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in dual readout calorimetry analysis

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Abstract

The DREAM collaboration has introduced the "Q/S Method" for obtaining the energy estimator from simultaneous Cherenkov and scintillator readouts of individual hadronic events. We show that the algorithm is equivalent to an elementary method.

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1. Introduction

The response of a hadronic calorimeter to an incident pion or jet of energy E can be written as

$$\text{hadronic response} = E[\, f_{em} + (1 - f_{em})(h/e)] \tag{1}$$

where a fraction f_{em} is deposited in electromagnetic (EM) cascades, mostly initiated by π^0 decay gamma rays, and h/e is the energy-independent ratio of detection efficiencies for the hadronic and EM energy deposits.[1] (Here and elsewhere the energy E is normalized to the electron response.) In the case of a dual readout calorimeter, in which a Cherenkov signal Q and scintillator signal S are read out for each event, Eq. 1 can be generalized:[1–4].

$$Q = E[f_{em} + (1 - f_{em})(h/e|_Q)] \tag{2}$$

$$S = E[f_{em} + (1 - f_{em})(h/e|_S)] \tag{3}$$

John Hauptman has suggested the less cumbersome notation $\eta_X \equiv (h/e|_X)$, which we use in this paper:

$$Q = E[f_{em} + (1 - f_{em})\,\eta_Q] \tag{4}$$

$$S = E[f_{em} + (1 - f_{em})\,\eta_S] \tag{5}$$

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[1]Whether one writes this ratio as e/h, as is conventional, or h/e is not important for our present purposes. But since $1/(e/h)$ shows up in most of the equations, we prefer to use the reciprocal of e/h.

The EM fraction f_{em} is a feature of the event, while the efficiency ratios η_Q and η_S are different for the two channels. Equations 4 and 5 are the starting point for any analysis of dual-readout hadron calorimetry data.

If f_{em}, η_Q, and η_S are known exactly, and if there are no photoelectron or other statistical contributions, then Q and S are uniquely determined by the incident hadron energy E. If, on the other hand, all of these quantities are subject to statistical fluctuations, then E as determined from the equations must be regarded as the *estimator* of the hadron (or jet) energy for a particular event.

These equations appear explicitly in Fig. 11 of the first DREAM paper at the Perugia Conference on Calorimetry in High Energy Physics[1] and appear either explicitly or implicitly in subsequent DREAM papers. The most complete description of the DREAM analysis is given by Akchurin, et al.[2] (henceforth Ak05), and it is the basic reference for this paper. Several data reduction schemes are presented, but the algorithm considered most basic is the fairly convoluted "Energy-independent Q/S correction method." We show here that it can be obtained in a few lines from Eqns. 4 and 5.

2. The energy-independent Q/S correction method.

The estimator E, whose determination is the object of the analysis, can be eliminated by dividing Eq. 4 by Eq. 5, to obtain

$$\frac{Q}{S} = \frac{f_{em} + (1 - f_{em})\eta_Q}{f_{em} + (1 - f_{em})\eta_S} \ . \tag{6}$$

This is Eq. 2 in Ak05, except that in that paper values of h/e special to the DREAM experiment are inserted for η_Q and η_S. It can be solved for f_{em}. Although the result,

$$f_{em} = \frac{(Q/S)\eta_S - \eta_Q}{(1 - \eta_Q) - (Q/S)(1 - \eta_S)} \ , \tag{7}$$

is not given in the paper, its availability is assumed in the rest of its discussion.

Leakage corrections are incorporated as part of the Method. They are obviously important, but here we assume they have already been made to Q and S as given in Eqns. 4 and 5.

The final estimator of the energy, called S_{final}, is given by Ak05's Eq. 7:

$$S_{\text{final}} = S_{\text{corr}} \left[\frac{1 + p_1/p_0}{1 + f_{em}\, p_1/p_0} \right] \ , \tag{8}$$

where $p_1/p_0 = e/h - 1$. We identify S_{final} with the energy estimator E, and replace S_{corr} by S because the leakage correction has already been made. From context, e/h is $e/h|_S$. The equation then becomes

$$\begin{aligned} E &= S \, \frac{e/h|_S}{1 + f_{em}(e/h|_S - 1)} && (9) \\ &= \frac{S}{\eta_S + f_{em}(1 - \eta_S)} \ , && (10) \end{aligned}$$

which we recognize as just a rearrangement of Eq. 5.

It remains to insert the expression for f_{em} into this equation. Simplification of the result is fairly tedious, but finally yields

$$E = S\left[\frac{(1-\eta_Q)-(Q/S)\eta_S}{\eta_S-\eta_Q}\right] \; . \tag{11}$$

3. Direct solution

We can write the simultaneous equations 4 and 5 as

$$\begin{pmatrix} Q & -(1-\eta_Q) \\ S & -(1-\eta_S) \end{pmatrix}\begin{pmatrix} 1/E \\ f_{em} \end{pmatrix} = \begin{pmatrix} \eta_Q \\ \eta_S \end{pmatrix} \tag{12}$$

with immediate solutions

$$E = S\left[\frac{(1-\eta_Q)-(Q/S)\eta_S}{\eta_S-\eta_S}\right] \tag{13}$$

$$f_{em} = \frac{(Q/S)\eta_S-\eta_Q}{(1-\eta_S)-(Q/S)(1-\eta_Q)} \tag{14}$$

for the *estimators* of E and f_{em} on an event-by-event basis. This method has been published elsewhere[3; 4], but the identity of the approach with the "Q/S method" was not previously recognized.

4. Discussion

In part because of the relatively small number of particles involved early in a hadronic cascade, the efficiency with which the hadronic energy deposit is visible in either the Cherenkov or scintillator channel varies from event to event. In contrast, the efficiency with which the EM deposit is detected varies little. The result is that η_Q and η_S are stochastic variables, mostly reflecting the variation of h. The values of η_Q and η_S required to compute the energy estimator for each event via Eq. 11 (or Eq. 13) are thus not only unknown but unknowable, give "only" dual readout. In actual data reduction, there is little choice but to replace them by their mean values:

$$E = S\left[\frac{(1-\langle\eta_Q\rangle)-(Q/S)\langle\eta_S\rangle}{\langle\eta_S\rangle-\langle\eta_S\rangle}\right] \tag{15}$$

It is also useful to rewrite Eqs. 4 and 5:

$$\langle Q/E\rangle = f_{em}+(1-f_{em})\,\langle\eta_Q\rangle \tag{16}$$

$$\langle S/E\rangle = f_{em}+(1-f_{em})\,\langle\eta_S\rangle \tag{17}$$

Since $\langle Q/E\rangle$ and $\langle S/E\rangle$ are linear in f_{em}, $\langle Q/E\rangle$ is a linear function of $\langle S/E\rangle$, describing a line segment from $(\langle Q/E\rangle\,,\langle S/E\rangle) = (\langle\eta_Q\rangle\,,\langle\eta_S\rangle)$ at the all-hadronic

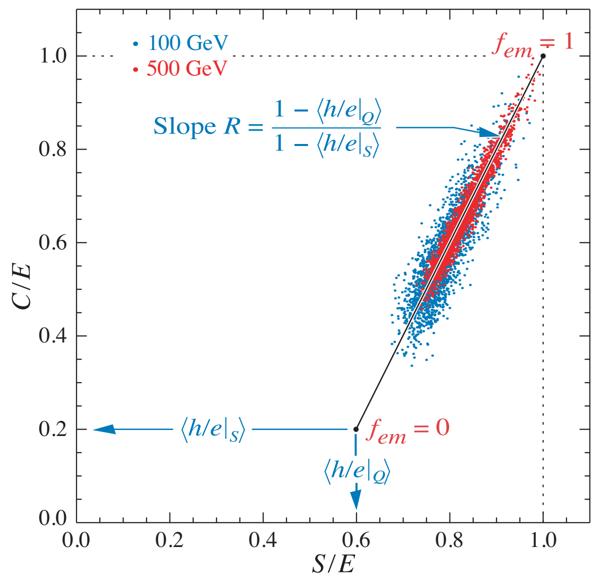

Figure 1: Energy-independent event locus in the Q/E–S/E plane. With increased energy, resolution improves and the mean moves upward along the locus.

extreme, $f_{em} = 0$, to $(\langle Q/E \rangle, \langle S/E \rangle) = (1, 1)$, at the all-EM extreme, $f_{em} = 1$. This event locus is shown in Fig. 1. As the energy increases, the Monte Carlo event scatter shown in the figure moves upward and becomes more clustered as the resolution improves.

The energy-independent event locus has slope

$$R = \frac{1 - \langle \eta_Q \rangle}{1 - \langle \eta_S \rangle} \ . \tag{18}$$

This slope can be determined either by linear fits to monoenergetic (test beam) event distributions in the Q/E–S/E plane, or, perhaps more accurately, by separately finding $\langle \eta_Q \rangle$ and $\langle \eta_S \rangle$ via π/e measurements as a function of energy. It can be used to cast Eq. 13 into a more tractable form[3; 4]:

$$E = \frac{RS - Q}{R - 1} \tag{19}$$

Since f_{em} is not needed in data reduction, it is only of academic interest. Experimental distributions based on DREAM data are shown in Ak05 and earlier publications by the collaboration. The experimental distribution is broadened by resolution effects, and so does not necessarily conform to $0 \leq f_{em} \leq 1$.

5. Acknowlegments

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References

[1] R. Wigmans, "First results of the DREAM project," Proc. 11th Inter. Conf. on on Calorimetry in Particle Physics, Perugia, Italy, 29 March–02 April, 2004, ed. C. Cecchi, P. Cenci, P. Lumbrano, & M. Pepe, World Scientific (2005) 241–257.

[2] N. Akchurin, et al., Nucl. Instr. and Meth. A 537 (2005) 537–561.

[3] D.E. Groom, Nucl. Instr. and Meth. A 572 (2007) 633–653.

[4] J. Beringer et al. (Particle Data Group), Phys. Rev. D86 (2012) 010001.