which cannot in general be distinguished from the ion fluorescence. Secondly, spontaneous emission results in eventual loss of the ion state, preventing arbitrary improvement of the detection fidelity by signal integration. Together, these two factors set a fundamental limit on the detection fidelity.

Given the inherently probabilistic nature of fluorescence counts [BZ88], state detection can be formalized as a hypothesis testing problem: given a set of detector counts, we want

⁴The Doppler limit, however, is lowered in terms of the mean phonon number, balancing this effect.

to know whether the ion was most likely in the bright state $|B\rangle$, or the dark state $|D\rangle$. In this experiment, two detection techniques are used: thresholding, and maximum likelihood estimation (MLE) [Bur10]. Thresholding is a simple method for state discrimination that considers a single measurement of the total counts for a given period [Bur10,Roo00]. The simplicity of thresholding makes it easy to implement on low-latency systems, and can be straightforwardly computed for arbitrarily many ions.

The maximum likelihood method uses the additional information encoded in the arrival time of the photon counts to improve the achievable readout fidelities. The time-resolution of the photon counts further enables the incorporation of spontaneous emission using lightweight and less cumbersome expressions than were required for eq. (4.42). This technique is studied thoroughly in [Gam+07,HRW07,Bur10,Lan06]. A major advantage of the MLE method for this experiment is that it can be calculated recursively, facilitating implementation as an adaptive routine [Bur10]. In the adaptive routine, the additional readout fidelity provided by MLE is traded for speed: the likelihoods are calculated in real time until they cross a set threshold, at which point the readout is terminated. This is integral to experimental sequences where timing constraints impede “sciencing,” such as projective motional state preparation on decoherence-limited modes [Kie+16].

4.3.1 Detection Probabilities

Formally, we consider a measurement over a single readout period τ that yields a total number of counts n . The probability that the ion in state $|B\rangle$ or $|D\rangle$ produces the measurement n over τ is given by $B(n) = \mathbb{P}(n \mid B)$ and $D(n) = \mathbb{P}(n \mid D)$, respectively. As we are interested in detector counts, $B(n)$ and $D(n)$ can reasonably be assumed to be Poisson-distributed with

parameters $\tau(R_B + R_D)$ and τR_D , respectively, i.e.

$$B(n) = e^{-\tau(R_B + R_D)} \frac{(\tau(R_B + R_D))^n}{n!} \quad (4.39)$$

$$D(n) = e^{-\tau R_D} \frac{(\tau R_D)^n}{n!} \quad (4.40)$$

Dark to Bright Errors Spontaneous emission from $|D\rangle \rightarrow |B\rangle$ can be modeled by an exponential distribution with a probability density function (PDF)

$$f(t) = \frac{e^{-t/t_c}}{t_c}, \quad (4.41)$$

where $\lambda = \frac{1}{t_c}$ is the decay constant. The count distribution for a $|D\rangle \rightarrow |B\rangle$ decay at some time $t < \tau$ is given by another Poisson distribution $g(t, \tau)$ with parameter $\gamma = tR_D + (\tau - t)(R_D + R_B) = \tau R_D + (\tau - t)R_B$ ⁵. The modified dark-state count rate $D_0(n)$ can then be described using the law of total expectation [Bur10, Lan06]:

$$D_0(n) = e^{-\tau/t_c} D(n) + X_0(n) \quad (4.42)$$

$$X_0(n) = \int_0^\tau \left(\frac{e^{-t/t_c}}{t_c} \right) \frac{e^{-(\tau R_D + (\tau - t)R_B)} (\tau R_D + (\tau - t)R_B)^n}{n!} dt. \quad (4.43)$$

The first term in $D_{(n)}$ represents the count rate given no decay during τ , while the second term $X_0(n)$ represents the count rate given some decay during τ . Intuitively, the form of $X_0(n)$ can thus be understood as the average of the possible count distributions weighted by the probability of the decay event. In practice, values of $X_0(n)$ are best extracted numerically.

⁵The sum of two Poisson random variables with parameters λ_1, λ_2 is itself Poisson with parameter $\lambda_1 + \lambda_2$ [BT08].

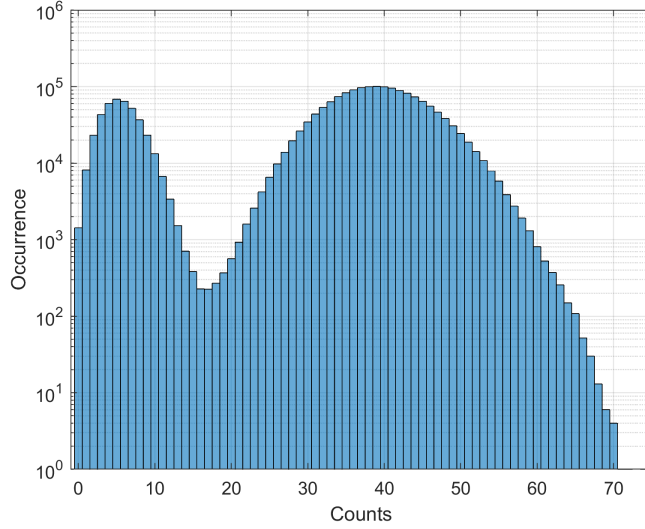


Figure 4.5: **Histogram of fluorescence counts during state detection.**

Bimodal distribution of detected photon counts over a $750\ \mu\text{s}$ readout window, accumulated over 2×10^6 experimental shots. The 397 nm beam is applied at high power and detuning $\delta \approx -\Gamma/2$, together with the 866 nm repump. Distributions for the dark $|D\rangle$ and bright $|B\rangle$ states are peaked at $n = 5, 39$, respectively, and give a discrimination threshold $n_T = 16.6$ (eq. (4.44)).

4.3.2 Thresholding

In thresholding, a threshold n_T is set such that, if $n < n_T$, the ion is assumed to be in $|D\rangle$, and if $n \geq n_T$, then the ion is assumed to be in $|B\rangle$. In general, the optimal value for n_T is found when $B(n_T | \tau) = D(n_T | \tau)$ [Bur10]. If spontaneous emission is ignored, then, using eq. (4.39), this is:

$$n_T = \frac{\tau R_B}{\ln\left(1 + \frac{R_B}{R_D}\right)} \quad (4.44)$$

If the contribution of spontaneous emission is included (i.e. dark-to-bright errors), then n_T cannot be expressed in closed-form and must be found numerically.

4.3.3 Maximum Likelihood Estimation (MLE)

We consider a series of N count observations $\{n_m \mid 1 \leq m \leq N\}$ each measured at time $\tau_m = m\tau$, with an overall measurement time $N\tau$. The likelihoods of the observations $\{n_m\}$

are then

$$p_B = \mathbb{P}(\{n_m\} \mid B) \quad (4.45)$$

$$p_D = \mathbb{P}(\{n_m\} \mid D). \quad (4.46)$$

Neglecting $|B\rangle \rightarrow |D\rangle$ effects, the bright state likelihood can be given explicitly in terms of $B(n)$:

$$\begin{aligned} p_B &= \mathbb{P}(\{n_m\} \mid B) = \mathbb{P}(n_1 \cap \dots \cap n_N \mid B) \\ &= \mathbb{P}(n_1 \mid B) \dots \mathbb{P}(n_N \mid B) \\ &= \prod_{i=1}^N B(n_i), \end{aligned} \quad (4.47)$$

where we have used the fact that the observations $\{n_m\}$ are iid.

For p_D , $|D\rangle \rightarrow |B\rangle$ must be included, which we assume can occur at most once. To simplify the analysis, we assume that the counts of a bin with a $|D\rangle \rightarrow |B\rangle$ event are simply given by $B(n)$, eliminating any dependence on $X_0(n)$; this approximation is valid when the bin times are short relative to the decay timescale, i.e. $\tau \ll t_c$ [Bur10]. The probability of a decay event over a time interval $[\tau_p, \tau_{p+1})$ for the PDF given by eq. (4.41) is

$$\begin{aligned} \mathbb{P}(\tau_p < t < \tau_{p+1}) &= \int_{\tau_p}^{\tau_{p+1}} \frac{e^{-t/t_c}}{t_c} dt \\ &= e^{-\tau_p/t_c} (1 - e^{-\tau/t_c}) \equiv g(t_p). \end{aligned} \quad (4.48)$$

Similarly, the probability of *no* decay events is $g(t > t_p) = e^{-t_p/t_c}$. Given a $|D\rangle \rightarrow |B\rangle$ event

at time τ_p , the likelihood is described by the conditional probability

$$\begin{aligned}
p_D(\{n_m\} \mid \tau_p) &= \mathbb{P}(n_1 \cap \dots \cap n_N \mid D, \tau_p) \\
&= \mathbb{P}(n_1 \cap \dots \cap n_p \mid D) \mathbb{P}(n_{p+1} \cap \dots \cap n_N \mid B) \\
&= \prod_{i=1}^p D(n_i) \prod_{j=p+1}^N B(n_j).
\end{aligned} \tag{4.49}$$

The total likelihood is given by the law of total probability, where the sum is over all possible decay times τ_m :

$$\begin{aligned}
p_D &= \sum_{p=0}^{\infty} p_D(\{n_m\} \mid t = \tau_p) g(t_p) \\
&= g(t > t_p) \prod_{i=1}^N D(n_i) + \sum_{p=0}^{N-1} g(t_p) \prod_{i=1}^p D(n_i) \prod_{j=p+1}^N B(n_j).
\end{aligned} \tag{4.50}$$

A similar approach can be used to account for the less frequent $|B\rangle \rightarrow |D\rangle$ errors.

Recurrence Relations

The forms of eqs. (4.47) and (4.50) can be simplified as a recurrence relation to make them tractable in real-time implementations. For eq. (4.47), this is straightforward:

$$p_B(N) = p_B(N-1)B(n_N), \quad p_B(N=0) = 1. \tag{4.51}$$

A similar expression for eq. (4.50) is enabled by seeing that $e^{-\tau_p/t_c} = e^{-p\tau/t_c}$, which allows eq. (4.48) to be expressed recursively

$$\begin{aligned}
p_D(N) &= F_N + AG_N, \quad \text{where} \\
F_0 &= 1, & F_N &= e^{-\delta} F_{N-1} D(n_N) \\
G_0 &= 0, & G_N &= (F_{N-1} + G_{N-1}) B(n_N),
\end{aligned} \tag{4.52}$$

and for compactness we have defined $A \equiv 1 - e^{-\delta}$, $\delta \equiv \tau/t_c$.

4.4 Implementation in $^{40}\text{Ca}^+$

We choose to trap $^{40}\text{Ca}^+$ not because it is hard, but because it is easy.

As the original aim of this thesis was to conduct QLS on a molecular ion with a co-trapped atomic ion, we are interested in as straightforward and as simple an atomic ion as possible to ease the burden of QLS. For experiments not interested in spectroscopy or novel qubit platforms, the list of atomic ion candidates is effectively restricted to the alkaline earth metals (Group 2 of the periodic table) since these have simple, Hydrogen-like structure when singly ionized. $^{40}\text{Ca}^+$ was therefore chosen due to its relative simplicity, as determined by its litany of helpful properties and institutional experience at UCLA, but also due to its very amenable mass — it is comparable to that of most simple diatomic molecules, facilitating stable trapping and efficient sympathetic cooling (see section 2.4.4). The relatively low mass of $^{40}\text{Ca}^+$ further simplifies trapping: stable Mathieu parameters are accessible at reasonable trap voltages and geometries, enabling ease of manufacture and reduced decoherence [Bro+15].

4.4.1 Neutral ^{40}Ca

Before $^{40}\text{Ca}^+$ ions can be explored, they must first be loaded and trapped. Calcium ions can be safely and easily produced since the ^{40}Ca isotope of Calcium is 96.941% abundant in nature [SM05], non-radioactive⁶ [Kon+21], and non-toxic [HP06]⁶.

Neutral ^{40}Ca can be photoionized using a resonance-enhanced two-photon process [Gul+01]: neutral ^{40}Ca is first excited from the $4s^2\ ^1\text{S}_0 \rightarrow 4s4p\ ^1\text{P}_1$ state by a resonant 423nm photon, then photoionized by excitation into the continuum by a photon with $\lambda < 389\text{ nm}$ (i.e. above the ionization threshold). This protocol is isotope selective as the isotope shift of the 423nm transition is 394 MHz for the nearest isotope, ^{42}Ca [Luc+04].

⁶For which my mother is no doubt thankful.

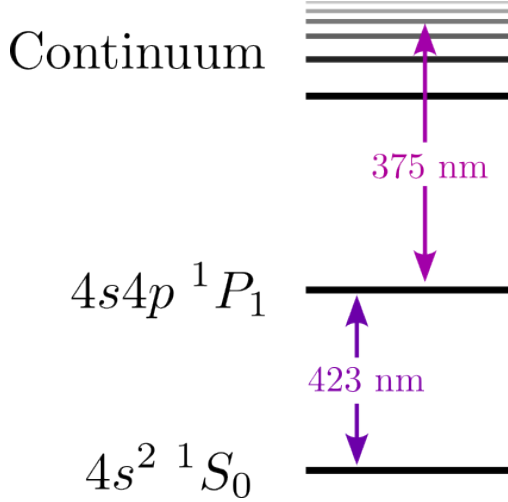


Figure 4.6: **Two-step resonance-enhanced photoionization of neutral ^{40}Ca .**

A 423 nm photon resonantly drives the $4s^2 \ ^1S_0 \rightarrow 4s4p \ ^1P_1$ transition, after which a second photon at 375 nm (above the 389 nm ionization threshold) excites the electron into the continuum.

4.4.2 Level Structure of $^{40}\text{Ca}^+$

$^{40}\text{Ca}^+$ has an electronic configuration $[\text{Ar}] 4s$ and is isoelectronic with Potassium. The atomic structure of $^{40}\text{Ca}^+$ (fig. 4.7 and table 4.1) is simplified by a lack of nuclear spin and thus hyperfine structure.

Key parameters of interest are the transition wavelengths, lifetimes, branching ratios for each state (Table 4.1). In the low magnetic-field regime, the Zeeman splittings of each state are separated within an [Acousto-Optic Modulator \(AOM\)](#) range (~ 100 MHz), allowing all optical transitions to be addressed by a single AOM. The key advantages of the $^{40}\text{Ca}^+$ level structure are its ease of access by (relatively) cheap and robust diode lasers and its practical ease of use in the laboratory. Prêt-à-porter laser systems for $^{40}\text{Ca}^+$ are common and available from e.g. Toptica [14]. With the exception of the transitions at 866nm and 854nm, all frequencies are within the visible spectrum and can thus be easily aligned by eye⁷, and are not too far in the UV that they result in excessive charging of the trap or damage optics.

The qubit is defined on the $|^2S_{1/2}, m_J = -1/2\rangle \leftrightarrow |^2D_{5/2}, m_J = -5/2\rangle$ transition, selected as it has the largest Clebsch-Gordan factor of all transitions in the $^2S_{1/2} \leftrightarrow ^2D_{5/2}$ manifold. This transition has the advantage over the $|^2S_{1/2}, m_J = +1/2\rangle \leftrightarrow |^2D_{5/2}, m_J = +5/2\rangle$ transition in that any sidebands from the other m_J states in the manifold will be red-

⁷The “reasonably visible” spectrum is between 350nm - 780nm, though visibility of wavelengths below 380nm is generally due to downconverted fluorescence.

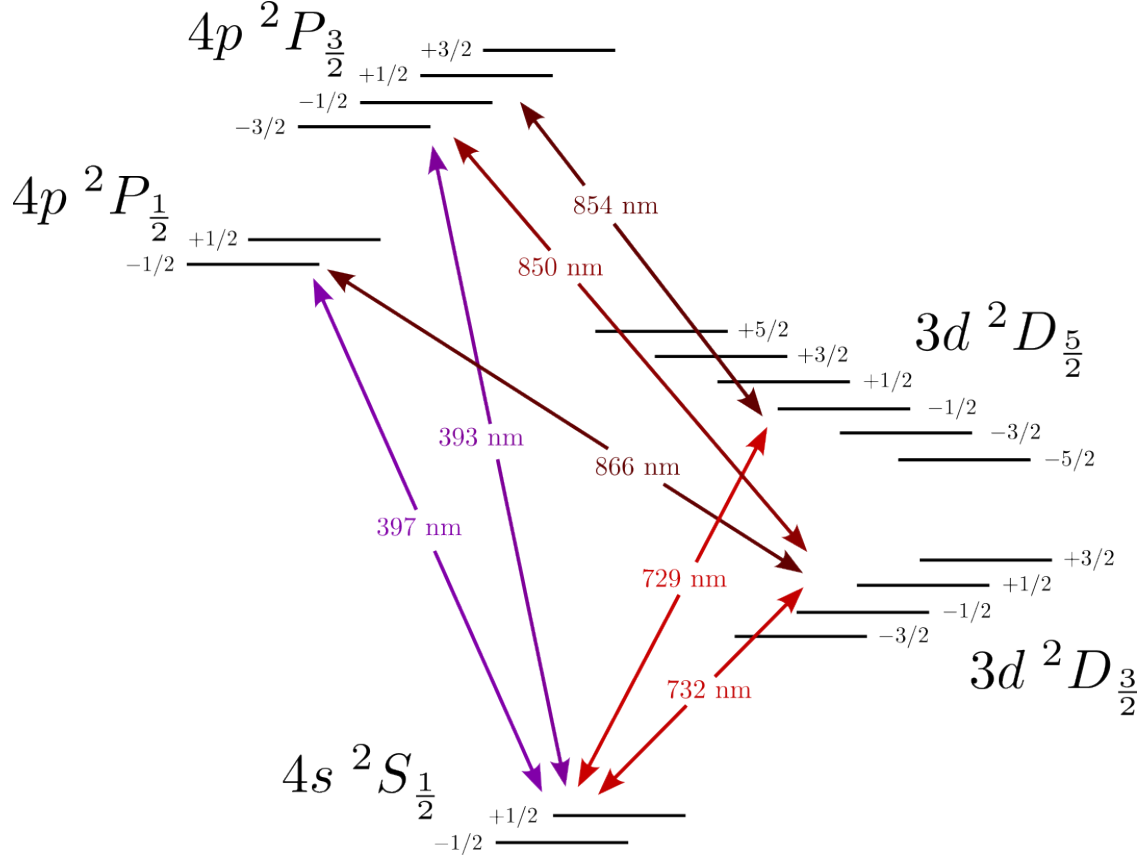


Figure 4.7: **Level structure of $^{40}\text{Ca}^+$.**

The $^2S_{1/2} \leftrightarrow ^2P_{1/2}$ (397 nm) dipole transition is used for cooling, detection, and state preparation; the $^2D_{3/2} \leftrightarrow ^2P_{1/2}$ (866 nm) and $^2D_{5/2} \leftrightarrow ^2P_{3/2}$ (854 nm) transitions are used for repumping/quenching; and the narrow $^2S_{1/2} \leftrightarrow ^2D_{5/2}$ (729 nm) quadrupole transition is used to define the optical qubit. For completeness, the $^2S_{1/2} \leftrightarrow ^2P_{3/2}$ (393 nm), $^2D_{3/2} \leftrightarrow ^2P_{3/2}$ (850 nm), and $^2S_{1/2} \leftrightarrow ^2D_{3/2}$ (732 nm) transitions are shown, though not used in this experiment. Key values are provided in table 4.1.

detuned, preventing their excitation once the ion has been cooled into the ground state of motion [Roo00].

⁻¹[Mei+20]

⁻²[Ger+08]

⁻³[Het+15]

⁻⁴[Ram+13]

⁻⁵[Sha+17]

⁻⁶[Sha+16]

⁻⁷The remainder decays to the $^2D_{3/2}$ via a heavily suppressed M1 transition.

State	Lifetime	Transition	Wavelength (air, nm)	Branching Ratio (%)
$^2P_{3/2}$	6.639(42) ns ⁻¹	$^2D_{5/2}$	854.209	5.87(2) ⁻²
		$^2D_{3/2}$	849.802	0.661(4) ⁻²
		$^2S_{1/2}$	393.366	93.47(3) ⁻²
$^2P_{1/2}$	6.904(26) ns ⁻³	$^2D_{3/2}$	866.214	6.435(7) ⁻⁴
		$^2S_{1/2}$	396.847	93.565(7) ⁻⁴
$^2D_{5/2}$	1.1649(44) s ⁻⁵	$^2S_{1/2}$	729.147	99.8(12) ⁻⁵⁻⁷
$^2D_{3/2}$	1.195(8) s ⁻⁶	$^2S_{1/2}$	732.289	N/A

Table 4.1: **Level structure values for $^{40}\text{Ca}^+$.**

Wavelengths presented are for propagation in air. All values are experimentally measured in the literature. Negative exponents here are a L^AT_EX workaround for intra-table footnotes, and do not indicate numerical exponents.

4.4.3 State Preparation of $^{40}\text{Ca}^+$

Preparation of the $|^2S_{1/2}, m_J = -1/2\rangle$ state for the $|^2S_{1/2}, m_J = -1/2\rangle \leftrightarrow |^2D_{5/2}, m_J = -5/2\rangle$ optical qubit transition can be implemented using either the 729nm pathway, or via a σ^- -polarized 397nm beam.

729nm Optical Pumping

The 729nm pathway excites the $|^2S_{1/2}, m_J = +1/2\rangle \leftrightarrow |^2D_{5/2}, m_J = -3/2\rangle$ transition, which is simultaneously quenched into the $^2P_{3/2}$ state via an 854nm repump. This state population then decays into either $^2S_{1/2}$ sublevels (though preferentially into the $m_J = -1/2$ due to selection rules and Clebsch-Gordan coefficients) gradually “pumping” population into the desired $|^2S_{1/2}, m_J = -1/2\rangle$ state. The primary advantages of the 729nm pathway are that it is frequency-resolved, which suppresses effects from imperfect polarization control, and that it operates in the resolved-sideband and Lamb-Dicke regimes, minimizing spurious heating. Practically, the 729nm pathway has minimal overhead, as it can be implemented using the same beam used to address the $|^2S_{1/2}, m_J = -1/2\rangle \leftrightarrow |^2D_{5/2}, m_J = -5/2\rangle$ qubit transition. However, excitation to the $^2D_{5/2}$ state is slow, such that the overall procedure operates on

the millisecond timescale [Ram14], and is therefore not implemented in this experiment.

σ^- -397nm Optical Pumping

The σ^- -polarized 397nm pathway is used in our experiment. The $|^2S_{1/2}, m_J = +1/2\rangle \leftrightarrow |^2P_{1/2}, m_J = -1/2\rangle$ transition is excited by a σ^- -polarized 397nm beam to gradually “pump” population into the desired $m_J = -1/2$ sublevel. Use of the dipole $^2S_{1/2} \leftrightarrow ^2P_{1/2}$ transition enables faster optical pumping at the $\sim 10 \mu s$ timescale. As Zeeman sublevels of the $^2S_{1/2}$ are poorly resolved at low quantization fields (such as in our experiment), the procedure relies on polarization to selectively excite the $|^2S_{1/2}, m_J = +1/2\rangle$ sublevel, which is complicated by window birefringence⁸. To achieve the requisite polarization control, the σ^- -397nm beam is directed along the quantization axis to suppress π -transitions; this prepares the $|^2S_{1/2}, m_J = -1/2\rangle$ in $\approx 20 \mu s$ with fidelity $F \approx 0.99$.

4.4.4 Cooling of $^{40}\text{Ca}^+$

Doppler Cooling

Doppler cooling in $^{40}\text{Ca}^+$ is conducted on the $^2S_{1/2} \leftrightarrow ^2P_{1/2}$ transition at 397nm. Due to the $\approx 7\%$ branching ratio from $^2P_{1/2} \rightarrow ^2D_{3/2}$, a repump at 866nm is simultaneously applied to repump population out of the $^2D_{3/2}$ state. This 866nm repump is slightly blue-detuned by a few MHz to prevent coherent population trapping (CPT) due to interference between the probability amplitudes of the $^2S_{1/2}$, $^2P_{1/2}$, and $^2D_{3/2}$ states. Additionally, to prevent unintended pumping into the $^2D_{5/2}$ state by 393nm components in the 397nm beam, the 854nm beam is similarly applied.

The $^2P_{1/2}$ state has a decay rate $\Gamma = 23.05(17) \times 2\pi \text{ MHz}$ [Het+15], which sets a Doppler limit $T_D = 0.387 \text{ mK}$. For a typical trap frequency $\omega_0 \approx 1 \times 2\pi \text{ MHz}$ used in our experiment, this equates to a mean phonon number $\bar{n} \approx 8.1$, and a Lamb-Dicke parameter⁹

⁸These are numerous in our experiment due to the cryogenic shields.

⁹Assuming the 729nm beam is parallel with the mode of interest

$\eta^2 \bar{n} = \left(|\mathbf{k}_{729}| \sqrt{\frac{\hbar}{2m_{\text{Ca}} \omega_0}} \right)^2 \bar{n} \approx 0.097^2 \times 8.1 = 0.076$ for the 729 nm $^2\text{S}_{1/2} \rightarrow ^2\text{D}_{5/2}$ transition in $^{40}\text{Ca}^+$. Doppler cooling therefore initializes the ion into the Lamb-Dicke regime $\eta^2 \bar{n} \ll 1$ and is sufficient for coherent operations on the carrier.

In experiment, Doppler cooling was typically applied for 1 ms with a detuning between $[-1, -0.5] \Gamma$ and a saturation parameter $s \approx 0.1$ for the 397nm beam. All cooling parameters were tuned by fitting Rabi oscillations on the carrier transition to extract the temperature (see section 3.5.3).

Sideband Cooling

Sideband cooling is conducted on the same $|^2\text{S}_{1/2}, m_J = -1/2\rangle \leftrightarrow |^2\text{D}_{5/2}, m_J = -5/2\rangle$ transition as for qubit operations. Again, the red detuning of this transition relative to the other Zeeman sublevels in the manifold is helpful to prevent accidental heating on a blue-detuned transition. Quenching is done using the $^2\text{D}_{5/2} \leftrightarrow ^2\text{P}_{3/2}$ transition at 854nm, where selection rules dictate that population is transferred into the $|^2\text{P}_{3/2}, m_J = -3/2\rangle$ sublevel. Likewise, the $|^2\text{P}_{3/2}, m_J = -3/2\rangle$ sublevel decays into the $|^2\text{S}_{1/2}, m_J = -1/2\rangle$ sublevel, forming a closed cycling transition. Using eq. (4.36) with typical values $\tilde{\Gamma} = \Omega_0 \eta_{20} \sqrt{2}$, $\Omega_0 \eta_{20} = 50 \times 2\pi$ kHz, $\omega_0 \approx 1 \times 2\pi$ MHz, we find a minimum SBC temperature of $\bar{n}_s \approx 4 \times 10^{-3}$, which is below the reasonable detection threshold in experiment.

Leakage During SBC Leakages outside the SBC manifold are primarily due to decays of the $^2\text{P}_{3/2}$ state into both the D-states. When population is transferred into the $^2\text{P}_{3/2}$ state, a small fraction decays into the $^2\text{D}_{3/2}$ state, requiring use of the 866nm repump to return population into the SBC manifold. During repumping by the 866nm, population is returned to the $^2\text{P}_{1/2}$ state, which can decay into the $|^2\text{S}_{1/2}, m_J = +1/2\rangle$ state and halt cooling. However, this leakage is suppressed by the small value of the $^2\text{P}_{3/2} \rightarrow ^2\text{D}_{3/2}$ branching ratio.

A more significant source of leakage is the $^2\text{P}_{3/2} \rightarrow ^2\text{D}_{5/2}$ decay due to its non-negligible branching ratio. Though decay into the $^2\text{D}_{5/2}$ state is returned to the $^2\text{P}_{3/2}$ state by the

854nm quench beam, sublevels outside the cooling manifold can again be populated, enabling decays to the $|^2S_{1/2}, m_J = +1/2\rangle$ state.

The leakage problem is fixed by pumping the $|^2S_{1/2}, m_J = +1/2\rangle$ sublevel back into the $|^2S_{1/2}, m_J = -1/2\rangle$. This process is most comfortably done, though not in this experiment, by spin polarization with the 729nm on the $|^2S_{1/2}, m_J = +1/2\rangle \leftrightarrow |^2D_{5/2}, m_J = -3/2\rangle$ optical qubit transition, preferred for its resolved sidebands which mitigate spontaneous emission-induced heating [Ram14]. However, we find that spin polarization with the σ^- -polarized 397nm beam does not result in significant heating despite imperfections in the polarization and has the added advantage of being fast enough to add negligible timing overheads.

Pulse Sequence Sideband cooling was implemented using a continuous protocol. While a pulsed protocol was initially used, it was later deprecated due to its slower cooling rate.

Our implementation of continuous **SBC** consisted of two stages: a first stage designed to maximize cooling efficiency, and a second “polish” stage that cooled at low power. The first stage consisted of a series of cooling cycles; within each cycle, the 729 nm and 854 nm beams were applied to cool each mode for a specified fraction of the cycle time. The 729 nm frequency and 854 nm power were set independently for each mode to optimize the cooling efficiency. To return leakage into the $S_{1/2} \leftrightarrow D_{5/2} \rightarrow P_{3/2}$ sideband cooling manifold, the σ^- -397 nm and 866 nm beams were applied periodically. The 729 nm and 854 nm beams remained on during repumping; the repump periodicity was independent of the cycle time to prevent complex failure modes¹⁰. To ensure that unintended off-resonant heating caused by the spin polarization beams was cooled away, no spin polarization pulses were scheduled within 100 μ s of the end of the first stage.

In the second stage, the 729 nm was applied at low power to cool a single mode, minimizing off-resonant excitation of spectator transitions and thereby optimizing the final temperature

¹⁰For example, if spin polarization was only applied at the end of every cycle, then the last mode of the cycle would likely have a higher temperature due to unintended off-resonant heating from the σ^- -397 nm beam.

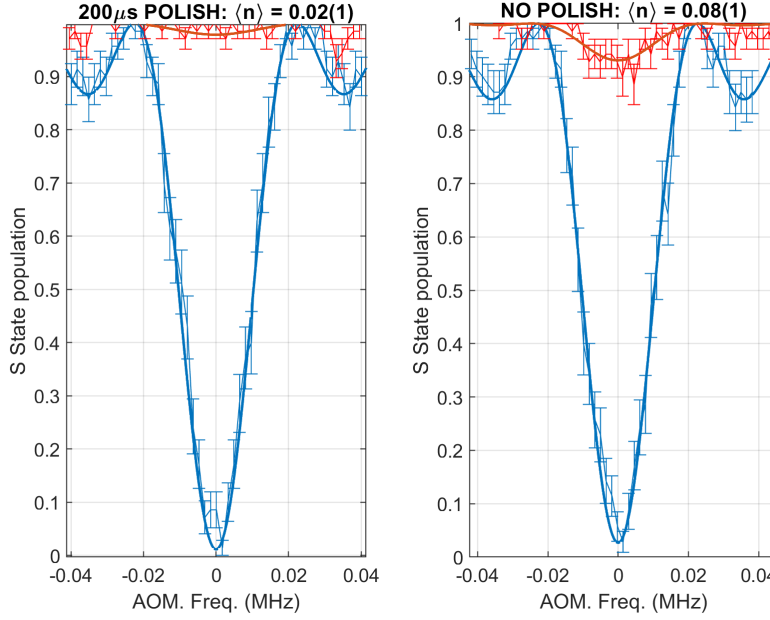


Figure 4.8: **Effect of low-power “polish” Sideband Cooling on final temperature.** Sideband Ratio thermometry on the axial mode at $\omega_z = 702 \times 2\pi$ kHz with (left) and without (right) a 200 μ s low-power “polish” stage following the main, maximum power SBC sequence. The Red Sideband and Blue Sideband spectra are fitted (solid lines) and compared to extract the mean phonon number, following eq. (3.41). In the polish stage, the 729 nm Rabi frequency is halved and the quench amplitude reoptimized for minimum temperature (see eq. (4.36)), suppressing off-resonant spectator excitation to reach $\langle n \rangle = 0.02(1)$, compared to 0.08(1) without. Error bars denote one standard error.

(see eq. (4.36)); the 729 nm frequency and 854 nm power were independently tuned in this stage. A final spin polarization sequence was applied after the polish stage; this was found necessary to achieve state-preparation fidelities $\gtrsim 95\%$.

Optimization Parameters for SBC were tuned using thermometry with either the Sideband Ratio (SBR) or Rapid Adiabatic Passage (RAP) readout techniques. Though low temperature phonon measurements are typically done with SBR, we favored RAP for its robustness, single-shot operation, and higher sensitivities over SBR (applicable only in the small $\bar{n}$ regime). When tuning the parameters, SBC was applied only for a short time (~ 1 ms) to maximize the readout sensitivity. Since the temperature decays exponentially as cooling is applied, application of SBC for too long resulted in poor sensitivities.

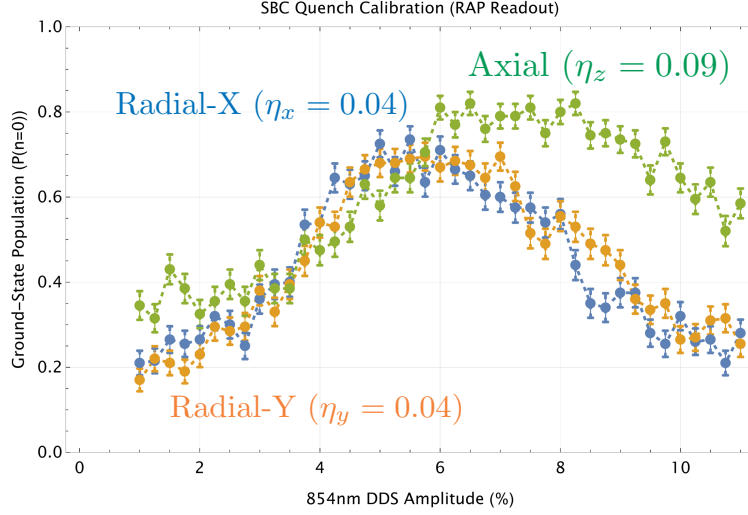


Figure 4.9: **Optimization of the 854nm quench for Sideband Cooling (SBC).** Ground-state population $P(n=0)$, measured by Red Sideband RAP readout, as a function of the 854nm DDS amplitude for the three motional modes. The cooling rate (eq. (4.38)) is optimized on each mode by tuning the quench amplitude to their different Lamb-Dicke Parameters.

Optimization of the 854 nm power was critical for *efficient* sideband cooling — from eqs. (4.36) and (4.38), the cooling rate is maximized at $\tilde{\Gamma} = \Omega_0 \eta_{20} \sqrt{2}$, in contrast to the minimum temperature $\bar{n}$, which is a minimum only at $\tilde{\Gamma} = 0$. The 854nm power was separately optimized for each mode due to their different Lamb-Dicke parameters; typical calibrations for the three motional modes are shown in Figure 4.9. Optimal 729nm frequencies were slightly red-detuned from their measured values (~ -2 kHz) due to the stark shift from the 854nm.

When cooling large numbers of modes (e.g. of a multi-ion chain), faster spin polarization periodicity was necessary, as cooling of more modes involved more emission from the $^2P_{3/2}$ state, increasing the likelihood of leakage outside the cooling manifold. While use of sideband cycles is preferred by [Roo00] to prevent heating of modes while others are being cooled, this was not a significant factor in our experiment due to the low heating rates ($\dot{\bar{n}} \approx 3$ quanta/s).

“Taste classifies, and it classifies the classifier.”

Pierre Bourdieu, *A Social Critique of the*

Judgement of Taste

Chapter 5

Experimental Implementation

5.1 Main Apparatus

Contributions The ion trap, cryogenic system, vacuum, and loading systems were almost entirely assembled by [GDM](#)¹, with supervision by [HW](#). Initial technical advice was provided by [CS](#). [HMK](#) and [RA](#) were sources of technical discussion for cryogenics.

Further details can be found in [\[Mit26\]](#).

5.1.1 The Ion Trap

The starting design for the trap was sourced from Hartmut Häffner’s group at Berkeley, which was in turn sourced from Rainer Blatt’s group at the University of Innsbruck. Significant modifications were made to reduce the complexity of manufacturing and assembly with the aim of improving finished quality² in the final trap. The result was a trap thematically similar to the [Linear Quadrupole Trap \(LQT\)](#) previously used in the MOTion experiment at UCLA [\[Sch+16,Sch+14a\]](#), but with rods replaced by trap blades archetypal of the Innsbruck design [\[SQ17,Gul03\]](#) for tighter trapping and better-resolved sidebands.

Material and component selection was guided by cryogenic compatibility (thermal con-

¹With lightning speed and deftness as well.

²And, presciently, cost-effectiveness.

ductivity and thermal contraction) and machinability. The trap was composed of two key components: a trap frame and a set of modular trap blades, both machined from Beryllium Copper (BeCu) (fig. 5.1). Blades were fastened to the frame using Nickel-coated M2 stainless steel screws, with MACOR insulators and sapphire spacers³ providing electrical insulation between the blades and frame. As in the [Linear Quadrupole Trap \(LQT\)](#), axial confinement was provided by a pair of endcap blades, geometrically identical to the radial blades and positioned on opposite ends of the trap axis; the use of a single blade geometry for both radial and axial electrodes significantly reduced manufacturing cost and lead times compared to conventional conical endcap electrodes.

The trap was quoted to be machined using a wire-EDM; however, given the quoted cost and part quality, the actual fabrication method is uncertain. The trap was manually assembled with the radial ion-electrode distances (r_0) matched within 2.6% (fig. 5.1).

5.1.2 Cryogenics

A Stinger⁴ closed-cycle cryogenic system was used to cool the trap to $\sim 4 - 10$ K. The core aims of the Stinger system were to achieve sufficient cooling capacity without introducing vibrations, and to preserve Helium (i.e. closed-cycle) for cost-effective long-term operation. The Stinger met these aims by using a Gifford-McMahon (GM) cryocooler for high-capacity (0.9 W at 4 K), closed-cycle operation, and Helium exchange gas for vibrationless cooling of a coldplate (i.e. the thermal load – the ion trap mount).

The Stinger system was composed of 4 subsystems⁵, depicted in Figure 5.2: (a) a cryocooler⁶, (b) a low-pressure (100 psi) Helium exchange gas loop, driven by a recirculator⁷, (c) a high-pressure (350 psi) Helium compressor⁸, and (d) the Stinger bayonet, a bellows assembly which carried Helium from the GM cryocooler into the chamber. High-pressure

³Selected for its favorable thermal conductivity *at low temperatures*.

⁴ColdEdge Technologies

⁵Though the Stinger bayonet is technically part of the cryocooler, its operation is best treated separately.

⁶SHI Model SRDE-418 Gifford-McMahon

⁷Sumitomo F-20

⁸Sumitomo F-70

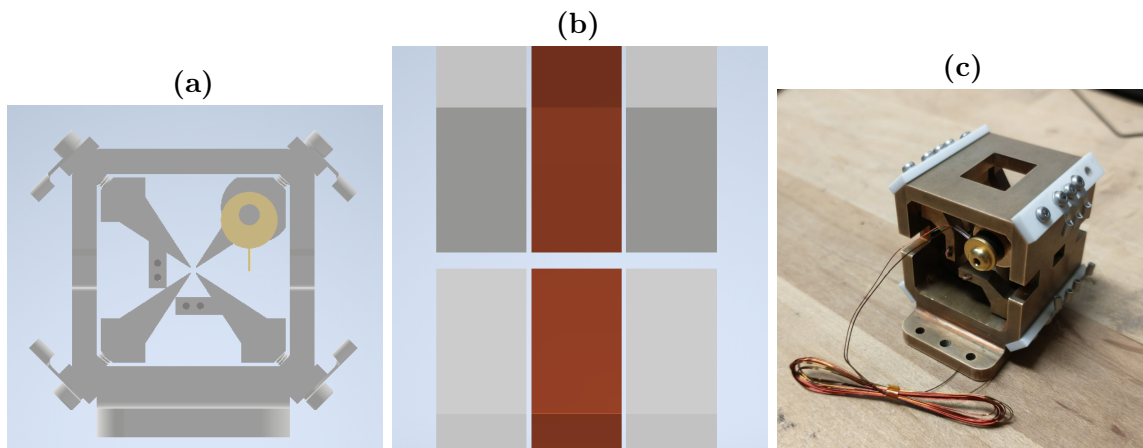


Figure 5.1: **Innsbruck-style blade trap used in experiment.**

(a, b) CAD render of the ion trap (front view, side view). (c) Assembled ion trap. Trap blades are mounted directly into a monolithic enclosure with sapphire spacers for electrical insulation; all trap components are machined from Beryllium Copper. The trap RF is applied on two diametrically opposed radial electrodes (a: upper right, bottom left) that are shorted together; the EGGS RF is applied on the other pair of radial electrodes (a: upper left, bottom right) and are separately addressable. Axial confinement is generated by a set of blades geometrically identical to the radial blades, and positioned at both ends of the trap (b: grey). Two perpendicular shim electrodes (visible in a) are used to translate the ion within the trap for micromotion compensation. A temperature diode (yellow; visible in a, c) is mounted directly onto the trap for cryogenic diagnostics.

Helium from the compressor was used by the GM cryocooler to cool the low-pressure Helium gas from the recirculator to ≈ 6 K. The cold, low-pressure Helium was then sent into the mechanically-isolated Stinger bayonet for delivery into the lower experiment chamber via a custom insertion flange (see section 5.1.3). At the tip of the Stinger, a capillary tube further cooled the gas to 4 K via Joule-Thomson expansion, which in turn cooled the coldplate to ≈ 4 K and a radiation shield to ≈ 20 K. Measurements of Stinger-induced vibrations with an interferometer revealed an RMS displacement ≈ 25 nm when the system received sufficient mechanical support⁹.

The system was monitored by a temperature controller¹⁰ using resistance temperature detectors (RTDs). Temperature readings during experimental operation were found to be unreliable due to significant RF pickup and grounding issues. Furthermore, in-chamber RTDs

⁹See GDM's forthcoming thesis.

¹⁰Lakeshore Model 336

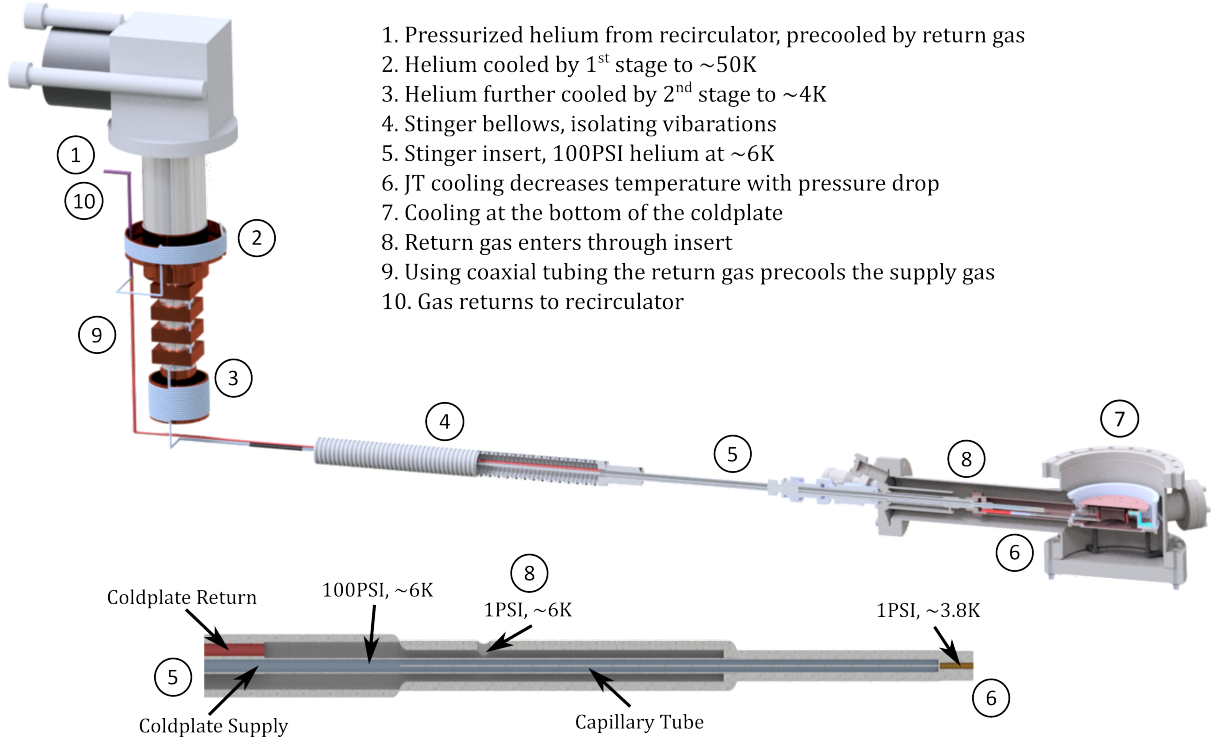


Figure 5.2: **Overview of the Stinger cryogenic system.**

were disconnected from the controller when they were discovered to be a significant vector of coherence-destroying disease¹¹ (see section 3.4). The temperature controller was also used to power resistive, $50\ \Omega$ ¹² heaters¹³, which were used to expedite system warmup. During warmup, both heaters were operated at 0.3 A, which reduced the required warmup times from approximately 3 days to half a day. However, prolonged use of the heaters saw an increase in their resistance over time, resulting in their eventual failure. Additionally, connection of the heaters to the controller was found to result in a $> 10\ \text{dB}$ attenuation for $\approx 80\ \text{MHz}$ signals coupled to the EGGs electrodes despite electrical isolation of the heaters from the chamber system; the cause of these issues are currently uncertain.

Shielding Two layers of shielding were used to thermally and optically isolate the ion trap from the room-temperature vacuum chamber: a radiation shield, which formed the outermost

¹¹For which they are given the very appropriate name, *Anopheles*.

¹²The heaters were $50\ \Omega$ as the controller was intended for cryogenic temperature servos.

¹³Lakeshore HTR-50

layer, and the 4K stage, which housed the ion trap. To minimize direct line of sight from the chamber walls to the ions, through-holes in the shields for wiring and vacuum conductance were laterally offset between layers where possible.

5.1.3 Vacuum Apparatus

The vacuum system was composed of two chambers: a custom base chamber fabricated as part of the cryogenic system¹⁴, and an upper chamber used to provide the experimental volume for the cryogenic shields and ion trap.

The base chamber had a welded 4.5" CF nipple to accept the Stinger cryogenic insert, as well as four 2.75" CF flanges, which were used to accommodate all vacuum pumps and gauges: a titanium-sublimation pump¹⁵, a combined NEG/ion pump¹⁶, a [Residual Gas Analyzer \(RGA\)](#)¹⁷, and an angle valve connected to a CF 2.75" vacuum tee (for turbopump access) with a magnetron pressure gauge¹⁸.

The upper chamber was an 8" spherical octagon¹⁹ connected to the 8" CF flange atop the base chamber, which made available one 8", eight 2.75", and sixteen 1.33" CF flanges. The top 8" flange was fitted with a reentrant viewport for imaging (see section 5.2.3), while seven of the eight 2.75" flanges were fitted with viewports for laser access. The remaining viewport was connected to the loading systems via a 2.75" tee, with one arm connected to the H^{35}Cl^+ leak valve system (see section 5.1.4) via a VCR bellows, and the other arm connected to the $^{40}\text{Ca}^+$ oven²⁰ via a gate valve to protect the oven from exposure to atmosphere (from e.g. opening the vacuum chamber).

Seven of the sixteen 1.33" CF flanges were used for electrical feedthroughs: two were used to power resistive heaters²¹ for expediting warmups, two were used for resistance temperature

¹⁴ColdEdge Stinger

¹⁵Varian & Associates

¹⁶SAES NexTorr Z200

¹⁷Stanford Research Systems RGA200

¹⁸Agilent FRG-702

¹⁹[Kimball Physics MCF800-ExtOct-G2C8A16](#)

²⁰[AlfaVakuo AlfaSource](#)

²¹Lakeshore Cryotronics HTR-50

detectors (RTDs)²², and the remaining three were used to provided the trap voltages. DC electrode voltages were delivered at a single feedthrough via a custom feedthrough PCB²³ (see section 5.3.5).

External heaters were mounted at the top imaging viewport and the DC voltage feedthrough to prevent condensation.

Pumping

Following initial assembly, the chamber was baked at low temperature (120 °C) in a homebuilt oven for a period of 12 days; the maximum temperature was practically limited by the magnetron gauge (120 °C) and its cables. Bakeouts were not performed despite later openings of the vacuum chamber due to the powerful cryopumping from the cryogenics. If necessary, a bakeout can be performed using the same resistive heaters used to expedite warmup — over the weekend of 2025/10/24, the heaters were accidentally²⁴ left on after reaching room temperature during a system warmup, which persisted until the system reached ≈ 100 °C. Despite the great panic this caused, no lasting effects are believed to have resulted.

The chamber was pumped to UHV ($\approx 10^{-8}$ mbar) using a turbomolecular pump system²⁵ comprised of a turbomolecular pump²⁶ backed by a scroll pump²⁷, which was removed during experiment operation (i.e. when cold). Continuous passive pumping was provided by the combined NEG/ion pump²⁸. The Ti-sub pump was used only when vacuum pressure was high and the system was actively turbopumped due to concerns about Hydrogen outgassing from its filaments; this invariably was the case only at room temperature, as opposed to when cryogenic, which limited its possible effectiveness. However, the dominant source of pumping was the cryogenics (cryopumping), though this often became saturated following

²²Lakeshore Cryotronics 3062

²³Designed by CS, assembled by GDM

²⁴It was my fault.

²⁵Agilent Turbo Pump System (TPS) Flexy 74

²⁶Agilent TwisTorr 304FS

²⁷Agilent SH-100

²⁸SAES NexTorr Z200

loading of H^{35}Cl^+ .

5.1.4 Loading (Sourcery)²⁹

$^{40}\text{Ca}^+$ Oven A 50 mg high-purity (99.5%) $^{40}\text{Ca}^+$ oven³⁰ (known otherwise as a “getter” in our group [Pur19]) was mounted in a custom insulating MACOR holder³¹ inside a 2.75" CF tee to serve as an atomic source. The MACOR mount was necessary to electrically isolate the oven from the vacuum chamber and direct the oven output towards the trap via a small hole in the radiation shield.

For loading, the oven was resistively heated by a linear regulated power supply³² to create a stream of neutral $^{40}\text{Ca}^+$ for photoionization. Thick, 12 AWG cables used to connect the oven to the power supply. The cables were fused and twisted to prevent signal pickup and overcurrent events from overheating the oven and degrading the background pressure³³.

The oven arrived filled with Argon and sealed with Indium to prevent exposure to atmosphere. To prepare it for experimental use, it was resistively heated at high current (> 4 A) until the seal was “popped,” as detected by a pressure spike on the RGA.

H^{35}Cl^+ Leak Valve The leak valve system was used to introduce dilute HCl into the experiment chamber for loading (Figure 5.3). The system comprised a secondary reservoir (a 2.5" CF nipple) for holding HCl gas in reserve, and a primary loading region (a 2.5" CF cross). A cylinder of dilute HCl gas³⁴ (50ppm) held in a Krypton balance gas³⁵ was connected by a VCR valve to the reservoir. A leak valve then connected the reservoir to the cross, and was used to load HCl at a controlled pressure into the cross, which was monitored by a

²⁹[Hud06], Chapter 4.

³⁰AlfaVakuo AlfaSource AS-Ca25-2L-050, (2mm Type L, 50 mg)

³¹Designed and manufactured by GDM.

³²GW-Instek GPP3060

³³Such an event occurred on 2022/04 and required a week-long evacuation of the chamber with a turbopump.

³⁴Airgas

³⁵More like blood gas — global conflict in Eastern Europe has created a minor crisis for Krypton gas, which is now something of a blood diamond.

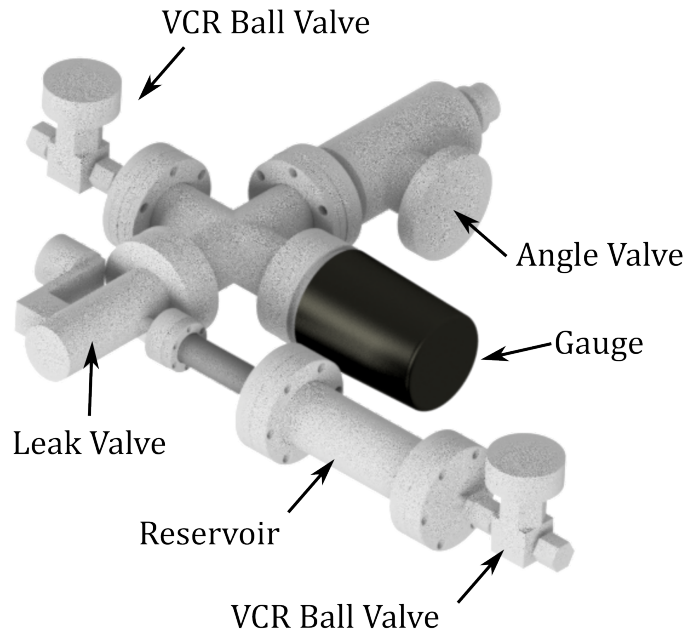


Figure 5.3: H^{35}Cl^+ leak valve system.

Dilute H^{35}Cl^+ is held in the reservoir and leaked into the cross using the leak valve and monitored on a magnetron gauge. The system connects to the main experiment chamber through the VCR ball valve (upper left), and is evacuated using a turbopump connected to the angle valve.

magnetron³⁶ gauge. An angle valve connected the cross to a turbopump pumping system³⁷, which was used to evacuate the cross and terminate loading.

5.2 Optics and Light Delivery

Contributions [DK](#) built the motorized laser shutters for the 375 nm and 423 nm lasers. [FRH](#) and [PA](#) worked on a 393 nm beampath and helped test the laser box design. [HW](#), [GDM](#), and I contributed equally to the optical beampaths. The imaging system was assembled by [GDM](#). [HW](#) designed and implemented the 243nm generation scheme and maintained it with help from [GDM](#). [HH](#) assisted in the creation of the magnetic field apparatus and extended the laboratory architecture.

³⁶Agilent FRG702

³⁷Agilent Turbo Pump System (TPS) Flexy 74

5.2.1 Overview

Each laser is physically implemented in two areas: an optical conditioning stage on the optical table(s), where light is prepared for experiment delivery, and an experiment stage where the light is delivered to the ion. The optical conditioning stages are outlined in sections 5.2.4 to 5.2.6. This section provides an overview of the experiment stage.

Here, all optics are mounted on breadboards³⁸ and raised on 1" posts to meet the height of the trap. Beampaths are matched to chamber viewports due to the optical coatings used. For reference purposes, chamber windows are numbered clockwise with each edge.

397nm Delivery The 397nm main and σ^- lasers are outcoupled³⁹ and combined on a 50/50 beamsplitter⁴⁰ prior to chamber entry via window 4 (EUV-coated, Edmund Optics). The rejected light from the 50/50 beamsplitter is directed onto a photodiode⁴¹ for monitoring and experiment sequence diagnostics; for its diagnostic prowess, this photodiode is lovingly referred to as “Gregory House, M.D.”⁴²

This beampath is aligned with the quantization axis of the trap to enable robust state preparation into the $|^2S_{\frac{1}{2}}, m_J = -\frac{1}{2}\rangle$ state by the σ^- 397nm beam. The polarization of the σ^- pass is sanitized prior to combination by an HWP and PBS with an additional HWP and QWP to create the required σ^- polarization relative to the quantization axis. To compensate for the birefringence of the viewport windows, a polarimeter⁴³ was used to tune the waveplates by comparing the polarization before and after the chamber (fig. 5.4).

The combined beams are focused using their outcouplers together with a single lens then aligned to the ion position. A flip mirror is installed prior to the lens to create an alternative beampath for alignment with the 243nm beam; this pass is blocked during normal operation

³⁸Thorlabs

³⁹Thorlabs CFC5-A

⁴⁰Union-Optic

⁴¹Thorlabs DET36A2

⁴²The oscilloscope used to monitor the photodiodes is referred to with similar affection as the “Princeton-Plainsboro Teaching Hospital.”

⁴³Thorlabs PAX1000VIS

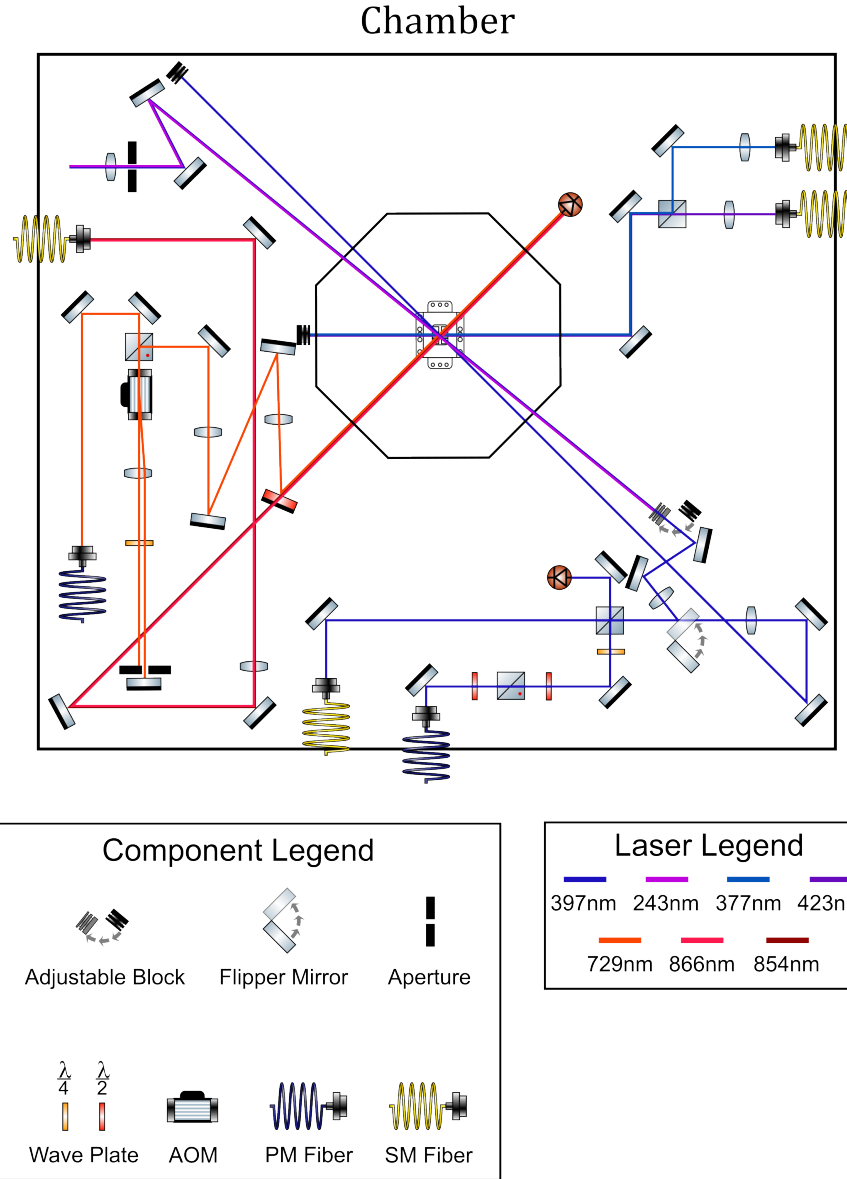


Figure 5.4: **Beampaths for delivery of light into the chamber.**

Top view of optics used to align beams into the chamber; beampaths are matched to viewport coatings (see Figure 5.5). All optics are mounted on a mezzanine level for vertical alignment into the trap.

to safely dump the 243nm beam.

729nm + 854nm + 866nm Delivery The 729nm light is combined with the IR (854nm + 866nm) beams on a dichroic mirror⁴⁴ immediately before chamber entry via viewport 6

⁴⁴Union Optic

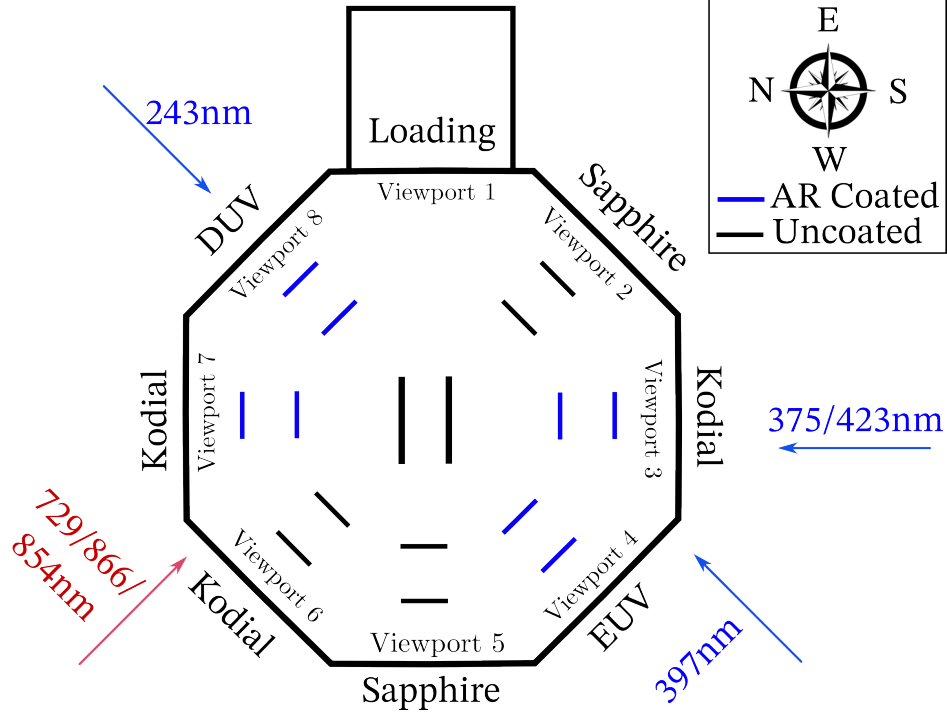


Figure 5.5: **Layout of optical viewports in the experiment chamber.** Viewports are numbered clockwise for reference throughout this chapter.

(Kodial). The IR beams are focused using the outcoupler and a single lens and aligned to the trap center prior to combination on the dichroic.

The 729nm beam is passed through a motorized⁴⁵ HWP then sent into a double-pass AOM. Light reflected from the PBS in the double-pass is measured on a photodiode⁴⁶ to monitor the polarization stability of the 729nm light. Though this double-pass AOM implementation is now redundant due to the double-pass present in the injection lock system, it remains active due to experimental priorities. Following the double-pass AOM, a two-lens telescope minimizes the beam waist at the ion and increases the achievable Rabi frequencies. A 729nm beam waist (measured as the FWHM) of $w_0 = 57 \mu\text{m}$ at the ion position is obtained by measuring the Rabi frequency on the optical qubit transition as a function of ion position. This is suboptimal, though robust to ion position drifts; improvements to the focusing optics would require removal of the double-pass AOM to maximize the available optical real estate.

⁴⁵Thorlabs ELL14

⁴⁶Thorlabs S121C

Light transmitted through the chamber at viewport 2 (SAP) is collected onto a photodiode⁴⁷ for monitoring and diagnostics; this photodiode is referred to as “Robert Chase, M.D.”

Photoionization Delivery The 375nm and 423nm beams are outcoupled and combined on a pellicle beamsplitter for delivery via viewport 3 (Kodial). As $^{40}\text{Ca}^+$ photoionization is robust and a great surfeit of power is available in these beams, a specially-coated window was deemed unnecessary. These beams are joined near the chamber as their Rayleigh lengths $z_R = \frac{\pi w_0^2}{\lambda}$ are sufficiently different that we are unable to focus them at the trap from a single fiber. The beams are individually focused by their outcouplers⁴⁸ and a single lens each. Light transmitted through the chamber at viewport 7 (Kodial) is blocked for safety.

243nm Delivery The pulsed 243nm light enters through viewport 8 (DUV-coated). Light is coupled over free-space (inside a tube, for health and safety) from the dye laser to the table. A spatial filter is implemented by a lens⁴⁹ and aperture⁵⁰ to improve the mode quality and prevent trap charging from scatter. This beampath is aligned using a special alignment path implemented with the 397nm (see [397nm Delivery](#)), with the transmission at viewport 4 blocked for safety.

5.2.2 Magnetic Fields

The quantization field was defined by a pair of permanent magnet arrays in a Helmholtz configuration mounted on viewports 4 and 8 (Figure [5.6](#)). The magnet arrays were formed by Samarium Cobalt (SmCo) magnets⁵¹ mounted in a circular 3D-printed holder.

Two pairs of current-carrying compensation coils were used in Helmholtz configurations to null residual stray magnetic fields relative to the quantization axis. These coils were mounted

⁴⁷[Thorlabs DET36A2](#)

⁴⁸CFC5-A

⁴⁹Union-Optic

⁵⁰Thorlabs

⁵¹[CMS Magnetics](#)

above and below the chamber, and on viewports 4 and 8. The compensation coils were constructed by wrapping 200 turns of 18AWG magnet wire⁵² around a 3D-printed holder and affixed with epoxy⁵³. All coil pairs were wired in series to suppress common-mode current fluctuations, and were powered by adjustable linear regulated power supplies⁵⁴, with typical current draws of < 1 A. An extra set of compensation coils were mounted on viewports 2 and 6 for later implementation of a magnetic field servo system.

A field strength of $B = 3.63197(2)$ G at the ion position was obtained by measurement of the Zeeman splitting of the $^{40}\text{Ca}^+ \ ^2\text{D}_{\frac{5}{2}}$ states (Figure 5.6(d)).

5.2.3 Imaging

Ion fluorescence was collected through a cutout at the top of the trap frame; fluorescence collection from below the trap was not possible due to the cryogenics. A custom objective from Sill Optics⁵⁵ with focal length $f_o = 66.9$ mm was placed at a working distance $d_w \approx 56$ mm away from the trap center, enabled by the reentrant viewport on the top flange of the chamber. The objective had 94% transmission at 397nm with an additional passband at 729nm; a similar design from Sill Optics is used by several notable Calcium-based experiments [Har13,Pru14,Ben08]. A 2" mirror was used to align the fluorescence onto a tube lens with $f = 1000$ mm for focusing onto the detector plane. The magnification of the system is $M = \frac{f_e}{f_o} \approx \times 15$. The reentrant viewport was surrounded by desiccant and resistively heated to prevent condensation buildup, exacerbated by the cryogenic Stinger arm, which caused aberrations in the image.

To avoid the challenge of mounting the optics directly above the experiment, as well as to increase the portability of the imaging system (e.g. in case the vacuum chamber had to be routinely opened), a periscope with 2" mirrors was used to redirect the fluorescence to a detection system mounted firmly on the table. Following the periscope, the fluorescence

⁵²MWS Wire Polyester A/I Topcoat

⁵³Henkel Loctite STYCAST

⁵⁴GW-Instek GPP3060, Keithley 2231A-30

⁵⁵Sill Optics S6ASS2241

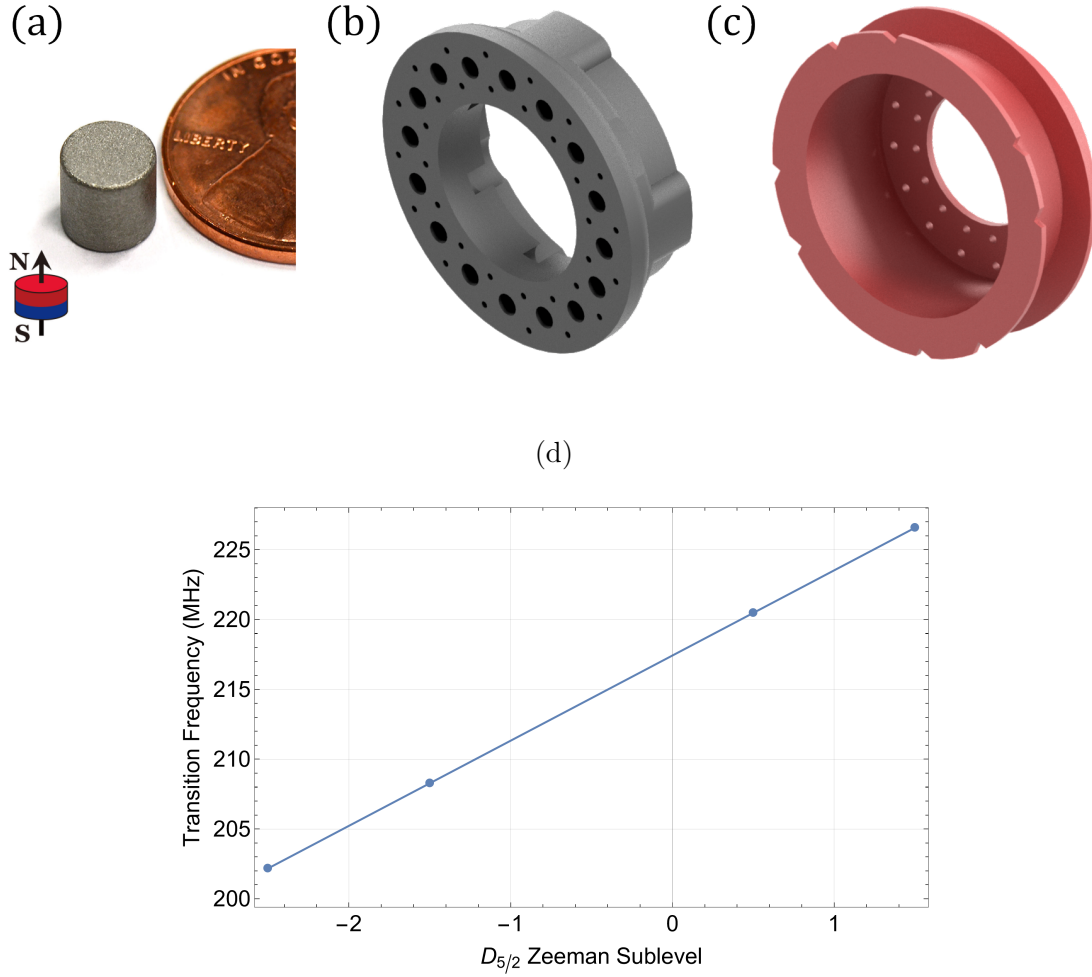


Figure 5.6: **Magnetic field apparatus used to define the quantization axis.**

(a, b) SmCo magnets (US penny for scale) are mounted into 3D-printed holders that arrange the magnets into arrays to generate the quantization field. (c) 3D-printed holder for current-carrying compensation coils, used for fine adjustment of the quantization field. (d) B-field calibration using the transition frequencies of the $^{40}\text{Ca}^+ 2D_{5/2}$ Zeeman sublevels; a linear fit gives $B = 3.631\,97(2)$ G at the ion.

entered a darkened box housing a motorized flipper mirror mount⁵⁶ to direct the fluorescence towards either a [Photo-Multiplier Tube \(PMT\)](#) or an [Electron-Multiplying Charged-Coupled Device \(EMCCD\)](#) camera. 397nm bandpass optical filters were mounted in front of both detectors to reduce spurious background. A set of spatial filters⁵⁷ were placed in front of the PMT to filter laser scatter from the trap blades.

⁵⁶Thorlabs MFF101

⁵⁷Thorlabs VA100

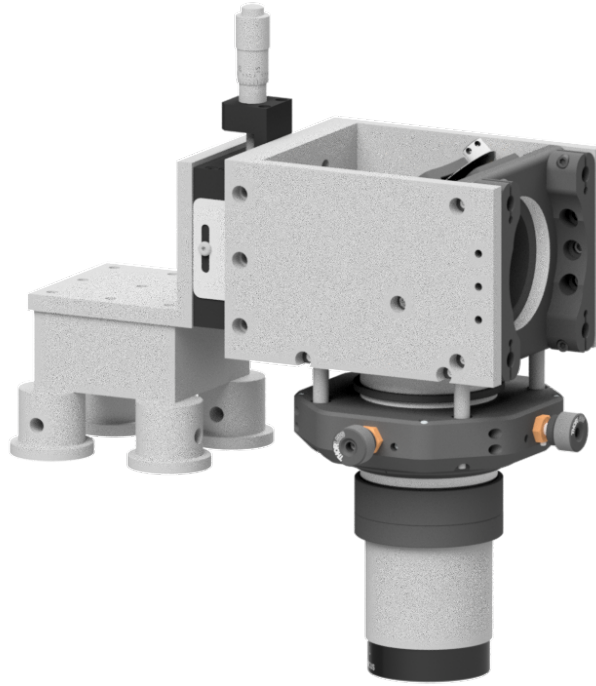


Figure 5.7: **Imaging assembly for the objective lens.**

CAD render of the mount holding the Sill Optics objective (lens tube, bottom) above the reentrant top viewport of the chamber; translation stages enable alignment to the ion. Collected fluorescence is relayed by a periscope (not shown) to a detection box housing the [PMT](#) and an [EMCCD](#) camera.

Optical Detectors

PMT The PMT⁵⁸ included an analog front-end consisting of an internal high-voltage power supply and photon-counting circuit with TTL output, and was powered by the +5V channel of a linear regulated power supply⁵⁹. Experiment data was exclusively taken with the PMT, which was connected to ARTIQ for photon counting; the PMT's specified 5ns pulse-pair resolution was resolvable by the ARTIQ TTL hardware (3ns resolution).

EMCCD Camera The EMCCD camera⁶⁰ was used for coarse, spatially-resolved detection (to e.g. detect chain configuration) and for operator diagnostics. An internal [Thermo-Electric Cooler \(TEC\)](#), powered by a +7.5V [SMPS](#), kept the [EMCCD](#) detector at $-70\text{ }^{\circ}\text{C}$ to minimize

⁵⁸Hamamatsu 10682-210

⁵⁹GW-Instek, GPP3060

⁶⁰Andor iXon3

dark counts. Though the TEC offered a minimum temperature of $-80\text{ }^{\circ}\text{C}$, operation at this temperature was found to be unstable. Camera data was read by a PCI FPGA card connected to a computer and processed using custom LabRAD software using the Andor SDK2 API. The pixel-to-distance ratio of the camera was calibrated to be $\approx 1\text{ pix}/\mu\text{m}$ by comparing the visually determined ion chain spacing to that calculated from the secular frequency. As the camera required obligate control by its accompanied PCI card, fast, synchronized control by ARTIQ was not possible⁶¹, and was used only for asynchronous (i.e. user-facing) operation.

5.2.4 Diode Lasers

All diode lasers in this experiment are Toptica DL Pro [External-Cavity Laser Diodes \(ECDLs\)](#) with current, temperature, and piezo control provided by Toptica DLC Pro controllers (with the exception of the 375 nm laser). A summary of the relevant lasers can be found in table 5.1. This section discusses the only the “standard” diode lasers; an overview of the stable 729 nm systems can be found in sections 5.2.5 and 5.2.6.

The original design intent for the diode lasers was to implement their beampaths on custom breadboards⁶² to standardize their implementation, and house them inside sturdy enclosures⁶³ to minimize the effect of ambient perturbations and fend against drift. These enclosures were then to be rack-mounted, Hong Kong-style, to provide an easily controllable environment⁶⁴ while conserving precious optical table real estate; this is today implemented at UCLA in the $^{133}\text{Ba}^+$ experiment and the Leibbrandt group.

However, temporary often has a way of staying permanent: for a first test, coarse and compact beampaths for the lasers were set up to verify that the beams could be successfully shaped, mode-matched, and coupled in the allotted space. Since these beampaths, though suboptimal, were operational and integral to trapping, the political justification for further

⁶¹Not all camera interfaces are compatible with CameraLink, particularly true for older cameras; this was learned too late in our experiment.

⁶²Similar to that done in the [Jayich Group at UCSB](#)

⁶³These were ludicrous $\sim 100\text{ kg}$ 19" aluminum enclosures with commensurately ludicrous $\sim 30\text{ kg}$ lids.

⁶⁴Climate-controlled and vibration-isolated racks are widely available.

λ (nm)	Type	Model	Purpose	Power (Typical)
243	Dye Laser	Sirah CobraStretch	H^{35}Cl^+ REMPI	100 μJ
375	Diode Laser	Nichia	$^{40}\text{Ca}^+$ PI	1.6 mW
397	ECDL	Toptica DL Pro HP	Doppler Readout Loading State Prep. (σ^-)	20 μW 300 μW 2 mW 20 μW
423	ECDL	Toptica DL Pro HP	$^{40}\text{Ca}^+$ PI	2 mW
854	ECDL	Toptica DL Pro HP	Repump (Qubit)	20 μW
866	ECDL	Toptica DL Pro HP	Repump (Doppler)	160 μW
729	ECDL	Toptica DL Pro	Qubit	36 mW
1064	Nd:YAG	Spectra-Physics Quanta-Ray Lab Series	Pump (dye laser)	N/A

Table 5.1: **Summary of lasers used in our experiment.**

All Toptica DL Pro lasers are stabilized by locking to a multiplexed wavemeter (HighFinesse Wavelength Meter, Ångstrom WS Ultimate 2) with 16 channels. PI (i.e. $^{40}\text{Ca}^+$ PI) is shorthand for photoionization. The 1064 nm Nd:YAG is frequency-tripled to 355 nm to pump the Sirah [Pulsed Dye Laser \(PDL\)](#) (Coumarin 102). The Sirah PDL in turn emits 486 nm light, which is frequency-doubled to 243 nm. The 375 nm laser is a free-running laser diode mounted in a simple Aluminum housing. Note that the powers listed are those used in experiment, and not the laser output power.

construction was felt to be absent; as such, these beampaths remain in their original, draft configurations, with some minor modifications⁶⁵. A superstructure with acrylic panels was constructed to act as a guardrail and to prevent dust accumulation.

397nm (Main + σ^-)

The table beampath for the 397 nm is shown in fig. 5.8, with an initial ≈ 61 mW of light available at the laser head. Initial beam-shaping optics were required for collimation as the DL Pro was purchased without integrated shaping optics.

⁶⁵Actually, you see, what happened was, I had to *deliberately* do it wrong – on purpose! I had to test my powers — any run-of-the-mill, leave-at-5PM graduate student can succeed when optics are well aligned.

Following beam-shaping, the beam path was divided into 3 separate passes: an unpolarized main pass (for cooling and readout), a σ^- -polarized pass (for state preparation), and a wavemeter pass (for monitoring and locking). Approximately 4 mW of light was split off into the wavemeter pass using a window⁶⁶, then fiber-coupled into an SM300 fiber for frequency stabilization.

For the main pass, approximately 38 mW of light was split off and sent into a double-pass AOM⁶⁷ for waveform control and shuttering. A motorized aperture⁶⁸ was used to block the 0th-order beam, and could be opened to increase the light available for loading or ion rescue. After the double-pass AOM, the main pass was fiber-coupled⁶⁹ into an unpolarized SM300 fiber for experimental use [Pru14].

The remaining 8 mW of light was sent into the σ^- pass and similarly sent into a double-pass AOM⁷⁰. Light was then coupled into a PM fiber⁷¹, with an HWP prior to the coupler for polarization-matching to the fiber axes.

397nm Mode Stability The mode stability of the 397 nm laser presented a serious obstacle to experimental operation and was a key contributor to H^{35}Cl^+ ejection. A sufficiently stable mode at the desired wavelength was nontrivial to find and regardless suffered significant mode competition. Declining mode quality was diagnosed by anomalously broad $^2\text{S}_{\frac{1}{2}} \leftrightarrow ^2\text{P}_{\frac{1}{2}}$ spectra and increased mode-hopping. Initially, this mode quality problem was addressed by making incremental adjustments to the temperature, current, and piezo values — an approach that treated symptoms, rather than the underlying disease. A later and substantially more effective approach was to “shake the tree”: laser parameters were scanned across their full safe operating range to map out regions of stability” [Hob21], from which the most favorable mode was selected. Insufficient optical isolation is currently suspected to be the root cause of

⁶⁶For cost reasons.

⁶⁷Brimrose TEM-110-5-380-900

⁶⁸Thorlabs ELL15

⁶⁹Thorlabs CFCA-A, $f = 4.6$ mm

⁷⁰Brimrose TEM-110-5-380-900

⁷¹Thorlabs PM-S405

Calcium Cooling

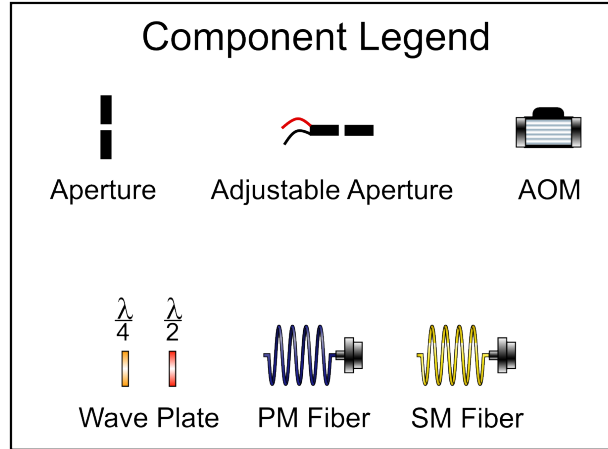
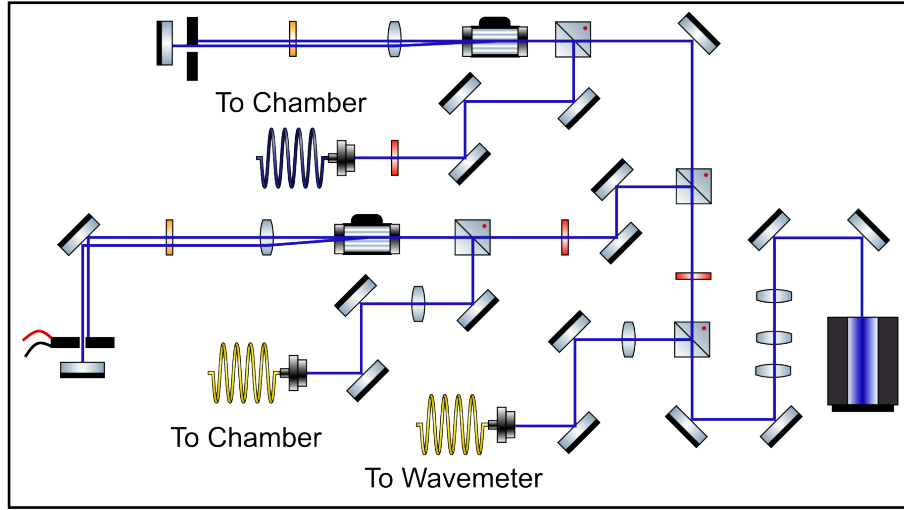


Figure 5.8: **Calcium 397 nm beam path (on table).**

Following beam shaping, the light is split into a σ^- pass for state preparation (top), an unpolarized pass for cooling (center), and a wavemeter pass (bottom). The main and σ^- passes are sent through a double-pass AOM for fast switching prior to fiber-coupling.

the mode instability problem .

393nm Background The 397 nm diode was found to have a nonzero component at 393nm [Pru14,Bur10,Sha+16], which can be observed straightforwardly in experiment by shuttering the 854 nm repump and looking for quantum jumps. Initially, an optical filter was used to remove the 393nm background. Later, this optical filter was removed due to a significant reduction in the 393nm background, which is believed to be linked to the mode

stability.

Photoionization (375nm + 423nm)

Due to the weak dependence of $^{40}\text{Ca}^+$ photoionization dynamics on the frequency of the 375nm photon, the source for the 375nm laser is a free-running diode⁷² without thermal stabilization. A current controller⁷³ is used to adjust the intensity. A minimal layout is used to couple light into an SM300 fiber (fig. 5.9). Beam shaping optics are absent due to the abundance of power available.

The 423nm beampath is similar to that of the 397nm (Figure 5.8). Apart from the wavemeter pass, only a single pass is used, which lacks an AOM and is directly fiber coupled.

Due to the UV wavelength and high power of the PI lasers, shutters preceding the fiber couplers are necessary to prevent long term damage to the fibers. Motorized shutters [Zha+15] controlled by a LabJack T7-Pro were used to automatically block the PI lasers when not loading.

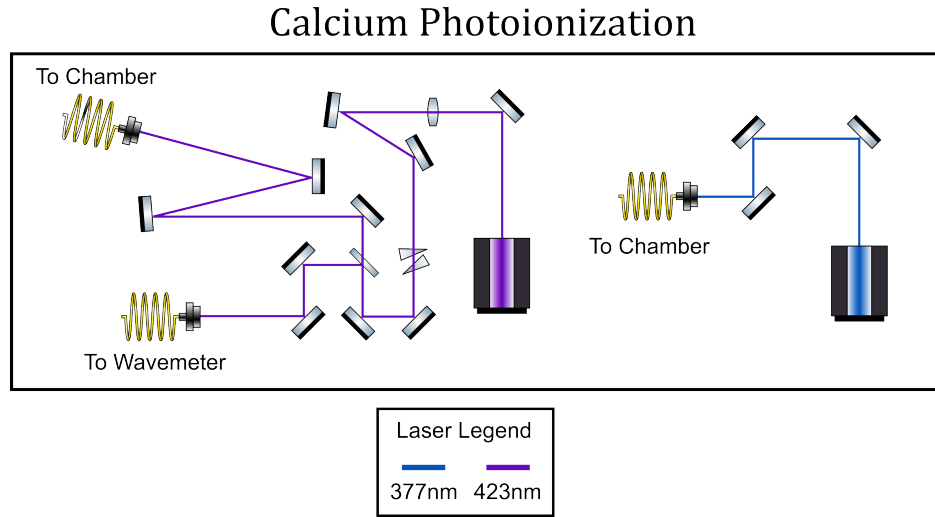


Figure 5.9: **Calcium photoionization beampath (on table).**

The 423 nm laser (left) is split into experiment (top) and wavemeter (bottom) passes; the free-running 375 nm diode (right) is directly fiber-coupled due to the insensitivity of the ionization pathway to the laser frequency and an abundance of available power. The beams are combined on a pellicle beamsplitter prior to delivery to the trap (see Figure 5.4).

⁷²Nichia NDU4116, also used by [Pru14]

⁷³AMO1 (the very first prototype, in fact), built by PY.

Repumps (854nm + 866nm)

The beampaths for the 854nm and 866nm lasers are shown in fig. 5.10 and demonstrate a very prescient exercise of experimental austerity. The beams at the output of both laser heads were both highly elliptic and astigmatic and required significant (though futile) beamshaping. Glass microscope slides were epoxied⁷⁴ to 1" mirror mounts⁷⁵ to sample each beam to be fiber-coupled and sent to the wavemeter for monitoring. To reduce experimental “time-to-market,” the AOMs for the repumps were used only in the single-pass configuration. Both beams were combined on a 50/50 beamsplitter cube and coupled into a 780HP⁷⁶ fiber for experiment delivery.

The insufficient extinction provided by the single-pass AOMs posed a problem for the 854nm beam. During experimental sequences, RF signals from other DDS channels were picked up upstream of the amplifier, causing leakage of the 854nm beam and deshelling population in the $^2D_{\frac{5}{2}}$ state. This was most efficiently remedied by a deliberate misalignment of the 854nm beam into the coupler, which reduced the effect of RF pickup at the cost of higher required drive powers. Additional safeguards against unintended pickup were incorporated into the DDS signal chain (see section 5.2.4).

AOM Signal Paths

ARTIQ AD9910 DDS channels (see section 5.4.1) were amplified⁷⁷ to drive all AOMs. A 24V linear regulated power supply⁷⁸ with 30A capacity was connected to a breakout box to distribute power to the AOM amplifiers. A 9 nF ceramic and 10 μ F electrolytic capacitor was connected internally at each port of the breakout box to improve decoupling between ports and suppress transients in power draw during pulse sequences. External RF switches⁷⁹

⁷⁴Thorlabs NOA60

⁷⁵Thorlabs KM100

⁷⁶Thorlabs

⁷⁷Mini-Circuits ZHL-03-5WF+, Mini-Circuits ZHL-2-8-S+

⁷⁸Acopian V24PH30AFH

⁷⁹Mini-Circuits ZASWA2-50DR-FA+

Calcium Repumps

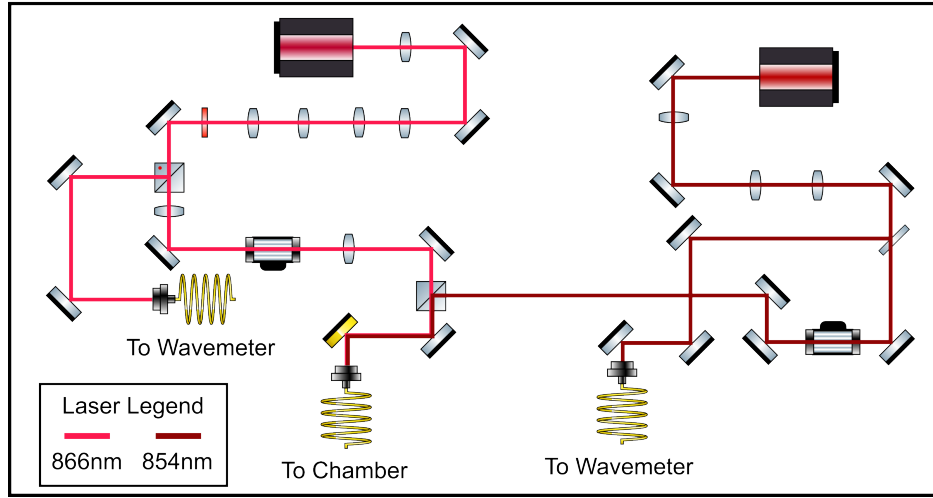


Figure 5.10: **Calcium repumps (854 nm + 866 nm) beam path (on table).**

Following beam shaping, the light is split into a wavemeter pass (left, center right) via glass pickoffs, passed through single-pass AOMs, then combined on a 50/50 beamsplitter and fiber-coupled to the experiment (center).

placed immediately before the amplifiers were used to prevent unintended RF pickup from driving the AOMs. The RF switches were powered by a bipolar 5V linear regulated power supply⁸⁰ and driven by TTLs from ARTIQ. RF pickup was further passively suppressed by the use of double-shielded coaxial cable⁸¹.

Wavemeter Locking

With the exception of the 729 nm narrow linewidth lasers, which are only *monitored* on the wavemeter, all diode lasers are monitored and locked to a multiplexed wavemeter⁸². Light from the diode lasers are fiber-coupled over to the optical table housing the wavemeter and coupled into one of two optical switches, depending the wavelength: blue ($\lambda < 600$ nm) light is coupled into an 8-channel Photonic Crystal Fiber (PCF) switch⁸³, while red light ($\lambda > 600$ nm) is directed into an 8-channel Single-Mode Switch⁸³. As a single channel on each switch is reserved as a “dark” channel to prevent crosstalk when the other switch is active,

⁸⁰Acopian

⁸¹Hubert+Suhner, RG223/U

⁸²HighFinesse Wavelength Meter, Ångstrom WS Ultimate 2

⁸³HighFinesse

only 14 channels are available.

Computer Operation A dedicated computer⁸⁴ is used to interface with the wavemeter and the PCF switch (both over RS232), run the wavemeter configuration program⁸⁵, house the PCI DAC card⁸³ that accompanies the wavemeter for laser feedback, and manage communication with client computers. As noted in the manual⁸⁶, it is important that the computer controlling the wavemeter is dedicated to the task: since the polling of frequency values by the computer is synchronized to switch operation over RS232, any non-deterministic CPU delays (from e.g. interrupts from other programs) will result in acquisition of a frequency reading for an incorrect channel. Additionally, as the wavemeter is shared across 3 different experiments and is polled continuously by many clients (with some implementing their own software servos), significant network traffic is generated. To prevent excessive bandwidth consumption on our experiment's network, the wavemeter computer traffic is instead routed through the building's network.

Feedback To enable laser frequency stabilization, the wavemeter software uses the measured frequency to compute an error relative to a programmed setpoint, converts it into a control signal using a similarly programmed transfer function, then outputs an analog control signal via an 8-channel, 16-bit PCI DAC card. The DAC output is connected to analog input ports on the DLC Pro controllers, which implement the feedback via piezo control. Simple PID feedback is used for wavemeter locking of all diode lasers in this experiment, with loop parameters manually tuned using the Ziegler-Nichols method.

Due to the multiplexed nature of the wavemeter, the measurement time is shared across all 14 channels, which results in slow (on the ~ 1 s timescale) loop bandwidths. As the wavemeter-locked diode lasers are used only to drive dipole transitions, these small bandwidths (and thus large laser linewidths) are sufficient; the lock serves only to prevent long-term frequency

⁸⁴Assembled by [GDM](#).

⁸⁵HighFinesse Wavelength Meter

⁸⁶As well as from our lived experience.

drifts.

To prevent long term drifts in the wavemeter’s absolute accuracy, the cavity-stabilized 729 nm light is used as a relative reference for periodic wavemeter autocalibration. However, due to a lack of available channels⁸⁷, this functionality is used sparingly.

Wavemeter Switch Damage A persistent bugbear has been damage to the PCF switch from exposure to the UV lasers, which produces significant crosstalk between switch channels — over the course of this experiment, the blue switch has been replaced four times. While the documentation for the PCF switch (as well as for other HighFinesse blue switches) specifies wavelength acceptance ranges down to 350 nm, it has been intimated that the switches are nevertheless damaged by light below 400 nm. To prolong its useful lifespan, the UV channels (i.e. the 397 and 423 nm lasers) use increased exposure times (to reduce the input power) of ≈ 150 ms, and are blocked when not in use.

5.2.5 729nm Stable Laser System

To enable coherent operations on the $^{40}\text{Ca}^+$ optical qubit transition, a narrow linewidth laser was required, for which a bespoke 729 nm laser cavity with specified <1 Hz linewidth and accompanying stabilization system⁸⁸ were purchased from [Stable Laser Systems \(SLS\)](#).

This purchase included all relevant components, of which the most relevant here are: an FPGA-based servo unit with offset-locking functionality, and a [Fiber Noise Cancellation System \(FNCS\)](#) system.

Contributions The 729nm stable laser (henceforth given the metonym, the “SLS”) was purchased from [Stable Laser Systems \(SLS\)](#) and maintained by [HW](#). Initial assembly of the SLS was performed by Nate Phillips (then with SLS), with later modifications implemented by [HW](#).

⁸⁷Which are *manually* multiplexed, in addition to being *switch* multiplexed.

⁸⁸This excludes the laser head itself (Toptica DL Pro), though this was shipped directly to SLS as the laser head was required for cavity characterization.

Operation

Light at 729nm is provided by an [External-Cavity Laser Diode \(ECDL\)](#)⁸⁹ controlled by a Toptica DLCpro. The output is directed onto a beampath to condition the light, a portion of which is sampled for [PDH](#)-locking to a [Ultra-Low Expansion \(ULE\)](#) Fabry-Pérot optical cavity with an [Free Spectral Range \(FSR\)](#) of 1.5 GHz. The cavity is temperature-stabilized⁹⁰ at the zero-crossing temperature at 33.51 °C and housed inside a [UHV](#) vacuum chamber at $\approx 10^{-8}$ mbar, continuously pumped by an ion pump⁹¹. The system is housed inside an Aluminum enclosure with heat-reflective and vibration-damping insulation, which is mounted on its own optical table with active vibration isolation⁹². An uninterruptible power supply for the ion pump and temperature controller is necessary for both power conditioning (to isolate the system from mains ripple/transients) and for backup power.

Approximately 13 mW of power is available at the main output, with $\approx 300\mu\text{W}$ available at a second output for monitoring on a wavemeter. Originally, this main output was directed to the experiment via a 13 m PM-630-HP⁹³ fiber. Later, this light was directed instead to the injection lock system (section 5.2.6) for conditioning and amplification.

Despite the presence of a [HWP](#) prior to the PM-fiber at the output, polarization drifts formed a persistent and significant source of deep consternation. Due to the [PBS](#) at experiment-side to sanitize the [PER](#), these polarization drifts were converted to power drifts, obstructing data taking. Though many candidate causes were identified and treated, none were successful in treating the underlying disease; this problem was later bypassed using the injection lock system (see section 5.2.6).

⁸⁹Toptica DL Pro

⁹⁰Wavelength Electronics LFI-3751

⁹¹Gamma Vacuum

⁹²Herzan AVI Series

⁹³Thorlabs

SLS FPGA Servo & Lock Acquisition

The SLS servo is FPGA-based with support for PDH offset locking and [Multiple Input, Multiple Output \(MIMO\)](#) transfer functions. Fast feedback (~ 1 MHz) on the cavity reflection is applied via the laser current, while slow feedback is applied via the laser piezo (~ 1 kHz). An additional servo for the sideband power is available when offset-locking, though this has not yet been configured in our experiment. The $J(-1)$ sideband is used for locking at an offset of 187 MHz. The servo parameters were initially tuned using the cavity transmission and PDH signals. A later attempt at servo parameter optimization via Ramsey measurements on the ion was unable to resolve the effect of parameter changes due to the presence of extraneous noise, which limited the interrogation time to < 4 ms (see [Section 6.3.4](#)).

Absent extraordinary circumstances, we have found that the SLS can stay locked indefinitely⁹⁴ without intervention; the longest measured lock period has been ~ 3 weeks. However, acquisition of the initial lock is often challenging due to the use of offset locking. Due to the lack of optical power in the sidebands relative to the carrier, the cavity transmission photodiode may not detect sufficient light, preventing lock acquisition. While a straightforward remedy is to increase the incident power by e.g. realignment of the FiberDock on the laser head, a more expedient⁹⁵ technique is to use “optical chopping”: by using a lens tissue to “chop” the light in front of the cavity, sufficient light can be scattered into the cavity transmission photodiode to trigger a lock event. Locking relies on synchronization of this false “lock event” to the true lock point, and so this procedure is stochastic; the “chopping” is necessary to increase the likelihood of a successful lock.

Cavity Drift Upon its reception in 2021, the cavity was specified to drift at an approximately linear rate of 0.26 Hz/s (22.5 kHz/day, or 0.94 kHz/hour)⁹⁶. As the Rabi frequency on the motional sidebands of the qubit transition were of order 1 kHz, this drift limited

⁹⁴Though this is wasteful, as the laser diode has a finite lifetime.

⁹⁵And secret.

⁹⁶Over time, this was found to stabilize to < 0.1 Hz/s.

the maximum experiment runtime possible without operator intervention (for recalibration). While automatic calibration sequences were possible [Sri20,McC+19a], the benefit was deemed incommensurate with the effort required.

In 2024, a closer reading of the servo manual found that a feedforward drift compensation feature (called drift cancellation) was available on the servo. With drift cancellation enabled, the servo continuously applies a feedforward adjustment to the offset lock frequency. Currently, a drift compensation of 0.06 Hz/s (0.22 kHz/hour) is successfully applied to stabilize long-term drifts of the SLS, enabling longer continuous experiment runtimes and reducing operator fatigue.

5.2.6 729nm Injection Lock

Due to the expanding scope of the EGGS experiment, more laser power at 729nm was needed to accelerate spin-motion experiments and expand our science arsenal. To meet these burgeoning demands, a filter cavity–injection lock–tapered amplifier system was assembled (henceforth referred to as the “injection lock system” or thereabouts). The filter cavity⁹⁷ was used to remove deleterious servo bumps on the existing 729nm light, preventing their later amplification. This filtered light was then used to injection lock a laser diode to ensure sufficient power to seed the tapered amplifier⁹⁸, the output of which was directed onto a beampath used to condition the light for experimental use.

Since its construction, the high power provided by the injection lock system has enabled a growth in our experimental capabilities so great that it, with all its connotations, might be called tumorous. For this reason, we have bestowed upon the injection lock system the weighty honorific of: “Rubicon.”

Contributions The 729nm injection lock system was primarily assembled by JAR. Initial discussions were supported by Evan Reed and Lu Qi from Kenneth Brown’s DPQL experiment

⁹⁷Stable Laser Systems FCC-TBD

⁹⁸Optica TAprio

at Duke. [GDM](#) supervised the initial design and purchase. [PY](#) built the dual analog notch filter at ≈ 8 kHz and ≈ 13 kHz. A significant amount of hardware for implementation, testing, and characterization of the system was generously lent by the UCLA AMO groups.

Layout & Operation

The injection lock system was assembled on a honeycomb optical breadboard⁹⁹ set up on the dedicated laser table — an overview is shown in fig. 5.11. A custom enclosure, modeled after that used for the SLS, was machined from sheet Aluminum and lined with acoustic insulation and heat-reflecting insulation to suppress ambient vibrations and thermal drifts.

Prior to filtering, a 60 dB dual-stage optical fiber isolator¹⁰⁰ upstream of the cavity was needed to prevent backreflections from disturbing the [PDH](#) lock of the SLS source laser. A fiber isolator was preferred over a free-space isolator to reduce the number of mating sleeves and optics required, as these degrade the polarization extinction ratio (PER, i.e. polarization purity). A second, 60 dB dual-stage isolator¹⁰¹ was needed following the filter cavity to isolate it from the injection-locked laser diode. Though a 30 dB isolator was initially used for this purpose, the quality of isolation was found to be a significant determinant of lock stability for the filter cavity, which motivated an increase in isolation. Accumulation of residue on fiber faces was found to be a significant contributor to diminished long-term performance (in both isolation and power transmission); ambient humidity is believed to be the source of this residue, though this remains unconfirmed.

The injection lock diode was housed in a temperature-stabilized¹⁰² enclosure and controlled by a custom current controller¹⁰³. The injection-lock diode was seeded by ~ 3 mW and produced ~ 30 mW at its output, which was mode-matched and fiber-coupled¹⁰⁴ into the [TA](#). Following amplification by the TA, the light was outcoupled¹⁰⁴ and sampled by a beamsplitter

⁹⁹Thorlabs

¹⁰⁰Agiltron OIUW-21012C333

¹⁰¹Toptica TPI-OPT ISOL DSR 740

¹⁰²AMO2, custom built by [PY](#)

¹⁰³AMO1, custom built by [PY](#)

¹⁰⁴Toptica FiberDock

for monitoring on a wavemeter¹⁰⁵.

The remaining light was frequency shifted by a double-pass AOM¹⁰⁶ that was additionally used as an actuator for intensity stabilization, then sent into a single-pass AOM¹⁰⁷ to enable the creation of multi-chromatic light (for e.g. cat-state creation) [Kie15]. Both the double- and single-pass AOMs were driven by AD9910 channels from ARTIQ, which were subsequently amplified¹⁰⁸. To generate phase-coherent multi-tone RF signals for the single-pass AOM, three AD9910 channels were combined using a 3-way power splitter¹⁰⁹ prior to amplification. The AOM amplifiers are powered by a linear regulated power supply¹¹⁰.

For transfer to the experiment, the light was fiber-coupled¹¹¹ with $\approx 65\%$ efficiency into a 15 m high power APC-PC PM-780HP fiber¹¹². A minor increase in fiber-coupling efficiency to $\approx 80\%$ is likely possible by instead using a PM-630HP fiber.

Filter Cavity Stabilization

The filter cavity had a linewidth of 176 kHz and an FSR of 2 GHz, selected to remove the servo bumps at $\approx 2\pi \times 1.2$ MHz [Pru14,Ake+15]. As these bumps overlap with the motional sideband frequencies, they can resonantly excite the carrier during sideband excitation, setting a floor on the achievable SBC temperature (see section 4.2.2). An internal heater and piezo were provided for temperature stabilization¹¹³ and cavity length control, respectively, with the latter used to PDH lock the cavity to the SLS source. Light entering the filter cavity was phase-modulated at 10 MHz by an EOM¹¹⁴ to generate sidebands that reflected off the cavity and were directed onto a fast photodiode by an internal circulator for demodulation into a PDH signal.

¹⁰⁵HighFinesse Wavelength Meter, Ångstrom WS Ultimate 2

¹⁰⁶AeroDiode 780FSAOM, 200 MHz center frequency

¹⁰⁷Brimrose TEM-110-5-380-900

¹⁰⁸ZHL-03-5WF+

¹⁰⁹Mini-Circuits ZA3CS-400-3W+

¹¹⁰Acopian VA24H850FM

¹¹¹Thorlabs CFC5A-B

¹¹²Coastal Connections

¹¹³Wavelength Electronics LFI-3751

¹¹⁴QUBIG PM7-NIR

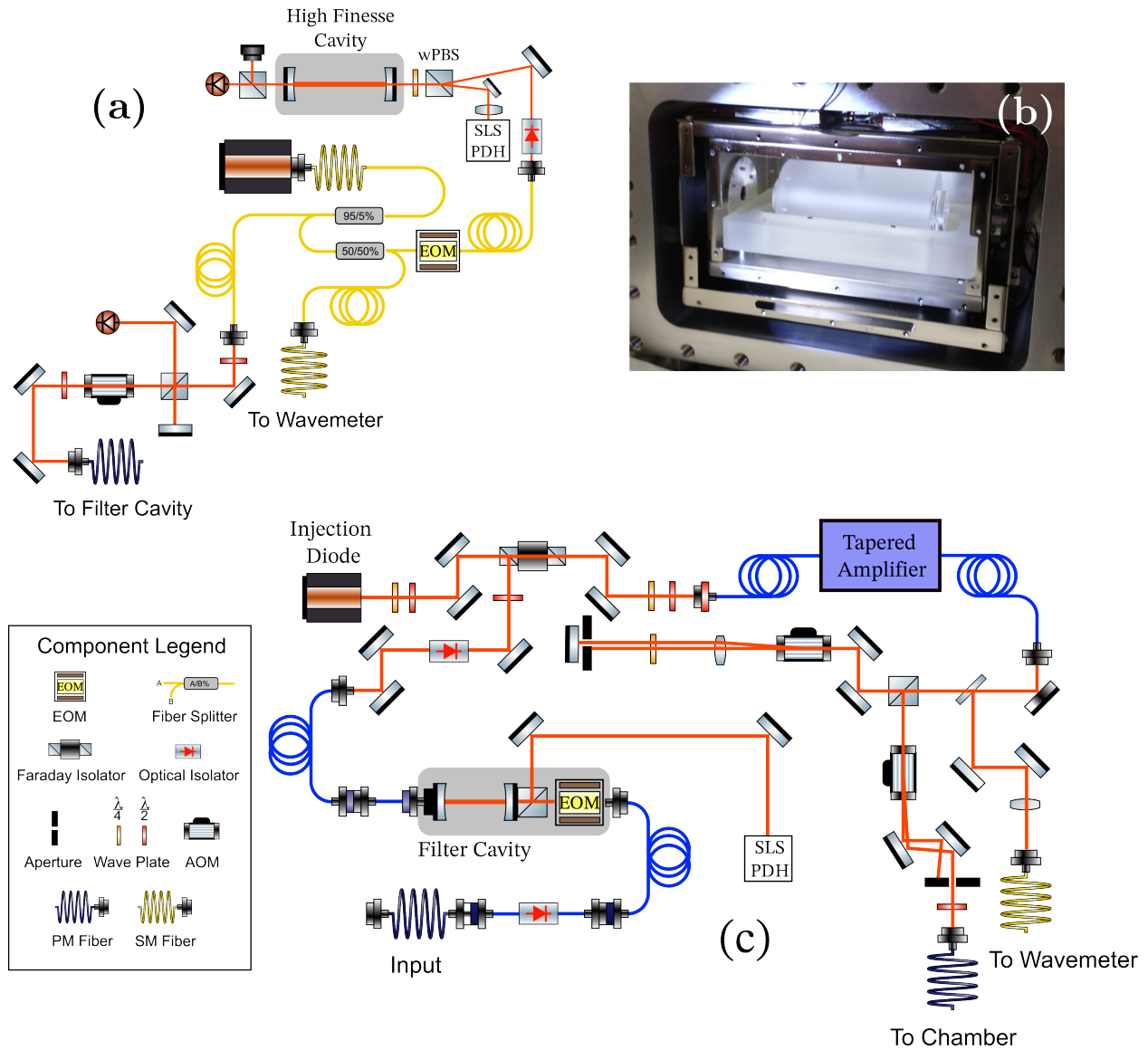


Figure 5.11: **Complete 729nm laser system.**

(a): 729nm **Stable Laser Systems (SLS)** for **PDH**-stabilization to a **ULE** reference cavity (b). (c): Light from the **SLS** (a) is directed into an injection-lock system where the light is filtered, amplified, and conditioned for delivery to experiment. See sections 5.2.5 and 5.2.6 for details.

PDH locking of the cavity to the incoming **SLS** source was needed to stabilize it against slower drifts; as such, bandwidth and latency were not significant concerns for stabilization¹¹⁵.

The many acoustic resonances of the piezo presented a significant challenge to robust

¹¹⁵With sufficient operator skill, a manual **GSL** of the filter cavity is possible.

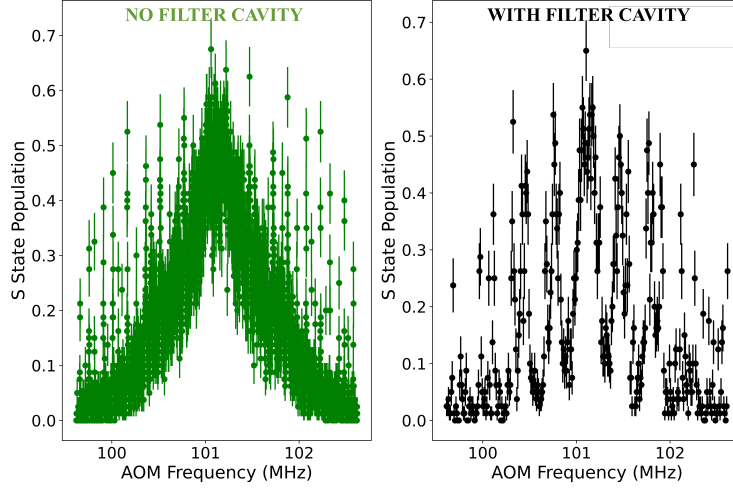


Figure 5.12: **729nm $^{40}\text{Ca}^+$ optical qubit spectrum with the filtered injection lock system.**

Optical qubit spectrum using 729nm injection-locked light without (left) and with (right) the filter cavity; without the filter cavity, off-resonant excitation from servo bumps at $\approx 2\pi \times 1.2\text{ MHz}$ broadens the carrier. Note that the x-axis is the **AOM** drive frequency; the actual frequency is doubled due to the double-pass configuration.

stabilization of the cavity [RS17,Hob21]. An initial attempt at locking using an analog PI servo¹¹⁶ was unable to achieve any reduction in the error signal when locked, and imprinted visible sidebands at the piezo resonance frequencies on the $^{40}\text{Ca}^+$ optical qubit transition. Characterization of the system revealed several piezo resonances, the largest of which were at $\approx 8, 13\text{ kHz}$ (Figure 5.13). Lowpass filtering¹¹⁷ was attempted to eliminate servo response to these resonances; however, this was unsuccessful due to insufficient phase margin at lower frequencies. Following these difficulties, it was found that amplification without the filter cavity did not significantly affect the coherence when exciting the $^{40}\text{Ca}^+$ optical qubit transition, allowing the filter cavity to be bypassed.

Locking was significantly improved following the use of a digital servo¹¹⁸ and the construction of a dual notch filter at $\approx 8, 13\text{ kHz}$ by PY. The digital servo allowed specification of arbitrary 1st- and 2nd-order IIR filters and thus in turn allowed placement of additional notch filters tuned to piezo resonances. With the many notch filters in place, a 1st-order PI

¹¹⁶Newport LB-1005

¹¹⁷Thorlabs EF110, EF112, EF114

¹¹⁸**NIST Digital Servo, designed by David Leibrandt & Jason Heidecker, assembled by Creotech**

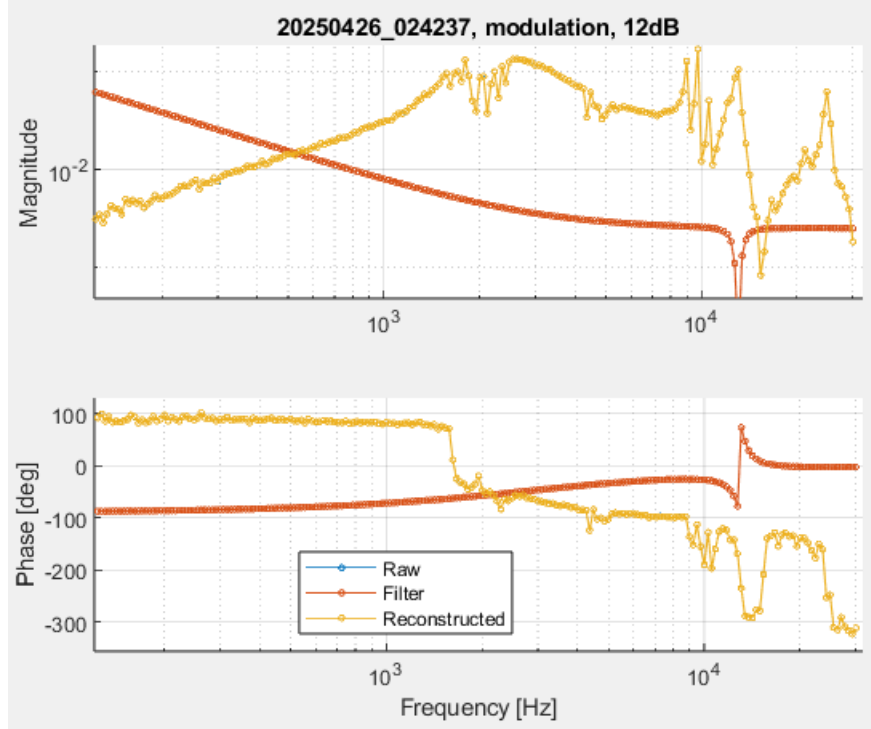


Figure 5.13: **Transfer function of the 729nm filter cavity piezo feedback system.**

A modulation signal added to the digital servo output is demodulated at the error signal input and deconvolved from the loop filter (red) to extract the system response. The digital servo uses a PI filter with 2.8 kHz corner frequency, and a $Q = 8$ notch filter at 12 kHz; sharp resonances around 1×10^4 Hz are further mitigated by external notch filters at 8 kHz and 13 kHz.

filter was found to be sufficient for frequency stabilization.

Laser Intensity Stabilization

A second channel of the NIST digital servo stabilized the 729nm intensity via a pickoff near the vacuum chamber, with feedback applied by servoing the RF power applied to the double-pass AOM via a [VVA](#)¹¹⁹. Due to space constraints and a desire to place it as downstream as possible, this pickoff was placed between the final coupling mirrors into the chamber. As this pickoff was downstream of the main 729nm AOM, the monitored light was shuttered during experimental sequences, necessitating an integrator-hold scheme to maintain stable feedback. To enable stabilization during active experiment operation, dedicated “relock” periods were

¹¹⁹[Mini-Circuits ZX73-2500-S+](#)

5.3.1 Helical Resonator

The helical resonator was used to step up the trap RF to the ~ 100 V required for confinement while providing impedance matching and filtering. Two versions of the helical resonator were constructed: a first prototype version following [Siv11], and a second version with improved usability based on designs from the Monroe group¹²⁰.

Resonator Design

The impedance of the trap system (including e.g. wires, parasitics) can be straightforwardly modelled as an RLC resonator, with characteristic equations [Siv11,Siv+12]

$$\Omega_{\text{RF}} = \frac{1}{\sqrt{LC}}, \quad Q = \frac{1}{R} \sqrt{\frac{L}{C}}, \quad (5.1)$$

where Ω_{RF} is the resonance frequency and Q is the resonance quality factor. Resonator design was determined by the fixed resistance and capacitance of the trap and a target resonance $\Omega_{\text{RF}} \approx 20 \times 2\pi\text{MHz}$, selected to achieve stable confinement with a secular frequency $\omega_0 \approx 1 \times 2\pi\text{MHz}$. A trap resistance and capacitance of $R \approx 2.2 \Omega$ and $C \approx 36 \text{ pF}$, respectively, were measured with an Agilent U1733C LRC Meter, which constrained a coil inductance $L \approx 1.9 \mu\text{H}$. The trap capacitance was larger than typical for similar blade traps ($\sim 10 \text{ pF}$ [Ben08,Gul03]) due to the large surface area of the electrodes, while the large trap resistance was due to the thin wiring (28AWG, Copper) needed to minimize the cryogenic heat load; parasitic inductances were negligible.

The resonator shield was constructed from a single block of Copper and contained 3D-printed supports to hold the resonator coils for improved mechanical stability. The antenna coil was mounted on a similar 3D-printed support and fastened to a hex adjustment knob¹²¹ on the lid for precise and repeatable coil alignment.

Connections to the resonator were made using [SubMiniature Version A \(SMA\)](#) connectors

¹²⁰Designs were initially procured by [MB](#).

¹²¹Thorlabs FAS300, LN2580

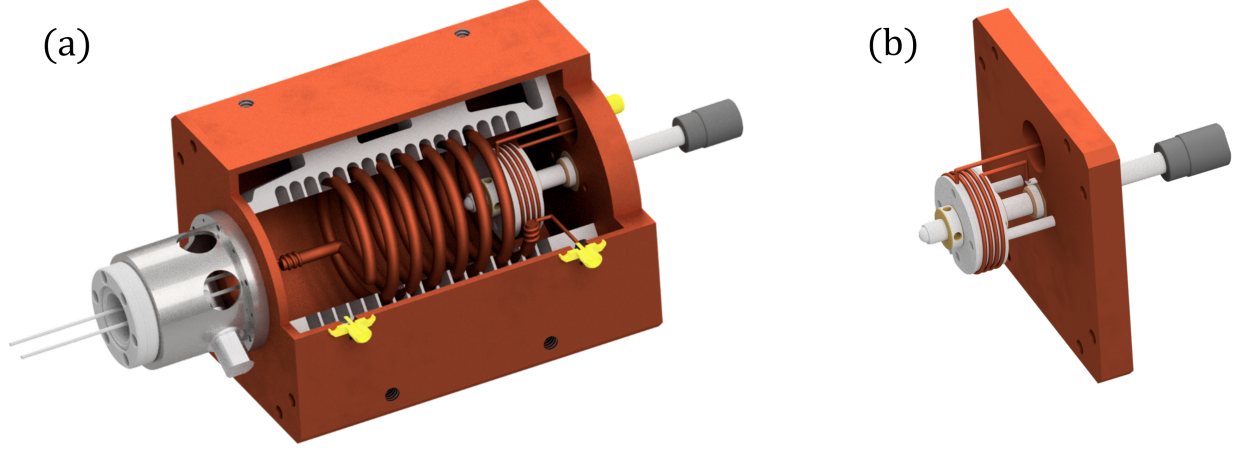


Figure 5.15: **Monroe-style helical resonator for the trap RF.**

(a) Cutaway view; the main coil is mounted on 3D-printed supports inside the copper shield; pickoffs on the side are connected to [SMA](#) feedthroughs for monitoring. **(b)** The coupling coil is mounted onto the resonator lid with an adjustment knob for axial translation of the coil.

mounted to the shield. A capacitive pickoff¹²² with division ratio $\frac{C_1}{C_1+C_2} = \frac{0.2}{0.2+20} \approx 0.01$ inside the shield was connected to the main coil for monitoring, characterization, and feedback (see section [5.3.2](#)).

The resonator shield was temperature-stabilized by a [TEC](#) driven with a servo controller¹²³ to suppress thermal drift of the secular frequency, as recommended in [[Har13](#)]. An initial concern was that thermal expansion of the resonator coils would shift the secular frequency; however, this mechanism was calculated to contribute only ~ 1 Hz/K, which was negligible. In our experiment, the primary benefit of temperature stabilization was the indirect thermal regulation of the capacitive divider and rectifier, which had thermal coefficients ≈ 60 ppm/K [[Joh+16](#)]; the divider and rectifier were mounted in locations that precluded direct thermal control. Stabilization was further complicated by the large thermal mass of the helical resonator and physical constraints that necessitated the use of an undersized heatsink. To avoid thermal runaway, the resonator system was stabilized at 26°C — well above the range of daily laboratory temperature fluctuations.

¹²²Vishay Quad Hi-Freq, $C_1 = 0.2$ pF (VJ0805D0R2BXPAJHT), $C_2 = 20$ pF (VJ1111D200FXRQJ)

¹²³AMO2, built by [PY](#)

Resonator Characterization & Alignment

The resonator was aligned by adjusting the antenna coil while monitoring the reflected power (S_{11}) on a [Vector Network Analyzer \(VNA\)](#)¹²⁴. Alignment was performed with the same cables and connections as during normal operation to ensure that parasitic capacitances from cables and connectors were accounted for. The resonator quality factor reported by the [VNA](#) was typically $Q \approx 150$. A direct, in-situ measurement of $Q = 131.4(4)$ was independently obtained by fitting the resonator ringdown recorded from an oscilloscope.

Manual VNA Optimization Tuning by VNA is necessarily ex-situ, as it requires disconnecting the system from the trap RF amplifier; this procedure assumes that the amplifier presents a $50\ \Omega$ output impedance. However, this assumption is generally invalid, as amplifiers typically exhibit a non-unity [VSWR](#).

To enable in-situ tuning, a manual [VNA](#) technique was employed. The rectifier connected to the capacitive divider was monitored on an oscilloscope to provide a scalar measurement of transmitted power (S_{21}) as the trap RF frequency was swept with a sawtooth waveform. A trigger output from the signal generator was used to synchronize the oscilloscope in XY mode, yielding a real-time display of the resonator’s transfer function.

5.3.2 Amplitude Stabilization

A trap amplitude stabilization following [\[Joh+16\]](#) was implemented in an attempt to improve the motional coherence time and secular frequency stability for robust QLS interrogation. We outline the implementation and hardware measurements here; relevant experimental background and experimental characterizations are discussed in section [3.4.2](#).

Servo Overview Our implementation (Figure [5.16](#)), closely follows [\[Joh+16\]](#). The trap RF was sampled at a pickoff on the helical resonator connected to a capacitive divider¹²⁵

¹²⁴Hewlett-Packard 8714ES

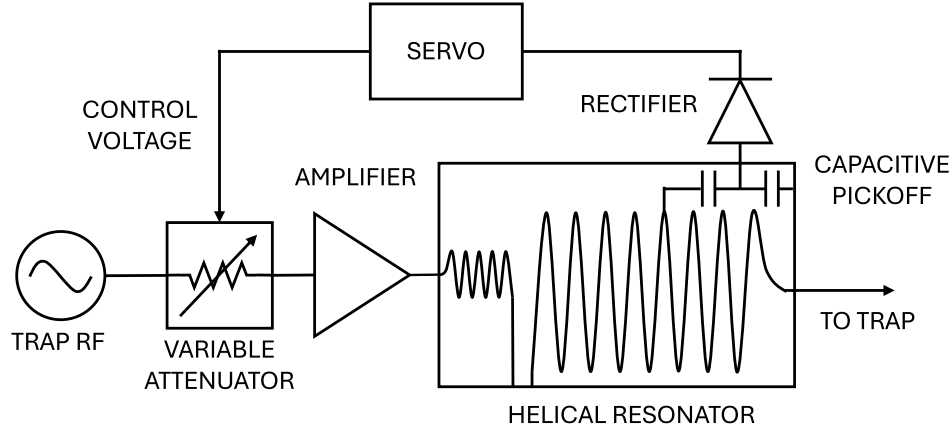


Figure 5.16: **Block diagram of the trap RF amplitude servo.**

The trap RF is sampled at a capacitive pickoff on the resonator then rectified to DC for comparison against a setpoint by the servo; the servo feeds back a control signal to a voltage variable attenuator to stabilize the RF amplitude.

with division ratio $\approx 1\%$. A rectifier connected to the pickoff converted the sampled trap RF into a DC signal, which was then fed into a PI servo controller¹²⁶ and compared to a setpoint provided by a precision DAC (see paragraph [Voltage Setpoint](#)) to generate an error signal. The rectifier followed the design in [Joh+16] and was assembled by MP and PY. To avoid signal attenuation from parasitic cable capacitances and suppress microphonics, the rectifier was connected directly to the pickoff. The servo output was used drive a [Voltage-Variable Attenuator \(VVA\)](#)¹²⁷ to stabilize the level of the trap RF. The VVA was powered by a +5V channel of a benchtop power supply¹²⁸. The VVA was used for feedback despite the benefits of passive mixers due to the difficulty of passively meeting LO power requirements in our densely-integrated system.

Voltage Setpoint Originally, a DAC following [Joh+16] was constructed to provide the voltage setpoint. The setpoint was composed of a 20-bit DAC¹²⁹ referenced to a 5V stable

¹²⁵Vishay Quad Hi-Freq, 0.2pF and 20pF

¹²⁶Newport LB-1005

¹²⁷Mini-Circuits ZX73-2500-S+

¹²⁸Keithley 2231A-30-3

¹²⁹Analog Devices EVAL-AD5791SDZ

reference¹³⁰ and controlled by an MCU board¹³¹. HP E3600 Series power supplies were used to power the DAC and 5V reference. However, due to the challenge of interfacing with the MCU board¹³², the DAC was later deprecated in favor of the integrated setpoint on the LB-1005 servo; this was not measured to affect the coherence of the radial modes.

Servo Controller The servo controller was tuned by minimizing the RMS voltage of the error signal, as measured on an oscilloscope. However, later measurements with the ion using a motional Ramsey sequence (see Section 3.4.2) revealed the lack of a clear dependence on the servo parameters. For this reason, the loop filter of the servo was set to Integral-only, using only integral gain up to a corner frequency of 3 kHz, after which the servo response was truncated.

The integrator hold function of the servo was triggered by ARTIQ to prevent spurious responses during experimental sequences which applied high power RF to the trap electrodes. To reduce the effect of turn-on glitches on the ion motion, a holdoff period of 100 μ s was inserted into experimental sequences following activation and preceding deactivation of integrator hold. Though integrator hold was known to cause an unexpected long-term drift in the control voltage (see section 5.3.3), this did not initially appear to impact the mode frequency on a ~ 1 ms timescale.

Later, motional coherence measurements found that use of integrator hold shortened the motional coherence from ≈ 2 ms to ≈ 0.7 ms. Due to the simultaneous need for both integrator hold and long coherence times, the rectifier was moved immediately before the helical resonator to filter spurious pickup during experiment sequences. In this new configuration, the rectifier instead samples the trap RF using a coupler in the forward configuration¹³³.

¹³⁰Analog Devices DC2095A-C

¹³¹Analog Devices EVAL-SDP-CB1Z

¹³²The packaged software for the ADI SDP was prone to crashes and lacked an open API, precluding custom software fixes.

¹³³Mini-Circuits ZMDC-10-1+

Squareatron A Squareatron¹³⁴ unit designed by TH [Har13], was generously sent to us by DTCA and JM at the University of Oregon. Squareatron is an active RF limiter that uses a logic level driver¹³⁵ to convert the incoming trap RF waveform into logic-level pulses, effectively “squaring” the input signal. As the output is effectively clamped to the driver’s supply rails, amplitude fluctuations are strongly suppressed. However, due to the presence of coherent noise faster than our dephasing timescales (see Section 3.4.2), the improvements offered by Squareatron have sadly gone unrealized.

5.3.3 Parametric Modulation

The outsize importance of micromotion to the QLS search with EGGS requires a faster and more robust method of micromotion compensation than is available with typical laser-based techniques, a role fulfilled by parametric modulation [Nad+21,ITU11]. The parametric modulation system was additionally repurposed to create motional squeezed state for quantum enhancement (see Figure 8.8). As a scientific discussion of the micromotion compensation protocol can be found in section 6.2.4, the discussion here is limited to the technical details.

Our implementation follows the procedure outlined in [Nad+21], though with some modifications. We first outline the basic apparatus, then describe practical considerations and their appropriate remedies.

Parametric modulation is implemented by the block diagram in Figure 5.17. This implements passive, phase-stable amplitude modulation of the trap RF with (ideally) minimal impact on its level. The trap RF signal is split¹³⁶ into two arms, in which one feeds the LO port of a mixer¹³⁷. An excitation signal, generated by an AD9910 channel from an Urukul, is applied to the IF port of the same mixer to create amplitude-modulated (AM) sidebands, which are then recombined with the carrier using a coupler¹³⁸ in the reverse configuration.

¹³⁴<https://github.com/OregonIons/Squareatron-5000>

¹³⁵Analog Devices LTC6957

¹³⁶Mini-Circuits ZMSC-2-1W+

¹³⁷Mini-Circuits ZX05-1L-S+

¹³⁸Mini-Circuits ZFBDC20-61HP-S+

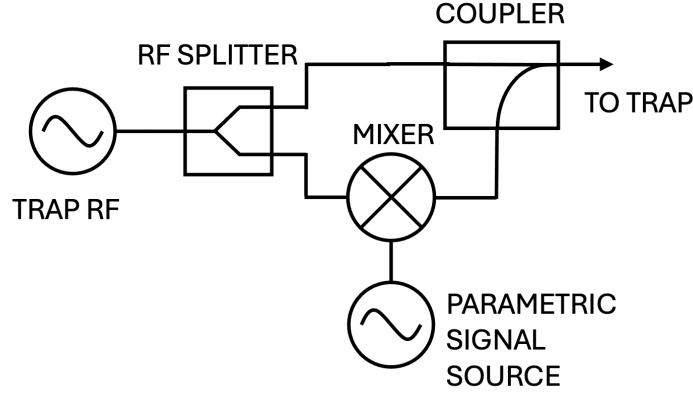


Figure 5.17: **Block diagram of the trap RF parametric modulation system.**

The trap RF is split into two arms to provide tunable, phase-stable parametric modulation with minimal effect on the carrier; one arm drives the LO port of a mixer to upconvert the parametric signal at the IF port and generate AM sidebands. Parametric sidebands are recombined with the carrier via coupler in reverse configuration prior to trap delivery; extraneous components are omitted for clarity.

For detection, photon counts are detected by the PMT and demodulated in software with respect to the applied excitation signal. Though the excitation signal can be applied with arbitrary initial phase, this phase must be known to enable the demodulation process.

Baluns¹³⁹ electrically isolate the trap RF system from external signal grounds, while switches improve signal extinction.

Power Considerations To achieve reasonable excitation for compensation, a modulation depth $m > 1\%$ is desired, which corresponds to a relative power of -46dBc in each sideband [Nad+21,Kna24]. Achieving this modulation depth is practically challenging due to the sideband attenuation from a high-Q helical resonator: a helical resonator with center frequency $\Omega_{\text{RF}} \approx 19 \times 2\pi\text{MHz}$ and quality factor $Q \approx 150$ attenuates sidebands at $\omega_0 \approx 2 \times 2\pi\text{MHz}$ by $\approx 24\text{dB}$, raising the sideband power requirement to -22dBc .

If RF realities are ignored, then the circuit in Figure 5.17 can be implemented passively: typical losses of non-resistive splitters, mixers, and couplers are -4 , -6 , and -10 dB , respectively, resulting in a modulated signal level of -20dBc , fulfilling the sideband power

¹³⁹Mini-Circuits FTB-1-6+

requirement. However, mixers generally require LO levels $> +3\text{dBm}$ to sufficiently drive the underlying diodes (which have forward voltages of $\approx 0.5\text{V} \rightarrow +7\text{dBm}$), though higher level mixers are desired to improve linearity and port isolation. This is further complicated by the fact that the performance of most commercially available signal generators become unspecified above $\approx +10\text{dBm}$. To meet these power requirements while avoiding the introduction of noise into the trap RF arm in Figure 5.17, an amplifier was placed immediately before the LO port of the mixer¹⁴⁰.

Additionally, the reflection of the sideband power by the helical resonator can result in damage to the trap amplifier. As RF isolators and circulators are generally unavailable in the trap RF band ($\approx 20\text{MHz}$), a robust amplifier with high directivity is required.

Intermodulation & Spurs Judicious component selection is required to minimize the unintended third-order intermodulation (IM3) products that are created at $\Omega_{\text{RF}} \pm 2\omega_0$ and destabilize the ion motion [Nad+21,ITU11]. Straightforwardly, nonlinear elements (switches, mixers, amplifiers) should be selected for high 3rd order intercept point (IP3) to prevent creation of IM products. The linearity of signal sources should also be considered: the second order harmonic of the AD9910 used to generate the excitation signal is prominent at -25dBc if used at high power, which improves to -40dBc by a 20dB reduction in the output level¹⁴¹. Usage of the AD9910 near its IP3 point can also inhibit efficient filtering due to the power-dependence of its output impedance; this was found to be the case with a Mini-Circuits BLP-1.9+ filter. Surprisingly, filtering was recovered by instead using a filter¹⁴² with worse impedance match characteristics.

Mode Frequency Drifts Operation at high modulation depths has been found to cause a power-dependent shift in the mode frequency, as noted by [Nad+21] and [Har13]¹⁴³. This is

¹⁴⁰This was done in an earlier version of the system; however, as the IM3 levels were found to be tolerable, the amplifier was moved after the mixer to buffer its output.

¹⁴¹<https://github.com/sinara-hw/Urukul/issues/29>

¹⁴²Thorlabs EF508

¹⁴³Though in a different context, see pg115.

particularly challenging for automated compensation, which adjusts the modulation depth to optimize the strength of the correlation signal¹⁴⁴. Moreover, this presents a unique difficulty in our experiment due to the poor COM-mode participation of $^{40}\text{Ca}^+$ when co-trapped with H^{35}Cl^+ , requiring greater modulation depths to achieve sufficient excitation. A later need to use parametric modulation towards coherent motional squeezing presented a further impetus for a solution of this issue.

The first cause is due to use of an amplitude stabilization servo for the trap RF (see section 5.3.2). The modulation signal was detected by the rectifier that created the error signal for the servo, which generated a spurious response in the RF amplitude. Though the servo has an integrator hold feature available, this has been found to result in significant drifts of the servo control signal, which we attribute to capacitor discharging effects in the analog servo. This effect was previously mitigated by placing the rectifier after the helical resonator to attenuate the detected modulation to a manageable level.

Secondly, testing of the modulation hardware with a spectrum analyzer¹⁴⁵ revealed a 0.3dB change in the RF power whenever the modulation signal was applied, corresponding to an approximate 3.5% change in the radial secular frequency. The bulk of the 0.3dB change was traced to LO leakage due to a DC offset at the IF port, introduced by the function generator used at the time to provide the excitation signal; this was fixed by addition of a DC block directly at the mixer’s IF port.

A remaining 0.2% change in the secular frequency was measured experimentally (corresponding to a $\approx 0.02\text{dB}$ change in the trap RF power) and attributed to changes in the VSWR of the mixer¹⁴⁶ as the modulation signal was applied. A 3dB decrease from -14dBc to -17dBc in the reflected wave at the LO port of the mixer was measured. Following the specified 30dB isolation of the splitter¹⁴⁷, a 0.016dB change in the trap RF amplitude was

¹⁴⁴Optimization of the correlation amplitude via laser damping is not viable in our system for automated compensation due to the lowpass behavior of our shim electrodes, which drift on the second timescale.

¹⁴⁵Keysight N9010A EXA Signal Analyzer

¹⁴⁶At the time, this was Mini-Circuits ZX05-1HW-S+.

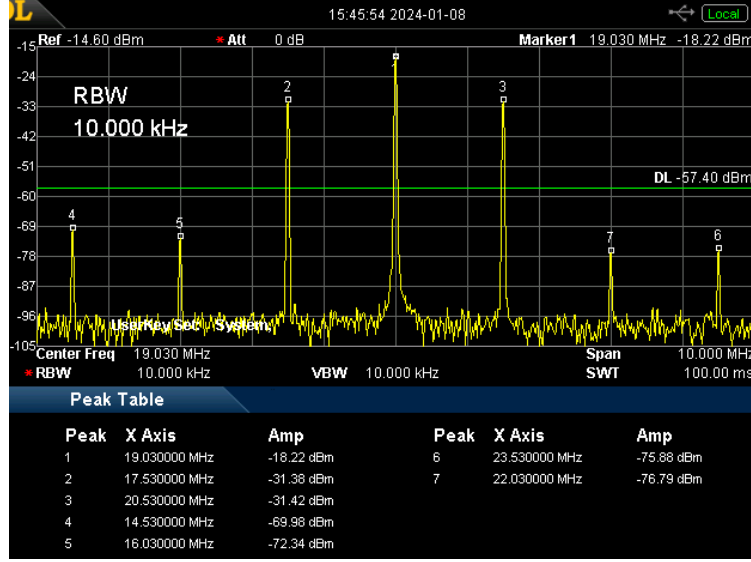


Figure 5.18: **Parametric modulation of the trap RF.**

Spectrum analyzer trace of the trap RF prior to trap delivery, centered on the carrier at $f_{\text{RF}} = 19.03$ MHz. The desired first-order amplitude-modulation sidebands at $\pm\omega_0 \approx 2\pi \times 1.5$ MHz (peaks 2–3) are at -13 dBc; these are attenuated by a further ≈ 20 dB following the helical resonator. Higher-order intermodulation spurs from the mixer at $\pm 2\omega_0$ and $\pm 3\omega_0$ (peaks 4–7) are suppressed by < -37 dB relative to the first-order sidebands, with further attenuation ≈ 40 dB following the helical resonator. Spurs at $\pm 2\omega_0$ must be minimized to avoid destabilization of the ion motion during parametric excitation.

calculated, in good agreement with the experimentally measured value. At the time, this issue was treated by adding attenuators at the LO and IF ports of the mixer to improve the impedance match; a later redesign of the trap RF electronics included amplifiers in both arms of the splitter, as well as after the RF port of the mixer, to buffer any potential impedance changes.

5.3.4 Trap Cancellation

Due to concerns about the low HCl loading rate, [HW](#) considered that stark shifts of the Rydberg states in HCl from the trap RF were preventing efficient REMPI¹⁴⁸, and proposed that the Gordian trap RF should be cancelled, following [\[BK20,BSK25\]](#). To do so, the

¹⁴⁷At the time, this was [Mini-Circuits Z99SC-62-S+](#).

¹⁴⁸A later reexamination of the theory by [HW](#) found that this was unlikely, as HCl is resonantly ionized from the ground state, where the effect of the trap RF-induced stark shifts are likely minor.

procedure outlined in [BK20,Bla22] was implemented and synchronized with the loading pulses from the 243nm loading beam for HCl.

Cancellation Sequence

Our implementation of trap cancellation, shown in Figure 5.19, closely follows [BK20,Bla22]. Normal trap RF (Ω_{RF} at low power was provided by a signal generator¹⁴⁹ connected to one port of an SPDT switch¹⁵⁰ (Switch 1). Cancellation was started in advance of a spectroscopy pulse by the “Q-Switch Sync” TTL signal of the 532nm pump laser¹⁵¹, which triggered the main experiment FPGA¹⁵² (Kasli) via a TTL card¹⁵³ to initiate the pulse sequence.

At the next phase of the trap RF, the RF source was switched to the RAMP-DOWN source, provided by one channel of a function generator¹⁵⁴ at high power and π -out of phase with Ω_{RF} , to quickly destructively interfere with and cancel residual energy stored in the helical resonator. Near the zero-crossing of the RF amplitude, the source was switched to RAMP-UP (provided by the other function generator channel) to quickly restore the trap RF, then switched back to Ω_{RF} .

The function generator was synchronized to the trap RF via the 10MHz SYNC reference signal provided by Ω_{RF} . Due to the experimental cost¹⁵⁵ of synchronizing ARTIQ/Kasli with a 10MHz reference, we instead synchronized to the trap RF by amplifying and biasing a pickoff to meet TTL thresholds, then feeding it to a TTL input on Kasli. A variable delay was inserted after triggering by the Q-Switch Sync TTL to match latencies to the 243nm spectroscopy pulse.

¹⁴⁹Rohde & Schwarz SMY-01, option SMY-B1

¹⁵⁰Mini-Circuits ZASWA2-50DR-FA+

¹⁵¹Spectra-Physics Quanta-Ray Nd:YAG Laser, Lab Series

¹⁵²Sinara Kasli v2, manufactured by M-Labs

¹⁵³Sinara 2118 BNC-TTL, manufactured by M-Labs

¹⁵⁴Rigol DG2052

¹⁵⁵ARTIQ/Kasli is clocked internally (Si5324) at 125MHz. While it can be instead clocked to either an internal or external 10MHz reference, this results in degraded phase noise performance.

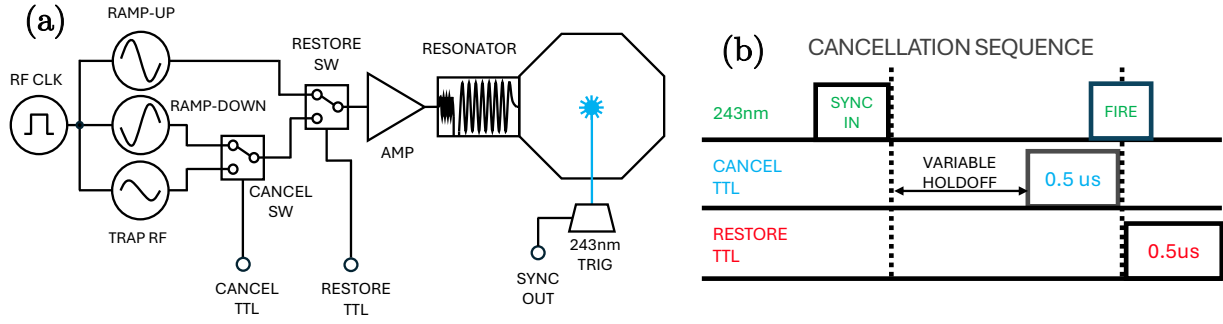


Figure 5.19: **Trap RF cancellation system.**

(a) The normal trap RF, a high-power out-of-phase cancellation drive, and a high-power in-phase restoration drive are selected by a pair of fast RF switches; all signal sources are synchronously clocked. The cancellation sequence is triggered by a SYNC pulse from the 243 nm dye laser. (b) Cancellation sequence: following the SYNC trigger, a variable holdoff is used to ensure the 243 nm spectroscopy pulse fires at the RF minimum. A TTL selects the out-of-phase drive to dissipate energy stored in the resonator, followed by the in-phase drive to restore the trap RF.

Calibration Calibration was performed by visually examining on an oscilloscope¹⁵⁶ an RF pickoff placed after the helical resonator and the output of a fast photodiode used to sample the 243 nm beam. The phases and amplitudes of RAMP-UP/DOWN were manually tuned in an iterative fashion to optimize the minimum trap RF voltage (Figure 5.20). Variable delays were tuned to align the minimum voltage point with the photodiode signal.

Cancellation Results

Results of the procedure are shown in Figure 5.21. The natural ring-down (ring-up) time of 4.92 μ s (5.30 μ s) was shortened to 1.25 μ s using cancellation and restoration powers of +39 dBm relative to a normal trap RF power of +26 dBm, resulting in a voltage extinction ratio of $\approx 66\times$.

The fastest cancellation time achieved was 1 μ s, which was limited by the damage threshold of the amplifier¹⁵⁷ and the minimum trap RF power required to trap ions; this could be straightforwardly improved by use of an amplifier with higher output IP3 (OIP3). Empirically,

¹⁵⁶Keysight D-SOX2024A

¹⁵⁷Mini-Circuits ZHL-5W-1+

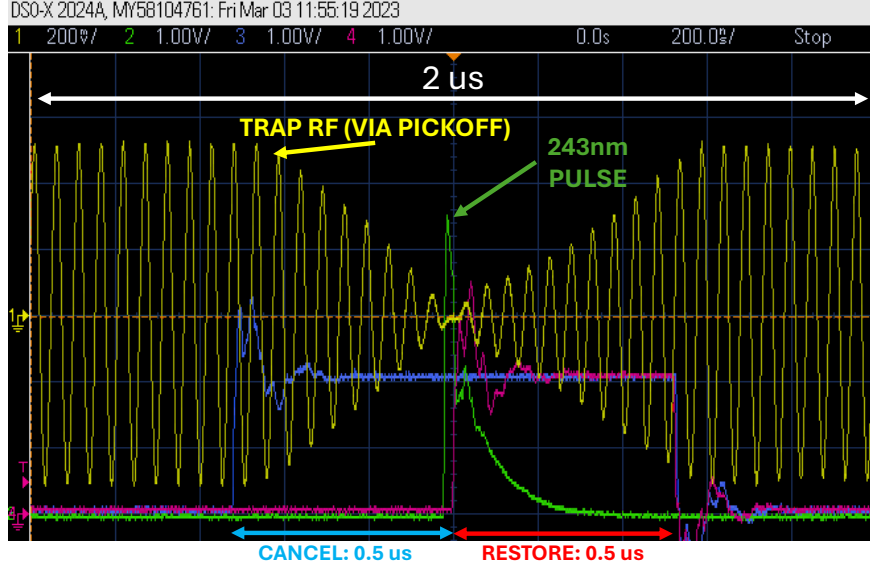


Figure 5.20: **Oscilloscope trace of the trap RF cancellation signals.**

The trap RF (yellow, via pickoff) is driven to a minimum during cancellation and restored immediately after; TTL signals for each phase are in blue and red, respectively. The sequence is triggered by a SYNC pulse (not shown) to fire with the 243 nm spectroscopy pulse (green).

we found that cancellation times below $1.5\mu\text{s}$ were required to prevent ion ejection at secular frequencies of 597kHz, 880kHz, 1103kHz along the axial and both radial directions, respectively. Regardless, ions were occasionally ejected during operation; the cause of this issue was unclear and not investigated further due to lack of experimental priority.

A key problem encountered was jitter of the cancellation minimum relative to the 243nm pulse, which we ascribe to the non-deterministic delays introduced by the poorly-implemented external synchronization between ARTIQ and the trap RF. A much improved implementation could simply generate the normal trap RF, cancellation, and restoration waveforms through ARTIQ, greatly reducing synchronization problems and calibration times required; this approach was not initially pursued due to overly rigid adherence to [BK20].

5.3.5 Trap DC Voltage Generation

Due to the small axial geometric factor of the trap, high DC voltages in the $> 300\text{V}$ range were required to provide sufficient confinement. Comparably high voltages ($\sim 50\text{V}$) were

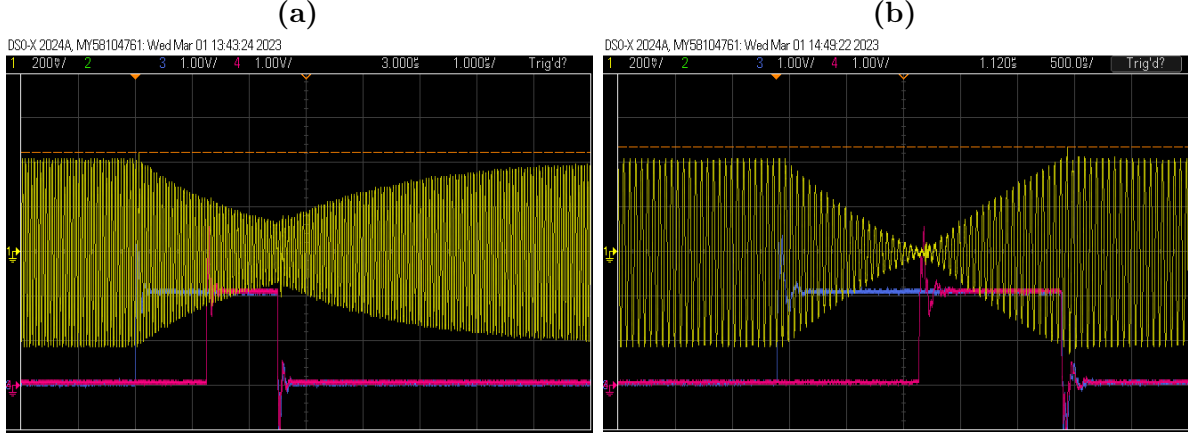


Figure 5.21: **Trap RF cancellation efficacy.**

Oscilloscope traces of the trap RF (yellow, via pickoff) with switch trigger signals (blue, red). **(a)** If the drive is directly switched, the RF rings down and back up with time constants $t_{90\%} = 4.92 \mu\text{s}$, $5.30 \mu\text{s}$, respectively. **(b)** Active cancellation shortens the ring-up/ring-down times to $1.25 \mu\text{s}$ and achieves a voltage extinction ratio of $\times 66$ at the trigger point (green trace).

required on the shim electrodes for micromotion compensation due to their distance from the trap center and their partial screening by the radial blades.

As fast ion transport or fast axial frequency changes were not required for the experiment, the speed of the voltage updates was non-critical, allowing control outside ARTIQ. To provide these voltages, a high-voltage (HV) DAC unit (AMO8), originally intended for control of micro-channel plate (MCP) bias voltages, was repurposed for this experiment. AMO8 was used to supply all DC voltages required for trapping, i.e. for the axial electrodes, both shim electrodes (Vertical Shim and Horizontal Shim), and the A-Ramp bias (used to add a DC bias to the radial electrodes). A breakout box to convert the D-Subminiature 37 output connector on AMO8 to individual SMA connections was designed and built by DK.

The output was connected to the vacuum chamber via a DC feedthrough with surface mount capacitors for AC decoupling. The DC feedthrough PCB was designed by CS and assembled by GDM. However, as the decoupling capacitors were found to attenuate high-frequency ($\sim 10^2 \text{MHz}$) signals delivered to the trap, these were later removed; this was not found to impact the axial coherence or long-term stability.

AMO8 HV DAC

AMO8 was designed and manufactured by [PY](#) from the UCLA Physics & Astronomy Electronics Shop (eShop). It uses a 32-channel 16-bit DAC¹⁵⁸ which is conditioned then amplified by custom low-noise amplifiers. The DAC is controlled over SPI by a custom microcontroller (MCU) board (designed by CS and PY and manufactured by PY) based on the ATmega2560¹⁵⁹. The MCU is controlled by the experiment over USB and has an additional touchscreen¹⁶⁰ for manual control.

Custom firmware was developed to implement simultaneous adiabatic voltage ramping of multiple channels. Though the original intent of this feature was to enable [Dipole-Phonon Quantum Logic \(DPQL\)](#), the hardware implementation was deemed insufficient for this goal due to the nondeterministic timing of the MCU and lack of support for synchronization. However, this feature remains particularly useful in daily trap operation for smooth trap changes and transport.

DC Noise Sources

The HV source for AMO8 was originally provided by an internal [Switched-Mode Power Supply \(SMPS\)](#). Due to noise concerns, the radial A-Ramp bias was, for a time, instead provided by an AA 1.5V alkaline battery [[BDW95](#)]. As a more permanent solution, AMO8 was modified to accept an external HV source, which we supplied with a linear regulated PSU¹⁶¹. This reduced the axial mode heating rate to $\dot{n} \sim 1$ quanta/s. Further sources of noise were found to be the internal ADC (used for current monitoring) and the MCU board. As remedies, the internal ADC was disabled in firmware, and the MCU board was physically mounted outside the grounded AMO8 box.

¹⁵⁸[Analog Devices AD5372BSTZ](#)

¹⁵⁹[Microchip ATmega2560](#)

¹⁶⁰[FTDI CleO35](#)

¹⁶¹[Bertan NIM-AC 313B](#)

Condensation

Condensation near electrical junctions can result in interesting compound failure modes, particularly in the context of HV sources. Consequently, a buildup of condensation at the DC electrode feedthrough on the chamber created a path to ground with a resistance $< 1\text{M}\Omega$. As the trap electrodes have a dominantly capacitive impedance, the AMO8 voltage outputs were directly connected to the trap with only decoupling capacitors at the trap feedthroughs. The ground “short” created by the condensation therefore resulted in significant and weather-dependent voltage division of the AMO8 output voltage that obstructed experimental operation. Visually, the ion position and secular frequencies were seen to drift slowly over time, while micromotion compensation became non-deterministic.

Additionally, due to the deprecation (see section [DC Noise Sources](#)) of AMO8’s current monitoring ADC and its original limited design scope, overcurrent events from the short were not detected by AMO8. The ADC represented the sole source of overcurrent *monitoring* in AMO8; there were (are) no modes of overcurrent *protection* — this was, to be sure, a design *feature* to enable a freedom of maneuver in experiment not afforded by other HV sources.

At the component level, fixes consisted of adding dessicants and resistive heaters to the trap feedthrough, and $2\text{M}\Omega$ resistors were added in series to the AMO8 outputs to prevent overcurrent draws. At the laboratory level, moisture buildup near cryogenics was mitigated with absorptive media¹⁶², and dehumidifiers were permanently installed in the room¹⁶³.

5.3.6 EGGS RF Signal Path

Overview

The EGGS RF signal path is shown in Figure [5.22](#). To implement high-fidelity [EGGS](#), high power ($> +30\text{ dBm}$) signal delivery *to the trap* was needed across a wide frequency range ($[40\text{MHz}, 1\text{GHz}]$), with reasonable frequency agility ($> 10\text{ MHz}$) at a given range.

¹⁶²i.e. towels.

¹⁶³Though used only when necessary.

Signals were generated by two channels of an [AWG](#), which were bandpass-filtered¹⁶⁴ and amplified¹⁶⁵. Reflective SPDT switches¹⁶⁶ were connected to the amplifier outputs, with high-power terminators¹⁶⁷ connected to the unused ports to absorb reflections when the switch was closed. Linear regulated power supplies¹⁶⁸ were used to power the amplifiers and switches. The output was then passed through a second stage of filtering¹⁶⁴ configured to pass the range of interest, then conditioned by the “Peter Box”¹⁶⁹ connected directly to the trap feedthrough. The high powers involved in the second stage constrained the filter selection available, resulting in suboptimal filter characteristics (e.g. rolloff, passband ripple).

The “Peter Box” signal conditioning stage consisted of a high-power bias tee and a terminated notch filter for each electrode. Addition of a DC bias through the EGGS electrodes was needed to break the radial mode degeneracy; both electrodes were biased by the same DC source.

Parasitic Couplings

Connecting signal sources and RF components to the trap electrodes required careful attention to the system’s impedance, as was discovered when first installing filters on the EGGS signal paths. Highpass filters (> 30 MHz) were needed on the EGGS electrode paths to suppress noise and spurious output from the high gain (≈ 50 dB) EGGS amplifiers, which otherwise produced significant motional heating. Direct connection of these filters to the electrodes severely degraded the trap stability and induced excess micromotion, which in turn prevented sideband cooling.

The root cause was traced to the change in the impedance seen by the trap RF drive at $\Omega_{\text{RF}} \approx 19$ MHz and phase-mismatched reflections onto the EGGS electrodes. Nominally, the RF electrodes were electrically independent of the EGGS electrodes; however, the

¹⁶⁴Selected according to the frequency range of interest.

¹⁶⁵Mini-Circuits ZHL-20W-13

¹⁶⁶Mini-Circuits ZSW2-272VHDR+

¹⁶⁷Mini-Circuits TERM-25W-183S+

¹⁶⁸Acopian VA24MT550FM, 3EB50

¹⁶⁹Named for its constructor, [PY](#).

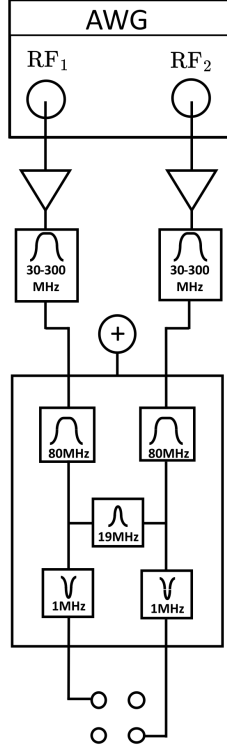


Figure 5.22: **Block diagram of the EGGs RF signal path.**

Each [AWG](#) output is amplified and bandpass-filtered (30 MHz to 300 MHz), then passed through a second conditioning stage (the “Peter Box”). The “Peter Box” DC biases the [AWG](#) signals to break the radial mode degeneracy, and contains notch filters at $f_{\text{RF}} \approx 19$ MHz to shunt to ground any trap RF pickup.

significant parasitic coupling between the RF electrodes and the surrounding conductors (EGGS electrodes, trap frame, shim electrodes) meant that the system transfer function was interdependent and sensitive to changes on ostensibly isolated paths. A first solution used lowpass filters¹⁷⁰ on each EGGs electrode to shunt the trap RF to ground and prevent it from propagating along the EGGs signal path and reflecting back — a similar problem was likely encountered in [[All11](#),[All+11](#)]¹⁷¹. A longer-term solution was the “Peter Box,”¹⁷² a custom diplexer with notch filters that selectively shorted the EGGs electrodes to ground at Ω_{RF} while passing the DC-biased EGGs RF.

In general, the large number of devices connected to the trap (as well as each other) created

¹⁷⁰Mini-Circuits SLP-21.4+

¹⁷¹See section 4.5 in DTCA’s thesis.

¹⁷²Designed and built by [PY](#).

parasitic couplings and loaded system impedances in unpredictable ways. Complications arose from the decoupling capacitors on the DC electrode feedthroughs (see section 5.3.5). Due to the strong parasitic coupling between electrodes, these capacitors provided an unintended low-impedance path to ground for the EGGS RF and resulted in significant wideband attenuation (~ 10 dB); this was solved by removal of the decoupling capacitors, relying instead on signal quality to mitigate decoherence. Connection of resistive heaters on the 40 K stage, located far from the trap and electrically isolated from the electrodes, shifted the EGGS RF coupling by ~ 2 dB. This was addressed by logging the complete trap configuration (e.g. all cable connections, powered devices) and defining a reference setup against which subsequent measurements could be compared.

Signal Integrity

Signal integrity was a priority for high-fidelity EGGS due to the ability of spurs to drive unintended higher-order motional transitions (see Chapters 8 and 9), as well as the sensitivity of H^{35}Cl^+ to broadband noise. This was complicated by the need for high power, phase-coherent multi-tone signals, and initially, the lack of a spectrally pure signal source.

Noise As the EGGS electrodes required independent control and thus separate amplifiers, the significant white noise from the +50 dB amplifiers was uncorrelated between the electrodes; lower gain amplifiers were non-viable due to their reduced IP3. Uncorrelated noise produces dipolar (rather than quadrupolar) field gradients at the ion, which couple more strongly to the motional mode by a factor¹⁷³ of $\eta^{-1} \approx 10^4$ [Tur+00b,Bro+15] and were thus capable of driving unintended transitions out of the H^{35}Cl^+ manifold of interest (see section 3.4.1 and eq. (7.10)). This noise was partially mitigated by minimizing signal leakage when the EGGS drive was inactive: the internal attenuation of the signal source was maximized, and RF switches were added following the amplifiers. The switches were further helpful in suppressing video feedthrough (see section 5.4.1). Implementation of these strategies reduced

¹⁷³This is the [Lamb-Dicke Parameter](#) for the [EGGS](#) field gradient.

the motional heating rate on the EGGS mode¹⁷⁴ from ≈ 50 quanta/s to ≈ 1 quanta/s. However, these strategies were applicable only when the EGGS RF was not actively applied (e.g. during sideband cooling, readout).

IMD Reduction of **IMD** products was achieved by using high **3rd-Order Intercept Point (IP3)** components. For the RF switches, this was not straightforward: we required fast (< 250 μ s) rise times to minimize sequence overhead, sufficient isolation ($\gtrsim 30$ dB) to mitigate the amplifier gain, and high **IP3** to prevent signal **IMD**. Generally, these goals are mutually exclusive: standard solid-state switches (i.e. PIN diodes) support fast switching and high isolation but low power handling, while the converse is true for electromechanical switches. The switches currently in use¹⁷⁵ represented a compromise between these requirements¹⁷⁶: it has a 25 μ s rise time, $\gtrsim 40$ dB isolation for signals below 500 MHz, and a 85 dBm **IP3**. To prevent pulse distortions from the non-negligible switch rise time, spectroscopy pulses involving the EGGS RF were padded with 25 μ s holdoff periods.

The spectral purity of the signal source was primarily limited by operation of the **AWG**¹⁷⁷ outside its specified frequency range (< 300 MHz), a limitation imposed by the internal **IQ**-upconversion mixers¹⁷⁸. This was mitigated by transitioning to a baseband variant of the **AWG**¹⁷⁹ without the **IQ** mixers, and by use of the **QMC** calibration features (see section 5.4.1).

5.4 Control

In rather a broad and generous sense, the challenge of experimental control finds a spiritual ancestor in the challenge of command and control following the escalation of conflict in Continental Europe in May of 1940. At the time, French command was unnecessarily

¹⁷⁴The motional mode defined by the direction of the EGGS electrodes.

¹⁷⁵**Mini-Circuits ZSW2-272VHDR+**

¹⁷⁶As well as our limited taste, evoking again Bourdieu (see chapter epigraph): switches such as Menlo Micro MM5120 offer improved characteristics.

¹⁷⁷**M-labs & Sinara, Phaser Upconverter**

¹⁷⁸Texas Instruments TRF372017

¹⁷⁹**M-labs & Sinara, Phaser Baseband**

geographically divided near Paris and without effective means of communication to coordinate their units across the Continent (not unlike managing instruments across an experiment), resulting in command cycles measured in days, as opposed to the hours in which entire battles transpired. In the field, this neglect of command and control persisted at the unit level — tank turrets were often staffed with a single person, requiring commanders to fulfill duties beyond commanding, a challenge compounded by the further burden of slow Morse radio [Lar05,96]. Together, these factors created an ossified control structure that subjugated even minor successes to a glacial operational tempo, forestalling any progress in the Allied campaign.

In developing the experimental control system of the experiment, a duty of great gravity thus falls to us, in commanding and controlling disparate instruments scattered across a full laboratory space that, in its prime, was spanned by *three* separate experiments¹⁸⁰. To avert the terrors of history, we should look to incorporate these historical lessons in our work.

Contributions [DK](#) built a breakout box for the LabJack T7-Pro.

5.4.1 Signals

Here, we discuss experimentally relevant features of the key signal sources used in this experiment.

AD9910 DDSs

These frequency agile, low-noise, and feature-rich sources form the workhorse of any ARTIQ-controlled experiment — and ours is no different. The AD9910 DDSs (Direct Digital Synthesizers) have integrated 14-bit Digital Analog Converters (DACs) and are clocked at 1 GHz internally with a 62.5 MSPS 4-wire SPI (Serial Peripheral Interface) for waveform updates. The AD9910s are integrated into the experiment via the “Urukul” DDS carrier

¹⁸⁰The sun never sets on the Holy Hudson Empire (because it’s in the *sub*-basement).

cards, purchased from M-labs and designed by the Sinara collaboration¹⁸¹. Communication with the AD9910s from the main Field Programmable Gate Array (FPGA)¹⁸² is multiplexed by a Complex Programmable Logic Device (CPLD) on the Urukul board.

RAM Mode The AD9910’s RAM mode allows it to bridge a gap between DDSs and AWGs (arbitrary waveform generator) that is poorly-serviced in the laboratory. Custom RF waveforms are needed to implement low-priority features (e.g. pulse shaping) in the laboratory which lack the political justification for more involved solutions, such as evaluation board assembly, or entail excessive cost, such as purchase of an AWG. Despite the use of an FPGA for experimental control, the specification of complex waveforms incurs a significant cost on its resources that is exacerbated by the bottleneck of the multiplexed SPI interface on Urukul.

In RAM mode, waveform data are supplied by a $1024 \times 32\text{bit}$ RAM block internal to the AD9910 with a minimum update rate of 4ns, allowing easy generation of fast, smooth waveforms. Two key uses are made of RAM mode in our experiment: pulse shaping and sideband cooling. For pulse shaping, the desired waveform is preprogrammed onto the AD9910 at the beginning of an experimental sequence, where timing and latency are non-critical. For a typical pulse length of 500ps, the 1024 word limitation enables an update rate of approximately 2 MHz, which suitably reduces off-resonant excitation and does not significantly deteriorate the spectral properties of the pulse shape. The relative update rate can be increased somewhat by exploiting waveform symmetries for more efficient use of the 1024 word depth.

For sideband cooling, the primary function of RAM mode is to reduce the resource pressure on the [FPGA](#) event scheduler (known as the [Scalable Event Dispatcher \(SED\)](#)). Our implementation of sideband cooling is particularly involved: the 729nm laser is periodically cycled through a set of frequencies¹⁸³ to cool multiple modes, requiring concomitant adjustment of the quench amplitude as well as periodic pulses from the spin polarization beams (see

¹⁸²a.k.a. Kasli.

sections sections 4.2.2 and 4.4.4 for the relevant background). For two ions (typically 6 modes, 20 cycles), enough events are submitted to saturate the SED, stalling experiment progress and resulting in a sequence error.

Digital Ramp Generation (DRG) The AD9910 further contains an internal Digital Ramp Generation (DRG) mode that allows linear waveform ramps. DRG can be simultaneously enabled with RAM mode, allowing generation by a single AD9910 channel of the shaped, chirped pulses required for Rapid Adiabatic Passage (RAP). Alternatively, the DRG can be applied towards amplitude ramping with the RAM instead directed towards nonlinear frequency chirps, such as for nonlinear-chirp RAP [Um+16].

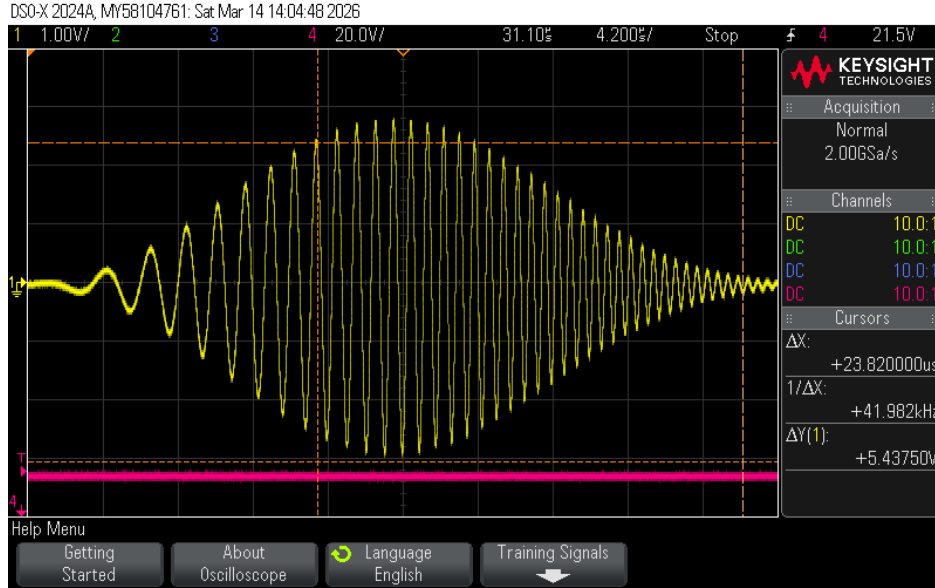


Figure 5.23: **Chirped and shaped waveforms from an AD9910 channel.**

A shaped (Blackman window) and frequency-chirped pulse of length $50 \mu\text{s}$ from an AD9910 channel of an ARTIQ Urukul DDS board. The Blackman window is created using 200 samples from the AD9910’s RAM modulation mode. The frequency is ramped linearly from $[0, 2]$ MHz over the length of the pulse ($50 \mu\text{s}$), and is generated by the Digital Ramp Generator on board the AD9910.

AD9910 Pitfalls Inevitably, use of the flexible AD9910 is accompanied by commensurate challenges.

¹⁸³Known as an “SBC schedule.”

If used to directly excite the ion motion (e.g. to apply a tickle), video feedthrough¹⁸⁴ from the internal switch¹⁸⁵ and attenuators¹⁸⁶ can result in significant unintended excitation of spectator modes. The timescale of the transient is of order $\sim 1\mu\text{s}$ and thus near the periods of the secular frequencies, while the sharp and aperiodic behavior of the transient results in a wide spectral occupancy. Together, these preclude straightforward frequency-domain mitigation (i.e. filtering). The solution of greatest parsimony is to attenuate the net signal while boosting the underlying DDS drive¹⁸⁷. As the video feedthrough transient is additive and is not inherent to the underlying signal, the SNR can be boosted by e.g. increasing the DDS [Amplitude Scaling Factor \(ASF\)](#), such that later attenuation recovers the original signal level. Alternatively, Motional Raman techniques can be used to spectrally separate the excitation signals far from the frequency band of the video feedthrough¹⁸⁸ (see chapter 8).

Changes between waveform profiles, as controlled by the PROFILE pins on the AD9910, can result in waveform glitches, which can in turn cause coherence-destroying phase slips or equipment-destroying overvoltage events. Waveform glitches due to profile changes are caused by the [CPLD](#)'s control over the 3-bit PROFILE pins. The [CPLD](#) controls the PROFILE pins via shift register and is thus unable to delay-tune the PROFILE pins to ensure that they meet setup-and-hold (S/H) requirements relative to SYNC_CLK. Unless the implementation is carefully considered, waveform changes between profiles should therefore be considered non-coherent. To demonstrate the impact of the PROFILE pin problem, we consider a change from profile 0 (0b000) to profile 7 (0b111): over 3 SYNC_CLK cycles, this could, in the worst case, be latched by the AD9910 as 1 (0b001), 5 (0b101), then 7 (0b111).

Where phase-coherent profile switching is necessary, such as to e.g. reduce lengthy waveform update latencies, this problem is best addressed by either setting all profiles to have the same frequency (preventing permanent changes to the phase accumulator from FTW

¹⁸⁴Video feedthrough is a sharp transient behavior at the output of a port due to the necessarily fast switching of control voltages.

¹⁸⁵[Analog Devices HMC349ALP4CE](#)

¹⁸⁶[Analog Devices HMC542BLP4E](#)

¹⁸⁷In RF circles, this is known as “*emphasis*.”

¹⁸⁸Though this may be excessive.

changes), or by changing only between profiles with a Hamming distance of one.

Phaser

The Phaser [AWG](#) served as the sole [AWG](#) in our laboratory and was therefore integral to the EGGS search (see chapter [7](#)) and the motional Raman experiments (see chapters [8](#) to [10](#)). The hardware was designed by the Sinara collaboration, the gateway designed by QUARTIQ and M-labs, and the physical device was manufactured by M-labs. Though the use of Phaser entailed nontrivial technical overhead and familiarity with myriad idiosyncrasies, it was tightly integrated into ARTIQ to enable the fast, dynamic changes required in an experimental sequence¹⁸⁹, and, perhaps more importantly, it was extremely cost effective¹⁹⁰, at a fraction¹⁹¹ of the cost of industry-standard [AWGs](#). For these reasons, mastery over waveform generation with Phaser was key to our experiment, and we here discuss its operation in some depth.

Phaser Operation A more thorough discussion of the Phaser architecture can be found in [\[Kra20\]](#). Signal generation is separated into three stages: a first, digital stage on the main FPGA (Kasli), a second digital stage on the Phaser onboard FPGA, and a final analog stage composed of a [Digital-Analog Converter \(DAC\)](#)¹⁹² and [IQ-upconverter](#)¹⁹³ (Figure [5.24](#)). As the first two stages are completely digital, the latencies there are deterministic.

Two output channels are available with Phaser. On Kasli, data streams for five [IQ Numerically-Controlled Oscillators \(NCOs\)](#) at 25MSPS with user-specified 15-bit amplitudes, 16-bit phases, and 32-bit frequencies ($[-12.5, +12.5]$ MHz) are sent to the [FPGA](#)¹⁹⁴ on Phaser per channel. On Phaser, these streams are interpolated to 500MSPS then sent to a [Digital Up-Converter/Conversion \(DUC\)](#) that shifts the phase and frequency of the oscillator waveforms over a range $[-250, +250]$ MHz. The interpolated data stream is sent to a 16-bit

¹⁸⁹Surprisingly, this is not always available: pulse sequencing on the Tektronix AWG70000 series is export controlled.

¹⁹⁰USD \$4260 as of 2026/03/22, see [M-labs](#).

¹⁹¹ $< 1/20$

¹⁹²Texas Instruments DAC34H84

¹⁹³Texas Instruments TRF372017

¹⁹⁴AMD Artix XC7A100T

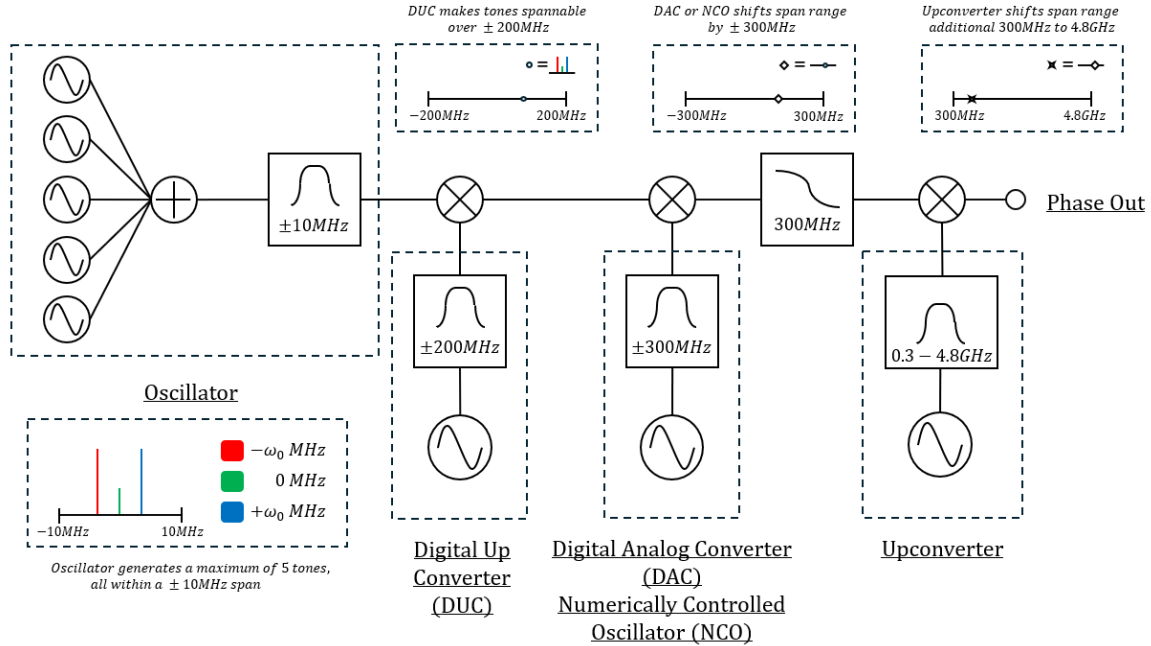


Figure 5.24: **Phaser gateway block diagram.**

Up to five **Numerically-Controlled Oscillators (NCOs)** with range $\pm 12.5\text{MHz}$ are implemented on Kasli and summed and sent to the Phaser board. An **FPGA-implemented Digital Up-Converter/Conversion (DUC)** on the phaser board shifts the tones across a $\pm 250\text{MHz}$ band, then streams the data to a **DAC-NCO**. The analog output of the **DAC-NCO** is passed through an **AA** filter. In the upconverter variant, the output is fed to a **PLL/VCO** for upconversion.

DAC¹⁹² that generates two analog **IQ** outputs at 1.25GSPS that are subsequently passed through **Anti-Aliasing (AA)** filters. In baseband Phaser, the in-phase is sent into a digital attenuator¹⁹⁵ prior to output.

Oscillator Latencies Due to the 25MSPS sample rate of the oscillators, deterministic waveform updates required alignment to the 40ns sample period. While deterministic, the relative latencies between oscillators were not matched^{196,197} and varied by parameter type; characterization of these latencies was not previously documented. For phase updates, the relative latencies of oscillators 0–4, when set sequentially and aligned to the 40ns sample period, were measured to be $0/40/80/80/120\text{ns}$.

¹⁹⁵Analog Devices HMC542B

¹⁹⁶<https://github.com/m-labs/artiq/issues/1659>

¹⁹⁷<https://forum.m-labs.hk/d/331-general-information-on-operation-of-phaser/4>

Waveform Generation Shaped waveforms required direct specification of all oscillator amplitudes and phases at each sample, representing significant technical overhead, particularly for the complex waveforms required by modulated gates and sensing (see sections 3.4.2 and 7.4.2 for applications in our experiment), as more compact modes of waveform generation standard on other platforms [Ris23,Bow+13] were not available. Smooth pulse shaping further required recording of the entire waveform sequence into DMA, which could not be modified at runtime; parameter scans therefore required separate recordings for each value. The cumulative effect was that waveform generation required lengthy calculation, uploading to the FPGA, and recording into RAM, resulting in initialization times of up to one minute.

To manage this complexity, a waveform compiler (SpinEchoWizard) was developed to allow specification of multipulse waveforms in terms of experimentally relevant primitives (e.g. pulse duration, amplitude, envelope shape) that were compiled into the per-oscillator samples required by Phaser while handling the phase bookkeeping required to ensure phase coherence.

Phaser (Upconverter)

On Phaser upconverter, the analog IQ outputs are connected into an IQ PLL/VCO upconverter¹⁹⁸ that centers the output frequency within a [300 MHz, 4.8 GHz] band, then sent into a digital attenuator¹⁹⁹ prior to output.

Unavoidable imbalances between I and Q arms in real IQ mixers result in quadrature modulation errors, the most significant of which are LO leakage, caused by relative DC offset, and incomplete sideband suppression, caused by relative phase skew and amplitude imbalance. LO leakage was suppressed by adjusting the DC offset registers on the TRF, while the image sideband was suppressed by adjusting the gain imbalance and phase offset registers on the DAC. Compensation values were optimized by iterative tuning of the relevant registers while monitoring spurs on a spectrum analyzer. This procedure achieved additional suppression

¹⁹⁸Texas Instruments TRF372017

¹⁹⁹Analog Devices HMC542B

of the LO leakage and the image sideband by 50 dB and 40 dB to ~ -80 dBc (from initial values of $\sim -30, -40$ dBc), respectively; further compensation beyond this level was found to be unstable. Optimal compensation values differed substantially between the two output channels but were stable across power cycles. Despite the significant improvement to the LO and image spurs, compensation was not found to reduce higher-order spurious products originating from the DAC.

5.4.2 Software Control

Contributions GDM wrote an ARTIQ experiment and added a button to the PMT client. JAR contributed *significantly*²⁰⁰ to the codebase. A collaboration with JP and LM (the LaSER project) made great contributions to LabRAD (see [Mil22] for further details). A mini-compute cluster running Slurm was set up by DK. Much LabRAD software was forked from the Häffner group at UC Berkeley and the Jayich group at UC Santa Barbara.

Overview

Experimental control needs are broadly divided into two regimes: the fast, nanosecond-scale, for running deterministic pulse sequences, and the slow, user-timescale, for asynchronous management of instruments. In our experiment, we use two software solutions to service both regimes. *LabRAD* handles the slower, user-facing regime, managing a network of services that exposes laboratory hardware via asynchronous remote-procedure calls. *ARTIQ*²⁰¹ handles the fast regime: it compiles experimental sequences directly to the ARTIQ hardware crates (the main control hardware, including the FPGA peripheral cards) where events are executed with nanosecond resolution over a deterministic real-time input/output (RTIO) bus. ARTIQ further supplies the ancillary infrastructure needed to run experimental sequences, such as a job scheduler, a key-value store, and a peripheral manager.

²⁰⁰Per JAR’s exhortations.

²⁰¹Advanced Real-Time Infrastructure for Quantum Physics (ARTIQ) was originally developed by M-Labs in collaboration with NIST. Development is now split across three arms: software (M-Labs), gateway

This division of labor is not rigorous: ARTIQ experiments can call LabRAD functions to configure slow hardware, and LabRAD can in turn drive ARTIQ hardware by exposing its device drivers in a dedicated server. Though ARTIQ provides a native, more user-friendly, efficient, and better-supported interface for slow peripherals ([Network Device Support Packages \(NDSPs\)](#)), its catalog of existing drivers is small and lacks generic support for multi-host systems. LabRAD’s stellar coverage on these fronts is the sole *experimental* reason²⁰² for its adoption in the laboratory; these advantages are somewhat diminished by the fact that LabRAD is no longer actively developed and is built atop the antiquated and effectively defunct Twisted library (used for asynchronous networking).

LabRAD

LabRAD, developed in 2007 by M. Ansmann and M. Neeley of the Martinis group [[Ans09](#)], follows a server-client architecture, with traffic routed by a central LabRAD manager, and packet contents defined by the LabRAD protocol. Broadly, LabRAD consists of only two components: servers, and clients. Servers expose services²⁰³ to the manager in a flat hierarchy (i.e. all servers sit at the same level) and are typically used to interface with a device, following a “one device, one server” convention. Clients are any programs that consume these services.²⁰⁴ LabRAD further implements *nodes*: special-purpose servers that manage operation of other servers running locally.

Core to LabRAD is the servers that compose its [Virtual Instrument Software Architecture \(VISA\)/General Purpose Interface Bus \(GPIB\)](#) and serial device interfaces — these allow devices connected to a host to be controlled by server on other hosts, accessible to clients anywhere on the network. Serial devices communicate over serial (typically adapted into

(quartiq), and hardware (Sinara). Funding is provided by M-Labs, alongside a consortium of trapped-ion groups and Oxford Ionics.

²⁰²Possible non-experimental reasons include cerebrovascular event, internalized shame, “systematic workflow inertia,” flagrant corruption, or divine punishment.

²⁰³Q: What is a tensor? A: A tensor is something that transforms like a tensor.

²⁰⁴For example, to record the chamber vacuum pressure, one could write e.g. an “ion pump server” exposing a function that queries the pump pressure, and a client that connects to the LabRAD manager to call this function and log the result.

USB)²⁰⁵ without a common protocol, foreclosing automatic discovery. [VISA/GPIB](#) is a metonym for devices communicating via [Standard Commands for Programmable Instruments \(SCPI\)](#), a protocol that allows for easy device discovery and identification²⁰⁶.

GPIB/VISA Interface The [VISA](#) interface comprises three servers: `GPIBBusServer`, `GPIBDeviceManager`, and `GPIBManagedDeviceServer`.

- `GPIBBusServer` runs on each host and exposes its peripheral bus, such that any device connected to the host is addressable over LabRAD.
- `GPIBDeviceManager` polls every `GPIBBusServer`, collects `*IDN?` responses to identify connected devices, and routes each to the appropriate `GPIBManagedDeviceServer`.
- `GPIBManagedDeviceServer` is a user-written virtual class that defines a bare interface for a family of devices, recognized via a user-specified list of valid `*IDN` responses. On recognition, the `GPIBManagedDeviceServer` instantiates a `GPIBManagedDeviceWrapper` that implements the device-specific drivers.

This allows a single `GPIBManagedDeviceServer` to service a broad family of devices (e.g. an “oscilloscope server”) despite varying command sets between vendors (e.g. Rigol vs. Tektronix).

Serial Interface Serial communication is less involved due to the lack of a common identification protocol. A `SerialBusServer` on each host exposes the local serial ports to LabRAD, and users write device-specific `SerialDeviceServers` which connect to hardware via the `SerialBusServer`.

²⁰⁶The specification of [VISA/GPIB](#) specifies the communication protocol rather than the physical interface; [VISA/GPIB](#) devices can use serial, for example. The term “serial device” is generally applied to devices that *lack* the [SCPI](#) protocol.

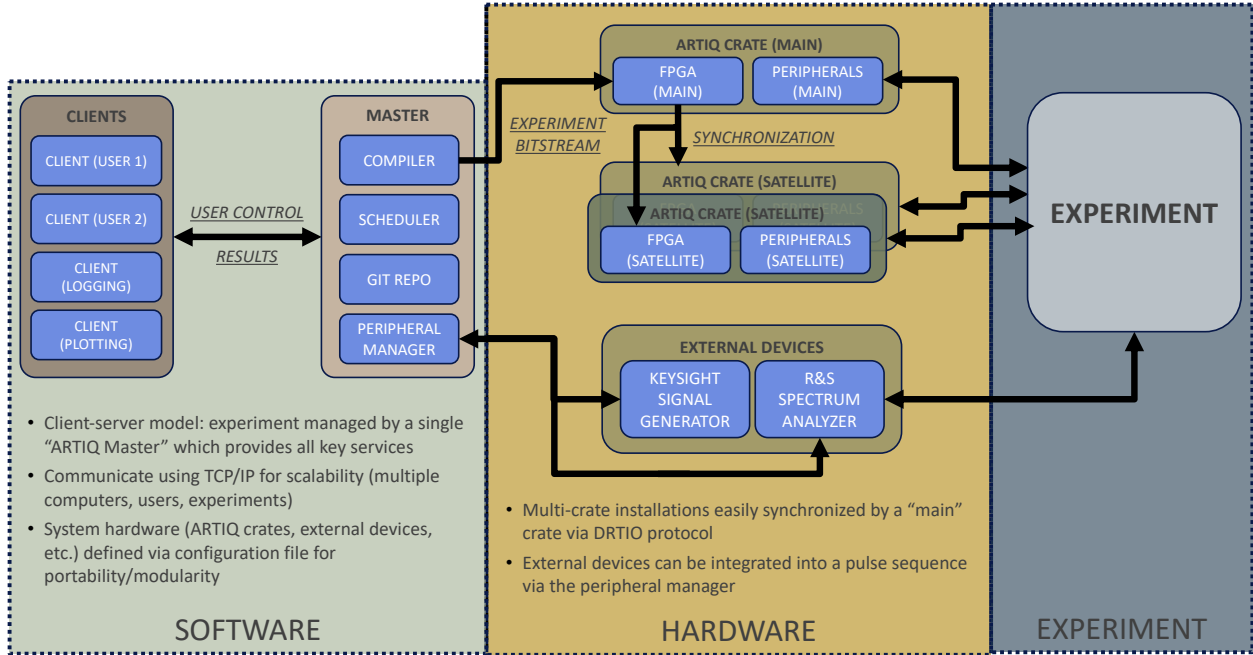


Figure 5.25: **ARTIQ control architecture.**

A single `artiq_master` (compiler, scheduler, Git repository, and peripheral manager) serves a set of clients over the network and compiles experiments to kernel binaries for the ARTIQ crates. A “main” crate synchronizes any satellite crates over DRTIO protocol, while external benchtop instruments are integrated into pulse sequences by a peripheral manager.

ARTIQ

ARTIQ executes fast, time-critical pulse sequences on an [FPGA](#)-controlled system of peripheral cards. User-written experiments are compiled to kernel binaries and transmitted over the network to the Kasli [FPGA](#) controller, where the kernel CPU schedules events into a [FIFO](#) queue tagged with their intended execution time; multiple [FIFO](#) lanes allow events to be scheduled in parallel (or in the past) relative to other events. Similarly to LabRAD, ARTIQ is organized around a central `artiq_master` that hosts a collection of services communicating using `SiPyCo`²⁰⁷.

LAX_exp ARTIQ is minimal by design, leaving the user to extend it as their experiment demands. We accordingly developed `LAX_exp`, an extension providing the abstractions needed for modularity and portability across experiments, while being unobtrusive and behaving as

²⁰⁷Simple Python Communications, ARTIQ’s in-house [RPC](#) protocol.

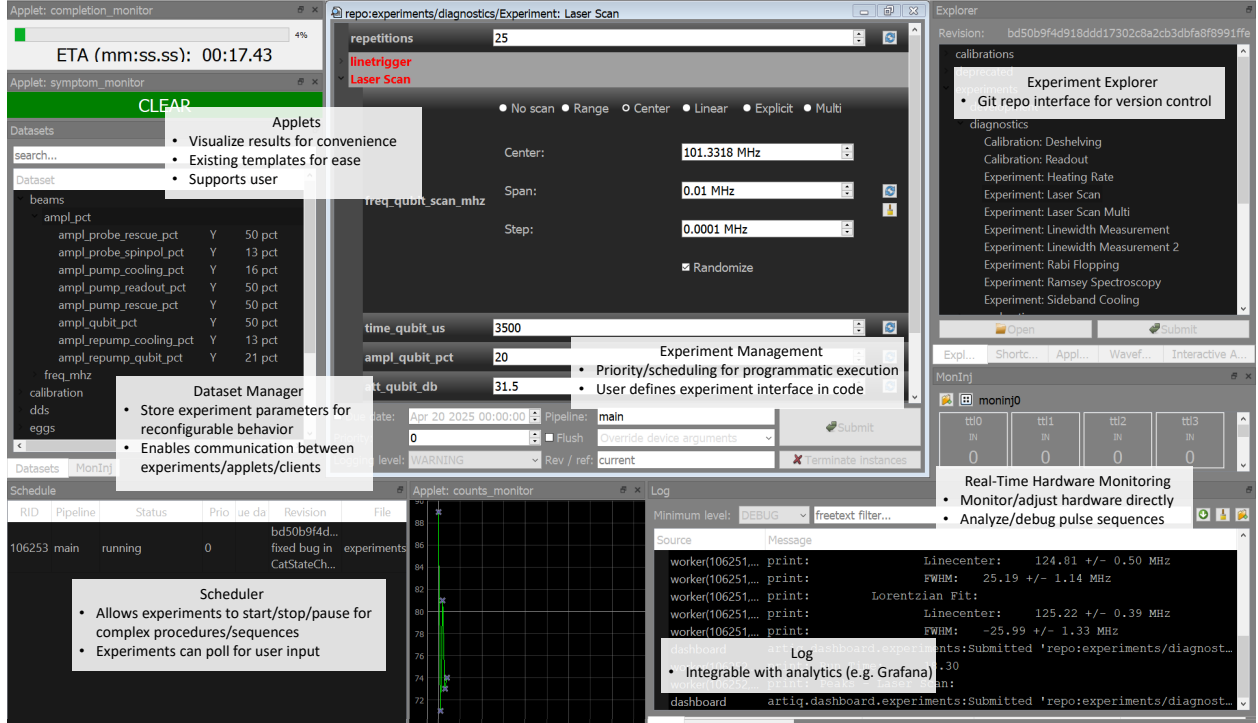


Figure 5.26: **ARTIQ** user interface.

Annotated view of the ARTIQ dashboard: the Explorer browses Git-versioned experiments, configured with a submission panel exposing experiment arguments; the scheduler queues and runs experiments; datasets serve as a centralized key-value store; applets visualize results; MonInj (Monitoring-Injection) enables real-time hardware monitoring and updates.

closely to base ARTIQ as possible.

LAX_exp is built around three object types: LAXDevice, which wraps an ARTIQ hardware interface with experiment-relevant defaults; LAXSequence, which encapsulates a reusable pulse-sequence fragment (e.g. Doppler cooling); and LAXExperiment, which handles the initialization and object management required to run a complete sequence — e.g. dataset registration, parameter registration, automatic pre-compilation to [Direct Memory Access \(DMA\)](#).

“Can [ten] years’ worth of
imaginary training be put to use?
Apparently, it can.”

Oh Dae-Su, *Oldboy*

Chapter 6

Experimental Techniques

6.1 Loading

6.1.1 $^{40}\text{Ca}^+$ Loading

Oven Operation

Loading of ^{40}Ca was initiated by resistive heating of the oven (see section 5.1.4), typically at 3.2 A, 1.2 V, to create an atomic beam that intersected the trap volume for photoionization. Following ionization, the ions were able to be trapped and were cooled until crystallization. Typical loading times were ~ 150 s, measured from oven activation until ion chain crystallization. This was likely dominated by the time required for the oven to reach the minimum evaporation temperature (492 K at 10^{-10} mbar). The oven current was selected to balance a need for short loading times to prevent charging from trap exposure to the high power (> 1 mW) PI and cooling beams, against a need to minimize the vapor pressure from the oven, which depends exponentially on temperature [Bal+18]. In the future, both needs could be simultaneously met by implementation of a temperature servo to quickly bring the oven to the evaporation temperature [Bal+18,Kna24].

Trap Parameters

The trap depth defines the maximum ion temperature that can be confined, which is proportional to the trap potential energy at the ion-electrode distance r_0 [Gho95, §2.5]:

$$U_D = \kappa m \omega_r^2 r_0^2, \quad (6.1)$$

where ω_r is the secular frequency along the r direction, and κ is a proportionality constant dependent on trap architecture; for typical LQTs, this is $\approx 10\%$ [Lei+03].

Loading was optimized by the use of a shallow trap: reduction of the secular frequencies from $\omega_{z,r} \sim 1000 \times 2\pi$ kHz to $\sim 300 \times 2\pi$ kHz reduced typical loading times from 20 to 3 minutes. This improvement was likely a result of the reduction in potential energy imparted to ions ionized off the trap null (due to the large waist and high power of the PI beams), as well as ejection of hot ions by the low trap depth [Roo+99, Gug+15]. These values yield a trap depth $U_D \simeq 0.05$ eV $\simeq 500$ K, sufficient to confine ions from the atomic beam at $T \approx 650$ K¹.

Following initial, post-loading calibration procedures (e.g. micromotion compensation, laser alignment), the minimum stable trapping voltages at the axial micromotion null were determined and noted for later loading. These loading “optima” were found to be stable day-to-day and generally persisted even following a cryogenic warmup cycle.

Laser Parameters

To maximize the cooling rate for hot ions, the aperture of the 397nm beam was opened to use the 0th-order beam, which was detuned from the $^2S_{1/2} \leftrightarrow ^2P_{1/2}$ by 230 MHz ($\sim -10\Gamma$). The power (~ 2 mW) in the 0th-order was maximized by turning off the RF drive to the 397nm AOM.

¹Per manufacturer specifications, the 3.2 A current placed our type 2L oven at ~ 650 K.

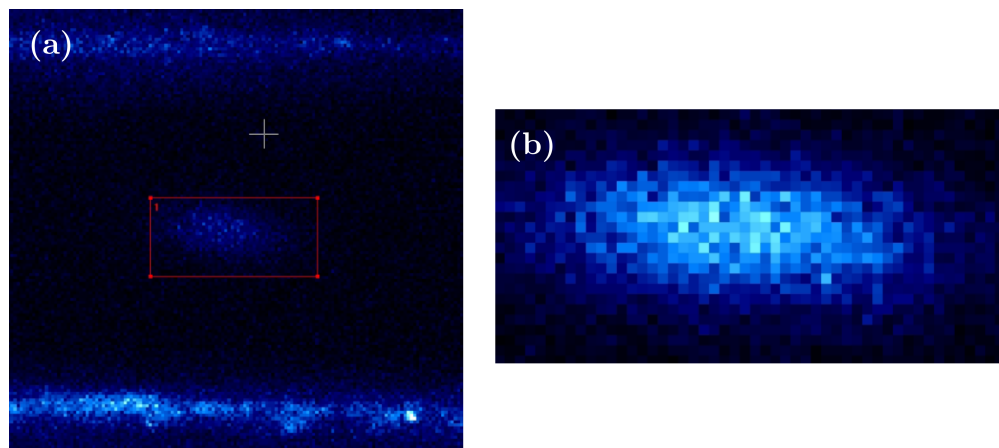


Figure 6.1: **Loading a cloud of $^{40}\text{Ca}^+$ ions.**

Andor iXon3 [EMCCD](#) camera images showing a large $^{40}\text{Ca}^+$ cloud loaded into a shallow trap at high oven current, used to recover beam alignments and trap optima when loading *ex nihilo*. (a) Both the ion cloud and scatter from the trap blades (bright horizontal bands). The cloud is weakly visible as the 397nm beam is off-resonant. (b) Zoomed-in image with frequency-optimized 397nm fluorescence.

Loading $^{40}\text{Ca}^+$ *ex Nihilo*

When loading *ex Nihilo* (such as after changes to the vacuum system), knowledge of beam alignments and trap optima were unable to be assumed, requiring a modified approach to loading. The oven was operated at high current (4 A to 5 A) and shallow trap parameters to produce a large neutral flux and load a large cloud, as opposed to a chain, of ions. Shallow trap parameters yielded additional benefit due to the larger cloud dimensions, increasing the likelihood of fluorescence. Initially, it was useful to confirm correct oven operation by observing the mass spectra of the background gas on the [RGA](#).

To improve the poor [SNR](#) of the fluorescence, the signal background was subtracted by periodically extinguishing the repump lasers, with further improvement by averaging. Similarly, ion positions were scanned by stepping the axial DC voltages to examine any position-dependence in the fluorescence; this was preferable to adjusting the laser alignment due to the repeatability of the DC voltages. As a last resort, the imaging filters could be removed to allow alignment of the [PI](#) beams by observing neutral fluorescence directly.

6.1.2 Chain Control

Chain Order

The order of ions in a two-ion chain ($^{40}\text{Ca}^+ - \text{H}^{35}\text{Cl}^+$ vs $\text{H}^{35}\text{Cl}^+ - ^{40}\text{Ca}^+$) affected the 729nm laser intensity experienced by the $^{40}\text{Ca}^+$ ion, resulting in a configuration-dependent Stark shift of the motional sidebands. This shift was sufficiently large to inhibit accurate state readout when sideband ratio thermometry was in use², necessitating a protocol for automatic and deterministic chain reordering.

In the absence of dedicated “twisting” electrodes [Hom13], chain reordering was instead performed dissipatively by a procedure we termed “shuffling”: the chain was repeatedly driven through a structural phase transition until the desired configuration was obtained, following the same principle as in [Hom13,Eri21]. Two methods were used to induce the transition: decrystallization via a strong motional tickle, and kinking via a DC bias on the radial electrodes. Decrystallization was implemented by extinguishing the cooling beams, applying a brief tickle pulse (~ 5 ms), and then restoring cooling to recrystallize the chain. This method was later deprecated due to the increased risk of ion loss during the decrystallized phase and the long recrystallization timescales. Kinking was implemented by applying a large DC bias to the radial electrodes until the chain adopted a zigzag configuration (see section 2.4.2). Unlike decrystallization, kinking did not destabilize the chain, provided the bias voltage was carefully selected: values too small were unable to induce the transition, while values too large resulted in ejection of the calcium ion.

As shuffling was inherently probabilistic, fast detection of the resulting chain configuration was essential to achieve preparation on a reasonable timescale (ideally < 10 s for experiment runtimes of ~ 100 s). This was first accomplished using the PMT: the spatial filter preceding the detector was sufficiently narrow that the two chain configurations produced distinguishable bright-state count rates (165 counts/3ms vs 135 counts/3ms), enabling discrimination within < 100 ms. Camera-based detection was later implemented and proved more robust, though

²This issue was resolved by later adoption of RAP-based readout.

at the cost of greater control complexity and significantly slower readout (~ 10 s).

Recrystallization

Molecular ion loading significantly degraded the vacuum pressure, resulting in frequent (~ 1 event/10 minutes) collisions that decrystallized the chain (a “melt”), an effect exacerbated by micromotion from drifts in stray fields (e.g. trap charging) incurred during the loading process. To reduce the likelihood of decrystallization, high-power (~ 1 mW), red-detuned (-3Γ) cooling (termed “resuscitation”) was applied during all available dead time in the experimental sequence (e.g. during initialization, following DMA recording, between successive shots). Configurable rescue periods were additionally provided at the end of each shot to allow recrystallization in the event of a mid-shot collision. However, these dedicated rescue periods, typically 1 ms to 5 ms, were often insufficient to recrystallize the chain; since even less cooling power was applied in a typical shot, the chain frequently remained decrystallized for the remainder of the sequence. Extending the rescue periods was impractical due to the resulting increase in experimental overhead.

To enable long rescue periods only when needed, a real-time collision detection system (“death detection”) was implemented on Kasli. Experiment photon counts were stored, thresholded, and processed in real time using a circular buffer and a [Finite-Impulse Response \(FIR\)](#) filter with a simple moving-average kernel, adding < 15 μ s of overhead per shot. When the filter output exceeded a predetermined threshold, the sequence halted and initiated an extended rescue period (~ 5 s); by reducing the frequency of rescue interventions, longer and more aggressive measures could be employed, such as opening the 397nm aperture (which took ~ 1 s) to admit additional high-power, red-detuned cooling light. The trigger threshold was made hysteretic to suppress false triggers in sequences with a high expected fraction of dark-state measurements. This framework was subsequently extended to detect additional error conditions, such as the inability to drive a transition following an H^{35}Cl^+ reaction. Future implementations could adapt the threshold dynamically to the count history, or use

more complex filter kernels to extract symptom information encoded in the frequency of dark/bright-state measurements.

6.2 Micromotion Compensation

EGGS places stringent demands on micromotion compensation due to the significant, $\sim$ MHz-order shifts it can produce in the QLS search for the yet-undetected H^{35}Cl^+ transition (see sections 7.3.2 and 7.4.3). For reference, an ion displacement of $x_0 = 1\text{ }\mu\text{m}$ is expected to produce a shift of approximately 300 kHz, compared to a maximum search linewidth of approximately 1 kHz. This precarious situation becomes dire when one considers the molecular loading procedure: gas is leaked into the chamber to be ionized with 243nm light, a procedure which often takes several *hours* and results in significant charging of the trap.

Stray fields were compensated radially via the shim electrodes (Figure 6.2) and axially via the endcap voltages. During normal operation (excluding molecular loading), compensated shim voltages fell in the range [40, 90] V and were stable to $\approx 5\text{ V}$ between days. Modulation depths of $\beta \approx 1/50$ were routinely achieved following compensation on all three axes, though we often compensated the radial modes only owing to the difficulty of axial compensation. Radial compensation was performed almost exclusively with the parametric technique for its speed and robustness; axial compensation used the Rabi ratio technique. Though the parametric technique had been implemented axially, it required sweeping a continuous range of compensation voltages, which was impractical axially: translating the ion required large voltage changes ($\sim 50\text{ V}$) that necessitated realignment of beams and imaging between measurements.

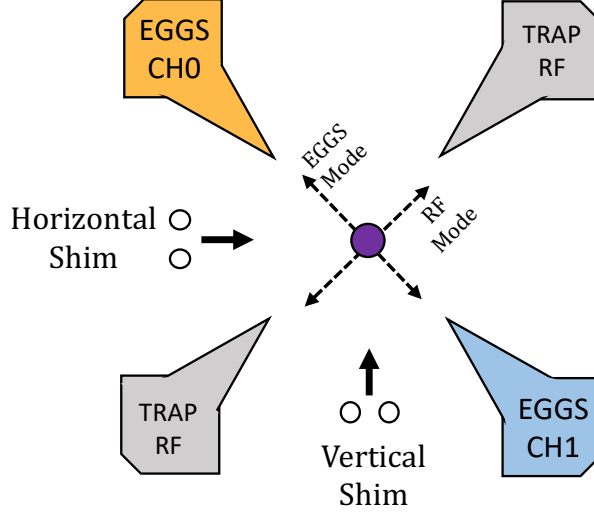


Figure 6.2: **Layout of trap electrodes for micromotion compensation.**

Axial trap cross-section showing electrode assignments. The trap RF is carried by an opposing pair of radial electrodes, with the other pair carrying the **EGGS** drives and a DC bias to break the radial mode degeneracy. The motional modes are oriented along these electrode pairs and thus referred to as the “RF” and “EGGS” modes. Voltages are applied to the horizontal and vertical shim (H- and V-shim) electrodes for micromotion compensation. Vertical ion position is visually unresolvable in our experiment as the imaging system is oriented vertically.

6.2.1 Coarse Compensation

The ion displacement from the RF null due to a stray field E is

$$x_0 = \frac{qE}{m\omega^2}. \quad (6.2)$$

Changes to the strength of the radial pseudopotential (i.e. ω) therefore resulted in observable shifts in the ion position — the minimum compensable x_0 was therefore determined by the camera resolution, which for one pixel was $1\text{ }\mu\text{m}$. To provide the additional degree of freedom needed for both radial modes, ion fluorescence was used as a heuristic measure of compensation quality along the vertical axis. Compensation proceeded by iterating between adjusting the horizontal shim via position shifts and the vertical shim via fluorescence. This procedure was used only at the outset of the experiment, when micromotion was severe enough to interfere with spectroscopy.

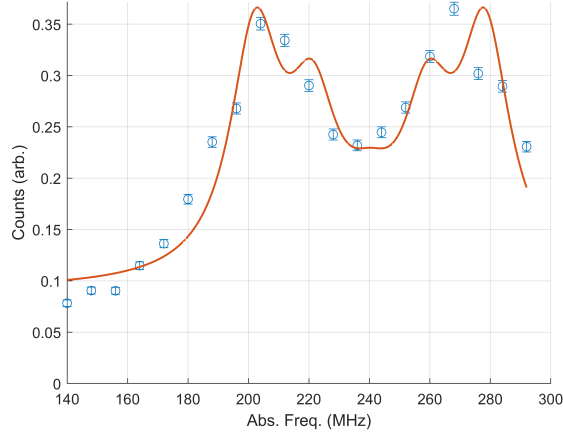


Figure 6.3: **Micromotion-modulated spectrum of the $S_{1/2} \leftrightarrow P_{1/2}$ transition in $^{40}\text{Ca}^+$.**

Background-subtracted ion fluorescence is measured against the 397 nm beam detuning for an ion with large uncompensated micromotion. The frequency-modulated lineshape is fitted (orange) with Bessel components up to J_2 , giving a modulation index $\beta = 2.9(1)$ at a trap RF frequency $f_{\text{RF}} = 19.026$ MHz. As the natural linewidth and trap RF frequency are comparable, the transition is sensitive only at large β and thus used only for verification.

6.2.2 397nm Doppler Linescan

The 397nm transition was insufficiently sensitive to micromotion since the trap RF frequency was comparable to the natural linewidth ($\Gamma_{\text{nat}} = 2\pi \times 23.05$ MHz, $\Omega_{\text{RF}} = 2\pi \times 19$ MHz), and was further complicated by unresolved Zeeman components separated by 13.65 MHz. However, as micromotion sidebands on the Doppler cooling transition are apparent only at large modulation depths, this was used only to confirm the quality of compensation, rather than for actual compensation.

6.2.3 729nm Rabi Ratio

Micromotion characterization on the $|^2S_{1/2}, m_J = -\frac{1}{2}\rangle \leftrightarrow |^2D_{5/2}, m_J = -\frac{5}{2}\rangle$ transition is identical to that on the 397 nm [Lei+03], but operates in the resolved-sideband regime. The modulation depth was extracted by comparing the Rabi frequency on the first micromotion

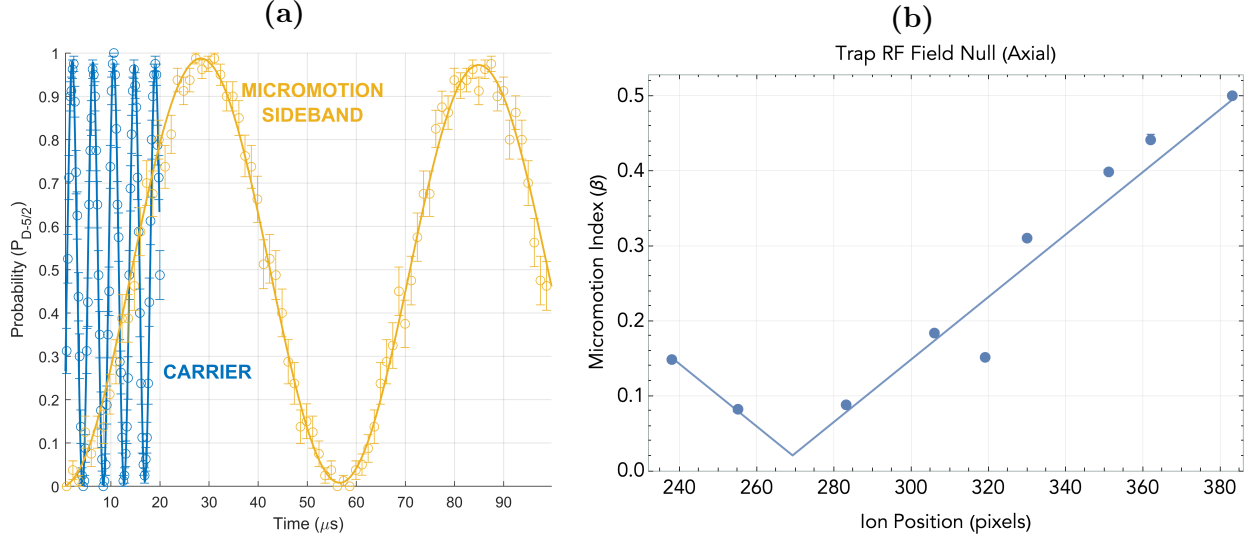


Figure 6.4: **Axial micromotion compensation by characterization of the micromotion sideband on the $S_{1/2} \leftrightarrow D_{5/2}$ transition in $^{40}\text{Ca}^+$.**

(a) Rabi oscillations on the carrier (blue) and first micromotion sideband (yellow); the modulation index β is extracted by the ratio of fitted Rabi frequencies (eq. (6.3)). (b) Modulation index β versus axial ion position, using the axial endcap voltages to translate the ion across the trap RF null. A function $y = a|x - x_0|$ is fitted to extract the optimum $x_0 = 269(7)$ pix. Error bars represent one standard error and are too small to see in (b). A minimum $\beta = 1/12.2(4)$ is shown, though values up to $\beta = 1/50$ are regularly achievable.

sideband with that on the carrier [Ber+98]:

$$\frac{\Omega_{\text{MM}}}{\Omega_0} = \frac{|J_1(\beta)|}{|J_0(\beta)|} \approx \frac{\beta}{2} \quad (6.3)$$

Ω_0 and Ω_{MM} were determined by first locating the carrier and sideband frequencies via linescan, then fitting Rabi oscillations at each frequency. Each change in compensation voltage required recalibration of the transition frequencies due to changes in intensity-induced AC Stark shifts. To deconvolve axial measurements from any axial dependence of the null, the radial micromotion was reoptimized at each axial position. Following the introduction of the parametric technique (see below), the Rabi ratio technique was relegated solely to axial compensation.

6.2.4 Parametric Modulation

Parametric modulation has long been used for micromotion compensation [Sud+00a,ZRS02] [ITU11,Leu15]. A unique contribution by [Nad+21] was the demodulation of the time-resolved ion fluorescence with respect to the parametric drive, which replaced the previous approach of measuring the mean fluorescence, which required large, destabilising motional excitation. Demodulation yields both amplitude and phase of the correlated response, enabling robust extraction of the compensation optimum by linear regression and making the technique well-suited to automated routines.

Parametric Dynamics Intuitively, parametric micromotion compensation can be understood as the continuous limit of the position shift technique used for coarse compensation section 6.2.1. The trap pseudopotential is amplitude-modulated at depth h and frequency $\omega \approx \omega_0$ near the secular frequency, such that the Mathieu q -parameter becomes $q \rightarrow q(1 + h \cos \omega t)$. Doppler cooling provides viscous damping at rate ξ . The modulation shifts the secular frequency to [Nad+21]:

$$\omega_1 \equiv \frac{\Omega_{\text{RF}}}{2} \sqrt{a + \frac{q^2}{2} \left(1 + \frac{h^2}{2}\right)}. \quad (6.4)$$

In the pseudopotential approximation (see section 2.1.3), the secular motion obeys [Nad+21]

$$\ddot{x} + 2\xi\dot{x} + \left(\omega_1^2 + 2\tilde{h} \cos(\omega t)\right) x = x_0\omega_0^2 \quad (6.5)$$

where $x_0 = \frac{qE}{m\omega_0^2}$ is the displacement from the RF null, and the effective modulation depth $\tilde{h}$ is:

$$\tilde{h} \equiv h \frac{q^2}{2} \left(\frac{\Omega_{\text{RF}}}{2\omega_1}\right)^2. \quad (6.6)$$

Effective Dynamics The steady-state solution takes the Floquet form (section 2.2.1) $x(t) = x_0 + C_1 e^{i\omega t}$, where C_1 is [Nad+21]:

$$|C_1| = \left| \frac{2x_0 \tilde{h}}{(f^2 - 1)^2 + (fd)^2} \right| \quad \arg(C_1) = \arctan\left(\frac{fd}{f^2 - 1}\right), \quad (6.7)$$

where $f \equiv \omega/\omega_1$ and $d \equiv \xi/\omega_1$ are the normalized frequency and damping parameters. eq. (6.7) describes the resonance of a damped driven harmonic oscillator: the resonance is at $\omega = \sqrt{\omega_1^2 - 2\xi^2}$, and the phase of the motion shifts by π at $\omega = \omega_1$ independent of ξ ; this enables direct, ξ -independent determination of the secular frequency. The effect of the applied parametric drive can therefore be understood as creating an offset-dependent driving term.

Sample lineshapes are shown in Figure 6.5. Excessive drive amplitude produces a sharp drop in correlated rate near resonance rather than a Lorentzian peak [Nad+21]; this nonlinear regime is diagnosed by a simultaneous drop in total fluorescence and is avoided in practice. The large phase fluctuation between $[-\pi, \pi]$ when off-resonant is due to the weakly driven motion, which correlates poorly and results in phase-wrapping. Fits to the phase lineshape are best performed using a general error function due to the noise created from phase-wrapping.

Modulated Fluorescence The parametrically driven motion Doppler-shifts the 397 nm cooling transition, modulating the fluorescence rate (see section 4.4.4). Linearization of the steady-state scattering rate gives eq. (4.15):

$$R_{\text{scatter}} = R_0 (1 + \kappa v) \quad \kappa = \frac{8\delta k}{\Gamma^2} \frac{1}{1 + s + \left(\frac{2\delta}{\Gamma}\right)^2}. \quad (6.8)$$

where $R_0 = R|_{v=0}$ is the background scattering rate, and the constant κ is governed only by fluorescence parameters. Since $v \propto \text{Im}(C e^{-i\omega t})$, the correlation strength $\eta \equiv \langle R_1/R_0 \rangle$ is

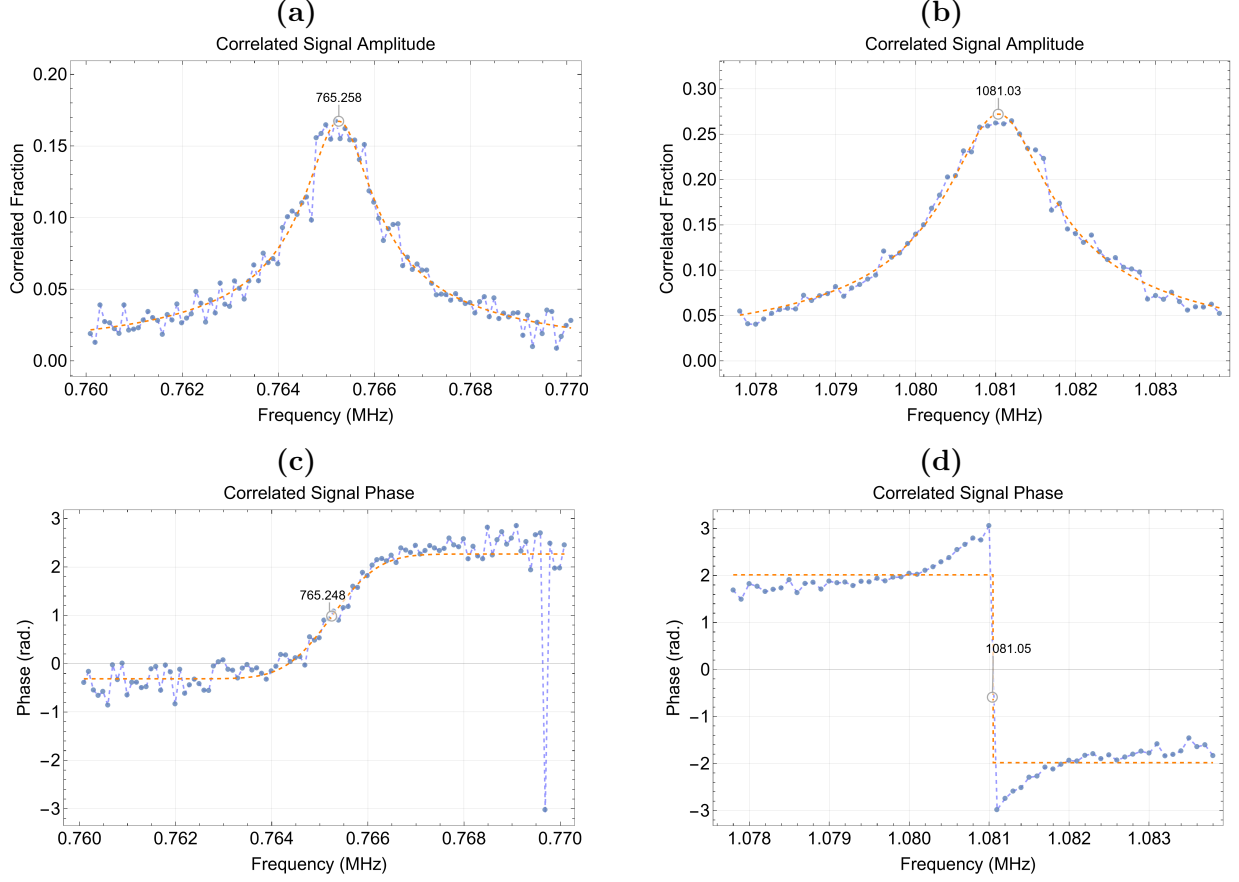


Figure 6.5: **Measured motional amplitude and phase response to parametric driving via time-resolved photon demodulation.**

(a,b) Correlated amplitude and (c,d) phase versus parametric drive frequency for the two radial modes at $2\pi \times 0.765$ MHz (RF mode; a,c) and $2\pi \times 1.081$ MHz (EGGS mode; b,d). The correlated signal is obtained by timestamping incident photons and demodulating with respect to the drive frequency. A Breit–Wigner profile is fitted to the amplitude, while an error function is fitted to the phase for robust extraction of the resonance (weak correlation when far from resonance results in noisy phase-wrapping). The π phase crossing (c,d) yields the secular frequency independently of the damping rate ξ , unlike the amplitude peak linecenter (see eq. (6.7)). 10^4 counts are demodulated for each data point, setting a $1/\sqrt{N}$ shot noise background of 1% in the correlated fraction.

estimated by demodulating the photon timestamp stream³:

$$\eta \equiv \left\langle \frac{R_1}{R_0} \right\rangle = \kappa\omega C_1 \langle \text{Im}(e^{-i\omega t})e^{i\omega t} \rangle = \frac{\kappa\omega C_1}{2}. \quad (6.9)$$

Since $\eta \propto C \propto x_0$ (eq. (6.7)), the signal is linear in the stray-field displacement and its zero-crossing defines the compensation optimum.

As in Doppler cooling, SNR is optimized for small saturation intensities $s \ll 1$ and a detuning $\delta = -\Gamma/2$. The background scattering rate R_0 defines the noise floor for this technique, and thus the number of counts required to achieve a given uncertainty in η . $N = 5000$ counts are typically obtained, which sets a noise floor $\eta = 1.4\%$.

Voltage Sweep The parametric excitation signal is plotted as a function of the vertical shim voltage and drive frequency in Figure 6.6 for both radial modes. The optimum compensation voltage V_{opt} for each mode is found at the center of the “triangular peak” (Figure 6.6(a,b)). As the horizontal shim voltage is adjusted, both voltage optima shift until they are coincident for a vertical shim voltage — the voltages at this point are the global micromotion optima. Unlike in [Nad+21], the resonance frequencies do not significantly depend on the shim voltages, simplifying compensation.

A more efficient protocol scans the shim voltages while parametrically exciting only at a fixed frequency near resonance. As the correlated signal η is linear in the micromotion offset (eq. (6.9)), it traces a line in the complex plane as a function of the shim voltage. The optimum is extracted via a complex linear least-squares fit: a line is fitted to the $(\text{Re } \eta, \text{Im } \eta)$ data with individual points projected onto this line, and the voltage corresponding to the minimum distance from the origin is taken as the compensation point [Nad+21]. While the magnitude of η alone can be fitted to extract V_{opt} , the complex linear fit is computationally cheap and robust to statistical noise.

³Equivalent to evaluation of the DFT of the photon counts at a single frequency point.

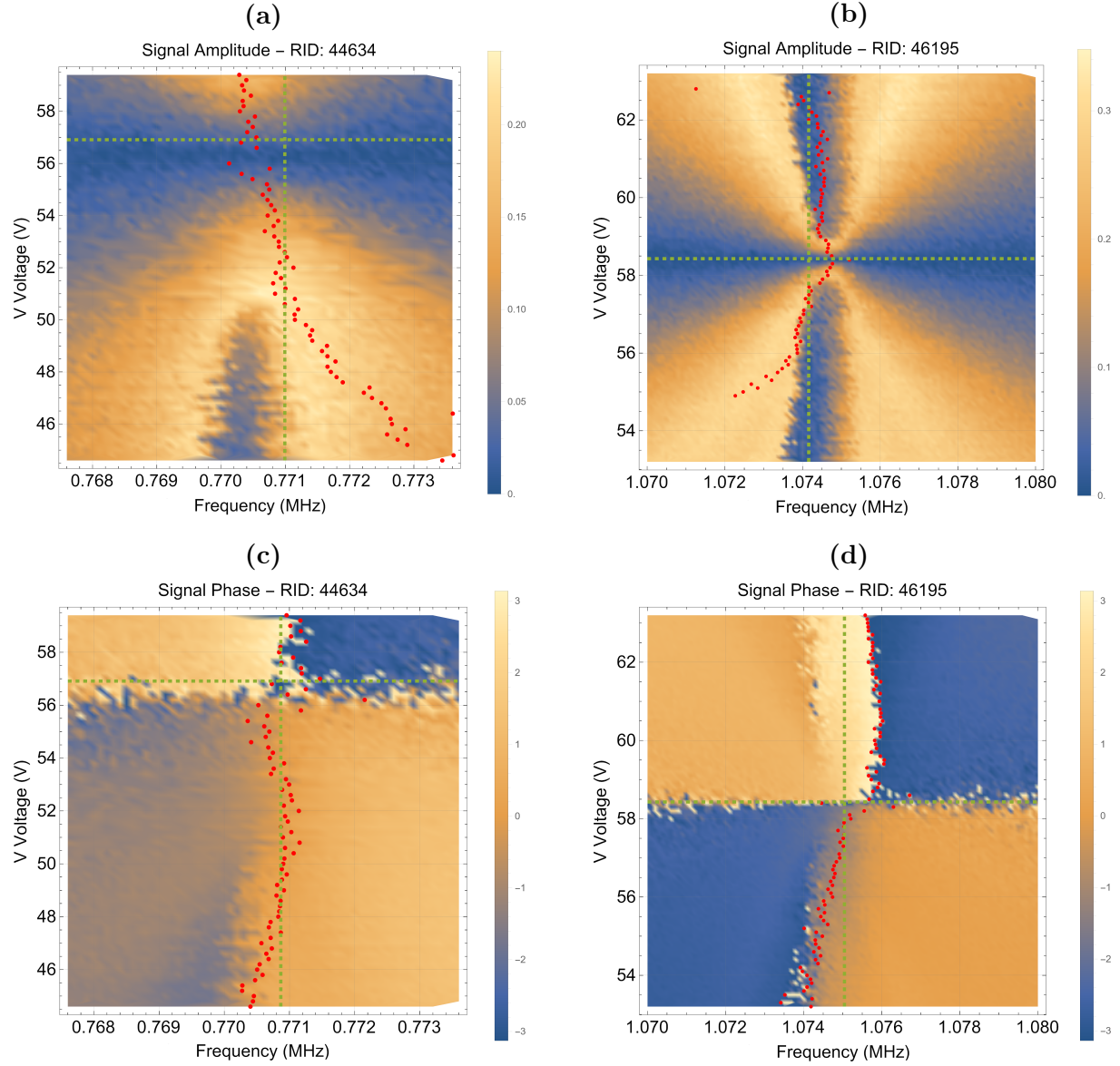


Figure 6.6: **Micromotion compensation along the vertical shim direction.** Two-dimensional parametric scans over drive frequency and vertical shim voltage for the **(a, c)** RF and **(b, d)** EGGs modes, showing the correlated amplitude (a, b) and phase (c, d). Red points track the fitted resonance at each voltage; the intersection of the dashed lines marks the predicted optimum, obtained by averaging the linecenters and by the voltage optima extracted from one-dimensional voltage scans at each secular frequency. The correlated amplitude vanishes at the trap RF null and grows with displacement, until the motion becomes nonlinear and demodulation fails, producing dips that broaden away from the null. The phase response flips as the null is crossed, providing a robust and efficient signal for compensation. 10^4 counts are demodulated per data point, setting a $1/\sqrt{N}$ shot-noise background of 1% in the correlated fraction.

Automation An automated procedure, demonstrated in Figure 6.7, is used in our experiment for fast compensation. First, parametric drive powers are configured to prevent nonlinear dips in the signal (visible from the colored inversions near resonance in Figure 6.6). These values are adjusted every iteration based on the previous excitation amplitude and desired scan range. The global compensation optimum is predicted based on the intersection of the mode-axes vectors, guesses for which are successively refined by linear fits to a running history of the previous optima.

The shim voltages are then set to the global optimum, and the voltage of one of the shims is scanned as the correlation signal η is measured for each radial modes. After the voltage scan, the photon timestamps are demodulated and the complex linear fit is applied to extract the optimum for each mode.

The routine converges when successive optima lie within a preset tolerance, and terminates early if this criterion is met to avoid overfitting. For typical parameters (eleven voltage steps, two modes, 5000 counts per point), a single iteration takes ~ 50 s; full convergence from a cold start typically requires ~ 3 iterations. The dominant overhead ($\sim 60\%$) is the 888 ms settling time required for each voltage update due to RC time constants of the electronics, though this could be mitigated using predistortion.

6.3 Calibrations

6.3.1 Beam Alignments

Coarse Alignment

During setup of the experiment (prior to trapping), beams were coarsely aligned into the chamber by maximizing transmission through the trap, as measured by an optical power meter⁴ placed at the opposing viewport. To verify that all beams were aligned onto a common position, the narrow 397nm optical filter on the camera was temporarily removed to enable

⁴Thorlabs PM100D, S121C

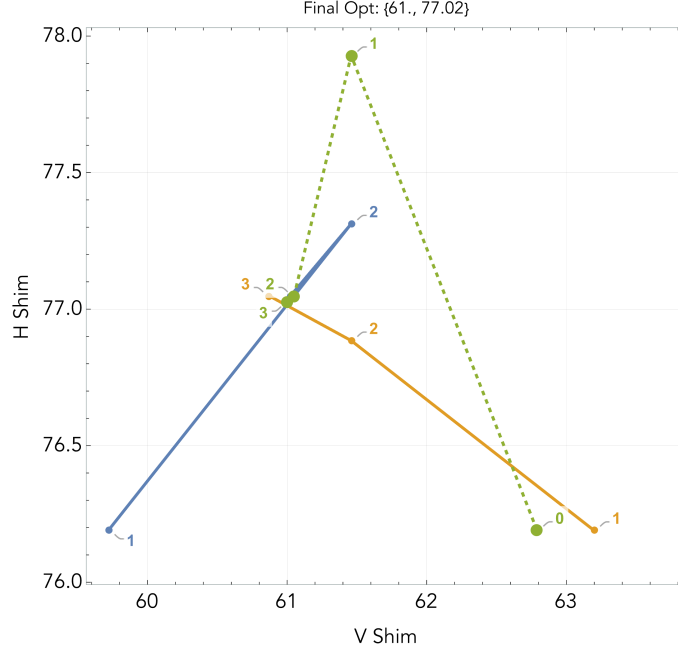


Figure 6.7: **Automated micromotion compensation along both radial modes.**

At each iteration (numbered) the single-mode optima for mode 0 ($2\pi \times 1.576$ MHz, EGGS mode, blue) and mode 1 ($2\pi \times 1.286$ MHz, RF mode, orange) are extracted by complex linear fits and extrapolated to their intersection (green) to predict the global optimum. Successive iterations refine the mode-axis estimates until the predicted optima converge, here terminating at $\{V, H\} = \{61.0, 77.0\}$ after three iterations.

visualization of the other wavelengths. Each beam was then walked onto the trap surface, and the resulting scatter was used to reference the beam to the trap center. In general, alignments could have been more reproducible by mounting the steering mirrors on a translation stage; this was not done in our experiment, and beam realignment required significant operator skill.

Coarse alignment of the 729 nm beam was best performed with a trapped ion by maximizing the rate of quantum jumps into the $^2D_{5/2}$ state. As this state decayed slowly ($\tau \approx 1$ s), the alignment process was expedited by applying a small amount of 854 nm light, which deshelled the ion to convert quantum jumps into visible fluorescence dips. In general, the camera was preferred over the PMT during coarse alignment tasks due to its spatial resolution.

Doppler (397nm + 854nm + 866nm) & Imaging Alignment

Alignment of the Doppler beams (main 397nm, 866nm, and 854nm) and imaging system was performed using a background-subtracted “Imaging Alignment” experiment run in real time. A signal measurement of the ion fluorescence was obtained from the PMT with all Doppler beams applied, while a background measurement was obtained with the infrared repumps extinguished, shelving the ion into the dark $^2D_{3/2}$ state such that only laser scatter was collected. Signal and background exposures of 3 ms each were interleaved in a 10:1 ratio to improve the duty cycle, and the background-subtracted counts were processed and plotted in real time to enable rapid optimization. Alignment of the infrared repumps was performed at powers well below saturation to increase sensitivity to beam pointing.

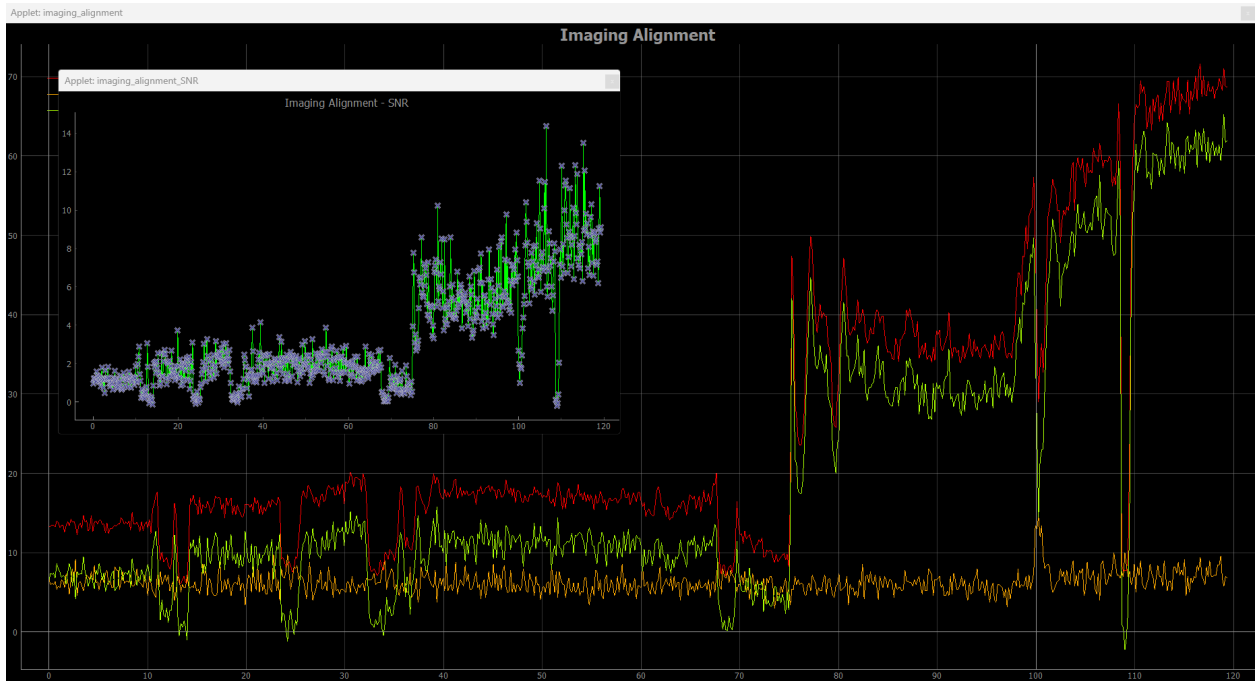


Figure 6.8: **Real-time “Imaging Alignment” diagnostic for Doppler beams and imaging optics.**

Background-subtracted **PMT** counts and the derived **SNR** (inset) are displayed in real time as the final 397 nm steering mirror is walked. Total, background, and background-subtracted (total – background) counts are shown in red, orange, and green, respectively.

397nm (σ^-) Alignment

The polarization of the σ^- 397nm beam was coarsely set using a polarimeter⁵. The beam was then spatially overlapped with the unpolarized 397nm Doppler cooling beam by driving both AOMs at the same frequency and optimizing the contrast of the resulting interference fringes. For fine alignment of the σ^- polarization with the quantization axis, the magnetic field direction was adjusted via the shim coils (see section 5.2.2). Following [Roo00], Doppler fluorescence due to the σ^- beam was minimized by iteratively tuning the current through each pair of shim coils. This procedure could be automated as a pulse sequence for fast and regular recalibration: the ion would first be Doppler cooled by the main 397nm beam then the σ^- -induced fluorescence measured by the PMT; this sequence would then be repeated as the coil currents scanned across a search range.

For a typical spin polarization sequence, the σ^- beam was applied for 20 μ s with detuning $\delta \approx -\Gamma$ at a saturation intensity $s \approx 0.1$ (19 μ W); a pumping efficiency of $> 99\%$ was inferred from spectroscopy on the $|^2S_{1/2}, m_J = -\frac{1}{2}\rangle \leftrightarrow |^2D_{5/2}, m_J = -\frac{5}{2}\rangle$ transition using these parameters. Limitations on the pumping efficiency were set by excitation of the π and σ^+ transitions. In addition to the σ^- polarization of the beam, the π transition was further suppressed by alignment along the quantization axis, while the σ^+ was further suppressed by frequency selection (the $\sigma^\pm$ transitions were separated by 13.65 MHz). As this state preparation fidelity remained robust over the course of the experiment, routine or thorough characterization of spin polarization was not a priority.

729nm (Qubit) Alignment

It is disrespectful to waste optical qubit laser power: it is disrespectful to the laser diode, it is disrespectful to the funding agencies, and it is disrespectful to other researchers starved of power. As the optical power of the 729nm laser is often the limiting factor in experiments (e.g. duty cycle, spin-motion gate fidelities), one has a further responsibility to oneself not to

⁵Thorlabs PAX1000VIS

waste one’s own precious time. Optimization of the 729nm alignment is therefore an endeavor that requires our every effort; here, we give it thus.

Initial Procedure Precise alignment of the 729nm qubit laser was performed by monitoring the Rabi frequency Ω on the $|^2S_{1/2}, m_J = -\frac{1}{2}\rangle \leftrightarrow |^2D_{5/2}, m_J = -\frac{5}{2}\rangle$ transition. To measure Ω , we Rabi flopped on the carrier for a single interrogation time t , then read out the qubit state. The interrogation time was typically set to implement a $\pi/2$ pulse, which maximizes the signal-to-noise ratio by placing the measurement at the point of steepest slope. Measurements were thresholded and averaged in groups of 40 to form a single data point and plotted in real time as the beam was aligned by adjusting the final steering mirror to maximize Ω . The adaptive maximum-likelihood readout technique of [Bur10] (see section 4.3.3) was employed to reduce readout duration and thus the cycle time from 3.5 ms to 2.1 ms.

Optimal Measurements We now consider optimization of the alignment sensitivity. The Fisher information for estimating Ω from a binary-outcome measurement is $\mathbb{F}(\Omega) = (\partial_\Omega \mathbb{P})^2 / (\mathbb{P}(1 - \mathbb{P}))$, where $\mathbb{P}$ is the excitation probability (population in the $^2D_{5/2}$ state). For decoherence-free Rabi flopping, $\mathbb{P} = \cos^2(\Omega t/2)$, which yields $\mathbb{F}(\Omega) = t^2$. Since the interrogation time is negligible relative to the experiment cycle time (which is dominated by state preparation and readout), the Fisher information per shot is maximized by interrogating at the longest possible t .

In practice, thermal occupation outside the Lamb-Dicke regime couples the carrier transition to the motional state, causing the oscillations to decohere (see section 4.1.1). Approximating this thermal decoherence as Markovian dephasing with decay time τ , the Rabi oscillation envelope then decays as $e^{-t/\tau}$, and the Fisher information becomes $\mathbb{F}(\Omega) = t^2 e^{-2t/\tau}$. This is maximized at $t = \tau$, yielding $\mathbb{F}_{\max} = \tau^2/e^2$ [Hue+97] — thermal decoherence thus sets a fundamental limit on the achievable alignment sensitivity.

To push this limit, the ion was prepared into the Lamb-Dicke regime using a short, multi-mode sideband cooling stage of 1 ms, cooling the ion to $\bar{n} \sim 1$ and extending τ from

$2\text{ }\mu\text{s}$ to $15\text{ }\mu\text{s}$, requiring $\sim \times 56$ fewer measurements for the same sensitivity.

Implementation In practice, changes of the Rabi frequency by $\gtrsim \pi/t$ caused the alignment signal to alias, making its interpretation ambiguous. To prevent this, the optimal (long t) measurements were interleaved and processed with additional short- t measurements to provide an unambiguous, coarse estimate of Ω . Use of shaped pulses suppressed off-resonant excitation and improved signal quality.

Polarization drifts of the 729 nm beam were exacerbated during alignment by thermal stress on the [PM](#) fiber induced by proximity of the human body; polarization should therefore be monitored continuously during alignment. The multiple optical windows in our system (vacuum chamber, radiation shield, and 4 K shield) produced significant ghosting which complicated beam alignment. To avoid accidental alignment to a ghost reflection, the beam was scanned across the full travel of the mirror mount during initial alignment.

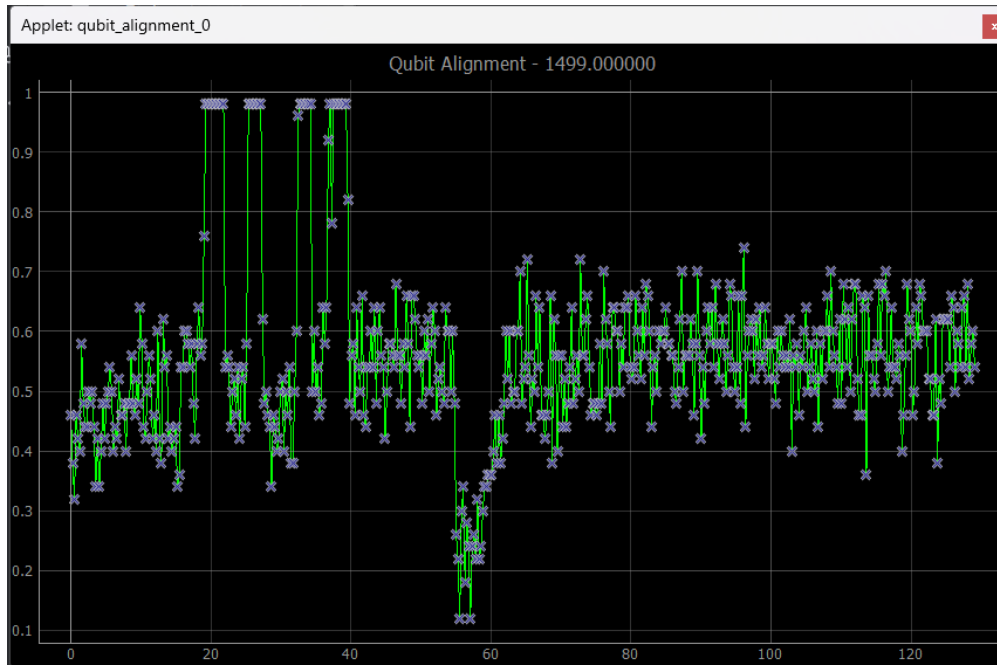


Figure 6.9: **Qubit (729nm) alignment sequence.** See main text for details.

6.3.2 397nm Spectroscopy

Spectroscopy on the 397 nm transition was needed to calibrate frequencies for Doppler cooling and readout. Two pulse sequences were used: a standard interleaved protocol, and a probe-reset protocol.

Intensity Equalization Probing the 397nm dipole transition required low saturation intensities to prevent power broadening, which increased the dependence of the count rate on the laser intensity. To mitigate this, an amplitude calibration procedure was used to equalize the beam intensity at each AOM frequency (Figure 6.10).

AOM drive powers were calibrated using a binary search at each frequency to bring the beam intensity within 10% of a target value (typically 5 μW), as measured by a photodiode⁶ on an ADC. Frequencies within the calibration range were randomized to average over beam-pointing drifts. Additional holdoffs of 1 ms were inserted to account for the lowpass response of the photodiode capacitance and ADC input resistance, and room lights were turned off to eliminate systematic errors from stray light. The calibrated power values were later retrieved by the experiment sequence and interpolated to the desired probe frequencies.

Pulse Sequences

In the standard interleaved protocol⁷ (fig. 6.11(a,b)), the ion was first Doppler cooled (397nm, 866nm, 854nm), followed by the spectroscopy pulse (397nm, 866nm) and count collection, then a background measurement (397 nm only) for later subtraction. To prevent heating (e.g. during interrogation the blue-detuned portion of the spectrum) and power broadening, the 397nm pulse was kept short (20 μs) and at low power (5 μW); the 866nm beam was applied with $\sim 170 \mu\text{W}$. The presence of the 866nm repump during the spectroscopy pulse can complicate

⁶Thorlabs DET36A2

⁷Surprisingly, interleaved protocols were not always the norm due to a lack of suitable control electronics — see for example [Roo00,Chr20]. A study of theses from antiquity (early 2010s and before, see e.g. [Neg18,Sta19,Pru14]) shows that the ancients were often required to *build* their own control electronics and systems — from scratch! Standardized experimental control systems only became the norm around the mid-2010s, with the development of ARTIQ at NIST (to replace the old hfgui [Lan06]).

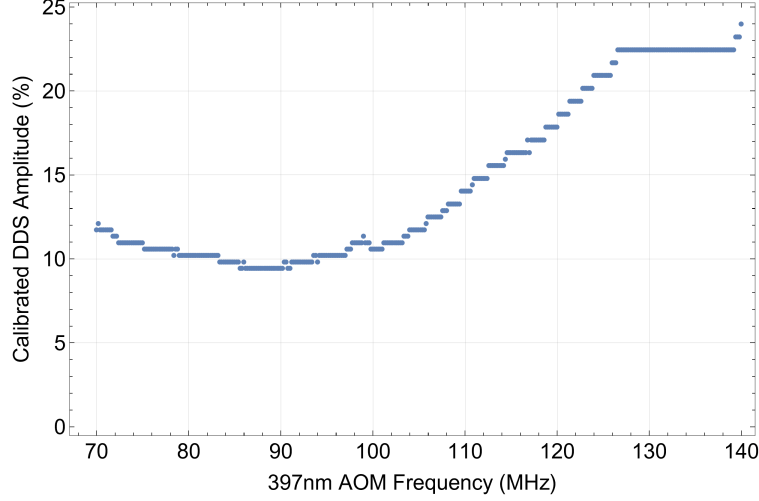


Figure 6.10: **397nm intensity calibration for AOM frequency response compensation.**

Calibrated DDS amplitude versus AOM frequency required to hold the optical power at the photodiode at a target of $5.0(5) \mu\text{W}$. A binary search is used at each frequency for efficient extraction of the target amplitude. Akima interpolation maps the calibrated values onto the desired probe detunings during spectroscopy.

the lineshape by coupling the $^2\text{D}_{3/2}$ state, introducing potential [Electromagnetically-Induced Transparency \(EIT\)](#) features and a general dependence on the 866nm beam parameters (polarization, saturation intensity); this contributes systematic uncertainties to the extracted transition linecenter [[Jan15](#),[Pru14](#)].

A second technique from [[Pru14](#)] (fig. 6.11(a,c)) bypassed these issues by extinguishing the 866nm repump during the spectroscopy pulse. Following Doppler cooling, spectroscopy was instead performed using a series of probe-reset pulses: the 397nm probe was applied alone as a short pulse ($3 \mu\text{s}$), followed by the 866nm repump ($10 \mu\text{s}$) to return population to the $^2\text{S}_{1/2}$ manifold. To improve the duty cycle, a large number of probe-reset cycles were applied consecutively, with Doppler cooling inserted only once every ~ 30 cycles. From [[Pru14](#)], the accumulated count rate with only the 397nm applied is:

$$N(t) = \int_0^t \rho_{ee} \Gamma(\tau) d\tau = \frac{1}{1-p} (1 - e^{-(1-p)Rt}) \quad (6.10)$$

where R is the $^2\text{S}_{1/2} \leftrightarrow ^2\text{P}_{1/2}$ scattering rate and p is the $^2\text{P}_{1/2} \rightarrow ^2\text{S}_{1/2}$ branching ratio.

In the short-pulse limit $t \ll 1/[R(1-p)]$, eq. (6.10) reduces to $N(t) \approx Rt$, such that the measured counts are directly proportional to the scattering rate.

Linescans

Results from both sequences are shown in Figure 6.11. A Gaussian profile was fitted to the data for simplicity, in place of the full Voigt profile. To suppress EIT features in the interleaved protocol, the 866nm beam was blue-detuned by ~ 3 MHz from its resonance.

Linewidths of $\Gamma \approx 2\pi \times 36$ MHz were observed, in excess of the natural linewidth $\Gamma_{\text{nat}} = 2\pi \times 23.05$ MHz. This broadening is attributed to the unpolarized 397nm beam simultaneously driving the σ^+ transition, which is unresolved at our magnetic field. From the measured field strength $B = 3.63197(2)$ G (see section 5.2.2), the $\sigma^\pm$ components are separated by 13.65 MHz, consistent with the observed excess linewidth.

Differences between the two protocols are obscured by insufficient averaging but agree to within a few MHz, which is sufficient for our purposes. The slightly larger linewidth from the probe-reset sequence can likely be attributed to insufficient Doppler cooling between probe-reset cycles. In practice, the interleaved sequence is used due to its relative simplicity.

6.3.3 729nm Spectroscopy

Spectroscopy on the $^2S_{1/2} \leftrightarrow ^2D_{5/2}$ electric quadrupole transition at 729 nm provides the workhorse diagnostic of the experiment, simultaneously calibrating the magnetic field, secular frequencies, beam geometry, and AC Stark shifts of the qubit [Roo00, Gug17].

Zeeman Sublevels

Following [Roo00], the $|^2S_{1/2}, m_J = -1/2\rangle \leftrightarrow |^2D_{5/2}, m_J = -5/2\rangle$ transition is used as the carrier for almost all coherent operations as it is the lowest-frequency line in the manifold, minimizing the likelihood of spurious transitions near its motional sidebands and enabling robust Sideband Cooling — at the cost of increased magnetic-field sensitivity. A linear fit

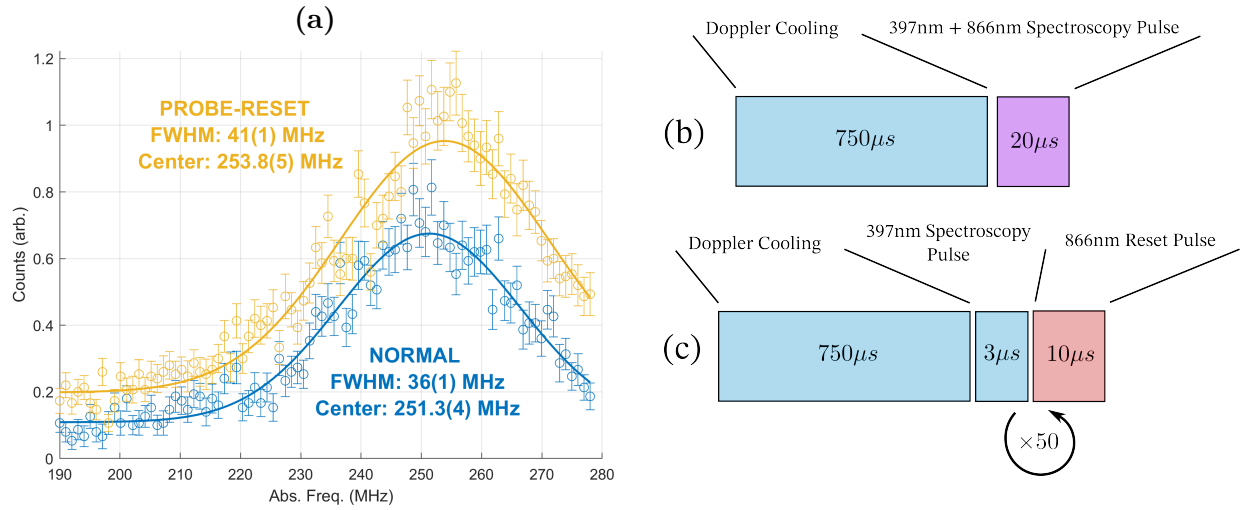


Figure 6.11: Spectroscopy on the $S_{1/2} \leftrightarrow P_{1/2}$ transition in $^{40}\text{Ca}^+$ using a standard interrogation protocol and a probe-reset protocol.

(a) Background-subtracted ion fluorescence versus 397 nm probe detuning for both protocols, each fitted to a Gaussian profile; linecenters agree to within ~ 2 MHz. The increase in the FWHM linewidth for the probe-reset sequence is attributed to thermal broadening from less frequent Doppler cooling. (b, c) Pulse sequences for the interleaved (b) and probe-reset (c) protocols: the 866 nm repump is applied throughout the spectroscopy pulse in (b), but switched off in (c). The fitted $\Gamma \approx 2\pi \times 36$ MHz exceeds the natural linewidth $\Gamma_{\text{nat}} = 2\pi \times 23.05$ MHz, consistent with unresolved σ^+ excitation at the 13.65 MHz Zeeman splitting.

to the five⁸ Zeeman transitions accessible from the ground $|^2S_{1/2}, m_J = -1/2\rangle$ sublevel via the quadrupole selection rule $\Delta m_J \in \{0, \pm 1, \pm 2\}$ yields $B = 3.631\,97(2)$ G, serving as the primary calibration of the magnetic field at the ion.

Motional Sidebands

At our trap frequencies $(\omega_x, \omega_y, \omega_z) = 2\pi \times (1.285\text{ MHz}, 1.575\text{ MHz}, 0.701\text{ MHz})$, three first-order sidebands appear on either side of the stretch carrier, with Lamb–Dicke parameters $\eta_q = \Omega_{\text{BSB},q}/\Omega_0$ extracted from the ratio of sideband to carrier Rabi frequency at matched probe power; the resulting values $\eta_z \approx 0.08$, $\eta_x \approx \eta_y \approx 0.04$ are consistent with the calculated projections $\cos \theta_z = 1/\sqrt{2}$ and $\cos \theta_{x,y} = 1/2$ (see section 4.4.2).

Unlike an RF tickle, the 729 nm laser drives higher-order combination sidebands at $\pm(n\omega_x \pm m\omega_y \pm p\omega_z)$ via the anharmonic terms in the plane-wave expansion of the laser field. This complicates mass spectrometry during molecular ion loading, as the proliferation of sidebands and their varying Rabi frequencies complicates the identification of spectral lines, motivating a careful choice of trap parameters such that transitions of interest with the target species (i.e. H^{35}Cl^+) remain well separated.

These spectator transitions are also a source of decoherence during coherent operations (e.g. Fock state decomposition, see section 3.5.2), as off-resonant excitation of nearby combination modes induces small AC Stark shifts and population leakage that limits fidelity [Rog+25b]; this can be straightforwardly mitigated by cooling all spectator motional modes to near the ground state of motion ($n \sim 0.2$).

⁸The $\Delta m_J = 0$ transition to $|^2D_{5/2}, m_J = -1/2\rangle$ is omitted from the fit, as its Rabi frequency was insufficient for reliable excitation.

Stark Shifts

Off-resonant coupling to spectator transitions (e.g. the carrier when trying to excite a motional sideband) induces an AC Stark shift given by

$$\Delta_{\text{AC}} \approx \frac{\Omega^2}{2\omega} \quad (6.11)$$

where ω is the detuning from the transition of interest. In experiment, this AC stark shift is most prominent when exciting the axial mode due to its low secular frequency $\sim 700 \times 2\pi$ kHz and the large carrier Rabi frequency $\Omega \sim 250 \times 2\pi$ kHz, which results in a shift of the transition by $\Delta_{\text{AC}} \sim 44 \times 2\pi$ kHz. Precise spectroscopy of $\text{S} \leftrightarrow \text{D}$ transitions thus requires low optical powers to prevent uncertain systematic shifts.

6.3.4 Spin Decoherence

Though not critical to our experiment⁹, spin coherence serves as a general laboratory diagnostic, particularly for the magnetic field stability and the 729 nm laser. We measure $T_2 \approx 24$ ms with XY-8 dynamical decoupling and $T_2^* \approx 1$ ms to 2 ms in free evolution; as the $\text{D}_{5/2}$ decay of $T_1 \approx 1$ s is far longer than either timescale, we restrict our discussion to dephasing.

Spin Dephasing

We describe pure spin dephasing with the Lindbladian (see section 3.4), where it is represented by the jump operator $\hat{L} = \sqrt{\Gamma_\phi/2} \hat{\sigma}_z$. Under free evolution, the Lindbladian is

$$\frac{d\hat{\rho}}{dt} = \frac{\Gamma_\phi}{2} (\hat{\sigma}_z \hat{\rho} \hat{\sigma}_z - \hat{\rho}), \quad (6.12)$$

with an evolution given by

$$\rho(t) = \rho(0) e^{-\Gamma_\phi t}. \quad (6.13)$$

⁹Both [EGGS](#) and our sensing projects require only that the $^{40}\text{Ca}^+$ be coherently manipulated for cooling and readout

As in the bosonic case, the populations are preserved while off-diagonal coherences decay, leaving a completely mixed density matrix when fully dephased.

T_2^* measurements used a similar protocol as in section 3.4.2, where a phase scan was used to extract the coherence via Ramsey at each delay time. Fits of the decay to a Gaussian envelope (as opposed to the expected exponential decay) are in better agreement with the data, suggesting significant contribution of slow, non-Markovian noise [Rus+16]; this is supported by the marked difference between the measured T_2^* and T_2 values. Three approaches were made to improve T_2^* :

- **729nm PDH tuning:** we tuned the PDH lock parameters of the narrow 729nm linewidth laser; no improvement to T_2^* was found.
- **60 Hz line triggering:** experimental shots were triggered by an AC line trigger (built following [Ram14]); this improved $T_2^* \sim 2 \text{ ms} \rightarrow 3 \text{ ms}$.
- **Use of the 729nm FNCS:** implemented prior to the injection lock system, this improved $T_2^* \sim 3 \text{ ms} \rightarrow 4 \text{ ms}$.

Measurements of T_2 were obtained via a Free-Induction Decay (FID)-style protocol (see section 3.4.2 for further details) (Figure 6.13), where the Ramsey delay T was scanned at fixed phase, with concatenated XY-8-N sequences for dynamical decoupling. The XY-8-N sequence was selected (over e.g. XY-4-N) for its improved robustness to pulse errors [SÁS12]. A single application of the XY-8 sequence improved $T_2^* \sim 3 \text{ ms} \rightarrow 24 \text{ ms}$; further concatenation (e.g. XY-8-2) yielded minimal improvement, suggesting it to be the true T_2 . Beyond this, pulse errors, rather than environmental noise, often limit further improvement [SÁS12]; this could be mitigated by higher-order decoupling sequences (e.g. XY-16) or by replacing the π -pulses with composite robust pulses, such as the Wimperis F1 [Wim91] or UBB [TKV15] sequences. More informative diagnostics, such as correlation spectroscopy [Bos+17, Sch+17], could further resolve the residual noise spectrum to guide targeted mitigation.

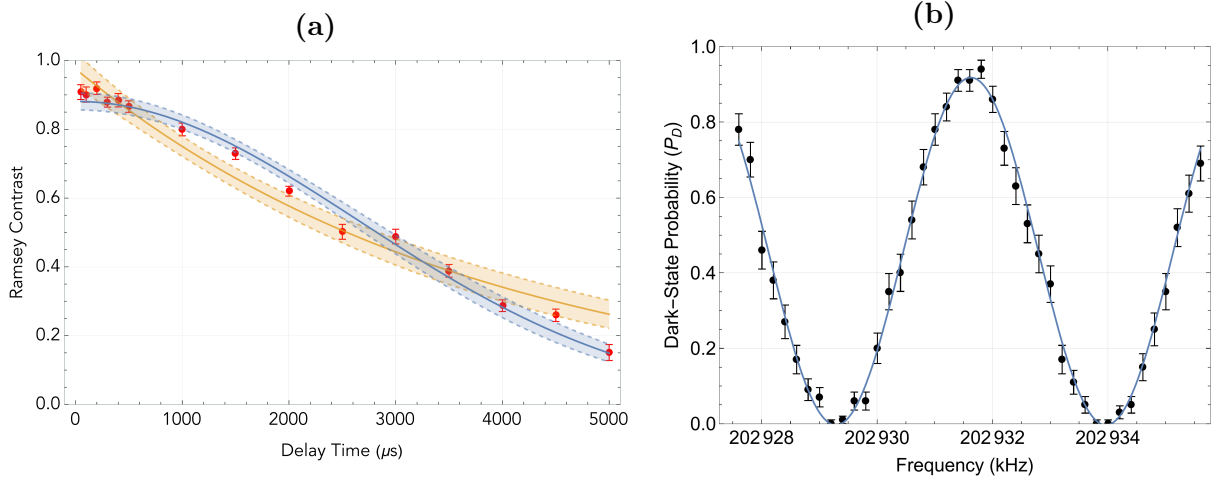


Figure 6.12: **Inhomogeneous spin dephasing (T_2^*) characterized with a Ramsey sequence.**

(a) Ramsey contrast versus free-evolution delay T fitted to exponential (orange) and Gaussian (blue) decay profiles. The exponential and Gaussian fits yield contrasts $A = 0.976(22)$, $0.881(12)$ and decay constants $\tau = 3810(240) \mu\text{s}$, $3752(89) \mu\text{s}$, respectively. The closer agreement of the data with the Gaussian profile suggests a dominant contribution from slow, non-Markovian noise. (b) Representative Ramsey fringe at $T = 200 \mu\text{s}$ obtained by a frequency scan. A sinusoidal profile is fitted to extract the contrast for (a). Each shot is triggered off the AC line to mitigate 60 Hz pickup.

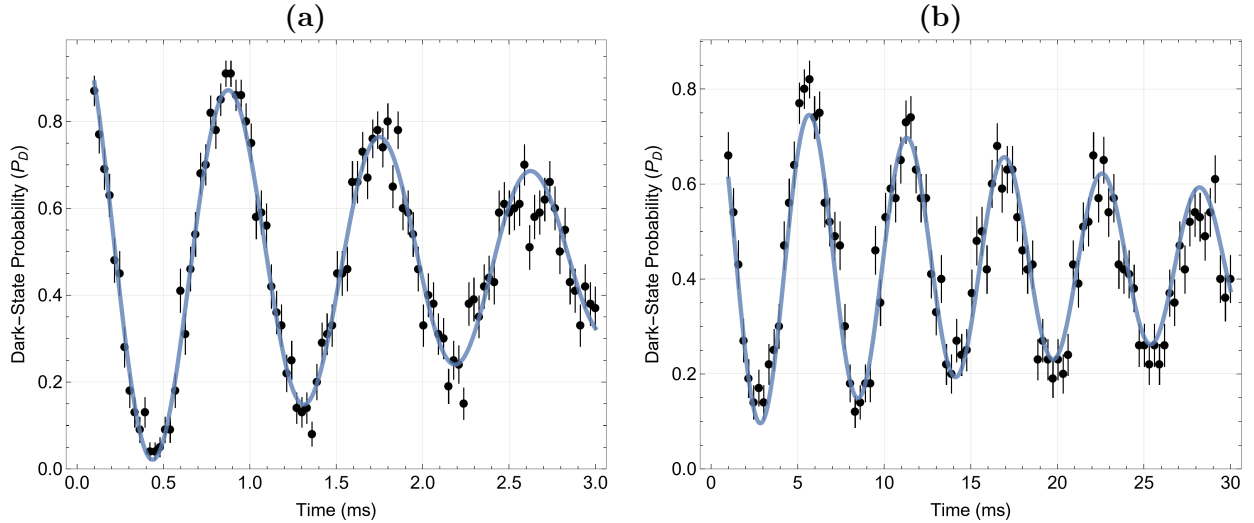


Figure 6.13: **Suppression of homogeneous spin dephasing via dynamical decoupling.** Coherence measured by a [Free-Induction Decay](#) protocol with inter-pulse delay T scanned at fixed detuning. Dephasing times are extracted as the decay constant of an exponentially damped sinusoidal fit. (a) A baseline measurement without decoupling gives $T_2^* \approx 3.6(2) \text{ ms}$. (b) Using a concatenated XY-8-2 sequence, the coherence persists to $T_2 \approx 24.6(16) \text{ ms}$; further concatenation gives little improvement, consistent with approaching the true T_2 . Note the change in the time axis between (a) and (b).

To “deconvolve” T_2/T_2^* measurements from drift-induced contrast decays and enable fair comparison of T_2/T_2^* values, total run durations should be matched to prevent preferential exposure to spurious drifts. This effect is characterized in Figure 6.14 by acquiring a single long T_2^* measurement with a free-induction-decay-style protocol and re-fitting it over progressively decimated subsets of the data¹⁰. This lengthens the total run time by adding an effective delay between retained samples, while the total sample number is kept constant.

Laser Amplitude Noise

Noise in the Rabi frequency Ω limits the maximum number of coherent oscillations and thus the fidelity of Rabi-based protocols such as Fock-state decomposition (see section 3.5.2). Following [Roo00], we model the Rabi frequency noise as a Gaussian random variable with standard deviation σ_Ω and assume it to be quasi-static within a pulse and uncorrelated between shots. The maximum number of coherent oscillations N is then set by the condition that the accumulated uncertainty exceeds a half period¹¹ $\delta T = N \cdot (2\pi/\Omega) \cdot (\sigma_\Omega/\Omega) = \pi/\Omega$, giving $N = \Omega/(2\sigma_\Omega)$.

We observe $N \approx 23$ coherent oscillations, implying a fractional Rabi-frequency uncertainty $\sigma_\Omega/\Omega \approx 2.2\%$. An improved diagnostic would be to Rabi flop at a fixed time corresponding to a slope (as opposed to an extremum) and Fourier-transform the resulting time series to recover the noise spectrum. Dominant sources of intensity noise are expected to be the injection-locked diode seed, the tapered amplifier output, AOM beam-pointing drift, and polarization drift. To mitigate these, an active amplitude stabilization system (see section 5.2.6) was installed with a pickoff placed close to the ion¹² to maximize the noise sources integrated into the feedback loop (including polarization noise converted to amplitude noise); this was able to improve the contrast decay by $\times 3.38$ (from the ratio of the fitted contrast decays in Figure 6.15).

¹⁰i.e. every first point (1,2,3,...,N), then every second point (1,3,5,...,2N), etc.

¹¹This is because the signal becomes non-injective and aliases.

¹²2 mirrors upstream of the chamber.

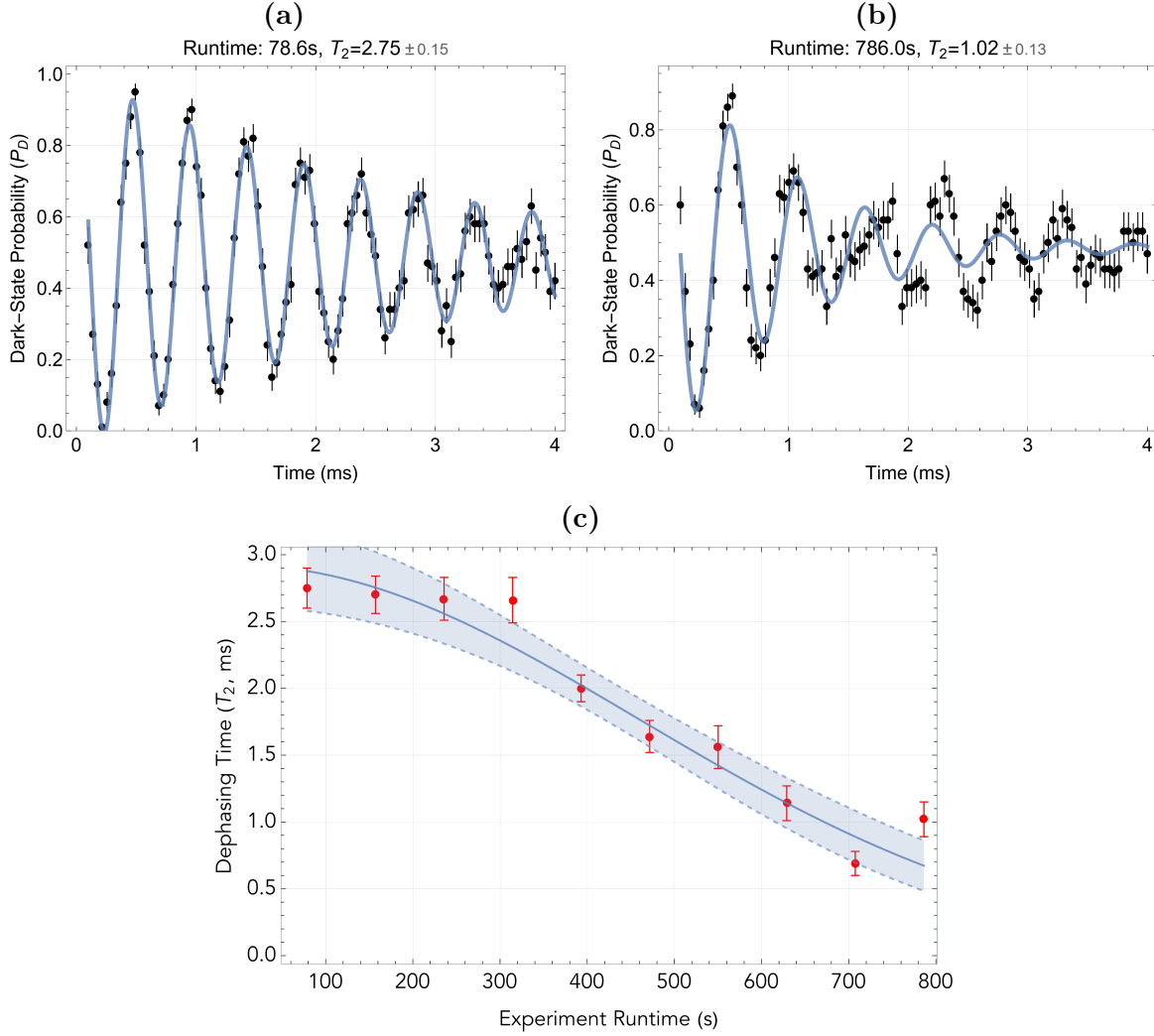


Figure 6.14: **Long experiment runtimes expose measurements to drift-induced dephasing.**

Apparent T_2^* versus total runtime, extracted from a [FID](#) dataset over progressively decimated subsets (retaining every n^{th} point) to lengthen effective run duration at fixed sample count. T_2^* is extracted as the decay constant of an exponentially damped sinusoidal fit. **(a)** No decimation (runtime 78.6s), $T_2 = 2.75(15)$ ms. **(b)** Every tenth point (runtime 786s), $T_2 = 1.02(13)$ ms. A beatnote visible near $T \approx 3$ ms suggests the excess dephasing here arises from a discrete shift in the transition frequency. **(c)** T_2^* versus effective runtime fitted to a Gaussian profile; the apparent coherence degrades as longer runs accumulate exposure to slow drift.

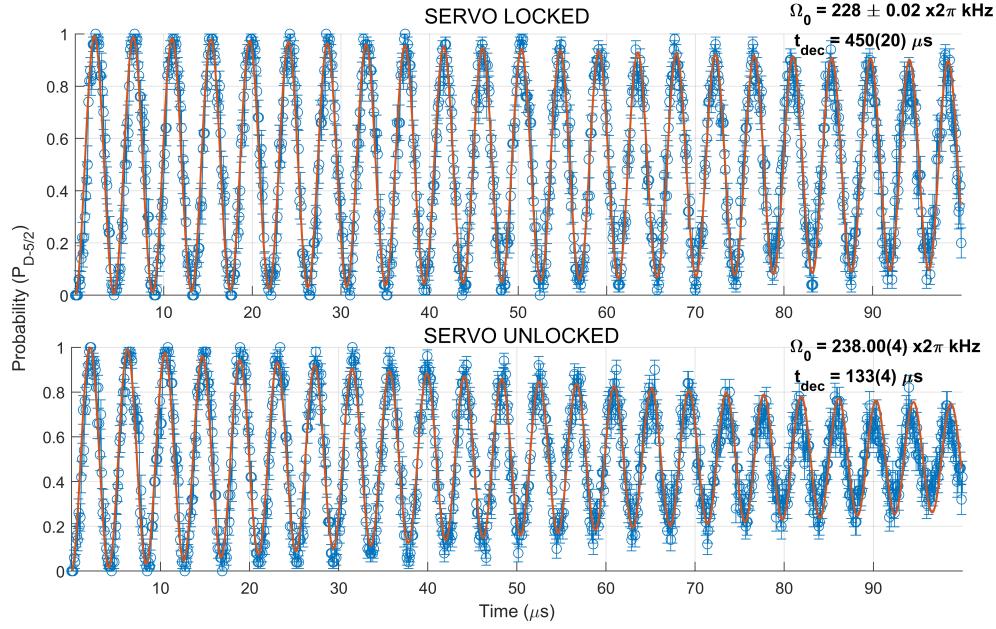


Figure 6.15: **Laser intensity stabilization improves maximum optical qubit interrogation time.**

Carrier Rabi oscillations on the $|^2S_{1/2}, m_J = -\frac{1}{2}\rangle \leftrightarrow |^2D_{5/2}, m_J = -\frac{5}{2}\rangle$ transition with the 729 nm intensity servo locked (top) and unlocked (bottom). Stabilization extends the coherent interrogation time by $\times 3.38$ from $t_{dec} = 133(4) \mu s$ to $450(20) \mu s$. The coherence time is extracted as the decay constant of an exponentially damped sinusoidal fit.

Line Noise

Magnetic field noise — dominantly from 60 Hz mains pickup — modulates the $S \leftrightarrow D$ transition frequency, introducing coherent dephasing. This is mitigated by triggering experimental sequences to the AC line such that spectroscopy pulses execute at a reproducible phase of the AC line. However, not all phases are equal: the sinusoidal AC line is sharpest at a zero-crossing, but is second-order insensitive at an extremum.

The holdoff following the trigger was made configurable, and was calibrated by measuring the $S \leftrightarrow D$ frequency and dephasing as a function of the holdoff time. This holdoff extends the duration of each shot by up to $1/60 \text{ Hz} \approx 16.7 \text{ ms}$, exposing longer runs to slow drifts and transient glitches; this could be mitigated by building a line trigger with an adjustable trigger point. Experimentally, line triggering was found to only modestly improve T_2^* ; this is likely due to attenuation of line noise by the cryogenic copper radiation shields surrounding the trap.

Tesla: “Have you considered the cost of such a machine?”

Angier: “Why, hasn’t good come of your obsessions?”

Tesla: “Well, at first. But I followed them too long. I’m their slave . . . and one day they’ll choose to destroy me.”

On the cost of real magic, *The Prestige*

Chapter 7

Electric-Field Gradient Gates

7.1 Introduction

[Electric-Field Gradient Gates \(EGGS\)](#) is the original aim of this thesis: to construct an alternative toolbox for trapped-ion quantum logic — single-qubit rotations, two-qubit entanglement, state preparation and detection — entirely from electric-field gradients, in place of the lasers or magnetic-field gradients used in established architectures. Importantly, [EGGS](#) is enabled only with molecular ions, as only molecules, by virtue of their additional degrees of freedom, possess the required low-lying, electric-dipole-allowed transitions. Lambda-doublet states, in particular, are opposite parity states with transitions in the radio-frequency range, allowing excitation by electric fields. Though atoms possess similarly low-lying transitions — e.g. the hyperfine ~ 1.8 GHz $|^2S_{1/2}, F=2\rangle \leftrightarrow |^2S_{1/2}, F=3\rangle$ transition in $^{25}\text{Mg}^+$ — these connect states of the same parity and are M1 magnetic-dipole transitions, requiring instead magnetic-field gradients to excite.

7.1.1 Motivation

Challenges for Atomic Trapped Ion Quantum Computers

[EGGS](#) seeks to address limitations of existing laser-driven trapped-ion quantum computers. First, delivery of laser light to ions remains an active challenge at the $\sim 10^3$ qubit-level required for [Noisy, Intermediate-Scale Quantum Computers](#) (NISQs): current use of bulk, free-space optics [[Ran+25](#)] is limited to the $\sim 10^3$ qubit regime, while integrated photonics [[Meh+16](#)] remain challenged to deliver at the low-noise and power levels required at UV wavelengths. [EGGS](#), in contrast, is inherently scalable, as it piggybacks on the existing trapping RF infrastructure, as [EGGS](#) drives can be applied to those same trap electrodes. Second, laser-driven gates suffer infidelities from photon scattering, with greater power required for improved fidelities, further complicating optical delivery [[Oze+07](#)]. As spontaneous-emission rates scale as f^3 via Fermi's golden rule, spontaneous emission errors on the radio-frequency-band transitions of [EGGS](#) are negligible. Third, laser-driven entangling gates typically require the system to be cooled into the Lamb-Dicke regime to mitigate the anharmonicities of the laser drive, again complicating optical delivery. Due to the uniform field gradient generated by the trap electrodes, a need for a similar cooling is absent for [EGGS](#), allowing operation even with ions in thermal motion [[HC21](#)].

The Molecular Moment

Developments over the past decade have made it a ripe, molecular moment to hatch [EGGS](#). A decade of work at [NIST](#) [[Cho+17](#),[Lin+20](#),[Cho+20](#)] and [PTB](#) [[Wol+16](#)] have demonstrated coherent manipulation and non-destructive readout of single trapped molecular ions with [Quantum Logic Spectroscopy \(QLS\)](#), with similar work at the Odom group demonstrating molecular state preparation via optical pumping [[Sto+20](#)]. Prêt-à-porter closed-cycle 4 K cryostats with $\sim$ Watt-class cooling power (sufficient to cool an ion trap system), are now mature, commercially available, and routinely integrated with trapped-ion systems [[Kna24](#),[Lab+08](#),[Mos+23](#)],

enabling preparation of molecules into their ground rovibrational state. Field-gradient quantum logic, proposed in [MW01] and first demonstrated in [Osp+11], has similarly matured into a competitive candidate for a scalable trapped ion quantum computer, enabling the highest reported two-qubit gate fidelities on any architecture [Hug+25].

7.1.2 Quantum Logic Spectroscopy (QLS)

The additional degrees of freedom of molecules present, again, a Faustian bargain: the same rich internal structure of molecules that enables low-lying E1 transitions also precludes the direct optical cycling required for state readout. When a molecule decays from an electronically excited state, the resulting vibrational distribution is set by the Franck-Condon factors between the excited and ground potential energy surfaces, which for a typical Hund’s case (a) diatomic results in significant vibrational branching. Since transition energies vary with vibrational quantum number, the molecule quickly reaches a dark state, and repumping is impractical as the $v \leftrightarrow v'$ transition energies are of order THz.

Instead, Quantum Logic Spectroscopy (QLS) [Sch+05] is used for *indirect* readout of the molecular state by co-trapping the molecular “spectroscopy” ion (i.e. H^{35}Cl^+) with an atomic “logic” ion ($^{40}\text{Ca}^+$ in our case) to supply the cooling and detection capabilities lacked by the molecule. By exciting the molecule with a state-dependent force, the shared motional bus (provided by the Coulomb potential) is conditionally excited, allowing readout by the logic ion. The traditional, clock-style QLS scheme used for interrogation of the spectroscopy ion involves a first π -pulse followed by a subsequent RSB pulse to convert spin excitation into motion for readout¹. As this conditional excitation must be distinguished from a thermal background, robust QLS generally requires the chain to be cooled near the ground motional state².

Due to the rise of sympathetic, mixed-species chains, QLS has become ubiquitous: it

¹A different scheme (see section 7.3.2) is used in this thesis to enable robust detection of a yet-unknown transition.

²This requirement is not strict: geometric-phase gates have been demonstrated for QLS with Doppler-cooled ions [Hem+13].

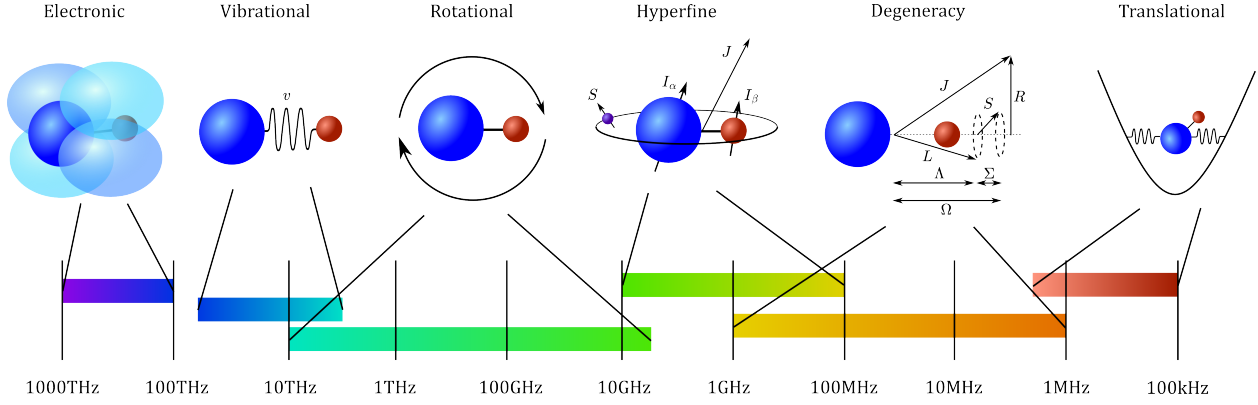


Figure 7.1: **Energy scales of the degrees of freedom of a diatomic molecule.**

is used in Quantinuum’s Ba–Yb architecture [Mos+23,Ran+25] and in precision ion clock schemes [Mar+25]. Of course, QLS is not without its challenges — see section 2.4.4 for a discussion of the mode-decoupling problem in mixed-species chains.

7.2 Molecular Structure

The presence of multiple bound nuclei in a molecule gives rise to rotation and vibration, which underlie the explosion of available states as compared to an atom. These additional degrees of freedom are grouped following Figure 7.1. Here, we provide an abbreviated overview of *diatomic* molecular structure as it pertains to H^{35}Cl^+ .

7.2.1 Hund’s Case (a)

As H^{35}Cl^+ has a ground state $X^2\Pi_{3/2}$ with nonzero Λ and strong spin-orbit coupling relative to rotation (see Figure 7.2), it is well-described by Hund’s case (a).

In Hund’s case (a), the orbital and spin angular momenta $\mathbf{L}$, $\mathbf{S}$ are strongly coupled to the internuclear axis by the electrostatic potential and the spin-orbit coupling, respectively. Both $\mathbf{L}$, $\mathbf{S}$ have well defined projections onto the internuclear axis Λ , Σ , respectively, and combine to form $\mathbf{\Omega}$ with total axial projection $\Omega = |\Lambda + \Sigma|$. Including the rotational angular momentum $\mathbf{R}$, the total angular momentum is $\mathbf{J} = \mathbf{\Omega} + \mathbf{R}$. For case (a), the good quantum

numbers are $\Lambda, \mathbf{S}, \Sigma, \Omega, \mathbf{J}, M_J$.

7.2.2 Fine-Rotational Structure

For Hund's case (a), the fine-rotational structure can be described by the effective Hamiltonian [Saw10]

$$\hat{H}_{\text{eff}} = \hat{H}_{\text{rot}} + \hat{H}_{\text{FS}} \quad (7.1)$$

where $\hat{H}_{\text{rot}}$ and $\hat{H}_{\text{FS}}$ are the rotational and fine-structure Hamiltonians, respectively.

Rotation Rotation of the molecule about a principal axis is described by the Hamiltonian

$$\hat{H}_{\text{rot}} = B \mathbf{R}^2 \quad (7.2)$$

where $B \propto 1/I$ is the rotational constant, and I is the moment of inertia along the given axis. Monohydrides, e.g. H^{35}Cl^+ , have large rotational constants as compared to other heteronuclear diatomics due to the small atomic radius of Hydrogen, which results in a short bond length and concomitantly small moment of inertia [Cha25]. In the Hund's case (a) basis, eq. (7.2) becomes

$$\hat{H}_{\text{rot}} = B (J(J+1) + S(S+1) - 2\Omega\Sigma - \Lambda^2) . \quad (7.3)$$

Fine Structure In addition to the spin-orbit interaction, fine structure in molecules consists of an additional spin-rotation term

$$\hat{H}_{\text{FS}} = \underbrace{A \mathbf{L} \cdot \mathbf{S}}_{\text{Spin-Orbit}} + \underbrace{\gamma \mathbf{R} \cdot \mathbf{S}}_{\text{Spin-Rotation}} . \quad (7.4)$$

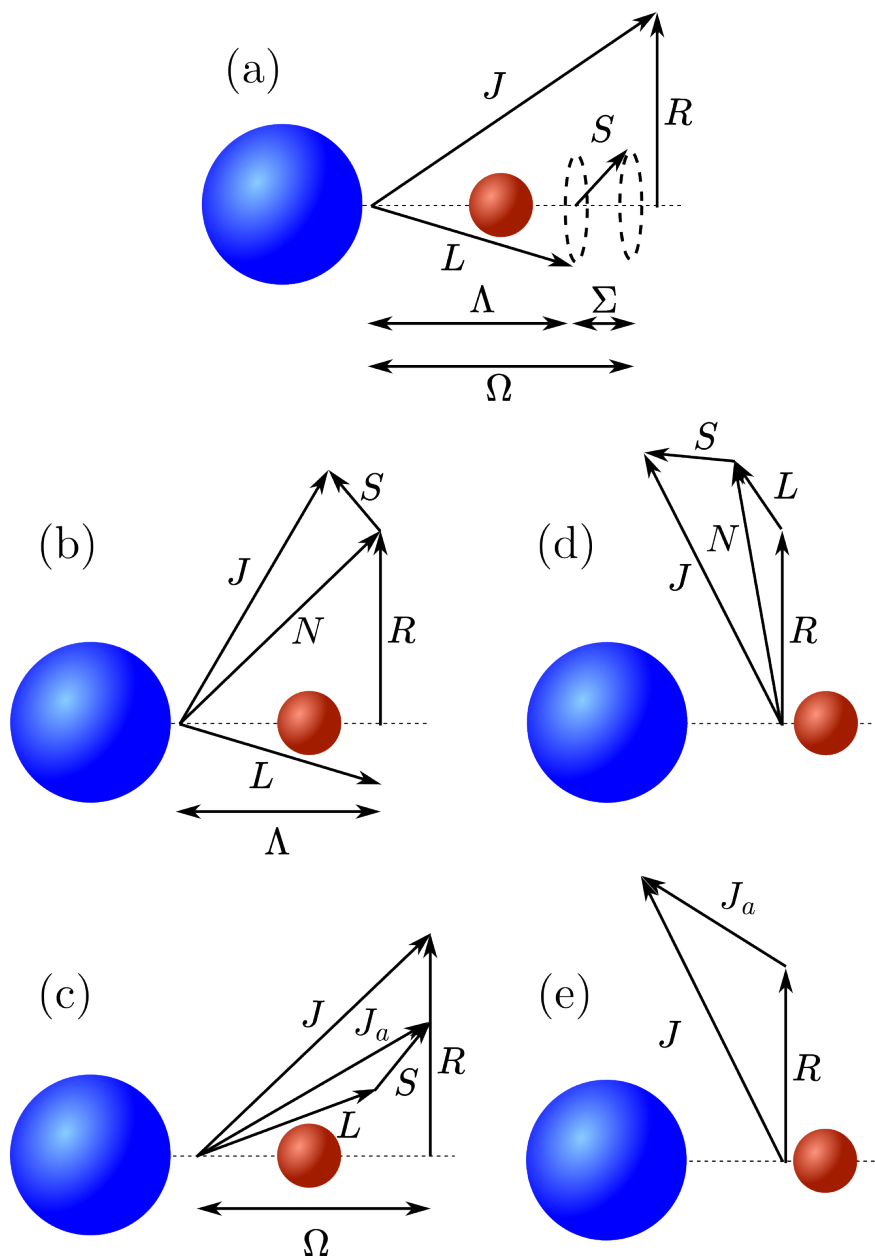


Figure 7.2: **Hund's cases describing diatomic molecular structure in different regimes.**

Hund's cases enumerate limiting angular-momentum coupling schemes for a diatomic molecule and are ordered by the relative strengths of the electrostatic coupling of $\mathbf{L}$ to the internuclear axis, the spin-orbit coupling, and the rotational coupling. In case (a), $\mathbf{L}$ and $\mathbf{S}$ are both strongly coupled to the internuclear axis with well-defined projections Λ and Σ . In case (b) the spin-orbit coupling is weak, and $\mathbf{S}$ couples instead to the rotational angular momentum $\mathbf{N}$. In cases (c) and (d), the spin-orbit and rotational couplings, respectively, dominate over the electrostatic coupling to the axis. The $X^2\Pi_{3/2}$ ground state of H^{35}Cl^+ , with nonzero Λ and large spin-orbit coupling relative to its rotational constant, is well described by case (a).

7.2.3 Hyperfine Structure

The molecular hyperfine Hamiltonian arises from the coupling of the nuclear spin to the system, and contains the following interactions:

$$\begin{aligned} \hat{H}_{\text{HFS}} = & \underbrace{a \mathbf{I} \cdot \mathbf{L}}_{\text{Nuclear Spin - Orbit}} + \underbrace{b_F \mathbf{I} \cdot \mathbf{S}}_{\text{Fermi Contact}} + \overbrace{\sqrt{10} g_S \mu_B g_N \mu_N \frac{\mu_0}{4\pi} \mathbf{I} \cdot \mathbf{S}}^{\text{Nuclear Spin - Electron Spin}} \\ & \underbrace{- \frac{e T^{(2)}(\mathbf{Q}) \cdot T^{(2)}(\nabla \mathbf{E})}{\text{Nuclear Electric Quadrupole - Electrostatic}}}_{\text{Dipole - Dipole}}, \end{aligned} \quad (7.5)$$

where a, b_F are hyperfine constants, and e is the electron charge. The quadrupole term is possible as nuclei with $I \geq 1$ possess electric quadrupole moments.

7.2.4 Lambda-Doubling

For Hund's case (a), the energy is degenerate under reflection of the axial projections $\Omega, \Lambda, \Sigma \leftrightarrow -\Omega, -\Lambda, -\Sigma$. These projections can be formed into eigenstates of parity to fulfill the Laporte rule (allowing E1 transitions), known as lambda-doublet states. For a $^2\Pi$ molecule, this is [BC03, Saw10]

$$\begin{aligned} |J, M_J, \Omega, e\rangle &= \frac{1}{\sqrt{2}} (|J, M_J, \Omega, \Lambda, \Sigma\rangle + |J, M_J, -\Omega, -\Lambda, -\Sigma\rangle) \\ |J, M_J, \Omega, f\rangle &= \frac{1}{\sqrt{2}} (|J, M_J, \Omega, \Lambda, \Sigma\rangle - |J, M_J, -\Omega, -\Lambda, -\Sigma\rangle), \end{aligned} \quad (7.6)$$

where, for compactness, we have neglected the hyperfine structure.

This degeneracy is lifted by the spin-orbit and rotational Hamiltonians of the molecule via second order perturbation. This results in the lambda-doublet Hamiltonian [BC03]

$$\hat{H}_{\text{AD}} = \sum_{k=\pm 1} e^{-2ik\phi} \left(-\frac{q}{2} T_{2k}^{(2)}(\mathbf{J}, \mathbf{J}) + \frac{(p+2q)}{2} T_{2k}^{(2)}(\mathbf{J}, \mathbf{S}) \right) \quad (7.7)$$

where p, q are Λ -doubling constants.

7.2.5 H³⁵Cl⁺ Structure

As neutral H ³⁵Cl has a fully closed shell, ionization removes a nonbonding electron from a π orbital, leaving an unpaired electron in a $\Lambda = 1$ shell³ to give a ground configuration $X\ ^2\Pi$ for H³⁵Cl⁺. For $\Lambda = 1$, Ω takes possible values $3/2, 1/2$. As the outer shell is more than half-filled, the spin-orbit constant A is negative (per Hund’s *rules*), making the $X\ ^2\Pi_{3/2}$ the ground state of H³⁵Cl⁺.

The structure of H³⁵Cl⁺ in the ground rotational state ($X\ ^2\Pi_{3/2} (J = 3/2)$) was calculated by [HW](#) and [ERH](#) using the molecular structure constants in [\[GDP12\]](#) with PGOPHER; further details can be found in [GDM](#)’s forthcoming thesis. The Zeeman shifts were computed separately by [HW](#) and combined with the zero-field PGOPHER output to yield the full structure in [Figure 7.3](#).

Following standard atomic selection rules, [EGGS](#) can be used to drive $\Delta F = 0, \pm 1$ transitions so long as they are accompanied by a change in lambda-doublet state to provide the odd parity required to fulfill the Laporte rule. Due to the weak coupling of the $I = 1/2$ Hydrogen nuclear spin, it can be treated as a perturbation to the final system, generating an effective Zeeman shift which we colloquially refer to as “super-hyperfine”:

$$E_{\text{S-HFS}} = -g_H \mu_N m_H. \quad (7.8)$$

The M1 operator required to flip the $I_H = 1/2$ nuclear spin is parity-even and thus inaccessible to [EGGS](#), which couples through the parity-odd molecular dipole. The “super-hyperfine” structure is consequently best treated by partitioning the H³⁵Cl⁺ manifold into two non-interacting manifolds — $F = F_1 + 1/2$ and $F = F_1 - 1/2$, where $F_1 = J + I_{\text{Cl}}$. For the [EGGS QLS](#) search, this implies that the active manifold is effectively selected at random upon loading and remains fixed for the lifetime of the ion; detected [QLS](#) peaks may accordingly

³Here, Λ refers to orbital angular momentum, and is the molecular analog to L for atomic term symbols. Yes, this is confusing as Λ is also used to signify the projection of the orbital angular momentum onto the internuclear axis — yet another example of the doublespeak that the graduate student must deftly navigate; see epigraph in [chapter 10](#).

shift discretely by $E_{\text{S-HFS}}$ between loading attempts.

Transitions between the two manifolds would instead require an oscillating magnetic field to drive the M1 transition — via e.g. a microwave horn.

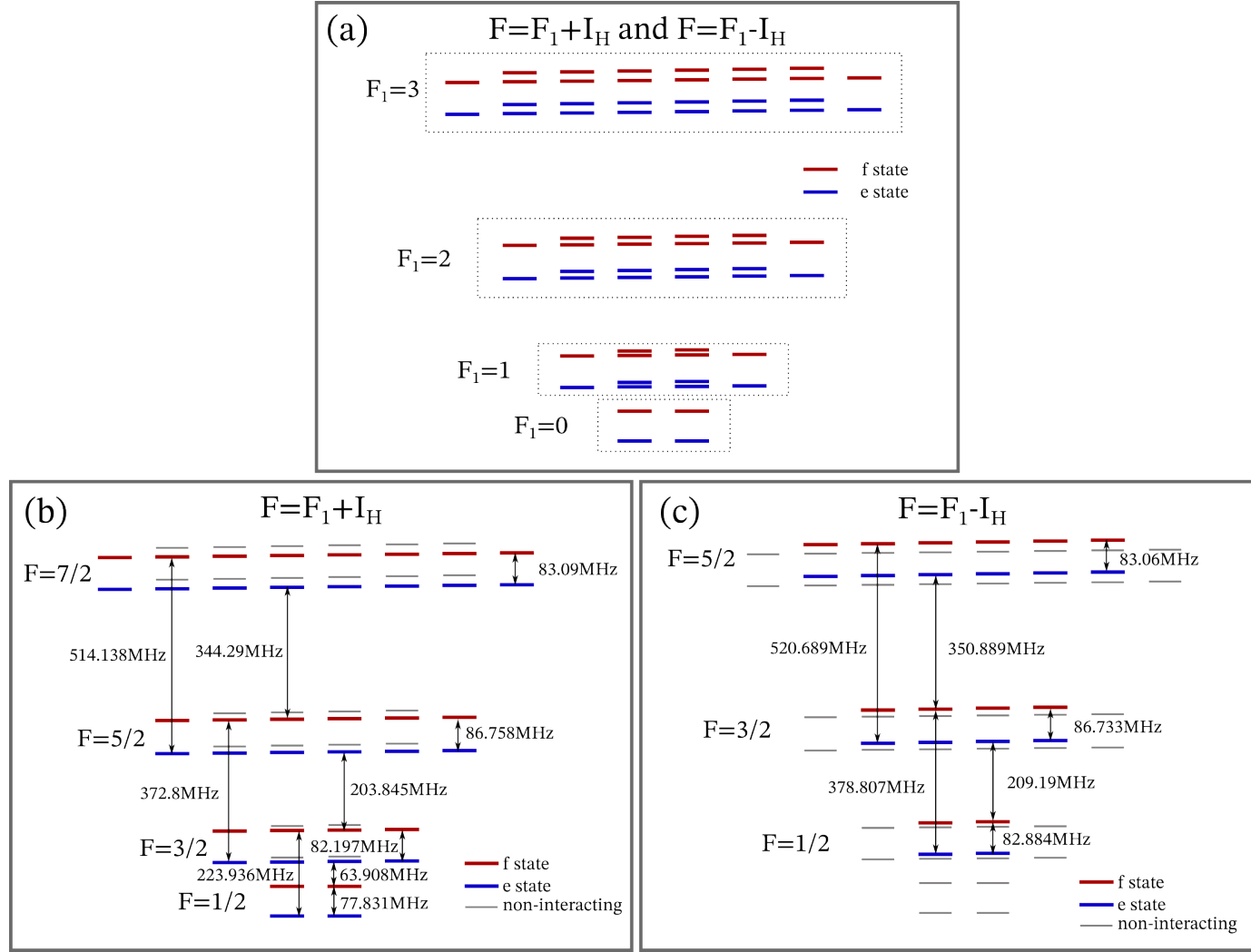


Figure 7.3: **Energy level diagram for the ground state ($X^2\Pi_{J=3/2}$) of H^{35}Cl^+ .** (a) The $J = 3/2$ ground rotational level splits into hyperfine manifolds $F = 3, 2, 1, 0$, formed from $F_1 = J + I_{\text{Cl}}$ with the $I_{\text{Cl}} = 3/2$ chlorine nuclear spin, and further resolved into Zeeman sublevels by a $B = 3.631\,97(2)$ G bias field. Each level is split by lambda-doubling into opposite-parity e, f states. (b, c) The weakly coupled $I_H = 1/2$ hydrogen spin partitions the structure into two non-interacting (via E1 transition) “super-hyperfine” manifolds ($F = F_1 \pm 1/2$). The molecule is initialized into either manifold upon loading; transitions between manifolds (b) and (c) requires an M1 transition and is inaccessible with EGGS.

7.3 EGGS Basics⁴

In this section, we outline how [EGGS](#) voltages applied to the trap electrodes generate the complete toolbox required for quantum logic.

7.3.1 Gate Primitives

To focus on the underlying principles, we abstract the molecular ion into two dipole-connected states $\{|g\rangle, |e\rangle\}$; in our experiment, these are any two opposite-parity lambda-doublet states. This base system is described by the two-level Hamiltonian

$$\hat{H}_{\text{mol}} = \frac{\Delta}{2} \hat{\sigma}_Z, \quad (7.9)$$

where Δ is the transition frequency. Coupling to the applied electric field is governed by the transition dipole moment $d = \langle g | \hat{\mathbf{d}} \cdot \hat{\mathbf{x}} | e \rangle$, with $\hat{\mathbf{x}}$ defined along the magnetic-field quantization axis; for H^{35}Cl^+ , $d \sim 1$ debye.

Single-Qubit Manipulation

A homogeneous dipole field is generated by driving two diametrically opposed electrodes out of phase, taking the form [\[HC21\]](#)

$$\vec{\mathbf{E}}_{E1} = -\frac{V}{2r_0} \cos(\omega t + \phi) \hat{\mathbf{x}}, \quad (7.10)$$

where for simplicity we have assumed the field is aligned with the quantization axis. eq. [\(7.10\)](#) yields the lab-frame Hamiltonian

$$\begin{aligned} \hat{H}_{E1}/\hbar &= -\frac{\mathbf{d} \cdot \mathbf{E}_{E1}}{\hbar} = \frac{dV}{2r_0\hbar} \hat{\sigma}_X \cos(\omega t + \phi) \\ &= \Omega \hat{\sigma}_X \cos(\omega t + \phi) \end{aligned} \quad (7.11)$$

⁴or “Raw [EGGS](#).”

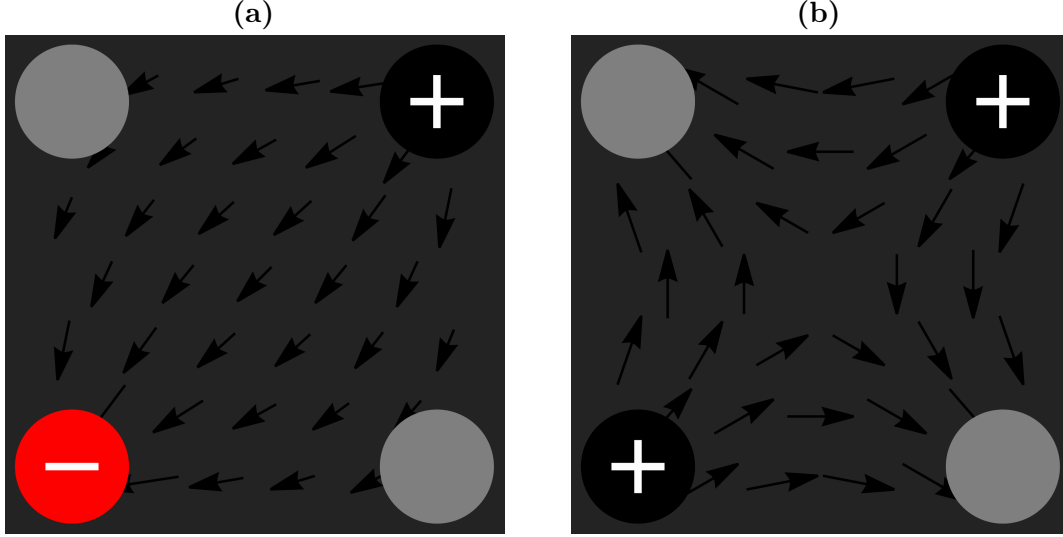


Figure 7.4: **Generation of electric field gradients for molecular ion quantum logic.** **(a)** Driving two diametrically opposed electrodes out of phase produces a dipolar field (eq. (7.10)) that is approximately uniform over the ion and couples to the molecular dipole to drive carrier (spin-only) transitions. **(b)** Driving the same electrodes in phase produces a quadrupolar field with a spatial gradient (eq. (7.12)) that varies over the extent of the ion's dipole moment to exert a net force. The quadrupole gradient couples spin to motion and enables entangling operations. As the wavelength of the radiation used for EGGs ($\lambda \sim \text{m}$ at $\sim 100 \text{ MHz}$) far exceeds the trap dimensions, the field gradient at the ion is set by electrode boundary conditions ($\lambda \sim r_0$) rather than the free-space wavelength.

where we have defined $\Omega = dV/(2r_0\hbar)$. This is the standard carrier interaction and enables the arbitrary single-qubit rotations required for control over the spin state.

Spin-Motion Coupling

A quadrupole field is generated by driving two diametrically opposed electrodes in-phase, taking the form [HC21]

$$\vec{\mathbf{E}}_{E2} = -\frac{V}{r_0^2} \cos(\omega t + \phi) \vec{\mathbf{x}}. \quad (7.12)$$

This yields the lab-frame Hamiltonian

$$\hat{H}_{E2}/\hbar = 2\Omega\hat{\sigma}_X \cos(\omega t + \phi) \left(\frac{x_{\text{eq}}}{r_0} + \sum_i \eta_{i,m} (\hat{a}_m^\dagger + \hat{a}_m) \right) \quad (7.13)$$

where x_{eq} is the equilibrium position of the ion with respect to the *field null*, and $\eta_{i,m}$ is the Lamb-Dicke parameter of the i -th ion for the m -th motional mode

$$\eta_{i,m} = \frac{1}{r_0} \sqrt{\frac{\hbar}{2m_i\omega_m}} b_{i,m}. \quad (7.14)$$

This interaction couples spin to motion, enabling the two-qubit operations required for entanglement. For a single H^{35}Cl^+ and typical values $\omega_0 \sim 1$ MHz, $r_0 = 550$ μm , $m = m_{\text{H}^{35}\text{Cl}^+}$, we have $\eta \sim 2.15 \times 10^{-5}$.

Dipole Field Component The x_{eq} term in eq. (7.13) drives spin-only carrier transitions which results in an effective stark shift that depends on the field null offset x_{eq} [JJ07]:

$$\hat{H}_{\text{eff}}/\hbar = \left(\frac{\Omega x_{\text{eq}}}{r_0} \right)^2 \left(\frac{2\Delta}{\Delta^2 - \omega^2} \right) \hat{\sigma}_Z \quad (7.15)$$

where ω is the drive frequency. This carrier component is generic to gradient-gate schemes and is typically suppressed by adjusting the field itself, rather than the ion position, to minimize carrier excitation, which effectively induces an additional dipole term to cancel the x_{eq} term [Sri20,Sri+19,Sri+21]. A similar compensation strategy to that in [Sri+21] is not yet possible in our experiment as the molecular transition has yet to be found. Additionally, as the trap RF field null is not necessarily coincident with the null of the EGGS fields at a given drive frequency, both the dipole components from the trap RF and EGGS fields must be compensated.

7.3.2 Bichromatic Heating Readout

Readout is the most important step in our experiment (thus far), as it is the first step to QLS and is needed before any other gate primitives can be demonstrated. In contrast to the carrier + RSB schemes typical of QLS [Sch+05], we implement readout by simultaneous application of Red Sideband (RSB) and Blue Sideband (BSB) quadrupole tones to generate

the bichromatic Hamiltonian

$$\hat{H}/\hbar = \Omega\eta\hat{\sigma}_X (\hat{a}^\dagger + \hat{a}) . \quad (7.16)$$

This exerts a state-dependent force $\hat{U}(t) = \hat{D}(\alpha\hat{\sigma}_X)$, with displacement $\alpha = \Omega\eta t$, identical to the bichromatic Hamiltonian used to generate cat states. Bichromatic readout is thus preferred over traditional, single-tone QLS readout for our search as it results in a displacement that grows linearly with time, allowing arbitrarily large signal, and thus SNR, against a given background, and does not require foreknowledge of pulse times or coupling strengths. Similar protocols have been demonstrated in [Kie+20,Cui+22], though both employ geometric-phase gates in the style of [Hem+13] rather than open-trajectory excitation.

Search Linewidths

The linewidth of the bichromatic interaction (for carrier detuning/error) determines the search parameters, and is straightforwardly $\Gamma \sim \eta\Omega$ (eq. (7.16)), which for typical values $\omega_0 \sim 1$ MHz, $V_{\text{EGGS}} \sim 1$ V, $r_0 = 550$ μm is $\Gamma = 93.53 \times 2\pi$ Hz.

Though this value is exceedingly small relative to a typical search range of ~ 1 MHz, in practice, we are not interested in the relative *width* of the peak, but rather, any excitation at all, and so define an effective QLS linewidth by any excitation above some threshold (e.g. one phonon). Longer interrogation times, as well as larger drive voltages, therefore increase this effective linewidth. Intuitively, when the carrier is detuned, the spin and motion approximately separate, such that the motion is still driven resonantly despite the carrier detuning.

Dipole Cancellation

When the two quadrupole tones are placed symmetrically about the transition for bichromatic readout, their dipole-induced Stark shifts are opposite and approximately equal, and thus

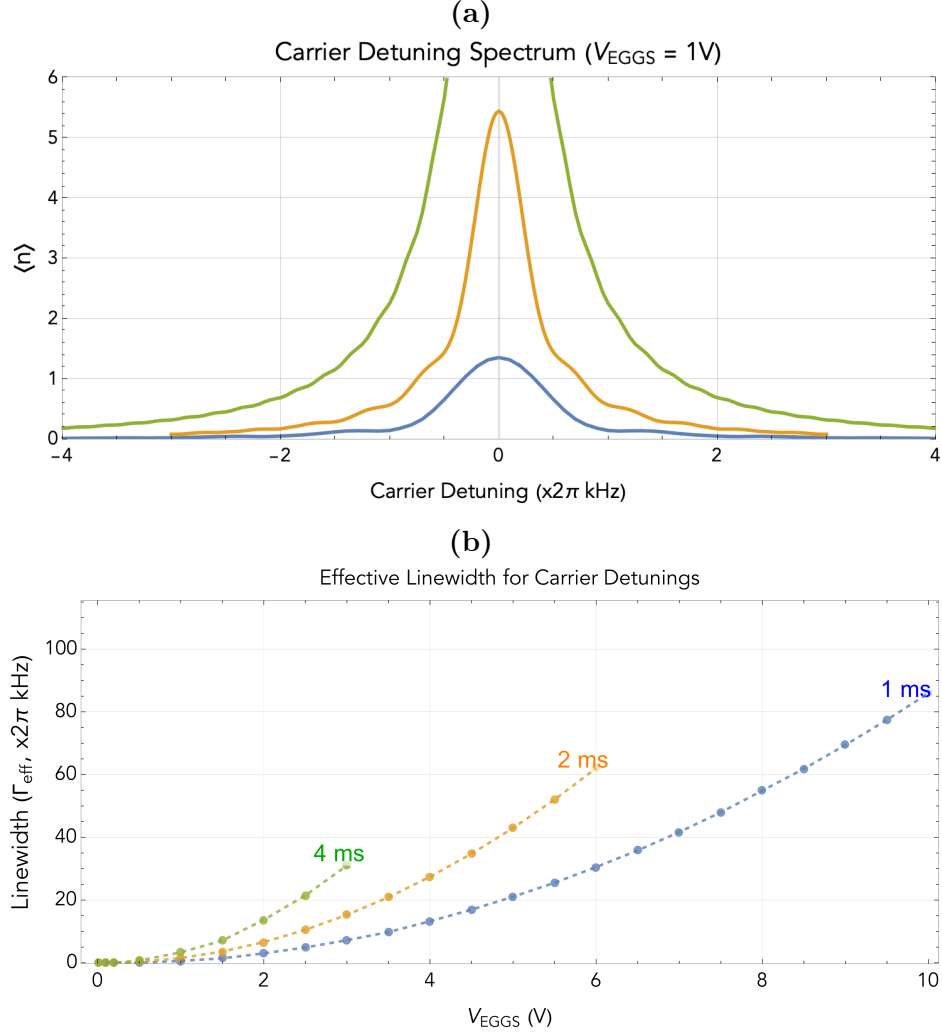


Figure 7.5: **Effective search linewidths for EGGs.**

An effective linewidth is defined by motional excitation above a detection threshold $\bar{n} > 1$ and grows with both interrogation time and drive voltage; simulations use values $V_{\text{EGGS}} = 1\text{ V}$, $\eta\Omega_{\text{EGGS}} = 190 \times 2\pi\text{ Hz}$. **(a)** Simulated mean phonon number versus carrier detuning for interrogation times $\tau = 1$ ms (blue), 2 ms (orange), and 4 ms (green). For a bichromatic drive, the motional dynamics are insensitive to carrier detuning, such that the phase-space trajectory remains open and motional excitation accumulates with interrogation time. **(b)** Effective linewidth Γ_{eff} versus drive voltage for 1 ms, 2 ms and 4 ms interrogation times.

cancel to first order:

$$\hat{H}_{\text{eff}}/\hbar \approx \left(\frac{\Omega x_{\text{eq}}}{r_0} \right)^2 \left(\frac{4\Delta}{4\Delta^2 - \omega_0^2} \right) \hat{\sigma}_Z. \quad (7.17)$$

Remaining stark shifts can be further suppressed by smooth amplitude ramps on the tone envelopes [HC21] (see section 3.3.2 for a discussion of pulse shaping).

7.3.3 Dynamical Decoupling (Green EGGS)

To extend the [EGGS QLS](#) linewidth and protect against carrier-frequency errors during the search, we apply continuous dynamical decoupling, a technique developed for [MS](#) entangling gates. Continuous [DD](#) of the [MS](#) gate was proposed by [\[Ber+12\]](#) and first demonstrated by [\[Tan+13,LBP13\]](#). These works implemented the interaction as an [Single-Sideband \(SSB\)](#) gate — a carrier [DD](#) tone driven alongside a single motion-coupled sideband — which left the gate susceptible to carrier-frequency errors, as the gate dynamics themselves shift with the carrier detuning. This was improved by [\[Har+16\]](#), who extended continuous [DD](#) to the full bichromatic [MS](#) configuration (carrier plus both motional sidebands), known as a [DDMS](#) (dynamically decoupled [Mølmer-Sørensen](#)) gate, decoupling the carrier from the gate dynamics; this same work also implemented the gate via field gradients, in close analogy to our setting. We adapt this technique to [EGGS QLS](#) to broaden the search linewidth and improve our odds of finding the transition; pulsed [DD](#) is unsuited to our case, as the requisite π -pulse timings cannot be calibrated before the molecular transition is found.

The carrier is applied with the lab-frame Hamiltonian (eq. [\(7.10\)](#))

$$\hat{H}_{\text{DD}}/\hbar = \Omega_0 \hat{\sigma}_X \cos((\Delta + \delta)t) \quad (7.18)$$

together with the bichromatic interaction of eq. [\(7.16\)](#), where δ is the carrier detuning from the molecular transition. Transforming into the interaction picture with respect to eq. [\(7.9\)](#) yields (with the appropriate [Rotating Wave Approximations](#))

$$\begin{aligned} \hat{H}/\hbar = \frac{\eta\Omega_{\text{EGGS}}}{\Omega_0^2 + \delta^2} (\hat{a}^\dagger + \hat{a}) & \left(-\hat{\sigma}_Z \delta\Omega_0 \left(1 - \cos\left(\sqrt{\Omega_0^2 + \delta^2}t\right) \right) \right. \\ & + \hat{\sigma}_X \left(\Omega_0^2 + \delta^2 \cos\left(\sqrt{\Omega_0^2 + \delta^2}t\right) \right) \\ & \left. + \hat{\sigma}_Y \delta\sqrt{\Omega_0^2 + \delta^2} \sin\left(\sqrt{\Omega_0^2 + \delta^2}t\right) \right). \end{aligned} \quad (7.19)$$

eq. [\(7.19\)](#) retains the spin-dependent displacement of eq. [\(7.16\)](#), but with a “robustness”

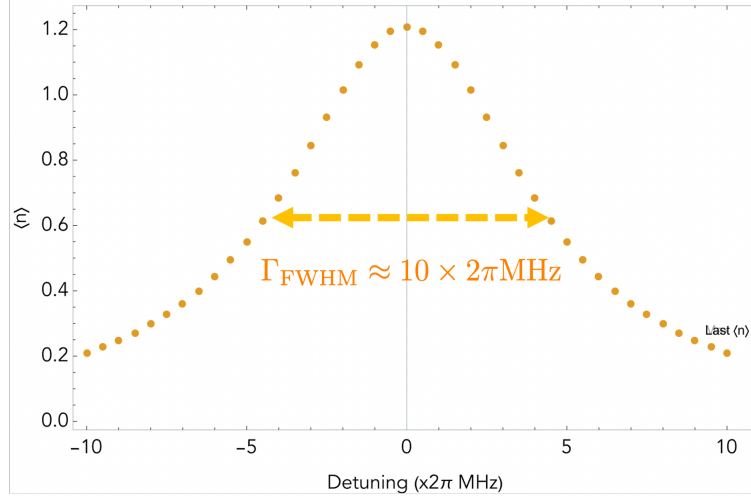


Figure 7.6: **Linewidth broadening via dynamically-decoupled EGGS.**

Simulated mean phonon number versus carrier detuning for interrogation time $\tau = 1$ ms and 1 V ($\Omega_0 \sim 10 \times 2\pi$ MHz) on the carrier for dynamical decoupling and 1 V ($\eta\Omega_{\text{EGGS}} = 190 \times 2\pi$ Hz) on each sideband. The strong carrier tone dynamically-decouples the interaction by defining a $\hat{\sigma}_X$ axis about which $\hat{\sigma}_Z$ errors (i.e. carrier detuning) are rotated away; this yields a **FWHM** linewidth $\Gamma_{\text{DD}} = \Omega_0 \sim 10 \times 2\pi$ MHz, orders of magnitude wider than the primitive **EGGS** linewidth. As the bichromatic **EGGS** interaction commutes with the dynamical decoupling term, the interaction strength Ω_{EGGS} is preserved (eq. (7.20)) and the maximum phonon number is unchanged.

tunable by Ω_0 . For $\Omega_0 \gg \delta$, the resulting **QLS FWHM** linewidth is approximately

$$\Gamma_{\text{DD}} = \eta\Omega_{\text{EGGS}} \frac{\Omega_0^2}{\Omega_0^2 + \delta^2}. \quad (7.20)$$

Intuitively, the strong carrier defines an axis along $\hat{\sigma}_X$ about which the $\hat{\sigma}_Z$ carrier frequency errors are rotated to zero, while the bichromatic heating, also along $\hat{\sigma}_X$, is unaffected. This improvement is contingent on phase alignment of the tones: a carrier term applied out-of-phase with the bichromatic tones will instead rotate the heating away.

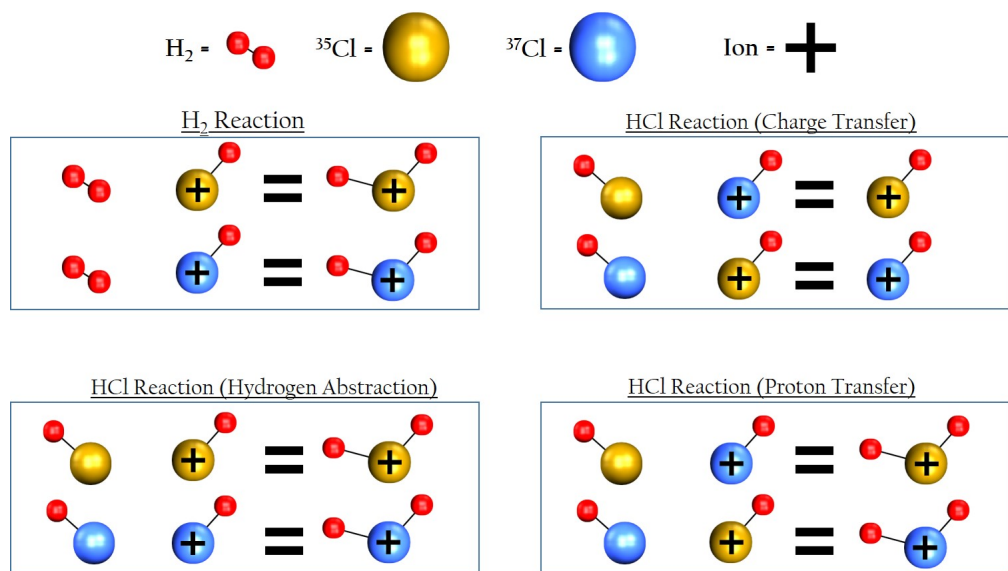


Figure 7.7: H^{35}Cl^+ reaction loss channels.

Loss channels are inferred from changes in the secular frequency; this method is unable to distinguish H_2 reaction from Hydrogen abstraction. The dominant loss channel is reaction of H^{35}Cl^+ into $\text{H}_2^{35}\text{Cl}^+$; $\text{H}_2^{35}\text{Cl}^+$ is dissociated back to H^{35}Cl^+ by 397 nm light via a 1 + 1 [Resonance-Enhanced Multi-Photon Dissociation \(REMPD\)](#) pathway, allowing recovery without reloading.

7.4 Challenges to EGGS⁵

7.4.1 Reactions

Though background reactions are suppressed by cryogenic operation, we find that H^{35}Cl^+ nonetheless reacts roughly every 5 min to 20 min. This is in part due to our molecular loading procedure, which degrades the background pressure by leaking in gas. Reaction loss channels for H^{35}Cl^+ were determined via motional sideband spectroscopy and are enumerated in Figure 7.7; of these, the dominant channel is reaction with H_2 . Fortuitously, preliminary measurements suggest that $\text{H}_2^{35}\text{Cl}^+$ is dissociated by 397 nm light through a 1+1 REMPD pathway. Following reactions during data acquisition, the experiment is therefore paused and the 397 nm beam is applied at maximum power to encourage dissociation to H^{35}Cl^+ .

⁵or “Scrambling [EGGS](#).”

7.4.2 Motional Raman

QVSA Background Suppression

With great irony, the dipole tone required for dynamical decoupling excites an unintended motional Raman transition — a QVSA interaction arising from the coupling of the applied fields to the ion charge — that dominates the EGGS interaction. For typical parameters $V_q = V_d = 1$ V, $\eta \sim 2 \times 10^{-5}$, $\omega = 2\pi \times 1$ MHz, and transition frequency $\Delta = 2\pi \times 80$ MHz, the effective QVSA Rabi frequency $\Omega_{\text{QVSA}} = 2\pi \times 78$ kHz far exceeds $\eta\Omega_{\text{EGGS}} = 2\pi \times 0.14$ kHz by nearly three orders of magnitude. To suppress this background while preserving DD, periodic π phase shifts are applied to the dipole tone (see section 3.3.3). The bichromatic interaction commutes with the phase-shifted carrier (see section 7.3.3) and is therefore unaffected, while the QVSA trajectory, referenced to the carrier phase, is reversed, suppressing the background, though this introduces a distortion of the DD envelope that grows with the number of phase shifts.

As the drive strength (and thus the dynamically-decoupled linewidth) is increased, the QVSA displacement grows in turn, and pulse errors and decoherence preclude complete background suppression, requiring more frequent phase shifts to prevent accumulation of the displacement. More phase shifts also improves robustness to secular frequency detuning errors by reducing the pulse segment length τ : the filter function of the sequence has a resonance at $\delta_{\omega_0} = 1/\tau$, such that additional phase shifts push the resonance out to higher detunings (see section 3.3.3 for an overview of phase space trajectories). This introduces a tradeoff between the drive strength and the number of phase shifts required for adequate suppression.

For the EGGS search, a $(0, \pi)_N$ phase waveform was used, though this choice is suboptimal: an e.g. $(0, \pi, \pi, 0)_N$ sequence (i.e. Walsh(2)) offers better noise robustness for a similar number of phase shifts (see Figure 3.8).

Intrinsic Squeezing

Intrinsic squeezing is a further motional Raman interaction that limits [EGGS](#): two quadrupole tones separated by $2\omega_0$ drive a two-phonon interaction that squeezes the motional state (see section [8.2.3](#) for the theory). As this tone configuration is identical to that for bichromatic heating, intrinsic squeezing is an unavoidable source of background for [EGGS QLS](#). At 80 MHz, intrinsic squeezing is the dominant source of background and limits the usable signal voltages and pulse lengths, with the latter compounded by the rapid dephasing of squeezed states.

Three strategies were considered to mitigate intrinsic squeezing: moving to a higher-frequency [EGGS](#) transition, where the squeezing parameter scales steeply with ω (suppressing the phonon population exponentially); abandoning the bichromatic heating protocol; or applying antisqueezing to reverse the unwanted squeezing.

While baseband antisqueezing at $2\omega_0$ is simpler to implement, it is challenged by variations in the [EGGS](#) RF coupling across the search range, which rotate the phase-space orientation of the intrinsic squeezing. We therefore considered *intrinsic antisqueezing*: at the end of the [EGGS](#) pulse, the carrier dipole tone is turned off, the phase of one quadrupole sideband is shifted by π , and the pulse is again applied for the same duration as the original pulse. As the tones are applied at the same frequencies, the transfer function does not change, ensuring the antisqueezing is correctly aligned in phase space. Though this procedure cancels the [EGGS](#) bichromatic heating on resonance, without dynamical decoupling, the [QLS](#) lineshape is sufficiently narrow that this loss of signal is negligible. In practice, intrinsic antisqueezing proved difficult to implement: the high powers required and the resulting limitations on motional coherence forced a trade between the [EGGS](#) signal strength and the antisqueezing background reduction.

7.4.3 Trap RF

The trap RF itself generates a significant stark shift due to its large ~ 150 V voltage and relatively small detuning from the ~ 80 MHz [EGGS](#) transitions (the trap RF oscillates at

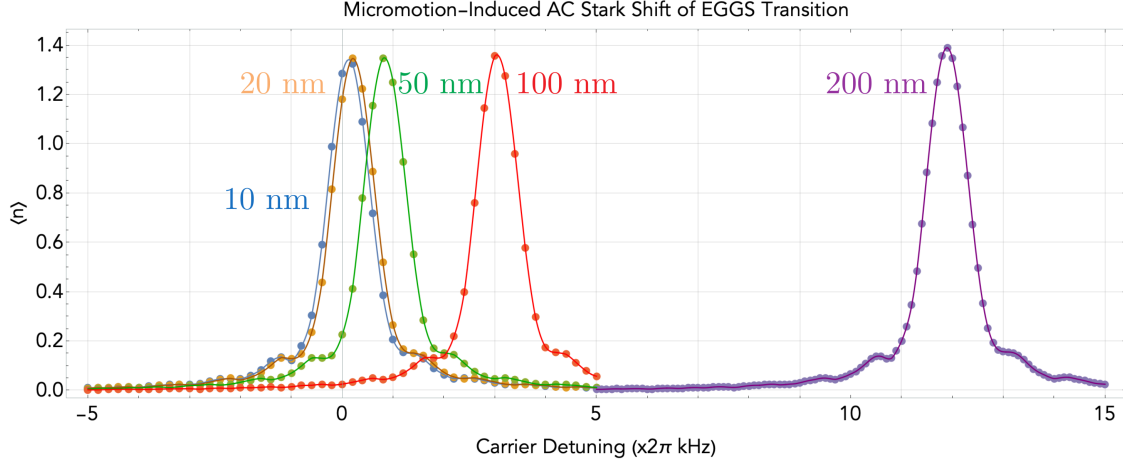


Figure 7.8: **Stark shifts induced by trap RF-null offsets.**

Simulated bichromatic heating spectra (mean phonon number versus carrier detuning) for different ion offsets x_{eq} from the trap RF null. The off-null dipole component of the trap RF produces an AC Stark shift $\propto x_{\text{eq}}^2$ (eq. (7.21)) that shifts the transition in excess of the primitive EGGS linewidth $\eta\Omega_{\text{EGGS}} \sim 100 \times 2\pi$ Hz. Typical values $V_{\text{RF}} = 150$ V, $\omega_{\text{RF}} = 19 \times 2\pi$ MHz, $\Delta_{\text{EGGS}} = 80 \times 2\pi$ MHz are used for the simulation.

$\omega_{\text{RF}} \sim 2\pi \times 19$ MHz). The trap RF presents two sources of stark shifts: a large dipole component when the ion is offset from the RF null by x_{eq} , and a quadrupole component that results in a phonon-dependent stark shift. Unlike the EGGS tones, the stark shifts produced by the trap RF cannot be mitigated by pulse-shaping, balancing (in e.g. a bichromatic configuration), or gradient compensation (see section 7.5).

Trap RF – Dipole Component

From eq. (7.15), an ion offset from the field null experiences a large dipole component of the field, with stark shift

$$\hat{H}_{\text{eff}}/\hbar = \left(\frac{\Omega_{\text{RF}} x_{\text{eq}}}{r_0} \right)^2 \left(\frac{2\Delta}{\Delta^2 - \omega_{\text{RF}}^2} \right) \hat{\sigma}_Z. \quad (7.21)$$

This shift is of order $\sim 50 \times 2\pi$ kHz/ μm^2 , requiring micromotion compensation better than ~ 50 nm to remain within a typical EGGS linewidth $\eta\Omega_{\text{EGGS}} \sim 100 \times 2\pi$ Hz. This is challenging experimentally due to the large change in stray fields from molecular ion loading, which then dissipate over hours; regular and robust micromotion calibration is thus required during EGGS data acquisition.

Trap RF – Quadrupole Component

As long as the ion remains trapped, the quadrupole term is omnipresent and cannot be compensated by shimming. Using eq. (7.13) with the James & Jerke effective Hamiltonian [JJ07] results in a phonon-dependent stark-shift

$$\hat{H}_{\text{eff}}/\hbar = (2\eta\Omega_{\text{RF}})^2 \Delta \left(\frac{\Delta^2 - \omega_0^2 - \omega_{\text{RF}}^2}{(\Delta^2 + \omega_0^2 - \omega_{\text{RF}}^2)^2 - 4\Delta^2\omega_0^2} \right) \hat{\sigma}_Z \left(\hat{a}\hat{a}^\dagger - \frac{1}{2} \right) \quad (7.22)$$

where Δ is the transition frequency, ω_0 is the secular frequency, and ω_{RF} is the trap RF frequency. For $\Delta \gtrsim \omega_{\text{RF}} \gg \omega_0$, this reduces to

$$\hat{H}_{\text{eff}}/\hbar \approx (2\eta\Omega_{\text{RF}})^2 \left(\frac{\Delta}{\Delta^2 + \omega_0^2 - \omega_{\text{RF}}^2} \right) \hat{\sigma}_Z \left(\hat{a}\hat{a}^\dagger - \frac{1}{2} \right) \quad (7.23)$$

For typical values $\Delta \sim 80 \text{ MHz}$, $\omega_0 \sim 1 \times 2\pi \text{ MHz}$, $\omega_{\text{RF}} \sim 19 \times 2\pi \text{ MHz}$, $\eta \sim 2 \times 10^{-5}$, this yields a motional stark shift of $\sim 11 \times 2\pi \text{ Hz}$ per phonon. This sets a maximum phonon number $\bar{n} \sim 10$ before the transition is shifted outside the $\Omega_{\text{EGGS}} \sim 100 \times 2\pi \text{ Hz}$ linewidth. This level of excitation is robustly detectable in the Doppler regime but precludes coarser diagnostics such as fluorescence counting or Doppler recoiling [Eps+07].

Because the shift originates from the trap RF itself, its evolution is deterministic and reproducible so long as the trap RF amplitude is held constant. A quadratic chirp (i.e. frequency ramp) of the EGGS drive frequencies can therefore track the instantaneous shift and extend coherent operation past the $\bar{n} \sim 10$ ceiling. A simpler route is continuous dynamical decoupling (section 7.3.3), which broadens the EGGS linewidth and absorbs the shift directly.

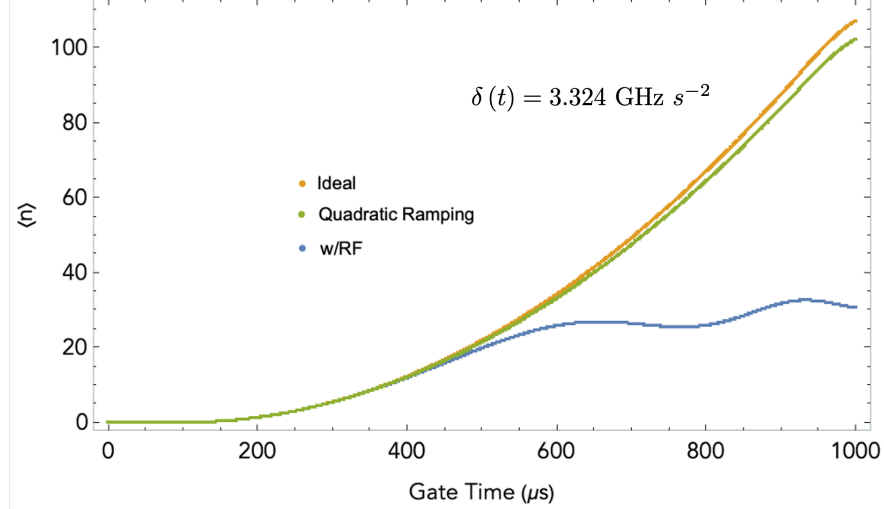


Figure 7.9: **Phonon-dependent stark shift compensation with a quadratic frequency chirp.**

Simulated mean phonon number during bichromatic heating with (blue, green) and without (orange) the trap RF-induced phonon-dependent stark shift (eq. (7.22)). The phonon-dependent shift detunes the interaction as $\langle n \rangle$ grows and stalls the excitation (blue) near $\langle n \rangle \approx 25$; this shift can be compensated by a quadratic frequency chirp at $\dot{\delta} = 3.324 \text{ GHz s}^{-2}$ that tracks the cumulative shift (green), recovering the ideal behavior (orange). Typical values $V_{\text{RF}} = 150 \text{ V}$, $\omega_{\text{RF}} = 19 \times 2\pi \text{ MHz}$, $\Delta_{\text{EGGS}} = 80 \times 2\pi \text{ MHz}$ are used for the simulation. Note that a larger value for $V_{\text{EGGS}} = 10 \text{ V}$, $\eta\Omega_{\text{EGGS}} = 1.9 \times 2\pi \text{ kHz}$ is used in this figure (cf. figs. 7.5 and 7.6).

7.5 Field Gradient Calibration⁶

Control over the field gradients required for EGGS was a subtle but major obstacle to realizing EGGS. This control is critical for maintaining phase alignment during dynamical decoupling, suppressing residual field-induced Stark shifts, and preventing leakage through off-resonant excitation. Though this discussion belongs conceptually with the broader challenges to EGGS (see section 7.4), its outsize importance warrants a dedicated section.

Underlying the problem is the small Lamb-Dicke parameter: for 1V applied nominally in quadrupolar configuration, a relative phase error of 0.01 turns between the two electrodes results in a dipole component of $\approx 62.8 \text{ mV}$, which has a carrier Rabi frequency $\Omega \sim 290 \times 2\pi \text{ kHz}$ — sufficient to drive spectator transitions or Stark shift the transition outside

⁶or “Deus EGGS Machina.”

the narrow [EGGS](#) linewidth.

7.5.1 Comparison to Existing Architectures

Residual carrier fields from gradient gates are typically suppressed by tuning the amplitude and phase of drives across multiple electrodes to shift the gradient null onto the ion [[Sri20](#), §5.5][[Har13](#), §6.4.1][[All+13](#),[Osp+11](#),[War+13](#),[Web+24](#)]. This calibration is delicate as the null can shift from impedance changes at high power or changes to the ion’s equilibrium position [[Car+13](#)]. This challenge can be alleviated with an optimized meander electrode in surface traps [[Hah+19](#)] (though at significant technical overhead), or by introducing dynamical decoupling to decouple the interaction from residual Stark shifts [[Har+16](#),[Sri+19](#)]⁷.

Though field gradient calibration is evidently not novel, the problem is vastly magnified with [EGGS](#). With [EGGS](#), direct qubit-based calibration is not possible as the transition frequencies have yet to be found, forcing indirect calibration via motional Raman excitation, which is harder to interpret. Additionally, calibration must be performed *continuously* across the ~ 10 MHz [QLS](#) search range, as opposed to a single frequency. The low transition frequencies of H^{35}Cl^+ compound the problem, since the dipole Stark shifts are increased at lower transition frequencies (eq. (7.15)). Finally, the dense Zeeman structure of H^{35}Cl^+ places spectator transitions separated by ~ 1 MHz, sufficiently close to be driven by the residual carrier field, resulting in effective spin dephasing or dark state pumping.

7.5.2 Gradient Calibration via Motional Raman

Experimentally, the [EGGS](#) drives were generated by two synchronously clocked outputs (referred to as CH0 and CH1) from an [AWG](#) (Phaser, Sinara/M-Labs), with the relative distribution of the field gradients (i.e. dipole vs quadrupole) controlled by the relative phase and amplitude of the signals on CH1 relative to CH0 (see section 5.3.6 for further details on [EGGS](#) signal generation).

⁷The [NIST](#) near-motional oscillating gradient scheme is intrinsically dynamically decoupled.

Use of motional Raman (i.e. the 2-tone [QVSA](#) effect, see chapter 8 for further details) for field gradient calibration is complicated by its use of multiple tones — a dipole and a quadrupole tone, spaced by the secular frequency ω_0 . As the [QVSA](#) interaction relies on the interaction between both tones, it is sensitive only to the relative field geometry between the two tones (e.g. whether the tones are configured as dipole-dipole vs dipole-quadrupole); this can be calibrated by scanning the CH1 phase of only a single tone.

CH1 Phase Calibration Calibration of the absolute field geometry (e.g. whether the tones are dipole-quadrupole vs quadrupole-dipole) requires an absolute reference, i.e. an interaction sensitive only to a single multipole component. For this, we use the intrinsic squeezing effect (see section 8.2.3), which requires two *quadrupole* tones separated by $2\omega_0$. Intrinsic squeezing is further robust to residual off-null dipole fields, as these generate only a [QVSA](#) interaction, which requires a tone at ω_0 absent from the applied drives. The field geometry can thus be calibrated to a pure quadrupole by tuning the tone CH1 phases to maximize the intrinsic squeezing. As a secondary diagnostic for the field geometry, a single tone was applied at high power to off-resonantly drive the ion motion. As squeezing effects and off-null dipole fields are suppressed by factor $\eta \sim 2 \times 10^{-5}$ and x_{eq}/r_0 , respectively, the off-resonant heating is maximized for a pure dipole.

Field Gradient Purity By iteratively tuning the amplitudes and phases of the tones on each electrode, residual dipole components can be suppressed by > 25 dB. However, this requires control of the amplitudes and phases at the 10^{-4} fractional level, leaving the calibration unstable and frequency-dependent. The resulting overhead makes this procedure impractical for the continuous wideband calibrations required for the [EGGS QLS](#) search.

7.5.3 Spatial Field Characterization

The combination of the off-axis dipole Stark shift and the unknown frequency dependence of the electrode coupling motivated a direct mapping of the EGGS field gradient. We

demonstrate a proof-of-concept measurement by displacing the ion along all three spatial directions within the trap and measuring the strength of the [QVSA](#) interaction along each motional mode⁸. For an overview of trap geometry and relevant electrodes, see Figure 6.2.

Protocol

To ensure a robustly interpreted result, a straightforward, equal-amplitude 2-tone [QVSA](#) interaction was separately applied on the EGGS CH0 and CH1 electrodes to characterize their relative coupling strengths. For comparison, the trap RF field null was characterized along all three motional modes using micromotion compensation techniques.

The ion was translated radially using the vertical and horizontal shim electrodes, which move it at 45° relative to the modes (Figure 6.2). As an approximate reference, a change of 110 V on the horizontal shim corresponds to $\sim 7 \mu\text{m}$ of physical displacement (as measured by the [EMCCD](#) camera) at secular frequencies $(\omega_z, \omega_y, \omega_x) = (702, 1287, 1582) \times 2\pi \text{ kHz}$. Axial translations were performed by adjusting both axial electrode voltages in combination, maintaining a constant axial secular frequency at each position. As the imaging system is directed vertically, only horizontal and axial translations are visible; vertical translations cannot be resolved.

For each data point, a linescan was obtained by scanning the tone separation frequency, then fitted using a sinc-squared function to yield the mean phonon number $\bar{n}$. This mean phonon number was processed into a geometric factor β using $\bar{n} \propto (\beta V_a)(\beta V_b)/(\omega^2 - \omega_0^2)$ (see eq. (8.7)), where V_a, V_b are the voltages of the two [QVSA](#) tones; this assumes that both tones experience the same insertion loss. As we are interested only in the *relative* change in the geometric factor β between the CH0 and CH1 electrodes, results are normalized to the minima within each figure. Similarly, the relative voltages V_a, V_b can be extracted analytically from the foreknown applied signals. Field gradient data are finally fitted to a function $y = k \times |x - c|$ to extract the field null for each electrode.

⁸Motional excitation along modes perpendicular to the [EGGS](#) electrodes (on which the [EGGS](#) drives are applied) is possible due to fringing components of the applied field.

Results

Results for the projections onto the radial EGGS and RF modes are presented in Figure 7.10. The null of the combined field (i.e. if signals were applied to both the CH0 and CH1 electrodes simultaneously) occurs where the field strengths from each electrode are balanced (i.e. where the curves intersect), which does not appear to, in general, be coincident with the trap RF null, nor necessarily accessible by the shim electrodes. Excess micromotion constrained the accessible range of shim voltages, though this range could in principle be extended by reducing the trap RF voltage.

The applied dipole field, delivered directly on the EGGS electrodes, far exceeds the position-dependent residual off-null dipole field along the EGGS mode. Along the RF mode, the applied dipole field reaches the ion only through fringing and is correspondingly weaker, such that the residual off-null contribution dominates. As the residual off-null dipole field originates from the quadrupole field, it is unaffected by fringing. Repeating the procedure at $40\,2\pi\times\text{MHz}$ and $100\,2\pi\times\text{MHz}$ confirms that the combined null is frequency-dependent — the EGGS QLS search range therefore cannot rely on a single calibration, and will require rigorous calibration across its full extent.

The behavior of the axial field gradient is similar to that of the RF mode, with clear overlap between the CH0 and CH1 nulls (Figure 7.10). Axial micromotion is also generated by fringing and is thus correspondingly weak, enabling translation over a much larger scan range $\sim 100\,\mu\text{m}$, compared to the $\sim 10\,\mu\text{m}$ achievable radially.

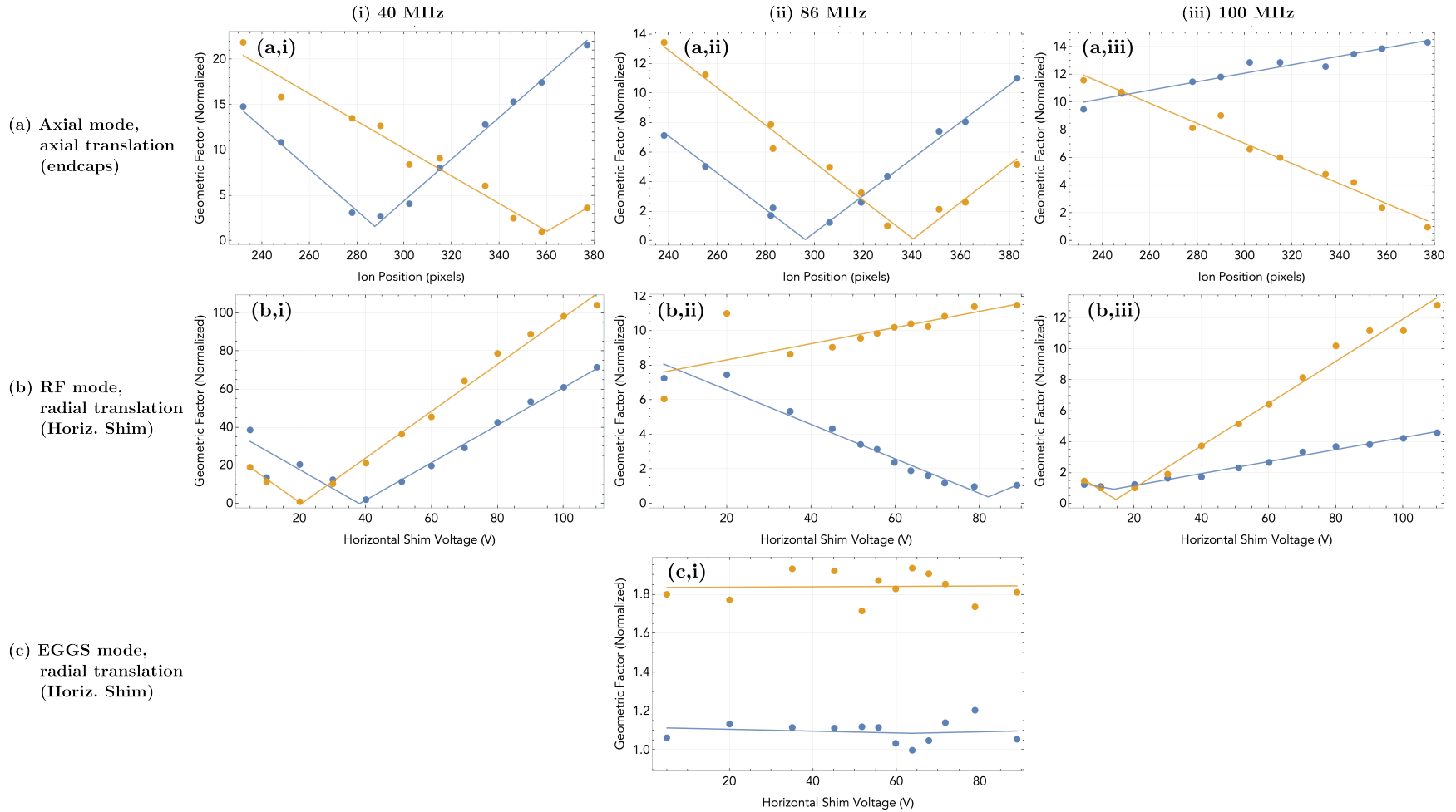


Figure 7.10: **Spatial characterization of the field gradients from the EGGS drive at 40, 86, and 100 MHz.** Geometric coupling factor β of the EGGS CH0 (blue) and CH1 (orange) electrodes are extracted from the QVSA excitation strength as the ion is translated through the trap. At each point, a two-tone QVSA interaction with equal tone voltages is applied and a linescan yields the peak phonon number (eq. (8.9)); each channel's coupling is modeled as a geometric factor linear in voltage, i.e. $V_q, V_d \rightarrow \beta V_q, \beta V_d$, and normalized to the dataset minimum. Each curve is fitted to $\beta = k|x - c|$ (solid lines); the combined null lies at the intersection of the curves and is not necessarily coincident with the trap RF null nor accessible within the shim range, and shifts markedly across 40 MHz, 86 MHz and 100 MHz. **(a)** Probing the axial mode under axial translation. **(b, c)** Probing the RF and EGGS modes under horizontal translation via the horizontal shim, which displaces the ion along both modes as they lie at an angle to the horizontal plane. See fig. 6.2 for the electrode layout. Note that the horizontal axis differs between datasets: axially 1 pix $\approx$ 1 μ m, radially 110 V $\approx$ 7 μ m. Further details are found in the text.

7.6 Searching for EGGS⁹

7.6.1 Search Protocol

Throughout 2024, the bulk of the EGGS QLS search was conducted in two bands: the 80 MHz to 90 MHz band, where almost all $\Delta F = 0$ transitions are clustered, and the 510 MHz to 520 MHz band of $F = 2, I = \pm 1/2 \leftrightarrow F = 3, I = \pm 1/2$ transitions, chosen due to the suppression of the QVSA background at high frequency. The search used the bichromatic heating protocol of section 7.3.2, with a dynamically-decoupled carrier (section 7.3.3) to broaden the effective linewidth and periodic π phase shifts (section 7.4.2) to suppress the QVSA background. The predicted spectrum for the 83 MHz band is shown in Figure 7.11, calculated for typical search parameters and without the phase-shift reversals.

In general, peak detection was a rare event, occurring on the order of once every few days. To avoid missing a transition due to redistribution between samples, the range was scanned progressively from left to right, with all averages completed at each frequency before advancing. To distinguish detected peaks from the background, the following diagnostics were applied: shimming the transition off resonance (see section 7.4.3); exchanging the quadrupolar and dipolar field gradients (to which the QVSA background is insensitive); and varying the strength of the dipolar dynamical-decoupling carrier. In practice, peaks did not persist sufficiently long (typically 2 min to 3 min) for any of these tests to be conclusive.

Averaging across the many datasets to improve the Signal-to-Noise Ratio was challenged by redistribution. As the molecule is continually redistributed, the bulk of datasets will be noise, creating a prohibitive noise floor — this is compounded by the contribution of the hydrogen nuclear spin to the H^{35}Cl^+ structure, as the ion is in a different manifold for roughly half of all samples (see Figure 7.3).

⁹or “Iacta Alea EGGS.”

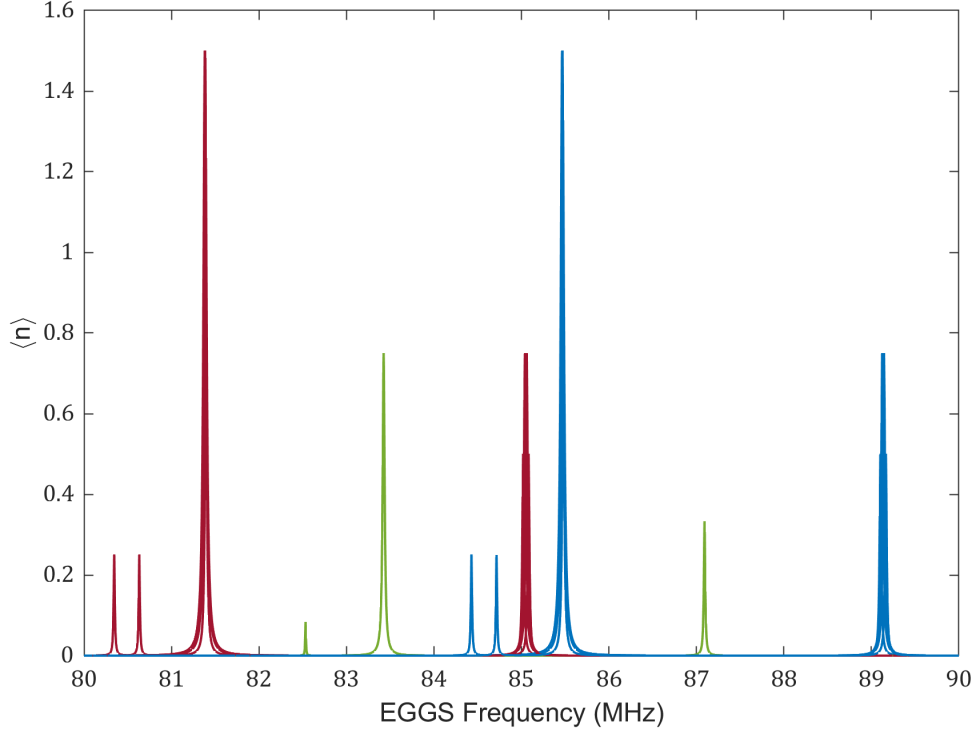


Figure 7.11: **Predicted EGGS spectrum in the 80 MHz to 90 MHz band.**

Bichromatic EGGS is applied with $\eta\Omega_{\text{EGGS}} = 190 \times 2\pi$ Hz and a carrier tone with $\Omega_0 = 10 \times 2\pi$ kHz for [Dynamical Decoupling](#); a bias field $B = 3.631\,97(2)$ G is used to calculate the Zeeman splittings. Transition dipole moments were calculated in PGOPHER by [HW](#); see [GDM's thesis](#) for further details. $\Delta F = -1, 0, +1$ transitions are shown in red, green, and blue, respectively.

7.6.2 Search Results

The most promising peaks recovered from several hundred hours of data are shown in [Figure 7.12](#). Though these peaks are close to the predicted transition frequencies, they present several inconsistencies: the measured peak separations of [fig. 7.12](#) are incompatible with the expected Zeeman splittings in [fig. 7.11](#), and the peaks recur more rarely than expected from the predicted 5 min to 20 min [Black-Body Radiation](#)-redistribution lifetime (section 7.4.1). The 518 MHz-band peaks exhibit further disagreement with the larger expected peak strength and linewidth (larger voltages are used for the 518 MHz band due to the reduced [QVSA](#) background).

If the veracity of these peaks is to be believed, these inconsistencies then challenge our

understanding of our apparatus. Their scarcity requires the ion to redistribute far faster than predicted, requiring either blackbody radiation in excess of that measured by the temperature diodes on the trap, or a different mechanism of redistribution altogether. A potential mechanism is incomplete control over the field gradients, which could leave a large residual dipole field capable of pumping the ion into other manifolds — this is likely due to poor characterization of the trap field gradients at the time of the search, and could further explain the disagreement between the measured and expected peak heights and linewidths.

It was found that the peaks appeared even when scanning a “control” band (i.e. a frequency range outside the potential H^{35}Cl^+ transition range), which suggests another explanation for the peaks as QVSA background converted into spurs by drift in the secular frequency (see Figure 7.13). If the QVSA background is sufficiently detuned, then the periodic phase-shifts used for background removal instead accumulate the background excitation, appearing as a peak. For small secular frequency errors, the background excitation sits at a sharp slope and can thus itself be negligible while leaving it susceptible to outsize amplification with further drifts in the secular frequency (Figure 7.13(d)).

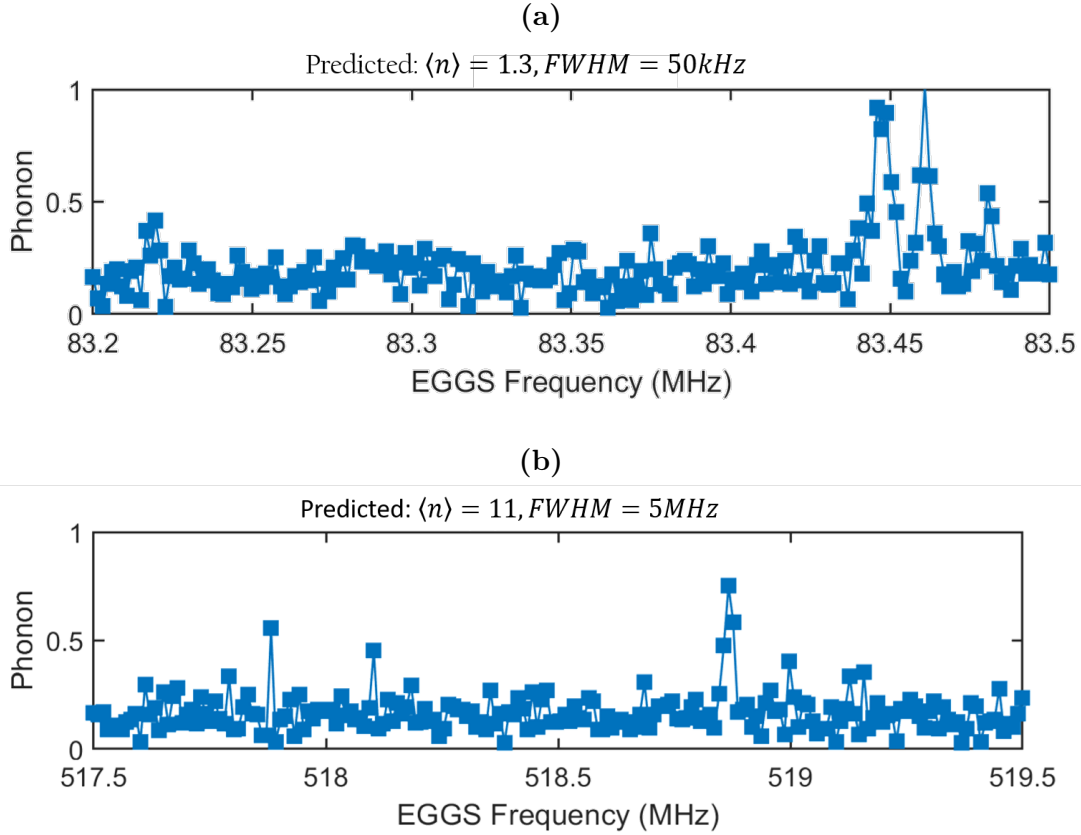


Figure 7.12: **Peak data from the EGGS QLS search.**

The most promising EGGS QLS signals recovered from several hundred hours of search data, annotated with predicted peak $\langle n \rangle$ and FWHM linewidths for the search parameters. **(a)** 83 MHz-band peaks near 83.46 MHz. The peak frequency suggests excitation of the $\Delta F = 0, \Delta m_F = 0$ transitions within the $F = 3$ manifold; the peak separation suggests excitation of the $\Delta F = 0, \Delta m_F = -1$ transitions within the $F = 2$ manifold. **(b)** 518 MHz-band peaks near 519 MHz. Excitation at higher frequencies enables use of larger drive voltages due to reduced QVSA background. The peak frequency suggests excitation of a $|F = 2, m_F\rangle \leftrightarrow |F = 3, m_F - 1\rangle$ transition. The peak identities inferred from the transition frequencies show inconsistencies: the peak separations are incompatible with expected Zeeman splittings, the peak recurrence rates are far below the 5 min to 20 min redistribution lifetime, and the measured strengths and widths are well below the annotated predictions.

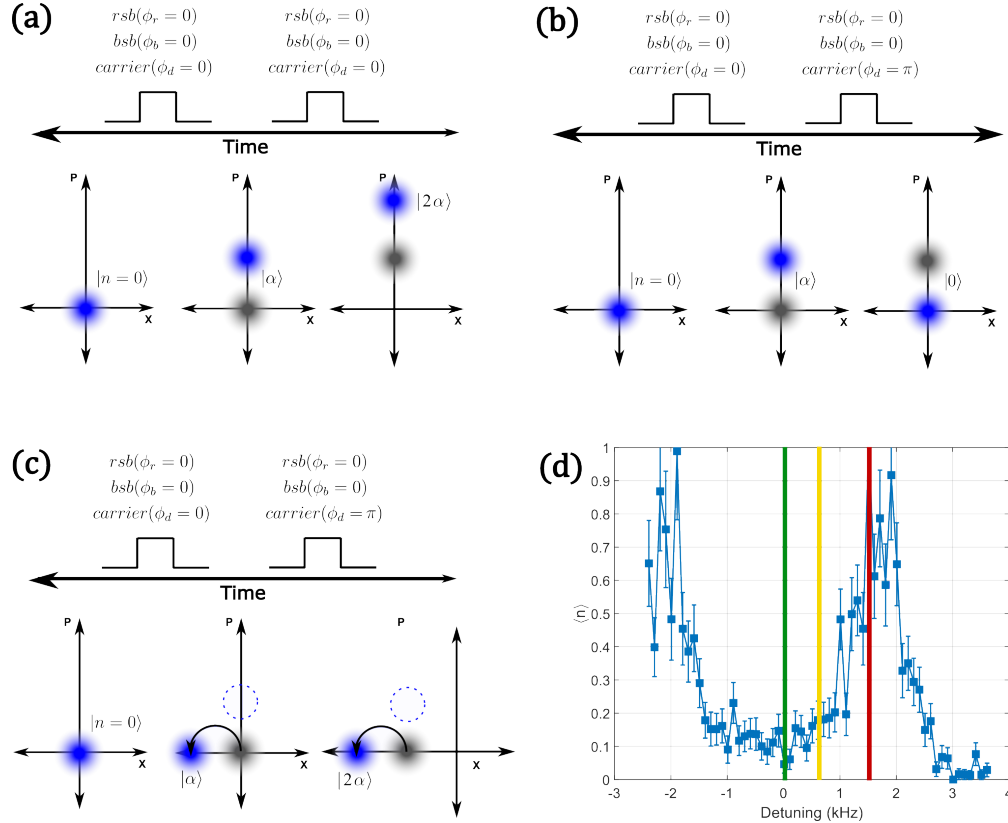


Figure 7.13: **QVSA** background suppression converts secular frequency errors into spurious EGGS peaks.

(a) The carrier dipole tone required for dynamical decoupling drives an unwanted motional Raman (**QVSA**) displacement, which, without phase reversal, accumulates over time. (b) Ideally, reversal of the dipole phase by π halfway through the sequence reverses the **QVSA** displacement and suppresses the background. (c) Off-resonance, the phase-space trajectory is curved and the displacement instead accumulates with the phase-reversal. (a-c) Current (previous) phase-space positions are shown in blue (grey). (d) Measured **QVSA** background versus secular frequency detuning. Background negligible at small drift (yellow) is susceptible to outsize amplification under further drift (red); ideal, resonant behavior is denoted by the green line.

Chapter 8

Motional Raman

Abstract

Ultrasensitive detection of the frequency, phase, and amplitude of radio frequency electric fields is central for applications in radio communication, cosmology, dark matter searches, and high-fidelity qubit control. Quantum harmonic oscillator systems, especially trapped ions, offer high-sensitivity electric field sensing with nanometer spatial resolution, but are typically restricted to narrow frequency ranges centered around the motional frequency of the trapped ion oscillator or the frequency of an optical transition in the ion. We present a procedure enabling precise electric field detection over an expanded frequency range. Specifically, we use motional Raman transitions in a single trapped ion to achieve sensitivity across a frequency range over 800 times larger than previous approaches. We show that the method is compatible with both quantum amplification via squeezing and measurement in the Fock basis, allowing a demonstration of performance 3.4(20) dB below the standard quantum limit. The approach can be extended to other quantum harmonic oscillator systems, such as superconducting qubit-resonator systems.

Contributions [HW](#) and [ERH](#) conceived the experiments. [HW](#), [GDM](#), and [CZCH](#) built and maintained the experimental setup. [HW](#), [GDM](#), [CZCH](#) and [JAR](#) carried out the

measurements. [ERH](#) supervised all work. All authors discussed the results and contributed to the manuscript. We thank [SCB](#), [PSH](#), and [WCC](#) for helpful discussion. This work was supported by NSF (PHY-2110421 and OMA-2016245), AFOSR (130427-5114546), and ARO (W911NF-19-1-0297).

8.1 Introduction

A Survey of Electric Field Sensing

Well-controlled quantum systems offer quantum-limited metrology capabilities [[GLM04](#)] [[DRC17](#),[Pez+18](#)]. Quantum harmonic oscillators (QHOs) controlled at the single-quanta level provide excellent platforms for high-sensitivity detection of radio frequency (RF) electric fields. For a single QHO mode, non-classical Fock states provide optimal precision and sensitivity for phase-insensitive measurements of displacement, and were implemented with trapped ions to measure the amplitude [[Wol+19](#)] and frequency [[Wol+19](#),[McC+19a](#)] of an oscillator below the standard quantum limit (SQL). Additionally, a Fock state with up to $N = 100$ quanta was used with superconducting microwave cavities to demonstrate displacement and oscillator frequency sensitivity approaching the Heisenberg limit [[Den+24](#)].

Squeezing to achieve “quantum amplification” has enhanced the sensitivity of a QHO to resonant forces by an order of magnitude [[Bur+19](#)]. Additionally, measurements entangling collective spin states of trapped ions with a shared motional mode have shown sensitivity to displacements ≈ 9 dB below the SQL [[Gil+21](#)]. Similarly, NOON states have facilitated frequency sensitivities near the Heisenberg limit [[Zha+18](#)], while important work with motional cat states [[Mil+21](#)] and amplitude-modulated displacements of Fock states [[Kel+21](#)] have been used for oscillator noise spectroscopy from near DC to the oscillator’s natural frequency. Meanwhile, to achieve highly sensitive off-resonant measurements, motional oscillations detuned from the oscillator’s natural frequency by $\sim$ MHz have been mapped onto the internal spin through spin-dependent optical dipole forces (ODF) to detect displacements 40 times

smaller than the zero-point fluctuations of the oscillator [Gil+17]. Stabilization of phase-coherence between the displacement and ODF, can provide further enhancement by an order of magnitude [Aff+20].

The Narrowband Status Quo

Despite these state-of-the-art sensitivities [Gil+21], outperforming those achieved on other platforms such as NV centers [Sch+14b,Qiu+22] and Rydberg atoms [Liu+22,Hol+22], challenges remain. First, these techniques are often insensitive to the phase of the oscillator’s displacement. Techniques with phase sensitivity, such as the phase-sensitive red sideband (PSRSB) method [Bur+19,Hem+13], phase-dependent energy shift induced by spin-dependent ODF [Aff+20], and quantum amplification methods [Bur+19], require an RF electric field frequency that is near either an internal optical transition or the oscillator’s motional frequency, respectively. Implementations of the PSRSB method or phase-dependent energy shift with optical fields are compromised by difficult-to-control optical phase noise [Day+22], though this is overcome on dedicated platforms using microwave-field gradients [Bur+24], while the increased decoherence associated with quantum amplification limits interrogation times and degrade the achievable sensitivities [Bur+19]. Second, even for phase-insensitive techniques, the accessible frequency range is limited near the oscillator’s natural frequency, with achieved bandwidths of only several MHz.

A Wideband Post Bellum

Here, we introduce a phase-sensitive heterodyne detection scheme constructed from the interference of multiple motional Raman transitions that provides the ability to “continuously” sense the amplitude, frequency, and phase of an unknown field across a wide band with high precision. In what follows, we describe the underlying interaction and operation of a wideband quantum vector signal analyzer (QVSA) based on a single trapped ion. We benchmark the method by measuring the known transfer function of a commercial filter and

demonstrate *in situ* calibration of a qubit control line. Further, we characterize the sensitivity and precision of the QVSA at 82 MHz for use in future molecular ion qubit gates [HC21] and measure an amplitude sensitivity (without the use of a pre-amplifier) of $51(10) \mu\text{V}/\sqrt{\text{Hz}}$, corresponding to an electric field sensitivity of $66(13) \text{ mV m}^{-1}/\sqrt{\text{Hz}}$ with a minimum detectable voltage of $3.8(8) \mu\text{V}$ ($4.9(1.0) \text{ mV/m}$); a frequency sensitivity of $39(9) \text{ Hz}/\sqrt{\text{Hz}}$ with minimum resolution $3.8(1.6) \text{ Hz}$; and a phase sensitivity of $117(26) \text{ mrad}/\sqrt{\text{Hz}}$ with minimum phase resolution $9.1(2.0) \text{ mrad}$. We show compatibility with quantum amplification [Bur+19,Bur+21,Bur+24,Ge+19b] and use motional squeezing for a $9.0(4) \text{ dB}$ gain in voltage sensitivity and achieve an amplitude sensitivity $3.4(2.0) \text{ dB}$ below the SQL. Finally, we demonstrate the technique across the low-frequency RF to the UHF microwave frequency bands (100 kHz to 1 GHz), a range limited only by our synthesis capabilities, compare our electric field sensitivity to other platforms, and discuss straightforward modifications for improving the technique. This advancement paves the way for utilizing QHOs for wideband electric field sensing.

We demonstrate this technique using the $\omega \approx 2\pi \cdot 1 \text{ MHz}$ radial motional mode of a single $^{40}\text{Ca}^+$ ion in a cryogenic (4.5 K) linear Paul trap with an ion-electrode distance $r_o = 550 \mu\text{m}$. The QHO is initialized and read out via a Jaynes-Cummings Hamiltonian to the optical $\{|^2D_{\frac{5}{2}}, m_J = -\frac{5}{2}\rangle, |^2S_{\frac{1}{2}}, m_J = -\frac{1}{2}\rangle\}$ qubit manifold of $^{40}\text{Ca}^+$. Doppler cooling of the $^{40}\text{Ca}^+$ ion followed by resolved-sideband cooling on the optical qubit transition [Roo+99,Lei+03] prepares the motional ground state with $\langle n \rangle = 0.01(1)$.

8.2 Motional Raman Theory

We first provide a concise overview of Motional Raman sufficient to understand the key results of this chapter, followed by a more complete treatment of the theory.

Overview To understand the operation of the QVSA, suppose an unknown oscillatory signal with voltage V_d at frequency ω_d and phase ϕ_d is applied to the trap in a dipole

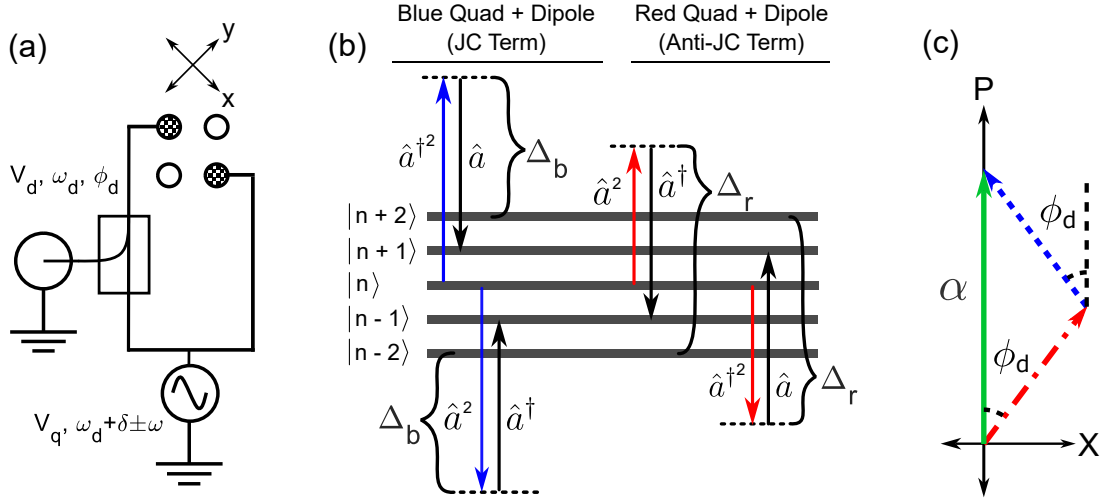


Figure 8.1: **Implementation of the quantum vector signal analyzer (QVSA).**

(a) Schematic of the **QVSA** drive coupling to the quadrupole ion trap. Two quadrupole tones ($\omega_d + \delta \pm \omega$) are coupled onto the electrodes to characterize an unknown dipole tone (ω_d). The dipole tone is easily supplied using either a BNC connector or a microwave horn. (b) Level diagrams for motional Raman transitions. The blue (red) arrow indicates the blue (red) quadrupole field, while the black arrow represents the dipole field. The combination of the blue quadrupole ($\omega_d + \delta + \omega$) and dipole fields results in displacement following the **Jaynes-Cummings Hamiltonian (JCH)**, whereas the combination of the red quadrupole ($\omega_d + \delta - \omega$) and dipole fields results in displacement according to the anti-JCH. $\Delta_b = \omega_d - \omega$; $\Delta_r = \omega_d + \omega$; (c) Phase space diagram showing that the phase sensitivity of the **QVSA** displacement α arises from the interference between the displacements generated by the multiple Raman transitions; each transition is excited by one quadrupole and one dipole tone. We assume the initial phase of the quadrupole tones to be zero.

configuration (the dipole tone), while two voltages at V_q and frequencies $\omega_d + \delta \pm \omega$ are applied in a quadrupole configuration (the quadrupole tones), where ω is the ion's motional mode frequency and δ is the frequency detuning as shown in Figure 8.1(a). We define the initial phase of the quadrupole tones as zero. The Hamiltonian of the ion motion is then:

$$\begin{aligned}
 \frac{\hat{H}}{\hbar} = & \omega \hat{a}^\dagger \hat{a} + \frac{\Omega_q}{2} (\hat{a} + \hat{a}^\dagger)^2 \cos((\omega_d + \delta + \omega)t) \\
 & + \frac{\Omega_q}{2} (\hat{a} + \hat{a}^\dagger)^2 \cos((\omega_d + \delta - \omega)t) \\
 & + \Omega_d (\hat{a} + \hat{a}^\dagger) \cos(\omega_d t + \phi_d),
 \end{aligned} \tag{8.1}$$

where $\Omega_q = eV_q x_o^2 / (\hbar r_o^2)$, $\Omega_d = \kappa eV_d x_o / (\hbar r_o)$, r_o is the ion-electrode distance, $x_o = \sqrt{\hbar / (2m\omega)}$

is the zero-point wavefunction of the QHO, and κ is a mode-dependent geometrical factor.

The resulting evolution can be understood by noticing that the dipole tone off-resonantly drives $\Delta n = \pm 1$ transitions, while the quadrupole tones off-resonantly drive $\Delta n = \pm 2$ transitions (Figure 8.1(b)). For $\delta < \Omega_q \Omega_d / (\omega_d - 2\omega)$, the dipole tone interacts with either quadrupole tone to resonantly drive a three-phonon Raman transition, resulting in an overall $\Delta n = \pm 1$. Undesirable off-resonant interactions can be neglected in the regime $\omega_d, \omega, |\omega_d - \omega| \gg \delta$, or alternatively suppressed by pulse-shaping the applied tones [Zar+19]. Formally, the dominant dynamics are given by the second-order Magnus expansion, which yields a displacement operator with displacement

$$\alpha = i\Omega_q \Omega_d \frac{\omega}{\omega_d^2 - \omega^2} \frac{\sin(\delta t - \phi_d) + \sin(\phi_d)}{\delta}, \quad (8.2)$$

which depends on the amplitude V_d , frequency ω_d , interaction time t , and phase ϕ_d of the dipole tone.

The dependence on ϕ_d is surprising. Typically, the drive phase ϕ_d dictates the phase-space quadrature of the displacement and not its magnitude; this underlies the challenge of phase-sensitive measurement in QHO systems. Here, phase sensitivity arises from interference between the displacement generated by the dipole and the red quadrupole tone, and the displacement generated by the dipole and the blue quadrupole tones (Figure 8.1(c)). If phase sensitivity is undesirable, like with shot-to-shot dipole phase fluctuations, the electric field can still be measured by applying only one quadrupole tone.

8.2.1 The QVSA Propagator

Suppose an unknown, oscillatory signal at frequency ω_d is applied to the trap in a dipole configuration while two signals at frequencies at $\omega_d + \delta \pm \omega$ are applied to the trap in a quadrupole configuration, as shown in Figure 8.1(a). We assume $\omega_d, \omega, |\omega_d - \omega| \gg \delta$ to avoid direct “tickling” at the secular frequency. The motion of the ion can be described by the

Hamiltonian:

$$\hat{H}/\hbar = \omega \hat{a}^\dagger \hat{a} + \Omega_q (\hat{a} + \hat{a}^\dagger)^2 \cos((\omega_d + \delta)t) \cos(\omega t) + \Omega_d (\hat{a} + \hat{a}^\dagger) \cos(\omega_d t + \phi_d), \quad (8.3)$$

where $\Omega_q = eV_q x_o^2 / (\hbar r_o^2)$, $\Omega_d = \kappa eV_d x_o / (\hbar r_o)$, r_o is the ion-electrode distance, $x_o = \sqrt{\frac{\hbar}{2m\omega}}$ is the length scale of the QHO, and κ is a geometrical factor. In the interaction picture with respect to the harmonic oscillator we have:

$$\begin{aligned} \hat{H}_I/\hbar = & \Omega_q \left(\hat{a}^2 e^{-2i\omega t} + 2 \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \hat{a}^{\dagger 2} e^{2i\omega t} \right) \cos((\omega_d + \delta)t) \cos(\omega t) \\ & + \Omega_d (\hat{a} e^{-i\omega t} + \hat{a}^\dagger e^{i\omega t}) \cos(\omega_d t + \phi_d). \end{aligned} \quad (8.4)$$

The propagator is evaluated using the Magnus expansion $\hat{U} = e^{\sum_i \hat{A}_i}$. The first order Magnus term $\hat{A}_1$ consists of off-resonant terms which we ignore as they do not affect the long term dynamics, which can also be fixed by pulse shaping [Zar+19]. The second order Magnus term is given as:

$$\hat{A}_2 = \frac{1}{2} \left(-\frac{i}{\hbar} \right)^2 \int_0^t \int_0^{t'} [\hat{H}(t'), \hat{H}(t'')] dt'' dt'. \quad (8.5)$$

We focus on terms that lead to displacements (see sections 8.2.1 and 8.2.3 for a more complete treatment). We also disregard the secular frequency shift terms arising from the second-order Magnus expansion. These shifts are typically negligible and can be corrected by remeasuring the secular frequency in the presence of the QVSA field. This leaves only the terms that arise from the dipole-quadrupole interaction, which are responsible for the primary dynamics of interest.

After applying a [RWA](#) to drop terms that oscillate at 2ω and $2\omega_d$ and evaluating the time integrals, the propagator is evaluated as:

$$\hat{U} = \exp \left[i\Omega_q \Omega_d \frac{\omega}{\omega_d^2 - \omega^2} \frac{\sin(\delta t - \phi_d) + \sin(\phi_d)}{\delta} (\hat{a} + \hat{a}^\dagger) \right] = \hat{D}(\alpha), \quad (8.6)$$

which is the form of a displacement operator with $\alpha \equiv i\Omega_q\Omega_d\frac{\omega}{\omega_d^2-\omega^2}\frac{\sin(\delta t-\phi_d)+\sin(\phi_d)}{\delta}$. In the $\delta t \rightarrow 0$ limit, eq. (8.6) reduces to:

$$\alpha = i\Omega_q\Omega_d\frac{\omega}{\omega_d^2-\omega^2}t\cos(\phi_d). \quad (8.7)$$

8.2.2 The QVSA Propagator with Phase Delay

Defining the RSB (BSB) phase as ϕ_r (ϕ_b), the Hamiltonian of the ion motion is:

$$\begin{aligned} \hat{H}/\hbar = & \omega\hat{a}^\dagger\hat{a} + \Omega_d(\hat{a} + \hat{a}^\dagger)\cos(\omega_d t + \phi_d) \\ & + \frac{\Omega_q}{2}(\hat{a} + \hat{a}^\dagger)^2(\cos((\omega_d + \delta + \omega)t + \phi_b) + \cos((\omega_d + \delta - \omega)t + \phi_r)). \end{aligned} \quad (8.8)$$

As in section 8.2.1, the resulting propagator is a displacement operator with:

$$\alpha = i\Omega_q\Omega_d\frac{\omega}{\omega_d^2-\omega^2}e^{i\phi_-}\frac{\sin(\delta t - \phi_d + \phi_+) + \sin(\phi_d - \phi_+)}{\delta}. \quad (8.9)$$

where $\phi_\pm = \frac{\phi_b \pm \phi_r}{2}$ (cf. eq. (8.6)). Assuming $\delta t \ll 1$, this becomes

$$\alpha = i\Omega_q\Omega_d\frac{\omega t}{\omega_d^2-\omega^2}\frac{e^{i(\phi_d-\phi_r)} + e^{-i(\phi_d-\phi_b)}}{2}. \quad (8.10)$$

The first (second) exponential in eq. (8.10) is the phase term corresponding to the red (blue) sideband tone. Setting $\phi_r = \phi_b = 0$ motivates the choice of phase for Figure 8.1(c).

As the frequencies of the quadrupole tones are scanned across the dipole frequency, three peaks are observed (Figure 8.2). Two peaks arise at $\delta \approx \pm 2\omega$ from a phase-independent interaction between the dipole tone and each quadrupole tone individually, while a central peak at $\delta \approx 0$ arises from the phase-dependent interaction between the dipole and both quadrupole tones (Figure 8.3). While scalar spectrum analysis is therefore possible with only a single sideband tone, *vector* signal analysis requires the presence of all three tones to yield

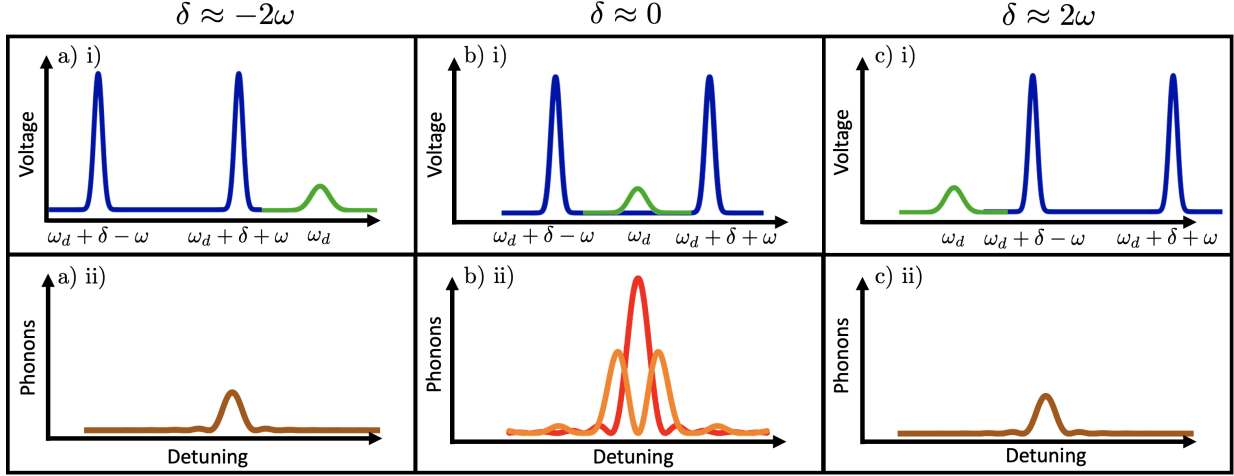


Figure 8.2: **Illustration of the spectrum sweep procedure** (see section 8.2.2).

(a-c, i) Two quadrupole tones (blue) are swept in frequency across a weaker dipole tone (green). (a-c, ii) The resulting mean phonon number $\langle n \rangle$ produced from the corresponding interaction in (i). (a, i-ii) The interaction is primarily determined by that between the blue quadrupole tone at $\omega_d + \delta + \omega$ and the dipole tone at ω_d . This interaction is phase-independent. (b, i-ii) Both quadrupole tones at $\omega_d + \delta \pm \omega$ interact with the dipole tone at ω_d . The resulting interaction is phase-dependent, and can produce a resonance (red) when the phase relation satisfies $\phi_d - \phi_+ = 0$, where $\phi_+ = \frac{\phi_b + \phi_r}{2}$, or an antiresonance (orange) when $\phi_d - \phi_+ = \frac{\pi}{2}$. (c, i-ii) Conversely to (a), the interaction is primarily determined by that between the red quadrupole tone at $\omega_d + \delta - \omega$ and the dipole tone at ω_d , and is phase-independent.

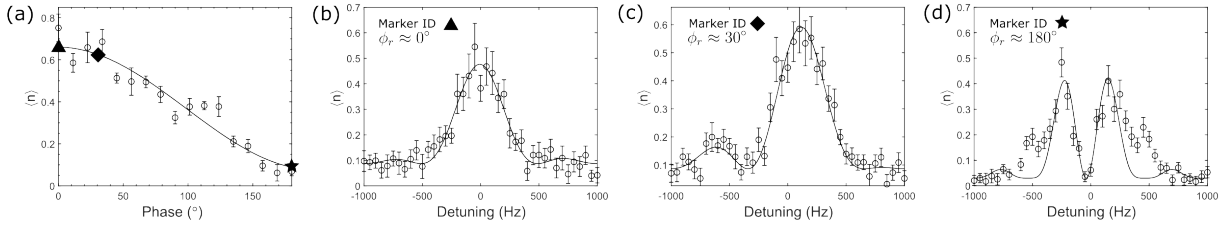


Figure 8.3: **Phase dependence of the spectrum sweep procedure.**

The phase relation between the QVSA tones fundamentally changes the lineshape of the resonance. By sweeping just ϕ_r , the phase dependence is reduced to $\langle n \rangle \propto \cos^2(\frac{\phi_r}{2})$. (a) shows a phase sweep at the resonance (i.e. $\delta = 0$), with three characteristic phases marked by a triangle (b), a diamond (c), and a star (d). Voltage parameters vary between marked points. (d) illustrates the production of an antiresonance, which occurs when the red sideband is completely out of phase with the blue sideband (see Figure 8.1(c) for a graphical representation). The amplitude discrepancy between (a) and (b-d) is due to a difference in the applied QVSA voltages. Error bars represent one standard error. The solid lines are fitted lines using eq. (8.9).

a phase dependence, and has the added benefit of a stronger interaction.

8.2.3 Intrinsic Squeezing

Consider the effect of the two quadrupole tones in isolation, i.e. $V_d = 0$. Without loss of generality, $\phi_r = \phi_b = 0$. The interaction Hamiltonian of the ion motion is then (cf. eq. (8.4)):

$$\hat{H}_I/\hbar = \Omega_q \left(\hat{a}^2 e^{-2i\omega t} + 2 \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \hat{a}^{\dagger 2} e^{2i\omega t} \right) \cos((\omega_d + \delta)t) \cos(\omega t). \quad (8.11)$$

As in section 8.2.1, we consider only the second-order Magnus term and apply a [Rotating Wave Approximation \(RWA\)](#) to find the propagator:

$$\hat{U} \approx \exp \left[\frac{i\Omega_q^2 t}{2} \left(\frac{\omega}{\omega_d^2 - \omega^2} \right) (\hat{a}^2 + \hat{a}^{\dagger 2}) \right] = \hat{S}(z), \quad (8.12)$$

which is the form of a squeezing operator with $z \equiv -i\Omega_q^2 \left(\frac{\omega}{\omega_d^2 - \omega^2} \right) t$.

We now consider the interaction of all three tones in full. Combination of eq. (8.6) and eq. (8.12) gives the second order Magnus term:

$$\hat{A}_2 = (\alpha \hat{a}^\dagger - \alpha^* \hat{a}) + \frac{1}{2} (z^* \hat{a}^2 - z \hat{a}^{\dagger 2}), \quad (8.13)$$

where α is defined in eq. (8.7). Using the Zassenhaus relation, the resulting propagator is:

$$\hat{U} = \hat{S}(z) \hat{D} \left(\sum_{j=1}^{\infty} \frac{i^j (-1)^{j+1} |\alpha| |z|^{j-1}}{j!} \right) = \hat{S}(z) \hat{D}(\alpha'), \quad (8.14)$$

$$\text{where } \alpha' \equiv \left| \frac{\alpha}{z} \right| (i \sinh |z| - \cosh |z| + 1). \quad (8.15)$$

The complete [QVSA](#) interaction thus consists of two parts: a displacement operator $\hat{D}(\alpha')$ and a residual squeezing operator $\hat{S}(z)$.

The resulting displacement α' is larger than that calculated in eq. (8.7) due to the intrinsic squeezing generated by the quadrupole tones (see eq. (8.12)), an effect we refer to as *intrinsic amplification*. For small quadrupole voltages, i.e. as $\Omega_q/\omega_q \rightarrow 0$, the squeezing parameter

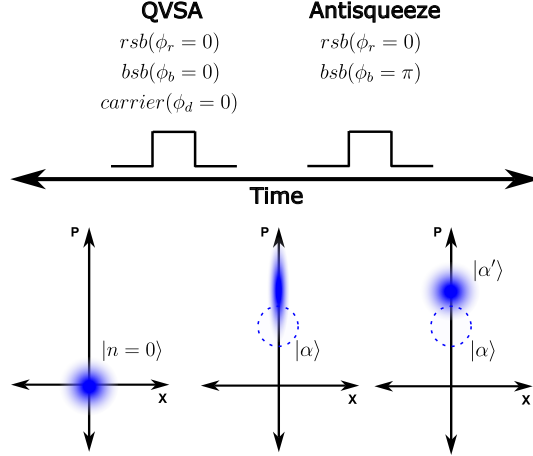


Figure 8.4: **Pulse sequence for intrinsic amplification and associated Wigner functions.**

The dashed lines on the bottom figures depict a pure displacement to $|\alpha\rangle$ as would be expected. The **QVSA** pulse is comprised of a displacement operation followed by a squeeze operation, as illustrated by the colored ellipse. To convert from a squeezed state back to a coherent state $|\alpha'\rangle$ we antisqueeze by shifting the blue sideband phase by π .

becomes negligible, i.e. $z \rightarrow 0$, and thus $\alpha' \rightarrow \alpha$, reproducing eq. (8.7) and eq. (8.6). The residual squeezing results in an undesirable increase in the measurement background, which becomes particularly pronounced for longer measurement times. However, this residual squeezing can be negated by application of an antisqueezing operation $\hat{S}^\dagger(z)$ following the **QVSA** interaction (*intrinsic antisqueezing*). This can be straightforwardly implemented by disabling the dipole tone and phase-shifting a single quadrupole tone by π , as shown in Figure 8.4.

8.3 Experimental Implementation

8.3.1 QVSA Sequence

Each QVSA experiment is comprised of four steps. A sample pulse sequence including quantum amplification is shown in Figure 8.5.

First, the ion is prepared in the motional ground state via sideband cooling. Second,

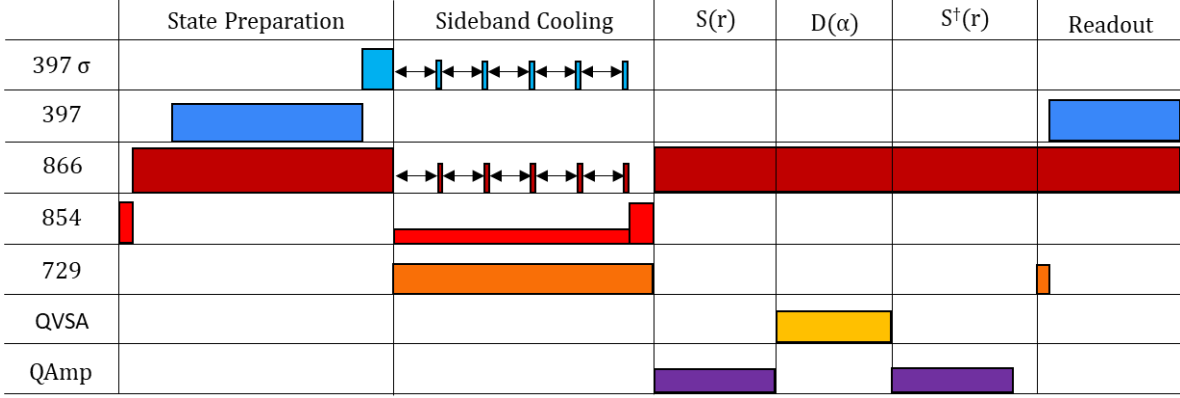


Figure 8.5: **Pulse sequence for the QVSA with Quantum Amplification.**

The ion is Doppler cooled and optically pumped into the $|^2S_{\frac{1}{2}}, m_J = -\frac{1}{2}\rangle$ state. Continuous sideband cooling subsequently prepares the ion near the ground motional state $|n = 0\rangle$. Quantum Amplification is achieved by preparing the ion in a squeezed state before applying the QVSA pulse, which amplifies the displacement and minimizes uncertainty along the squeezing axis. The squeezing is then reversed via an anti-squeezing pulse, returning the ion to a coherent state.

the QVSA interaction is applied to induce a displacement α . Third, the motional state is projected onto the $\{S, D\}$ spin manifold via excitation on the first red/blue motional sideband for some given time t . This procedure can be expressed as a unitary operation $\hat{U}_{r/b}$. For sideband ratio measurements, we choose $t = t_\pi = \pi/\Omega_{01}$, where Ω_{01} is the Rabi frequency for the $|S, n = 0\rangle$ to $|D, n = 1\rangle$ transition. The spin state can in turn be read out via state-dependent fluorescence, which are described by the projection operators $\{\hat{\Pi}_\mu\}$ where $\mu \in \{0 \text{ (dark)}, 1 \text{ (bright)}\}$. The final D-state probability can thus be expressed as $P_{r/b} = P_{r/b}(\mu|\alpha) = \text{Tr}\{\hat{\Pi}_\mu \hat{U}_{r/b}^\dagger \hat{\rho}(\alpha) \hat{U}_{r/b}\}$. Fourth, the ratio of the red and blue sideband D-state populations ($R = P_r/P_b$) is converted to a displacement and a mean phonon number at different detunings of the quadrupole tones (relative to the dipole tone), assuming a coherent state. The resulting profile is fitted with a sinc^2 function to extract the displacement. However, for Allan deviations, we instead measure only a single point at the secular frequency resonance.

8.3.2 Signal Generation

Signals for the QVSA interaction are generated using the Sinara “Phaser” [Arbitrary Waveform Generator \(AWG\)](#). The [AWG](#) has two coherent output channels which are used to separately address a pair of quadrupolar trap electrodes (Figure 8.6). Dipole tones (i.e. $\hat{x}$ terms) and quadrupole tones (i.e. $\hat{x}^2$ terms) can be simply generated from the same AWG by setting an inter-channel phase delay of π and 0, respectively.

In practice, the QVSA signals are applied to excite motion *perpendicular* to the QVSA electrodes (i.e. along the y mode), which is made possible due to imperfections in trap geometry, though the effective dipole voltage is reduced by an experimentally determined geometric factor $\kappa = 1/42$. This is desirable since the x-mode experiences a higher motional heating rate as a result of the electronics connected to the [QVSA](#) electrodes.

8.3.3 Calibration Procedures

Phase delays and latencies between the signals applied to the QVSA electrodes change the multipole moments of the field and can thus reduce the QVSA interaction strength and the motional coherence time. However, these can be simply calibrated by sweeping the inter-electrode phase delay at the RF source to maximize the resulting displacement. Similarly, the QVSA signals can be aligned in phase space (Figure 8.1(c)) by sweeping the phase of a single quadrupole tone (e.g. ϕ_r , the phase of the red quadrupole tone) to maximize the resulting displacement. From eq. (8.9), maximal displacement occurs when $\phi_d = \phi_+ = \frac{\phi_r + \phi_b}{2}$, where ϕ_r and ϕ_b are the red and blue quadrupole phases, respectively. Calibration of the quadrupole amplitudes is achieved by applying a dipole tone together with a single quadrupole tone and equalizing the resulting displacement (Figure 8.2(a, c)). The quadrupole voltages can in turn be determined absolutely by applying the two quadrupole tones at $\omega_d \pm \omega$ without the dipole tone to generate a motional squeezed state (see section 8.2.3), from which the squeezing coupling strength (and thus the voltage) can be extracted.

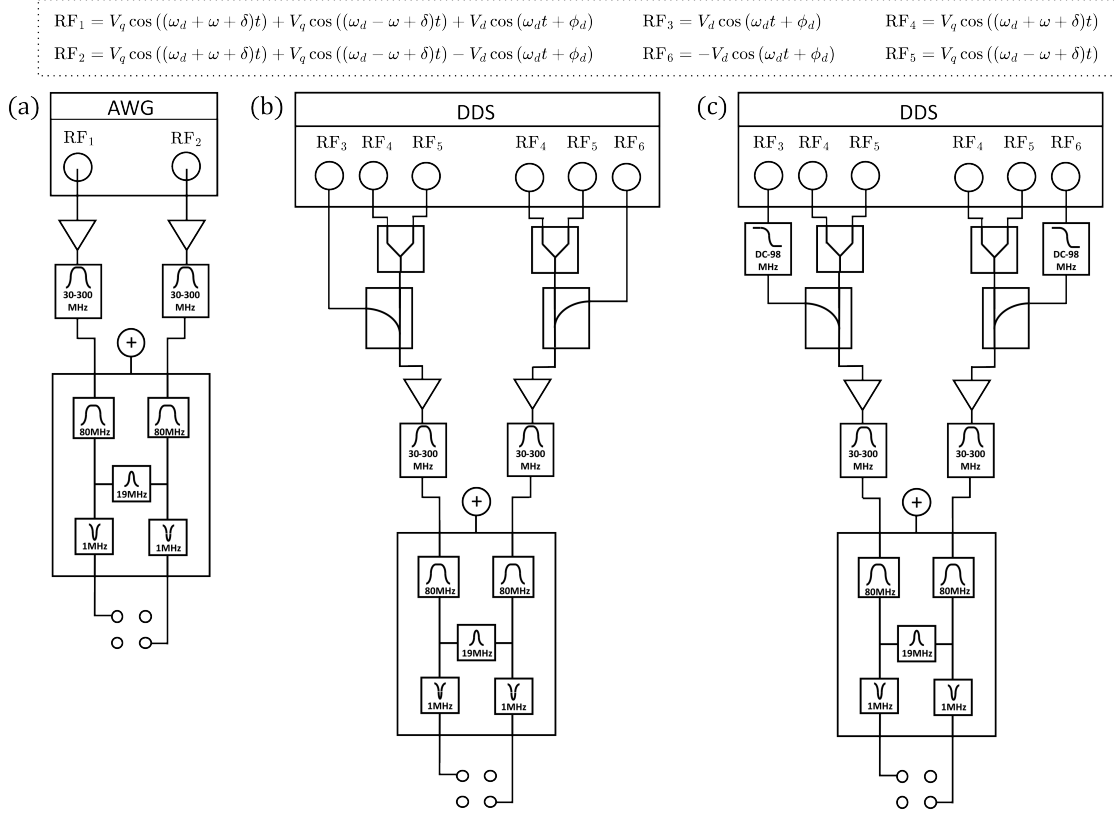


Figure 8.6: **Sample RF schematic of the QVSA signal generation apparatus.**

(a-c) Signals are amplified (Mini-Circuits ZHL-20W-13SW+) before filtering and DC biasing (to break the radial mode degeneracy). A narrow 19 MHz band-pass filter between the QVSA electrodes shorts any pickup from the 19 MHz trap electrodes, which improves trap stability. Different combinations of filters can be used for different frequency ranges. (a) Dipole and quadrupole tones are generated with two channels of an Arbitrary Waveform Generator (Sinara “Phaser”) by setting an inter-channel phase delay of π and 0, respectively. (b-c) QVSA signals are generated using two banks of synchronously clocked [Direct Digital Synthesis/Synthesizers \(DDSs\)](#) (Sinara AD9910 Urukul). This configuration allows QVSA function as a [Vector Network Analyzer \(VNA\)](#) as the dipole tones can be separately passed through a Device Under Test.

8.4 Characterization

8.4.1 Complete Waveform Sensing

We demonstrate wideband vector signal analysis of an unknown dipole tone (V_d, ω_d, ϕ_d) as follows. First, the [QHO](#) is prepared in the ground state $|0\rangle$. Next, the quadrupole tones are applied with amplitude Ω_q and detuning δ for time t , creating a coherent state $|\alpha\rangle$. The

mean phonon number $\langle n \rangle = |\alpha|^2$ of the QHO is measured from the relative intensity of the motional sidebands of the optical qubit transition. This process is repeated as the detuning δ is scanned to maximize the displacement and thus determine the unknown frequency ω_d (Figure 8.7(a)-(c)). Once ω_d is determined, ϕ_d is found by repeating the above protocol at $\delta = 0$ while varying the phase of the applied quadrupole tones. Illustrated in Figure 8.7(d)-(f), the displacement is maximized when $\phi_d = 0$. With ω_d and ϕ_d determined, measurement of $\langle n \rangle$ is used to determine V_d . A similar measurement scanning the dipole voltage V_d is shown in Figure 8.7(g)-(i).

Subfigure	κV_d	V_q	Total Time	Pulse Shaping?
(a)	$(0.239 \pm 0.002) \text{ mV}$	$(8.5 \pm 0.1) \text{ mV}$	1ms	Yes
(b)	$(0.489 \pm 0.006) \text{ mV}$	$(2.88 \pm 0.03) \text{ V}$	1ms	No
(c)	$(42 \pm 3) \text{ mV}$	$(3.8 \pm 0.2) \text{ V}$	1ms	No
(d)	$(0.226 \pm 0.002) \text{ mV}$	$(9.17 \pm 0.01) \text{ mV}$	1ms	Yes
(e)	$(2.20 \pm 0.02) \text{ mV}$	$(0.515 \pm 0.005) \text{ V}$	1ms	No
(f)	$(0.31 \pm 0.02) \text{ V}$	$(0.84 \pm 0.05) \text{ V}$	1ms	No
(g)	See Figure	$(8.62 \pm 0.09) \text{ mV}$	1ms	Yes
(h)	See Figure	$(1.36 \pm 0.02) \text{ V}$	1ms	No
(i)	See Figure	$(3.9 \pm 0.3) \text{ V}$	1ms	No

Table 8.1: **Parameters for Figure 8.7.**

8.4.2 Precision Benchmarks

We characterize the precision and sensitivity of the technique. The relevant statistical background is covered in section 8.4.4. Frequency precision requires a preliminary spectrum (Figure 8.7(a)-(c)) to determine the lineshape. To balance sensitivity with intrinsic noise in subsequent experiments, phonon measurements are repeatedly taken at two frequencies near the half-maxima to determine the center frequency. The resulting overlapping Allan deviation (Figure 8.8(a)) is compared to the **Cramér-Rao Bound (CRB)** and the **Standard Quantum Limit (SQL)**, given by $\Delta\delta \approx 2\pi \cdot 0.209 / (t\langle\alpha\rangle\sqrt{N})$, where $t = 2 \text{ ms}$ is the interaction time, and N is the number of measurements. The Allan deviation of the frequency estimator saturates the CR bound and approaches the SQL with sensitivity $39(9) \text{ Hz}/\sqrt{\text{Hz}}$ until

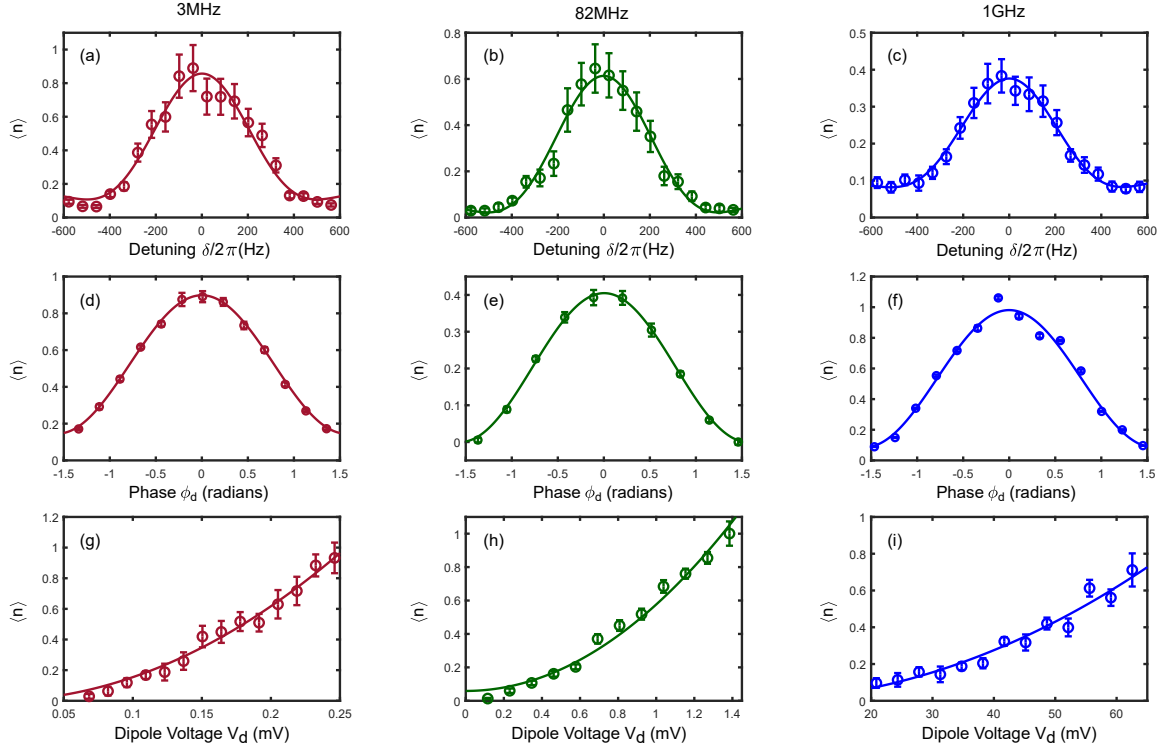


Figure 8.7: **Wideband demonstration of frequency, phase, and amplitude sensing using the QVSA.**

(a)-(c) Dipole signals are detected by sweeping a pair of quadrupole tones across $\omega_d/2\pi = 3$ MHz, 82 MHz, and 1 GHz, respectively. The lineshape $\langle n \rangle = |\alpha|^2$ is obtained by varying the detuning δ . (d)-(f) illustrate the dependence of $\langle n \rangle$ on ϕ_d for 3 MHz, 82 MHz, and 1 GHz, respectively. The phase dependence is measured with detuning $\delta = 0$. The quadratic dependence of $\langle n \rangle$ on V_d is measured in (g)-(i). The combination of frequency, phase, and amplitude sensing provide the means for vector signal analysis. Solid lines are fit using eq. (8.2) to extract the voltages experienced by the ion. Error bars represent one standard error. Exact parameters used for each panel are listed in Table 8.1.

$N \approx 6300$, achieving a minimum resolution of 3.8(16) Hz, corresponding to a fractional frequency resolution of $4.6(2.0) \times 10^{-8}$.

Similarly, phase precision is determined by a two-point measurement at 45° and 135° with resonant detuning (Figure 8.8(b)), and is compared to the CR bound and the SQL, $\Delta\phi_d = 1/(\langle\alpha\rangle\sqrt{2N})$. The minimum phase resolution reaches $\approx 9.1(2.0)$ mrad after $N \approx 9800$ experiments with sensitivity $117(26)$ mrad/ $\sqrt{\text{Hz}}$. Deviation from the CR bound is due to drifts of the secular frequency ω of up to $\approx 2\pi \cdot (200 - 600)$ Hz over the course of these

measurements and is induced by temperature and humidity changes of the lab environment.

To determine voltage sensitivity, a large quadrupole voltage V_q is used to measure the phonon number, maximizing sensitivity to V_d . Measurements are repeated until the noise floor is reached (typically $N \sim 10^4$). Figure 8.8(c) shows the minimum detectable voltage (ΔV_{min}) as a function of drive time t , with $V_q = 8.4(1)$ V (28 dBm), which is limited by component damage threshold and is extendable with straightforward upgrades. The data follows a $1/\sqrt{N}$ trend until ≈ 1 ms, where motional decoherence becomes significant. The voltage sensitivity is $51(10)$ $\mu\text{V}/\sqrt{\text{Hz}}$ with a minimum detectable voltage of $3.8(8)$ μV at $N = 11000$, corresponding to an electric field sensitivity of $66(13)$ $\text{mV}\text{m}^{-1}/\sqrt{\text{Hz}}$ with an electric field resolution of $4.9(1.0)$ mV/m .

8.4.3 Wideband Operation

To demonstrate wideband operation of the QVSA, we characterize the electric field sensitivity at additional frequencies from 100 kHz to 1 GHz (Figure 8.9), a frequency range $> 800\times$ larger than previous techniques. Performance closely matches that expected from equation (8.28) (solid red line). Phase sensitivity is maintained until $\omega_d \lesssim \omega$ regime, where only the blue quadrupole tone can be applied, preventing the interference that generates phase sensitivity. However, phase sensitivity could be recovered by interfering the QVSA displacement with a resonant displacement at ω . From equation (8.28), at large ω_d , the sensitivity and bandwidth of the QVSA is determined by the Mathieu q-parameter $q_q = \frac{2eV_q}{mr_0^2\omega_q^2}$ of the *quadrupole* fields. However, practical limitations constrain selection of q_q . Empirical results suggest that $q_q \leq 0.01$ is required on our experiment to prevent disturbance of trap dynamics, which limits the achievable V_q , and therefore sensitivity, at frequencies $\omega_d \lesssim 20$ MHz. For $\omega_d \gtrsim 20$ MHz, the voltage V_q required to maintain $q_q = 0.01$ exceeds component damage thresholds (28 dBm) of our experiment. Our practical inability to maintain $q_q = 0.01$ at higher frequencies causes a concomitant deterioration of sensitivity, reflected in the change in slope above 20 MHz in Figure 8.9. Importantly, this limit is not fundamental.

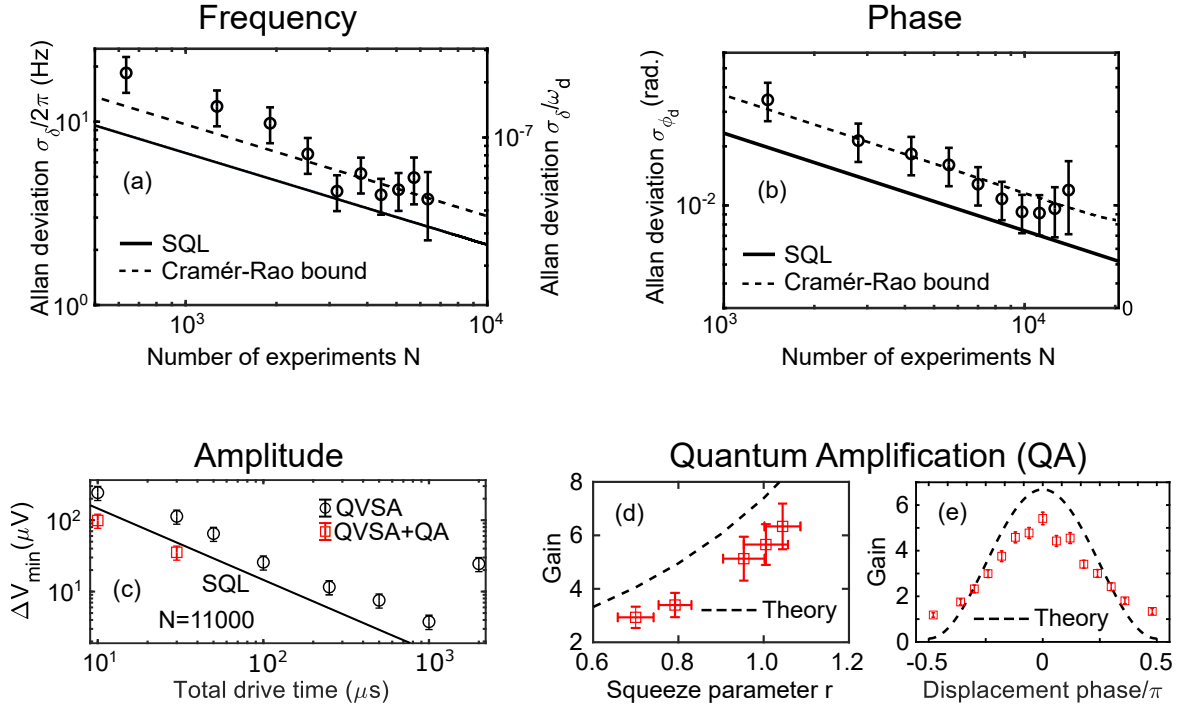


Figure 8.8: **Frequency, phase and amplitude sensitivity of QVSA and integration of quantum amplification (QA).**

(a-b) Overlapping Allan deviation of frequency and phase of a 82 MHz dipole tone. (a) The measured frequency sensitivity is $39(9) \text{ Hz}/\sqrt{\text{Hz}}$ with a minimum resolution of $3.8(16) \text{ Hz}$ after 6300 experiments. (b) The measured phase sensitivity is $117(26) \text{ mrad}/\sqrt{\text{Hz}}$ with a minimum resolution $9.1(20) \text{ mrad}$ after 9800 experiments. (a-b) The probe time is 2 ms and each experiment takes ~ 17 ms. Solid lines represent the corresponding standard quantum limit (SQL). Dashed lines represent the corresponding **Cramér-Rao Bounds**, both of which are saturated by our measurements. (c-e) Integration of quantum amplification with the **QVSA**. (c) The minimum detectable voltage ($\Delta V_{\min}$) on the electrodes with and without quantum amplification. Each data point represents 11000 experiments, and takes ~ 17 ms each. The measured sensitivity to the voltage coupled onto the electrodes is $51(10) \mu\text{V}/\sqrt{\text{Hz}}$ with a minimum detectable voltage of $3.8(8) \mu\text{V}$. The electric field sensitivity is $66(13) \text{ mV m}^{-1}/\sqrt{\text{Hz}}$ with an electric field resolution of $4.9(10) \text{ mV m}^{-1}$. Notably, with quantum amplification, the voltage sensitivity is $3.4(2.0) \text{ dB}$ and $2.8(2.0) \text{ dB}$ below the **SQL** for $10 \mu\text{s}$ and $30 \mu\text{s}$, respectively. (d-e) Quantum amplification through squeezing. By applying a squeeze operation $\hat{S}(r)$ before and an anti-squeeze operation $\hat{S}^\dagger(r)$ after $10 \mu\text{s}$ of QVSA operation, the mean phonon number $\langle n \rangle$ is amplified. The gain scales with the squeeze parameter r (d) and its phase relative to the **QVSA** displacement, with $r = 0.95$ (e). Dashed lines represent the maximum achievable gain from theory. Parametric squeezing is applied for $30 \mu\text{s}$ at roughly 75 mV . (a-e) Error bars represent one standard error.

Carrier Frequency (MHz)	0.1	0.6	3	30	100
E-field Sensitivity (mV/m/ $\sqrt{\text{Hz}}$)	8.3(1.1)	2.1(3)	2.2(4)	8.1(1.7)	66(14)
Phase Resolution (mrad)	-	-	9.4(7)	9.9(1.2)	15.7(2.5))

Carrier Frequency (MHz)	200	350	600	800	1000
E-field Sensitivity (mV/m/ $\sqrt{\text{Hz}}$)	$4.4(8) \cdot 10^2$	$1.3(3) \cdot 10^3$	$2.5(5) \cdot 10^3$	$3.6(8) \cdot 10^3$	$1.0(2) \cdot 10^4$
Phase Resolution (mrad)	8.6(1.3)	22.8(10.2)	9.3(1.7)	9.6(2.9)	11.7(2.2)

Table 8.2: **Data for measurements shown in Figure 8.9.**

“-” means not applicable. All values listed in the table are derived from an Allan deviation following the same procedure used in Figure 8.8.

To contextualize the QVSA performance, Figure 8.9 additionally compares electric field sensitivity across various quantum sensing techniques. Ref. [Qiu+22] reports phase-insensitive electric field measurements in NV centers from roughly 200 kHz to 1 MHz. Both Ref. [Wol+19] and Ref. [Gil+21] are phase-insensitive, using 150 entangled ions in a Penning trap and single ion in a Paul trap, respectively, and are limited to sensing at the oscillator’s secular frequency. Ref. [Aff+20] demonstrates the ability to perform highly sensitive measurements of the electric field strength, even for fields detuned by $\sim$ MHz from the oscillator’s secular frequency, with phase sensitivity. While Ref. [Mey+20] predicts that Rydberg atoms theoretically provide a tunable frequency range from kHz to THz, to our knowledge, the only experimental demonstrations within the MHz to GHz range are Ref. [Liu+22] and Ref. [Hol+22], which are phase-insensitive with frequency ranges of 10 MHz and 4 MHz, respectively, both shown in Figure 8.9. In comparison, the QVSA is the only method with phase sensitivity, demonstrating a frequency range across the low-frequency RF to UHF microwave bands, and sensitivities competitive with more specialized techniques.

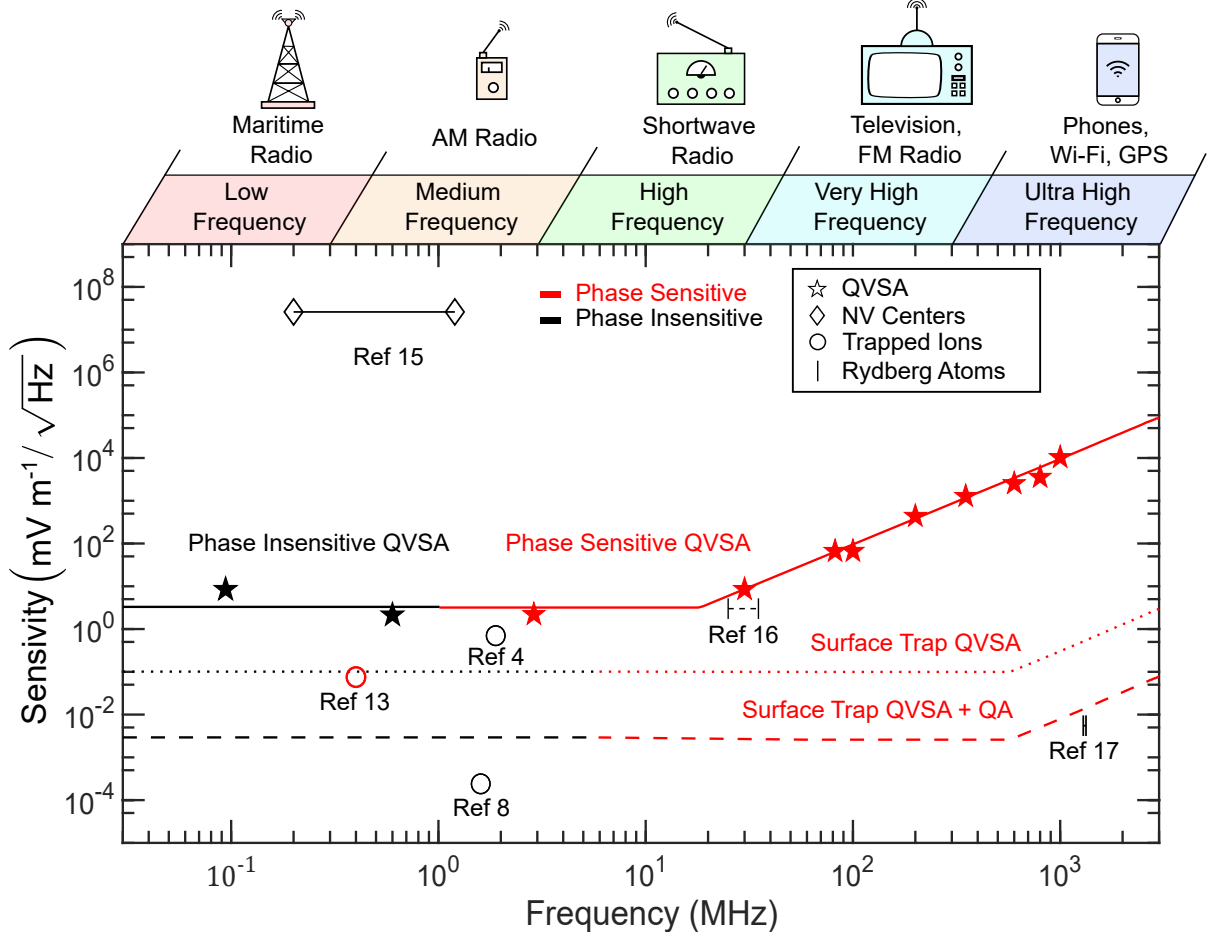


Figure 8.9: **Comparison of electric field sensitivity between different quantum sensing techniques.**

Phase insensitive methods are shown in black; phase sensitive methods are shown in red; $\diamond$: NV centers; $\circ$: ions; $|$: Rydberg atoms; $\star$: our measurement. The length of lines corresponds to the frequency range of operation. The solid red line represents the estimated QVSA performance in our linear trap, which aligns closely with our measurements (exact measurement results are listed in Table 8.2). Quadrupole tones are applied with a constant Mathieu q -parameter of $q_q \approx 0.01$ to yield a nearly constant sensitivity from 100 kHz until ≈ 20 MHz. Above ≈ 20 MHz, practical limitations on power prevent the constant Mathieu q -parameter of $q_q = 0.01$ from being applied, causing the minimum achievable sensitivity to increase with frequency. The dotted red line indicates the estimated QVSA sensitivity if implemented on a surface trap with an ion-electrode distance of $40 \mu\text{m}$, $^9\text{Be}^+$ as a trapped ion, a secular frequency of $2\pi \cdot 6$ MHz, and a maximum quadrupole power of 39.5 dBm. The dashed red line represents anticipated near-term enhanced QVSA sensitivity in a surface trap with Quantum Amplification (QA). This comparison did not account for scaling with the number of ions.

8.4.4 Statistical Analysis

Fisher Information and the Cramér-Rao Bound (CRB)

In parameter estimation, the Cramér-Rao Bound (CRB) sets a lower bound on the achievable variance of an estimator $\Delta\alpha$ in terms of its Fisher Information F , i.e. $\Delta\alpha \geq \Delta\alpha_{CR} = 1/\sqrt{F}$.

Classically, the Fisher Information is calculated:

$$F(\alpha) = \sum_x \frac{1}{P(x|\alpha)} \left(\frac{\partial P(x|\alpha)}{\partial \alpha} \right)^2, \quad (8.16)$$

where $P(x|\alpha)$ is the probability of outcome x given the parameter α . Fisher information is additive for repetitive independent measurements. The maximal F over the estimator space is called the [Quantum Fisher information \(QFI\)](#) ($\mathfrak{F}$), which in turn leads to the [Quantum Cramér-Rao Bound \(QCRB\)](#).

Saturating the [Cramér-Rao Bound](#)

Here, we show that the [CRB](#) of our sideband ratio measurement can be saturated by adjusting the relative number of red and blue sideband measurements to constitute R . We define the inversion estimator [\[Xu+19\]](#) $\hat{\alpha} = f^{-1}(R)$ to evaluate the uncertainty of the displacement α given that $R = P_r/P_b = f(\alpha)$. Assuming that the uncertainties of the red and blue sideband probabilities $\Delta P_{r/b}$ are dominated by quantum projection noise (which follows a binomial distribution and is thus $\Delta P_{r/b} = \sqrt{p_{r/b}(1 - p_{r/b})/N}$), propagation of uncertainty gives:

$$\Delta \hat{\alpha} = \sqrt{\frac{1}{F_r N_r} \left(\frac{\partial \alpha}{\partial R} \right)^2 \left(\frac{\partial R}{\partial P_r} \right)^2 \left(\frac{\partial P_r}{\partial \alpha} \right)^2 + \frac{1}{F_b N_b} \left(\frac{\partial \alpha}{\partial R} \right)^2 \left(\frac{\partial R}{\partial P_b} \right)^2 \left(\frac{\partial P_b}{\partial \alpha} \right)^2} \sqrt{N_r + N_b}, \quad (8.17)$$

where P_r (P_b) is the D-state population after a red (blue) sideband pulse, N_r (N_b) is the number of red (blue) sideband measurements, and F_r (F_b) is the Fisher Information per experiment for a red (blue) sideband measurement. To account for imperfections in the experimental procedure, we use an empirical formula $P_{r/b}^{real} = A P_{r/b}(\alpha) + B$, where $A \approx 0.94$ and accounts for decoherence processes, e.g. laser power fluctuations, while $B \approx 0.03$ and accounts for background noise, which for our experiment is dominated by the servo bump of the qubit addressing laser. For comparison, the [CRB](#) per experiment calculated from

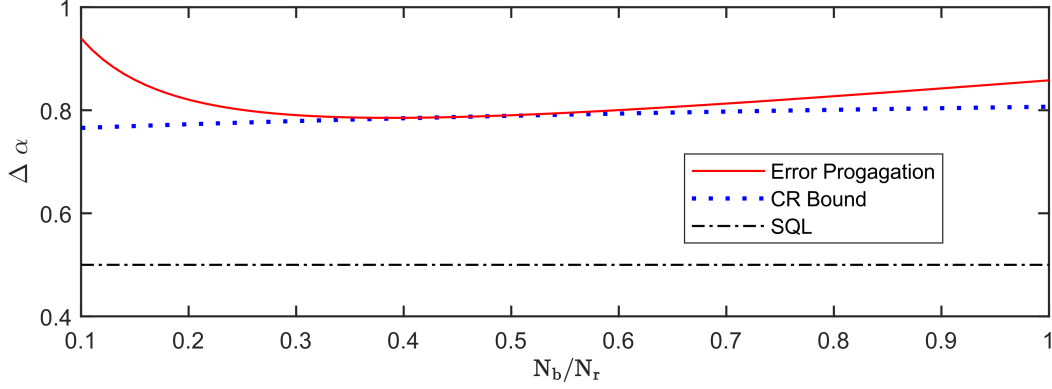


Figure 8.10: **Uncertainty in the measured displacement $\Delta\alpha$ as a function of the relative number of red and blue sideband measurements used for the phase measurement in Figure 8.8(b).** The **Cramér-Rao Bound (CRB)** is saturated when $N_b/N_r \approx 0.4$. Experimental imperfections are included in the estimation.

eq. (8.16) is:

$$\Delta\alpha_{CRB} = \sqrt{\frac{N_r + N_b}{N_r F_r + N_b F_b}}. \quad (8.18)$$

Figure 8.10 shows both the CRB and $\Delta\hat{\alpha}$ alongside the SQL as a function of N_b/N_r for $\alpha \approx 0.68$. The estimator performance is maximized for $N_b/N_r \approx 0.4$, where it also saturates the CRB.

The Standard Quantum Limit (SQL)

The standard quantum limit (SQL) of a measurement is the minimum achievable uncertainty when using classical states. Here, we derive the standard quantum limit (SQL) for amplitude, frequency and phase sensing as depicted in Figure 8.8.

For displacement sensing, the SQL is $\Delta\alpha_{SQL} = \frac{1}{2}$ [Wol+19,Gil+21]. The SQL for the mean phonon number $\langle n \rangle$ is then:

$$\Delta n_{SQL} = \frac{\Delta\alpha_{SQL}}{\frac{\partial\alpha}{\partial n}} = \sqrt{n}. \quad (8.19)$$

The Standard Quantum Limit (SQL) — Frequency Sensing

For frequency sensing with Rabi spectroscopy, the mean phonon number $\langle n \rangle$ expressed in terms of the dipole tone detuning δ (using eq. (8.2)) is:

$$\langle n \rangle = |\tilde{a}|^2 \frac{\sin^2(\delta t)}{(\delta t)^2}, \quad (8.20)$$

where $|\tilde{a}|$ is the displacement amplitude and t is the QVSA interaction time. Using eq. (8.19), the SQL becomes $\Delta n = |\tilde{a}| \sin(\delta t) / (\delta t \sqrt{N})$, where N is the number of experiments. The frequency uncertainty of the dipole tone $\Delta\delta$ at the SQL is therefore:

$$\Delta\delta = \frac{\Delta n}{\frac{\partial n}{\partial \delta}} = \frac{(\delta t)^2}{\delta t^2 \cos(\delta t) - t \sin(\delta t)} \frac{1}{2|\tilde{a}| \sqrt{N}}. \quad (8.21)$$

Unlike in Ramsey spectroscopy, where the uncertainty is independent of detuning [Ita+93], Rabi spectroscopy measurements require judicious selection of the readout points. For the Allan deviation in Figure 8.8(a), we use two points near the half-maxima, which balances the sensitivity with the signal-to-noise ratio (SNR). Under these conditions, $\sin(\delta t) / (\delta t) \approx 1/\sqrt{2}$. The SQL for frequency sensing is thus:

$$\Delta\delta \approx 2\pi \frac{0.209}{|\tilde{a}| t \sqrt{N}}. \quad (8.22)$$

The Standard Quantum Limit (SQL) — Phase Sensing

Following eq. (8.20), the mean phonon number $\langle n \rangle$ expressed in terms of the dipole tone phase ϕ_d is:

$$\langle n \rangle = |\tilde{a}|^2 \cdot \sin^2(\theta + \phi_d), \quad (8.23)$$

where θ is the variation of the initial phase. At the SQL, this becomes $\Delta n = |\tilde{a}| \cdot \sin(\theta + \phi_d)/\sqrt{N}$, which allows us to calculate the phase uncertainty at the SQL:

$$\Delta\phi_d = \frac{\Delta n}{\frac{\partial n}{\partial \phi_d}} = \frac{1}{\cos(\theta + \phi_d)} \frac{1}{2|\tilde{a}|\sqrt{N}}. \quad (8.24)$$

Similarly, for the Allan deviation in Figure 8.8(b), two points are measured at the half-maxima (i.e. $\cos(\theta + \phi_d) = 1/\sqrt{2}$). The SQL for phase sensing is thus:

$$\Delta\phi_d = \frac{1}{|\tilde{a}|\sqrt{2N}}. \quad (8.25)$$

The Standard Quantum Limit (SQL) — Amplitude Sensing

The mean phonon number $\langle n \rangle$ expressed in terms of the dipole tone amplitude V_d is:

$$\langle n \rangle = AV_d^2 t^2, \quad (8.26)$$

where A is a prefactor that depends on both the frequency and the quadrupole voltage. For example, at 82 MHz with $V_q = 8.4$ V, $A = 1.1 \times 10^{13} \text{V}^{-2} \text{s}^{-2}$. The SQL is thus:

$$\Delta V_d = \frac{\Delta n}{\frac{\partial n}{\partial V_d}} = \frac{1}{2t\sqrt{AN}}, \quad (8.27)$$

where N is the number of experiments. The dependence of the electric field E_d on the applied voltage V_d is $E_d = 0.715V_d/r_0$, as determined from simulations of our trap using COMSOL Multiphysics.

8.5 Applications

8.5.1 Vector Network Analysis

We demonstrate the functionality of the QVSA technique for vector network analysis by characterizing the transfer function of a commercial low-pass filter (Mini-Circuits SLP-100+) (Figure 8.11). For this measurement, V_d and ϕ_d are measured as a function of frequency with and without the filter inserted. The RF configuration used for these measurements is shown in Figure 8.6(b-c). For normalization, the transfer function of the dipole tone is measured without the **Device Under Test (DUT)** (i.e. the low-pass filters) in place (Figure 8.6(b)). At each frequency, the **RSB** phase ϕ_r is scanned, and the resulting displacement is fitted to eq. (8.2) to extract the voltage (V_n) and phase (ϕ_n) of the measured signal, as similarly shown in Figure 8.7. Next, the devices are connected in series with the relevant dipole tone DDS channels (RF3 and RF6) respectively as shown in (Figure 8.6(c)), which alter the amplitude and phase of the dipole field experienced by the ion. A similar measurement is repeated to extract the new voltage (V_f) and phase (ϕ_f) sensing by the ion. Finally, the phase delay from low pass filters can be expressed as $(\phi_f - \phi_n)$ and attenuation can be written as $20\log_{10}(V_f/V_n)$. The data are shown alongside the measurement from a standard network analyzer (Hewlett-Packard 8714ES) (Figure 8.11), demonstrating excellent agreement.

As the present experimental apparatus has been designed to demonstrate electric-field gradient gates on molecular ion qubits [HC21], we have used the same procedure to measure the coupling efficiency of qubit control signals to the ion (Figure 8.12). The measured coupling efficiency is compared against that from a traditional measurement using a capacitor divider connected to trapping RF electrodes and a vector network analyzer over the frequency range accessible with our **DDS** (20 MHz - 300 MHz). As shown in Figure 8.12, the traditional measurement fails to accurately report the filter function. Changes to the measurement conditions required to interface the network analyzer, e.g. impedance changes at measurement ports, likely account for this discrepancy, serving as an archetypal example of the difficulty of

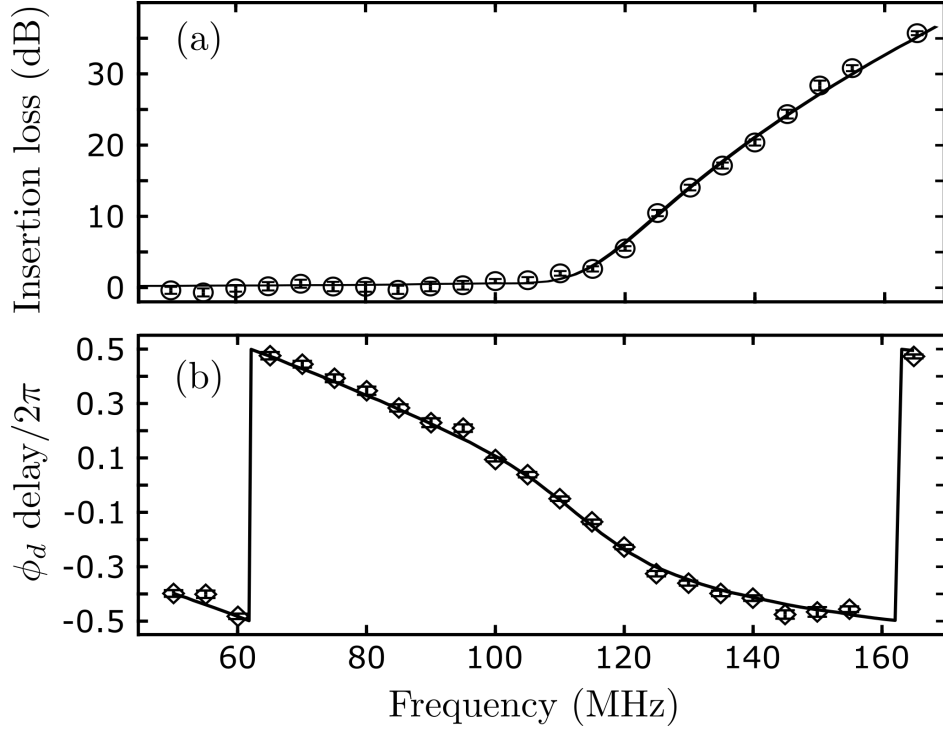


Figure 8.11: **Transfer function of a commercial low-pass filter (Mini-Circuits SLP-100+) measured by the QVSA technique (points) and compared to a commercial Vector Network Analyzer (Hewlett-Packard 8714ES).**

(a-b) $\circ$ and $\diamond$ are measured through the QVSA. Solid lines are measured via a commercial vector network analyzer. Each point is composed of $N \approx 13500$ experiments. Error bars represent one standard error. No re-scaling or offset is applied to the data.

accurately calibrating qubit signals. This technique could be used to measure the transfer function of shuttling electrodes in **Quantum Charged Coupled Device (QCCD)** trapped-ion quantum computers [Mos+23], and qubit control lines in superconducting qubits coupled to a microwave resonator.

8.5.2 Quantum Amplification

Since the QVSA senses a displacement, its voltage (electric field) sensitivity can be enhanced by leveraging quantum amplification [Bur+19, Gil+21]. Adapting quantum amplification requires generation of a squeezed state $\hat{S}(r)|0\rangle$ prior to application of the QVSA interaction. Assuming a squeeze parameter $r \in \mathbb{R}$, this reduces the position uncertainty of the **QHO** wavefunction e^{-r} , while increasing the momentum uncertainty by e^r [NDP95, Bur+19]. Application of the

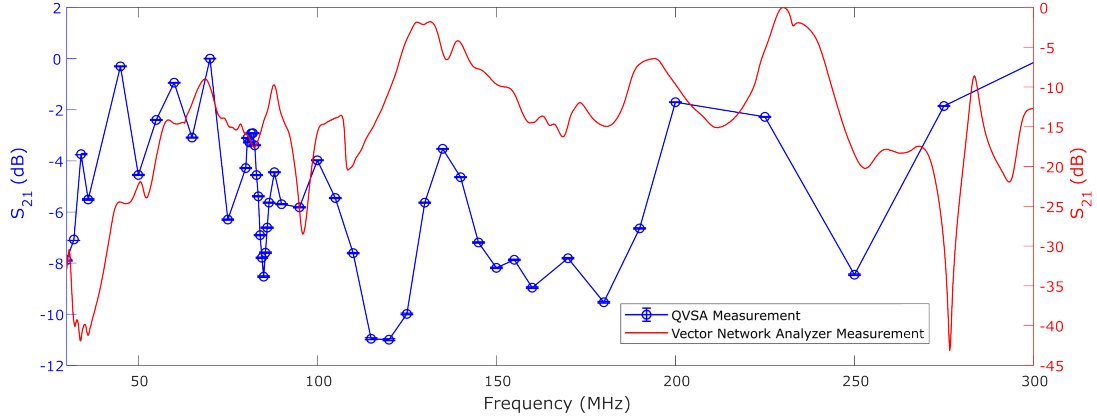


Figure 8.12: **Insertion loss of the ion trap system measured using the QVSA and a commercial vector network analyzer (VNA).**

The wideband transfer function of a qubit control line is compared against that from a traditional measurement using a capacitor divider connected to trapping RF electrodes and a vector network analyzer over the frequency range accessible with our DDS ([20, 300] MHz). The traditional measurement fails to accurately report the filter function. Changes to the measurement conditions required to interface the network analyzer, for example impedance changes at measurement ports, likely account for this discrepancy, serving as an archetypal example of the difficulty of accurately calibrating qubit signals. Blue (red) data shows the insertion loss measured by the QVSA (VNA). Error bars for the QVSA measurements represent one standard error and are too small to be seen.

QVSA interaction then produces a displaced-squeezed state $\hat{D}(\alpha)\hat{S}(r)|0\rangle$, where for simplicity we assume $\alpha \in \mathbb{R}$. Finally, anti-squeezing is applied to reverse the squeeze operation and return the ion to a coherent state for precise readout, and yields an amplified displacement $\hat{S}^\dagger(r)\hat{D}(\alpha)\hat{S}(r)|0\rangle = \hat{D}(e^r\alpha)|0\rangle$. See Figures 8.4 and 8.5 for further details.

We implement this amplification by parametrically modulating the trap potential, adding a relative phase offset π for anti-squeezing. To verify the anti-squeezing fidelity, we measure the phonon number after anti-squeezing immediately following squeezing. Using the maximum coupling strength $r/t = 2\pi \cdot 5.5(2)$ kHz applied for 30 μ s, we recover $\langle n \rangle = 0.03(2)$, which is indistinguishable from the initial state. Figure 8.8(d) shows quantum amplification of $\langle n \rangle$ as a function of r , with a maximum gain in $\langle n \rangle$ of 6.3(9). Maximal amplification requires orientation of the QVSA displacement along the squeezing axis in phase space (Figure 8.8(e)). This is achieved by adjusting the phases of the quadrupole tones by equal

and opposite amounts. Discrepancies between the achieved and predicted gains likely arise from phase instability between the squeezing and displacement operations. We characterize the voltage sensitivity in the manner previously described and find a metrological gain $10\log_{10}(\Delta V_{\text{QA}}/\Delta V_{\text{SQL}})^2 = 3.4$ (2.0) dB and 2.8(2.0) dB below the SQL for interaction times of 10 μs and 30 μs , respectively (Figure 8.8(c)). Our achievable amplification is currently limited by the increased motional decoherence of a squeezed state.

8.5.3 Phonon-Photon Transduction

Following quantization of the electromagnetic field, the QVSA interaction Hamiltonian in eq. (8.1) can be expressed as $\hat{a}\hat{b}_d\hat{b}_q^\dagger + \text{h.c.}$, where $\hat{a}$ is the annihilation operator for a motional phonon, $\hat{b}_d$ is the annihilation operator for a photon of the dipole field, and $\hat{b}_q$ is the annihilation operator for a photon of the quadrupole field.

Consider transduction from a mechanical phonon to a microwave photon, i.e. $|n_a, n_d, n_q\rangle = \sum_n P(n) |1, n, 0\rangle \rightarrow |n_a, n_d, n_q\rangle = \sum_n P'(n) |0, n-1, 1\rangle$. The ion is first prepared in the $|n=1\rangle$ Fock state via application of a [Blue Sideband](#) pulse following sideband cooling. A TE₂₁ microwave cavity, resonant at the quadrupole frequency with mode occupation near zero (via e.g. cooling by a dilution refrigerator), is constructed around a surface ion trap. The dipole tone is then applied classically via a surface trap electrode or a microwave horn. This configuration enables conversion of a single motional phonon into a single microwave photon, as well as the reverse process, with the number of cavity photons generated controlled by the Fock-state occupation of the motional mode.

8.6 Future Directions

8.6.1 Improvements

Generally, the sensitivity to an unknown parameter θ can be improved by either reducing the uncertainty of the measured quantity (e.g. Δn), which can be achieved via the use of

non-classical states, or by increasing the magnitude of the parameter response (e.g. $\partial n/\partial\theta$), which can be achieved by adjusting experimental parameters.

Use of Non-Classical States

For single QHOs, Fock states, squeezed states, and cat states are the most commonly used non-classical states to probe signals with uncertainty below the SQL. The Fisher information for a Fock state $|n\rangle$ is $F_\alpha^{\text{Fock}} = 8n + 4$ [Wol18]. Thus, application of the QVSA on a Fock state $|n = 12\rangle$ could improve the sensitivity by a factor of $5\times$. The maximum F for a squeezed state $|r\rangle$ is $F_\alpha^{\text{squeezed}} = 4e^{2|r|}$ [Wol18]. Our maximum achievable squeezing parameter r is limited by our control electronics, improvements to which could yield a gain in sensitivity of $9\times$ [Bur+19]. The maximum F for a Schrödinger cat state is $F_\alpha^{\text{Cat}} = 16(|\alpha|^2 + \frac{1}{4})$ [Wol18]. Use of a cat state with $\alpha = 3$ could thus improve the sensitivity by roughly $6\times$ [Hem+13].

Experimental Modifications

To understand the fundamental limits of the QVSA frequency range, eq. (8.2) can be made frequency (ω_d) independent by expression in terms of the Mathieu q -parameter:

$$\alpha = \beta \frac{q_q V_d}{r_0 \sqrt{m\omega}} \frac{\sin(\delta t - \phi_d) + \sin(\phi_d)}{\delta}, \quad (8.28)$$

where β is a constant prefactor and $q_q = \left\{ \frac{2eV_q}{mr_0^2\omega_q^2} \text{ for } \omega_q \approx \omega_d \gg \omega; \frac{2eV_q}{mr_0^2(\omega_d^2 - \omega^2)} \text{ for } \omega_d \approx \omega \right\}$ is the Mathieu q -parameter of the quadrupole tones. This can be applied to estimate the bandwidth of the QVSA and to evaluate the potential improvements achievable by transitioning to an upgraded platform. Importantly, the independence of eq. (8.28) from ω_d indicates that the bandwidth of our protocol is not fundamentally limited.

However, at larger frequencies, greater power is required to maintain the same q_q . On our experiment, we were unable to maintain a fixed q_q above ≈ 20 MHz due to damage thresholds of electronic components – this underlies the change in slope of the expected sensitivity

(solid red line) in Figure 8.9. Additionally, empirical evidence on our experiment suggests that trapping dynamics are disturbed once the q -parameter of the applied quadrupole tones reaches 10% of the trap's q -parameter – this effectively limits our implementation to $q_q < 0.01$. It should be noted that this constraint is specific to our system – a higher q_q may be feasible on other systems.

Consequently, there are several straightforward, readily available improvements, all using existing technology, that could greatly enhance the QVSA operation. The features and benefits of these are outlined below:

- Use of a surface trap [McC+19a]: a decrease in r_0 from 550 μm in our current trap to 40 μm .
- Trapping of a lighter species: e.g. ${}^9\text{Be}^+$, which is lighter than ${}^{40}\text{Ca}^+$ by ≈ 4 .
- Stabilization of the secular frequency to $\approx 2\pi \cdot 10$ Hz: an increase in interaction time from our current 1 ms to > 10 ms and greater quantum amplification (which is limited by motional decoherence). Currently, motional coherence times up to 55 ms have been achieved[Val+25].
- Operation at higher q_q : the use of a surface trap, with its lower r_0 would increase q_q even with lower input powers. For an r_0 of 40 μm , a 200 mV field at 82 MHz would result in $q_q = 0.01$, compared to the $q_q = 0.0005$ achieved in our trap with 8.4 V. Increasing the power damage thresholds of system electronics or the use of a resonator to minimize power requirements would allow higher powers to be applied to achieve large q_q even at high frequencies.
- Increase in the secular frequency from $2\pi \cdot 0.8$ MHz to $2\pi \cdot 6$ MHz: reduction in sideband cooling time from 13 ms for ${}^{40}\text{Ca}^+$ to 230 μs for ${}^9\text{Be}^+$ [McC+19a], and thus the duration of each experimental trial reduces from $(16 + t)$ ms to $(1 + t)$ ms, where t is the QVSA interrogation time. This reduction in duration of each experimental trial exceeds the reduction in sensitivity due to the higher secular frequency.

- Amplification of the dipole tone: A simple pre-amplifier, e.g. Mini-Circuits ZHL-20W-13SW+ would increase the amplitude sensitivity by 30 dB.

Improved Amplitude Sensitivity

Here, we provide detailed information regarding the estimation of electric field sensitivity enhancement, illustrated in Figure 8.9. From eq. (8.28), reduction in species mass from 40 AMU to 9 AMU, increase in q_q from 0.01 to 0.03, reduction in state preparation time from 16 ms to 1 ms, and increase in interrogation time from 1 ms to 10 ms, yield a collective sensitivity enhancement of

$$\sqrt{\frac{40\text{AMU}}{9\text{AMU}}} \frac{0.03}{0.01} \frac{10\text{ ms}}{1\text{ ms}} \sqrt{\frac{0.8\text{MHz}}{6\text{MHz}}} \sqrt{\frac{17\text{ ms}}{11\text{ ms}}} = 30\times, \quad (8.29)$$

resulting in an electric field sensitivity $0.1\text{ mVm}^{-1}/\sqrt{\text{Hz}}$. A maximum applied power of 9 W extends this sensitivity up to 600 MHz (red dotted lines, Figure 8.9).

A further increase in the QVSA interrogation time from 10 ms to 55 ms (by e.g. eliminating sources of motional dephasing), together with an $18\times$ gain from quantum amplification (QA) ($\times 2$ beyond that reported in [Bur+19]) leads to a further sensitivity enhancement of

$$\frac{55\text{ ms}}{10\text{ ms}} \sqrt{\frac{11\text{ ms}}{56\text{ ms}}} = 42\times. \quad (8.30)$$

This corresponds to an amplitude sensitivity as low as $0.2\text{ pV}/\sqrt{\text{Hz}}$, or equivalently, an electric-field sensitivity $\approx 2.8\text{ }\mu\text{V m}^{-1}/\sqrt{\text{Hz}}$ (red dotted line, Figure 8.9) — this surpasses the current state-of-the-art electric field measurements on Rydberg atoms [Jin+20]. While the sensitivities reported in [Gil+21] are below those measured here, those sensitivities were obtained with $N = 150$ trapped ions. To be conservative, values reported by others have not been scaled by the particle number N ; though use of our protocol with greater N , such as in a multiplexed surface trap, would similarly improve sensitivity by $\sqrt{N}$.

Improved Frequency and Phase Sensitivity

Unlike amplitude sensitivity, gains in the frequency and phase sensitivities are harder to realize due to their independence from experimental parameters, as evident from eq. (8.22) and eq. (8.25) (assuming $\alpha \leq 1$). With regards to frequency, a five-fold increase in the motional coherence time from 2 ms to 10 ms would yield a concomitant five-fold improvement in the sensitivity, per eq. (8.22). Implementation of a Ramsey spectroscopy detection scheme, as opposed to the current Rabi spectroscopy detection scheme, would further improve the sensitivity by ~ 2.5 , as shown in [Wol+19]. Improvements to our squeezed state generation could further amplify our measurement by a factor of 9. Together, these modifications could improve the sensitivity by an overall factor of roughly $110\times$ to reach $0.3 \text{ Hz}/\sqrt{\text{Hz}}$. By contrast, since the phase sensitivity is independent of the QVSA interaction time (see eq. (8.25)), the only avenue for improvement is through the reduction of the duration of each experimental trial. This time could be reduced from 17 ms to $700 \mu\text{s}$ for a sensitivity gain of 5. Again, together with the use of a squeezed state, a total gain of roughly $45\times$ in sensitivity could be realized to reach $3 \text{ mrad}/\sqrt{\text{Hz}}$.

8.6.2 Extension to Other Platforms

Our protocol is extendable to other QHO architectures with the ability to measure displacements and generate the necessary Raman transitions, such as the Penning trap of [Gil+21], where our scheme would broaden the frequency range for dark matter searches. Beyond trapped ions, our scheme could be implemented in solid-state platforms: coupling of a microwave tone to a superconducting Josephson parametric amplifier (JPA) at frequency ω_d , with inductive coupling of a pump tone to the DC superconducting quantum interference device (SQUID) of the JPA [Yam+08] at frequency ω_q , would realize the full QVSA Hamiltonian. This may allow sensitive electric field detection beyond $> 1 \text{ GHz}$ and *in situ* calibration of superconducting/ion qubit control lines which are notoriously challenging due to e.g. stray capacitances of measurement lines and insufficient generality or directness

of existing techniques [[Jer+19](#),[Mil+21](#),[Gel+24](#)]. Further, the [QVSA](#) could be leveraged to produce an effective beam-splitter-like Hamiltonian to entangle microwave photons with motional phonons, and could therefore be used for frequency transduction or electric field noise spectroscopy (see section [8.5.3](#)). Finally, the higher motional frequencies of trapped electrons [[Mat+21](#)] would extend the detection band to the THz, while use of superconducting resonators with cryogenic amplifiers may allow sensitivities significantly beyond the current state-of-the-art.

Theo: “A hundred years from now there won’t be one sad [student] to look at any of this. What keeps you going?”

Nigel: “You know what it is, Theo? I just don’t think about it.”

Chapter 9

On a world without graduate students, *Children of Men*

Subharmonic Motional Raman

Abstract

A key advantage of quantum metrology is the ability to surpass the [Standard Quantum Limit \(SQL\)](#) for measurement precision through the use of non-classical states. However, there is typically little to no improvement in precision with the use of non-classical states for measurements whose duration exceeds the decoherence time of the underlying quantum states. Measurements aimed at the ultimate possible precision are thus performed almost exclusively with classical states and, therefore, are constrained by the [SQL](#). Here, we demonstrate that by using the phenomenon of subharmonic excitation, in combination with a recently demonstrated technique of Raman excitation of a harmonic oscillator, the frequency of an electric field can be measured at a resolution below the standard quantum limit of the corresponding linear generator, which we term the linear standard quantum limit. With this method we measure a radio-frequency electrical signal with a fractional frequency uncertainty of $0.56 \text{ Hz}/80 \text{ MHz} = 7 \times 10^{-9}$, which to our knowledge is the most precise frequency measurement of a radio-frequency electrical signal using a quantum harmonic oscillator. Because the input states can be classical, the coherence time is not degraded by the enhanced decoherence typically associated with nonclassical states, thereby improving

the ultimate achievable precision. While we demonstrate this technique using motional Raman subharmonic excitation of a single $^{40}\text{Ca}^+$ ion through engineered Floquet states, this technique is expected to be extendable to other platforms, such as NV centers, solid-state qubits, and neutral atoms, where it can provide metrological gain for sensing across the radio frequency, microwave, and optical domains.

Contributions We thank [DK](#) for building a computer cluster used to simulate these results. This work was supported by NSF (PHY-2110421 and OMA-2016245), AFOSR (130427-5114546), and ARO (W911NF-19-1-0297).

9.1 Introduction

Background

[Quantum Harmonic Oscillators \(QHOs\)](#) are highly-sensitive platforms for detecting optical and radio-frequency (RF) electromagnetic fields near the natural frequency ω of the oscillator [[Hem+13](#),[Bur+19](#),[Wol+19](#)]. QHOs are thus powerful precision sensors in areas as diverse as ultra-precise measurements of force [[Knü+10](#),[Bie+10](#),[Gil+17](#),[Liu+21](#)] and acceleration [[CH17](#)], searches of axionic dark matter [[Bra+03](#)], and gravitational wave detection [[Col11](#)]. Importantly, the recent development of motional Raman protocols have unlocked the potential of QHOs for broadband field sensing by extending the usable frequency range by over three orders of magnitude across the RF and microwave bands [[Wu+25b](#)].

The sensitivity of a QHO-based measurement to a physical parameter θ is bounded by $\sigma_\theta = (\sqrt{F_q(\phi)}(\partial\phi/\partial\theta))^{-1}$, where $F_q(\phi)$ is the Fisher information for parameter ϕ of a QHO in state q . For coherent states $|\alpha\rangle$, the displacement α is typically a product of the interrogation time τ and some function f that depends on the frequency ω_s and strength Ω of an applied field (referred to hereafter as the signal field), i.e. $\alpha = \tau f(\omega_s, \Omega)$. Thus, given that the Fisher information for a protocol that measures QHO displacement (α) $F_\alpha \leq 4$, the [SQL](#) bounds

the sensitivity of measurements of the signal frequency to $\sigma_{\omega_s} = (2\tau(\partial f/\partial\omega_s))^{-1}$. Further limitations appear in systems with finite coherence time τ_c where sensitivity is optimized at $\tau \approx \tau_c/2$ [Hue+97].

To improve sensitivity, non-classical states are employed, such as Schrödinger cat states [Hem+13,Joh+17,Mon+96], squeezed states [Bur+19,Wu+25b], and high Fock states [Wol+19,McC+19a], to increase the Fisher information [Wol18]. However, these states are challenging to generate and control, as their coherence times scale inversely with metrological gain [Hue+97]. In practice, extracting gains > 10 dB with non-classical states, for even short timescales, has proven difficult [Pez+18].

Nonlinearity-Enhanced Sensing

Here, we instead leverage a nonlinear measurement as an alternative means to enhance sensitivity. It has been theoretically proposed that, by exploiting k -body (nonlinear) probe dynamics, the precision of parameter estimation can surpass the conventional $1/N$ “Heisenberg” scaling associated with linear (one-body) metrology, instead following a scale of $1/N^k$ with optimal entangled inputs, and $1/N^{k-1/2}$ scaling with product inputs [Boi+07,Boi+08,Boi+09,HZL24]. In contrast, we instead propose and experimentally demonstrate an alternative nonlinear approach. We show that the precision achieved with our nonlinear generator surpasses the standard quantum limit of the corresponding linear generator (hereafter referred to as the linear SQL ($K=2$)). Specifically, by parametrically exciting a QHO subharmonic resonance at $2\omega/K$, where ω is the QHO resonance and $K \in \mathbb{N}$, using a motional Raman protocol [Wu+25b], we demonstrate that the frequency precision of a signal field can be improved by $\sigma_{\omega_s} = (K\tau(\partial f/\partial\omega_s))^{-1}$. This $K/2$ sensitivity enhancement arises from a $K/2$ -photon process and requires only classical states, entailing no cost to the coherence time τ regardless of the order K . To be clear, our protocol employs classical probe states and therefore does not surpass the SQL of the nonlinear generator, which improves the scaling by a factor of $K/2$ relative to the linear SQL (see section 9.2.4).

We demonstrate this technique on the motional modes of a single, trapped $^{40}\text{Ca}^+$ ion and confirm that subharmonic excitation technique is a K -order nonlinear effect via motional Ramsey spectroscopy. We showcase wideband operation by sensing fields at various signal frequencies, observing the expected $2/K$ sensitivity scaling up to $K = 24$. Finally, we benchmark the ultimate sensitivities by conducting frequency Allan deviations at various K , finding improvement of 12.3(9) dB over the corresponding linear SQL; in all cases, the corresponding linear SQL is surpassed without use of non-classical states. We achieve a minimum frequency uncertainty of 0.56(32) Hz, representing a sixfold improvement over previous measurements [Wu+25b] to set a new bar for the state-of-the-art in QHO-based frequency sensing in the RF regime.

Nonlinear Subharmonics

Subharmonic excitation of a harmonic oscillator is driven by a force $|\vec{F}| = x(\Omega/2) \cos(\omega_p t)$, where the amplitude is proportional to the equilibrium displacement x . The resultant potential $V = x^2 \Omega \cos(\omega_p t)$ modulates the oscillator's natural frequency ω and excites a parametric resonance when $\omega_p = 2\omega/K$ for $K \in \mathbb{N}$ [LL76]. This process, equivalent to K -quanta absorption from the drive field to produce two phonons (see Figure 9.4(a)), results in a time evolution approximated by a QHO squeezing operator $\hat{S}(r)$, where $r \propto \Omega^K/(\omega/2)^{K-1}$ (see section 9.2.1). Subharmonics have been explored in mass spectrometry, where oscillating electric-field gradients were used to excite subharmonics and infer ion mass [Sud+00b,ZRS02,00a,Tom+03]. Resonance occurs at $2\omega = K(\omega_p^{(0)} + \delta)$, with $\omega_p^{(0)} = 2\omega/K$ and detuning δ . Since linewidth of a parametric resonance is a function of the drive frequency, it also becomes reduced by K . Though these past studies have observed linewidths below the Fourier transform limit (FTL), no improvement in mass resolution was observed. This is because, though the central frequency $\omega_p^{(0)}$ is determined with K -fold improved sensitivity, i.e. $\partial f/\partial \omega_p \rightarrow K \partial f/\partial \omega_p$, the parametric excitation is no more sensitive to ω , since $\sqrt{F(\omega_p)} = \sqrt{F(\omega)}/K$. Subharmonic excitation and its concomitant line-narrowing has

also been observed with multi-photon Raman transitions in ultracold atoms [TR87,Cat+01], albeit with a different underlying mechanism; as in the mass spectroscopy case, no metrological gain was achieved.

To overcome this limitation, we employ motional Raman transitions to heterodyne the signal tone with a probe tone and excite a subharmonic in the QHO baseband. Given the ability to choose the applied probe tone frequency ω_p , this technique allows broadband sensing of ω_s with sensitivity $K/2$ below the corresponding linear SQL.

Theoretical Overview

We first provide an overview of the relevant theory sufficient to follow the remainder of the chapter, sparing the reader the more involved theoretical treatment that follows (see¹ section 9.2). Suppose a “signal” tone with voltage V_s and phase ϕ_s at frequency ω_s in quadrupolar configuration (i.e. a linear field gradient) is applied to the ion. A “probe” tone at frequency $\omega_p = \omega_s + 2\omega_{r/z}/K + \delta$ is then applied with a dipole component V_d and quadrupole component V_q , where $\omega_{r/z}$ is the QHO resonance frequency on mode r, z . The Hamiltonian of the ion motion under these fields is:

$$\begin{aligned} \frac{\hat{H}}{\hbar} = & \omega_{r/z} \hat{a}^\dagger \hat{a} + \Omega_q (\hat{a} + \hat{a}^\dagger)^2 \cos(\omega_p t) \\ & + \Omega_d (\hat{a} + \hat{a}^\dagger) \cos(\omega_p t) \\ & + \Omega_s (\hat{a} + \hat{a}^\dagger)^2 \cos(\omega_s t + \phi_s), \end{aligned} \quad (9.1)$$

where $\Omega_q = eV_q\eta^2/\hbar$, $\Omega_d = eV_d\eta/\hbar$, $\Omega_s = eV_s\eta^2/\hbar$, and η is the effective Lamb-Dicke parameter. The dipole tone Ω_d is included as it carries a lower order of η than quadrupole tones and modifies the motional Raman interaction to produce a displacement, rather than a squeeze operator. This is beneficial as coherent states are more easily manipulated and, given our readout procedure, provide significantly higher Fisher information than squeezed

¹Avoid?

states (see section 9.2.1).

In the interaction picture with respect to $\omega \hat{a}^\dagger \hat{a}$, the Hamiltonian becomes:

$$\begin{aligned}
\frac{\hat{H}_I}{\hbar} = & + \frac{\Omega_q}{2} (\hat{a}^2 e^{-i(2\omega+\omega_p)t} + \hat{a}^{\dagger 2} e^{i(2\omega+\omega_p)t}) \\
& + \frac{\Omega_q}{2} (\hat{a}^2 e^{i(\omega_p-2\omega)t} + \hat{a}^{\dagger 2} e^{-i(\omega_p-2\omega)t}) \\
& + \frac{\Omega_q}{2} (2\hat{a}^\dagger \hat{a} + 1) (e^{i\omega_p t} + e^{-i\omega_p t}) \\
& + \frac{\Omega_d}{2} (\hat{a} e^{-i(\omega+\omega_p)t} + \hat{a}^\dagger e^{i(\omega+\omega_p)t}) \\
& + \frac{\Omega_d}{2} (\hat{a} e^{i(\omega_p-\omega)t} + \hat{a}^\dagger e^{-i(\omega_p-\omega)t}) \\
& + \frac{\Omega_s}{2} (\hat{a}^2 e^{-i(2\omega+\omega_s+\phi_s)t} + \hat{a}^{\dagger 2} e^{i(2\omega+\omega_s+\phi_s)t}) \\
& + \frac{\Omega_s}{2} (\hat{a}^2 e^{i(\omega_s-2\omega+\phi_s)t} + \hat{a}^{\dagger 2} e^{-i(\omega_s-2\omega+\phi_s)t}) \\
& + \frac{\Omega_s}{2} (2\hat{a}^\dagger \hat{a} + 1) (e^{i(\omega_s t+\phi_s)} + e^{-i(\omega_s t+\phi_s)}). \tag{9.2}
\end{aligned}$$

This can be expressed as a sum of eight Hermitian components, each comprised of a factor Ω_i , ladder operators, and time dependent phase $e^{\pm i\omega_i t}$ (see section 9.2.1, and eq. (9.12) for the corresponding Hamiltonian components):

$$\Omega_1 = \Omega_q, \quad \omega_1 = \omega_p + 2\omega_{r/z} \tag{9.3}$$

$$\Omega_2 = \Omega_q, \quad \omega_2 = \omega_p - 2\omega_{r/z} \tag{9.4}$$

$$\Omega_3 = \Omega_q, \quad \omega_3 = \omega_p \tag{9.5}$$

$$\Omega_4 = \Omega_d, \quad \omega_4 = \omega_p + \omega_{r/z} \tag{9.6}$$

$$\Omega_5 = \Omega_d, \quad \omega_5 = \omega_p - \omega_{r/z} \tag{9.7}$$

$$\Omega_6 = \Omega_s, \quad \omega_6 = \omega_s + 2\omega_{r/z} \tag{9.8}$$

$$\Omega_7 = \Omega_s, \quad \omega_7 = \omega_s - 2\omega_{r/z} \tag{9.9}$$

$$\Omega_8 = \Omega_s, \quad \omega_8 = \omega_s \tag{9.10}$$

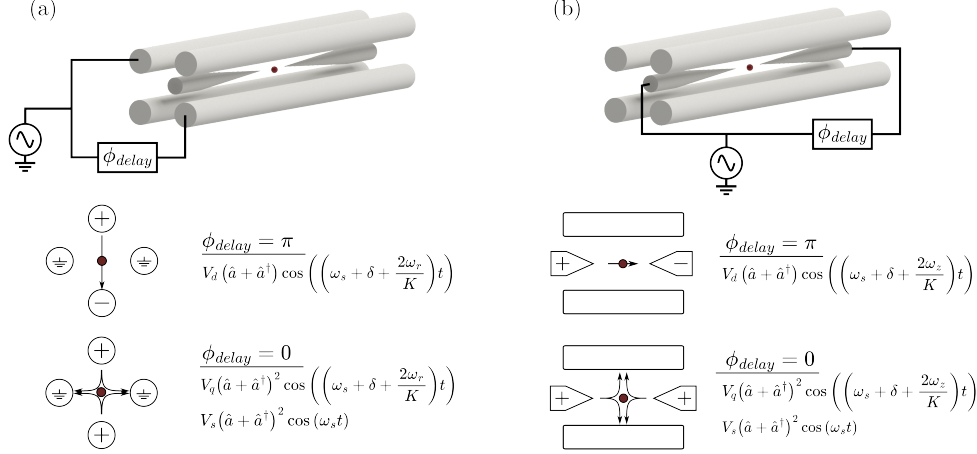


Figure 9.1: Implementation of subharmonic excitation on the radial and axial modes.

(a) Subharmonic excitation is applied to excite the radial motional mode by coupling an oscillating voltage onto a pair of opposing trap electrodes. (b) The axial motional mode can be excited by instead coupling the voltage onto the axial endcaps of the trap. A dipole field is generated for a relative phase $\phi_{\text{delay}} = \pi$ between the voltages coupled to the electrodes, while a quadrupole field is generated if the coupled voltages are exactly in phase (i.e. $\phi_{\text{delay}} = 0$).

Using multimode Floquet theory, the resulting effective Hamiltonian can be represented as an eight-mode Floquet Hamiltonian [ESM05, Wan+22]:

$$\hat{H} = \sum_{\vec{c}} \left(\hat{H}_1^{\vec{c}} + \cdots + \hat{H}_K^{\vec{c}} + \cdots \right) \times e^{-i((\vec{c} \cdot \vec{\omega})t + (c_6 + c_7 + c_8)\phi_s)}$$

$$\{\vec{c} \equiv (c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8)\}, \quad \{\vec{\omega} \equiv (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8)\} \quad (9.11)$$

where $\vec{c}$ and $\vec{\omega}$ denote the Floquet coefficients and frequencies for each mode, respectively, and the coefficient K denotes the perturbation order. Here, eq. (9.11) can be understood to drive virtual motional transitions in a multimode Floquet space.

While lower-order terms are readily obtained using methods from [ESM05, Wan+22], derivation of general analytical expressions for the K th-order term $\hat{H}_K^{\vec{c}}$ is nontrivial. Nonetheless, the essential behaviors are evident from eq. (9.11). A resonant displacement results when $\vec{c} \cdot \vec{\omega} = 0$, which, using $c_k \in \mathbb{Z}$, yields the following restrictions: for the probe tones, $\sum_{k=1}^5 c_k = \pm K/2$, and for the signal tone, $\sum_{k=6}^8 c_k = \mp K/2$.

9.2 Theoretical Analysis

In this section, we delve further into the dynamics of subharmonic motional excitation within the framework of multimode Floquet theory [ESM05, Wan+22].

9.2.1 Multimode Floquet Theory

We continue the brief theoretical overview in section 9.1, starting from eq. (9.11). The Fourier components corresponding to the eight selectable tones are given as follows:

$$\hat{H}^{(0,-1,0,0,0,0,0,0)} = \hat{H}^{(1,0,0,0,0,0,0,0)} = \frac{\Omega_q}{2} \hat{a}^{\dagger 2}, \quad (9.12)$$

$$\hat{H}^{(0,1,0,0,0,0,0,0)} = \hat{H}^{(-1,0,0,0,0,0,0,0)} = \frac{\Omega_q}{2} \hat{a}^2, \quad (9.13)$$

$$\hat{H}^{(0,0,-1,0,0,0,0,0)} = \hat{H}^{(0,0,1,0,0,0,0,0)} = \frac{\Omega_q}{2} (2\hat{a}^\dagger \hat{a} + 1), \quad (9.14)$$

$$\hat{H}^{(0,0,0,0,-1,0,0,0)} = \hat{H}^{(0,0,0,0,1,0,0,0)} = \frac{\Omega_d}{2} \hat{a}^\dagger, \quad (9.15)$$

$$\hat{H}^{(0,0,0,0,1,0,0,0)} = \hat{H}^{(0,0,0,0,-1,0,0,0)} = \frac{\Omega_d}{2} \hat{a}, \quad (9.16)$$

$$\hat{H}^{(0,0,0,0,0,0,-1,0)} = \hat{H}^{(0,0,0,0,0,0,1,0)} = \frac{\Omega_s}{2} \hat{a}^{\dagger 2}, \quad (9.17)$$

$$\hat{H}^{(0,0,0,0,0,0,1,0)} = \hat{H}^{(0,0,0,0,0,0,-1,0)} = \frac{\Omega_s}{2} \hat{a}^2, \quad (9.18)$$

$$\hat{H}^{(0,0,0,0,0,0,0,-1)} = \hat{H}^{(0,0,0,0,0,0,0,1)} = \frac{\Omega_s}{2} (2\hat{a}^\dagger \hat{a} + 1), \quad (9.19)$$

When the resonance condition $\sum_{k=1}^8 c_k \omega_k = 0$ is satisfied, a higher-order subharmonic is excited, generating a motional displacement. The resonance condition can be reduced to

$$\left(\sum_{k=1}^8 c_k \right) \omega_s + \left(\left(\sum_{k=1}^5 c_k \right) \frac{2}{K} + (2c_1 - 2c_2 + c_4 - c_5 + 2c_6 - 2c_7) \right) \omega + \left(\sum_{k=1}^5 c_k \right) \delta = 0, \quad (9.20)$$

which yields the constraints:

$$\begin{aligned}
c_1 + c_2 + c_3 + c_4 + c_5 + c_6 + c_7 + c_8 &= 0, \\
(2c_1 - 2c_2 + c_4 - c_5 + 2c_6 - 2c_7) &= -1, \\
c_1 + c_2 + c_3 + c_4 + c_5 &= K/2,
\end{aligned} \tag{9.21}$$

which can be intuitively understood as a multi-phonon Raman transition. Generation of a displacement resonant with the secular frequency ω requires that the number of probe tones matches the number of signal tones to cancel the carrier frequency ω_s . As derivation of a general analytical expression for the arbitrary K th-order Hamiltonian that includes all parameters is nontrivial; instead, we here derive an empirical formula.

Empirical Derivation for α

The K th-order Hamiltonian $H_{(K)}^{(c_1, \dots, c_8)}$ involves $K-1$ commutation operations [ESM05, Wan+22]. From (9.12), all Fourier components $H_{(K)}^{(c_1, \dots, c_8)}$ are comprised of only first-order ($\hat{a}$, $\hat{a}^\dagger$) or second-order ($\hat{a}^2$, $\hat{a}^{\dagger 2}$, $\hat{a}^\dagger \hat{a}$) ladder operators. The commutation between ladder operators is nonvanishing only when the number of first-order ladder operators is exactly one, which occurs only if the dipole tone contributes only one of K terms in the Hamiltonian. Together with the resonance constraints, we arrive at an empirical formula for subharmonic driving of a coherent state:

$$\alpha = \beta(\omega_s, K, \omega_{r/z}) \Omega_d \Omega_q^{(K-2)/2} \Omega_s^{K/2} e^{-i\phi_s K/2}, \tag{9.22}$$

where β is a function of the signal frequency ω_s , the secular frequency ω , and the subharmonic order K , and can be numerically calculated from QuTip simulations [JNN12, JNN13].

Verification With K=2

We confirm the Floquet theory in section 9.2.1 by evaluation of the $K = 2$ case. The second-order term in the eight-mode Floquet theory is:

$$H_{(2)}^{\vec{c}} = -\frac{1}{2} \sum_{\vec{c}'} \frac{[H^{\vec{c}}, H^{\vec{c}-\vec{c}'}]}{\vec{c} \cdot \vec{\omega}}. \quad (9.23)$$

Evaluation of (9.23) gives:

$$\begin{aligned} H_{(2)} &= 2 \left(H_{(2)}^{(0,0,0,1,0,-1,0,0)} + H_{(2)}^{(0,0,0,-1,0,1,0,0)} + H_{(2)}^{(0,0,0,0,1,0,0,-1)} + H_{(2)}^{(0,0,0,0,-1,0,0,1)} \right) \\ &= - \left(\frac{[H^{(0,0,0,0,0,-1,0,0)}, H^{(0,0,0,1,0,0,0,0)}]}{\omega_p + \omega} + \frac{[H^{(0,0,0,0,0,1,0,0)}, H^{(0,0,0,-1,0,0,0,0)}]}{-(\omega_p + \omega)} \right. \\ &\quad \left. + \frac{[H^{(0,0,0,0,0,0,0,-1)}, H^{(0,0,0,0,1,0,0,0)}]}{\omega_p - \omega} + \frac{[H^{(0,0,0,0,0,0,0,1)}, H^{(0,0,0,0,-1,0,0,0)}]}{-(\omega_p - \omega)} \right) \\ &= -\frac{\Omega_s \Omega_d}{4(\omega_p + \omega)} [\hat{a}^2, \hat{a}^\dagger] + \frac{\Omega_s \Omega_d}{4(\omega_p + \omega)} [\hat{a}^{\dagger 2}, \hat{a}] \\ &\quad - \frac{\Omega_s \Omega_d}{4(\omega_p - \omega)} [2\hat{a}^\dagger \hat{a}, \hat{a}] + \frac{\Omega_s \Omega_d}{4(\omega_p - \omega)} [2\hat{a}^\dagger \hat{a}, \hat{a}^\dagger] \\ &= \frac{\omega \Omega_s \Omega_d}{\omega_p^2 - \omega^2} (\hat{a}^\dagger + \hat{a}), \end{aligned} \quad (9.24)$$

which is consistent with the result obtained from the Magnus expansion [Wu+25b]. The QHO squeezing operator $\hat{S}(r)$, generated via parametric excitation, can also be analyzed using Floquet theory. Similarly, application of a single quadrupole tone at frequency ω_p results in a Hamiltonian:

$$\frac{\hat{H}}{\hbar} = \omega \hat{a}^\dagger \hat{a} + \Omega_q (\hat{a} + \hat{a}^\dagger)^2 \cos(\omega_p t), \quad (9.25)$$

where $\omega_p = 2\omega/K + \delta$ and $\Omega_q = eV_q\eta^2/\hbar$. Transforming into the interaction picture with respect to $\omega\hat{a}^\dagger\hat{a}$, the Hamiltonian (9.25) becomes:

$$\begin{aligned}\frac{\hat{H}_I}{\hbar} = & \frac{\Omega_q}{2} (\hat{a}^2 e^{-i(2\omega+\omega_p)t} + \hat{a}^{\dagger 2} e^{i(2\omega+\omega_p)t}) \\ & + \frac{\Omega_q}{2} (\hat{a}^2 e^{i(\omega_p-2\omega)t} + \hat{a}^{\dagger 2} e^{-i(\omega_p-2\omega)t}) \\ & + \frac{\Omega_q}{2} (2\hat{a}^\dagger\hat{a} + 1) (e^{i\omega_p t} + e^{-i\omega_p t})\end{aligned}\tag{9.26}$$

The Hamiltonian can be treated as a sum of three Hermitian components, each composed of a factor Ω_i , ladder operators, and a time dependence $e^{\pm i\omega_i t}$, where $\{\Omega_1 = \Omega_q, \omega_1 = (2/K + 2)\omega\}$, $\{\Omega_2 = \Omega_q, \omega_2 = (2/K - 2)\omega\}$ and $\{\Omega_3 = \Omega_q, \omega_3 = 2\omega/K\}$. Using Eqs. (A16) and (A17) in [Wan+22], the K^{th} -order term of the Hamiltonian can be approximately expressed as $\hat{H} = \Omega^K / (\omega/2)^{K-1} (\hat{a}^2 + \hat{a}^{\dagger 2})$, creating a squeezing operator $\hat{S}(r)$ where $r \propto \Omega^K / (\omega/2)^{K-1}$.

Role of the Probe Tone

Unlike previous schemes [Sud+00b,ZRS02,00a,Tom+03] which have exclusively employed quadrupole tones, the present approach applies an additional probe tone in the dipole configuration for several reasons:

- The effective Lamb-Dicke parameter η associated with a dipole tone is of a lower order than that of quadrupole tones, which reduces the power threshold required to excite higher-order subharmonics.
- Application of a dipole tone induces motional excitation that generates a coherent state rather than a squeezed state. Coherent states operate within a linear regime compared to squeezed states, making them more straightforward to manipulate compared to a squeezed state.
- Measurement of a coherent state provides higher Fisher information, leading to enhanced sensitivity in parameter estimation, such as for frequency measurements.

9.2.2 K -th order Dynamics

We are interested in the behavior near a subharmonic resonance, i.e. $\vec{c} \cdot \vec{\omega} = \gamma$, where γ is some small detuning from resonance and $\gamma \approx \delta \ll \omega \ll \omega_s, \omega_p$. These resonance conditions (eq. (9.21)) can be modified:

$$\left(\sum_{k=1}^8 c_k\right) \omega_s + \left(\left(\sum_{k=1}^5 c_k\right) \frac{2}{K} + (2c_1 - 2c_2 + c_4 - c_5 + 2c_6 - 2c_7)\right) \omega + \left(\sum_{k=1}^5 c_k\right) \delta = \gamma, \quad (9.27)$$

which in turn yields the constraints

$$\begin{aligned} c_1 + c_2 + c_3 + c_4 + c_5 + c_6 + c_7 + c_8 &= \gamma, \\ (2c_1 - 2c_2 + c_4 - c_5 + 2c_6 - 2c_7) &= -1, \\ c_1 + c_2 + c_3 + c_4 + c_5 &= K/2. \end{aligned} \quad (9.28)$$

Collecting terms of order $O(\delta)$, the resonance condition eq. (9.28) is fulfilled only for $(\sum_{k=1}^5 c_k) \delta = \gamma$ since $\gamma \approx \delta \ll \omega \ll \omega_s, \omega_p$. Use of eq. (9.20) with eq. (9.28) gives $K\delta/2 = \gamma$, such that the time-dependence of the K -th order Hamiltonian (9.11) is:

$$e^{i\gamma t} = e^{-i(K\delta t/2 + (c_6 + c_7 + c_8)\phi_s)}, \quad (9.29)$$

which modifies eq. (9.11) to become

$$\bar{H}_I(t) = \sum_{\vec{c}} (H_{(1)}^{\vec{c}} + \dots + H_{(K)}^{\vec{c}} + \dots) e^{-i(K\delta t/2 + (c_6 + c_7 + c_8)\phi_s)} + \text{h.c.}, \quad (9.30)$$

Using prior knowledge that the Hamiltonian yields a displacement, eq. (9.30) simplifies to:

$$\bar{H}_I(t) = C \cdot \hat{a} \cdot e^{-iK/2\delta t - i(c_6 + c_7 + c_8)\phi_s} + \text{h.c.} \quad (9.31)$$

where C is a δ -independent amplitude. Following Eqns. (14-20) of [Lee+05], eq. (9.31) takes the form of an oscillating displacement with:

$$\alpha = C \cdot \text{sinc}\left(\frac{K\delta t}{4}\right). \quad (9.32)$$

Together with eq. (9.22), the displacement α can be made explicitly as

$$\alpha = \beta(\omega_s, K, \omega_{r/z}) \Omega_d \Omega_q^{(K-2)/2} \Omega_s^{K/2} e^{-i\phi_s K/2} \text{sinc}\left(\frac{K\delta t}{4}\right). \quad (9.33)$$

9.2.3 Numerical Simulations

Simulations were conducted in QuTiP by time-evolving the initial state under the influence of the subharmonic excitation Hamiltonian (9.1) via the time-dependent Schrödinger equation. Constituent data points in Figure 9.2(a-b) were obtained by simulating the system at different detunings δ to extract a lineshape, which was fitted using a sinc-squared lineshape to extract the maximum mean phonon number, linewidth, and center frequency. This procedure was repeated at different values of the target variable to extract the dependence on the target variable. The subharmonic excitation Hamiltonian was pulse-shaped in simulation using a tapered-cosine window with $100\mu\text{s}$ rise and fall times and a total length of 1.2ms . The basis size was varied for each data point to reduce simulation overheads, though confirmed to be sufficiently large to contain all simulation dynamics. The Fock state populations and Wigner functions were extracted for each simulation to confirm that a coherent state was produced.

The mean phonon number scales as V_d^2 for all orders of subharmonic excitation, where V_d is the dipole voltage, indicating an overall linear dependence of the displacement on V_d , in agreement with the theoretical analysis and the empirical formulae (9.33), (9.22). Simulations instead varying the quadrupole voltages V_s , V_q show that the resultant phonon number scales as $(V_s V_q)^{2(K-1)}$, indicating that the displacement scales as $K - 1$, again in agreement with the theoretical analysis and the empirical formula (9.22). Simulations in Figure 9.3 examine

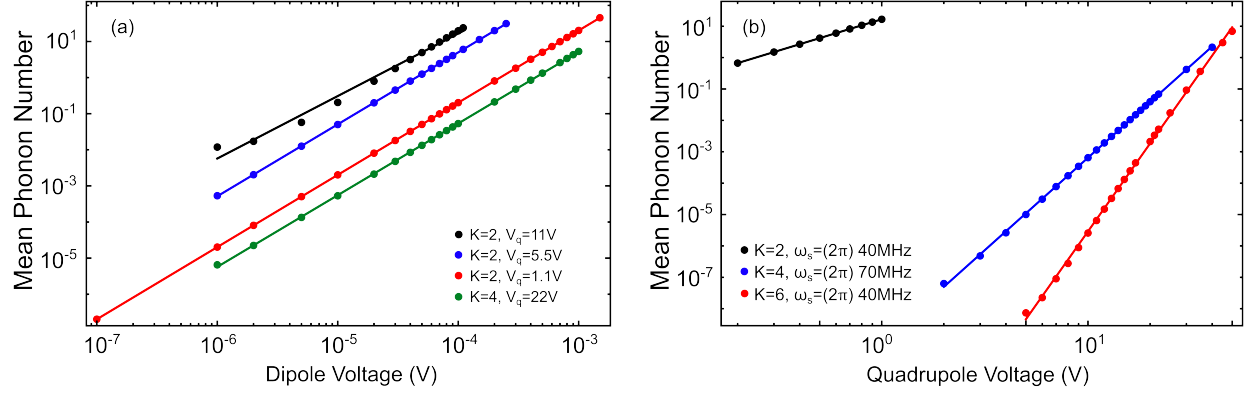


Figure 9.2: **Dependence of the subharmonic excitation on the applied dipole voltage V_d and the applied quadrupole voltages V_s, V_q , respectively.**

Results are simulated using a $\omega = 1$ MHz secular frequency. A linear model $y = mx + b$ is fitted to the log-log transform of the results to extract the power-law dependence at each subharmonic order. (a) Simulated results at all subharmonic orders exhibit an approximately quadratic dependence of mean phonon number on dipole voltage V_d , equivalent to a linear dependence of displacement on V_d . $\omega_s/2\pi = 40$ MHz. (b) Simulated results exhibit a power-law scaling of the mean phonon number on V_q , using a $V_d = 1$ mV, that agrees well with the predicted scaling of $2(K - 1)$.

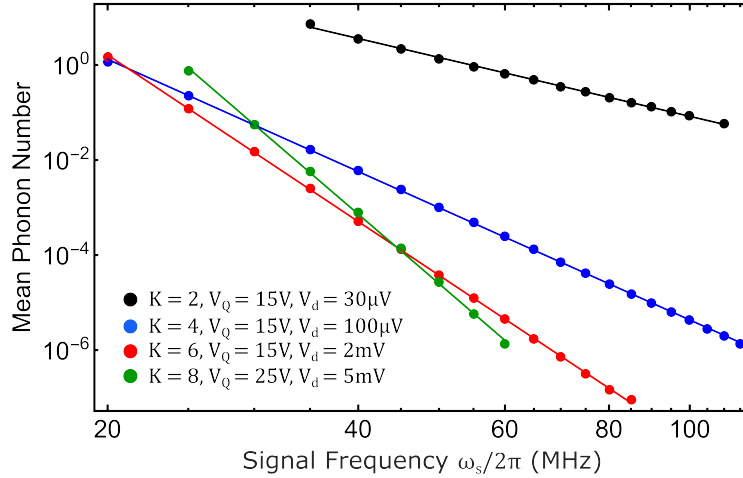


Figure 9.3: **Dependence of the subharmonic excitation on the signal tone frequency ω_s .**

Results are simulated using a $\omega = 1$ MHz secular frequency. A linear model $y = mx + b$ is fitted to the log-log transform of the results to extract the power-law dependence at each subharmonic order. Simulated results exhibit a power-law scaling of the mean phonon number on ω_s in good agreement with the predicted scaling of $\langle n \rangle \propto \omega_s^{2K}$, or equivalently, $\alpha \propto 1/\omega_s^K$.

the dependence of the mean phonon number $\langle n \rangle$ on the signal frequency ω_s . These results demonstrate that $\langle n \rangle \propto 1/\omega_s^{2K}$, indicating that the displacement scales as $\alpha \propto 1/\omega_s^K$.

9.2.4 The Nonlinear Standard Quantum Limit (SQL)

We here derive a general expression for the frequency uncertainty ($\Delta\delta$) for the K -th order subharmonic. Since only the δ -dependent term is relevant for deriving the uncertainty ($\Delta\delta$), eq. (9.33) can be simplified as:

$$, \alpha = |\tilde{a}| \frac{\sin(K\delta\tau/4)}{(K\delta\tau/4)} \quad (9.34)$$

where $\tilde{a}$ represents the product of all prefactors multiplying the Sinc function in eq. (9.33) and corresponds to the maximum displacement in phase space, τ is the interrogation time, and N is the number of experiments. $\tilde{a}$ is typically fixed at approximately 0.6 across different experiments and values of K .

The frequency uncertainty $\Delta\delta$ can be obtained through standard error propagation:

$$\Delta\delta = \frac{\Delta\alpha}{\frac{\partial\alpha}{\partial\delta}} = \frac{4\Delta\alpha}{K\tau} \frac{(K\delta\tau/4)^2}{(K\delta\tau/4 \cos(K\delta\tau/4) - \sin(K\delta\tau/4))} \frac{1}{|\tilde{a}|\sqrt{N}}, \quad (9.35)$$

where $\Delta\alpha$ denotes the uncertainty in α for a single measurement. Its value depends on the Fisher information associated with the measurement technique and is ultimately bounded by 0.5 for a classical state. The actual frequency uncertainty is determined from an Allan deviation analysis, denoted σ_{ω_s} . In this approach, two operating points close to the half-maximum of the signal are used, which optimally balance sensitivity and signal-to-noise ratio. Under these conditions, $\text{sinc}(\frac{K}{4}\delta\tau) = 1/\sqrt{2}$. Equation (9.35) thus becomes:

$$\Delta\delta \approx 2\pi \frac{4\Delta\alpha}{K} \frac{0.418}{|\tilde{a}|\tau\sqrt{N}}, \quad (9.36)$$

Increasing K thus leads to a smaller uncertainty of δ . When $\Delta\alpha$ reaches the shot-noise limit, i.e. $\Delta\alpha = 1/2$, the frequency uncertainty $\Delta\delta_{SQL}$ corresponding to the SQL of the K th-order nonlinear generator is therefore:

$$\Delta\delta_{SQL} \approx 2\pi \frac{2}{K} \frac{0.418}{|\tilde{a}|\tau\sqrt{N}}. \quad (9.37)$$

The linear standard quantum limit (SQL) corresponds to the special case of $K = 2$, which can be obtained directly from eq. (9.36). The linear SQL for frequency sensing is thus:

$$\Delta\delta_{SQL}^{K=2} \approx 2\pi \frac{0.418}{|\tilde{a}|\tau\sqrt{N}}. \quad (9.38)$$

9.3 Characterization

The theory in section 9.2 has several consequences. First, δ becomes amplified by a factor $\sum_{k=1}^5 c_k = K/2$, narrowing the linewidth relative to a linear measurement by $2/K$. Second, the K th-order Hamiltonian $H_K^{(i)}$ involves $K - 1$ commutators [ESM05,Wan+22]. Since all Fourier components $H_K^{(i)}$ are comprised of only first-order ($\hat{a}$, $\hat{a}^\dagger$) or second-order ($\hat{a}^2$, $\hat{a}^{\dagger 2}$, $\hat{a}^\dagger \hat{a}$) operators, the K th-order Hamiltonian is nonvanishing only when the number of first-order operators is exactly one, which occurs when the dipole tone contributes only one term (see section 9.2.3). These determine the number of tones needed at each frequency, such that the displacement α follows eq. (9.33) where β is a function of the signal frequency ω_s , the QHO frequency $\omega_{r/z}$, and subharmonic order K , independent of δ (see section 9.2.2). The multimode Floquet process can be illustrated as a multiphoton transition, shown in Figure 9.4(a). The dash-dotted line represents the subharmonic excitation for a quadrupole-only configuration (ii), while the dashed lines indicate excitation for a quadrupole/dipole configuration (iii).

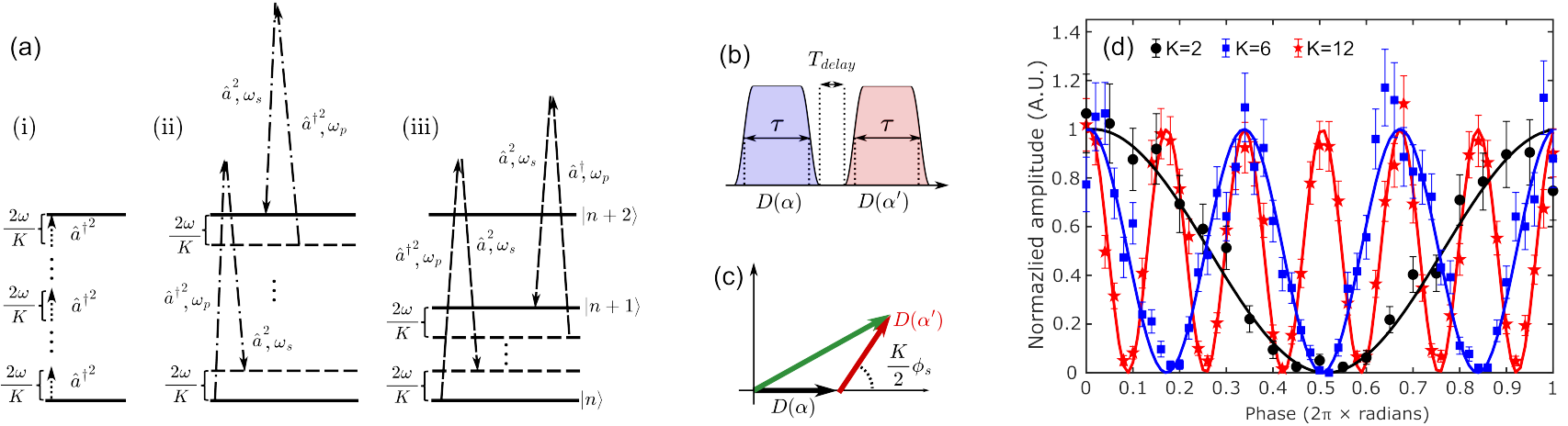


Figure 9.4: **Energy-level diagrams for motional subharmonic excitation and verification via motional Ramsey spectroscopy.**

(a) Level diagrams for a K th-order parametric excitation (i), quadrupole subharmonic Raman excitation (ii) and quadrupole/dipole subharmonic Raman excitation (iii). (b-d) Experimental verification of accelerated dynamics via a motional Ramsey sequence with a higher-order subharmonic excitation. (b) Simplified pulse sequence of the motional Ramsey scheme used to examine the phase response. A subharmonic excitation pulse is applied for interrogation time τ to generate a displacement $\hat{D}(\alpha)$. Following free evolution for time T_{delay} , a second subharmonic excitation pulse of equal area is applied with phase ϕ_s relative to the initial pulse to generate a second displacement $\hat{D}(\alpha')$, mapping the phase onto the ion's motional state. (c) Motional Ramsey protocol in phase space. The phase ϕ_s rotates the second displacement $\hat{D}(\alpha')$ relative to the first. The phase dependence at order K thus corresponds to the periodicity of the Ramsey fringes. (d) Ramsey fringes generated by scanning the phase ϕ_s of the second subharmonic excitation pulse at different orders K . Here, $\tau = 1$ ms with delay time is $T_{\text{delay}} = 50 \mu\text{s}$. Solid lines are fits of the data to a sine. Excitation amplitudes for all subharmonic orders K are normalized for comparison. Error bars represent one standard error.

Implementation To verify this result and demonstrate the technique, we employ the QHO of our trapped $^{40}\text{Ca}^+$ ion with radial (axial) secular frequency $\omega_r \approx 2\pi \cdot 1$ MHz ($\omega_z = 2\pi \cdot 0.7$ MHz). Doppler cooling of the ion followed by resolved-sideband cooling on the optical qubit transition prepares both the radial (axial) motional mode near the ground state with $\langle n \rangle \approx 0.01(1)$. The motional dephasing time τ_c of the radial (axial) mode without application of external fields is ≈ 2 ms (≈ 20 ms).

Subharmonic excitation is driven on a given QHO mode by applying to the trap electrodes both the signal tone at ω_s in a quadrupole configuration and the probe tone at $\omega_p = \omega_s + 2\omega_{r/z}/K + \delta$ with both dipole and quadrupole components (see section 9.1). Pulses are applied with a tapered-cosine envelope to mitigate off-resonant excitation [Zar+19]. In the remainder of this chapter, we assume that all AC stark shifts of the secular frequency from the quadrupole tones are absorbed into the resonance condition.

9.3.1 Phase Evolution

We first confirm that the dynamics of subharmonic excitation agree with the model in eq. (9.33). We employ a motional Ramsey sequence [Wol+19], depicted in Figure 9.4(b), to demonstrate that subharmonic excitation is a K -th order process. Following cooling near the motional ground state $|0\rangle$, a resonant subharmonic excitation pulse is applied for interrogation time τ to generate a displacement $\hat{D}(\alpha)$. Following free evolution for T_{Delay} , a second resonant subharmonic excitation is applied with relative phase ϕ_s to generate a secondary displacement $\hat{D}(\alpha')$. For a K th-order process, this leads to a phase shift $K\phi_s/2$ of the displacement, confirmed in Figure 9.4(d) for different orders. Rabi oscillations on the blue motional sideband confirm that subharmonic excitation induces the expected displacement (Figure 9.5(a, b), insets).

Line-narrowing of our protocol, relative to the linear case, is examined via spectroscopy at different subharmonic orders K . Specifically, we measure the frequency of a signal tone by sweeping the probe detuning δ across the subharmonic resonance (Figure 9.5 (a,b)), then

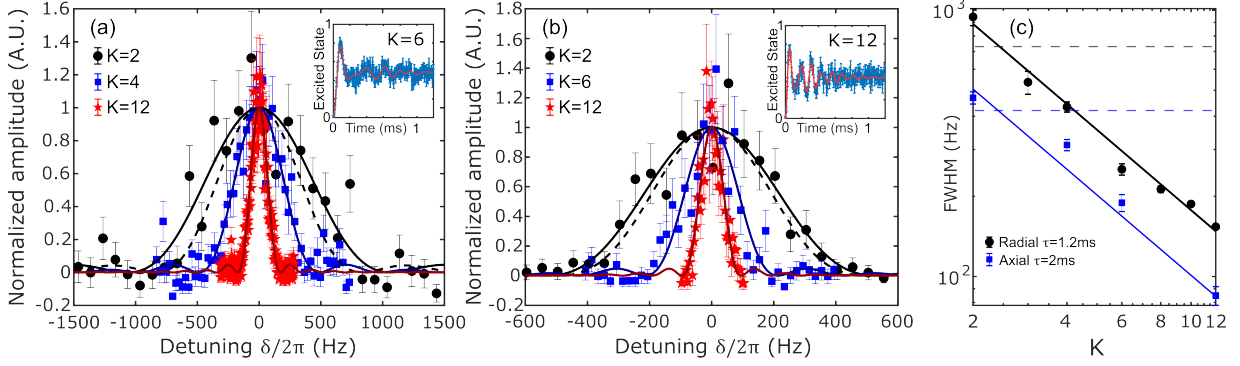


Figure 9.5: **Linewidth narrowing beyond the linear Standard Quantum Limit (SQL) using subharmonic excitation.**

(a, b) Detection of an “unknown” signal with subharmonic excitation at $\omega_s/2\pi = 70$ MHz on the radial mode (a) and $\omega_s/2\pi = 80$ MHz on the axial mode (b). The “unknown” frequency is determined by scanning the probe detuning to maximizing the signal amplitude. For visibility, fit amplitudes are normalized to unity, with corresponding data rescaled by the same factor. Solid lines are fits of the data to a sinc-squared lineshape. Insets in (a, b): Blue sideband Rabi oscillations characterize the motional state after subharmonic excitation. Solid red lines are fits assuming a motional coherent state. (c) Dependence of linewidth on subharmonic order K . The linewidth is defined as the Full-Width at Half-Maximum (FWHM). The solid line is a linear fit to the data with no offset. (a, black data in c) An effective interrogation time is $\tau = 1.2$ ms used at different values of K . (b, blue data in c) $\tau = 2.0$ ms at different values of K . τ is the pulse width of a square pulse of equivalent amplitude and area. (a, b, c) Dashed lines show the Fourier Transform Limit (FTL) of the applied tapered-cosine pulse envelope and corresponds to the linear SQL. Error bars represent one standard error.

measure the resultant mean phonon number $\langle n \rangle$ using the sideband ratio technique [Wu+25b]. Linewidths were extracted by fitting a sinc-squared profile $|\beta(\omega_p)|^2$ to the data (Figure 9.5(c)). At $K \geq 4$, the linewidth of the subharmonic surpasses the linear case (dashed lines in Figure 9.5 (c)), with improvement by approximately 6 times over the linear case at $K = 12$. To demonstrate arbitrary frequency operation, we excite the $K = 6$ subharmonic at $\omega_s/2\pi = 70$, 80 and 200 MHz (see section 9.3.3).

From Figure 9.5(c), the linewidths narrow as $2/K$. The $2/K$ improvement is the maximum achievable and requires a square pulse envelope; non-square pulse envelopes, e.g. Blackman or Gaussian envelopes, result in reduced narrowing due to the nonlinear dependence on amplitude. We use a tapered-cosine pulse (Figure 9.4(b)) to retain the narrowing effect while suppressing off-resonant excitation.

9.3.2 Metrological Gain

We next examine the *useful* metrological gain relative to the corresponding linear SQL. The subharmonic lineshape is first measured, as in Figure 9.5(a,b). Subsequent frequency measurements are made at the two half-maxima of the lineshape, then fitted to determine the center frequency. These are used to compute the overlapping Allan (σ_{ω_s}), shown in Figs. 9.6 (a,b) for an 80 MHz signal at various K . For comparison, the solid line represents the corresponding linear SQL ($\Delta\delta_{SQL}^{K=2}$) computed from the pulse area. A minimum frequency resolution of 0.85(18) Hz was achieved at $N \approx 16000$ with $K = 12$ on the axial mode. The maximum metrological gain [Pez+18] of $20\log_{10}(\Delta\delta_{SQL}^{K=2}/\sigma_{\omega_s}^{min}) = 12.3(9)$ dB over the linear SQL was realized at $N \approx 1600$ with interrogation time $\tau = 2$ ms (see section 9.2.4). Likewise, a gain of 4.9(8) dB was obtained on the radial mode with $K = 24$ and $\tau = 2$ ms, with a minimum resolution 2.3(2) Hz at $N \approx 7200$. Reduction in measurement performance on the radial mode results from increased off-resonant excitation and greater secular frequency instability. Higher power requirements at higher orders result in a practical limit of $K = 24$ ($K = 12$) on the radial (axial) mode; likewise, an increase in sensing frequency results in increased power requirements. This limit is not fundamental. These effects can be mitigated by optimizing the RF coupling efficiency to reduce the required input powers.

The achieved 12.3(9) dB gain is comparable to that with a linear measurement on a nonclassical $|n \approx 20\rangle$ Fock state [Den+24]. However, there is a crucial difference. Non-classical states enhance sensitivity in parameter estimation by increasing separation between superposed states of a system, which results in greater phase accumulation for improved Fisher information [DC17]. However, non-classical states decohere faster - for example, the coherence time of a Fock state $|n\rangle$ worsens as $1/n^p$, where $p \geq 1$ [Tur+00a,00b]. This reduces *usable* measurement times to largely eliminate any metrological gain over the SQL in the context of ultimate precision. As our protocol requires only classical input states with metrological gain derived from a *nonlinear* interaction, it eliminates the excess decoherence associated with non-classical states to improve *ultimate* precisions for frequency sensing.

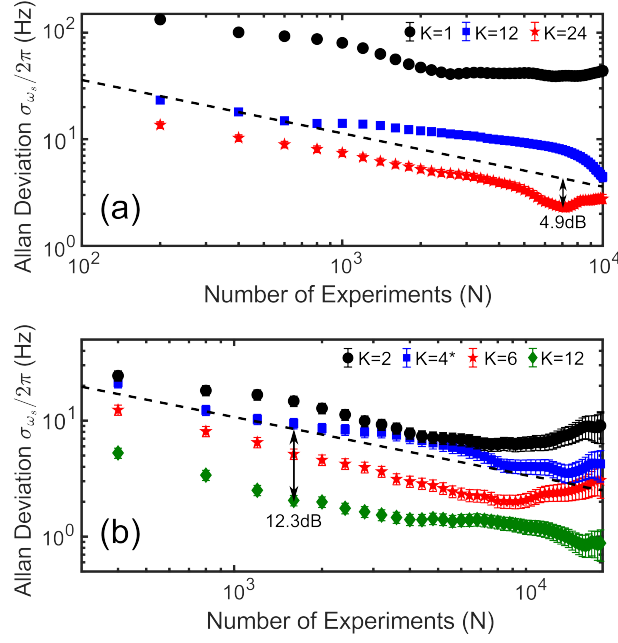


Figure 9.6: **Frequency resolution of the subharmonic protocol.**

(a-b) Overlapping frequency Allan deviations for an 80 MHz tone at different subharmonic orders K on the radial (a) and axial (b) modes. Each experiment takes ~ 17 ms. Solid lines represent the corresponding SQL for the given interrogation time, which correspond with the SQL of the $K = 2$ subharmonic. (a) Subharmonic excitation is probed on the radial mode with interrogation time $\tau = 2$ ms. A minimum frequency resolution $\sigma_{\omega_s}^{\min}/2\pi = 2.3(2)$ Hz is achieved with $K = 24$ order at $N \approx 7200$. A maximum metrological gain of 4.9(8) dB is similarly achieved with $K = 24$ at $N \approx 7000$. (b) Subharmonic excitation is probed on the axial mode with $\tau = 2$ ms. A minimum frequency resolution $\sigma_{\omega_s}^{\min}/2\pi = 0.85(18)$ Hz is achieved with $K = 12$ at $N \approx 16000$. A maximum metrological gain of 12.3(9) dB is achieved with the same $K = 12$ at $N \approx 1600$. Error bars represent one standard error.

Finally, a minimum frequency uncertainty $\sigma_{\omega_s}^{\min}/2\pi = 0.56(32)$ Hz was achieved at 80 MHz with $\tau = 5$ ms on the axial mode (see section 9.3.4). To our knowledge, this surpasses the precision of previous frequency measurements in the radio-frequency band using QHO-based system and sets a new state-of-the-art for these measurements. This measurement could easily be improved by approximately 4 times by improving damage thresholds of system electronics, with further gains achievable by extending motional coherence times.

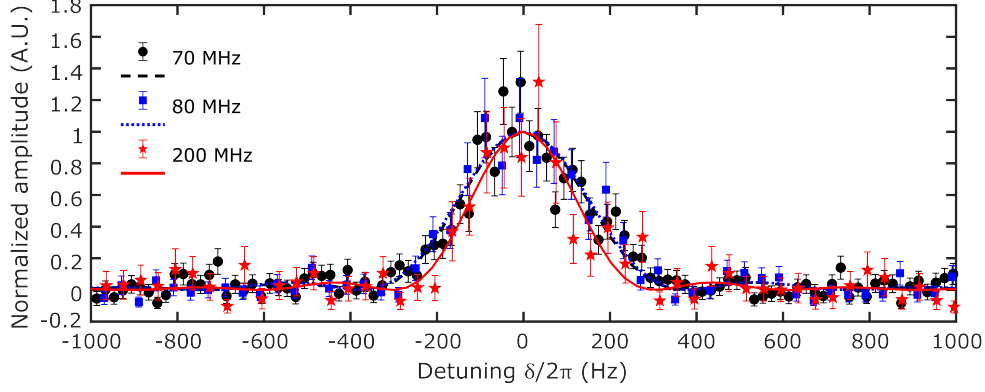


Figure 9.7: $K=6$ subharmonic excitation at different signal frequencies $(\omega_s/2\pi)=(70, 80 \text{ and } 200 \text{ MHz})$.

Lines are fits to the data using a sinc-squared profile. All data are normalized by the peak amplitudes of their fit. The interrogation time is fixed at $\tau = 1.2 \text{ ms}$. Error bars represent one standard error.

9.3.3 Wideband Demonstration

To confirm the wide frequency operation of our protocol, we characterized the $K=6$ subharmonic resonance at three different signal frequencies $\omega_s/2\pi = 70, 80$ and 200 MHz (Figure 9.7). Traces are normalized by maximum excitation and exhibit highly overlapping lineshapes, making our protocol uniquely suited to the challenges of wideband quantum sensing. The sensing with the $K = 6$ subharmonic at $\omega_s = 200 \text{ MHz}$ required an RF power of roughly 33 dBm . Hardware upgrades to e.g. improve signal coupling onto the trap electrodes should enable sensing up to $\approx 400 \text{ MHz}$.

9.3.4 Higher-Order Subharmonics

To achieve a state of art frequency resolution for an 80 MHz signal tone, we excited the $K = 12$ subharmonic resonance on the axial mode with an interrogation time up to $\tau = 5 \text{ ms}$, as illustrated in the inset of Figure 9.8. The Allan deviation was determined by measuring two points at the **Full-Width at Half-Maximum (FWHM)** of the motional spectrum. A minimum frequency resolution of $\sigma_{\omega_s}/2\pi = 0.56(32) \text{ Hz}$ after $N = 9600$ experiments. The motional coherence time for the axial mode is 20 ms . The primary limitation on the achievable minimum

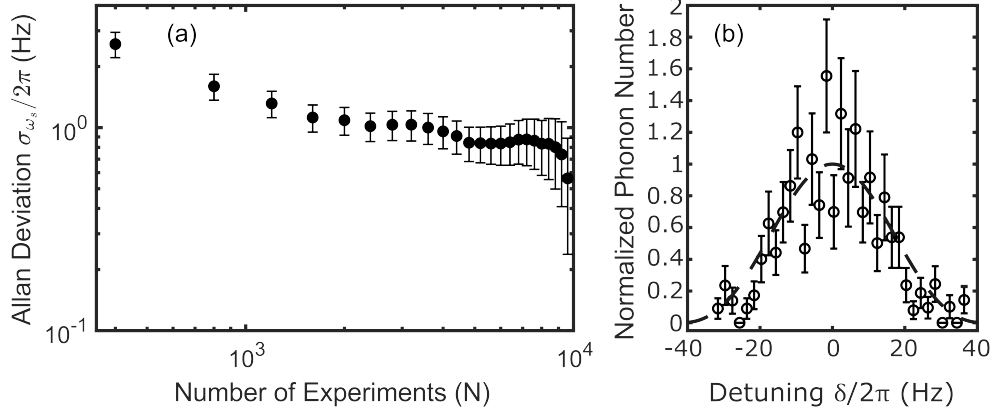


Figure 9.8: **High resolution scan of an 80 MHz probe tone using the $K = 12$ subharmonic resonance on the axial mode.**

(a) An overlapped Allan deviation yields a minimum frequency uncertainty $\sigma_{\omega_s}/2\pi = 0.56(32)$ Hz at $N = 9600$. The interrogation time is fixed at $\tau = 5$ ms. Each individual experiment takes 22 ms and is comprised of cooling pulses, the subharmonic excitation sequence, and state detection. (b) Motional excitation spectrum generated by scanning the frequency of the probe tone. Error bars represent one standard error.

frequency resolution is the damage threshold of trap electronics (5 W). Measurements at other values of K at $\tau = 5$ ms were not pursued to avoid risking damage to the electronic hardware. Upgrades to system electronics would allow excitation of subharmonic orders beyond $K = 12$, improving the minimum resolution by a factor of at least $> 4\times$.

9.4 Discussion

9.4.1 Coherent State Detection

A common method for determining the mean phonon number $\langle n \rangle$ of different quantum states involves measuring the excited state population (P_r) after a pulse applied for $t = \pi/\Omega_{01}$ the red motional sideband of the $|^2S_{\frac{1}{2}}, m_J = -\frac{1}{2}\rangle \Rightarrow |^2D_{\frac{5}{2}}, m_J = -\frac{5}{2}\rangle$ qubit transition, where Ω_{01} is the Rabi frequency on the blue motional sideband on the same transition between Fock state $n=0$ and $n=1$.

In Figure 9.9 (a), we plot the mapping of phonon number onto the excited state population for both a coherent state and a squeezed state using our protocol. As can be seen, due to

the form of a squeezed state in the Fock basis, the slope of the mapping is much less steep for a squeezed state than a coherent state. Intuitively, this means that squeezed states will provide worse performance during the readout step. More quantitatively, in the framework of quantum metrology, better sensitivity corresponds to a larger Fisher information. For the same mean phonon number, the Fisher information for readout of a squeezed state (F_s) is lower than for that of a coherent state (F_c) for our readout scheme. Specifically, the Fisher information of our readout is $F(n) = \sum \frac{1}{P_r} \left(\frac{\partial P_r}{\partial n} \right)^2 = \frac{1}{P_r(1-P_r)} \left(\frac{\partial P_r}{\partial n} \right)^2$. From the relation between excited state population (P_r) and phonon number shown in Figure 9.9(a), the ratio of the Fisher information between a coherent and a squeezed state is shown in Figure 9.9(b); the improved sensitivity of the coherent state is apparent as F_c/F_s exceeds unity for all $\langle n \rangle$, demonstrating that coherent states yield higher Fisher information and therefore improved detection sensitivity, making them advantageous for precision measurement.

9.4.2 Frequency Sensitivity under Dephasing with Different Strategies

Though nonclassical states are used to improve sensitivities [Bur+19,Wol+19,McC+19a][Gil+17,Hem+13], they incur a penalty of excess decoherence due to their sub-Planck structure [Zur01,Tur+00a]. This is well-noted with GHZ states [DRC17], which can be used to enhance a signal by a factor of M , where M is the number of probes in the GHZ state. However, in the presence of pure dephasing, GHZ states decohere M times faster, which negates their M -fold gain to yield *no* net metrological gain. Similarly, when sensing on bosonic modes (such as those of a trapped ion), Fock superposition states $|\psi_n\rangle = \frac{|0\rangle+|n\rangle}{\sqrt{2}}$ offer Heisenberg-limited n -fold gain, but decohere in excess by a factor n^2 for a net *reduction* in metrological potential [McC+19a,Tur+00a,00b]. In this context, our nonlinear protocol can be used to achieve *ultimate* metrological gain as it yields amplification while still creating a coherent state — as coherent states are classical and lack the sub-Planck structure of nonclassical states, they incur no excess decoherence, which, when combined with our nonlinear protocol, allows for

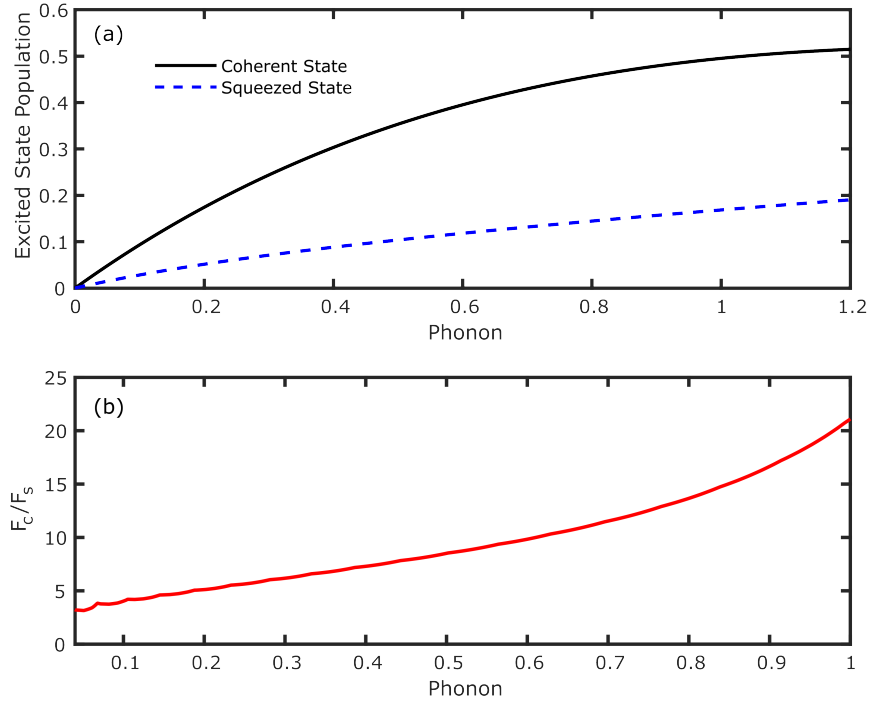


Figure 9.9: **Characterizing the sensitivity of phonon readout for coherent and squeezed states.**

(a) The relation between population probability in excited state $|^2D_{\frac{5}{2}}\rangle$ and phonon number for different motional states. The population is measured after applying red sideband Rabi oscillation for interrogation time $\tau = \pi/\Omega_{01}$, where Ω_{01} is the Rabi frequency for the $|^2S_{\frac{1}{2}}, m_J = -\frac{1}{2}, n = 0\rangle$ to $|^2D_{\frac{5}{2}}, m_J = -\frac{5}{2}, n = 1\rangle$ transition. (b) The Fisher information for a coherent state (F_c) is greater than the Fisher information of a squeezed state (F_s) for any mean phonon number, making them a more desirable quantum state for precision measurements.

net, *penalty-free* metrological gain.

To concretely compare the metrological potential of our nonlinear protocol in the presence of decoherence, we simulate the following motional Ramsey protocol for frequency sensing: the state is first displaced, then subject to free evolution for a delay time T_D , then antidisplaced [Wol+19, McC+19a, McC+19b]. The state is coupled to a phase reservoir over the entirety of the protocol to induce dephasing. This coupling is selected to give $T_2 = 2$ ms. Amplification strategies using nonclassical states typically apply this motional Ramsey on the state, then apply time-reversal dynamics (i.e. undo the nonclassical state) to remove any added noise

[Bur+19,Bur+21]. For example, quantum-enhanced frequency sensing with fock states uses a protocol of state creation ($|0\rangle \rightarrow |n\rangle$), displacement, free evolution, antidisplacement, and state reversal ($|n\rangle \rightarrow |0\rangle$) [Wol+19]. For simplicity, we assume the nonclassical state creation and reversal to be instantaneous. By contrast, our nonlinear subharmonic protocol involves no nonclassical state dynamics; instead, the displacement (and antidisplacement) is simply generated using a higher-order subharmonic resonance - here, we use the $K = 10$ subharmonic.. As a reference, we show the bare $K = 2$ linear protocol, which represents the standard quantum limit. For fair comparison between nonclassical protocols and our nonlinear protocol, the displacement of the motional Ramsey is kept constant at $\alpha = 0.1$, while the amplification of the nonclassical states (characterized by the mean phonon number $\bar{n}$ of the amplified state) is kept constant across the nonclassical states, i.e. $\bar{n} = 5$.

To determine the ultimate achievable metrological gain of each protocol, we simulate a frequency linescan at each delay time T_D . The maximum frequency Fisher information at each delay time is then calculated from the linescan using $F(\delta) = \frac{(\partial_\delta P)^2}{\text{var}(P)}$ [Wol+19], from which the sensitivity σ_δ can be extracted (via $\sigma_\delta = 1/\sqrt{F(\delta)/T}$).

From Figure 9.10, though Fock states achieve the best sensitivity when using a nonclassical strategy, this improvement is surpassed by our nonlinear protocol using the $K = 5$ subharmonic. Again, this is due to the fact that our nonlinear protocol results in the creation of a classical coherent state, which suffers no excess decoherence. In this sense, such a nonlinear protocol can be said to be *penalty-free*.

9.4.3 Conclusion

In summary, we have demonstrated high-resolution frequency measurements using a nonlinear generator that surpasses the corresponding linear SQL, enabled by driving Floquet-engineered motional Raman subharmonics. This technique is adaptable to platforms supporting Raman transitions, including nitrogen-vacancy centers, superconducting circuits, and neutral atom systems. With a stable reference, this method refines electromagnetic-field frequency sensing

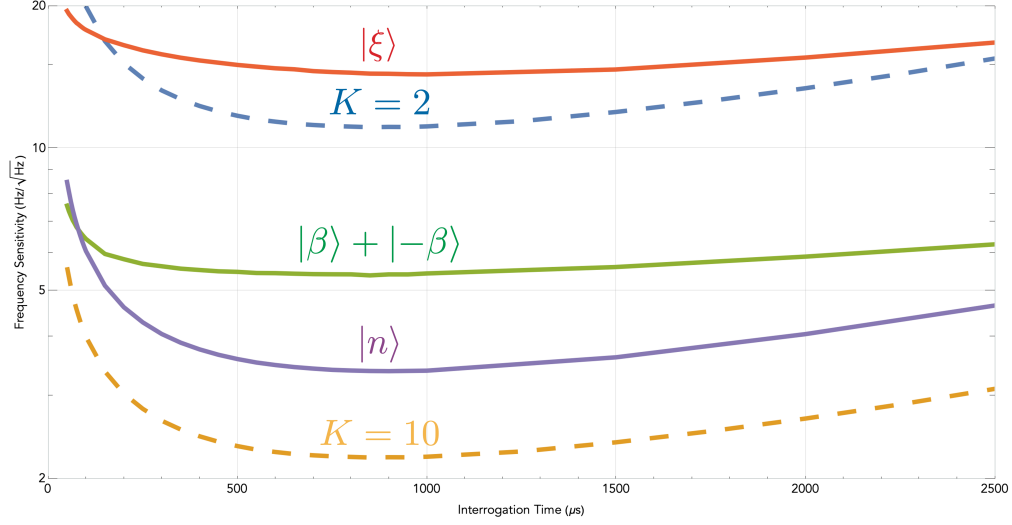


Figure 9.10: **Simulated frequency sensitivity using a motional Ramsey protocol (displace - delay - antidisplace) with different amplification strategies in the presence of dephasing ($T_2 = 2$ ms).**

The amplification-free case is represented by the $|K = 2\rangle$ linear protocol, which uses $|\alpha = 0.1\rangle$. Quantum amplification with nonclassical states is implemented by applying this $K = 2$, $|\alpha = 0.1\rangle$ linear protocol to the nonclassical state, then time-reversing the nonclassical dynamics. We simulate quantum amplification of the motional Ramsey using a Fock state ($|n\rangle$, $n = 5$), a squeezed state ($|\xi\rangle$, $\xi = 1.544$), and a cat state ($|\beta\rangle + |-\beta\rangle$, $\beta = 2.236$). The parameters of the nonclassical states have been chosen to keep the mean phonon number $\bar{n} = 5$. Amplification with our nonlinear protocol is implemented by instead generating the displacement of the motional Ramsey using a higher-order subharmonic, shown here as the $K = 10$ subharmonic. For fair comparison, the displacement is kept the same as that with the nonclassical protocols, i.e. $\alpha = 0.1$.

across the RF, microwave, and even optical domains. Importantly, our technique requires only classical input states, eliminating the excess decoherence that limits measurements with nonclassical states to improve *ultimate* precisions for frequency sensing. Despite this, our technique remains compatible with nonclassical states of motion [Wu+25b] (e.g. Fock states [Wol+19, WS21, McC+19a, Den+24]) for short-timescale metrological gain to surpass the SQL of the nonlinear generator.

Chapter 10

Super-Duper Resolution¹

“[Six] years it had taken him to learn what kind of smile was hidden beneath that [glistening dome]. O cruel, needless misunderstanding! O stubborn, self-willed exile from the loving breast! Two gin-scented tears trickled down the sides of his nose. But it was all right, everything was all right, the struggle was finished. He had won the victory over himself! He loved [experimental physics].”

Dr. Winston Smith, *1984*

Abstract

Non-orthogonal quantum states cannot be distinguished with complete certainty, making their underlying physical parameters difficult to resolve. Traditionally, it has been assumed that a system’s linewidth determines the minimum overlap between signals and thus sets the fundamental limit on frequency resolution when two electromagnetic fields are applied. Consequently, the measurement time T required to resolve a frequency difference $\Delta\omega$ was assumed to diverge as $\Delta\omega \rightarrow 0$. In this work, we show that Fourier-transform-limited linewidth does not play a defining role in resolving two closely spaced frequencies. Instead, the ultimate limit is set by parameter-independent quantum fluctuations, such as shot noise in our case. We propose and experimentally demonstrate the first general broadband protocol for

¹Name by [SC](#).

super-resolution spectroscopy in the radio-frequency and microwave domains. Specifically, our protocol uses a [Quantum Fluctuation Suppression \(QFS\)](#) sequence to encode the frequency separation $\Delta\omega$ between two unknown tones into a quantum state. This leverages phase information to suppress parameter-independent shot noise, thereby enhancing the signal-to-noise ratio and enabling super-resolution spectroscopy. With this approach, we resolve two randomly chosen electric fields of order 10^2 MHz separated by 5 Hz, with a measured frequency difference of 5.0(1.6) Hz. This represents an improvement of $200\times$ beyond the traditional spectral resolution limit, set by the 1 ms probe time of the measurement. As such, our technique accelerates data acquisition by more than five orders of magnitude compared to conventional methods. Crucially, as our protocol is rooted in the motional Raman [Quantum Vector Signal Analyzer/Analysis \(QVSA\)](#) framework, it is effective across an arbitrary frequency range and thus promises to enhance broadband sensing of electromagnetic fields and improve spectral efficiency of next-generation communication systems.

Contributions This work was supported by the NSF (PHY-2110421 and OMA-2016245), AFOSR (130427-5114546), and the ARO (W911NF-19-1-0297).

10.1 Introduction

Super-Resolved Metrology

The roots of modern quantum metrology can be traced to studies of the distinguishability of quantum states [Hel69,Woo81]. For two systems parameterized by θ and described by density operators $\hat{\rho}(\theta)$ and $\hat{\rho}(\theta + d\theta)$, a statistical distance $d = d\theta\sqrt{\mathcal{F}_Q(\theta)}$ can be defined [BC94]. This statistical distance can be understood as roughly the number of distinguishable states separating two quantum states in a single measurement. Here, $\mathcal{F}_Q(\theta)$ is the [Quantum Fisher information \(QFI\)](#), which sets a lower bound on the measured precision of θ in N measurements of the system $\hat{\rho}(\theta)$ via the [Quantum Cramér-Rao Bound \(QCRB\)](#) $\sigma_\theta \geq 1/\sqrt{N\mathcal{F}_Q(\theta)}$. Current

efforts aim to improve measurement precision by using quantum states that provide a larger $\mathcal{F}_Q$ and developing optimized protocols that allow the Fisher information F_c of a given measurement to saturate the bound set by $\mathcal{F}_Q$ [DRC17,Pez+18,BRM21,GLM04]. To reach the QCRB, most work has relied on Rabi or Ramsey style protocols to tie θ to a quantity of interest, e.g. a magnetic field, and a post-dynamics optimization of the measurement basis, e.g. using eigenstates of the symmetric logarithmic derivative [BC94].

Generally, a fundamental metrological challenge thus arises when the statistical distance between two quantum states goes toward zero [Woo81]. Such situations largely originate when quantum states resulting from the evolution due to two signals, e.g. closely spaced frequencies or spatial features, overlap substantially within the resolution limit (e.g. linewidth or diffraction limit) (see Figure 10.1(a)). As a result, their distinguishability is reduced - F_c vanishes - and the underlying parameters cannot be reliably measured. For decades, this phenomenon, known as Rayleigh’s curse [TNL16] (see section 10.2.1), was thought to set an ultimate limit in both frequency spectroscopy and imaging, as it was believed that the conventional resolution limit dictated the minimum overlap between quantum states.

Recent breakthroughs in super-resolution imaging (notably [TNL16]) have challenged this convention. For two incoherent sources, $\mathcal{F}_Q$ is, in fact, nonzero, regardless of the traditional resolution limit, indicating that a non-vanishing F_c is realizable. It was proposed in [TNL16] that, by strategically tailoring the measurement basis, the previously neglected phase information within an electromagnetic field can be extracted, allowing F_c to saturate $\mathcal{F}_Q$. This has been experimentally demonstrated in [Paú+16,TDL16,Yan+16,TFS17].

Frequency Super Resolution

The extension of super-resolution from spatial imaging to optical frequency domains is a natural progression, as advanced imaging techniques can be adapted to manipulate and measure the optical field [Bos+20,MLP22,LP24,Zha+25]. In contrast, achieving super-resolution in the radio-frequency (RF) and microwave (MW) regimes is nontrivial, owing to

the absence of direct analogues to these optical methods. To the best of our knowledge, the sole experimental demonstration of frequency super-resolution to date in the RF and MW domains was realized using nitrogen-vacancy (NV) centers in diamond [Rot+19]. It relied on time-correlated phase-sensitive measurements and a clever post-processing technique that averages over the phase and amplitude distributions of two closely spaced signals, achieving resolution an order of magnitude below the conventional spectral limit. The importance of signal incoherence was highlighted by [GRR19] as a necessary condition for super-resolution and proposed a general framework for achieving super-resolution by engineering a measurement process that nulls quantum projection noise, along with an implementation for magnetic-field detection with NV-centers which relies on precise and well-timed control sequences.

Our Solution: Super-Duper Resolution

It is highly desirable to extend super-resolution capabilities to other platforms and interactions with higher bandwidths and sensitivities, though without the strict requirement of correlated phase measurements. [Quantum Harmonic Oscillators \(QHOs\)](#) are a promising platform for sensing electric fields and related quantities [Hem+13,Bur+19,Gil+21,Den+24]. Notably, the recent advent of [Quantum Vector Signal Analyzer/Analysis \(QVSA\)](#) [Wu+25b] via motional Raman transitions has significantly expanded the potential of [QHO](#) sensing by enabling wideband operation for fundamental science (e.g., dark matter detection [Bra+03]), sensing (e.g. acceleration measurements [CH17], quantum radar [Bar+15,Ass+23], multidimensional field sensing [Säg+24]), and communication (e.g. quantum networking [Xia+17]).

Here, we describe and experimentally demonstrate a general broadband method for achieving frequency super-resolution applicable to any [QHO](#) system. Specifically, we use a trapped $^{40}\text{Ca}^+$ ion to detect a minimum resolvable frequency difference of $\Delta\omega/2\pi = 5.0(16)$ Hz between two initially “unknown” incoherent electric fields oscillating near 100 MHz with an interaction time of only 1 ms — a resolution over $200\times$ beyond than the spectral linewidth limit. The total measurement time required to resolve this 5 Hz difference is reduced by more

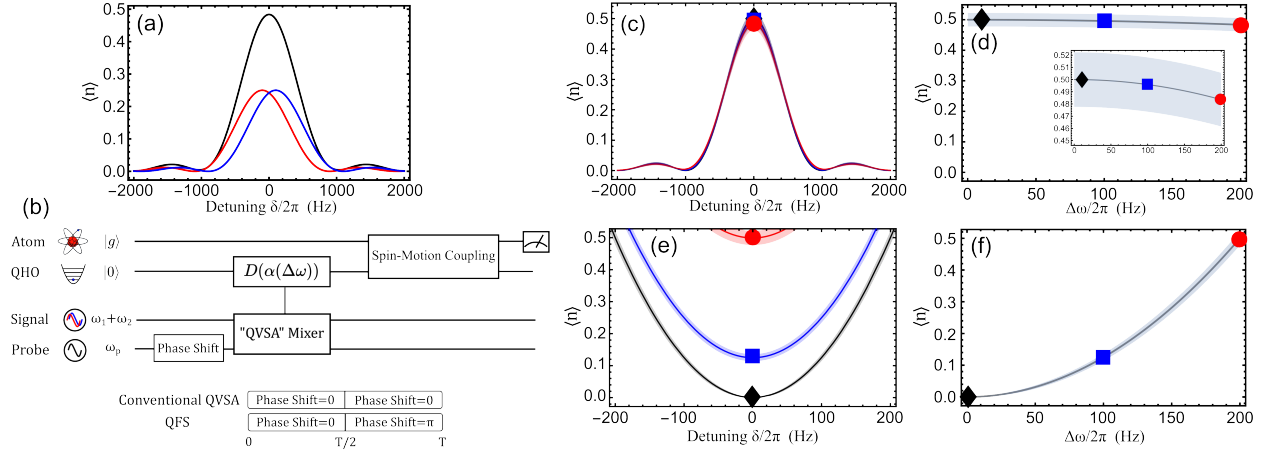


Figure 10.1: Spectroscopy of two incoherent tones using conventional **Quantum Vector Signal Analyzer/Analysis (QVSA)** and **Quantum Fluctuation Suppression (QFS)** techniques.

(a) Conventional spectroscopy with two incoherent signal tones. When the separation between the two tones falls below the traditional linewidth limit, their corresponding peaks (red and blue traces) merge (black trace) and become unresolvable. (b) Quantum circuits implementing conventional QVSA and QFS protocols for detecting the frequency difference $\Delta\omega = \omega_2 - \omega_1$. The QFS protocol is implemented by applying a π phase shift to the probe tone at time $T/2$. The “QVSA” mixer is realized by a motional Raman transition and heterodynes the quadrupolar probe and unknown signal tones, resulting in a displacement of the **Quantum Harmonic Oscillator (QHO)** [Wu+25b]. Expected (c) conventional and (e) QFS spectroscopy generated by scanning the probe detuning (δ) at several different $\Delta\omega$. (c, d, e, f) Expected mean phonon number $\langle n \rangle$ under the conventional protocol (d) and the QFS protocol (f) as a function of $\Delta\omega$ at zero detuning $\delta = 0$. Shaded regions represent the uncertainty $\sigma_{\langle n \rangle}$ due to phonon shot noise for $N = 1000$ trials. (d) Using the conventional protocol, the dominant source of shot noise in the measured $\langle n \rangle$ is independent of $\Delta\omega$, obscuring any dependence of $\langle n \rangle$ on $\Delta\omega$ and rendering $\Delta\omega$ unresolvable. (f) Using the QFS protocol, sensitivity to $\Delta\omega$ is recovered by nullifying the $\Delta\omega$ -independent shot noise at $\delta = 0$. Here, the interrogation time is $T = 1$ ms.

than five orders of magnitude (10^5) compared to conventional techniques.

This protocol implements a dynamic error suppression sequence [BB15] using motional Raman transitions to leverage phase information to null the $\Delta\omega$ -independent shot noise and realize nonzero F_c , even as $\Delta\omega \rightarrow 0$. Its ability to achieve super-resolution can thus be understood as a more effective encoding of $\Delta\omega$ into the quantum state for measurement. To the best of our knowledge, this protocol is the first on any platform to achieve broadband super-resolution of an oscillating electric field and sheds new light on quantum metrology, demonstrating sensing in regimes previously considered unmeasurable.

10.2 Theory

A thorough review of the motional Raman theory can be found in chapter 8.

10.2.1 Rayleigh's Curse

Conventional frequency sensing techniques typically operate by mapping frequency information onto the amplitude of a measured signal — for example, light intensity in absorption spectroscopy [Dem03], magnetic-field power spectral density with NV centers [Rot+19], or phonon detection for electric-field sensing with trapped ions [Wu+25b]. However, as the measurement outcomes associated with two distinct frequencies begin to overlap and merge into a single peak, the ability to resolve the constituent tones is significantly reduced and ultimately lost, as illustrated in Figure 10.1(a).

To understand how this protocol overcomes this limitation, suppose that two unknown oscillating, uniform electric fields are present at frequencies ω_1 and ω_2 . A probe tone at ω_p is applied in an electric quadrupole configuration to generate a motional Raman displacement and sense the unknown tones. The Hamiltonian of the ion motion under these fields is

$$\begin{aligned} \frac{\hat{H}}{\hbar} = & \omega_o \hat{a}^\dagger \hat{a} \\ & + \Omega_d (\hat{a} + \hat{a}^\dagger) (\cos(\omega_1 t) + \cos(\omega_2 t + \Delta\phi)) \\ & + \Omega_p (\hat{a} + \hat{a}^\dagger)^2 \cos(\omega_p t), \end{aligned} \quad (10.1)$$

where ω_o is the natural frequency of the QHO, $\Delta\phi$ is the relative phase between the two unknown tones, $\Omega_d = eV_d\eta/\hbar$, $\Omega_p = eV_p\eta^2/\hbar$, V_d is the voltage of the unknown tones, V_p is the probe voltage, and η is the effective Lamb-Dicke parameter [Wu+25b]. For simplicity, the amplitudes of the two unknown tones are assumed to be equal (see section 10.4.2). If the quadrupolar probe tone is applied at $\omega_p = \omega_\mu + \omega_o + \delta$, where $\omega_\mu = (\omega_1 + \omega_2)/2$ for some fixed $\Delta\phi$, a conventional QVSA protocol based on motional Raman effect results, creating a

coherent state $|\alpha\rangle$ with displacement:

$$\alpha(\Delta\omega, \Delta\phi, T, t_o) = i\Omega_{\text{eff}} \left(e^{i(\delta - \Delta\omega/2)t_o} \frac{1 - e^{i(\delta - \Delta\omega/2)T}}{\delta - \Delta\omega/2} + e^{-i\Delta\phi} e^{i(\delta + \Delta\omega/2)t_o} \frac{1 - e^{i(\delta + \Delta\omega/2)T}}{\delta + \Delta\omega/2} \right). \quad (10.2)$$

Here, $\Delta\omega = (\omega_2 - \omega_1)$ is the frequency difference between the unknown tones, $\Omega_{\text{eff}} = \Omega_d \Omega_p \omega_o / ((\omega_\mu \pm \Delta\omega/2)^2 - \omega_o^2)$ is the effective Rabi frequency, T is the interrogation time and t_o denotes the start time of the sequence. Assuming that $\Delta\phi$ fluctuates uniformly in $[0, 2\pi)$ between measurements but remains approximately constant during T , the state of the system is:

$$\hat{\rho} = \frac{1}{2\pi} \int |\alpha(\Delta\omega, \Delta\phi, T, 0)\rangle \langle\alpha(\Delta\omega, \Delta\phi, T, 0)| d(\Delta\phi). \quad (10.3)$$

By scanning the probe detuning δ , the spectrum of the unknown tones can be characterized at different $\Delta\omega$ (Figure 10.1(c)).

However, in the super-resolved limit ($\Delta\omega T \ll 1$), it is straightforward to show that $|\alpha(\Delta\omega, 0, T, 0)|^2 \approx 4\Omega_{\text{eff}}^2 T^2 - \frac{1}{3}\Omega_{\text{eff}}^2 T^4 \Delta\omega^2$ and $|\alpha(\Delta\omega, \pi, T, 0)|^2 \approx \frac{1}{4}\Omega_{\text{eff}}^2 T^4 \Delta\omega^2$. In the conventional protocol, the uncertainty of $\Delta\omega$ under the shot noise limit is thus $\sigma_{\Delta\omega} = \frac{\sigma_n}{\partial n / \partial \Delta\omega} = \frac{12\sqrt{2}}{\Omega_{\text{eff}} T^3 \Delta\omega}$, which diverges as $\Delta\omega \rightarrow 0$, leading to Rayleigh's curse. Fundamentally, this divergence results from the shot noise introduced by the $\Delta\omega$ -independent displacement, $2\Omega_{\text{eff}}T$, which dominates and obscures any dependence on $\Delta\omega$, shown in Figure 10.1(d). To achieve frequency super-resolution, this $\Delta\omega$ -independent displacement must therefore be removed.

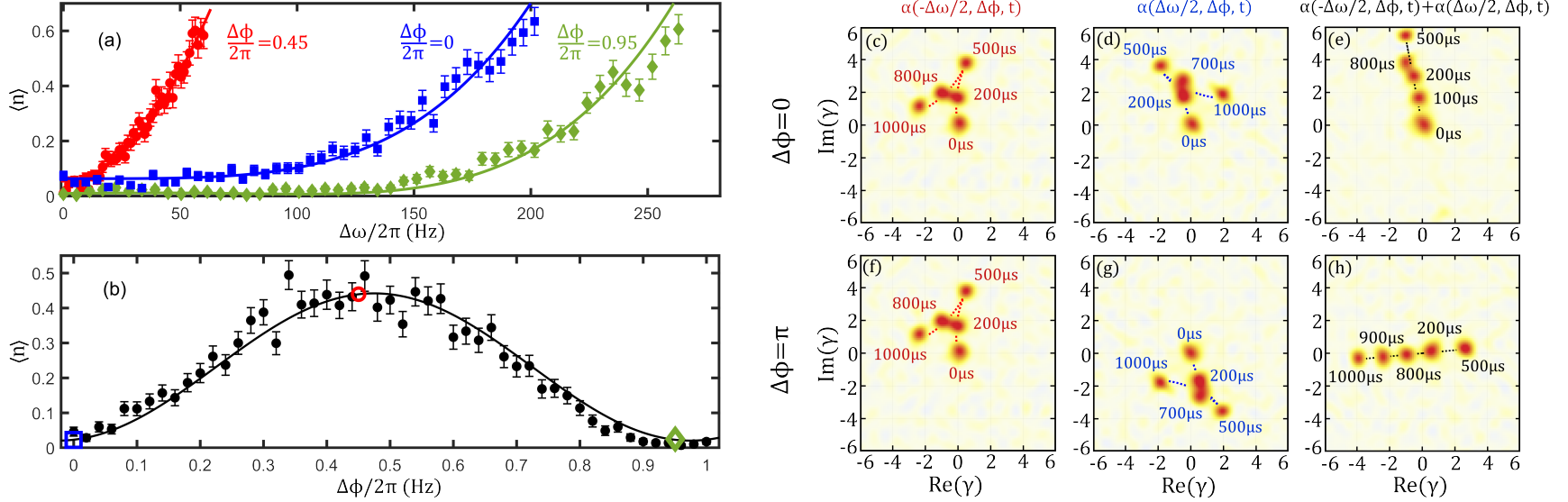


Figure 10.2: Dependence of the **QFS** protocol on the relative phase $\Delta\phi$ between two signal tones $\omega_1 \approx \omega_2 \approx 2\pi \cdot 86$ MHz.

(a) Super-resolution spectrum of $\Delta\omega = \omega_2 - \omega_1$ for $\Delta\phi = 0.9\pi$ (red, circle), $\Delta\phi = 1.9\pi$ (green, diamond), and $\Delta\phi = 0$ (blue, square). (b) Measured $\langle n \rangle$ as a function of $\Delta\phi$ at $\Delta\omega/2\pi = 50$ Hz. Experimental data (dotted points) are in excellent agreement with fits to eq. (10.5) (solid lines). Almost all parameters used in the fit are independently measured or controlled (e.g. Ω_{eff} and T), with a uniform vertical offset being the only fitting parameter. No additional vertical or horizontal scaling is applied. (a-b) A total interrogation time $T = 1$ ms was used. Error bars represent one standard error. Each point is averaged over $N = 600$ repetitions. (c-h) Phase space evolution of the QHO Wigner function under the **QFS** protocol for $\Delta\phi = 0$ (c, d, e) and $\Delta\phi = \pi$ (f, g, h) at $\Delta\omega/2\pi = 200$ Hz. The Wigner function is experimentally measured using motional state tomography. The blue (red) dashed line represent the analytical phase-space trajectory due to evolution under a single tone. The black dashed lines show the resultant trajectory when both tones are applied. (e,h) Following phase reversal at $T = 500\mu\text{s}$, the motional trajectory retraces itself to remove the $\Delta\omega$ -independent displacement, leaving only a $\Delta\omega$ -dependent displacement.

10.2.2 The Quantum Fluctuation Suppression Protocol

To effectively cancel this $\Delta\omega$ -independent displacement, we apply Quantum Fluctuation Suppression (QFS) by shifting the phase of the displacement by π at the midpoint of each interrogation time [BB15]. The total displacement under QFS is:

$$\alpha_{\text{SR}}(\Delta\omega, \Delta\phi, T, 0) = \alpha(\Delta\omega, \Delta\phi, T/2, 0) + e^{i\pi} \alpha(\Delta\omega, \Delta\phi, T/2, T/2) \quad (10.4)$$

The resultant mean phonon number $\langle n \rangle_{\text{SR}}$ for $\delta = 0$ is:

$$\langle n \rangle_{\text{SR}} = \frac{256\Omega_{\text{eff}}^2}{\Delta\omega^2} \sin^4\left(\frac{\Delta\omega T}{8}\right) \sin^2\left(\frac{\Delta\omega T/2 - \Delta\phi}{2}\right), \quad (10.5)$$

leading to Figure 10.1(e). In the super-resolved limit $\Delta\omega T \ll 1$ with random phase $\Delta\phi$, eq. (10.5) becomes

$$\langle n \rangle_{\text{SR}} = \langle |\alpha_{\text{SR}}(\Delta\omega)|^2 \rangle_{\Delta\phi} \approx \frac{\Omega_{\text{eff}}^2}{32} \Delta\omega^2 T^4. \quad (10.6)$$

As a result, $\Delta\omega$ becomes ‘encoded’ into the mean phonon number $\langle n \rangle_{\text{SR}}$, which can be efficiently measured in a QHO system, shown in Figure 10.1(f). The uncertainty in $\Delta\omega$ can be obtained from standard error propagation and eq. (10.6): $\sigma_{\Delta\omega}^{\text{SR}} = \frac{\sigma_{n_{\text{SR}}}}{\partial n_{\text{SR}} / \partial \Delta\omega} = \frac{2\sqrt{2}}{\Omega_{\text{eff}} T^2}$. The upper bound of the corresponding quantum Fisher information is $\mathcal{F}_Q(\Delta\omega) \leq (\Omega_{\text{eff}} T^2)^2 / 8$ (see section 10.2.4). According to the QCRB, $\sigma_{\Delta\omega}^Q = \frac{1}{\sqrt{\mathcal{F}_Q(\Delta\omega)}} \geq \frac{2\sqrt{2}}{\Omega_{\text{eff}} T^2}$. It is straightforward to see that these two expressions yield the same uncertainty, indicating that direct phonon-number measurements can saturate the QCRB. Notably, this result is independent of $\Delta\omega$, resulting in an uncertainty $\sigma_{\Delta\omega}$ that does not diverge as $\Delta\omega \rightarrow 0$, thereby enabling super-resolution measurement. Conceptually, QFS operates in phase space in a manner analogous to zero-delay Ramsey spectroscopy on the Bloch sphere. $\mathcal{F}_Q(\Delta\omega)$ effectively takes a form similar to that of Ramsey spectroscopy [Wol+19], suggesting a potential for ultra-performant frequency sensing with our approach.

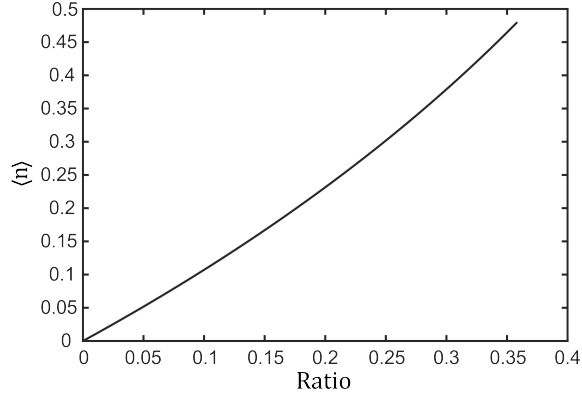


Figure 10.3: The conversion function $g(x)$ used to convert from a sideband ratio measurement (see section 3.5.1) to a mean phonon number $\langle n \rangle$ when the relative phase $\Delta\phi$ varies randomly between measurements.

10.2.3 Phase-Incoherent Sensing

The inversion estimator is employed to determine the mean phonon number $\langle n \rangle$ [Wu+25b]. Specifically, for a coherent state $|\alpha\rangle$, $\langle n \rangle$ can be inferred from the ratio of the motional sideband excitation probabilities P_r and P_b . Here, $P_{r/b}$ is the single-shot probability of a dark-state measurement following a red (blue) sideband Rabi pulse for duration $t_\pi = \pi/\Omega_{01}$, where Ω_{01} is the Rabi frequency of the $|^2S_{1/2}, n=0\rangle \leftrightarrow |^2D_{5/2}, n=1\rangle$ transition (see chapter 8).

To simulate incoherence between the signal tones, their relative phase $\Delta\phi$ is randomly drawn from a uniform distribution over $[0, 2\pi]$ for each measurement. Despite this phase randomization, precise estimation of $\langle n \rangle$ remains possible. The estimator can be adapted by instead first averaging the detection probabilities over all repetitions before applying the inversion estimator. This phase-averaged estimator for $\langle n \rangle$ is thus:

$$\tilde{n} = g\left(\frac{\langle P_r \rangle_{|\Delta\phi}}{\langle P_b \rangle_{|\Delta\phi}}\right) = g\left(\frac{k/N}{j/M}\right), \quad (10.7)$$

where k (j) is the number of dark-state measurements observed in N (M) trials after application of a Rabi pulse on the red (blue) motional sideband. The conversion function $g(x)$ from the averaged sideband ratio to $\langle n \rangle$ is shown in Figure 10.3.

Phase Coherent Detection Efficient spectral resolution can be recovered for coherent signals by leveraging time delay to engineering decoherence. Consider two coherent signal tones separated in frequency by $\Delta\omega = 1$ Hz: following a delay 0.5 s, the tones will accumulate a relative phase shift $\Delta\phi = 2\pi 1 \text{ Hz} \times 0.5 \text{ s} = \pi$. Uniform randomization of the dead time over $[0, 0.5\text{s}]$ is thus equivalent to randomizing the phase over $[0, \pi]$ between successive measurements, and provides a means to enhance spectral resolution of coherent tones.

10.2.4 Protocol Sensitivity

The density matrix for a pure state parameterized by θ is $\rho_\theta = |\psi_\theta\rangle \langle\psi_\theta|$ and has a [Quantum Fisher information](#)

$$\mathcal{F}_Q(|\psi_\theta\rangle) = 4 \left(\langle \dot{\psi}_\theta | \dot{\psi}_\theta \rangle - |\langle \dot{\psi}_\theta | \psi_\theta \rangle|^2 \right) \quad (10.8)$$

where $|\dot{\psi}_\theta\rangle = \frac{d|\psi_\theta\rangle}{d\theta}$. For frequency sensing, our protocol senses as a parameter the frequency separation $\Delta\omega$ using a coherent state $|\alpha\rangle = e^{i\alpha(\hat{a} + \hat{a}^\dagger)} |0\rangle$. Using $|\dot{\alpha}\rangle = i \frac{\partial \alpha}{\partial (\Delta\omega)} (\hat{a} + \hat{a}^\dagger) |\alpha\rangle$, the QFI becomes

$$\mathcal{F}_Q(|\alpha_{\Delta\omega}\rangle) = 4 \left(\langle \dot{\alpha}_{\Delta\omega} | \dot{\alpha}_{\Delta\omega} \rangle - |\langle \dot{\alpha}_{\Delta\omega} | \alpha_{\Delta\omega} \rangle|^2 \right) = 4 \left| \frac{\partial \alpha}{\partial (\Delta\omega)} \right|^2. \quad (10.9)$$

From eq. (10.5), the displacement following QFS is

$$\langle \alpha_{\text{SR}} \rangle = \frac{16\Omega_{\text{eff}}}{\Delta\omega} \sin^2 \left(\frac{\Delta\omega T}{8} \right) \sin \left(\frac{\Delta\omega T/2 - \Delta\phi}{2} \right). \quad (10.10)$$

In the super-resolved limit $\Delta\omega T \ll 1$, eq. (10.9) thus simplifies to $\langle \alpha_{\text{SR}} \rangle = \frac{\Omega_{\text{eff}} T^2 \Delta\omega}{4} \sin \left(\frac{\Delta\omega T/2 - \Delta\phi}{2} \right)$.

The density matrix of the system under the QFS protocol is then obtained by integrating over the relative phase, written as

$$\hat{\rho} = \frac{1}{2\pi} \int |\alpha_{\text{SR}}(\Delta\omega, \Delta\phi, T)\rangle \langle \alpha_{\text{SR}}(\Delta\omega, \Delta\phi, T)| d(\Delta\phi). \quad (10.11)$$

The QFI is similarly obtained using the same integration (convexity of the QFI [DJK15]) to produce

$$\mathcal{F}_Q(|\alpha_{\Delta\omega}\rangle) \leq \frac{\Omega_{\text{eff}}^2 T^4}{8} (1 + (\Delta\omega T)^2) \approx \frac{\Omega_{\text{eff}}^2 T^4}{8}. \quad (10.12)$$

According to the [Quantum Cramér-Rao Bound \(QCRB\)](#), $\sigma_{\Delta\omega}^Q = \frac{1}{\sqrt{\mathcal{F}_Q(\Delta\omega)}} \geq \frac{2\sqrt{2}}{\Omega_{\text{eff}} T^2}$. The QFS limit in Figure 10.5(c) is based on this expression.

10.3 Characterization

To verify and demonstrate our technique, we utilize a QHO realized on the axial motional mode of a single trapped $^{40}\text{Ca}^+$ ion with secular frequency $\omega_o \approx 2\pi \cdot 700$ kHz. The axial mode is chosen for its ≥ 20 ms coherence time. The ground motional state of the axial mode is initially prepared via Doppler cooling of the ion followed by resolved-sideband cooling on the optical $^2S_{1/2} \leftrightarrow ^2D_{5/2}$ qubit transition to yield a mean phonon number $\langle n \rangle \approx 0.01(1)$ (see chapter 8 for further detail).

For all experiments, the two “unknown” signal tones are applied as uniform electric fields by coupling voltages to the trap electrodes in a dipole configuration with frequencies $\omega_i = \omega_\mu \mp \Delta\omega/2$ for $i = 1, 2$ and relative phase $\Delta\phi$. Control operations are applied exclusively to the probe tone, which is similarly applied, though in a quadrupole configuration. Prior to application of the super-resolution protocol, ω_μ is first obtained using conventional QVSA spectroscopy (see section 10.4.3). During the super-resolution protocol, the probe frequency is set to $\omega_p = \omega_\mu + \omega_o$, with a phase shift of π applied to the probe tone at $T/2$. Following detection, the residual displacement is determined using either an inversion estimator for fixed $\Delta\phi$, or a phase-averaged estimator when $\Delta\phi$ varies randomly between measurements (see section 10.2.3).

Experimental Parameters Table 10.1 contains all experimental parameters used to produce Figures 10.1, 10.2, 10.4 and 10.5.

	$\omega_{1/2}/2\pi$ (MHz)	$\Omega_{\text{eff}}/2\pi$ (kHz)	$\Delta\phi$	T (ms)
Fig. 1(a)	Arbitrary	0.08	Uniformly random over $\in [0, 2\pi)$	1
Fig. 1(c)	Arbitrary	0.08	Uniformly random over $\in [0, 2\pi)$	1
Fig. 1(d)	arbitrary	0.08	Uniformly random over $\in [0, 2\pi)$	1
Fig. 1(e)	Arbitrary	0.5	Uniformly random over $\in [0, 2\pi)$	1
Fig. 1(f)	Arbitrary	0.5	Uniformly random over $\in [0, 2\pi)$	1
Fig. 2(a-b)	86	1.3(1)	See figure	1
Fig. 2(c)	86	1.3(1)	See figure	1
Fig. 3(a-d)	86	1.68(7)	Uniformly random over $\in [0, 2\pi)$	1
Fig. 4(a-c)	86	1.68(7)	Uniformly random over $\in [0, 2\pi)$	1

Table 10.1: **Experimental parameters used for each figure in section 10.3.**

10.3.1 Operational Principles

As a first test, the super-resolution protocol is applied for $\Delta\phi$ fixed at several values (0 , 0.9π , and 1.9π). The measured $\langle n \rangle$ for $\omega_\mu = 2\pi \cdot 86$ MHz and $T = 1$ ms is shown in Figure 10.2(a) as a function of $\Delta\omega$, where lines depict the analytical result given by eq. (10.5). All parameters in eq. (10.5) are either calibrated experimentally (see section 10.4.1) or set accurately in hardware. To account for instability of the QHO frequency between measurements, we add a uniform offset when fitting eq. (10.5) as the only free parameter. Figure 10.2(b) further shows the measured $\langle n \rangle$ as a function of $\Delta\phi$ at fixed $\Delta\omega = 2\pi \cdot 50$ Hz. The excellent agreement of experimental results with the theory indicate that, with only straightforward analysis, $\Delta\omega$ can be measured orders of magnitude below the traditional linewidth limit ($\sim 1/T = 1$ kHz) for fixed $\Delta\phi$. This phase relationship indicates that the protocol can be adapted to correlation spectroscopy to enable arbitrary resolution of $\Delta\omega$ [Bos+17,Sch+17].

The cancellation of excess shot noise, which enables the technique, can be understood by examining the evolution of the Wigner function in phase space. Figure 10.2(c-h) show *measured* Wigner functions of the ion motional state during its evolution under several variations of parameters. For these measurements, the QHO is initialized in the vacuum state $|0\rangle$, the displacement $\hat{D}(\gamma)$ applied, and the Wigner functions recorded using the characteristic function tomography method of [FH20]. Each constituent Wigner function was experimentally measured on a 26×26 square grid using ≈ 100 averages for each of the real and imaginary

parts, resulting in $\approx 1.3 \times 10^5$ data points per image. For $\Delta\omega = 0$, when the probe phase shifts by π at $T/2$, the motional trajectory retraces itself and returns to the origin, thereby canceling the $\Delta\omega$ -independent displacement. However, for nonzero $\Delta\omega$ the cancellation becomes incomplete due to the differing phase accumulation $\pm\Delta\omega T/2$ during the evolution for the displacements arising from ω_1 and ω_2 . This behavior can be seen in Figure 10.2(c-d) for $\Delta\phi = 0$ and (f-g) for $\Delta\phi = \pi$, with the first two columns showing the displacement of the individual signal tones and the last column showing the total displacement $\hat{D}(\alpha_{\text{SR}})$.

10.3.2 Sensitivity Benchmarks

Next, to test a more general scenario where the two signal tones are incoherent, $\Delta\phi$ is varied randomly between each measurement over a uniform distribution on $[0, 2\pi)$. Using the phase-averaged estimator (see section 10.2.3), the relationship between $\Delta\omega$ and the mean phonon number $\langle n \rangle$ for two incoherent signal tones is characterized, shown in Figure 10.4(a). Here, Ω_{eff} is independently measured and used in the fit. The solid line represents a fit of the data to inversion function of eq. (10.6) with a uniform horizontal offset as the sole free parameter. The shaded regions present the 68% prediction band for $\Delta\omega$ from the data. These narrow prediction bands, as in Figure 10.1(f) evidence clearly the ability of the protocol to achieve super-resolution on even incoherent signals.

Following this demonstration, an Allan deviation is performed at six different values of $\Delta\omega$ to evaluate the ultimate achievable resolution for $\Delta\omega$ with known Ω_{eff} from pre-calibration, presented in Figure 10.4(b-d). Here, the dashed lines indicate the expected $1/\sqrt{N}$ scaling. Deviations from this trend result from systematic sources of error, and are chiefly due to instability in the natural frequency of the QHO. After $N = 8160$ repetitions, the uncertainty in $\Delta\omega$ falls below the single-Hz level, and is nearly identical across all values of $\Delta\omega$, consistent with the theoretical prediction that, in the super-resolved measurement regime, the uncertainty is independent of $\Delta\omega$. To resolve signals separated by 5 Hz, our QFS protocol requires a measurement time of 110 s. By comparison, spectroscopy using

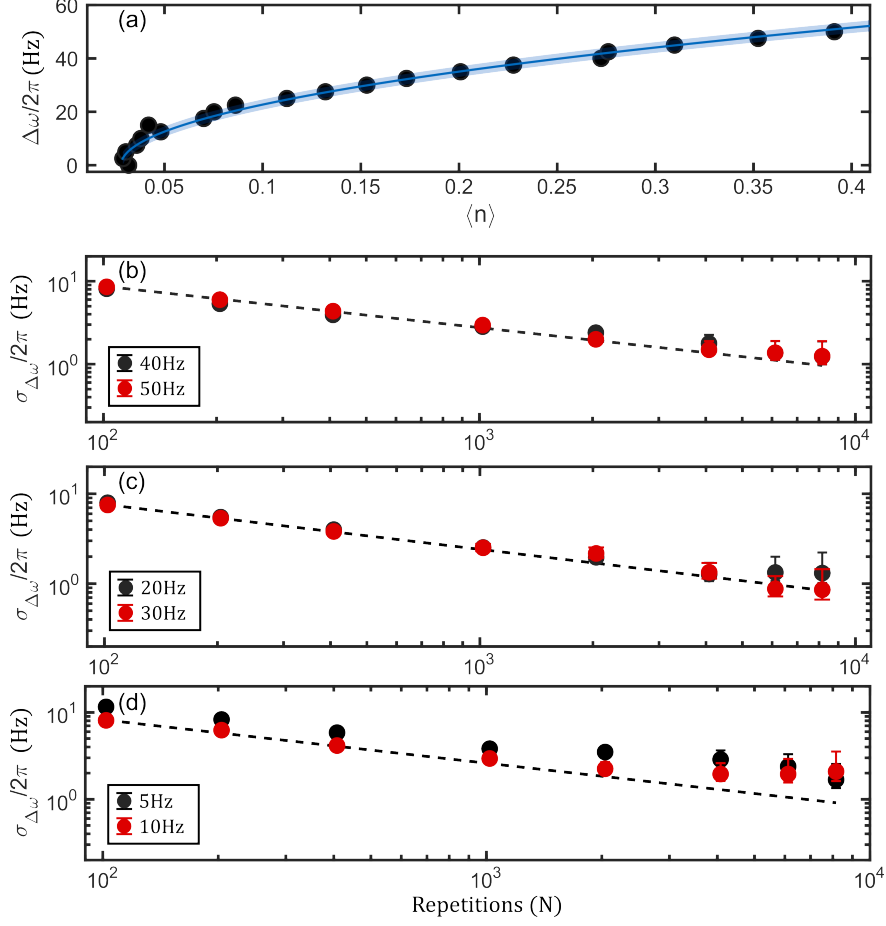


Figure 10.4: **Characterizing ultimate resolution of $\Delta\omega$ with two incoherent signal tones centered at $2\pi \cdot 86$ MHz.**

(a) Calibration curve used to convert the measured $\langle n \rangle$ to the parameter $\Delta\omega$. Experimental data are plotted as dotted points and fitted to an inversion of the analytical model (solid lines) in eq. (10.6) with only a constant horizontal offset as the sole fit parameter. The colored band indicates the 68% prediction band. Each point is averaged over $N = 5000$ measurements. There is no additional rescaling. (b-d) Overlapping Allan deviations of $\Delta\omega$ at different frequency separations $\Delta\omega$ with fixed “known” Rabi frequency Ω_{eff} . (b) $\Delta\omega/2\pi = 40, 50$ Hz. (c) $\Delta\omega/2\pi = 20, 30$ Hz. (d) $\Delta\omega/2\pi = 5, 10$ Hz. (a-d) Data was taken using a total interrogation time $T = 1$ ms. To simulate an incoherent signal, the relative phase between the two signal tones $\Delta\phi$ was varied randomly between each measurement from a uniform distribution on $[0, 2\pi)$. Error bars represent one standard error. The dashed lines represent a $1/\sqrt{N}$ trend. Each repetition takes ≈ 13 ms.

a conventional protocol would require roughly a measurement time of $3 \cdot 10^7$ s to achieve a similar resolution — this is a result that would be practically inachievable at any level without even factoring in systematic sources of error such as long-term drifts.

10.3.3 Detection of Incoherent Signals

As a final demonstration to characterize the precision and accuracy of our super-resolution protocol, we apply and sense two unknown, incoherent signals. We first optimize the probe frequency ω_p by scanning the probe detuning δ with our QFS protocol and minimizing the measured mean phonon number $\langle n \rangle$ (see section 10.4.3). The effective Rabi frequency Ω_{eff} is extracted by performing conventional spectroscopy, with a resulting prediction error ($\sigma_{\Omega_{\text{eff}}}$) $\approx 4\%$ based on the measured phonon number (see section 10.4.1). Next, we conduct a super-resolution measurement with interrogation time $T = 1$ ms at the optimized ω_p to determine $\Delta\omega$. To suppress the effect of drifts in ω_o , a reference signal with known $\Delta\omega$ is concurrently measured. The distribution of the measured $\omega_\mu \pm \Delta\omega/2$ is shown in Figure 10.5(a) for $\Delta\omega = 2\pi \cdot 30$ Hz. As expected, two incoherent spectral components (blue and red) are clearly resolved at frequencies $\omega_\mu \pm \Delta\omega/2$, in contrast to the conventional method, shown in purple. This process is repeated at several values of $\Delta\omega$ to compare the measured values $\Delta\omega$ to their applied true values in Figure 10.5(b). The excellent direct correspondence of our measurements to the true values in Figure 10.5(b) demonstrates the accuracy and reliability of our super-resolution protocol.

To contextualize the frequency resolution achievable with our super-resolution protocol, we compare our resolution limits against those of conventional protocol, depicted in Figure 10.5(c). The $\sigma_{\Delta\omega} = \Delta\omega$ line demarcates the resolvable from the unresolvable region. The uncertainty using conventional protocol crosses this bound at $\Delta\omega \approx 2\pi \cdot 66$ Hz using an interrogation time $T = 1$ ms and $N = 8160$ repetitions. In contrast, the QFS limit is $\sigma_{\Delta\omega}^{(\text{SR})} = 2\pi \cdot 0.65$ Hz for the same parameters, indicating that the spectrally resolvable frequency range can be extended by more than two orders of magnitude. The QFS limit corresponds to the lowest possible value set by both the corresponding Quantum Cramér-Rao Bound and the standard error propagation estimate. In practice, however, our super-resolution protocol has extended the resolvable frequency range by a factor of approximately $\approx 14\times$. Deviations of the measured uncertainty from $\sigma_{\Delta\omega}^{(\text{SR})}$ arise from a combination of unsaturated estimators [Wu+25b], uncertainty in

σ_n/n (2.5Hz)	σ_n/n (5Hz)	σ_n/n (10Hz)	σ_n/n (15Hz)
0.67	0.42	0.13	0.06
σ_n/n (20Hz)	σ_n/n (25Hz)	$\sigma_{\Omega_{\text{eff}}}/\Omega_{\text{eff}}$	σ_T/T
0.06	0.06	0.04	1e-4

Table 10.2: **Error budget for estimation of $\Delta\omega$ in Figure 10.5(b-c).**

The uncertainty in the phonon number σ_n is evaluated via an overlapping Allan deviation on the dataset in Figure 10.4(b-d). The uncertainty of ω_p leads to enhanced uncertainty in the phonon population and is incorporated into σ_n via the Allan deviation analysis. T can be accurately controlled in hardware.

Ω_{eff} and ω_μ , and systematic errors due to drifts in ω_o imperfections when applying the QFS protocol. Notably, for $\Delta\omega/2\pi = 5$ Hz, our protocol achieves a resolution that requires $> 10^5$ fewer experimental repetitions than the conventional approach, even when ignoring systematic sources of error in the latter. This enhanced resolution demonstrated by our protocol allows determination of $\Delta\omega$ at levels 200 times beyond the spectral linewidth limit ($\sim 1/T = 1$ kHz). The broadband ability of the protocol is showcased in section 10.3.4 by similarly resolving two signals separated by $\Delta\omega/2\pi = 20$ Hz across a range $[10^1, 10^3]$ MHz.

Error Budget

The total uncertainty in $\Delta\omega$ is given by:

$$\sigma_{\Delta\omega} = \Delta\omega \sqrt{\left(\frac{\sigma_n}{2n}\right)^2 + \left(\frac{\sigma_{\Omega_{\text{eff}}}}{\Omega_{\text{eff}}}\right)^2 + \left(\frac{2\sigma_T}{T}\right)^2}. \quad (10.13)$$

An error budget accompanying the measurements in Figure 10.5 is shown in Table 10.2.

10.3.4 Broadband Operation

Additional measurements following the protocol in section 10.3.3 across $\sim [10^1, 10^3]$ MHz demonstrate the unique ability of our protocol to achieve super-resolved linewidths (by $\approx 50\times$) across a range $[10^1, 10^3]$ MHz (Figure 10.6). The lowest frequency we measure at is 30 MHz, set by the 30 MHz cutoff frequency of the high-pass filters in our system used to suppress

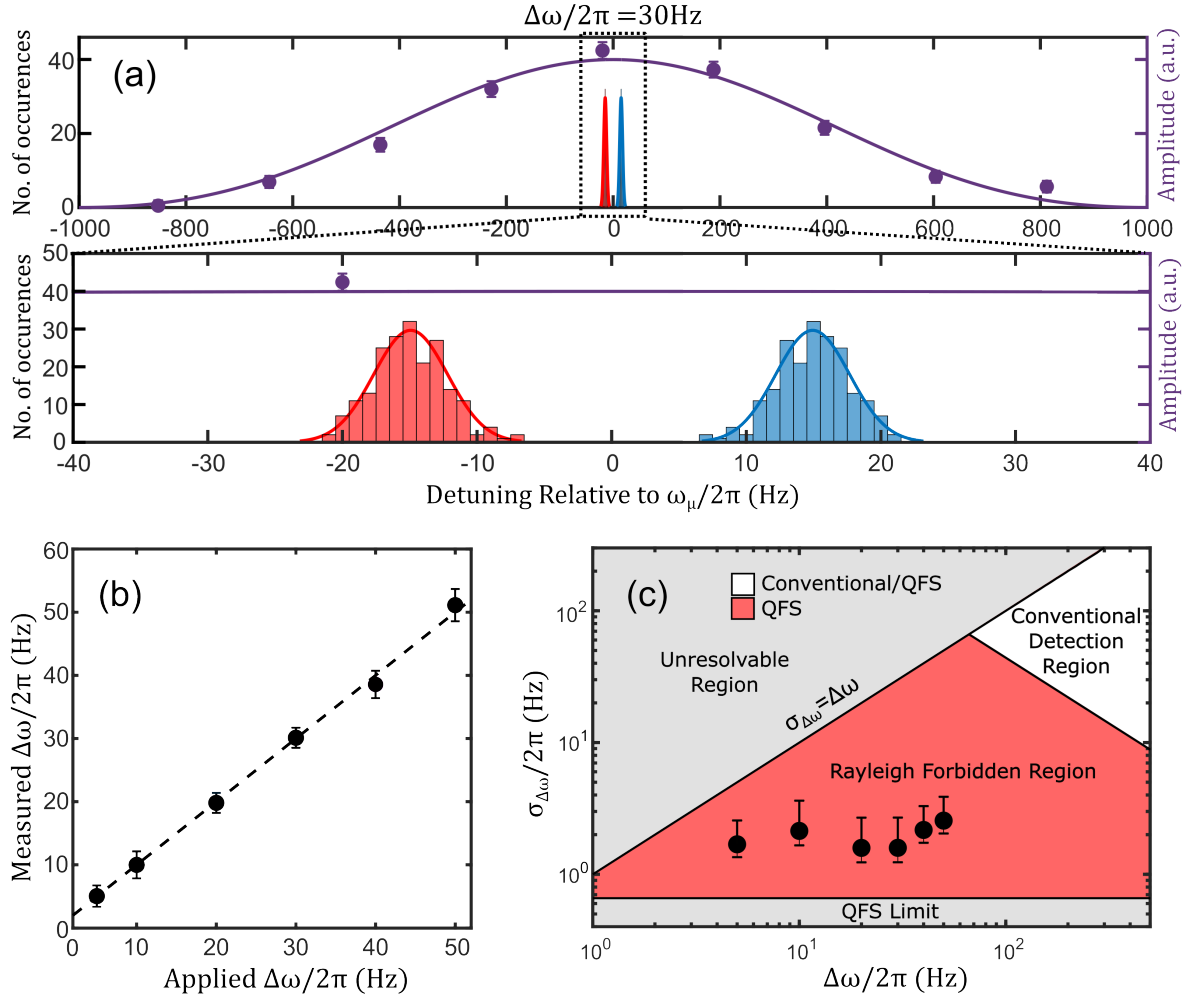


Figure 10.5: Measurement of “unknown” $\Delta\omega$ with two incoherent signal tones at $\omega_\mu = 2\pi \cdot 86$ MHz with budgeted errors.

(a) Distribution of measurements of $\omega_\mu \pm \Delta\omega/2$ for $\Delta\omega/2\pi = 30$ Hz with the QFS protocol (red and blue). For comparison, measurements (purple circles) and a corresponding fit (purple line) are shown for a conventional protocol. Solid red and blue lines depict Poisson fits to the measured distributions. Each occurrence is averaged over $N = 102$ repetitions. (b) Measured values of $\Delta\omega$ compared against true values. The dashed line represents an ideal 1:1 correspondence between the applied and measured values. (c) Comparison of resolvable regions between conventional and QFS protocols. The conventional detection region indicates the region resolvable by both conventional and QFS protocols. The QFS protocol extends the resolvable region into the Rayleigh-forbidden region—beyond the traditional limit imposed by linewidth—reaching the bound set by lowest possible value of the quantum Cramér–Rao limit, referred to here as the QFS limit. Beyond $\sigma_{\Delta\omega} = \Delta\omega$, $\Delta\omega$ becomes unresolvable (grey). At $\Delta\omega/2\pi = 5$ Hz, the QFS protocol requires 10^5 fewer repetitions than a conventional protocol to achieve the same uncertainty. (b-c) An interrogation time of $T = 1$ ms is used with $N = 8160$ repetitions. Error bars represent one standard error.

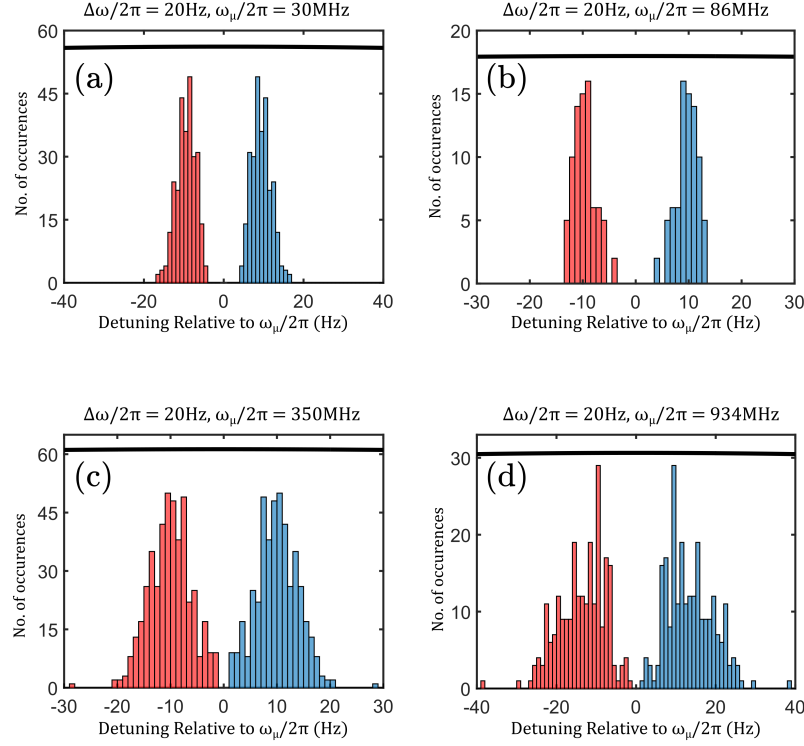


Figure 10.6: **Super-resolution of signal frequencies with separation $\Delta\omega/2\pi = 20$ Hz at different carrier frequencies ω_μ .**

Resolution of the signals at $\Delta\omega/2\pi = 20$ Hz represents surpasses the Fourier-transform limited linewidth ~ 1 kHz by a factor of $\approx 50\times$. For comparison, results are calculated for a conventional protocol (solid purple line). Each subfigure is averaged over M repetitions. (a) $M=102$, $\omega_\mu = (\omega_1 + \omega_2)/2 = 2\pi \cdot 30$ MHz; (b) $M=255$, $\omega_\mu = 2\pi \cdot 86$ MHz; (c) $M=51$, $\omega_\mu = 2\pi \cdot 350$ MHz; (d) $M=102$, $\omega_\mu = 2\pi \cdot 934$ MHz; $T = 1$ ms.

noise near the ion's secular frequency (≈ 1 MHz); this is easily extended to the kHz regime with more judicious signal conditioning. Super-resolution is demonstrated at a maximum frequency of 934 MHz, which is limited by the frequency response of system electronics, and is easily extendable with judicious system design.

The theoretical frequency range of our method can be determined by the [Quantum Fisher information \(QFI\)](#) bound $\mathcal{F}(\Delta\omega) \leq (\Omega_{\text{eff}} T^2)^2 / 8$ (see section 10.2.4), where the precision of $\Delta\omega$ is determined by T and $\Omega_{\text{eff}} = \Omega_d \Omega_p \omega_o / (\omega_{1/2}^2 - \omega_o^2)$. In this work, $\omega_{1/2} \gg \omega_o$. The precision $\Delta\omega$ thus scales with $\omega_{1/2}^2$, the same as the resolvable $\Delta\omega$. The ideal resolution is calculated for a probe power of 40 W and depicted by the blue curve in Figure 10.7. Under

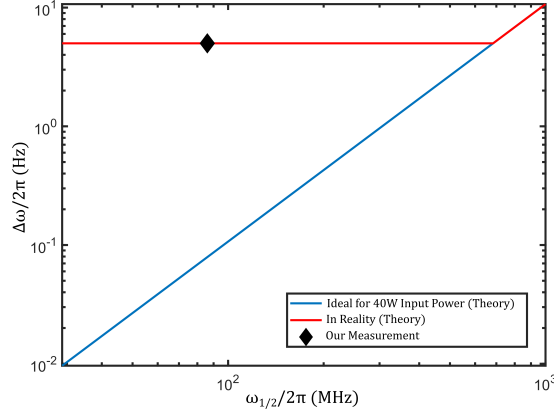


Figure 10.7: **Resolvable $\Delta\omega$ as a function of the signal frequency $\omega_{1/2}^2$.**

(Blue curve) Resolution in the ideal case, calculated with a probe power of 40W. As our super-resolution protocol relies on a motional Raman interaction, the resolution (Ω_{eff}) scales inversely with $\omega_{1/2}^2$. In practice, power constraints and trap anharmonicity limit the maximum coherent displacement and the concomitant protocol fidelity, which in turn limits the maximum Ω_{eff} . In our system, we find empirically that $\Omega_{\text{eff}}/2\pi < 1.7$ kHz is required to maintain high fidelity for the QFS protocol. (Red curve) Predicted achievable resolution when accounting for power and anharmonicity constraints, i.e. $\Omega_{\text{eff}}/2\pi < 1.7$ kHz. The resolution degrades at ≈ 700 MHz due to power constraints. (Black diamonds) Experimental results for best-case resolution at $\omega_{1/2}/2\pi = 86$ MHz using an interrogation time $T = 1$ ms.

these ideal conditions, our protocol is able to resolve two tones separated by 10 Hz at even the GHz-band — this is a factor $\times 100$ smaller than the Fourier-limited linewidth of ~ 1 kHz. In practice, however, practical power constraints and trap anharmonicities limit the maximum coherent displacement and therefore the usable Ω_{eff} . For $T = 1$ ms, we find empirically that $\Omega_{\text{eff}}/2\pi < 1.7$ kHz is required to maintain high QFS fidelity. Consequently, the resolvable $\Delta\omega$ remains roughly constant across a wide frequency range and increases only when the applied power reaches this limit at ≈ 700 MHz, shown by the red trace in Figure 10.7.

10.4 Experimental Considerations

10.4.1 Rabi Frequency Characterization

While coarse characterization of the power (or amplitude) of the input signals is achievable with standard methods (e.g. a spectrum analyzer), the ion itself can instead be employed as

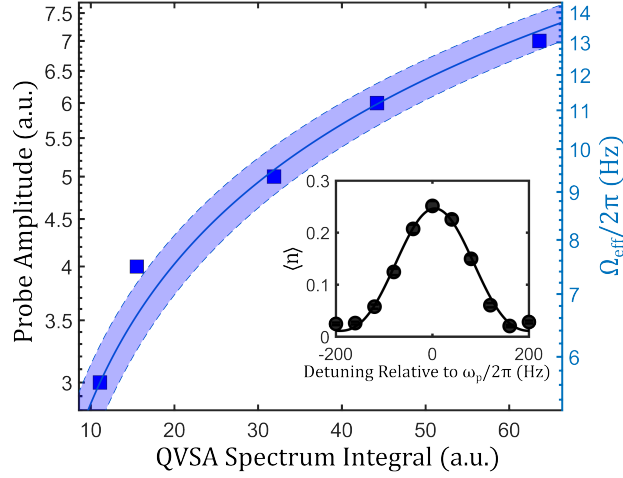


Figure 10.8: **Robust, *in situ* calibration of the effective Rabi frequency Ω_{eff} .**

Ω_{eff} is determined by integration of the total lineshape, obtained via conventional spectroscopy. Data (blue squared) are fitted to an analytical $\text{Area} \approx 1.8 T \Omega_{\text{eff}}^2$ (blue line). The shaded band indicates the $1\sigma \approx 68\%$ prediction band, which corresponds to a 4% prediction error in Ω_{eff} . The inset displays a sample spectrum used to calculate a constituent data point, here shown for an interrogation time $T = 5$ ms. Error bars in the inset are too small to see.

a *quantum* signal analyzer to effect a more accurate *in situ* calibration [Wu+25b]. We outline two approaches to determine the resulting effective Rabi frequency Ω_{eff} in the case of two incoherent signal tones with equal, but unknown, amplitudes. The straightforward approach is to simply measure the peak mean phonon number $\langle n \rangle$ of the spectroscopic lineshape with the probe frequency set to $\omega_{\mu} \pm \omega_o$. However, this is method susceptible to drifts in ω_o .

A more robust method extracts Ω_{eff} by integration of the total lineshape, illustrated in the inset of Figure 10.8 with a $1\sigma \approx 68\%$ prediction band depicted by the shaded band. A function

$$\text{Area} = \Omega_{\text{eff}}^2 \int_{-1/T}^{1/T} \langle |\alpha|^2 \rangle_{\Delta\phi} \frac{d\omega_p}{2\pi} \approx 1.8 \Omega_{\text{eff}}^2 T \quad (10.14)$$

is used for the fit, with interrogation time T set directly in hardware. This expression is accurate to within 2% as $T \Delta\omega/(2\pi) \leq 0.4 \text{ s } \Omega_{\text{eff}}$, as used in Fig.10.5, is then extracted via a calibration curve based on the measured phonon, shown in Figure 10.8. An interrogation time $T = 5$ ms is used. The resulting estimate of Ω_{eff} has a standard error of approximately 4%, as defined by the measured prediction band.

10.4.2 Amplitude Imbalance

When the amplitudes of the two signal tones are unequal, the spectroscopic peak shifts from the midpoint $(\omega_1 + \omega_2)/2$ to a weighted average:

$$\omega_{\text{peak}} = \frac{\Omega_1 \omega_1 + \Omega_2 \omega_2}{\Omega_1 + \Omega_2}, \quad (10.15)$$

where Ω_1 and Ω_2 are the respective effective Rabi frequencies. The integrated lineshape is thus:

$$\text{Area} \approx 0.9T (\Omega_1^2 + \Omega_2^2). \quad (10.16)$$

Similarly, the mean phonon number when the relative phase $\Delta\phi$ is randomized is:

$$\langle |\alpha|^2 \rangle_{\Delta\phi} \approx \frac{1}{8} \left(\frac{\Omega_1 \Omega_2}{\Omega_1 + \Omega_2} \right)^2 \Delta\omega^2 T^4. \quad (10.17)$$

Measurement of the ratio $R = \Omega_1/\Omega_2$ is thus required to determine $\Delta\omega$.

Three approaches can be used to determine the amplitude ratio R . The first is a classical, self-homodyne technique that uses typical laboratory test and measurement equipment (e.g. an oscilloscope, a spectrum analyzer). The input signals can be expressed:

$$f(t) = a_1 \cos(\omega_1 t) + a_2 \cos(\omega_2 t) \quad (10.18)$$

The inputs are split with equal amplitude using an RF splitter and fed into the LO and RF ports of a mixer, producing an output

$$\begin{aligned} f(t) = & \frac{a_1^2 + a_2^2}{2} + \frac{a_1^2}{2} \cos(2\omega_1 t) + \frac{a_2^2}{2} \cos(2\omega_2 t) \\ & + a_1 a_2 (\cos((\omega_1 - \omega_2)t) + \cos((\omega_1 + \omega_2)t)). \end{aligned} \quad (10.19)$$

The DC output $\frac{a_1^2 + a_2^2}{2}$ at the IF port can then be low-passed and measured on e.g. an oscilloscope. Similarly, in the super-resolution regime where $2\omega_1 \approx 2\omega_2 \approx \omega_1 + \omega_2$, the

$a_1^2 + a_2^2 + 2a_1a_2$ component can be measured on e.g. a spectrum analyzer. Together, these measurements can be processed to yield the desired ratio $R = \frac{a_1}{a_2} = \frac{\Omega_1}{\Omega_2}$.

The second approach combines quantum (ion-based) measurements with classical techniques. A key challenge with quantum systems is that the Schrödinger equation is inherently linear, making it difficult to obtain nonlinear cross-terms, e.g. a_1a_2 . This requisite nonlinearity can instead be introduced classically, by e.g. applying the inputs to an RF mixer, as in the classical method. However, instead of using traditional test and measurement equipment for detection, the mixed outputs can be detected using the quantum system at hand; here, the trapped ion. Detection of the mixed signal using the technique in Figure 10.8 now yields $\Omega_1^2 + \Omega_2^2 + 2\Omega_1\Omega_2$, which can be combined with the direct measurement of eq. (10.17) to yield the amplitude ratio R .

Finally, the amplitude ratio can be detected using only the QHO system at hand. Excitation of a nonlinear, second-order subharmonic generates an intermodulation term proportional to $\Omega_1\Omega_2$ and serves as the RF mixer in the classical method [Wu+26]. Specifically, integration of the lineshape (see section 10.4.1) obtained by a scan of ω_p about the second-order subharmonic gives:

$$\text{Area}_2 \propto \Omega_1^2 + \Omega_2^2 + 2\Omega_1\Omega_2, \quad (10.20)$$

where proportionality factors can be derived from the relevant theory. This complements the measurement in eq. (10.15) to allow exact calculation of the amplitude ratio R .

10.4.3 Probe Frequency Optimization

The probe frequency ω_p is optimized by scanning the probe detuning δ as the QFS protocol is applied. ω_p is optimized when the measured $\langle n \rangle$ is minimized, as shown in Figure 10.9. The theoretical fit estimates an uncertainty $\sigma_{\omega_p} \approx 2\pi \cdot 500$ mHz. However, suppose the applied ω_p deviates from the optimal value by some frequency offset ω_{offset} . The measured mean phonon

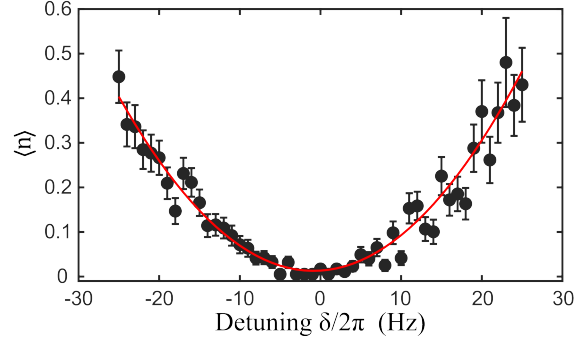


Figure 10.9: **Optimizing the probe frequency ω_p for the super-resolution protocol.** The mean phonon number $\langle n \rangle$ is measured while scanning the probe detuning δ as the QFS protocol is applied. The optimal probe frequency ω_p is determined by minimizing $\langle n \rangle$. The solid line is theoretical fit to the data, which gives an uncertainty $\sigma_{\omega_p} \approx 2\pi \cdot 500$ mHz.

number $\langle n \rangle$ is then:

$$\langle n \rangle = \frac{\Omega_{\text{eff}}^2}{8} \left(T^4 \Delta\omega^2/4 + T^4 \omega_{\text{offset}}^2 - T^6 \Delta\omega^2 \omega_{\text{offset}}^2/4 \right) \quad (10.21)$$

For small ω_{offset} , i.e. $\omega_{\text{offset}}^2 T^2 \ll 1$, this expression simplifies to $\langle n \rangle = \frac{\Omega_{\text{eff}}^2}{8} (T^4 \Delta\omega^2/4 + T^4 \omega_{\text{offset}}^2)$. Compared to the standard expression (Eq. (10.6)), this frequency error introduces an additional offset independent of $\Delta\omega$. To correct for this offset, a two-point measurement can be performed, measuring $\langle n \rangle$ using our protocol with shifting ω_{offset} by a known $\Delta\omega$. By jointly analyzing the two measurements, nonzero ω_{offset} can be compensated. In our experiment, the value of ω_{offset} can be controlled determined to $\approx 2\pi \cdot 500$ mHz, as demonstrated in Figure 10.9. However, the presence of a nonzero offset increases the uncertainty in $\langle n \rangle$ during an Allan deviation, artificially enlarging $\Delta\omega$ and degrading the measurement resolution.

10.4.4 Motional Background Compensation

To compensate for the drifts in the motional background between experimental runs, a coarse calibration (shown in Figure 10.4(a)) is performed before each measurement of $\Delta\omega$. We then measure $\langle n \rangle$ at different $\Delta\omega$ with the QFS sequence, using the same known Rabi frequency Ω_{eff} . Further background suppression is achieved by conducting concurrent calibrations, i.e.

measuring $\langle n \rangle$ at some known $\Delta\omega$. Motional background from e.g. system drifts can then be suppressed by subtracting the background, given by the calibration measurements, from the target measurement, prior to processing via Eq. (10.6).

10.4.5 Higher-Order Phase Reversal

Though the super-resolution protocol demonstrated in this work has used only a single π phase shift, it can in principle be extended to include multiple phase reversals (i.e. higher orders). Figure 10.10 plots the resulting mean phonon number for these higher-order implementations; only odd orders have been considered as only these achieve full suppression of the $\Delta\omega$ -independent background. The estimation efficiency of different protocol orders can be illustrated via standard error propagation. The uncertainty of $\Delta\omega$ is

$$\sigma_{\Delta\omega} = \frac{\Delta n}{\frac{\partial n}{\partial \Delta\omega}}. \quad (10.22)$$

If the resulting mean phonon number $\langle n \rangle$ is fixed, then the shot noise Δn is also fixed for all orders. In this case, a larger slope $\frac{\partial n}{\partial \Delta\omega}$ corresponds to a smaller uncertainty. As the first-order protocol yields the steepest slope and thus the smallest uncertainty (Figure 10.10), it has been used exclusively for this protocol.

10.5 Conclusion

In summary, we have shown that it is possible to resolve two oscillating electric-fields at a level $200\times$ beyond the spectral linewidth limit, a regime traditionally considered inaccessible. This technique relies on a heterodyned quantum interaction that leverages oft-neglected phase information to actively suppress excess shot noise via a dynamic error suppression sequence. Since this method is based on motional Raman interactions, it retains sensitivity across a wide frequency range. This approach offers a broadly applicable solution for achieving frequency super-resolution across both the RF and the MW domains, as demonstrated in [Wu+25b].

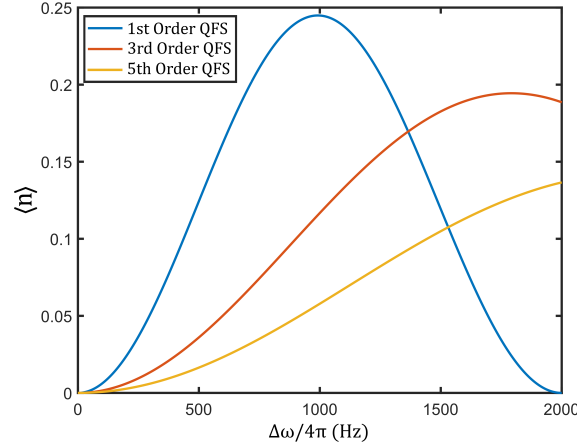


Figure 10.10: **Resultant mean phonon number $\langle n \rangle$ calculated as a function of the signal frequency separation $\Delta\omega$ using higher protocol orders (i.e. number of phase reversals).**

All curves have been calculated using identical parameters (Ω_{eff} , $T = 1$ ms).

Several straightforward improvements would yield further gains in resolution. By increasing both the interrogation time and the input power of the probe tone, the detection range can be extended by at least an additional order of magnitude. Our super-resolution protocol is further compatible with quantum amplification via non-classical states [Wol+19,McC+19a,Bur+19], offering enhancements to the precision of our super-resolution protocol, particularly on short timescales.

The impacts of our protocol are wide-ranging. Since the protocol is equivalent to motional Ramsey spectroscopy, it can be readily integrated into existing low-depth variational quantum circuits to enable further optimization of input states, allowing for more efficient and programmable quantum sensing [Kau+21,Mar+22], in even noisy environments [MKC21]. Further, the protocol is straightforwardly adaptable to a variety of QHO platforms, such as cQED [SKS22], and superconducting circuits [Kra+19]. Our protocol can be implemented using other Raman-based techniques; for instance, with NV centers for broadband magnetic field sensing as demonstrated in [Wan+22], or coupling with alternative two-level systems aimed at achieving frequency super-resolution. In addition, our super-resolution capability opens the possibility of increasing the number of communication channels and improving spec-

tral efficiency in bandwidth-limited communication environments, such as in next-generation wireless networks, satellite links, and quantum communication systems [Xia+17]. Finally, this method may serve as a tool for astrophysical applications, such as distinguishing RF/MW signals from distant regions of the universe. This could enable investigations of potential time variation in the fine-structure constant by resolving OH masers at varying cosmological distances [Hud+06].

“I’ve seen things you people wouldn’t believe. [Couches] on fire, off the shoulder of [Gayley]. I watched [243nm]-beams glitter in the dark near the Tannhäuser Gate. All those moments, will be lost, in time, like, tears — in rain. Time to [graduate].”

Roy Batty, *Blade Runner*

Chapter 11

Quantum-Enhanced Correlation Sampling

Abstract

Quantum sensing exploits nonclassical states to suppress quantum noise below the standard quantum limit (SQL), but the same fine, sub-Planck structure that grants their metrological advantage also accelerates their decoherence, limiting usable interrogation times and precluding improvement in *ultimate* sensitivity. Here, we restore persistent quantum advantage by using Fock-state quantum enhancement within a quantum heterodyne (Qdyne) protocol to decouple sensitivity from the dephasing-limited interrogation time τ . Measurements can thus be acquired at short τ where the quantum-enhanced gain is optimal, while precision accumulates with the total measurement time T . Using the motional mode of a single trapped $^{40}\text{Ca}^+$ ion, we observe quantum-enhanced precision that persists across seven orders of magnitude in measurement time beyond the dephasing-limited interrogation time with no indication of saturation, reaching $\sigma_\delta = 2.23 \times 10^{-6}$ Hz for a fractional uncertainty of $\sim 10^{-7}$. This work represents the first realization of Qdyne on any platform beyond NV centers, and the first on a quantum harmonic oscillator. Additionally, the use of motional Raman transitions extends the frequency range by a factor of $100\times$ into the GHz band by heterodyning the target signal to near the oscillator resonance. Our results recover quantum advantage at the

long timescales required to improve *ultimate* sensitivity, with direct implications for nanoscale [NMR](#), dark-matter searches, and molecular-ion quantum logic spectroscopy.

11.1 Introduction

11.1.1 Background

No matter the technological improvements in instrumentation or refinements in control, any measurement that uses only classical strategies will be constrained by the quantum zero-point fluctuations, which impose a fundamental limit on measurement sensitivity — this is the standard quantum limit (SQL). A core aim of quantum sensing is to suppress these quantum fluctuations through the use of nonclassical states [[Cav81](#)], thereby enabling ever greater sensitivities with which to probe areas such as fundamental physics — the exemplar being LIGO’s now-routine use of squeezed states to reduce shot- and radiation-pressure noise in gravitational wave detectors [[Col11](#),[Col19](#)].

A major effort in quantum sensing on quantum harmonic oscillator (QHO) platforms has been to generalize this quantum enhancement across applications by leveraging the distinct properties of different nonclassical states. In particular: Fock states have been shown to be optimal for phase-insensitive displacement measurements [[Wol+19](#),[Den+24](#)], Fock state superpositions have been used to demonstrate optimal frequency sensitivity [[McC+19a](#)], Schrödinger-cat states have been used to detect single-photon events for quantum logic spectroscopy [[Hem+13](#)], and GKP states have been used for multiparameter sensing [[Val+25](#)].

Despite these numerous achievements in quantum-enhanced protocols, they are largely unable to offer *ultimate* sensitivity improvements in the face of real-world decoherence. Nonclassical states, while offering sub-SQL scaling, suffer accelerated decoherence at a rate which can exceed their metrological gain, and often results in net *reductions* in ultimate sensitivity [[00b](#),[Tur+00a](#)]. This accelerated decoherence of nonclassical states can be understood as arising from their fine, sub-Planck structure, which, ironically, is the same source of their

metrological gain [Zur01]. This competition between gain and decoherence is well established in the context of multi-qubit GHZ states [Hue+97, SC07, Mon+11, Pez+18], as well as QHO platforms [00b, Tur+00a]. An illustrative example is quantum-enhanced frequency sensing using Fock state superpositions $|\psi_N\rangle = \frac{|0\rangle + |N\rangle}{\sqrt{2}}$, which offer an enhancement of N [McC+19a], but dephase as N^2 [00b, Tur+00a] to yield a net *reduction* in sensitivity by N .

Though some efforts have attempted to counter accelerated decoherence, such as the use of quantum memories to extend the usable sensing time, or by leveraging phase-space symmetry to design nonclassical states with robustness to decoherence [Wal21, Val+25], the bulk of demonstrations simply limit their interrogation times to below the decoherence limit. The result is that, in the long-timescale, decoherence-limited regime where ultimate sensitivities are reached, quantum enhancement offers little benefit over classical strategies. Restoring persistent quantum advantage at long timescales therefore requires a protocol that decouples the interrogation time from the precision.

11.1.2 Outline

In this work, we show that a quantum heterodyne (Qdyne, also known as CASR) protocol provides exactly this decoupling, and that the combination of Qdyne with quantum enhancement yields metrological gain that persists to arbitrarily long measurement times, limited only by the reference clock stability [Bos+17, Sch+17]. Qdyne, a method developed to acquire nanoscale NMR spectra with Nitrogen-vacancy (NV) centers in diamond, comprises a series of phase-sensitive measurements coherently synchronized to a reference clock. These features of Qdyne allow processing using Fourier analysis, which substitutes the conventional dependence of resolution on short T_2 -limited interrogation times for dependence on arbitrarily controllable total measurement times T . The frequency precision under Qdyne therefore scales with the total measurement time T rather than the interrogation time τ , decoupling the precision scaling from a dephasing-limited timescale.

The key contribution of this work is to demonstrate that, when quantum enhancement is

applied to the constituent measurements of a Qdyne protocol, metrological gain that persists to long measurement times is achieved, enabling improvement in *ultimate* sensitivity.

We begin by first outlining and demonstrating the novel implementation of the Qdyne protocol on the bosonic motional mode of a trapped ion. Our work represents, to the best of our knowledge, the first implementation of Qdyne on any platform other than NV centers, as well as the first using a QHO, showcasing the general relevance of our results to the broader quantum sensing community and paving the way for similar high-resolution sensing applications on trapped ions. A further innovation of our protocol is our use of motional Raman transitions, which enable wideband frequency sensing in the GHz range — far beyond the achievable Qdyne bandwidth — by heterodyning the target signal to near the QHO resonance ¹. We implement quantum enhancement using nonclassical Fock states [Wol+19] and demonstrate a gain of 3.6(5) dB with the $n = 3$ state over the classical, $n = 0$ state. For comparison, we benchmark the performance of quantum-enhanced spectroscopy without Qdyne and characterize dependence of the quantum-enhanced sensitivity on the interrogation time. Finally, we use quantum-enhanced Qdyne to sense the frequency of an unknown signal and achieve a frequency precision $\sigma_{\delta_0} = 2.23 \times 10^{-6}$ Hz at a total measurement time $T = 1.33 \times 10^4$ s.

11.2 Methods

11.2.1 Motional Raman Excitation

All relevant motional Hamiltonians are generated by coupling voltages from the Phaser AWG to the trap electrodes. A signal with voltage V_d at frequency ω_d and relative phase $\phi_{\text{AWG}} = \pi$

¹We recently became aware of a parallel work that similarly implements Quantum Frequency Mixing [Wan+22] on NV centers for wideband sensing [Yin+25].

between the two AWG outputs produces a dipolar field with lab-frame Hamiltonian

$$\hat{H}_d = -\frac{qV_d}{r_0} x_0 (\hat{a} + \hat{a}^\dagger) \cos(\omega_d t + \phi_d), \quad (11.1)$$

where ϕ_d is the phase of the dipole signal. Similarly, a signal with voltage V_q at frequency ω_q and relative phase $\phi_{\text{AWG}} = 0$ between the AWG outputs produces a quadrupolar field with lab-frame Hamiltonian

$$\hat{H}_q = -\frac{qV_q}{2r_0^2} x_0^2 (\hat{a} + \hat{a}^\dagger)^2 \cos(\omega_q t + \phi_q), \quad (11.2)$$

where ϕ_q is the phase of the quadrupole signal.

Simultaneous application of a dipole tone at frequency ω_d and two quadrupole tones at $\omega_d + \delta \pm \omega_0$ then drives a motional Raman transition, where the dipole and quadrupole tones off-resonantly drive $\Delta n = \pm 1$ and $\Delta n = \pm 2$ transitions, respectively, with their interference resulting in a three-phonon $\Delta n = \pm 1$ process that generates a displacement [Wu+25b]:

$$\alpha = i\Omega_q\Omega_d \frac{\omega_0}{\omega_d^2 - \omega_0^2} \frac{\sin(\delta t - \phi_d) + \sin(\phi_d)}{\delta}. \quad (11.3)$$

where $\Omega_d = qV_d x_0 / (\hbar r_0)$, $\Omega_q = qV_q x_0^2 / (2\hbar r_0^2)$ are the effective Rabi frequencies of the dipole and quadrupole tones, respectively. For convenience, we define an effective Rabi frequency

$$\Omega_{\text{eff}} = \Omega_q\Omega_d \frac{\omega_0}{\omega_d^2 - \omega_0^2} \quad (11.4)$$

Throughout this work, signals of interest are applied as dipole tones, while the quadrupole tones serve as the probe.

Stark shifts from the off-resonant tones are absorbed into the secular frequency ω_0 via in-situ calibration, and residual off-resonant excitation is suppressed by shaping the pulses with a tapered-cosine (Tukey) envelope.

11.2.2 Correlation Sampling with Motional Raman

The phase sensitivity of the motional Raman displacement ((11.3)) imprints the dipole phase ϕ_d onto the measured state, providing the phase sensitivity required for Qdyne. Provided the quadrupole tones are continuously phase-tracked across the experiment, a set of displacement measurements $\{\alpha_n\}$ at times $\{t_n\}$ remains mutually coherent and can be processed by Fourier analysis to extract the dipole signal frequency relative to the center frequency of the quadrupole tones ω_d . Phase-tracking is straightforwardly implemented by allowing the phase accumulators to evolve freely.

An overview of the protocol is provided in Figure 11.1. Each measurement uses a Rabi protocol, i.e. the motional Raman drive is applied continuously for an interrogation time τ . The resulting displacement is converted to an excitation probability via the Fock state overlap measurement, which takes the form [Wol+19]

$$P_n(\alpha) = |\langle n | \hat{D}(\alpha) | n \rangle|^2 = e^{-|\alpha|^2} L_n^0(|\alpha|^2)^2, \quad (11.5)$$

where L_n^0 is the n -th Laguerre polynomial. As the Laguerre polynomials are non-injective in $|\alpha|$, the displacement must be restricted to the monotonic range below the first zero of L_n^0 . For $n = 3$, the highest Fock state used in this work, this bounds the usable displacement to $|\alpha_{\max}| \approx 0.42$.

Since displacements are linear (neglecting the geometric phase), multiple signals within the sampling band can be simultaneously reconstructed from a single measurement set $\{\alpha_n\}$ with Nyquist-limited bandwidth $|\delta| < 1/(2T_s)$, where T_s is the sample period. For a single signal outside this window, the quadrupole tones can be re-centered on the signal of interest, returning the signal within the Nyquist bandwidth. For multiple signals separated beyond the Nyquist bandwidth, the spectrum can instead be reconstructed via compressive sampling [Bos+17,DRC17], or by varying the sampling rate and comparing the shifted peak frequencies [Sch+17].

When the signal detuning is small relative to the interrogation time $\delta \ll 1/\tau$, the displacement magnitude is independent of δ :

$$\alpha \approx i \Omega_q \Omega_d \frac{\omega_0}{\omega_d^2 - \omega_0^2} \tau \cos(\phi_d), \quad (11.6)$$

offering a flat frequency response over the signal band to aid spectral reconstruction.

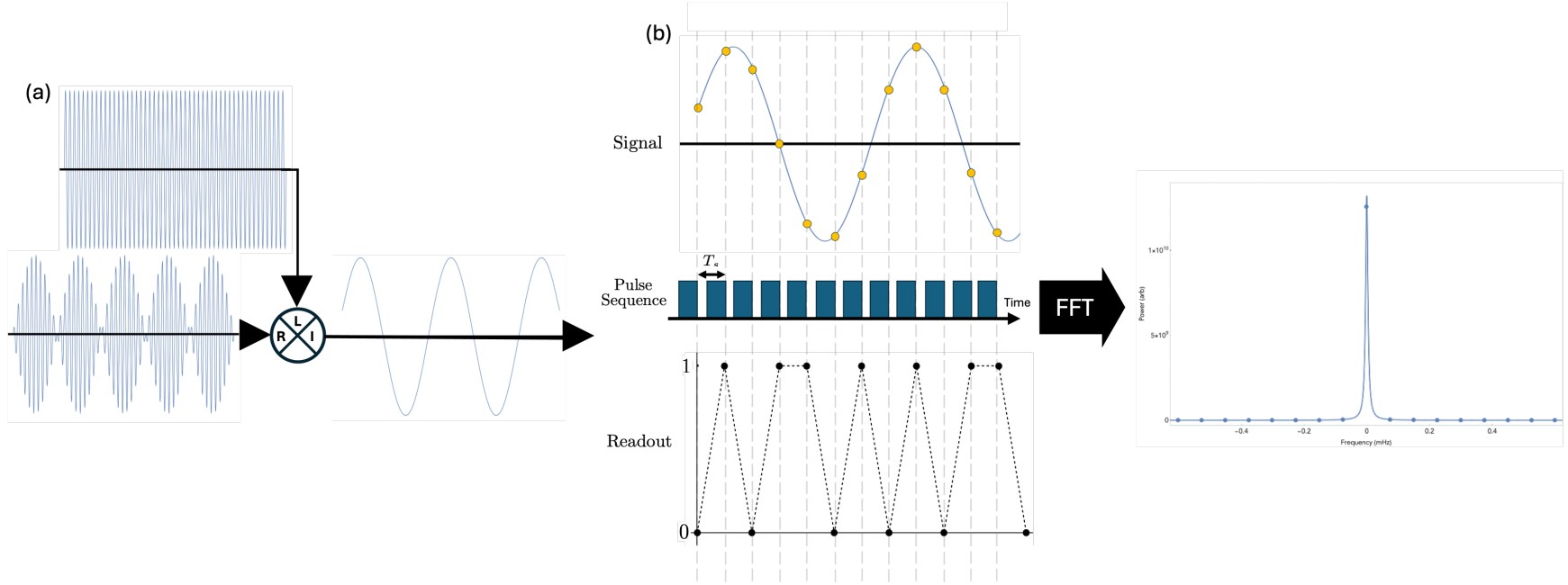


Figure 11.1: **Quantum-enhanced Qdyne on a trapped-ion quantum harmonic oscillator.** (a) Quantum heterodyne downconversion with motional Raman. Quadrupole tones at $\omega_d \pm \omega_0$ define a motional Raman resonance, mixing with a dipole signal tone at $\omega_d + \delta$ to produce a displacement $\hat{D}(\alpha)$ that oscillates at the detuning δ . (b) Qdyne operation. Phase-sensitive measurements consisting of an interrogation pulse of duration τ are repeated with fixed sample period T_s , with all oscillators continuously phase-tracked relative to a fiducial reference to ensure all measurements are mutually coherent. In our implementation, displacements $\hat{D}(\alpha)$ are mapped onto spin using Fock state overlap readout (see Methods); this distorts the signal due to nonlinear dependence of the measured probabilities on α , though this could be avoided using a readout protocol linear in α (e.g. Phase-Sensitive Red Sideband (PSRSB) [Hem+13,Bur+19]). (c) The resulting sequence is Fourier-transformed to reconstruct the signal spectrum with Fourier-limited resolution dictated by the total measurement time T , as opposed to the interrogation time τ as for a conventional protocol.

11.2.3 Fock State Enhancement

The Fock state enhancement protocol follows the procedure of [Wol+19]: an initial Fock state $|n\rangle$ is prepared, a displacement $\hat{D}(\alpha)$ is applied, and the overlap with the same state $|n\rangle$ is measured (see section 3.5.4 for further details). Fock states $|n\rangle$ are prepared by alternately exciting the blue and red motional sidebands of the narrow $S_{1/2} \leftrightarrow D_{5/2}$ qubit transition. For odd n , an additional carrier π -pulse follows the final sideband pulse to return the spin to the $S_{1/2}$ manifold and suppress spontaneous emission during the sequence.

The motional overlap readout is implemented by isolating the n -th motional state in the $S_{1/2}$ manifold and reading it out via state-selective fluorescence. For a given motional state distribution, a Rapid Adiabatic Passage (RAP) pulse on the red motional sideband transfers all population except that in $|n=0\rangle$ to the $D_{5/2}$ manifold, isolating it in the $S_{1/2}$ manifold. This population is shelved in an auxiliary state, after which the remaining $D_{5/2}$ population is returned to $S_{1/2}$. This sequence is repeated until the target Fock state population is isolated in the $S_{1/2}$ state, at which point state-selective fluorescence reads out the overlap ((11.5)).

To improve robustness to pulse area and detuning errors and to reduce calibration overhead, all sideband, carrier, and shelving pulses are implemented as RAP pulses, achieving individual pulse fidelities of approximately 97%. The use of RAP comes at a cost of approximately 1 ms per Fock state, accounting for $\approx 30\%$ of the total enhancement-sequence overhead. Shelving uses the $m_J = -3/2, +1/2, +3/2$ Zeeman sublevels of the $D_{5/2}$ manifold; the highest accessible Fock state is limited by the number of available auxiliary states, which in our implementation is thus $n \leq 3$. This could be substantially extended by adopting improved number-resolving motional-readout techniques, such as ultranarrowband pulse sequences [Mal+24].

11.3 Results

11.3.1 Wideband synchronized readout via motional Raman on a bosonic probe

We first demonstrate Qdyne with motional Raman by reconstructing the spectrum of a signal at 1050 MHz. A dipole signal tone at $\omega_d = 2\pi \times 1050$ MHz is heterodyned by the motional Raman interaction (see Methods) down to a detuning $\delta = 10$ Hz, producing a slow beatnote that is sampled at a fixed period $T_s = 20$ ms. This wideband demonstration used a Walsh-modulated quadrupole probe to suppress the effects of spurious noise; this was implemented only for the results in Figure 11.2. The recorded sequence of measurements is then Fourier-transformed to recover the signal, shown in Figure 11.2. The signal at 1050 MHz sits roughly three orders of magnitude above the motional resonance $\omega_0 \sim 2\pi \times 1$ MHz that limits the native Qdyne frequency range — using the motional Raman interaction for downconversion therefore extends the accessible range of Qdyne into the GHz band without a need for prohibitively fast probe pulses. The recovered linewidth is Fourier-limited at $1/T$, set by the total measurement time $T = 200$ s.

The main $\delta = 10$ Hz peak in Figure 11.2 is accompanied by a harmonic at 2δ — this is an artefact of the nonlinearity of the readout (eq. (11.5)) rather than the signal itself. For small displacements the readout probability is quadratic in the displacement, $P_0(\alpha) = e^{-|\alpha|^2} \approx 1 - |\alpha|^2$, such that a beatnote oscillating at δ is rectified to 2δ . This nonlinearity reduces the power in the fundamental and complicates the reconstruction of dense spectra, as spurious harmonics and intermodulation products can overlap genuine signals. This is not fundamental to the protocol: a readout linear in α , such as the phase-sensitive red sideband protocol [Hem+13,Bur+19], would suppress further spurious peaks.

The same nonlinearity shapes the response to multiple tones, examined in Figure 11.3 for two equal-amplitude dipole tones at detunings $\delta = 2\pi \times 5$ Hz and -4 Hz. Since the readout measures $|\alpha|^2$, squaring the displacement $\alpha(t) = a_1 \cos(\omega_1 t) + a_2 \cos(\omega_2 t)$ yields both the

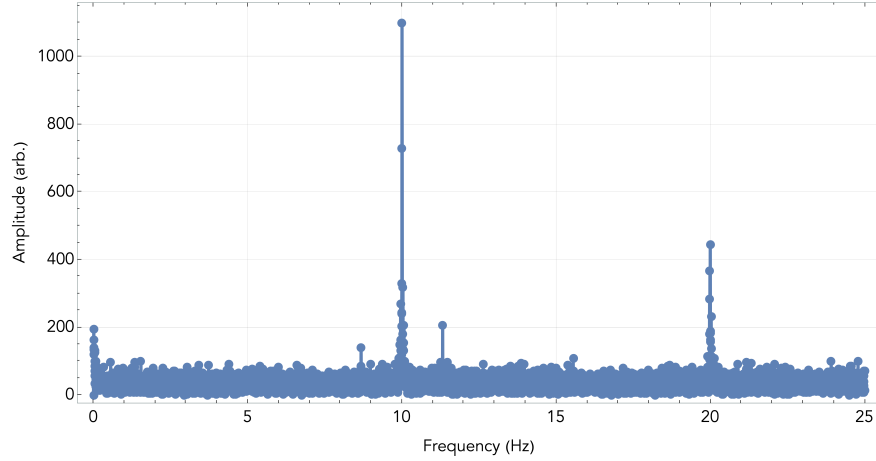


Figure 11.2: **Wideband synchronized readout with motional Raman.**

(a) Wideband synchronized readout of a GHz-band signal using motional Raman. FFT amplitude spectrum of a signal at $\omega_d = 2\pi \times 1050$ MHz, heterodyned by the motional Raman interaction to a detuning $\delta \approx 2\pi \times 10$ Hz and recovered by Qdyne without quantum enhancement (acquired with a phase-modulated quadrupole probe; see text) at a sample period $T_s = 20$ ms. The fundamental appears at δ ; the peak at 2δ is a harmonic of the nonlinear Fock state overlap readout ((11.5)). The recovered linewidth is Fourier-limited at $1/T$ and set by a total measurement time $T = 200$ s.

expected single-tone response at $2\omega_1 = 8 \times 2\pi$ Hz, $2\omega_2 = 10 \times 2\pi$ Hz, as well as cross terms at $\omega_1 \pm \omega_2 = 1.9 \times 2\pi$ Hz larger by a factor of 2 in amplitude.

11.3.2 Fock state Enhancement of Displacements

Fock states were chosen for their rotational symmetry in phase space, ensuring equal enhancement at all signal phases. Use of rotationally asymmetric states, such as cat states [Hem+13, WS21, Mil+21], would result in phase-dependent enhancement and thus distort the target signal, complicating dense spectra reconstruction [Gle+18] (Figure 11.3).

The Fock state enhancement protocol is benchmarked in Figure 11.4 to establish the metrological gains achievable at each displacement. The state overlap $P_n = |\langle n | \hat{D}(\alpha) | n \rangle|^2$ is measured as a function of the displacement α , where the displacement α is varied by sweeping the interrogation time $\tau \in (0, 250] \mu\text{s}$ at constant signal voltage (Fig. 11.4(a)). The interrogation time τ is swept up to a maximum value $\tau = 250 \mu\text{s}$, well below the dephasing

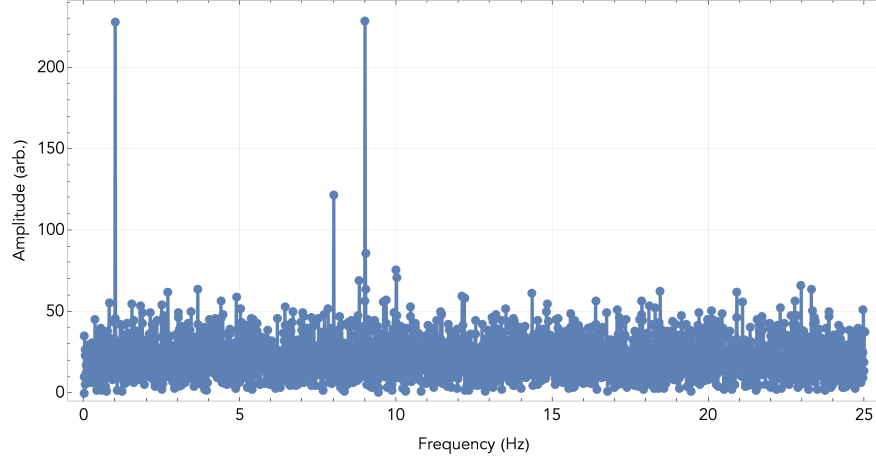


Figure 11.3: **Two-tone response of synchronized readout.**

FFT power spectrum of two equal-amplitude dipole tones injected at detunings $2\pi \times 5$ Hz and $2\pi \times -4$ Hz, heterodyned by the motional Raman interaction and recovered by Qdyne at probe displacement $\alpha \approx 0.4$ (sample period $T_s = 20$ ms). As the Fock state overlap readout (eq. (11.5)) measures $|\alpha|^2$, larger cross terms at 1, 9 Hz appear in addition to the expected single-tone terms at 8, 10 Hz.

time $T_2 \approx 1.5$ ms to ensure decoherence does not significantly affect these results. Analytical models of the form

$$P = C + Ae^{-|\dot{\alpha}\tau|^2} L_n^0(|\dot{\alpha}\tau|^2)^2 \quad (11.7)$$

are fitted to the data, where L_n^k are the associated Laguerre polynomials, and A , C , and $\dot{\alpha}$ are free parameters, with A , C are empirical factors to account for protocol infidelities. Each point in Fig. 11.4(a) is comprised of $N = 5000$ averages to smooth the data for the numerical differentiation required in (11.8) for Fig. 11.4(b). The classical measurement (i.e. without quantum enhancement) is represented by measurements with the $|n = 0\rangle$ state. The non-monotonic behavior for states $|n > 0\rangle$ arises due to negativity in the Fock state Wigner functions, which allows the overlap to vanish before the states are fully separated [Wol+19].

The metrological gain is determined by the Fisher Information (FI)

$$F_n(\alpha) = \frac{(\partial_\alpha P)^2}{P(1 - P)}, \quad (11.8)$$

calculated from the data in Fig. 11.4(a) and shown in Fig. 11.4(b). Solid curves in Fig. 11.4(b)

	F_n	$g_{n=0}$ (dB)	g_{SQL} (dB)
$ n=0\rangle$	4.4(5)	N/A	0.4(5)
$ n=1\rangle$	7.7(6)	2.4(6)	2.8(4)
$ n=2\rangle$	8.9(6)	3.0(6)	3.5(3)
$ n=3\rangle$	10.0(7)	3.6(5)	4.0(3)

Table 11.1: Quantum-enhanced Fock state gains from Figure 11.4 and maximized over α .

	F_n	$g_{n=0}$ (dB)	g_{SQL} (dB)
$ n=0\rangle$	2.4(4)	N/A	-2.2(6)
$ n=1\rangle$	6.8(6)	4.6(7)	2.3(4)
$ n=2\rangle$	8.5(6)	5.5(7)	3.3(3)
$ n=3\rangle$	5.1(5)	3.4(8)	1.1(4)

Table 11.2: Quantum-enhanced Fock state gains from Figure 11.4 at $\alpha = 0.44$.

are obtained by similarly computing the Fisher Information from curves fitted to the data in Fig. 11.4(a), thus incorporating infidelities from pulse errors and decoherence present in the measurement.

The maximum possible gain in the ideal case is represented by the Quantum Fisher Information (QFI), which for a Fock state is $\mathfrak{F}_n(\alpha) = 4(2n+1)$ [Wol+19,FRG25]; the QFI of the $|n=0\rangle$ state defines the Standard Quantum Limit (SQL). The maximum Fisher information is achieved with the $|n=3\rangle$ state at $\alpha = 0.27$, where $F_{n=3} = 10.0(7)$, representing an improvement beyond the SQL by $g_{\text{SQL}}(n=3) = 4.0(3)$ dB (Table 11.1). The measured $|n=0\rangle$ Fisher information of $F_0 = 4.4(5)$ exceeds the SQL $\mathfrak{F} = 4$ by 0.4(5) dB; however, this is within a 1σ -uncertainty of zero. Discrepancies between the achieved and ideal enhancements grow with Fock state number, reflecting the compounding of SPAM infidelities over the larger pulse counts required for higher- n preparation: for $n=3$, the 13 pulses required reduce an individual pulse fidelity of 97% to a final fidelity of 67%.

11.3.3 Dephasing Limits for Conventional Protocols

In Fig. 11.5, we measure the frequency sensitivity of each Fock state $|n\rangle$ as a function of τ , sweeping across the dephasing-limited regime relevant for Qdyne.

We implement quantum-enhanced spectroscopy to measure the frequency sensitivity to

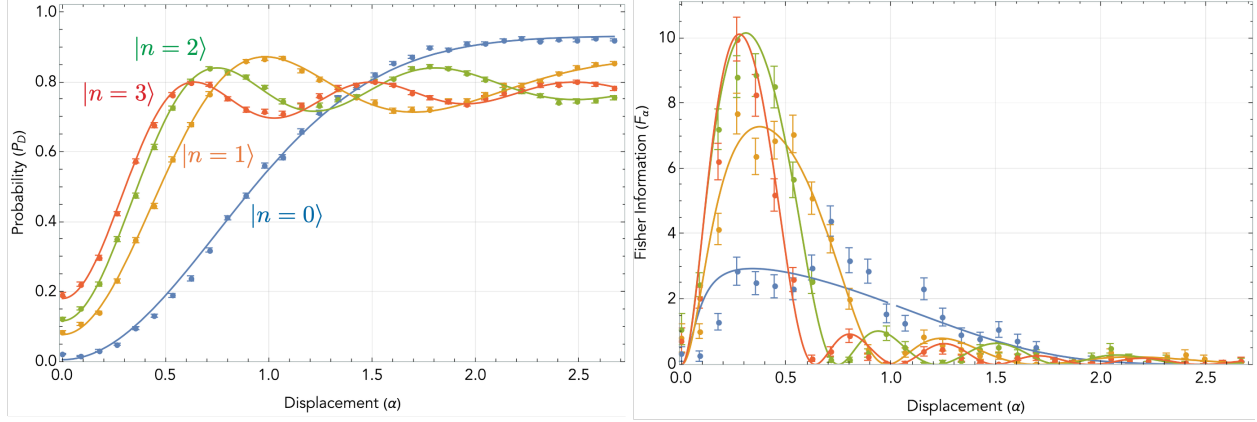


Figure 11.4: **Benchmarking quantum enhancement with motional Fock states.**

(a) Measured state overlap $P_n = |\langle n | \hat{D}(\alpha) | n \rangle|^2$ as a function of the displacement amplitude α , for Fock states $n = 0$ (blue), $n = 1$ (orange), $n = 2$ (green), and $n = 3$ (red). Displacements are produced via the motional Raman interaction with interrogation time swept over $\tau \in (0, 100] \mu\text{s}$ at fixed signal amplitude. $N = 5000$ repetitions are taken at each point to ensure smoothness for numerical differentiation in (b); error bars represent the 1σ standard error and are too small to see. Solid curves are fits to the data of the form $P = C + A e^{-|\dot{\alpha}\tau|^2} L_n^0(|\dot{\alpha}\tau|^2)^2$, with A , C , and $\dot{\alpha}$ as free parameters. A and C are used to account for protocol infidelities and decoherence. Non-monotonic behavior for $|n > 0\rangle$ reflects the negativity of the Fock state Wigner function, which allows the overlap to vanish before the states are fully separated.

(b) Fisher information $F_n(\alpha) = (\partial_\alpha P)^2 / (P(1 - P))$ extracted numerically from the data in (a) (points), with solid curves obtained by computing F_n from the analytical fits in (a). A maximum $F_{n=3} = 10.0(7)$ at $\alpha = 0.27$ represents an improvement of $4.0(3)$ dB beyond the SQL (Table 11.1); selected values are presented in Tables 11.2, 11.1. The $|n = 0\rangle$ measurement yields $F_0 = 4.4(5)$, within 1σ of the SQL value of $\mathfrak{F} = 4$. Discrepancies from the ideal Quantum Fisher Information $\mathfrak{F}_n = 4(2n + 1)$ reflect the compounding of pulse errors and decoherence over the many pulses required for higher- n preparation.

a known displacement, as in Fig. 11.4(a). For each τ , a Rabi lineshape is obtained by scanning the displacement detuning across a frequency range $\delta \sim O(1/\tau)$ and measuring the state overlap P_n , from which the frequency Fisher information $F_\delta(\tau)$ is numerically extracted; the maximum displacement $\alpha_{\max} = \Omega_{\text{eff}}\tau$ (see (11.4)) is held constant by scaling $\Omega_d, \Omega_q \propto 1/\tau$. Results are averaged over $N = 250$ ($N = 200$) repetitions for each value of δ in Fig. 11.5(a) (Fig. 11.5(b,c)). The lineshape is smoothed prior to calculation of the Fisher information by fitting to the analytical form eq. (11.7) where $\alpha = \alpha_{\max} \text{sinc}((\delta - \delta_0)\tau)$, with free parameters $\alpha_{\max}$, δ_0 and τ . For each τ , the maximum frequency Fisher information $F_\delta(\tau)$ is numerically calculated from the lineshape and multiplied by the ideal sample rate with zero dead time $\frac{1}{\tau}$, which is then related to the frequency sensitivity $v_{\delta_{\min}}$ by the Cramér-Rao bound $v_{\delta_{\min}} = \sqrt{\tau/F_\delta}$ [DRC17]. Note that, for considerations of the sensitivity and sample rate, the additional dead time required to generate higher Fock states is discounted as this overhead is an artefact of the chosen state generation protocol and is not fundamental; use of other nonclassical states require negligibly small dead times relative to the decoherence times, with highly squeezed states of $r = 2.54$ having been generated in $\sim 10 \mu\text{s}$ for quantum enhancement [Bur+19].

Results for sensitivity using $\alpha = 0.4$ are presented in Fig. 11.5(b). The ideal sensitivity $\delta_{\min} = \frac{k}{\alpha\sqrt{\tau}} \frac{1}{\sqrt{4(2n+1)}}$, where $k = 0.73$ is a constant for a Rabi protocol, is plotted (black dashed line) for $n = 0$, which serves as the SQL [Wu+25b]. The sensitivities for all states $|n\rangle$ follow the expected $1/\sqrt{\tau}$ scaling and are in good agreement with the analytical form of $\delta_{\min}$. For comparison, a linear model of the form $\ln(y) = \ln(k) - 0.5 \times \tau$ is fitted to the log-transformed data for times up to $\tau = 1000 \mu\text{s}$ (solid lines). The sensitivity gain relative to the $|n = 0\rangle$ measurement is obtained by comparison of the fit parameters, giving $g_{n=1} = 2.3(4) \text{ dB}$, $g_{n=2} = 4.9(5) \text{ dB}$, and $g_{n=3} = 5.4(4) \text{ dB}$, in reasonable agreement with the gains measured in Fig. 11.4(b) and reported in Table 11.2.

In Fig. 11.5(a), all Fock states reach their optimal sensitivity at the same interrogation time $\tau \approx 1000 \mu\text{s}$: the accelerated decoherence ordinarily expected to penalize higher-energy

states is absent in this small-displacement regime. This is in marked contrast to GHZ states [Hue+97,SC07] and Fock superpositions [Tur+00a], where dephasing accelerates as N^2 and erodes metrological gain. Intuitively, the rotational symmetry of Fock states in phase space makes them robust to small, anisotropic displacements; at $\langle n \rangle = |\alpha|^2 = 0.16$, the energy of the displacement is small relative to the Fock state and is too small to disrupt the fine, sub-Planck structure that ordinarily drives accelerated decoherence [Zur01]. No general closed-form expression exists for the dephasing of displaced Fock states due to the non-Gaussianity of the dephasing channel; we therefore rely on master-equation simulations to confirm the trends observed in Fig. 11.5. At larger displacements (Fig. 11.5(b)), this symmetry-based robustness breaks down and higher- n states do incur a sensitivity penalty; however, this regime lies outside the readout window set by the first zero of L_n^0 (see Methods), and is therefore not accessed in protocol operation. Additionally, due to our use of a motional Raman protocol, the magnitude of the displacement can be tuned to within the optimal range by adjusting the strength of the quadrupole sideband tones.

At $\tau \gtrsim 1000 \mu\text{s}$, dephasing exceeds the gain from the longer interrogation times and the sensitivity deteriorates, limiting the interrogation time to $\tau \sim T_2$. For a protocol to realize ultimate sensitivities with quantum enhancement, it must therefore be able to operate at short τ where the gain for constituent measurements persists, and accumulate that gain by some means other than extending τ itself. This requirement is met by Qdyne, demonstrated in its quantum-enhanced form with motional Raman in Fig. 11.6. Qdyne lifts this limit by rendering successive measurements mutually coherent, such that τ in $\delta_{\min}$ is substituted by the total measurement time T — this can be made arbitrarily long, limited only by the reference clock stability.

11.3.4 Quantum-enhanced Synchronized Readout

Figure 11.6 demonstrates that quantum enhancement, when applied with a Qdyne protocol, delivers metrological gain that persists across over three decades of total measurement time.

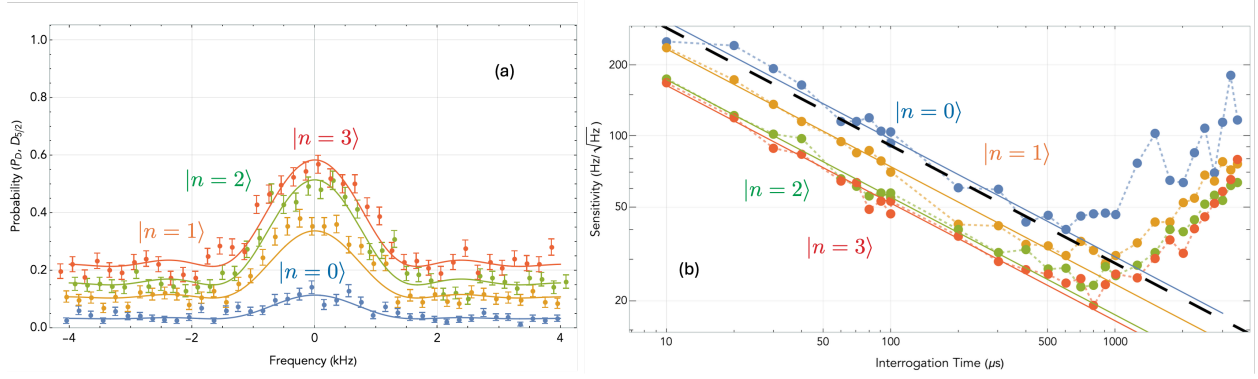


Figure 11.5: Sensitivity with conventional, quantum-enhanced spectroscopy is dephasing-limited.

Colors denote use of the $n = 0$ (blue), $n = 1$ (orange), $n = 2$ (green), and $n = 3$ (red) Fock states. **(a)** A Rabi lineshape at $\tau = 600 \mu\text{s}$ shows the state overlap P_n as a function of the signal detuning δ from a motional Raman resonance at $\omega_d = 86 \times 2\pi \text{ MHz}$. The contrast and baseline scale with n as in Fig. 11.4(a). Solid curves are nonlinear-least-squares fits to the data using the expected form $P_n = C + A e^{-|\alpha|^2} L_n^0(|\alpha|^2)^2$ (see Methods) where $\alpha = \alpha_{\max} \text{sinc}((\delta - \delta_0)\tau)$. A , C , and δ_0 are free parameters; $\alpha_{\max} = 0.293$ is fixed by calibration. Error bars represent the 1σ uncertainty; $N = 250$ repetitions are performed at each point. **(b)** Frequency sensitivity $\delta_{\min}(\tau) = \sqrt{\tau/F_\delta}$ as a function of interrogation time τ , at fixed displacement $\alpha = 0.4$. For each τ , a Rabi lineshape is acquired as in (a) (40 detuning points, $N = 200$ repetitions per point) and the maximum Fisher information F_δ is extracted numerically from the smoothed fit. Solid lines are linear fits to the log-transformed data for $\tau \leq 1000 \mu\text{s}$; the dashed black line is the ideal $n = 0$ (SQL) sensitivity $\delta_{\min} = (k/\alpha\sqrt{\tau})/\sqrt{4(2n+1)}$ with $k = 0.73$ (a fixed constant for a Rabi protocol [Wu+25b]). Trends for all Fock states follow the expected $\tau^{-1/2}$ scaling at short τ , with sensitivities improving over $|n = 0\rangle$ by 2.3(4) dB, 4.9(5) dB, and 5.4(4) dB for $n = 1, 2, 3$ respectively, in agreement with Table 11.2. At $\tau \gtrsim 1000 \mu\text{s}$, dephasing becomes significant and the sensitivity deteriorates, limiting the interrogation time to $\tau \sim T_2$. Qdyne lifts this limit by rendering successive measurements mutually coherent, such that τ in $\delta_{\min}$ is substituted by the total measurement time T — this can be made arbitrarily long, limited only by the reference clock stability.

This ability of quantum enhancement to persist to long timescales is, again, a feature of Qdyne: the short interrogation times prevent accelerated dephasing from eroding the metrological gain, while the precision scaling is transferred from τ to the total measurement time T , which can be made arbitrarily long.

A target signal at $\omega_c = 86 \times 2\pi$ MHz is heterodyned by the motional Raman protocol (see Methods) to an effective detuning $\delta = 20 \times 2\pi$ Hz. All measurements use an interrogation time $\tau = 300 \mu\text{s}$, displacement $\alpha = 0.4$, and sample period $T_s = 13.3$ ms, with all oscillators phase-coherent with respect to a fiducial reference (see Experimental Details). Fig. 11.6(a) shows the FFT spectra at $T = 1.33 \times 10^4$ for $n = 0$ to $n = 3$, with each spectrum fit to a Lorentzian via nonlinear least squares. The quantum-enhanced signal-to-noise ratio (SNR) with increasing n is directly visible and consistent with the measured gains of Table 11.2

Figure 11.6(b) shows the frequency precision σ_{δ_0} as a function of T , extracted as the 95% confidence interval of the fitted center frequency from FFTs of contiguous data subsets $[0, T]$. For all $n > 0$, σ_{δ_0} is improved over the $n = 0$ case across the full measured range of T and does not encounter the dephasing limits otherwise present in conventional quantum-enhanced spectroscopy at long interrogation times (Fig. 11.5(b)). The SQL for frequency precision under our protocol is

$$\sigma_{\delta_0}^{\text{SQL}} = \left(\frac{2}{3} \alpha^2 \frac{T^3}{T_s} \right)^{-1/2}, \quad (11.9)$$

demonstrating the $T^{-3/2}$ scaling in agreement with previous works on Qdyne [Sch+17, Bos+17]. Note that eq. (11.9) depends on α , unlike the quantum Fisher information for displacement sensing; this is due to the additional contribution to sensitivity from the slope of the linescan for frequency sensing. From eq. (11.9), the precision is independent of the interrogation time τ and is instead set by the total measurement time T — this underlies the persistence of metrological gain from quantum enhancement to long timescales.

Dashed lines are linear fits to the log-transformed data to extract the scaling of precision with measurement time T , which gives exponents of $-1.48(1)$, $-1.50(1)$, $-1.50(1)$, and $-1.50(1)$ for the $n = 0, 1, 2, 3$ states, respectively, in excellent agreement with the predicted

$T^{-3/2}$ scaling [Bos+17,Sch+17]. The fast oscillations in Figure 11.6(b,c) result from spectral leakage tied to the FFT bins; as the same bins are used for every trend, the oscillations appear synchronized.

Shown in Fig. 11.6(b), a minimum precision $\sigma_\delta = 2.23 \mu\text{Hz}$ is achieved with the $|n = 3\rangle$ state at $T = 1.33 \times 10^4$ s. The discrepancy between the precision improvements for the $n = 2, 3$ states measured in Figure 11.6(b) and the tabulated gains in Table 11.2 are attributed to changes in α between different runs.

The spectral resolution (via the Lorentzian-fitted linewidth) is Fourier-limited and scales as T^{-1} identically for all Fock states (Fig. 11.6(c)) — as expected, since Fock state enhancement operates by reducing quantum noise and improving the SNR (Fig. 11.6(a)) without accelerating the underlying dynamics [Wol+19,McC+19a].

The gain in Fig. 11.6(b) persists across measurement times that exceed the dephasing-limited interrogation time of Fig. 11.5 by approximately seven orders of magnitude, a regime entirely inaccessible to conventional quantum-enhanced spectroscopy. Further, unlike prior CASR demonstrations on NV centers [Bos+17,Sch+17], our data show no indication of saturation at the longest measured times. This is because all signals are generated from a single source and digitally upconverted, eliminating the dependence on external reference clock stability that limited prior work. A consequence of this architecture is that our implementation is sensitive only to the *relative* detuning between the dipole and quadrupole tones; the carrier frequency of the signals is upconverted and thus common-mode, which cannot itself be sensed. The fractional uncertainty is therefore defined relative to the detuning $\delta_0/(2\pi) = 20$ Hz rather than the carrier $\omega_c/(2\pi) = 86$ MHz, resulting in a fractional uncertainty $\sigma_{\delta_0}/\delta_0 = 1.12 \times 10^{-7}$ for a best-achieved precision of $\sigma_{\delta_0} = 2.23 \times 10^{-6}$ Hz with the $|n = 3\rangle$ state. Data acquisition was terminated at $T = 1.33 \times 10^4$ s due only to operator constraints; this limit is not fundamental. In principle, the precision, as well as the quantum-enhanced gain, can improve arbitrarily with measurement time, limited only by hardware drift (e.g. from crystal aging).

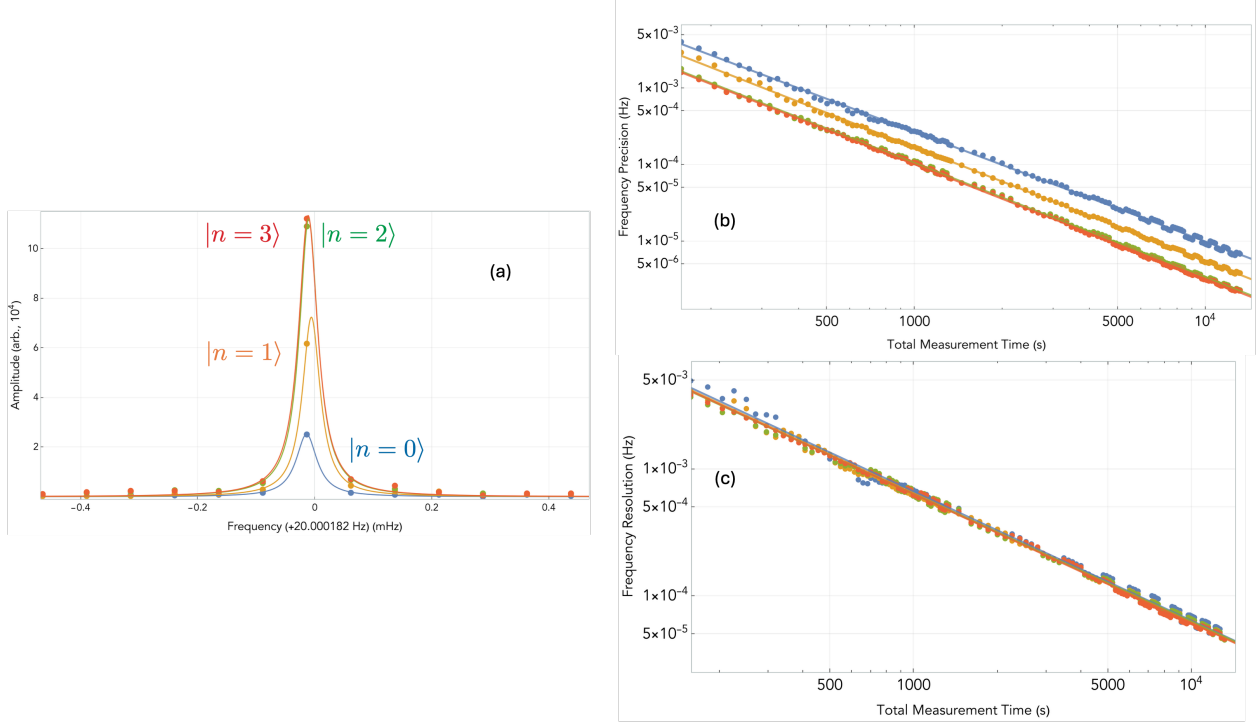


Figure 11.6: **Metrological gain persists to long measurement times with Quantum-enhanced Qdyne.**

The displacement from an $\omega_d = 86 \times 2\pi$ MHz dipole tone is heterodyned with a motional Raman interaction to yield an effective detuning $\delta = 10 \times 2\pi$ Hz; this oscillation frequency is doubled in the data due to the nonlinearity of the readout procedure (see Methods). Samples are measured using a Qdyne protocol on the $n = 0$ (blue), $n = 1$ (orange), $n = 2$ (green), and $n = 3$ (red) Fock states, and use an interrogation time $\tau = 300 \mu\text{s}$, displacement $\alpha = 0.4$, and sample period $T_s = 13.3$ ms. **(a)** FFT amplitude spectrum at total measurement time $T = 1.33 \times 10^4$ s. Solid curves are Lorentzian nonlinear-least-squares fits to the data. Fock state enhancement suppresses quantum noise and improves the signal-to-noise ratio (SNR), as seen in the peak amplitudes increasing with n , in agreement with the gains of Table 11.2. **(b)** Frequency precision σ_{δ_0} of the signal peak, defined as the 95% confidence interval on the fitted Lorentzian center frequency, as a function of total measurement time T . For all $n > 0$, σ_{δ_0} is improved over the $|n = 0\rangle$ case across the entire measured range of T , with no sign of the dephasing-limited deterioration in sensitivity observed in Fig. 11.5(b). Dashed lines are linear fits to the log-transformed data, yielding scaling exponents of $-1.48(1)$, $-1.50(1)$, $-1.50(1)$, and $-1.50(1)$ for $n = 0, 1, 2, 3$, respectively, and in good agreement with the predicted $T^{-3/2}$ precision scaling expected for Qdyne [Bos+17,Sch+17]. **(c)** Frequency resolution, defined as the linewidth of the fitted Lorentzian, as a function of T . Dashed lines are linear fits to the log-transformed data, yielding scaling exponents of $-1.03(1)$, $-1.01(1)$, $-1.00(1)$, and $-1.00(1)$ for $n = 0, 1, 2, 3$, respectively, in excellent agreement with the well-established T^{-1} scaling expected from Fourier analysis. The achieved resolutions are Fourier-limited as Fock state enhancement does not accelerate the underlying dynamics and thus cannot improve spectral resolution. Fast oscillations in (b,c) result from spectral leakage tied to the FFT bin times; as the same bin times are used for every measurement trend, the oscillations appear synchronized.

11.4 Discussion

Our experimental demonstration of a quantum-enhanced Qdyne protocol challenges limitations previously thought fundamental. The integration of quantum enhancement with the Qdyne protocol overcomes the accelerated decoherence of nonclassical resources by using short interrogation times. In this regime, nonclassical states are able to enhance the single-shot information without incurring accelerated decoherence, while Qdyne retains overall ultimate sensitivities by shifting the leading term for precision from the interrogation time to the measurement time. With Qdyne, the gains from quantum enhancement extend to long measurement times and high resolutions, regimes practically inaccessible with conventional, quantum-enhanced spectroscopy due to the typical $1/T_2$ dependence of resolution. This result builds on existing work that attempts to extract metrological gain using nonclassical resources in the presence of dephasing, making use of their oft-overlooked observations that quantum-enhanced measurements reach optimal sensitivity at the shorter interrogation times suitable for Qdyne [Hue+97,Mat+18,SC07,MBF11,Pez+18].

An interesting result from Figure 11.5 that the accelerated decoherence of nonclassical states does not, in general, prevent the improvement of ultimate sensitivities - we have demonstrated that Fock states, when used to sense small displacements, can in fact improve the ultimate frequency sensitivity. This result has not been previously recognized in the literature, since, to our knowledge, there is no general, closed-form expression for dephasing of bosonic states, in large part due to the non-Gaussianity of the dephasing channel. The robustness of displaced Fock states to dephasing is due to their phase-space symmetry, which small displacements are unable to sufficiently disrupt. Though Fock states couple more strongly to amplitude reservoirs and therefore heat at correspondingly higher rates, heating rates are typically orders of magnitude slower than the dephasing rate, and we therefore neglect heating throughout this work [Bro+15,Tal+16,Val+25]. Our finding thus makes quantum enhancement of renewed interest to the large body of precision sensing experiments which aim at improving ultimate sensitivities, such as molecular ion quantum-logic spectroscopy

experiments [Hem+13,HC21,Cui+22].

11.4.1 Applicability

Our work is broadly applicable across sensor platforms: it represents the first demonstration of Qdyne on any platform beyond NV centers, and is the first on a bosonic system. Quantum-enhanced Qdyne could be implemented on NV centers by using spin squeezing for quantum enhancement [Wu+25c]. As Qdyne with NV centers often operates with suboptimal interrogation times below the T_2 limit, quantum-enhancement serves as a knob with which the interaction can be tuned to recover ultimate sensitivities.

Conversely, for Qdyne implementations prioritizing ultimate sensitivity with T_2 -limited interrogation times, quantum-enhancement can be used to speed up interaction dynamics for shorter interrogation times, minimizing the sample period and extending the sensing bandwidth. These benefits are nontrivial: larger sensing bandwidths can improve the effective resolution, since larger magnetic bias fields can be used to further separate spectral features (e.g. chemical shifts in nanoNMR), while shortened interrogation times reduce susceptibility to non-fundamental sources of noise (e.g. $1/f$ noise) [ABB22]. Our use of a Quantum Frequency Mixing protocol further lifts the challenging low-field constraint of nano-NMR with Qdyne by enabling detection of signals above 1 GHz without commensurately fast pulses; this has also been demonstrated in a parallel work [Yin+25].

Our work promises similar improvements to QHO-based sensors. Quantum-enhanced spectrum analyzer protocols in trapped ions [Mil+21,Kel+21] suffer significantly reduced resolutions due to their use of nonclassical states, which could be countered by integration of Qdyne, enabling improved detection and mitigation of noise sources. Ions are also promising platforms for dark matter searches [Gil+17], where quantum-enhanced Qdyne promises significant benefits as it allows parallel, ultrasensitive measurement of all frequencies within extended bandwidths [Gil+21,Aff+20].

Though our work has used a simple model of uncorrelated, Markovian dephasing, it

is interesting to consider the effect of other noise models, which are likely to themselves become ultimate limitations as external noise sources are removed, and result in different tradeoffs between resolution, sensitivity, and bandwidth [Tur+00a,FRG25]. Future work could use engineered reservoirs [00b] to investigate truly ultimate performance limits for quantum-enhanced Qdyne.

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